

Assignment 2

AI1110 : Probability and Random Variables
Indian Institute of Technology Hyderabad

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Problem 11.16.4.6 : Three letters are dictated to three persons and an envelope is addressed to each of them, the letters are inserted into the envelopes at random so that each envelope contains exactly one letter. Find the probability that at least one letter is in its proper envelope.

Solution:

Let's define the random variable X_i as follows:

$$X_i = \begin{cases} 1, & \text{if the } i_{th} \text{ letter is in its proper envelope} \\ 0, & \text{if the } i_{th} \text{ letter is not in its proper envelope} \end{cases} \quad (1)$$

Pr(at least one letter is in its proper envelope)

$$\begin{aligned} & \Pr(X_1 = 1 \text{ or } X_2 = 1 \text{ or } X_3 = 1) \\ &= \Pr(X_1 = 1) + \Pr(X_2 = 1) + \Pr(X_3 = 1) - \Pr(X_1 = 1, X_2 = 1) - \Pr(X_1 = 1, X_3 = 1) - \Pr(X_2 = 1, X_3 = 1) \\ & \quad + \Pr(X_1 = 1, X_2 = 1, X_3 = 1) \end{aligned} \quad (2)$$

$$\Pr(X_1 = 1) = \frac{1}{3} \quad (3)$$

$$\Pr(X_2 = 1) = \frac{1}{3} \quad (4)$$

$$\Pr(X_3 = 1) = \frac{1}{3} \quad (5)$$

$$\Pr(X_1 = 1, X_2 = 1) = \frac{1}{6} \quad (6)$$

$$\Pr(X_1 = 1, X_3 = 1) = \frac{1}{6} \quad (7)$$

$$\Pr(X_2 = 1, X_3 = 1) = \frac{1}{6} \quad (8)$$

$$\Pr(X_1 = 1, X_2 = 1, X_3 = 1) = \frac{1}{6} \quad (9)$$

$$(10)$$

Substituting these values into the formula above, we get:

$$\Pr(\text{at least one letter is in its proper envelope}) = \frac{1}{3} + \frac{1}{3} + \frac{1}{3} - \frac{1}{6} - \frac{1}{6} - \frac{1}{6} + \frac{1}{6} \quad (11)$$

$$= \frac{2}{3} \quad (12)$$

Therefore, the probability that at least one letter is in its proper envelope is $\frac{2}{3}$.

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