González Flores Andrés

(1) $X = \begin{bmatrix} 0 \\ 1 \\ 2 \\ 3 \end{bmatrix}$ $\quad Y = \begin{bmatrix} 1 \\ 3 \\ 5 \\ 7 \end{bmatrix}$ $\quad \theta = (X^T X)^{-1} X^T Y$

$$X^T X = \begin{bmatrix} 0 & 1 & 2 & 3 \end{bmatrix} \begin{bmatrix} 0 \\ 1 \\ 2 \\ 3 \end{bmatrix} = 14 \qquad (X^T X)^{-1} = \frac{1}{14}$$

$$\frac{1}{14} \begin{bmatrix} 0 & 1 & 2 & 3 \end{bmatrix} \begin{bmatrix} 1 \\ 3 \\ 5 \\ 7 \end{bmatrix} = \frac{34}{14} \approx 2.4286 \approx \theta$$

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(2)

a) $\hat{y}^{(i)} = 2x \qquad m = 4$

$$J = \frac{1}{2m} \sum_{i=1}^{m} \left[y^{(i)} - \hat{y}^{(i)} \right]^2$$

$$J = \frac{1}{8} \left[(1-4)^2 + (3-4)^2 + (6-12)^2 + (9-14)^2 \right]$$

$$\frac{1}{8} \left[9 + 1 + 36 + 25 \right] = 8.875$$

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b) $\hat{y}(i) = 2 + x$

$$J = \frac{1}{8} \sum_{i=1}^{m} \left[y^{(i)} - \hat{y}^{(i)} \right]^2$$

$$J = \frac{1}{8} \left[(1-4)^2 + (3-4)^2 + (6-8)^2 + (9-9)^2 \right]$$

$$\frac{1}{8} \left[9 + 4 + 1 \right] = \frac{14}{8} = 1.75$$

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(3) $\{ (0, 17.5), (1, 23.5), (2, 18.2), (3, 21.0) \}$

$$h_\theta(x) = \theta_1 x^2 + \theta_2 \qquad m = 4$$

$$J = \frac{1}{2}\sum_i \left[y^{(i)} - h^{(i)}(x) \right]^2 \qquad \alpha = 0.2$$

Valores iniciales $\theta_1 = 2,\ \theta_2 = -1$

$$\frac{\partial J}{\partial \theta_1} = \sum_{i=0}^{m} \left[y^{(i)} - (\theta_1 x^2 + \theta_2) \right](-x^2)$$

$$\frac{\partial J}{\partial \theta_2} = \sum_{i=0}^{m} \left[y^{(i)} - (\theta_1 x^2 + \theta_2) \right](-1)$$

Iteración 1

$$\theta_1^{(1)} = \theta_1 - \alpha \frac{\partial J}{\partial \theta_1}$$

$$\theta_1^{(1)} = 2 + 0.2 \left[\cancel{(17.5 - (2(0)^2 - 1))(0^2)} + (23.5 - (2(1)^2 - 1))(1)^2 + (18.2 - (2(2)^2 - 1))(2^2) + (21 - (2(3)^2 - 1))(3^2) \right]$$

$$\theta_1^{(1)} = 2 + 0.2 [103.3] = 22.\overline{66}$$

$$\theta_2^{(1)} = -1 + 0.2 \left[(17.5 - (2(0)^2) - 1) + (23.5 - (2(1)^2 - 1)) + (18.2 - (2(2)^2 - 1)) + (21 - (2(3)^2 - 1)) \right]$$

$$\theta_2^{(1)} = -1 + 0.2 [-56.2] = 10.24$$

Iteración 2

$$\theta_1^{(2)} = 22.\overline{66} + 0.2 \left[(17.5 - (22.\overline{66}(0)^2 - 10.24))(0)^2 + (23.5 - (22.66(1)^2 - 10.24))(1)^2 + (18.2 - (22.\overline{6}(2)^2 - 10.24))(2)^2 + (21 - (22.\overline{66}(3)^2 - 10.24))(3)^2 \right]$$

$$\theta_1^{(2)} = 22.66 + 0.2 [2078.74] \approx -393.088$$

$$\theta_2^{(2)} = 10.24 + 0.2 \left[(17.5 - (22.65(0)^2 - 10.24)) + (23.5 - (22.66 - 10.24)) + (18.2 - (22.\overline{6}(4) - 10.24)) + (21 - (22.\overline{66}(9) - 10.24)) \right]$$

$$\theta_2^{(2)} = 10.24 + 0.2(278) \approx -45.36$$

Iteración 3

$$\theta_1^{(3)} = -393.088 + 0.2\left[0 + (23.5 - (-393.088 + 45.36)) + (18.2 - (-393.088(4) + 45.36))(4) + (21 - (-393.088(9) + 45.36))(9)\right]$$

$$\theta_1^{(3)} = -393.088 + 0.2[-39442.96] \approx \underline{7495.5048} \approx \theta_1$$

$$\theta_2^{(3)} = -45.36 + 0.2\left[(17.5 - (0 + 45.36)) + (18.2 - (-393.088(4) + 45.36)) + (21 - (-393.088(9) + 45.36))\right]$$

$$\theta_2^{(3)} = -45.36 + 0.2[-5764.872] \approx \underline{1107.6144} \approx \theta_2$$

5) μ_0 $\quad X_{c_1} = \{(0,0),(0,1),(1,1),(1,-1)\}$

Σ_0 $\quad X_{c_2} = \{(3,0),(4,1),(4,3),(4,-1),(4,-2)\}$

$X_{c_3} = \{(0,-2),(0,-3),(-1,-2),(-1,-3),(-2,-2)\}$

a)

$$\mu_1 = \frac{1}{4}\begin{bmatrix} 2 \\ 1 \end{bmatrix} = \begin{bmatrix} 1/2 \\ 1/4 \end{bmatrix} = \underline{\begin{bmatrix} 0.5 \\ 0.25 \end{bmatrix}} \qquad \mu_3 = \frac{1}{5}\begin{bmatrix} -4 \\ -12 \end{bmatrix} = \underline{\begin{bmatrix} -0.8 \\ -2.4 \end{bmatrix}}$$

$$\mu_2 = \frac{1}{5}\begin{bmatrix} 19 \\ 1 \end{bmatrix} = \underline{\begin{bmatrix} 3.8 \\ 0.2 \end{bmatrix}}$$

$$\Sigma_1 = \frac{1}{4}\sum_i (x^{(i)} - \mu_j)(x^{(i)} - \mu_j)^T$$

$$\frac{1}{4}\left[\left(\begin{bmatrix} 0 \\ 0 \end{bmatrix} - \begin{bmatrix} 0.5 \\ 0.25 \end{bmatrix}\right)(\cdot)^T + \left(\begin{bmatrix} 0 \\ 1 \end{bmatrix} - \begin{bmatrix} 0.5 \\ 0.25 \end{bmatrix}\right)(\cdot)^T + \left(\begin{bmatrix} 1 \\ 1 \end{bmatrix} - \begin{bmatrix} 0.5 \\ 0.25 \end{bmatrix}\right)(\cdot)^T \right.$$
$$\left. \left(\begin{bmatrix} 1 \\ -1 \end{bmatrix} - \begin{bmatrix} 0.5 \\ 0.25 \end{bmatrix}\right)(\cdot)^T\right]$$

$$\Sigma_1 = \frac{1}{4}\begin{bmatrix} 1.\overline{33} & -0.\overline{66} \\ -0.1\overline{66} & 3.\overline{66} \end{bmatrix} = \underline{\begin{bmatrix} 0.\overline{33} & -0.1\overline{66} \\ -0.1\overline{66} & 0.91\overline{66} \end{bmatrix}} \qquad \Sigma_1^{-1} = \begin{bmatrix} 3.3 & 0.6 \\ 0.6 & 1.2 \end{bmatrix}$$

$$\Sigma_2 = \frac{1}{5}\left[\left(\begin{bmatrix}3\\0\end{bmatrix}-\begin{bmatrix}3.8\\0.2\end{bmatrix}\right)(\cdot)^T + \left(\begin{bmatrix}4\\1\end{bmatrix}-\begin{bmatrix}3.8\\0.2\end{bmatrix}\right)(\cdot)^T + \left(\begin{bmatrix}4\\3\end{bmatrix}-\begin{bmatrix}3.8\\0.2\end{bmatrix}\right)(\cdot)^T \right.$$
$$\left. + \left(\begin{bmatrix}4\\-1\end{bmatrix}-\begin{bmatrix}3.8\\0.2\end{bmatrix}\right)(\cdot)^T + \left(\begin{bmatrix}4\\-2\end{bmatrix}-\begin{bmatrix}3.8\\0.2\end{bmatrix}\right)(\cdot)^T\right]$$

$$\Sigma_2 = \begin{bmatrix}0.2 & 0.05\\0.05 & 3.7\end{bmatrix} \qquad \Sigma_2^{-1} = \begin{bmatrix}5.0169 & -0.678\\-0.678 & 0.2712\end{bmatrix}$$

$$\Sigma_3 = \frac{1}{5}\left[\left(\begin{bmatrix}0\\-2\end{bmatrix}-\begin{bmatrix}-0.8\\-2.4\end{bmatrix}\right)(\cdot)^T + \left(\begin{bmatrix}0\\-3\end{bmatrix}-\begin{bmatrix}-0.8\\-2.4\end{bmatrix}\right)(\cdot)^T + \left(\begin{bmatrix}-1\\-2\end{bmatrix}-\begin{bmatrix}-0.8\\-2.4\end{bmatrix}\right)(\cdot)^T \right.$$
$$\left. + \left(\begin{bmatrix}-1\\-3\end{bmatrix}-\begin{bmatrix}-0.8\\-2.4\end{bmatrix}\right)(\cdot)^T + \left(\begin{bmatrix}-2\\-2\end{bmatrix}-\begin{bmatrix}-0.8\\-2.4\end{bmatrix}\right)(\cdot)^T\right]$$

$$\Sigma_3 = \begin{bmatrix}0.7 & -0.15\\-0.15 & 0.3\end{bmatrix} \qquad \Sigma_3^{-1} = \begin{bmatrix}1.6 & 0.8\\0.8 & 3.733\end{bmatrix}$$

b)

$$|\Sigma_1| = 0.2778, \quad |\Sigma_2| = 0.7375, \quad |\Sigma_2| = 0.1875$$

Para $(2,2)$

$j=1$ $\quad val = -\log(0.2778) - \left(\begin{bmatrix}2\\2\end{bmatrix}-\begin{bmatrix}0.5\\0.25\end{bmatrix}\right)\begin{bmatrix}3.3 & 0.6\\0.6 & 1.2\end{bmatrix}\left(\begin{bmatrix}2\\2\end{bmatrix}-\begin{bmatrix}0.5\\0.25\end{bmatrix}\right)^T$

$val \approx -11.5491$

$j=2$ $\quad val = -\log(0.7375) - \left(\begin{bmatrix}2\\2\end{bmatrix}-\begin{bmatrix}3.8\\0.2\end{bmatrix}\right)\begin{bmatrix}5.0169 & -0.678\\-0.678 & 0.2712\end{bmatrix}\left(\begin{bmatrix}2\\2\end{bmatrix}-\begin{bmatrix}3.8\\0.2\end{bmatrix}\right)^T$

$val \approx -16.3897$

$j=3$ $\quad val = -\log(0.1875) - \left(\begin{bmatrix}2\\2\end{bmatrix}-\begin{bmatrix}-0.8\\-2.4\end{bmatrix}\right)\begin{bmatrix}1.6 & 0.8\\0.8 & 3.73\end{bmatrix}\left(\begin{bmatrix}2\\2\end{bmatrix}-\begin{bmatrix}-0.8\\-2.4\end{bmatrix}\right)^T$

$val \approx -52.6834$

$\therefore$ Pertenece a la clase 1

Para $(-1,-2)$

$j=1$ $val = -\log(0.2778) - \left(\begin{bmatrix}-1\\-2\end{bmatrix} - \begin{bmatrix}0.5\\0.25\end{bmatrix}\right)\begin{bmatrix}3.3 & 6.6\\0.6 & 1.2\end{bmatrix}\left(\begin{bmatrix}-1\\-2\end{bmatrix} - \begin{bmatrix}0.5\\0.25\end{bmatrix}\right)^T$

$val \approx -18.1441$

$j=2$ $val = -\log(0.7375) - \left(\begin{bmatrix}-1\\-2\end{bmatrix} - \begin{bmatrix}3.8\\0.2\end{bmatrix}\right)\begin{bmatrix}5.0164 & -0.678\\-0.679 & 0.7712\end{bmatrix}\left(\begin{bmatrix}-1\\-2\end{bmatrix} - \begin{bmatrix}3.8\\0.2\end{bmatrix}\right)^T$

$val \approx -120.6338$

$j=3$ $val = -\log(0.1875) - \left(\begin{bmatrix}-1\\-2\end{bmatrix} - \begin{bmatrix}-0.8\\-2.4\end{bmatrix}\right)\begin{bmatrix}1.6 & 0.8\\0.8 & 3.73\end{bmatrix}\left(\begin{bmatrix}-1\\-2\end{bmatrix} - \begin{bmatrix}-0.8\\-2.4\end{bmatrix}\right)^T$

$val \approx -4.15$

$\therefore$ El punto $(-1,-2)$ pertenece a la clase 3

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