THE ALGEBRAIC STRUCTURE OF CARNATIC INDIAN MUSIC

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1 INTRODUCTION

125 years ago Dmitri Mendeleev and Lothar Meyer borrowed the idea of an octave from music to enrich the sciences and elucidate the systematic structure in chemistry. It is time for the physical sciences to return the favour. The essential idea we use is the notion of symmetry, which has been used with great success in the fields of molecular structure and elementary particle physics.

The most obvious symmetry in music is the resonance between overtones, which leads directly to the use of modulo 2 arithmetic. It is useful to imagine the notes of an octave arranged along the circumference of a circle, starting with

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the fundamental *sa* and proceeding along the circumference of the circle, so that the arc from the *sa* to any note gives the value of the note. When we move all around the circle and traverse 360 degrees, we come to the *Tara Sa*, which is represented by the same point on the circle as the *sa*. This is the structure that generates the modulo 2 arithmetic, i.e., for any note n,

$$2 \times n = n$$

The fractional scale corresponds to frequency relationships. Thus *Tara Shadja* is an overtone of *shadja*, with exactly twice the frequency; *Panchama* is 1.5 times the frequency of *shadja*.

In Indian musical scales, the notes are arranged with unequal spacings around the circumference of our imaginary circle, giving rise to a rich profusion of ragas. If we divide the circle into 1200 cents (instead of into 360 degrees), we have the scale of cyclic cents. A cyclic cent is a unit on a logarithmic scale to the base 2 with an amplitude factor of 1200, such that $sa = 1$ has 0 cents and $Tara\ Sa = 2$ has 1200 cents. Thus notes multiplied on the fractional scale correspond to intervals added on the scale of cyclic cents.

2 The Conventional Carnatic Musical Scale with 22 Srutis

The commonly accepted values[1] of the *srutis* used in Carnatic music are listed below :

No.	Note	Value in Cyclic Cents	Fractional Value
1	Shadja	0	1
2	Ekasruti Rishabha	90	256/243
3	Dwisruti Rishabha	112	16/15
4	Trisruti Rishabha	182	10/9
5	Chatusruti Rishabha	204	9/8
6	Shudha Gandhara	294	32/27
7	Sadharana Gandhara	316	6/5
8	Antara Gandhara	386	5/4
9	Chyuta madhyama Gandhara	408	81/64
10	Shudha Madhyama	498	4/3
11	Tivra shudha Madhyama	520	27/20
12	Prati Madhyama	590	45/32
13	Chyuta panchama Madhyama	612	729/512 (or 64/45)

[1] S. Bhagyalekshmy, '*Ragas in Carnatic Music*' (CBH Publications, Trivandrum, 1990); P. Sambamoorthy, '*South Indian Music*' (Indian Music Publishing House, Madras)

14	Panchama	702	3/2
15	Ekasruti Dhaivata	792	128/81
16	Dwisruti Dhaivata	814	8/5
17	Trisruti Dhaivata	884	5/3
18	Chatusruti Dhaivata	906	27/16
19	Shudha Nishada	996	16/9
20	Kaishiki Nishada	1018	9/5
21	Kakali Nishada	1088	15/8
22	Chyuta shadja Nishada	1110	243/128
1	Tara Shadja	1200	2

The rational numbers 3/2 (*Panchama*) and 4/3 (*Shudha Madhyama*) give rise to the so-called Cycles of Fifths and Fourths, respectively, from which all 22 *srutis* can be generated :

THE CYCLES OF FIFTHS

is obtained by taking $(3/2)^{Cycle}$ modulo 2

Cycle	Value in Cyclic Cents	Location in Musical Scale
1	702	Panchama
2	204	Chatusruti Rishabha
3	906	Chatusruti Dhaivata
4	408	Chyuta madhyama Gandhara
5	1100	Chyuta shadja Nishada
6	612	Chyuta panchama Madhyama
7	112	Dwisruti Rishabha
8	814	Dwisruti Dhaivata
9	318	Sadharana Gandhara
10	1018	Kaishiki Nishada
11	520	Tivra shudha Madhyama

THE CYCLES OF FOURTHS

is obtained by taking $(4/3)^{Cycle}$ modulo 2

Cycle	Value in Cyclic Cents	Location in Musical Scale
1	498	Shudha Madhyama
2	996	Shudha Nishada
3	294	Shudha Gandhara
4	792	Ekasruti Dhaivata
5	90	Ekasruti Rishabha
6	590	Prati Madhyama

7	1088	Kakali Nishada
8	386	Antara Gandhara
9	884	Trisruti Dhaivata
10	112	Dwisruti Rishabha

It is also well known, and easily verified, that the cycle of *Shudha Madhyama* is inverse to the cycle of *Panchama*, i.e. the Cycle of Fourths applied to the *Tara Shadja* yields the notes obtained from the Cycle of Fifths above. This is due to the fact that *Shudha Madhyama* (4/3) is algebraically the inverse of *Panchama* (3/2) modulo 2.

3 GENERATORS OF THE ALGEBRA

Further all the 22 *srutis* can be generated from a set of three 'atomic' frequency intervals :

1. *Pramana Sruti* = a = 81/80 (22 cents)
2. *Nyuna Sruti* = b = 25/24 (70 cents)
3. *Purna Sruti* = c = 256/243 (90 cents)

Thus we generate the structure :

	Ekasruti	Dwisruti	Trisruti	Chatusruti
Shadja	1			
Rishabha	c	ac	abc	a^2bc
Gandhara	a^2bc^2	a^3bc^2	$a^3b^2c^2$	$a^4b^2c^2$
Madhyama	$a^4b^2c^3$	$a^5b^2c^3$	$a^5b^3c^3$	$a^6b^3c^3$
Panchama	$a^6b^3c^4$ (=3/2)			
Dhaivata	$a^6b^3c^5$	$a^7b^3c^5$	$a^7b^4c^5$	$a^8b^4c^5$
Nishada	$a^8b^4c^6$	$a^9b^4c^6$	$a^9b^5c^6$	$a^{10}b^5c^6$
Tara Shadja	$a^{10}b^5c^7$ (=2)			

Several musicologists have tried to expand the musical scale of 22 *srutis*. Our investigations have shown that a systematic expansion is possible, using the cycles of *Sadharana Gandhara* and *Antara Gandhara* to generate further notes and smaller 'atomic' *sruti* intervals.

For instance, we have constructed an expanded musical scale with 44 *Srutis* including all notes upto 12 cycles of *Panchama, Shudha Madhyama, Sadharana Gandhara* and *Antara Gandhara*. A new 'atomic' *sruti* interval is thus generated, with a value of about 8 Cyclic Cents (fractional value = 15625/15552) which is less than half that of the *Pramana Sruti* (22 cents).

In fact, the process can be continued indefinitely, to higher and higher cycles, generating further notes lying in between the existing notes (these intermediate notes, however, are in all likelihood, indistinguishable by ear). When this process is continued to an infinite number of cycles, we get an infinitely dense 'dust' of points along the circumference of our imaginary circle, while yet not forming a continuum. Mathematically, this is the well-known Cantor dust, an example of a fractal (or scale-invariant) pattern. In Indian music we get a continuum from the dust by using the concept of *gamakas*.

Contrast this with the situation obtained from the equal-temperament scales of European classical music. Here the notes of the Major scale have the values 0, 200, 400, 500, 700, 900, 1100 cyclic cents; while the Minor keys have the values 100, 300, 600, 800, 1000 cyclic cents. No other notes are generated from any number of cycles or combinations of notes. In other words, the set of all notes in these scales forms a closed Group.

In a given *raga*, some of these *srutis* attain the status of *swaras*. The *melakarta ragas* of Carnatic music are obtained by permuting 7 *swaras* among 12 *swarasthanas*. Keeping the *shadja* and *panchama* invariant and admitting only two varieties for the *madhyama* and six combinations each for *rishabha-gandhara* and *dhaivata-nishada* yields 72 *mela ragas*.

So what is the significance of the 22 *srutis* ? With 7 *swaras* there are 21 possible pairs of notes (6+5+4+3+2+1) in any *sampoorna raga*. Add one null interval (between identical notes) and there are thus 22 possible frequency intervals between successive notes. These frequency intervals are the 22 *srutis*.

4 MODAL SHIFT OF TONIC

Modal Shift of Tonic is the process by which one can generate new *ragas* from existing ones and represents a second type of symmetry. These are derived from the *Murchanas* of ancient Indian music and are known as *Thaat* in contemperory Hindustani music. This is the symmetry that maps a polygon (inscribed in our imaginary circle, with its vertices at the *swaras* used to form the *raga*) onto a rotated version of itself (the new *raga*). Thus *ragas* obtained by Modal Shift of Tonic are related to each other by rotations in the space of our imaginary circle, as can be seen by performing the appropriate transformations (modulo 2) upon the cyclical cent values. For instance, starting with *raga Shankarabharanam*, we get :

Raga	sa	ri	ga	ma	pa	dha	ni	approx.Rotation to Shankarabharanam in cyclic cents
Shankarabharanam	0	204	386	498	702	906	1088	
Kharaharapriya	0	204	316	498	702	906	1018	200
Todi	0	112	316	498	702	814	1018	400
Kalyani	0	204	386	590	702	906	1088	500
Harikamboji	0	204	386	498	702	906	1018	700
Nata Bhairavi	0	204	316	498	702	814	1018	900

This is an example of what is known as a global symmetry transformation, since to obtain a given *raga*, the same transformation is applied at all points (*swaras*) of the parent *raga*. *Mela ragas* admitting of one or more such global symmetry transformations are called *Murchanakaraka melas*. A little reflection reveals that the special algebra which makes these transformations possible is the approximate relationship

$$a \times b = c$$

(also obtained by adding the respective cyclic cent values).

In contrast, local symmetry transformations are those that where the transformation applied differs from point to point on the scale. Such local transformations (known as local gauge transformations) are very important in quantum physics, leading to the class of theories known as Yang-Mills theories and supersymmetry, and they are also possible in music, leading to the rich *raga* structure obtained by replacing one *swara* with another.

5 SUMMARY

We have explored some consequences of the algebraic structure of Indian music and pointed out some aspects of the symmetry transformations resulting from this algebra. A very important and interesting question we are now investigating is whether the ability of the human brain to (accoustically) appreciate the beauty of this abstract algebra (i.e., music) implies a specific geometric structure of the connectivity patterns of the brain cells. We hope we have been able to convince you that it is a fascinating study.