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Assignment 2

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1. 1. given : $[2, 5, 1]^T$

① rotated $\pi/2$ about Y-axis
② rotated $-\pi/2$ about X axis
③ translated $[-1, 3, 2]^T$

$$R_x(\theta) = \begin{bmatrix} 1 & 0 & 0 \\ 0 & \cos\theta & -\sin\theta \\ 0 & \sin\theta & \cos\theta \end{bmatrix} \qquad R_x(\pi/2) = \begin{bmatrix} 1 & 0 & 0 \\ 0 & 0 & -1 \\ 0 & 1 & 0 \end{bmatrix}$$

$$R_y(\theta) = \begin{bmatrix} \cos\theta & 0 & \sin\theta \\ 0 & 1 & 0 \\ -\sin\theta & 0 & \cos\theta \end{bmatrix} \qquad R_y(\pi/2) = \begin{bmatrix} 0 & 0 & 1 \\ 0 & 1 & 0 \\ -1 & 0 & 0 \end{bmatrix}$$

translation
$t : \begin{bmatrix} -1 \\ 3 \\ 2 \end{bmatrix}$

Rotation $= R_x(-\pi/2) \times R_y(\pi/2)$

$$= \begin{bmatrix} 1 & 0 & 0 \\ 0 & 0 & -1 \\ 0 & 1 & 0 \end{bmatrix} \times \begin{bmatrix} 0 & 0 & 1 \\ 0 & 1 & 0 \\ -1 & 0 & 0 \end{bmatrix}$$

$$= \begin{bmatrix} 0 & 0 & 1 \\ -1 & 0 & 0 \\ 0 & -1 & 0 \end{bmatrix}$$

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making Transformation matrix :

$$\begin{bmatrix} R & t \\ 0 & 1 \end{bmatrix}$$

Trans. matrix :

$$\begin{bmatrix} 0 & 0 & 1 & -1 \\ -1 & 0 & 0 & 3 \\ 0 & -1 & 0 & 2 \\ 0 & 0 & 0 & 1 \end{bmatrix}$$

2. homogenising :

$$\begin{bmatrix} 2 \\ 5 \\ 1 \\ 1 \end{bmatrix}$$

new co-ord.

$$\begin{bmatrix} x \\ y \\ z \\ 1 \end{bmatrix} = \overset{T}{\begin{bmatrix} 0 & 0 & 1 & -1 \\ -1 & 0 & 0 & 3 \\ 0 & -1 & 0 & 2 \\ 0 & 0 & 0 & 1 \end{bmatrix}} \cdot \overset{h}{\begin{bmatrix} 2 \\ 5 \\ 1 \\ 1 \end{bmatrix}}$$

$$= \begin{bmatrix} 0 \\ 1 \\ -3 \\ 1 \end{bmatrix}$$

so the new co-ordinates are $\begin{bmatrix} 0 & 1 & -3 \end{bmatrix}^T$
$\quad x \quad y \quad z$

Mapping origin $\begin{bmatrix} 0 \\ 0 \\ 0 \\ 1 \end{bmatrix}$

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$$\begin{bmatrix} x \\ y \\ z \\ 1 \end{bmatrix} = \begin{bmatrix} 0 & 0 & 1 & -1 \\ -1 & 0 & 0 & 3 \\ 0 & -1 & 0 & 2 \\ 0 & 0 & 0 & 1 \end{bmatrix} \begin{bmatrix} 0 \\ 0 \\ 0 \\ 1 \end{bmatrix}$$

$$= \begin{bmatrix} -1 \\ 3 \\ 2 \\ 1 \end{bmatrix}$$

$$= \begin{array}{ccc} [-1 & 3 & 2]^T \\ x & y & z \end{array}$$

origin gets mapped to $[-1, 3, 2]$

3. R was $\begin{bmatrix} 0 & 0 & 1 \\ -1 & 0 & 0 \\ 0 & -1 & 0 \end{bmatrix}$

θ : angle of rotation

$$\cos\theta = \frac{\text{trace}(R) - 1}{2}$$

$$\text{trace}(R) = R_{11} + R_{22} + R_{33} = 0$$

$$\theta = \cos^{-1}\left(\frac{0-1}{2}\right) = \cos^{-1}\left(\frac{-1}{2}\right)$$

$$\theta = \frac{2\pi}{3} \text{ or } 120^\circ$$

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rotation vector n

$$n = \frac{1}{2\sin\theta}\begin{bmatrix} R_{32} - R_{23} \\ R_{13} - R_{31} \\ R_{21} - R_{12} \end{bmatrix}$$

$$n = \frac{2}{2\cdot\sqrt{3}}\begin{bmatrix} (-1) - 0 \\ 1 - 0 \\ (-1) - 0 \end{bmatrix}$$

$$n = \frac{1}{\sqrt{3}}\begin{bmatrix} -1 \\ 1 \\ -1 \end{bmatrix} \qquad n = \frac{1}{\sqrt{3}}(-\hat{i} + \hat{j} - \hat{k})$$

4.

$$R = I + \sin\theta \cdot N + (1-\cos\theta)\cdot N^2$$

We have $n = \frac{1}{\sqrt{3}}(-\hat{i} + \hat{j} - k)$

$$N = \begin{bmatrix} 0 & -n_n & n_y \\ n_z & 0 & -n_x \\ -n_y & n_n & 0 \end{bmatrix} = \begin{bmatrix} 0 & 1/\sqrt{3} & 1/\sqrt{3} \\ -1/\sqrt{3} & 0 & -1/\sqrt{3} \\ -1/\sqrt{3} & -1/\sqrt{3} & 0 \end{bmatrix}$$

$$N^2 = \begin{bmatrix} 0 & 1/\sqrt{3} & 1/\sqrt{3} \\ -1/\sqrt{3} & 0 & -1/\sqrt{3} \\ -1/\sqrt{3} & -1/\sqrt{3} & 0 \end{bmatrix} \times \begin{bmatrix} 0 & 1/\sqrt{3} & 1/\sqrt{3} \\ -1/\sqrt{3} & 0 & -1/\sqrt{3} \\ -1/\sqrt{3} & -1/\sqrt{3} & 0 \end{bmatrix}$$

$$N^2 = \begin{bmatrix} -2/3 & -1/3 & 1/3 \\ -1/3 & -2/3 & -1/3 \\ 1/3 & -1/3 & -2/3 \end{bmatrix}$$

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finding

$$R = \begin{bmatrix} 1 & 0 & 0 \\ 0 & 1 & 0 \\ 0 & 0 & 1 \end{bmatrix} + \frac{\sqrt{3}}{2}\begin{bmatrix} 0 & 1/\sqrt{3} & 1/\sqrt{3} \\ -1/\sqrt{3} & 0 & -1/\sqrt{3} \\ -1/\sqrt{3} & -1/\sqrt{3} & 0 \end{bmatrix} + \frac{1}{2}\begin{bmatrix} -2/3 & -1/3 & 1/3 \\ -1/3 & -2/3 & -1/3 \\ 1/3 & -1/3 & -2/3 \end{bmatrix}$$

$$= \begin{bmatrix} 0 & 0 & 1 \\ -1 & 0 & 0 \\ 0 & -1 & 0 \end{bmatrix}$$

Which is the same as obtained in the 1st part of question

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To prove :

2. $Rx = \cos\theta \cdot x + \sin\theta\,(u \times x) + (1-\cos\theta)(u^T x)\,u$

given : R is a rotation of angle θ about the axis u (unit vector)

Let :

(1) x, y, z : right handed orthonormal vectors

(2) $v = ax + by + cz$

vector to be rotated on u by θ

u in the terms of orthornal basis

(3) $u = \alpha x + \beta y + \gamma z$

$\alpha^2 + \beta^2 + \gamma^2 = 1$ (unit vector)

(4) v' = vector after rotation

$v' = ax' + by' + cu$

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using rotation formula

$x' = \cos\theta\, x + \sin\theta\,(u \times x)$

$y' = \cos\theta\, y + \sin\theta\,(u \times y)$

$$v' = a(\cos\theta\, x + \sin\theta\,(u \times x)) + b(\cos\theta \cdot y + \sin\theta\,(u \times y)) + cu$$

$$= \cos\theta\,(ax + by) + \sin\theta\,(a(u \times x) + b(u \times y)) + cu$$

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$$v' = \cos\theta(ax + by) + \sin\theta(u \times (ax + by)) + cu$$

$$= \cos\theta\, v + \sin\theta\,(u \times v) + cu$$

v in terms of $\parallel$ & $\perp$

$$v = (v \cdot u)u + (v - (v \cdot u)u)$$

$$v' = \cos\theta((v \cdot u)u + (v - (v \cdot u)u)) + \sin\theta(u \times (v - (v \cdot u)u)) + (v \cdot u)u$$

$$= \cos\theta\, v + \sin\theta\,(u \times v) + (1 - \cos\theta)(v \cdot u)u$$

Replacing v with x

$$Rx = \cos\theta \cdot x + \sin\theta\,(u \times x) + (1 - \cos\theta)(u^T x)u$$

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3. $x = K[R|t]X$

K: intrinsic parameter
$[R|t]$: extrinsic parameters
X : world co-ordinates

$C_1 : K_1 : x_1$
$C_2 : K_2 : x_2$

To show :
$$x_1 = H x_2$$
where H is 3×3 invertible matrix

$$x_1 = K_1 [I|0] X$$

$$x_2 = K_2 [R|0] X$$

We know second camera orientation is purely R applied to first

$$R_2 = R \cdot R_1$$

$$X = (K_2[R|0])^{-1} x_2$$

$$\Rightarrow \quad x_1 = K_1[I|0] (K_2[R|0])^{-1} x_2$$

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$$x_1 = K_1 [I|0] ([R|0]^{-1} K_2^{-1}) x_2$$

since $[I|0]$ is 3×4 & $[R|0]$ is 4×3

$$x_1 = K_1 R^{-1} K_2^{-1} x_2$$

$$x_1 = H x_2$$

$$H = K_1 R^{-1} K_2^{-1}$$

since K_1, R^{-1} & K_2^{-1} are all 3×3 matrices
H is 3×3 matrix

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5. I have P_c and P_e → 3D pt. observed by laser

P_c ↓ 3D pt. observed by camera

I can represent

① $P_c = R_1 \cdot P_e + t_1$

I want to find an R_1 & t_1 such that when I map $P_{c,i}$ & ~~$P_{e,i}$~~ $P_{c,i}$

$P_{c,i}$ ↓ actual

$P_{c,i}$ ↓ calculated by $R_1 \cdot P_e + t_1$

The error should be minimised

② Relating Plane observations to points

$$P_c = \alpha_c \cdot \theta_c$$

$$P_e = \alpha_e \cdot \theta_e$$

③ Substituting ② into ①

~~$P_c = R_1($~~

$$(\alpha_c \cdot \theta_c) = R_1(\alpha_e \cdot \theta_e) + t_1$$

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(4) Minimizing error across all observations

$$o_c = [o_{c,1} \quad o_{c,2} \quad \ldots \quad o_{c,n}]^T$$

$$o_e : [o_{e,1} \quad o_{e,2} \quad \ldots \quad o_{e,n}]^T$$

$$\alpha_c : [\alpha_{c,1} \quad \alpha_{c,2} \quad \ldots \quad \alpha_{c,n}]^T$$

$$\alpha_e : [\alpha_{e,1} \quad \alpha_{e,2} \quad \ldots \quad \alpha_{e,n}]^T$$

$$t_1 : (o_c^T o_c)^{-1} o_c^T (\alpha_c - \alpha_e)$$

& $R_1 = V U^T$

where $o_e\, o_c^T = U S V^T$

$\downarrow$

associated Singular Value Decomposition