

Assignment 2

AI1110 : Probability and Random Variables
Indian Institute of Technology Hyderabad

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Problem 11.16.4.6 : Three letters are dictated to three persons and an envelope is addressed to each of them, the letters are inserted into the envelopes at random so that each envelope contains exactly one letter. Find the probability that at least one letter is in its proper envelope.

Solution:

Let l_1, l_2, l_3 denote the three letters, and E_1, E_2, E_3 denote the three corresponding envelopes respectively.

'S' = Sample space = Randomly distributing the three letters in the three letters.

'A' = Event that atleast one letter is in correct envelope

Let X and Y be random variables such that,

$$X = \begin{cases} 1, & \text{if letter } l_1 \text{ is inserted} \\ 2, & \text{if letter } l_2 \text{ is inserted} \\ 3, & \text{if letter } l_3 \text{ is inserted} \end{cases} \quad (1)$$

$$Y = \begin{cases} 1, & \text{if envelope } E_1 \text{ is used} \\ 2, & \text{if envelope } E_2 \text{ is used} \\ 3, & \text{if envelope } E_3 \text{ is used} \end{cases} \quad (2)$$

Let R_{xy} denote that letter $X=x$ is inserted into envelope $Y=y$. The sample space is,

- 1) R_{11}, R_{32}, R_{23}
- 2) R_{22}, R_{31}, R_{13}
- 3) R_{33}, R_{12}, R_{21}
- 4) R_{11}, R_{22}, R_{33}
- 5) R_{12}, R_{23}, R_{31}
- 6) R_{13}, R_{21}, R_{32}

There are 4 ways in which at least one letter is correctly placed, i.e. at least one R_{ii} for $i=\{1, 2, 3\}$,

Hence,

$$n(S) = 3! = 6 \quad (3)$$

$$n(A) = 4 \quad (4)$$

Hence,

$$\Pr(\text{at least once } X = Y = i) = \frac{n(A)}{n(S)} \quad (5)$$

$$= \frac{4}{6} \quad (6)$$

$$= \frac{2}{3} \quad (7)$$