

Assignment 2

AI1110 : Probability and Random Variables
Indian Institute of Technology Hyderabad

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Problem 11.16.4.6 : Three letters are dictated to three persons and an envelope is addressed to each of them, the letters are inserted into the envelopes at random so that each envelope contains exactly one letter. Find the probability that at least one letter is in its proper envelope.

Solution:

Let L_1, L_2, L_3 denote the three letters, and E_1, E_2, E_3 denote the three corresponding envelopes respectively.

S= Sample space = Randomly distributing the three letters in the three letters.

E= Event that atleast one letter is in correct envelope

Let X and Y be random variables such that,

TABLE 0
RANDOM VARIABLE DECLARATION

Random Variable	Value of the random variable	Event
X	1	letter L_1 is inserted
	2	letter L_2 is inserted
	3	letter L_3 is inserted
Y	1	letter E_1 is used
	2	letter E_2 is used
	3	letter E_3 is used

Let (x, y) denote that letter $X=x$ is inserted into envelope $Y=y$. The sample space is,

- 1) $(1, 1), (3, 2), (2, 3)$
- 2) $(2, 2), (3, 1), (1, 3)$
- 3) $(3, 3), (1, 2), (2, 1)$
- 4) $(1, 1), (2, 2), (3, 3)$
- 5) $(1, 2), (2, 3), (3, 1)$
- 6) $(1, 3), (2, 1), (3, 2)$

There are 4 ways in which at least one letter is inserted in a proper envelope , i.e. at least one (i, i) for $i=\{1, 2, 3\}$.

Hence,

$$n(S) = 3! = 6 \quad (1)$$

$$n(E) = 4 \quad (2)$$

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Hence,

$$\Pr(X = i, Y = i) = \frac{n(E)}{n(S)} \qquad \exists i \in (1, 2, 3) \qquad (3)$$

$$= \frac{4}{6} \qquad (4)$$

$$= \frac{2}{3} \qquad (5)$$