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Unit 6
Kinetics of Particles

By

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Kinetics of Particles

- Introduction
- Newton’s Second Law
- Equations of Motion

Introduction

- Kinetics is the study of the relations between the forces and the motion. Here we will not seriously concern whether the forces cause the motion or the motion generates the forces (causality).

- In this chapter, the focus is on the particles. That is the body whose physical dimensions are so small compared with the radius of curvature of its path.

- There are at least 3 approaches toe the solution of kinetic problems: (a) Newton’s second law (b) work and energy method (c) impulse and momentum method.

Newton's Second Law

- $\mathbf{F} = m\mathbf{a}$
- m = mass (resistance to rate of change of velocity) of the *particle*
- $\mathbf{F}$ = *resultant force* acting on the particle
- $\mathbf{a}$ = resulting acceleration measured in a *nonaccelerating frame* of reference

- For most engineering problems on earth, the acceleration measured w.r.t. reference frame fixed to the earth's surface may be treated as absolute. And Newton's 2nd law of motion holds.

- Newton's 2nd law breaks when the velocities of the order of the speed of light are involved → theory of relativity

Equation of Motion and Solution of Problems

$$\sum \mathbf{F} = m\mathbf{a}$$ --- equation of motion

scalar components decomposition according to a specified coordinate

Two problems of dynamics

(1) specified kinematic conditions, find forces →
straightforward application of Newton's law as algebraic equations

(2) specified forces, find motion →
Difficulty depends on the form of force function (t, s, v, a), as the solutions are found by solving a system of differential equations.
For simple functions, we can find closed form solutions of motion as in rectilinear motion

Rectilinear Motion

If the x-axis is the direction of the rectilinear motion,

$$\sum F_x = ma_x \qquad \sum F_y = 0 \qquad \sum F_z = 0$$

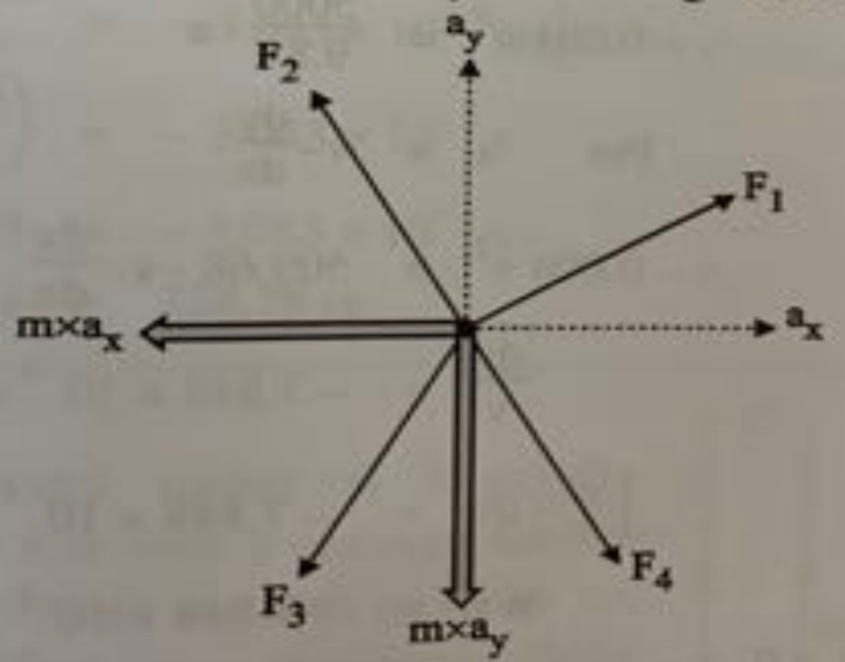
If we are not free to choose a coordinate direction along the motion,

the nonzero acceleration component will be shown up in all equations:

$$\sum F_x = ma_x \qquad \sum F_y = ma_y \qquad \sum F_z = ma_z$$

14.5 D'Alembert's Principle Applied to Rectangular Co-ordinate System

Consider a particle of mass m, moving with acceleration 'a'. If an imaginary force **m × a** is applied in opposite direction to that of acceleration, the particle is considered to be in dynamic equilibrium. If a_x and a_y are two components of acceleration in x and y direction respectively, apply $(m \times a_x)$ in opposite direction of a_x and then use $\Sigma F_x = 0$. Similarly apply $(m \times a_y)$ in opposite direction of a_y and then use $\Sigma F_y = 0$ along with D'Alembert's forces.



14.6 D'Alembert's Principle for Curvilinear Motion

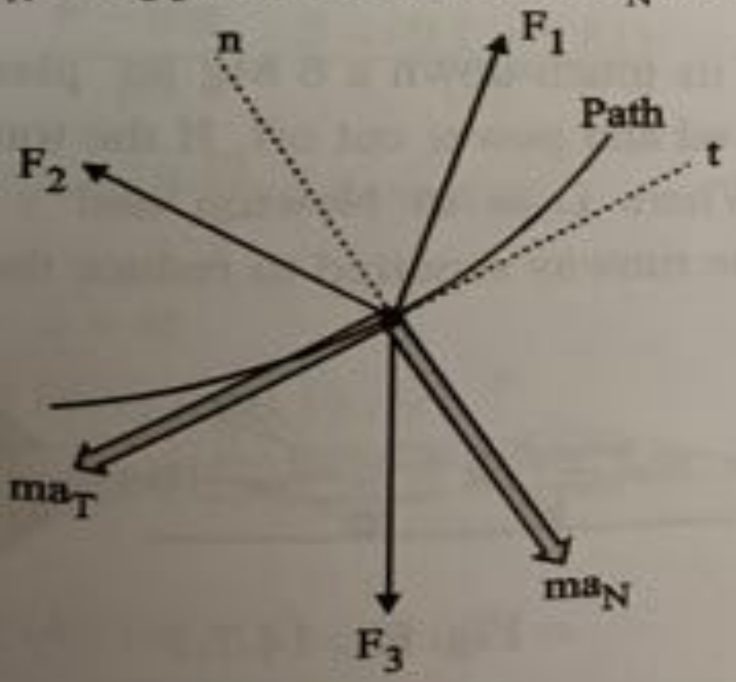
When a particle is moving along a curve it is subjected to two accelerations.

(1) Tangential acceleration a_T (2) Normal acceleration a_N.

- **For dynamic equilibrium,**

(1) Apply $m \times a_T$ in opposite direction of a_T and then use $\Sigma F_t = 0$

(2) Apply $m \times a_N$ in opposite direction of a_N and then use $\Sigma F_n = 0$



14.7 Working Rules for Applications of D'Alembert's Principle

14.7.1 For Rectilinear Motion

Draw Free body diagram of each particle separately showing all the forces acting on it. Also locate x and y-axis.

Decide or assume direction of acceleration.

Apply D'Alembert's force in opposite direction of acceleration, to create dynamic equilibrium.

Apply conditions of equilibrium $\sum F_x = 0$ and $\sum F_y = 0$

14.7.2 For Curvilinear Motion

Draw free body diagram of each particle separately showing all the forces acting on it. Also locate n and t axis.

Decide direction of a_T and a_N.

Apply D'Alembert's forces $(m \times a_T)$ and $(m \times a_N)$ in opposite directions of accelerations a_T and a_N respectively to create dynamic equilibrium.

Apply conditions of equilibrium in normal and tangential direction $\sum F_n = 0$ and $\sum F_t = 0$.

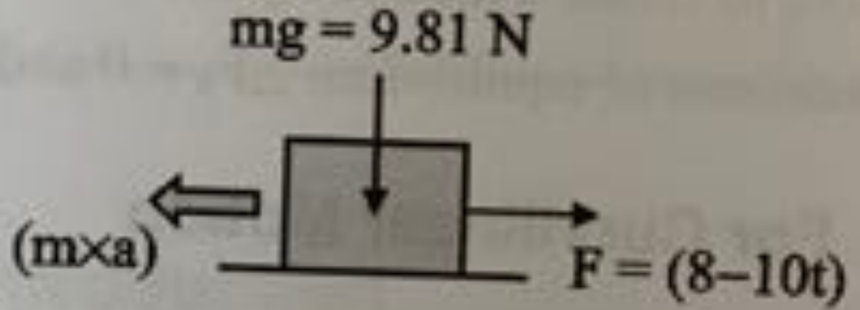
te : (1) If the particle starts from rest, $a_N = 0$ at $t = 0$

(2) If the particle moves with constant speed

$$\frac{dv}{dt} = 0 \text{ hence } a_T = 0$$

A box of mass 1 kg is placed on a smooth horizontal plane horizontal force F (N) acting which varies as F = 8 – 10 t. Find max. velocity attained by the body. Also find correspond displacement.

☑ **Soln. :**



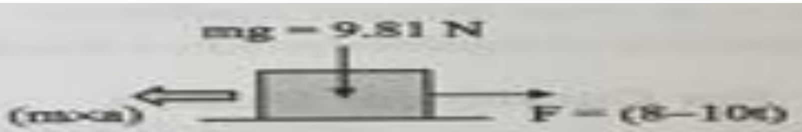


Fig. Ex. 14.7.1

By D'Alembert's principle, for dynamic equilibrium,

$\Sigma F_x = 0$ $\quad F - ma = 0$

$\therefore \quad 8 - 10t - ma = 0$

but m = 1 kg

$\therefore \quad a = 8 - 10t \qquad$ using $\quad a = \dfrac{dv}{dt}$

$\therefore \quad \dfrac{dv}{dt} = 8 - 10t \qquad \therefore \quad dv = (8 - 10t)\, dt$

Integrating,

$\int dv = \int (8 - 10t)\, dt \qquad v = 8t - \dfrac{10t^2}{2} + c_1$

when $t = 0$, $v = 0$, $\qquad \therefore \quad c_1 = 0$

$\therefore \quad v = 8t - 5t^2$ ...(1)

using $\quad v = \dfrac{ds}{dt} \qquad \therefore \quad \dfrac{ds}{dt} = 8t - 5t^2$

$ds = (8t - 5t^2)\, dt$

Integrating

$\int ds = \int (8t - 5t^2)\, dt \qquad \therefore \quad s = \dfrac{8t^2}{2} - \dfrac{5t^3}{3} + c_2$

when $t = 0$, $s = 0$

$\therefore \quad c_2 = 0 \qquad \therefore \quad s = 4t^2 - \dfrac{5t^3}{3}$ ...(2)

Max. Velocity : We know that for velocity to be maximum,

$\dfrac{dv}{dt} = a = 0$

$\therefore \quad 0 = 8 - 10t \qquad t = 0.8$ sec.

Substituting this value in Equations (1) and (2),

$v_{max} = 8(0.8) - 5(0.8)^2$

$\therefore \quad v_{max} = \mathbf{3.2\ m/s}$ ...Ans.

Displacement :

$s = 4(0.8)^2 - \dfrac{5}{3}(0.8)^3 \qquad s = \mathbf{1.706\ m}$ ...Ans.

For what values of angle θ will the acceleration of 2.5 kg block be 3 m/s^2 ($\rightarrow$) when subjected to a force of 30 N. $\mu_s = 0.6$, $\mu_k = 0.5$.

Consider F.B.D. of block. (Refer Fig. Ex. 14.7.7(a))

For Dynamic equilibrium,

$\Sigma F_x = 0 \qquad 30 \cos \theta - 0.5\,R - m \times a = 0$

$$\therefore \quad 2.5 \times 8 = 30 \cos \theta - 0.5\,R$$

$$20 = 30 \cos \theta - 0.5\,R \qquad \text{...(1)}$$

$\Sigma F_y = 0 \qquad 30 \sin \theta - (2.5 \times 9.81) + R = 0$

$$\therefore \quad R = (24.525 - 30 \sin \theta)$$

Substituting in Equation (1),

$$20 = 30 \cos \theta - 0.5\,(24.525 - 30 \sin \theta)$$

$$32.26 - 30 \cos \theta = 15 \sin \theta$$

$$\therefore 2.15 - 2 \cos \theta = \sin \theta$$

Squaring both sides,

$$(2.15 - 2 \cos \theta)^2 = \sin^2 \theta$$

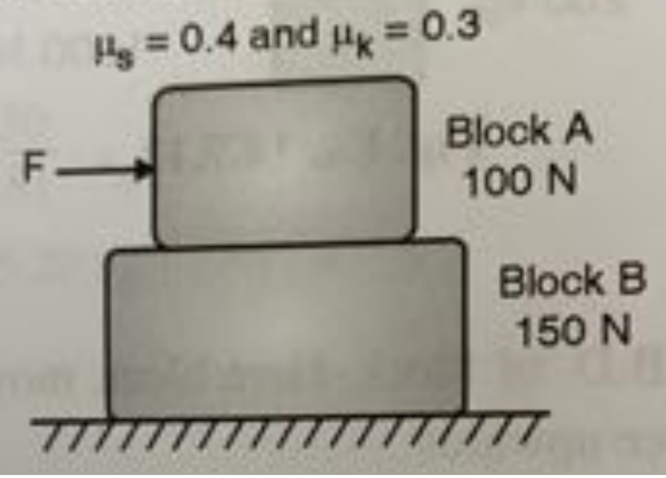
$$4.622 - 8.6 \cos \theta + 4 \cos^2 \theta = 1 - \cos^2 \theta$$

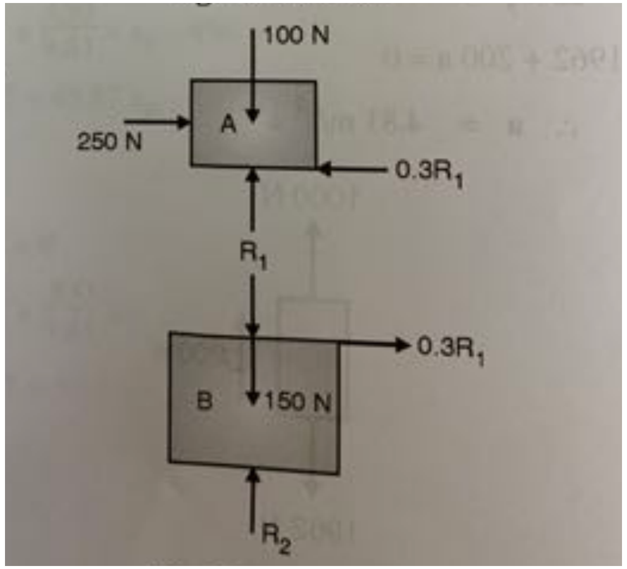
$$\therefore 5 \cos^2 \theta - 8.6 \cos \theta + 3.622 = 0$$

$$\therefore \cos \theta = 0.98 \qquad \text{and} \qquad \cos \theta = 0.736$$

$$\theta = \mathbf{11.48^\circ} \qquad \textbf{and} \qquad \boldsymbol{\theta = 42.61^\circ} \qquad \text{...Ans.}$$

Block B rest on smooth surface. If the coefficient of static and kinetic friction between A and B are $\mu_s = 0.4$ and $\mu_k = 0.3$ respectively, determine the acceleration of each block if a block A is push with a force F : (a) 30 N (b) 250 N. Refer Fig. Ex. 14.7.12.





Case I : When F = 30 N

The minimum force required to move block A on B is

$$\mu_S R = 0.4 \times 100 = 40 \text{ N}$$

So, when 30 N force is applied, both blocks will move together on surface.

Using,
$$\Sigma F = ma$$

$$30 = \frac{250}{9.81} \times a$$

$$\therefore \quad a = 1.177 \text{ m/s}^2 \rightarrow$$

Case II : when F = 250 N

For A

$$\Sigma F = ma_A$$

$$250 - 0.3\,R_1 = \frac{100}{9.81} \times a_A$$

$$250 - 0.3\,(100) = 10.91\,a_A$$

$$\therefore \quad a_A = 21.59 \text{ m/s}^2 \rightarrow$$

For B

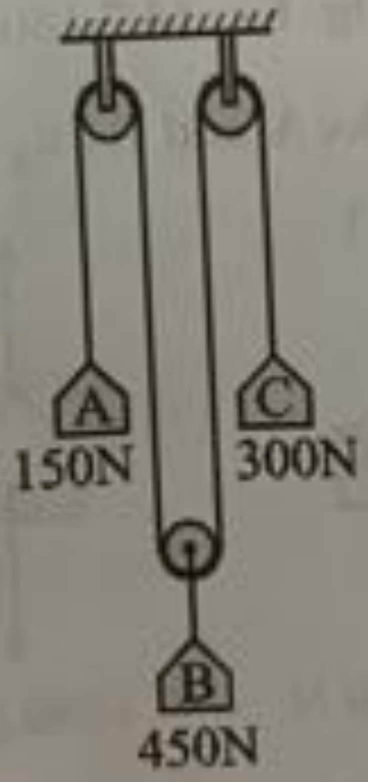
$$\Sigma F = ma_B$$

$$0.3\,R_1 = \frac{150}{9.81} \times a_B$$

$$\therefore \quad a_B = 1.96 \text{ m/s}^2 \rightarrow$$

Ex. 14.7.17

The system shown is released from rest. Neglecting size of pulleys and friction find acceleration of each block.



Soln. : For given system first find relation of acce...
ing concept of dependent motion.

...e length of rope, $L = x_A + 2\,x_B + x_C$

...ifferentiate twice we get,

$$a_A + 2\,a_B + a_C = 0 \quad \ldots(1)$$

Now, assuming directions of accelerations for block A, B and ... as downward (Assume positive sense with reference to datum)

$\therefore$ Assuming $a_A \downarrow$, $a_B \downarrow$ and $a_C \downarrow$.

F.B.D. of blocks and pulley with D'Alembert force acting in ...pposite direction of acceleration (i.e. upwards).

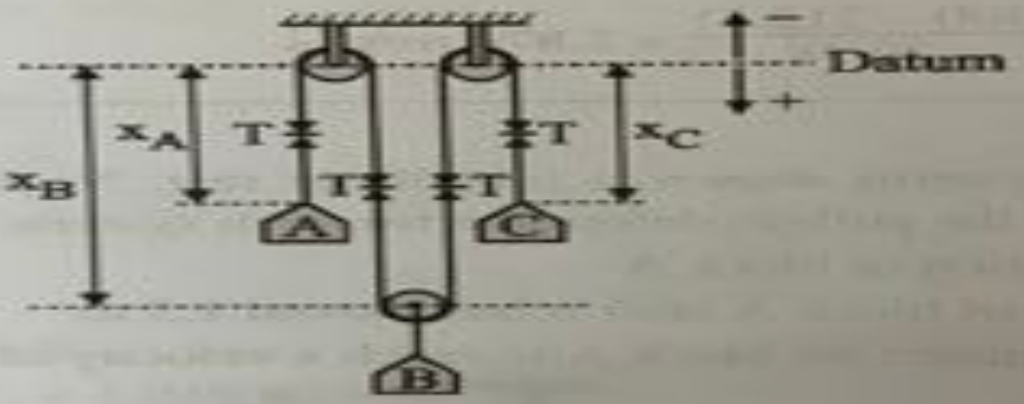
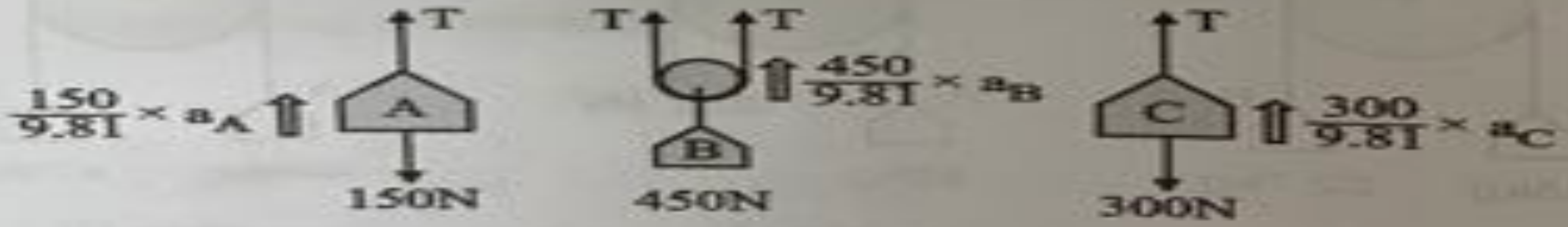


Fig. Ex. 14.7.17(a)



For A : $\Sigma F_y = 0$

$$T + \frac{150}{9.81} \cdot a_A - 150 = 0$$

$$\therefore \quad T + 15.29\, a_A - 150 = 0$$

$$\therefore \quad a_A = \frac{150 - T}{15.29}$$

For B : $\Sigma F_y = 0$

$$2T + \frac{450}{9.81} \times a_B - 450 = 0$$

$$\therefore \quad 2T + 45.87\, a_B - 450 = 0$$

$$\therefore \quad a_B = \frac{450 - 2T}{45.87}$$

For C : $\Sigma F_y = 0$

$$T + \frac{300}{9.81} \times a_C - 300 = 0$$

$$\therefore \quad T + 30.58\, a_C - 300 = 0$$

$$\therefore \quad a_C = \frac{300 - T}{30.58}$$

Substituting values of a_A, a_B and a_C in Equation (1),

$$a_A + 2a_B + a_C = 0$$

$$\frac{150 - T}{15.29} + 2\left[\frac{450 - 2T}{45.87}\right] + \frac{300 - T}{30.58} = 0$$

$$\therefore \quad 39.24 = 0.185\ T$$

$$\therefore \quad \mathbf{T = 212.11\ N}$$ **...Ans.**

Substituting this value of T in Equations (2), (3) and (4).

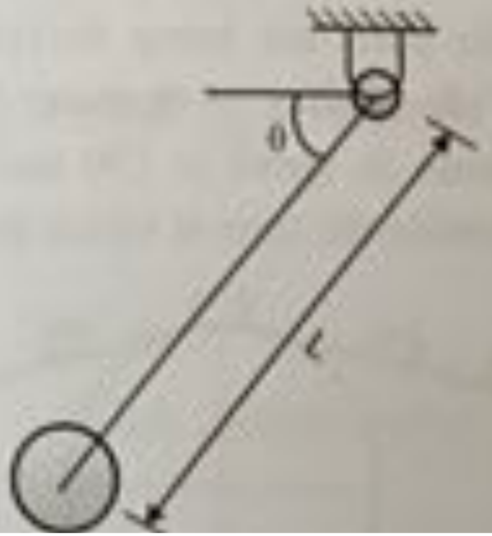
$$a_A = \frac{150 - 212.11}{15.29} = -4.062\ m/s^2 = \mathbf{4.062\ m/s^2 \uparrow}$$ **...Ans.**

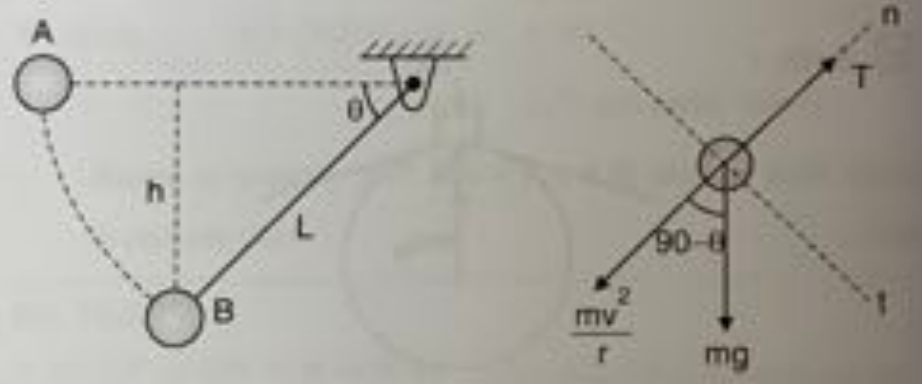
$$a_B = \frac{450 - 2\,(212.11)}{45.87} = \mathbf{0.56\ m/s^2 \downarrow}$$ **...Ans.**

$$a_C = \frac{300 - 212.11}{30.58} = \mathbf{2.874\ m/s^2 \downarrow}$$ **...Ans.**

Ex. 14.7.30 ... Dec. 15, 4 Marks

A pendulum bob has a mass of 10 kg and is released from rest when $\theta = 0°$ as shown in Fig. Ex. 14.7.30. Determine the tension in the cord at $\theta = 30°$. Neglect the size of bob.





(a) (b)

Fig. Ex. 14.7.30

F.B.D at position B.

$\therefore \Sigma F_N = 0$

$$T - \frac{mv^2}{r} - mg \sin\theta = 0$$

$$\therefore T = \frac{mv^2}{r} + mg \sin\theta$$

Here velocity at B is

$$V_B = \sqrt{2g\,L \sin\theta}$$

$$\therefore T = \frac{m\,2gL \sin\theta}{L} + mg \sin\theta = 3\,mg \sin\theta$$

But when $\theta = 30^\circ$

$$T = 3\,mg \sin 30^\circ = \frac{3mg}{2}$$

$$\frac{V^2}{r} = g$$

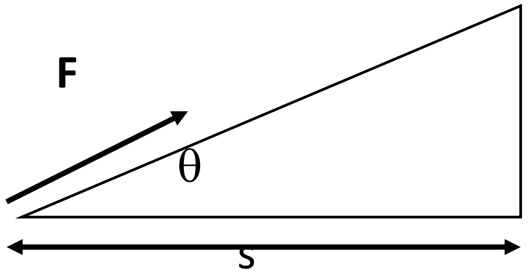
$$\therefore V_{max} = \sqrt{gr} = \sqrt{9.81 \times 60} = 24.26 \text{ m/s}$$

$$\mathbf{V_{max} = 87.34 \text{ kmph}}$$

Work Done By a Constant Force

Work is defined as the product of the constant force and the distance through which the point of application moves in the direction of the force.

Work done = $F \times s$ joules (J)



Work done = $s \times F\cos\theta$ J

Work and Energy

Energy

The energy of a body is its capacity for doing work.

Types of energy

1. Mechanical energy
2. Thermal energy
3. Electrical energy

The **Kinetic energy (KE)** of a body is the energy it possesses by virtue of its motion. It is measured by the amount of work that the body does in coming to rest.

$$KE = \frac{1}{2}mv^2$$

The **Potential energy (PE)** of a body is the energy it possesses by virtue of its position. It is measured by the amount of work that the body would do in moving from its actual position to some standard position.

$$PE = mgh$$

Work and Energy

The work – energy principle

If a constant force acts on an object over a certain distance, the work done by the force is equal to the gain in the kinetic energy of the object.

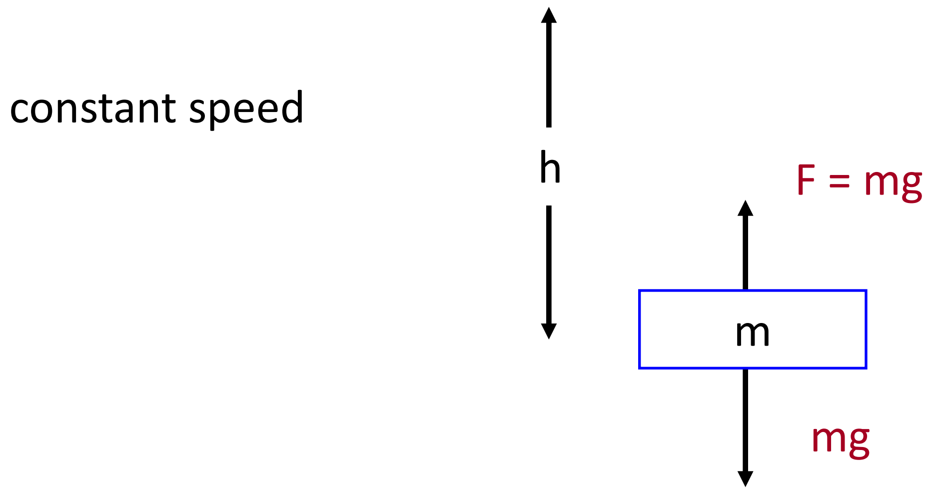
$$Work\ done = \frac{1}{2}mv^2 - \frac{1}{2}mu^2$$

Work Done Against a Resistance = R × Distance moved

Work Done by a force perpendicular to the direction of motion is zero.

Work Done Against Gravity

If a body of mass m is to raised vertically at a constant speed, then a force of mg N must be applied to the body. If the body is raised a distance h metres, then the work done against gravity = mgh J.



Example 1

A car and driver have total mass of 1200 kg. The gains speed from 5 ms^{-1} to 10 ms^{-1} with constant acceleration over a horizontal distance 200 m. Calculate the driving force.

$$Work\ done = \tfrac{1}{2}mv^2 - \tfrac{1}{2}mu^2$$

$$Work\ done = \tfrac{1}{2}\times 1200\times 10^2 - \tfrac{1}{2}\times 1200\times 5^2 = 45000\,J$$

$$Work\ done = F\times 200 = 45000$$

Therefore F = 225 N

Work and Energy

Example 5

A tractor of mass 600 kg pulls a trailer of mass 200 kg up a rough road inclined 10^0 to the horizontal. The resistance to the motion is 5 N per kg. Calculate the work done by the tractor engine , given that the vehicle travels at a constant speed of 1.5 ms^{-1} for 3 minutes.

$$F - 800g \times \sin 10 - 5 \times 800 = 0$$

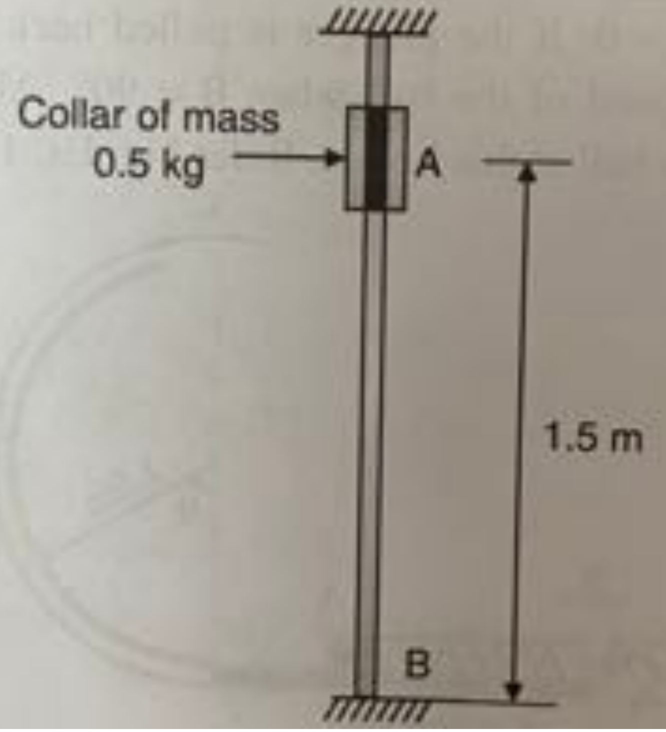
F = 5361.4 N

$$s = 1.5 \times 180 = 270$$

WD = F.s = 1448 kJ

W E

The small collar of mass 0.5 kg is released from rest at A and strikes the base B with velocity 4.7 m/s as shown in Fig. Ex. 15.11.10 determines the work done by frictional force using work energy principle.



W E

By work – energy principle,

$$U_{A\to B} = kE_B - kE_A$$

$$\text{W.D. by gravity} = mgh = 0.5 \times 9.81 \times 1.5 = 7.36 \text{ Joules}$$

$$\text{W.D. by friction} = U_F \text{ (say)}$$

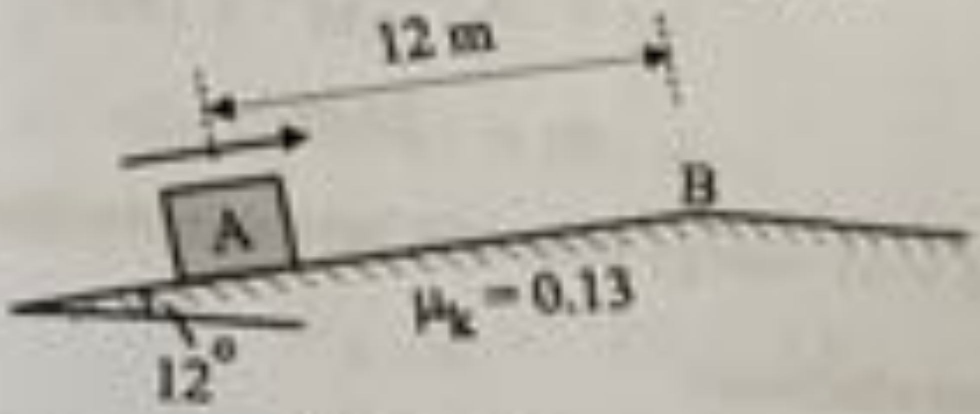
$$\therefore 7.36 + U_F = \frac{1}{2}(0.5)(4.7)^2 - 0$$

$$U_F = -1.8375 \text{ Joules}$$

$\therefore$ W.D. by friction

W E

A box is projected 12 m up a 12° incline so that it just reaches the top of incline with zero velocity. If $\mu_k = 0.13$ between box and incline, determine (i) initial velocity of box at A (ii) velocity of box as it returns back to 'A'. Refer Fig. Ex. 15.11.11.



W E

Apply work energy principle from $A \to B$.

Work-done calculations : (from $A \to B$)

1) W.D. by gravity $= -mgh$ (moving upward so negative)

$$= -mg \cdot S \sin\theta$$

$$= -m \times 9.81 \times 12 \sin(12^\circ)$$

$$= -24.475\ m \text{ joules}$$

2) W.D. by frictional force $= -\mu_k \cdot R \cdot S = -\mu_k \cdot mg\cos\theta \cdot S$

$$= -0.13 \times m \times 9.81 \cos(12^\circ) \times 12$$

$$= -14.97\ m \text{ ...joules}$$

Energy calculations

1) $KE_A = \frac{1}{2} m v_A^2$

$KE_B = 0$ (it reaches B with zero velocity)

$\therefore$ by work energy principle,

$$U_{A\to B} = KE_B - KE_A$$

$$-24.475\ m - 14.97\ m = 0 - \frac{1}{2} m v_A^2$$

$$v_A = \mathbf{8.882\ m/s} \qquad \text{...Ans.}$$

This is initial velocity of projection at A.

ii) **When box returns again from (B to A)**

1) W.D. by gravity $= 24.475\ m$

[Note : W.D. is same in magnitude but it is positive from $B \to A$]

2) W.D. by frictional force $= -14.97\ m$

[Note : This W.D. is always negative]

$\therefore$ Total W.D. $U_{B\to A} = 24.475\ m - 14.97\ m$

W

E

Energy calculations

$KE_B = 0 \qquad KE_A = \frac{1}{2} m v^2$

$\therefore$ by work-energy principle,

$U_{B \to A} = KE_A - KE_B \qquad 9.505\ m = \frac{1}{2} m v^2 - 0$

$\therefore \quad \mathbf{v = 4.36\ m/s.}$ **...Ans.**

This is velocity of box when it returns to original position 'A'.

Conservation of energy

Energy can never be created or destroyed. It can only be changed from one form to another.

Consider the situation of a moving body where no work is done against friction and that gravity is the only other force present then:

Total Energy = Kinetic Energy + Potential Energy = Constant

Or in other terms:

Total Initial Energy = Total Final Energy

Power

- Power is defined as the rate at which energy is transferred with respect to time

$$\text{Power} = \frac{\text{energy transfer}}{\text{time taken}} = \text{force} \times \text{speed} \quad \therefore \text{P} = \frac{\text{E}}{\text{t}} = \text{Fv}$$

$$\text{Unit : joule per second (J s}^{-1}\text{) or watt (W)} \quad \therefore 1\text{ W} = 1\text{ J s}^{-1}$$

- A car is moving on a rough road

W E

Power

Efficiency

- percentage of useful work done compared to the input energy

$$\text{Efficiency} = \frac{\text{Useful work done by the machine}}{\text{Energy used by the machine}} \times 100\%$$

Efficiency < 100% due to loss of friction

Efficiency = output power/ inputpoer

W

E

Impulse

- When a single, constant force acts on the object, there is an **impulse** delivered to the object is defined as the *impulse*
- The equality is true even if the force is not constant
- Vector quantity, the direction is the same as the direction of the force

$$\vec{I} = \vec{F}\Delta t$$

$$\frac{\Delta \vec{p}}{\Delta t} = \frac{\text{change in momentum}}{\text{time interval}} = \vec{F}_{net}$$

Impulse is a vector quantity and has the same direction as the average force.

SI Unit of Impulse: newton · second = (N · s)

Momentum, **p**

The linear momentum **p** of an object is the product of the object's mass *m* and velocity **v**:

$$\mathbf{p} = m\mathbf{v}$$

Linear momentum is a vector quantity that points in the same direction as the velocity.

SI Unit of Linear Momentum:

kilogram · meter/second = (kg · m/s)

Linear Momentum

- This is a new fundamental quantity, like force, energy. It is a vector quantity (points in same direction as velocity).
- The linear momentum **p** of an object of mass *m* moving with a velocity **v** is defined to be the product of the mass and velocity:

$$\vec{p} = m\vec{v}$$

- The terms momentum and linear momentum will be used interchangeably in the text
- Momentum depend on an object's mass and velocity

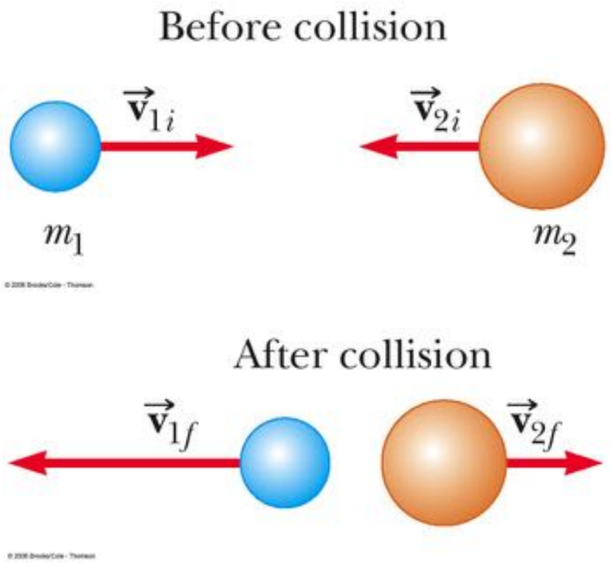
IMPULSE–MOMENTUM THEOREM

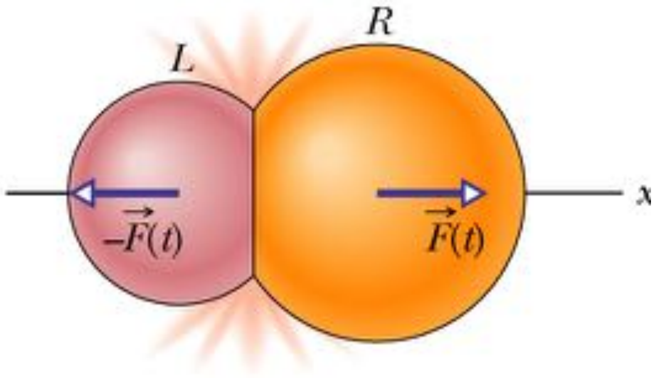
When a net force acts on an object, the impulse of the net force is equal to the change in momentum of the object:

$$\underbrace{\overline{\mathbf{F}}\Delta t}_{\text{Impulse}} = \underbrace{m\mathbf{v}_\mathbf{f}}_{\substack{\text{Final}\\\text{momentum}}} - \underbrace{m\mathbf{v}_\mathbf{0}}_{\substack{\text{Initial}\\\text{momentum}}}$$

final momentum = initial momentum plus impulse of the force during the time interval.

Conservation of Momentum





- Start from impulse-momentum theorem

$$\vec{F}_{21}\Delta t = m_1\vec{v}_{1f} - m_1\vec{v}_{1i}$$

$$\vec{F}_{12}\Delta t = m_2\vec{v}_{2f} - m_2\vec{v}_{2i}$$

- Since $\vec{F}_{21}\Delta t = -\vec{F}_{12}\Delta t$

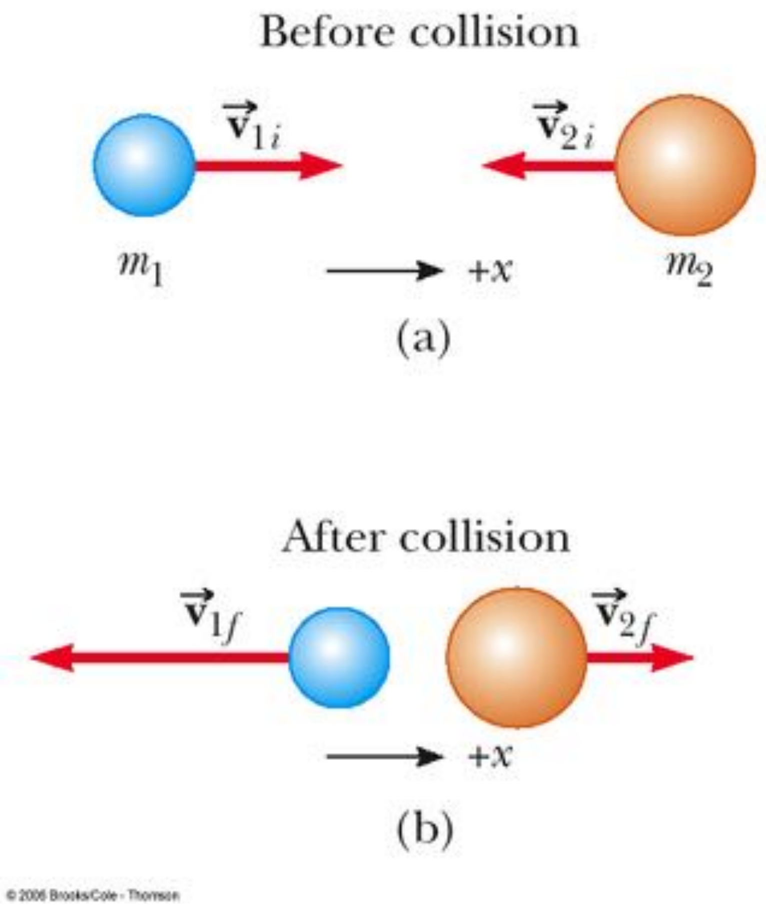
- Then $m_1\vec{v}_{1f} - m_1\vec{v}_{1i} = -(m_2\vec{v}_{2f} - m_2\vec{v}_{2i})$

- So $m_1\vec{v}_{1i} + m_2\vec{v}_{2i} = m_1\vec{v}_{1f} + m_2\vec{v}_{2f}$

- **Conservation of Energy:** If the collision is elastic, write a second equation for conservation of KE, or the alternative equation
  - This only applies to perfectly elastic collisions

$$\boxed{v_{1i} + v_{1f} = v_{2f} + v_{2i}}$$

- **Solve:** the resulting equations simultaneously



Notes : The momentum is conserved,

1. when resultant of forces is zero.
2. when time interval Δt is very small,
3. when all the external forces are non-impulsive.

Fire point : The total momentum is conserved only in one direction but not in another direction.

Impact/Collisions

It is the common normal to the plane of contact durin
impact.

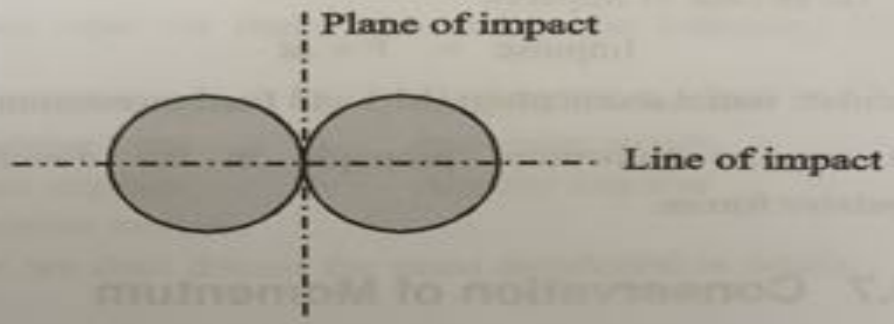


Fig. 16.9.1

16.9.2 Direct Impact

If the velocities of two particles are along the line of impac
the impact is called as direct impact.

Fig. 16.9.2

16.9.3 Oblique Impact

If both particles move along a line other than the line of
impact the type of impact is called oblique impact.

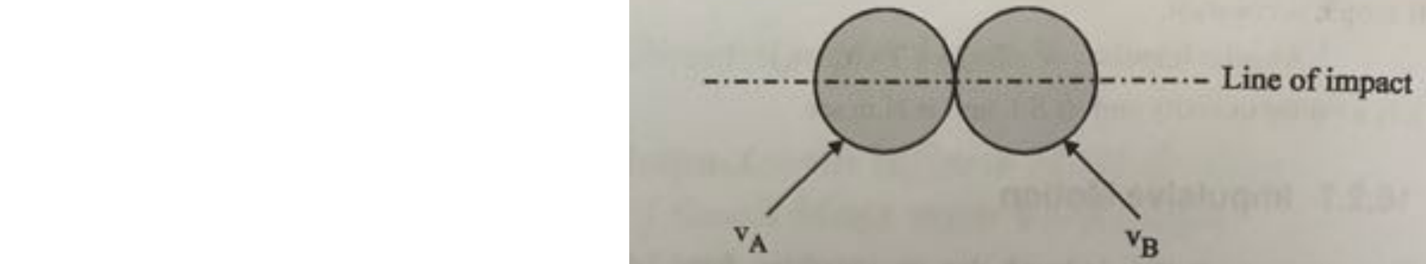
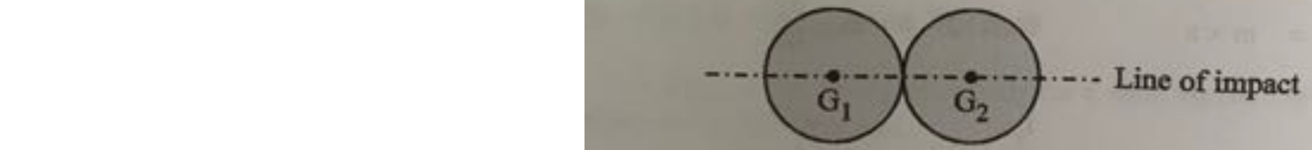


Fig. 16.9.3

16.9.4 Central Impact

If the mass centers of the two colliding bodies lie on line of impact, the impact is central impact.

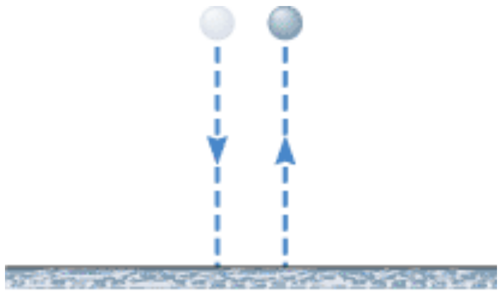


Impact/Collisions

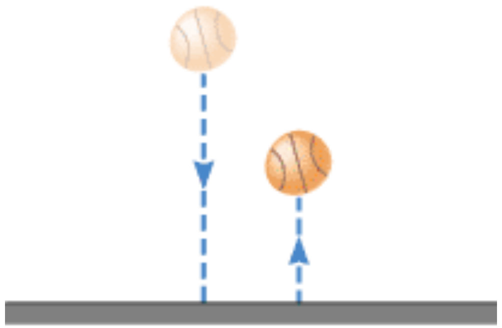
Collisions are often classified according to whether the total kinetic energy changes during the collision:

1.*Elastic collision*—One in which the total kinetic energy of the system after the collision is equal to the total kinetic energy before the collision.

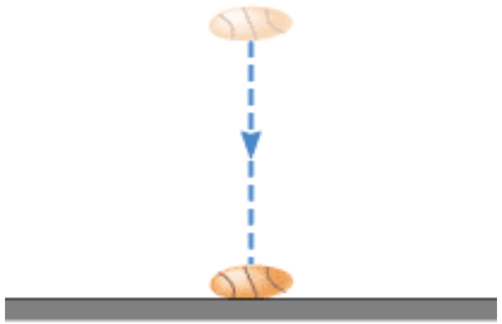
2.*Inelastic collision*—One in which the total kinetic energy of the system is not the same before and after the collision; if the objects stick together after colliding, the collision is said to be completely inelastic.



(*a*) Elastic collision

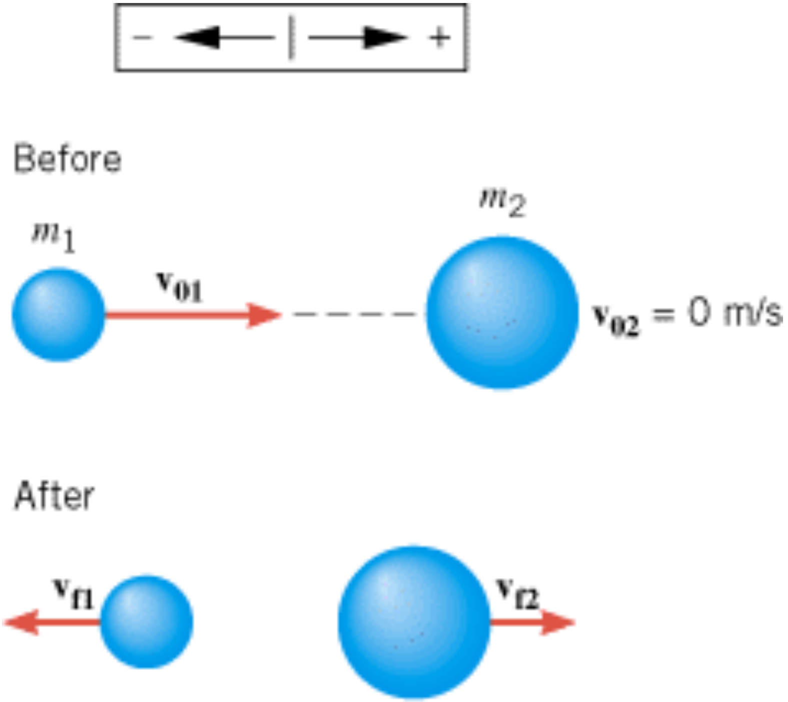


(*b*) Inelastic collision



(*c*) Completely inelastic collision

Collisions in One Dimension



1. Apply the conservation of momentum.
2. If the collision is elastic, apply the conservation of energy.

16.12 Types of Impact

There are four cases of impact which are of special interest.

1. Perfectly elastic impact.
2. Partially elastic impact (semi elastic impact)
3. Perfectly plastic impact.
4. Impact with very large (infinite) mass.

Now, we shall discuss these types and their properties in detail.

16.12.1 Perfectly Elastic Impact

In this type of impact there is a complete restoration. Both bodies regain their original shape and size completely.

Properties

(a) The momentum is conserved.

$$m_A u_A + m_B u_B = m_A v_A + m_B v_B$$

16.12.2 Semi Elastic Impact

In this type of impact there is a partial restoration. Both bodies do not regain their original shape and size completely.

Properties

(a) The momentum is conserved.

$$\therefore \quad m_A u_A + m_B u_B = m_A v_A + m_B v_B$$

(b) The kinetic energy is not conserved. There is some loss in K.E. during impact.

$$\therefore \quad \frac{1}{2} m_A u_A^2 + \frac{1}{2} m_B u_B^2 > \frac{1}{2} m_A v_A^2 + \frac{1}{2} m_B v_B^2$$

(c) Coefficient of restitution is more than zero but less than one

$$0 < e < 1$$

(d) Both bodies separate after impact.

16.12.3 Plastic Impact

In this type, both bodies coupled together after impact and move together with common velocity and due to permanent deformation there is no period of restoration.

Properties

1. The momentum is conserved.

$$\therefore \quad m_A u_A + m_B u_B = v (m_A + m_B)$$

where v = common velocity of both the bodies.

2. Kinetic energy is not conserved. There is considerable loss in kinetic energy during impact.

3. Co-efficient of restitution e = 0.

4. Both bodies move together after impact.

16.12.4 Impact with Infinite Mass

When a body of small mass collides with a body of very large mass as compared to first, the impact is considered as impact with infinite mass.

For example when a ball is dropped on a hard floor.

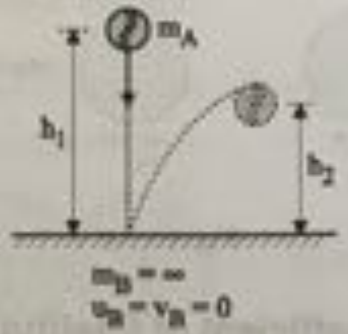


Fig. 16.12.1

Properties

1. Law of conservation of momentum is not applicable.
2. There is loss in K.E. due to impact.
3. Co-efficient of restitution.

$$e = \frac{v_B - v_A}{u_A - u_B}$$

but velocity of floor before and after impact is zero

$$\therefore \quad u_B = v_B = 0$$

$$\therefore \quad e = \frac{-v_A}{u_A}$$

but velocity of ball just before impact when it is dropped through height h_1 is $u_A = \sqrt{2 g h_1}\ (\downarrow)$

If after impact ball rises back to height h_2, its velocity after impact is given by, $v_A = \sqrt{2 g h_2}\ (\uparrow)$

$$\therefore \quad e = -\frac{v_A}{u_A}$$

$$= -\left[\frac{-\sqrt{2 g h_2}}{\sqrt{2 g h_1}}\right] \downarrow + \uparrow -$$

$$\therefore \quad e = \sqrt{\frac{h_2}{h_1}}$$

where h_1 = Height just before impact ; h_2 = Height after impact

16.12.5 Height after n^{th} Bounce

$$\boxed{d^{n^{th}} = \text{original height} \times e^{2n}}$$

Collisions Summary

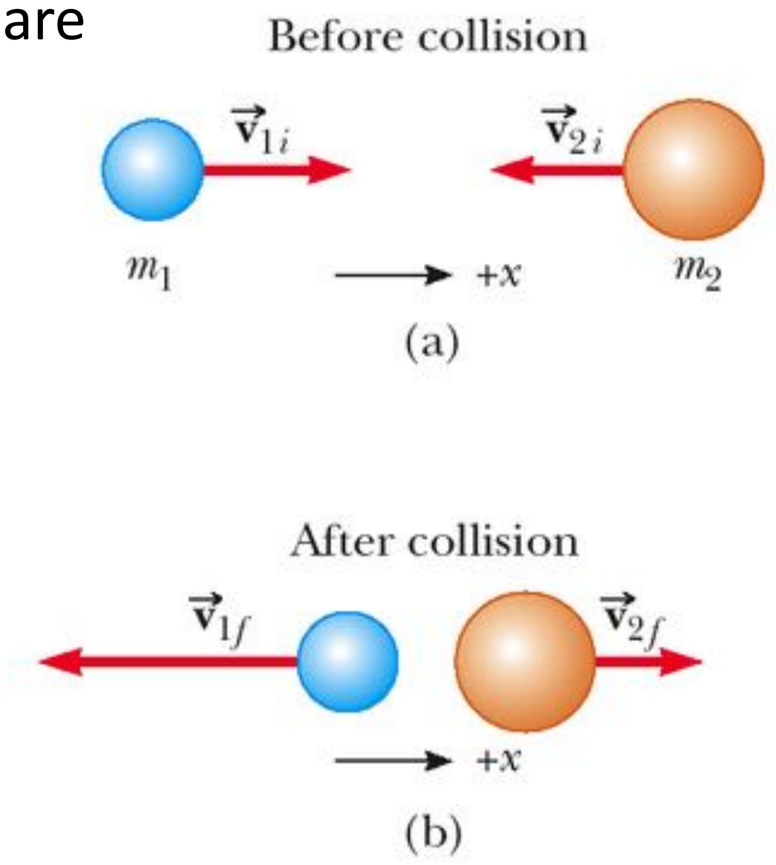
- In an elastic collision, both momentum and kinetic energy are conserved
- In a non-perfect inelastic collision, momentum is conserved but kinetic energy is not. Moreover, the objects do not stick together
- In a perfectly inelastic collision, momentum is conserved, kinetic energy is not, and the two objects stick together after the collision, so their final velocities are the same
- Elastic and perfectly inelastic collisions are limiting cases, most actual collisions fall in between these two types
- Momentum is conserved in all collisions

Elastic Collisions

- Both momentum and kinetic energy are conserved

$$m_1v_{1i} + m_2v_{2i} = m_1v_{1f} + m_2v_{2f}$$

$$\frac{1}{2}m_1v_{1i}^2 + \frac{1}{2}m_2v_{2i}^2 = \frac{1}{2}m_1v_{1f}^2 + \frac{1}{2}m_2v_{2f}^2$$



- Typically have two unknowns
- Momentum is a vector quantity
  - Direction is important
  - Be sure to have the correct signs
- Solve the equations simultaneously

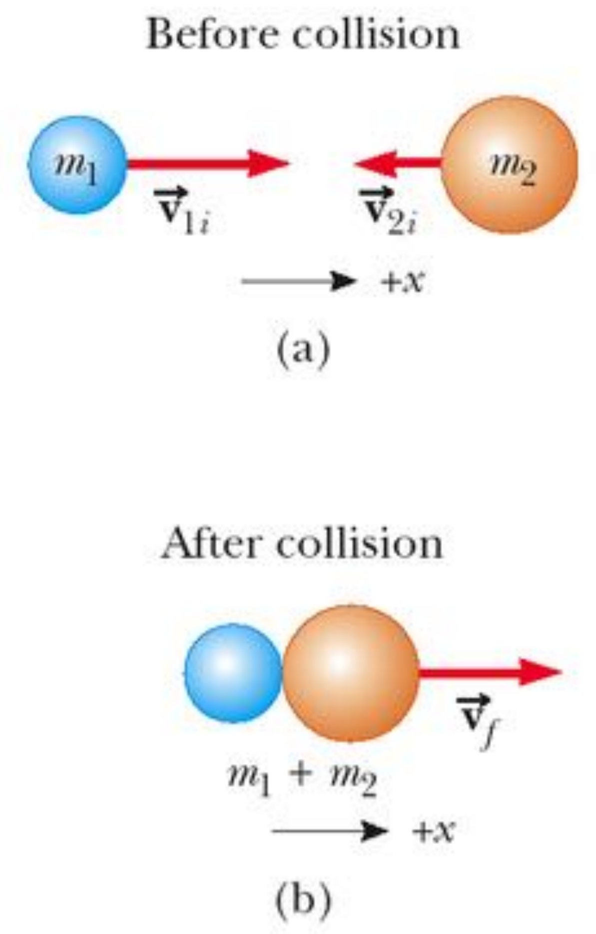
Perfectly Inelastic Collisions

- When two objects stick together after the collision, they have undergone a perfectly inelastic collision
- Conservation of momentum

$$\boxed{m_1 v_{1i} + m_2 v_{2i} = (m_1 + m_2) v_f}$$

$$\boxed{v_f = \frac{m_1 v_{1i} + m_2 v_{2i}}{m_1 + m_2}}$$

- Kinetic energy is NOT conserved



Summary of Types of Collisions

- In an elastic collision, both momentum and kinetic energy are conserved

$$\boxed{v_{1i} + v_{1f} = v_{2f} + v_{2i}} \qquad \boxed{m_1 v_{1i} + m_2 v_{2i} = m_1 v_{1f} + m_2 v_{2f}}$$

- In an inelastic collision, momentum is conserved but kinetic energy is not

$$\boxed{m_1 v_{1i} + m_2 v_{2i} = m_1 v_{1f} + m_2 v_{2f}}$$

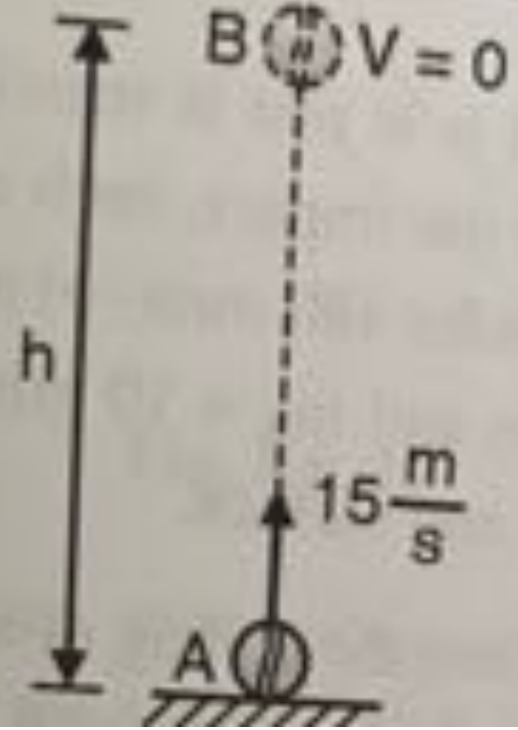
- In a *perfectly* inelastic collision, momentum is conserved, kinetic energy is not, and the two objects stick together after the collision, so their final velocities are the same

$$\boxed{m_1 v_{1i} + m_2 v_{2i} = (m_1 + m_2) v_f}$$

4 Marks

A ball has a mass of 30 kg and is thrown upward with a speed of 15 m/s. Determine the time to attain maximum height using impulse momentum principle. Also find the maximum height.

☑ **Soln. :**



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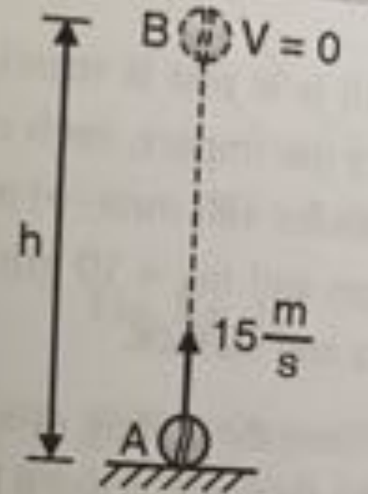

Fig. Ex. 16.12.33

From A → B

Using work-energy principle,

$$U_{A \to B} = KE_B - KE_A$$

$$-mgh = 0 - \frac{1}{2} m v_A^2$$

$$9.81\ h = \frac{1}{2}(15)^2$$

$$\therefore\ \mathbf{h = 11.47\ m}$$

A ball is dropped from an unknown height on a horizontal floor from which it rebounds to height of 8 m. If $e = 0.667$, calculate the height from which the ball was dropped.

A ball is dropped from an unknown height on a horizontal floor from which it rebounds to height of 8 m. If $e = 0.667$, calculate the height from which the ball was dropped.

☑ **Soln. :** Let, h_1 be the height of dropping

$$\text{Now,}\quad e = \sqrt{\frac{h_2}{h_1}}$$

$$0.667 = \sqrt{\frac{8}{h_1}}$$

$$\therefore h_1 = 17.98 \text{ m}$$

A 20 Mg railroad car moving with 0.5 m/s speed to the right collides with a 35 Mg car which is at rest. If after the collision the 35 Mg car is observed to move right with a speed of 0.3 m/s, determine the coefficient of restitution between the two cars.

☑ **Soln. :**

$m_A = 20 \times 10^3$ kg

$m_B = 35 \times 10^3$ kg

$u_A = 0.5 \frac{m}{s}$ $u_B = 0$

$v_A = ?$ $v_B = 0.3$ m/s

A 20 Mg railroad car moving with 0.5 m/s speed to the right collides with a 35 Mg car which is at rest. If after the collision the 35 Mg car is observed to move right with a speed of 0.3 m/s, determine the coefficient of restitution between the two cars.

☑ **Soln. :**

$m_A = 20 \times 10^3$ kg	$m_B = 35 \times 10^3$ kg
$u_A = 0.5 \frac{m}{s}$	$u_B = 0$
$v_A = ?$	$v_B = 0.3$ m/s

using law of conservation of momentum

$$m_A u_A + m_B u_B = m_A v_A + m_B v_B$$

$$(20 \times 10^3 \times 0.5) + 0 = (20 \times 10^3)\, v_A + (35 \times 10^3 \times 0.3)$$

$$\therefore v_A = -0.025 \frac{m}{s}$$

$\therefore$ Coefficient of restitution

$$e = \frac{v_B - v_A}{u_A - u_B}$$

$$= \frac{0.3 - (-0.02)}{0.5 - 0} = \mathbf{0.65}$$