

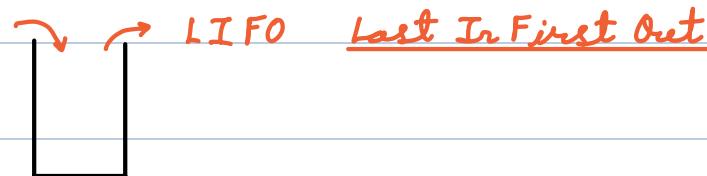
Stacks 1

TABLE OF CONTENTS

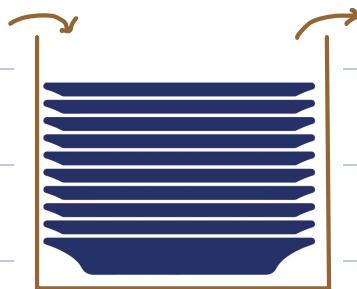
1. Stacks
2. Operations in Stacks
3. Implementing Stacks using Arrays
4. Implementing Stacks using Linked List
5. Double Character Trouble



Stacks



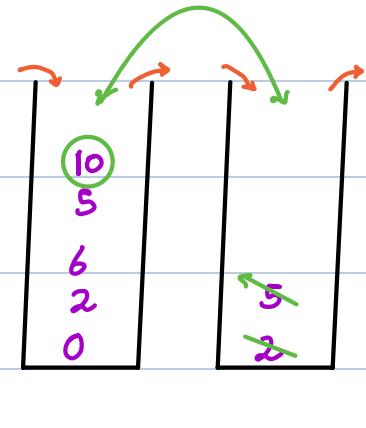
1. Stack of Plates



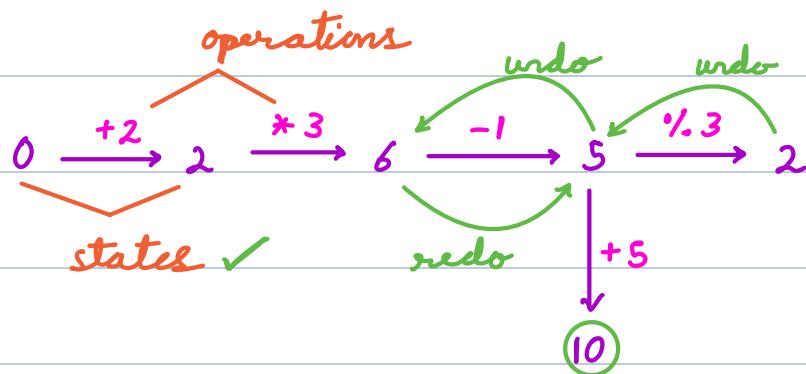
2. Stack of Chairs



3. Stack of Functions *(Recursion)*



4. Undo / Redo



If we perform a new operation → the redo stack becomes empty.



Scenario

In a Flipkart warehouse, boxes are stacked one over another. Each box is tagged with a weight, and for safety, heavier boxes must be placed below lighter ones. Workers need a system to check if adding a new box on top is safe.

Workers want to perform two kinds of operations:-

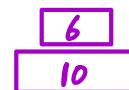
Type ADD : a new box of some weight on the top

Type REMOVE : the topmost box



Example :

Query type	Value	Answer (True/False)
ADD	10Kg	true ✓
ADD	5Kg	true ✓
ADD	12Kg	false (not allowed) ✗
REMOVE	-	true ✓
ADD	6Kg	true



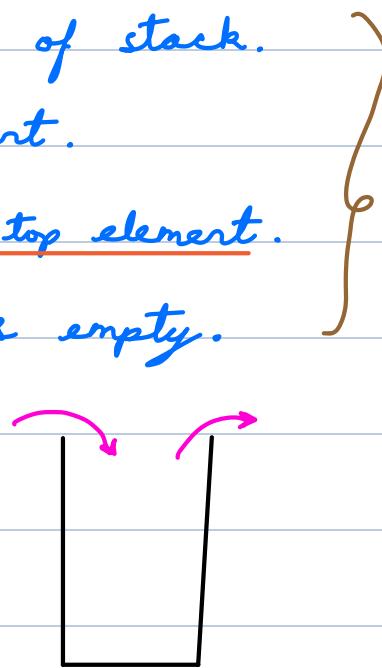
insert → if (wt of new box > wt of top box)
return false

else return true



Operations in Stack

1. **Push (x)** - Insert x on the top of stack.
 2. **Pop ()** - Remove the top element.
 3. **Top () / Peek ()** - check / get the top element.
 4. **isEmpty ()** - checks if stack is empty.
- $TC = O(1)$



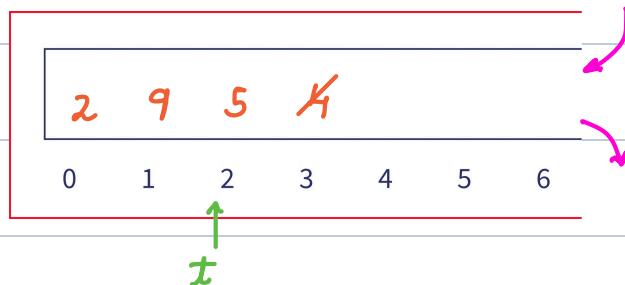
Implementing Stacks using Arrays

push (2)
push (9)
push (3)
peek () $\rightarrow 3$
push (4)

Stack

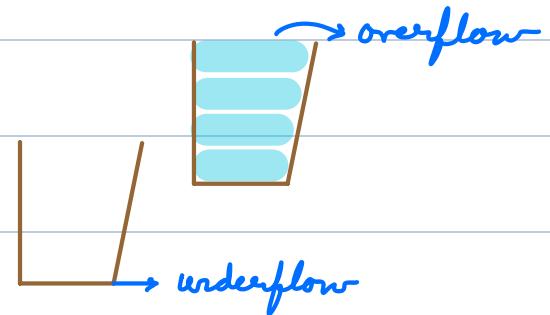
0 $__$ t

$t = -1$



pop () $\rightarrow 4$
pop () $\rightarrow 3$
push (5)

boolean isEmpty () {
 returns ($t == -1$)
}





```
void push (x) { // overflow
    t ++
    A [t] = x
}
```

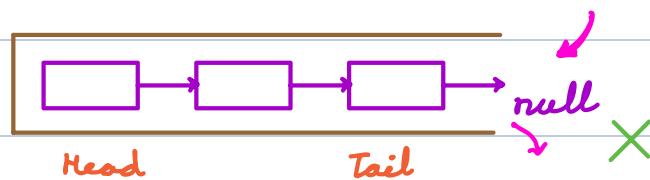
1) Use dynamic array ✓
2) Do not insert if array is full. ($t == N-1$)

```
int peek () {
    if (isEmpty ()) return -1 // underflow
    return A [t]
}
```

```
int pop () {
    if (isEmpty ()) return -1
    top = A [t]
    t --
    return top
}
```

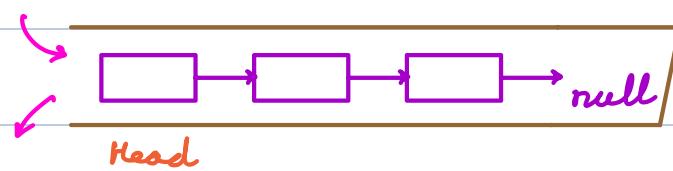
TC Operations $\rightarrow \underline{O(1)}$

Implementing Stacks using Linked List



insert as last node \rightarrow $TC = O(N)$
 $O(1)$

removing last node \rightarrow $TC = O(N)$



insert head \rightarrow $TC = O(1)$

remove head \rightarrow $TC = O(1)$

push (x) \rightarrow insert x as head

peek () \rightarrow return head data } check underflow

pop () \rightarrow remove head node }

isEmpty () \rightarrow check if head is null

$TC = \underline{O(1)}$



< Question > : Check whether the given sequence of parenthesis $[]$, $\{ \}$, $()$ is valid or not?

Example : $() [\{ \} ()]$ ✓

$() []$ ✓

$(\{ \})$ ✓

$(\{ \}) \}$ ✗

$([\{])$ ✗

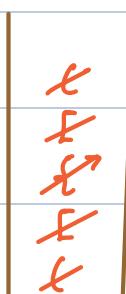
$([\{] []] ())$ ✓

$([\{] []] ())$ ()

store data but use
latest data first \Rightarrow LIFO
 \Rightarrow stack

open bracket \rightarrow store
close bracket \rightarrow check &
remove pair.

$([\{] []] ())$



$\{ }$ ✓
 $[]$ ✓
 $[]$ ✓
 $()$ ✓
 $()$ ✓



// stack \rightarrow st

for $i \rightarrow 0$ to $(N-1)$ {

$ch = s[i]$

 if ($ch == '{'$ || $ch == '('$ || $ch == '['$) {

 st.push(ch)

 } else if ($ch == '}'$) {

 if (st.peak() != '{') return false

 st.pop()

 } else if ($ch == ']'$) {

 if (st.peak() != '[') return false

 st.pop()

 } else {

 if (st.peak() != '(') return false

 st.pop()

}

}

return st.isEmpty()

TC = O(N) SC = O(N)

Double Character Trouble

Given a string str. Remove equal pair of consecutive elements till possible.

Return the strings without adjacent duplicates.

1. ~~a b b d~~ → ad

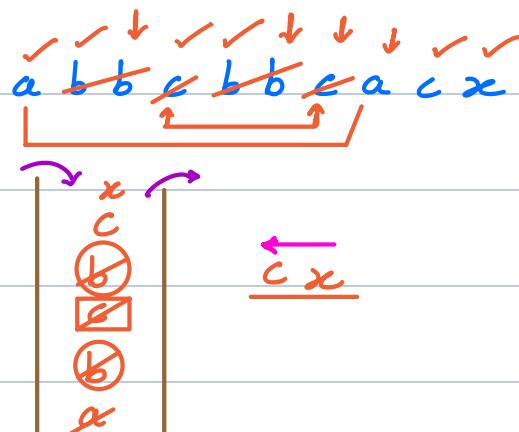
2. ~~a b c c b d e~~ → ~~ab~~~~b~~~~d e~~ → ade

3. ~~a b b b d~~ → abd

4. ~~a b b c b b c a c x~~ → ~~ac~~~~c~~~~a c x~~ → ~~a~~~~c~~~~c x~~ → cx

a b c d d

store data but use
latest data first ⇒ LIFO
⇒ stack





// stack → st

for $i \rightarrow 0$ to $(N-1)$ {

 ch = s[i]

 if (st.isEmpty() || (st.peak() != ch))

 st.push(ch)

 else st.pop()

}

ans = "

while (!st.isEmpty()) {

 ans = st.pop() + ans

}

return ans

TC = O(N)

SC = O(N)



infix

 $2 + 3$ $2 + (5 * 3)$

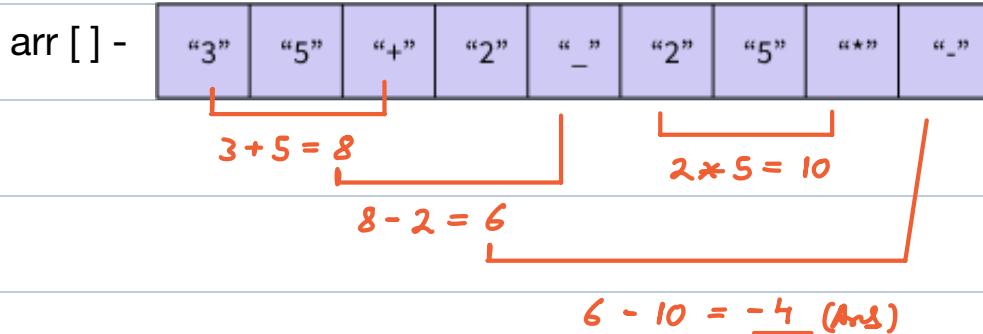
postfix

2 3 +

2 5 3 * +

operand1 operator operand2 | operand1 operand2 operator

< Question > : Evaluate given valid post fix expression.



Quiz : $5, 2, *, 3, -$

$\underline{5 * 2 = 10}$

$10 - 3 = 7$ (Ans)

operator \rightarrow select latest 2

operands & evaluate.

$\checkmark \checkmark \downarrow \checkmark \downarrow \checkmark \checkmark \downarrow \downarrow$
 $3 \ 5 \ + \ 2 \ - \ 2 \ 5 \ * \ -$



$y = st.pop()$

$x = st.pop()$

x operator y

$3 + 5 = 8$

$8 - 2 = 6$

$2 * 5 = 10$

$6 - 10 = -4$ (Ans)



H.W \rightarrow code using stack.

TC = $O(N)$ SC = $O(N)$

Stacks - 2

TABLE OF CONTENTS

1. Nearest Smaller Element [on L.H.S & on R.H.S]
2. Nearest Greater Element (NGE) - [on L.H.S & on R.H.S]
3. Index of Nearest Smaller
4. Largest Rectangle in Histogram
5. Sum of (max-min) for all Subarrays



Nearest Smaller Element

Q1 → Given an integer array find the index of nearest smaller element on left of i $\forall A[i]$.

$$A = [\begin{matrix} 8 & 2 & 4 & 9 & 7 & 5 & 3 & 10 \end{matrix}]$$

$$-1 \ -1 \ 1 \ 2 \ 2 \ 2 \ 1 \ 6$$

$\forall i$, find $\max j$ s.t $A[j] < A[i]$ & $j < i$.

$$A = [\begin{matrix} 4 & 6 & 10 & 11 & 7 & 8 & 3 & 5 \end{matrix}]$$

$$-1 \ 0 \ 1 \ 2 \ 1 \ 4 \ -1 \ 6$$

$$A = [\begin{matrix} 4 & 5 & 2 & 10 & 8 & 2 \end{matrix}]$$

$$-1 \ 0 \ -1 \ 2 \ 2 \ -1$$

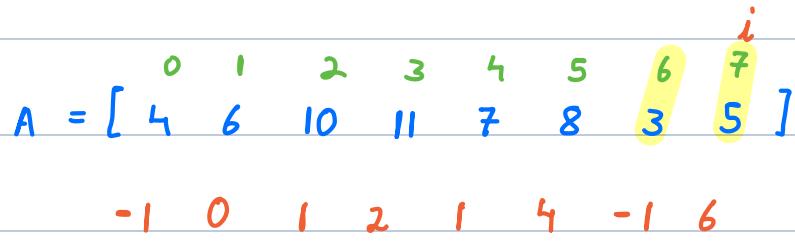
Bruteforce → $TC = \underline{O(N^2)}$ $SC = \underline{O(1)}$

$$A = [\begin{matrix} 8 & \underline{\quad} & \underline{\quad} & \underline{\quad} & \underline{\quad} & 5 & \underline{\quad} & \underline{\quad} \end{matrix}]$$

For any $i > 5$, can index 0 be the ans?

No, $\forall i > 5$, if $8 < A[i] \Rightarrow 5 < A[i]$

& closer index will be 5.



88
24
17
06

checking \rightarrow latest index first \Rightarrow LIFO \therefore use stack.

// stack st

for $i \rightarrow 0$ to $(N-1)$ {

 while (!st. isEmpty () $\&\&$ A [st. peek ()] $\geq A[i]$) {

 st. pop ()

 }

 if (st. isEmpty ()) ans[i] = -1

 else ans[i] = st. peek ()

 st. push (i)

}

$TC = O(N)$ $SC = O(N)$

Q2 \rightarrow Find nearest smaller or equal on left.

Q3 \rightarrow Find nearest greater element on left.

Q4 \rightarrow Find nearest greater or equal on left.





Q5 → Find nearest smaller element on right.

// stack st

for $i \rightarrow (N-1)$ to 0 {

 while (!st.isEmpty() && A[st.peek()] $\geq A[i]$) {

 st.pop()

}

 if (st.isEmpty()) ans[i] = -1

 else ans[i] = st.peek()

 st.push(i)

}

TC = $O(N)$ SC = $O(N)$

Q6 → Find nearest smaller or equal on right.

Q7 → Find nearest greater element on right.

Q8 → Find nearest greater or equal on right.



- A person uses Google Maps to find the nearest restaurants and picks one based on its proximity. Unfortunately, after visiting, they realised that the restaurant didn't meet their expectations.

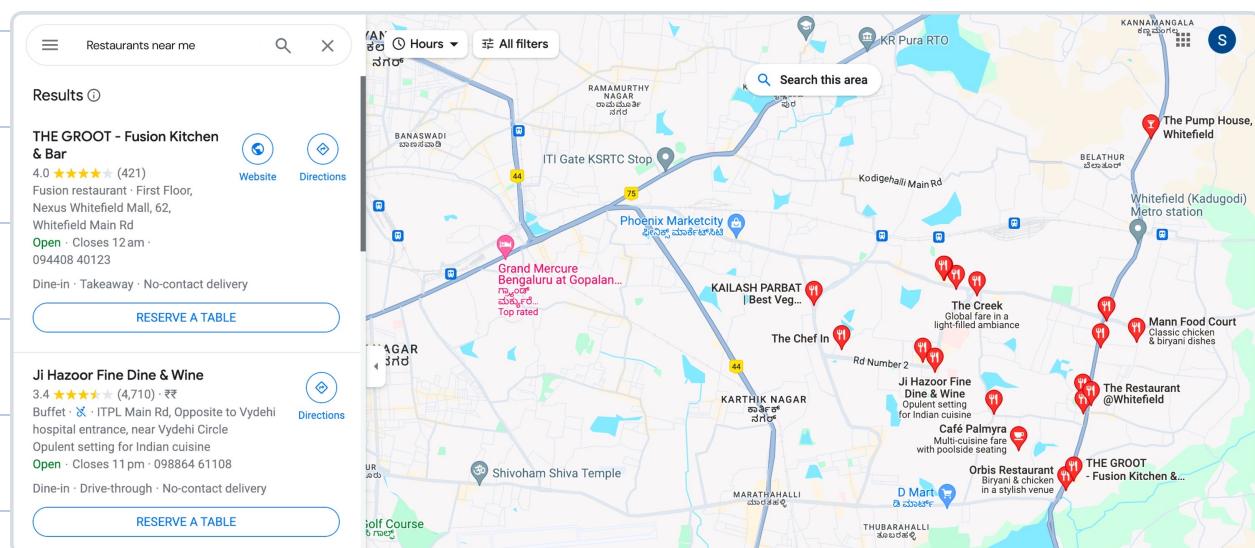
Task

Let's break it down with a simple example. You have a list of restaurants and their ratings. For each restaurant, we're going to find the next restaurant to the right on the list that's not just close but also has a higher rating than the current one. If there's no better option on the list, we'll say there's none available.

Problem

Given a sequence of restaurants listed on Google Maps with their ratings, create a tool that helps users discover the rating of the next higher-rated restaurant to the right for each listed establishment.

Nearest greater on right.



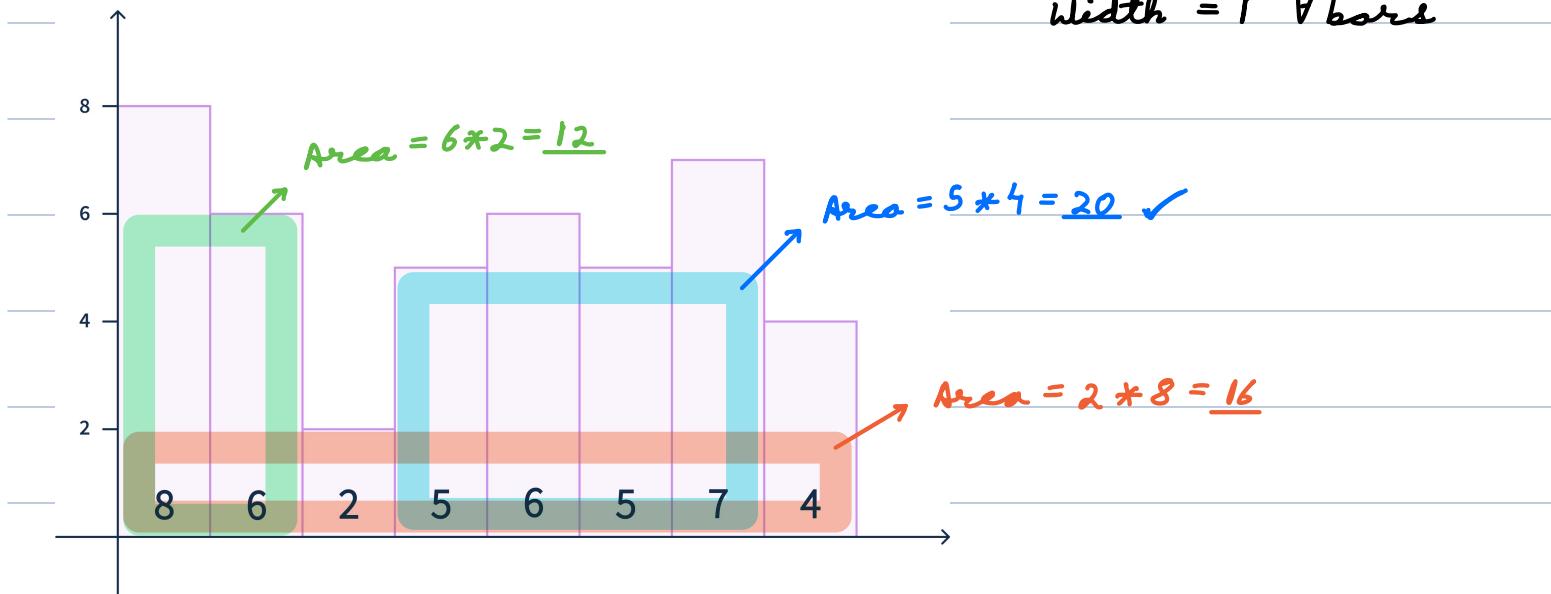
< Question > : Find the largest area of rectangle (formed by continuous bars) in histogram.

\downarrow

$H \neq W$

$A[i] \rightarrow$ Height of i^{th} bar

Width = 1 \forall bars



$$A = [1 \ 2 \ 3 \ 2 \ 1]$$



Idea \rightarrow Find continuous bars that give largest rectangle.

Breutforce \rightarrow \forall subarray, calculate

$$\text{area} = (\min A[i]) * \text{length of subarray}.$$

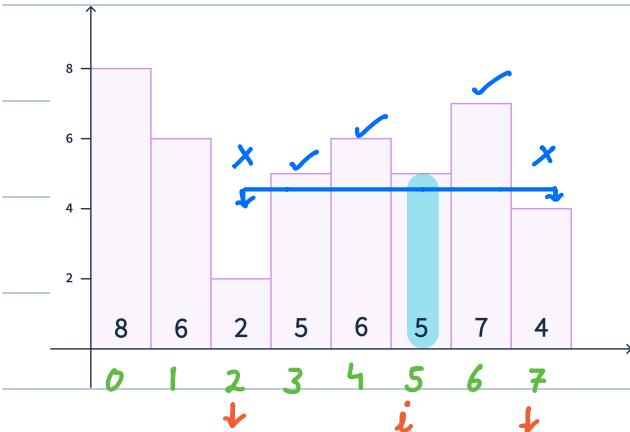
carry forward

$$l-r \Rightarrow r-l+1$$

$$TC = O(N^3) \rightarrow O(N^2) \quad SC = O(1)$$

$TC < O(N^2) \Rightarrow$ we cannot travel all subarrays i.e width.

Idea 2 → ∀ height find max width.



$[j+1 \quad k-1]$

$$\begin{aligned} \text{width} &= k - j - (j+1) + j \\ &= \underline{k - j - 1} \end{aligned}$$

Nearest smaller on left = j Nearest smaller on right = k

$$\text{Ans} = \max_i (A[i] * (\text{nearestSmallerRight}[i] - \text{nearestSmallerLeft}[i] - 1))$$

$$TC = O(N + N + N) = O(N)$$

$$SC = O(N)$$



< Question > : Find sum of (max-min) for all subarrays.

$1 \leq N \leq 10^5$

0 1 2
arr : [1 2 3]

	max	min	max-min
[1]	1	1	0
[1, 2]	2	1	1
[1, 2, 3]	3	1	2
[2]	2	2	0
[2, 3]	3	2	1
[3]	3	3	0
			<u>4</u> (Ans)

Bruteforce \rightarrow $TC = O(N^3) \longrightarrow O(N^2)$ (carry forward)

$SC = O(1)$

$A = [\textcolor{green}{2} \textcolor{yellow}{5} \textcolor{brown}{3}]$

0 1 2
3 7 8 9 11
x x ✓ x x

$$\begin{array}{l}
 2 \rightarrow 2-2 = 0 \\
 2 5 \rightarrow 5-2 = 3 \\
 2 5 3 \rightarrow 5-2 = 3 \\
 5 \rightarrow 5-5 = 0 \\
 5 3 \rightarrow 5-3 = 2 \\
 3 \rightarrow 3-3 = 0
 \end{array}$$

$$\begin{array}{l}
 2*(1-3) = -4 \\
 5*(4-1) = 15 \\
 3*(1-2) = -3 \\
 8
 \end{array}$$

8 (Ans)

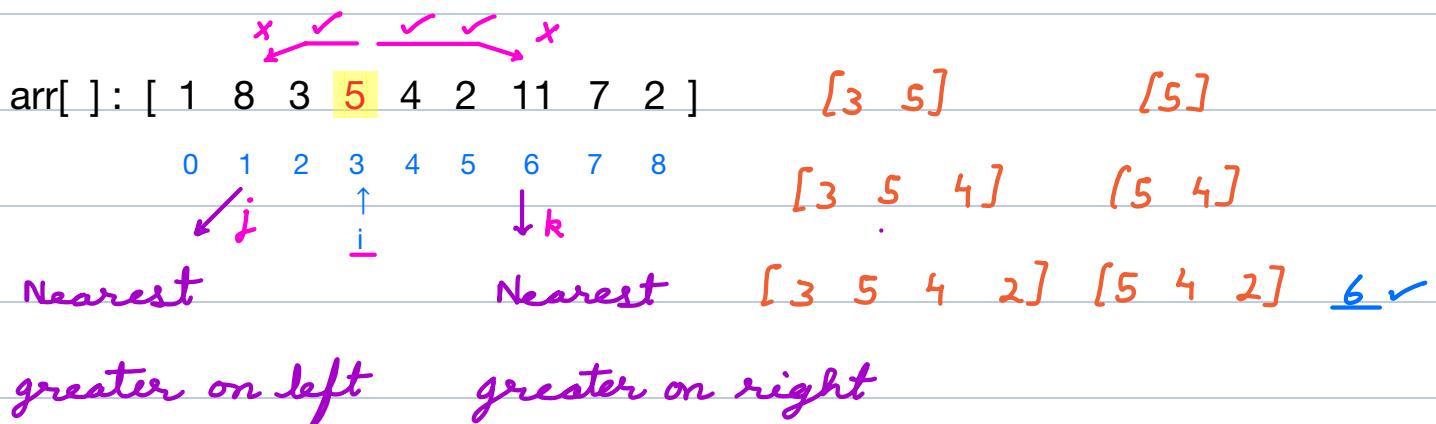
Contribution Technique

$$\text{Ans} = \sum_{i=1}^n \text{contribution of } A[i]$$

$$A[i] * \left(\# \text{subarrays} - \# \text{subarrays} \right)$$

$A[i]$ is max $A[i]$ is min

< Question > : In how many subarrays, ith element is the maximum element?



$$\# \text{subarrays} \rightarrow \underbrace{(\# \text{start}) * (\# \text{end})}_{[j+1 \dots i]} * [i \dots k-1] = (i-j) * (k-i)$$

$$i - (j+1) + 1 = \underline{i-j}$$

$$k-1 - i + 1 = \underline{k-i}$$

subarrays $A[i]$ is min ?

$j \rightarrow$ Nearest smaller on left

$k \rightarrow$ Nearest smaller on right



$$\text{Ans} = \sum_{\forall i} A[i] * ((i - \text{nearestGreaterLeft}[i]) * (\text{nearestGreaterRight}[i] - i) - (i - \text{nearestSmallerLeft}[i]) * (\text{nearestSmallerRight}[i] - i))$$

$$TC = O(N + N + N + N + N) = \underline{O(N)}$$

$$SC = \underline{O(N)}$$

H.W \rightarrow How to solve if duplicates are present.

Queues

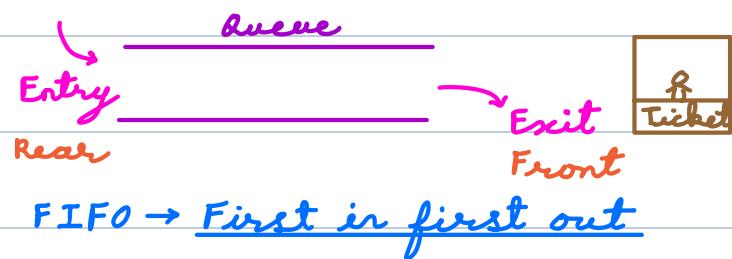
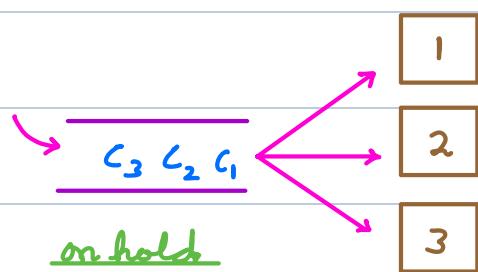
TABLE OF CONTENTS

1. Implementation of the queue using array
2. Implementation of the queue using stack
3. Perfect Number Question
4. Doubly Ended Queue
5. Sliding Window Maximum



Notes

Queue

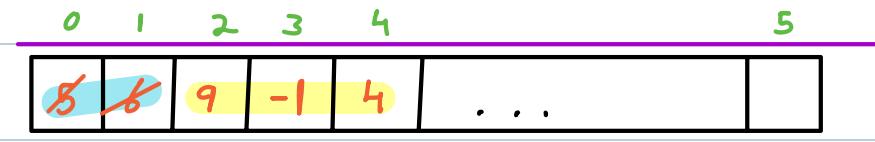


Operations in Queues

1. **Enqueue(x)** → Insert x at rear end.
2. **Dequeue()** → Remove element from front end.
3. **front() / rear()** → Get element of front/rear end.
4. **isEmpty()** → Check if the queue is empty. $TC = O(1)$

Array Implementation of Queues

Example: $\text{Eq}(5)$, $\text{Eq}(6)$, $\text{Eq}(9)$, $\text{Eq}(-1)$, $\text{Dq}()$, $\text{Dq}()$, front , $\text{Eq}(4)$



$$f = 0 \times 2$$

$$r = -1 \times 2 \times 3 \times 4$$

Queue $\rightarrow [f \dots r]$

void enqueue (x) {

$r++$

$A[r] = x$

}

overflow \rightarrow \triangleright Limit insertion.

\Rightarrow Use dynamic array. ✓

boolean isEmpty () {

$ler = r - f + 1$

$\text{return } (ler == 0) \quad // f > r$

}

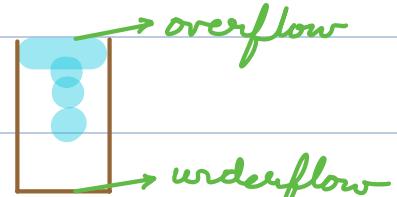
int dequeue () {

$\text{if } (\text{isEmpty}()) \text{ return } -1$

$f++$

$\text{return } A[f-1]$

}



\forall operations, $TC = \underline{O(1)}$



Implementation of Queue using LinkedList

Example: $\text{Eq}(5)$, $\text{Eq}(6)$, $\text{Eq}(9)$, $\text{Eq}(-1)$, $\text{Dq}()$, $\text{Dq}()$, front , $\text{Eq}(4)$



$\text{Tail.next} = \text{new Node}(x)$

$\text{Tail} = \text{Tail.next}$

$\text{Head} = \text{null}$

$\text{Tail} = \text{null}$

- 1) Enqueue (x) \rightarrow Insert at tail
 - 2) Dequeue () \rightarrow Remove head.
 - 3) front () \rightarrow Head . data
 - 4) rear () \rightarrow Tail . data
 - 5) isEmpty () \rightarrow Head == null
- underflow ✓
- $TC = \underline{O(1)}$



What will be the state of the queue after these operations $\text{enqueue}(3)$, $\text{enqueue}(7)$, $\text{enqueue}(12)$, $\text{dqueue}()$, $\text{dqueue}()$, $\text{enqueue}(8)$, $\text{enqueue}(3)$



What will be the state of the queue after these operations $\text{enqueue}(4)$, $\text{dqueue}()$, $\text{enqueue}(9)$, $\text{enqueue}(3)$, $\text{enqueue}(7)$, $\text{enqueue}(11)$, $\text{enqueue}(20)$, $\text{dqueue}()$





2

Implementation of Queue using Stack - [Direct]

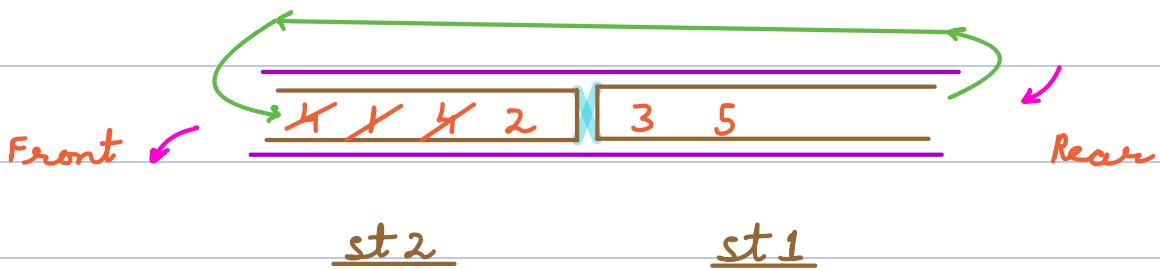
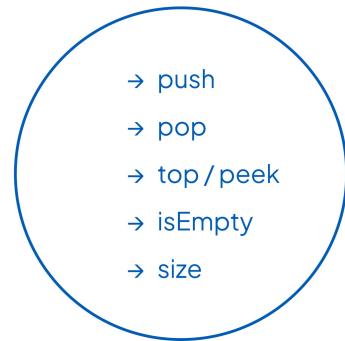
→ Dequeue

behind the scene only these functions can be used

→ Enqueue

→ front() / ~~rear()~~

→ isEmpty()



```
void enqueue (x) {  
    st1.push (x)  
}
```

✓

```
boolean isEmpty () {  
    return st1.isEmpty ()  
    && st2.isEmpty ()  
}
```

```
void move () {  
    while (! st1.isEmpty ()) {  
        st2.push (st1.pop ())  
    }  
    TC = O (K)
```



int dequeue () {

 if (isEmpty ()) return -1

 if (st2.isEmpty ()) {

 move () // st1 → st2 K elements

}

 → for next K dequeue

 return st2.pop ()

operations → $TC = \underline{O(1)}$

}

 → 2 steps per element.

$O(2) \rightarrow \underline{O(1)}$

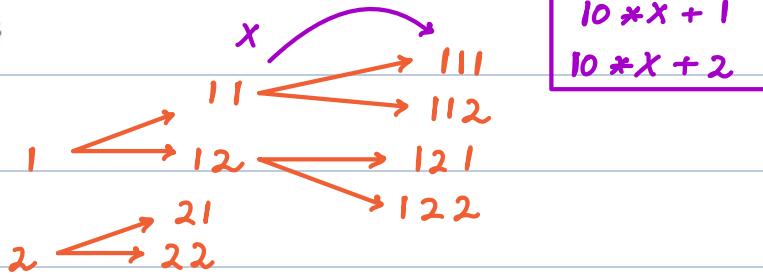
Perfect Numbers

Q → Find N^{th} number formed using digits 1 & 2.

$N \rightarrow 1 \quad 2 \quad 3 \quad 4 \quad 5 \quad 6 \quad 7 \quad 8$

1 2 11 12 21 22 111 112

0	10	20
1	11	21
2	12	22
⋮	⋮	⋮
9	19	29



x is generated before y

$\Rightarrow x$ will generate next numbers before y .

FIFO

$N = 5$

$\leftarrow x \ x \ 11 \ 12 \rightarrow$

$i = \cancel{x} 5 \quad x = \cancel{x} 2 \quad a = \textcircled{21} \quad b = 22$

if ($N \leq 2$) return N

$q.\text{enqueue}(1) \checkmark \quad q.\text{enqueue}(2) \checkmark$

$i = 3$

while ($i \leq N$) {

$x = q.\text{dequeue}()$

$a = x * 10 + 1 \checkmark \quad b = x * 10 + 2 \checkmark$

if ($i == N$) return $a \leftarrow$

if ($i + 1 == N$) return b

$q.\text{enqueue}(a) \quad q.\text{enqueue}(b)$

$i += 2$

}

$TC = \underline{O(N)}$

$SC = \underline{O(N)}$



Doubly Ended Queue

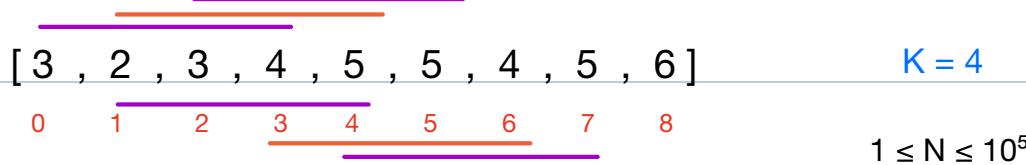


1. **Insertion at Front (push_front):** Add an element to the front (head) of the deque. ✓
2. **Insertion at Back (push_back):** Add an element to the back (tail) of the deque. ✓
3. **Removal from Front (pop_front):** Remove and return the element from the front of the deque. ✓
4. **Removal from Back (pop_back):** Remove and return the element from the back of the deque. ✓
5. **Front Element Access (front):** Get the element at the front of the deque without removing it. ✓
6. **Back Element Access (back):** Get the element at the back of the deque without removing it. ✓

Stack + Queue → Doubly Ended Queue

Sliding Window Maximum

< Question > : Find max of every subarray of size K.



$$[0 - 3] \rightarrow 4$$

$$\underline{\text{Bruteforce}} \rightarrow \text{TC} = O(N^3) \rightarrow \underline{O(N^2)}$$

$$[1 - 4] \rightarrow 5$$

$$SC = \underline{\mathcal{O}(1)}$$

$$[2 - 5] \rightarrow 5$$

$$[3 - 6] \rightarrow 5$$

[3, 2, 3, 14, 18, 5, 4, 5, 6]

$$[4 - 7] \rightarrow 5$$

$$[5 - 8] \rightarrow 6$$

H.W → Sliding Window + Deque.



3	2	9	4	-1	16	1	7	-2	5	-5
0	1	2	3	4	5	6	7	8	9	10

$K = 4$