Assignment 2
AI1110 : Probability and Random Variables
Indian Institute of Technology Hyderabad

Manpurwar Ganesh*

Problem 11.16.4.6 : Three letters are dictated to three persons and an envelope is addressed to each of them, the letters are inserted into the envelopes at random so that each envelope contains exactly one letter. Find the probability that at least one letter is in its proper envelope.

Solution:
Let L_1, L_2, L_3 denote the three letters, and E_1, E_2, E_3 denote the three corresponding envelopes respectively.
'S'= Sample space = Randomly distributing the three letters in the three letters.
'E'= Event that atleast one letter is in correct envelope
Let X and Y be random variables such that,

$$X = \begin{cases} 1, & \text{if letter } L_1 \text{ is inserted} \\ 2, & \text{if letter } L_2 \text{ is inserted} \\ 3, & \text{if letter } L_3 \text{ is inserted} \end{cases} \tag{1}$$

$$Y = \begin{cases} 1, & \text{if envelope } E_1 \text{ is used} \\ 2, & \text{if envelope } E_2 \text{ is used} \\ 3, & \text{if envelope } E_3 \text{ is used} \end{cases} \tag{2}$$

Let (x, y) denote that letter X=x is inserted into envelope Y=y. The sample space is,
1) $(1, 1), (3, 2), (2, 3)$
2) $(2, 2), (3, 1), (1, 3)$
3) $(3, 3), (1, 2), (2, 1)$
4) $(1, 1), (2, 2), (3, 3)$
5) $(1, 2), (2, 3), (3, 1)$
6) $(1, 3), (2, 1), (3, 2)$

There are 4 ways in which at least one letter is inserted in a proper envelope , i.e. at least one (i, i) for i={1, 2, 3},
Hence,

$$n(S) = 3! = 6 \tag{3}$$

$$n(A) = 4 \tag{4}$$

Hence,

$$\Pr(\text{at least once } X = Y = i) = \frac{n(E)}{n(S)} \tag{5}$$

$$= \frac{4}{6} \tag{6}$$

$$= \frac{2}{3} \tag{7}$$

*The studnent is with the Department of AI , Indian Institute of Technology, Hyderabad 502285 India e-mail: ai22btech11017@iith.ac.in.