STAT S 670 – Exploratory Data Analysis – Homework #3

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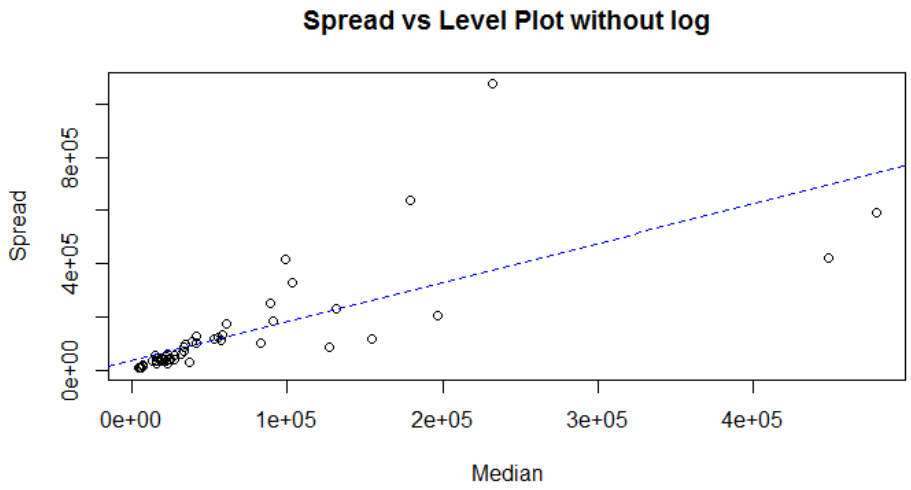
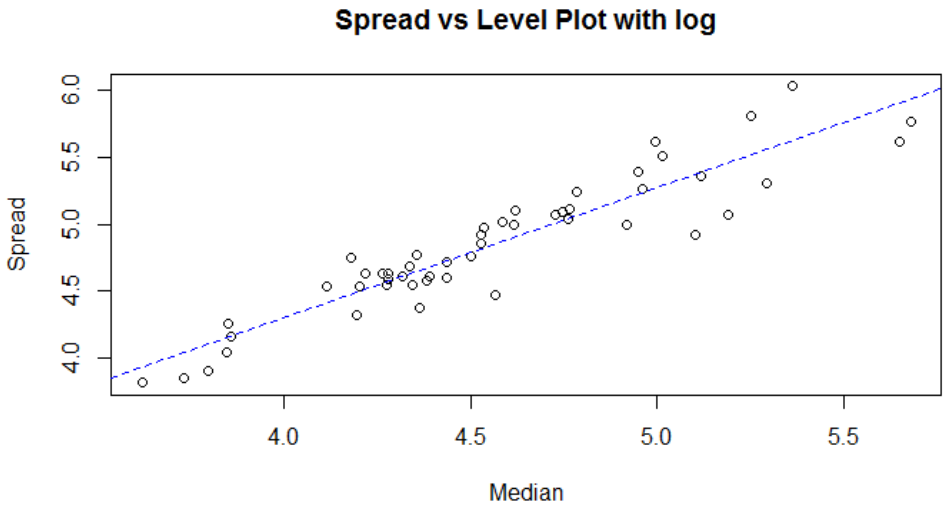
Solutions

1. a) Level VS Spread plot without any transformation



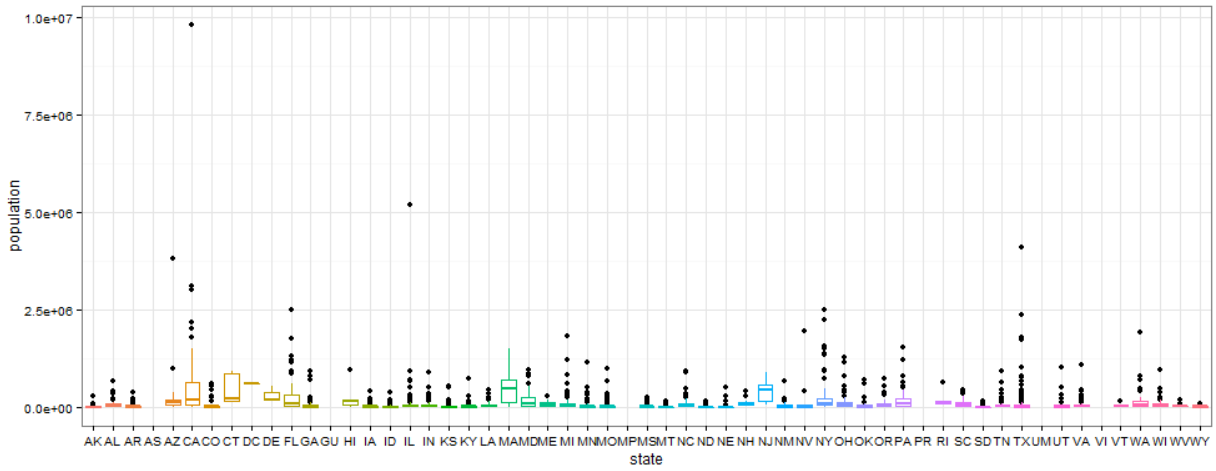
Level Vs Spread Plot with log transformation



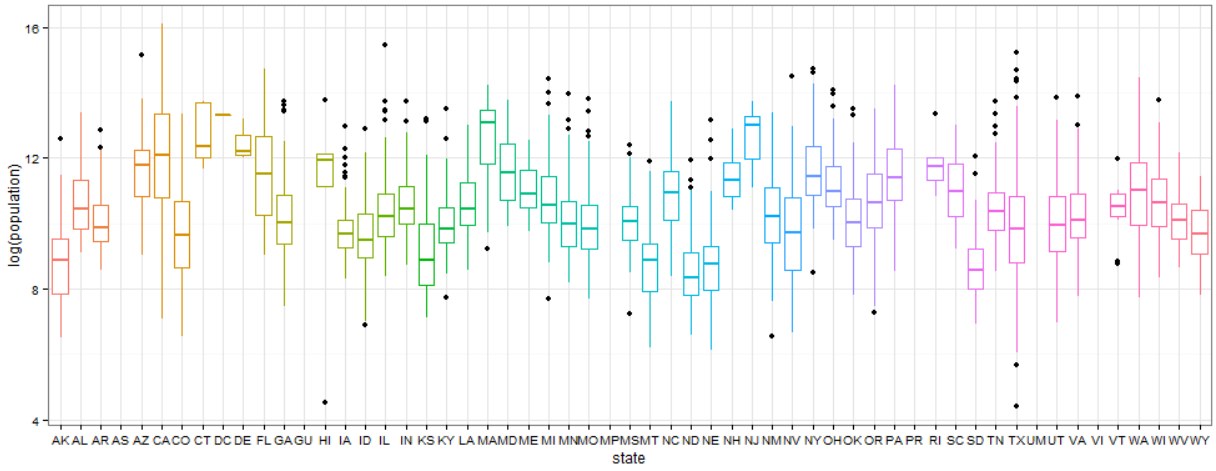
b) From the R code, 1-slope is 0.02806874, slope=0.9719
lm equation log(df) = 0.9719*log(m)+0.028

Hence from p formula, the most appropriate transformation would be log transformation.

c) Box plot without log transformation,



It can be seen that there are lot of outliers and outliers distort the interpretation of the box plot. Hence as suggested by the 1-p rule, following is the box plot with log transformation applied.



A clear visual comparison from the box plot with and without transformation supports the effectiveness of the transformation. It can be clearly seen that the box plot with log transformation has lesser outlier effects and better interpretable.

d) Transformation for symmetry table:

Since the transformation of California subset comes under transformation of data from multiple batches, this becomes a problem for transformation of symmetry.

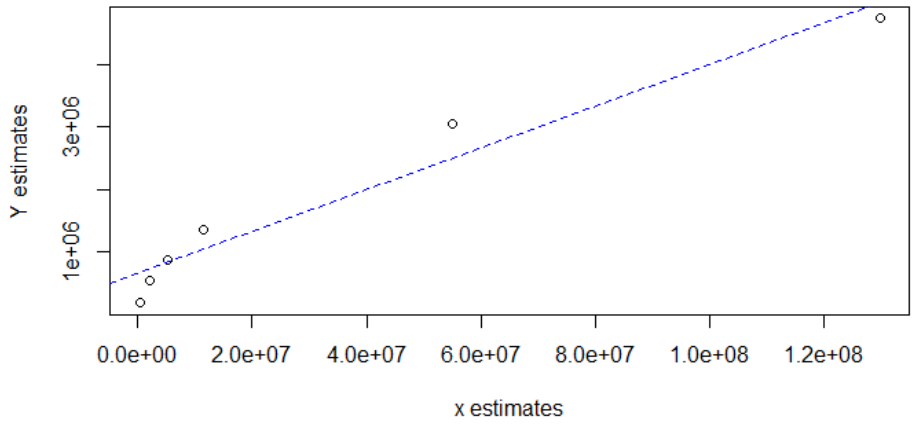
Also, since the transformation is for a single batch, the estimate of p is calculated from the slope of the lm fit line to the x and y axis columns in symmetry table.

Depth	X_L	X_U	Mid Summary	Spread	$\frac{(X_L - M)^2 + (M - X_U)^2}{4M}$	$\frac{XL + XU}{2} - M$	P estimate
F 15.0	45578	685306	365442	639728	382440.7	186301.5	0.5128617
E 8.0	20007	1418788	719397.5	1398781	2179922.2	540257	0.7521668
D 4.5	13994	2112426	1063209.8	2098432	5254066.3	884069.2	0.8317362
C 2.5	6463	3052773	1529617.8	3046310	11565752	1350477.2	0.8832348
B 1.5	2207.5	6456959	3229583.2	6454752	55043820.9	3050442.8	0.9445816
A 1.0	1175	9818605	4909890	9817430	129717941.5	4730749.5	0.9635305

Also from the R code, (Via Linear modeling)

```
[1] "The power is  0.966537662120274"
[1] "The slope is  0.0334623378797263"
```

e) The median of the p-estimate is also 0.857485 which suggests that there is no need for any transform ation. Also the spread vs level plot for the above table is as follows,



For the entire dataset, generating the p-estimate by considering it to be one single batch,

Depth	X_L	X_U	Mid Summary	Spread	$\frac{(X_L - M)^2 + (M - X_U)^2}{4M}$	$\frac{XL + XU}{2} - M$	P estimate
786.5	11104.5	66699	38901.75	55594.5	18232.06	13044.75	0.2845157
393.5	6157	157906.5	82031.75	151749.5	172343.66	56174.75	0.6740539
197	3423	321520	162471.5	318097	850058.92	136614.5	0.8392882
99	2071	622263	312167	620192	3444578.75	286310	0.916881
50	1321	919040	460180.5	917719	7719165.86	434323.5	0.9437344
25.5	813	1401948	701380.5	1401135	18314708.22	675523.5	0.9631158
13	662	1951269	975965.5	1950607	35849539.37	950108.5	0.9734973
7	494	2504700	1252597	2504206	59416269.29	1226740	0.9793535
4	416	3817117	1908766.5	3816701	138978803	1882909.5	0.9864518

2.5	188	4643567	2321877.5	4643379	206171486.9	2296020.5	0.9888635
1.5	86	7506640	3753363	7506554	541079576.5	3727506	0.993111
1	82	9818605	4909343.5	9818523	927201316.3	4883486.5	0.9947331

From the above table median p estimate is 0.9683065.

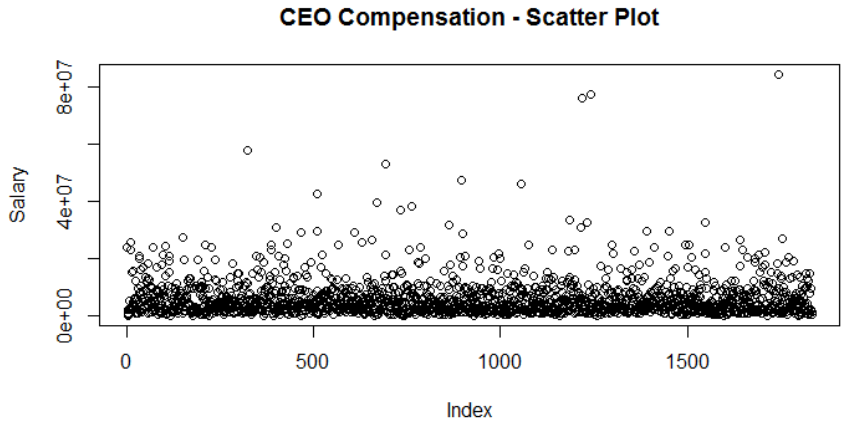
Hence from the above value it can be inferred that when considering the dataset as a set of different batches, the suggested transformation is log transform as shown in 1(a). However when we consider the entire dataset to be in one single batch, the p estimate comes to 0.9683. Hence when rounded to nearest integer, it is one and no transformation is suggested.
This seems to be intuitive considering the fact that when we separate data into batches, we would have to handle more outliers than when we have one single batch data.

From previous lecture, outliers are given by 0.4+0.07n for one single batch, considering additional 50 batches, outliers become high and requires more stringent transformation techniques.

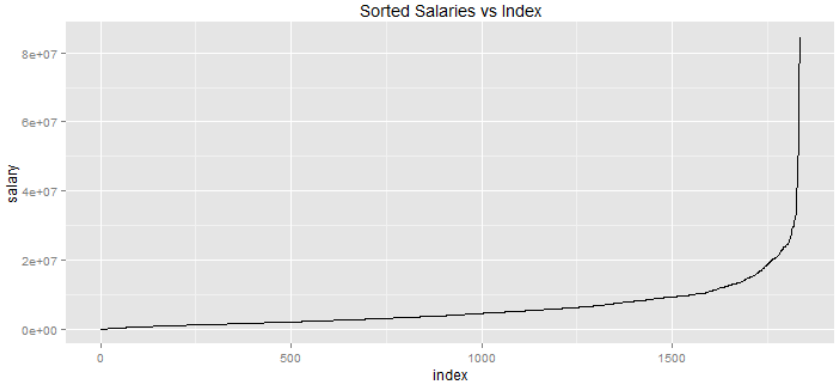
f) Find a and b of the matched transform:

Objective: Do a matched transform for the log transformation for the California Dataset, such that function $z = a \log_{10} x + b$
From the California data subset, the median is 179140.5.
Step 1: Calculate T′, Given $T(x) = log(x) = log_{10}e * log_e x$
$T' = log_{10}e * \frac{1}{x} = \frac{0.4343}{x}$
Median $= x_0 = 179140.5$ (Given)
Step 2: Calculate b
From lecture notes, b=$\frac{1}{T'(x_0)} = \frac{x_0}{0.4343} = \frac{179140.5}{0.4343} = 412481$
Step 3: Calculate a
Also from Lecture Notes, $a = x_0 - b * log_{10} x_0$
$= 179140.5 - 412481 * log_{10} 179140.5$
$\approx$ -1987702
Hence rounding a and b, the matched value transformation is, $z = 412481 * log(x) - 1987702$

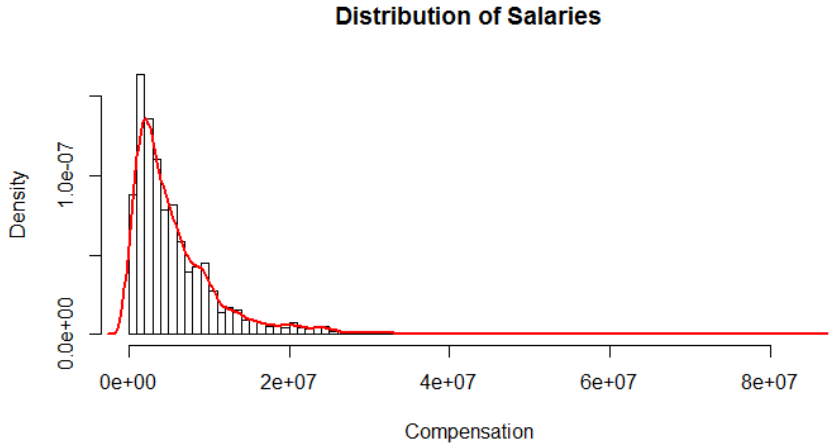
2. CEO Compensation Dataset
   a. There are 1835 rows in the dataset. The highest CEO is paid 84515000. The unusual things is that dataset is that,
      a. 8 CEOs draw zero salary
      b. The data is dense in the sorted lower value segments and sparse in the higher end.



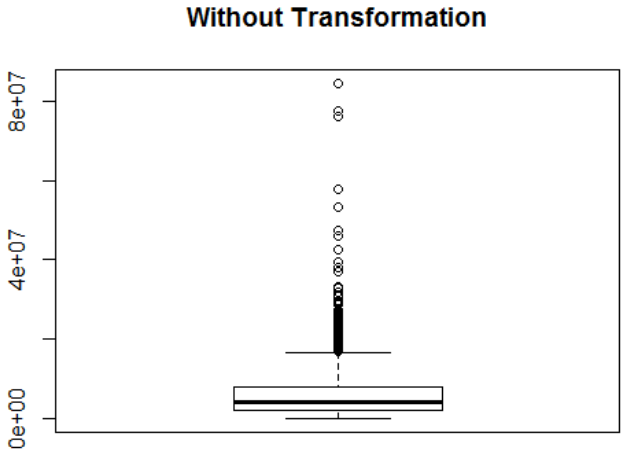
b. Following is the sorted salaries plotted by position, it can be seen that the linear is linear for lower and middle salaries, however it turns exponential!



With respect to distribution, the distribution is skewed right as shown below.



Following with our discussion the data symmetry, following is the box spot

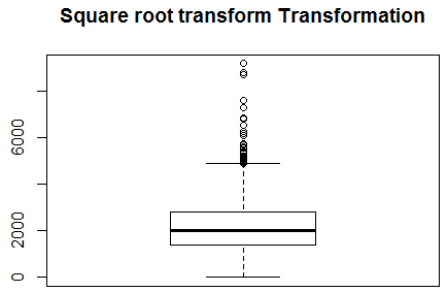


It can be seen that outliers present in the data distort the data representation, hence requires a transformation for readability and further data processing.

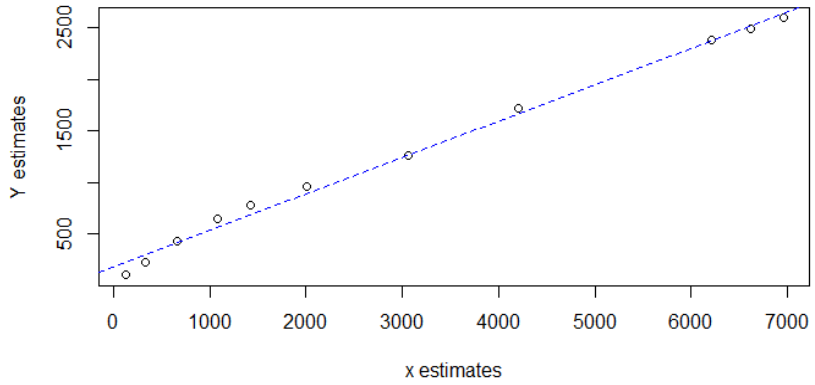
c. Following is the transform for symmetry table

Depth	X_L	X_U	Mid Summary	Spread	$\frac{(X_L - M)^2 + (M - X_U)^2}{4M}$	$\frac{XL + XU}{2} - M$	P estimate
F	1987500	7857000	4922250	5869500	1177155	911250	0.2258876
E	1219000	11190000	6204500	9971000	3698162	2193500	0.4068675
D	754500	16102500	8428500	15348000	9773695	4417500	0.5480215
C	459000	21370000	10914500	20911000	19568162	6903500	0.6472075
B	279500	25320500	12800000	25041000	29170960	8789000	0.6987072
A	86000	31719000	15902500	31633000	48811948	11891500	0.7563814
Z	0	42589000	21294500	42589000	93764037	17283500	0.8156703
Y	0	55502500	27751250	55502500	166259206	23740250	0.8572094
X	0	76831500	38415750	76831500	331520403	34404750	0.8962213
W	0	81035500	40517750	81035500	370784201	36506750	0.9015418
V	0	84515000	42257500	84515000	404947777	38246500	0.905552

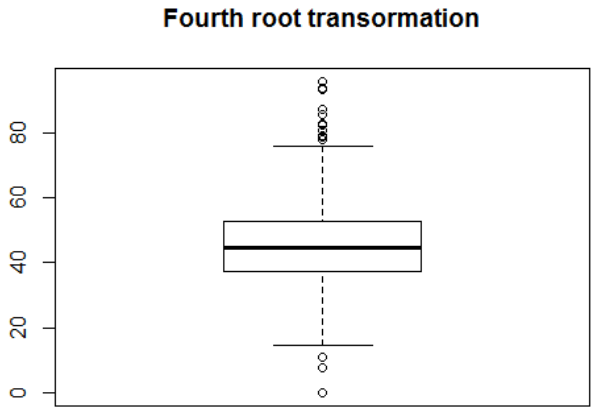
The median of the estimated p values is 0.75, hence plot square root transformation



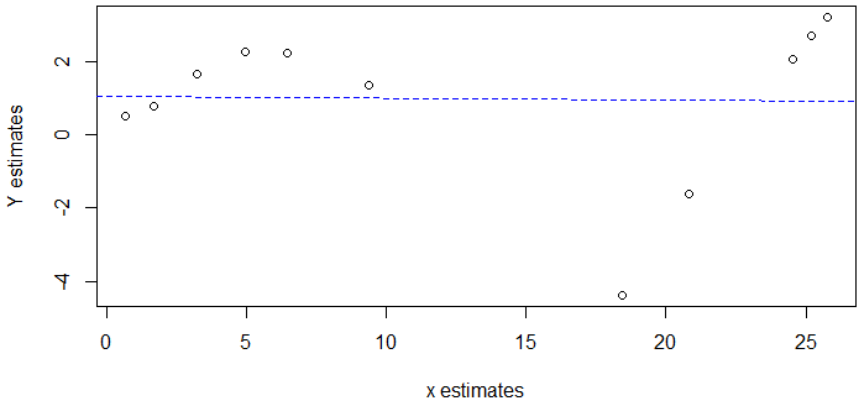
The Symmetry plot is,



Since the diagnostic plot did not give any optimal result, Consider another transform, fourth root transform, the box plot and symmetry plot is as follows,



The symmetry plot is as follows,



As seen from the above plot, since fourth root creates a straight line slope, as per the discussion in class, is more a proper transformation than the square root transform.

d. Since we cannot delete a data point, we cannot remove irregular values. However as shown above a proper transformation plot hugely increases the readability of the plot.

e. The two transformations are square root transformation and the fourth root transformation are already discussed in section 2.c

f. If one has to choose between the above two, a more apt one would be the fourth root. This can verified by plotting the transform plot for the fourth root of the dataset. We get a straight line, implying the transformation is correct.

	DEPTH	LOWER	UPPER	MID	SPREAD	XAXIS	YAXIS	PESTIMATE
F	459.5	37.5471	52.9437	45.2454	15.3965	0.6648534	0.4933	0.2580319
E	230	33.2277	57.8372	45.5325	24.6095	1.6984211	0.78035	0.5405439
D	115.5	29.4723	63.3465	46.4094	33.8741	3.2357364	1.6573	0.4878137
C	58	26.0287	67.991	47.0098	41.9622	4.9752536	2.25775	0.546204
B	29.5	22.9927	70.9356	46.9641	47.943	6.47482	2.21205	0.6583612
A	15	17.1248	75.0464	46.0856	57.9216	9.3906896	1.3335	0.8579976
Z	8	0	80.7838	40.3919	80.7838	18.440665	-4.3602	1.2364448
Y	4.5	0	86.3004	43.1502	86.3004	20.83149	-1.6019	1.076898
X	2.5	0	93.6227	46.8114	93.6227	24.530056	2.05925	0.916052
W	1.5	0	94.8623	47.4312	94.8623	25.215479	2.67905	0.8937538
V	1	0	95.8812	47.9406	95.8812	25.791724	3.1885	0.8763751

	Depth	Lower	Upper	Mid	Spread	xaxis	yaxis	pestimate
F	459.5	1409.7872	2803.033	2106.41	1393.246	123.8372	103.6621	0.162916
E	230	1104.0833	3345.146	2224.615	2241.062	325.7561	221.8664	0.3189187
D	115.5	868.6192	4012.782	2440.7	3144.163	664.8969	437.9524	0.3413228
C	58	677.4954	4622.77	2650.133	3945.274	1076.1223	647.3844	0.39841
B	29.5	528.6672	5031.89	2780.278	4503.222	1416.6302	777.5304	0.4511409
A	15	293.2576	5631.962	2962.61	5338.705	2008.9339	959.8619	0.5222033
Z	8	0	6526.025	3263.012	6526.025	3054.6817	1260.2643	0.5874319
Y	4.5	0	7448.501	3724.25	7448.501	4202.6279	1721.5022	0.5903748
X	2.5	0	8765.261	4382.63	8765.261	6209.2899	2379.8822	0.6167223
W	1.5	0	8999.896	4499.948	8999.896	6612.2996	2497.2	0.6223402
V	1	0	9193.204	4596.602	9193.204	6954.651	2593.8539	0.6270332

From the above LV plots, it can be seen that mid summaries of the first LV plot is more stable than the mid summaries of the second LV plot. The first LV plot corresponds to fourth root and second LV plot corresponds to square root. These LV plots clearly indicates as specified above how it stabilizes the mid summaries.

3.

(a) Objective: Do a matched transform and a cubed transformation for function $ax^{1/3} + b$
For cubed transformation, $T(x) = x^{1/3}$. Substututing this value to equation in the lecture notes, we get
$z = ax^{1/3} + b$
From the data given at table 4-1, the median is 3480.
Step 1: Calculate T′, Given $T(x) = x^{\frac{1}{3}}$
$T' = \frac{1}{3}x^{\frac{1}{3}-1} = \frac{1}{3x^{\frac{2}{3}}}$
Since median $= x_0 = 3480$ (Given)
Step 2: Calculate a
From lecture notes, a=$\frac{1}{T'(x_0)} = 3x_0^{\frac{2}{3}} = 3 * 3480^{\frac{2}{3}} = 688.92$
Step 3: Calculate b
Also from Lecture Notes, $b = 3480 - 688.92 * (3480)^{\frac{1}{3}}$
=3480-688.92(15.15)
= 3480 - 10437.14
≈ -6957.14
Hence rounding a and b, the matched value transformation is, $z = 690x^{\frac{1}{3}} - 6960$

(b) Proof: Proof the given equation is the exact transform, is given by below R code and Letter value Pair

```
FL<-c(2142,1788,1517,1248,963.5,727.5,579,345,114)
M<-c(3678,4115.5,4400.5,4799,4978.75,5241,5394.5,5510.25,5494)
FU<-c(4944,6643,7284,8350,8994,9754.5,10210,10675.5,1087)
Letters<-c("F","E","D","C","B","A","Z","Y","")
z<-cbind(FL,M,FU)
z<-(690*(z)^(1/3))-6960
z<-cbind(Letters,z)
```

A slightly formatted output of the above R command is as follows,

```
     Letters FL M FU
F  1934.514475  3690.900618 4794.619571
E  1414.745064  4097.491803 6010.91186
D   968.255138  4347.0618  6415.367899
C  468.8334047  4678.562435 7038.384422
B -144.9926211 4822.095282 7389.388208
A -754.2621839 5025.435267 7782.939899
Z -1209.018834 5141.322171 8008.938523
Y -2120.630432 5227.26324 8233.056948
-3614.322766 5215.271163 134.5622783
```

By comparing the above table with table 4-1 in UREDA book, the medians are approximately similar, thus the matched transform does its job

(c) The Mid summaries in the matched transform is approximately equivalent to the mid summaries in the 4-1 table. Thus the transformation does its job good.

Also, it can be seen that from the table 4-5, matched cube root transform does better than other transforms, especially the mid summaries is near to the true mid summaries.

Attachments: All R programs used for data analysis.

Firstprogram.r

```
library(noncensus)
data(counties)
findSpread <- function(nList){
  sortedInput<-sort(nList)
  medianFlg<-(1+length(sortedInput))/2
  median <-
ifelse(medianFlg==floor(medianFlg),sortedInput[medianFlg],(sortedInput[median
Flg-0.5]+sortedInput[medianFlg+0.5])/2)
  #print(median)
  flFlg<-(1+floor(medianFlg))/2
  fl<-ifelse(flFlg==floor(flFlg),sortedInput[flFlg],(sortedInput[flFlg-
0.5]+sortedInput[flFlg+0.5]/2))
  #print(fl)
  fuFlg<-length(sortedInput)-flFlg+1
  fu<-ifelse(fuFlg==floor(fuFlg),sortedInput[fuFlg],(sortedInput[fuFlg-
0.5]+sortedInput[fuFlg+0.5]/2))
  #print(fu)
  return(fu-fl)
}
states <- levels(counties$state)
medianDS <- as.numeric()
spreadDS <- as.numeric()

for (i in states){
  inState <- subset(counties,state==i)
  inPop <- sort(inState$population[!is.na(inState$population)])
  median <- median(inState$population)
  medianDS <- c(medianDS,median)
  spread <- findSpread(inPop)
  spreadDS <- c(spreadDS,spread)
}
statesSpread <- cbind.data.frame(states,medianDS,spreadDS)
statesSpread <- statesSpread[complete.cases(statesSpread),]
statesSpread <- subset(statesSpread,medianDS>0 & spreadDS >0)
statesSpread[statesSpread$states %!in% c("PR","GU","VI"),]

plot(statesSpread$medianDS,statesSpread$spreadDS,main="Spread vs Level Plot
without log",xlab = "Median",ylab="Spread")
linear<-lm(statesSpread$spreadDS~statesSpread$medianDS)
abline(linear,lty=2,col="blue")
1-linear$coefficients[2]

plot(log10(statesSpread$medianDS),log10(statesSpread$spreadDS),main="Spread
vs Level Plot with log",xlab = "Median",ylab="Spread")
linear<-lm(log10(statesSpread$spreadDS)~log10(statesSpread$medianDS))
abline(linear,lty=2,col="blue")
linear$coefficients[2]
1-linear$coefficients[2]

CA <- subset(counties,state=="CA")
source("lvalprogs.r")
```

```
lval(CA$population)
floors<-c(45578.0,20007.0,13994.0,6463.0,2207.5,1175.0)
ceils<-c(685306.0,1418788.0,2112425.5,3052772.5,6456959.0,9818605.0)

y<-
ggplot(counties,aes(state,population,color=state))+geom_boxplot()+theme_bw()+
guides(color=FALSE)
print(y)

z<-
ggplot(counties,aes(state,log(population),color=state,guides=FALSE))+geom_box
plot()+theme_bw()+guides(color=FALSE)
print(z)

y<-ggplot(CA,aes(state,-
(population)^5,guides=FALSE))+geom_boxplot()+theme_bw()+guides(color=FALSE)
print(y)
```

---

plotbatchtransform.r

```
plotTransformBatch<-function(inlist){
  #inlist<-(CA$population)
  source("lvalprogs.r")
  tab <- as.data.frame(lval(inlist))
  M <- tab$Lower[1];
  spreads <- tab[tab$Spread>0,1:5]
  xaxis <- as.numeric()
  yaxis <- as.numeric()
  pestimate <- as.numeric()
  for (i in seq(1:length(spreads$Mid))){
    xaxis<-c(xaxis,((spreads$Upper[i]-M)^2+(M-spreads$Lower[i])^2)/(4*M))
    yaxis<-c(yaxis,(((spreads$Upper[i]+spreads$Lower[i])/2)-M))
    pestimate <- c(pestimate,1-(yaxis[i]/xaxis[i]))
  }
  spreads["xaxis"]<-xaxis
  spreads["yaxis"]<-yaxis
  spreads["pestimate"]<-pestimate
  spreads<-spreads[complete.cases(spreads),]
  print(spreads)
  plot(spreads$xaxis,spreads$yaxis,main="Spread vs Level Plot",xlab="x
estimates",ylab="Y estimates")
  y<-lm(spreads$yaxis~spreads$xaxis)
  abline(y,lty=2,col="blue")
  print(paste("The power is ",1-coefficients(y)[2]))
  print(paste("The slope is ",coefficients(y)[2]))
  #return (spreads)
}
```

Question2.r

```
library(ggplot2)
setwd("C:/Users/Ganesh/Google Drive/Courses/STAT S 670/Homework 3")
source("plotTransformBatch.R")
CEOCompensation<-read.table("ceo.txt",header = TRUE)
summary(CEOCompensation)
nrow(CEOCompensation)
max(CEOCompensation)
plot(CEOCompensation$TotalCompensation,main="CEO Compensation - Scatter
Plot",ylab = "Salary")
plotC<-cbind(sort(CEOCompensation$TotalCompensation),1:nrow(CEOCompensation))
colnames(plotC)<-c("salary","index")
plotC<-as.data.frame(plotC)
qplot(index,salary,data=plotC,geom="line",main = "Sorted Salaries vs Index")
hist(CEOCompensation$TotalCompensation,breaks = 100,freq = F,main =
"Distribution of Salaries",xlab = "Compensation")
lines(density(CEOCompensation$TotalCompensation,kernel =
"epanechnikov"),lty=1,col="red",lwd=2)
qqnorm(CEOCompensation$TotalCompensation,main="QQ Plot for Salary
Distribution")
qqline(CEOCompensation$TotalCompensation)
plotTransformBatch((CEOCompensation$TotalCompensation))
#Box plot without any transfo
boxplot((CEOCompensation$TotalCompensation),main="Without Transformation")

#One Transformation 4th root that equals out
plotTransformBatch((CEOCompensation$TotalCompensation)^0.25)
boxplot((CEOCompensation$TotalCompensation)^0.25,main="Fourth root
transormation")
#One Transformation log that equals out
plotTransformBatch((CEOCompensation$TotalCompensation)^0.5)
boxplot((CEOCompensation$TotalCompensation)^0.5,main="Square root transform
Transformation")

FL<-c(2142,1788,1517,1248,963.5,727.5,579,345,114)
M<-c(3678,4115.5,4400.5,4799,4978.75,5241,5394.5,5510.25,5494)
FU<-c(4944,6643,7284,8350,8994,9754.5,10210,10675.5,1087)
Letters<-c("F","E","D","C","B","A","Z","Y","")
z<-cbind(FL,M,FU)
z<-(690*(z)^(1/3))-7000
z<-cbind(Letters,z)
```

---

question3.r

```
#Transformation plots
xlf<-function(xl,xu,median){
  return (((xl+xu)/2)-median)
}
ylf<-function(xl,xu,median){
  return (((xu-median)^2+(median-xl)^2)/(4*median))
```

```
}

median<-c(179140.5)
letters<-c("F","E","D","C","B","A")
xl<-c(45578.0,20007.0,13994.0,6463.0,2207.5,1175.0)
xu<-c(685306.0,1418788.0,2112425.5,3052772.5,6456959.0,9818605.0)
xlab <- as.numeric()
ylab <- as.numeric()
p <- as.numeric()
for (i in seq(1:length(letters))){
  ytemp<-ylf(xl[i],xu[i],median)
  xtemp<-xlf(xl[i],xu[i],median)
  ylab <- c(ylab,ytemp)
  xlab <- c(xlab,xtemp)
  p<-c(p,(1-(ytemp/xtemp)))
}
estimator<-cbind(letters,xl,xu,xlab,ylab,p)
print(estimator)
```