Date: __/__/____

14/04/24

CV Assignment 3

Ans 1. $E = [t]_\times R$

$$[t]_\times = \begin{bmatrix} 0 & -z & y \\ z & 0 & -x \\ -y & x & 0 \end{bmatrix} \quad \text{if} \quad t = \begin{bmatrix} x \\ y \\ z \end{bmatrix}$$

Let e be right nullspace
e' be left "

1. For Right Nullspace

$Ee = 0$

since $[t_\times] t = 0$

$[t_\times] R e = 0$

if $e = R^T t$ the $Ee = 0$ is satisfied

so $\boxed{e = R^T t}$

2. For Left Nullspace

$e'^T E = 0^T$

$e'^T [t_\times] R = 0$

if $e'^T = t$ then $e^T E = 0$ is satisfied

$e'^T = t$

since $(AB)^T = B^T A^T$

~~$e' = t.R$~~

Date: __/__/____

Since

$$t^T [t_\times] = 0$$

$\Rightarrow$ if $e' = t$

it also satisfies the condition

$e'^T E = 0$

so

$$\underline{e' = t}$$

Ans 2 $\quad R = I \qquad t = \begin{bmatrix} t_x \\ 0 \\ 0 \end{bmatrix}$

$$[t_\times] = \begin{bmatrix} 0 & 0 & 0 \\ 0 & 0 & t_x \\ 0 & t_x & 0 \end{bmatrix}$$

$$E = [t_\times] R = [t_\times] I = [t_\times]$$

Let P be a 3D homogenous coordinate

$$P = \begin{bmatrix} x \\ y \\ z \\ 1 \end{bmatrix}$$

Date: __/__/____

P_1, P_2 be image points by left & right cam

The skew symetric $[t]$

$$= \begin{bmatrix} 0 & -t_z & t_y \\ t_z & 0 & -t_x \\ -t_y & t_x & 0 \end{bmatrix}$$

① $P_1 = P$

$\Rightarrow P_2^T E P_1 = 0$

where $P_1 = \begin{bmatrix} x_1 \\ y_1 \\ 1 \end{bmatrix}$ $P_2 = \begin{bmatrix} x_2 \\ y_2 \\ 1 \end{bmatrix}$

$$[x_2 \ y_2 \ 1] \underbrace{\begin{bmatrix} 0 & 0 & 0 \\ 0 & 0 & -t_x \\ 0 & t_x & 0 \end{bmatrix}}_{E} \begin{bmatrix} x_1 \\ y_1 \\ 1 \end{bmatrix} = 0$$

$$\Rightarrow [x_2 \ y_2 \ 1] \begin{bmatrix} 0 \\ -t_x \\ t_x y_1 \end{bmatrix} = 0$$

$$-t_x y_2 + t_x y_1 = 0$$

$$\Rightarrow y_1 = y_2$$

Hence y co-ordinates are same

Since translation is purely on x. axis & Rotation is None
y. co-ordinate is same in both images

#learnthesmarterway

Date: __/__/___

Note # $E = [t]_\times R$

$$[t]_\times = \begin{bmatrix} 0 & -t_z & t_y \\ t_z & 0 & -t_x \\ -t_y & t_x & 0 \end{bmatrix} \quad \text{for } t = \begin{bmatrix} t_x \\ t_y \\ t_z \end{bmatrix}$$

since $t_y = t_z = 0$ & $R = I$

$$E = \begin{bmatrix} 0 & 0 & 0 \\ 0 & 0 & -t_x \\ 0 & t_x & 0 \end{bmatrix}$$

Using E in the question

Ans 3. To make epipolar lines horizontal and parallel T should map the epipoles to infinity in x direction

Let e : right epipole
e' : left epipole

general form of points at $+\infty$ x is $\begin{bmatrix} 1 \\ 0 \\ 0 \end{bmatrix}$

#learnthesmarterway

Date: __/__/____

We have to map $\begin{bmatrix} e_x \\ e_y \\ e_z \end{bmatrix}$ to $\begin{bmatrix} 1 \\ 0 \\ 0 \end{bmatrix}$

1) Rotating ~~about~~ around z- axis transforming e into x- y plane

Let θ be the angle e makes with x-axis in x-y plane

$\theta = \tan^{-1} \frac{e_y}{e_x}$ $\quad R_z(\theta) = \begin{bmatrix} \cos\theta & -\sin\theta & 0 \\ \sin\theta & \cos\theta & 0 \\ 0 & 0 & 1 \end{bmatrix}$

2) Rotate around y- axis transforming e into x-z plane

$$R_y(\phi) = \begin{bmatrix} \cos\phi & 0 & \sin\phi \\ 0 & 1 & 0 \\ -\sin\phi & 0 & \cos\phi \end{bmatrix}$$

$\phi = \tan^{-1} \frac{e_z}{e_x \cos\theta}$

$R_{rect} = R_y(\phi) \cdot R_z(\theta)$

$$R_{rect} = \begin{bmatrix} \cos\phi & 0 & \sin\phi \\ 0 & 1 & 0 \\ -\sin\phi & 0 & \cos\phi \end{bmatrix} \begin{bmatrix} \cos\theta & -\sin\theta & 0 \\ \sin\theta & \cos\theta & 0 \\ 0 & 0 & 1 \end{bmatrix}$$

Date: _/_/_

$$R_{rect} = \begin{bmatrix} \cos\phi \cos\theta & -\cos\phi \sin\theta & \sin\phi \\ \sin\theta & \cos\theta & 0 \\ -\sin\phi \cos\theta & \sin\phi \cos\theta & \cos\phi \end{bmatrix}$$

where $\theta = \tan^{-1} \dfrac{e_y}{e_x}$ $\quad \phi = \tan^{-1} \dfrac{e_z}{e_x \cos\theta}$