

# Trees - 1

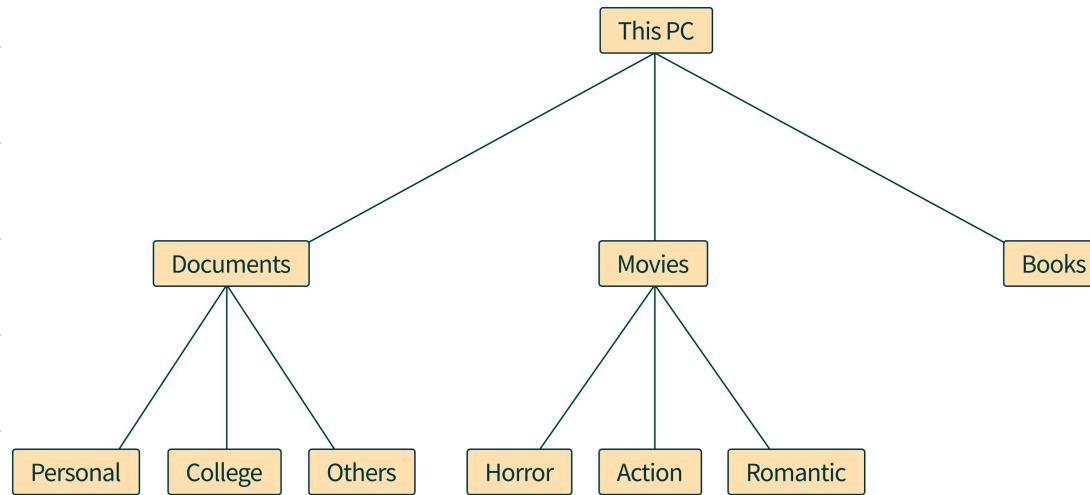
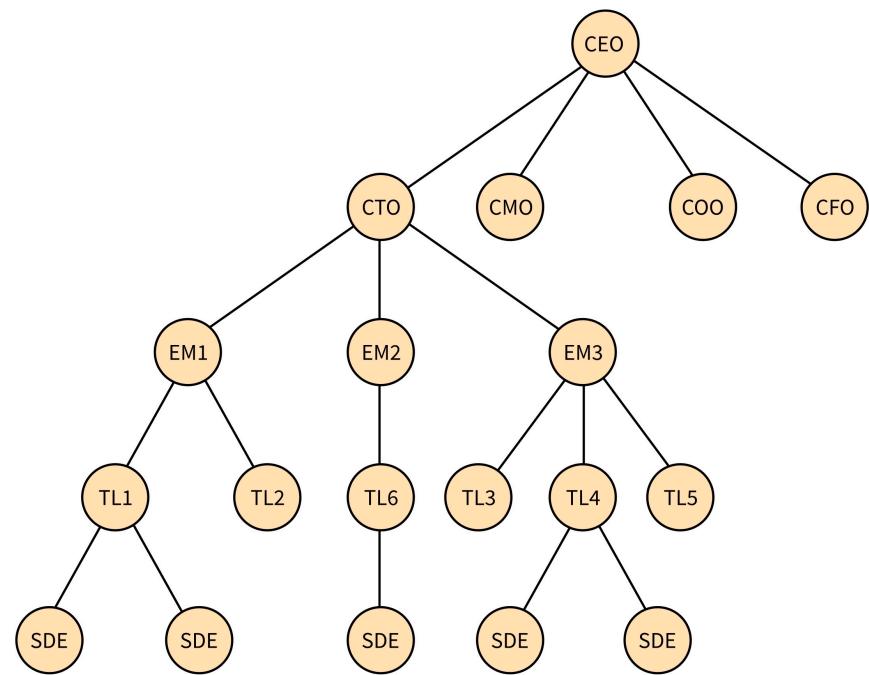
## TABLE OF CONTENTS

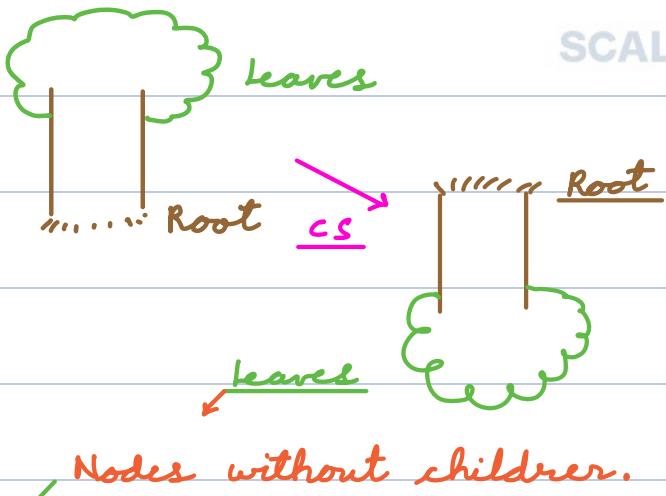
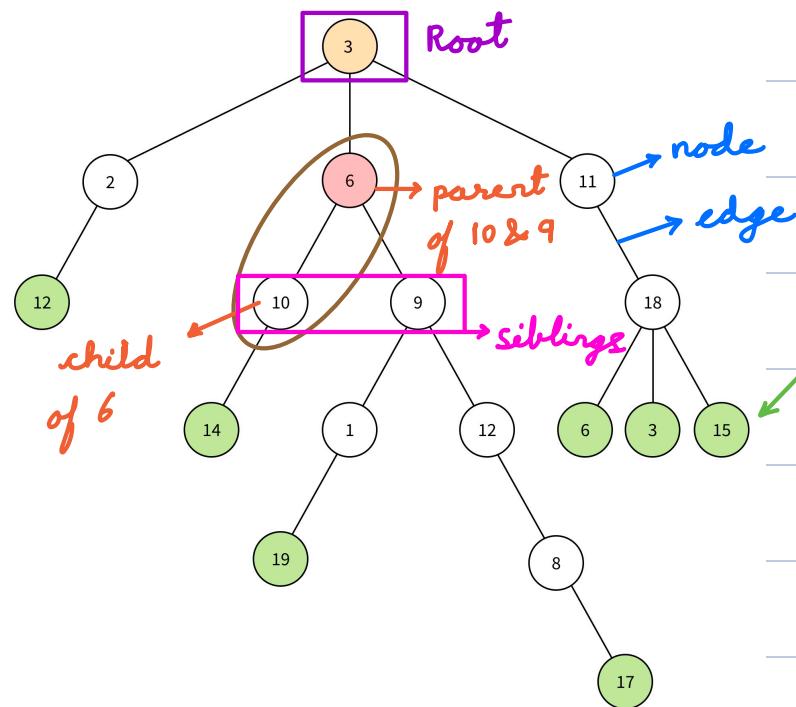
1. Introduction to Trees
2. Binary Tree
3. Traversal in Binary Tree
  - Pre-order
  - In-order
  - Post-order
4. Iterative In-order
5. Construct Tree from In-order and Post-order



# Trees

## Hierarchical Data





Height of node  $\rightarrow$  count of edges (distance) to travel from current node to farthest leaf.



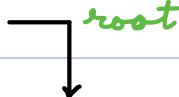
$$\text{height}(18) = 1$$

$$\text{height}(9) = 3$$

$$\text{height}(\text{leaf}) = 0$$

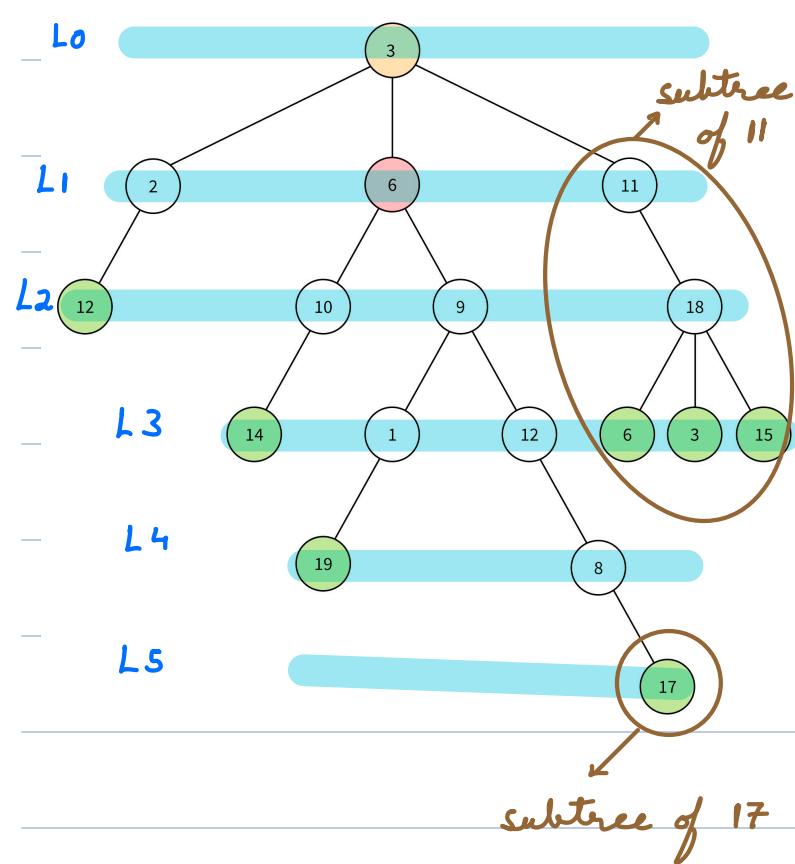
v. imp  $\rightarrow$  Height of tree = Height (root) = 5

Depth / Level of node  $\rightarrow$  count of edges (distance) to travel from root to reach the current node.



$$\text{depth}(9) = 2$$

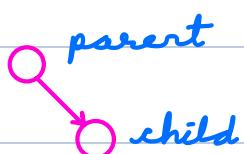
$$\text{depth}(14) = 3$$



Subtree of a node  $x \rightarrow$

All the nodes we can

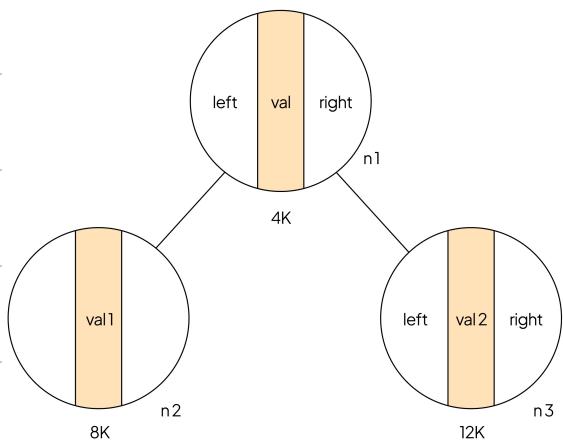
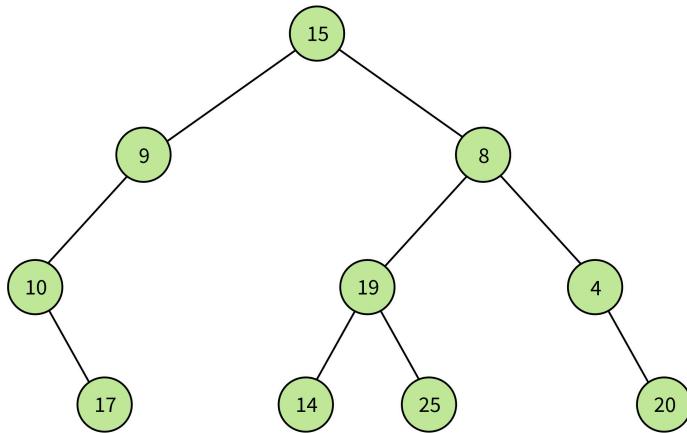
travel from node  $x$  are  
part of subtree of  $x$ .



subtree (root) = complete tree

# Binary Tree

✓ nodes, max # children = 2  
 $\# \text{children} = \{0, 1, 2\}$



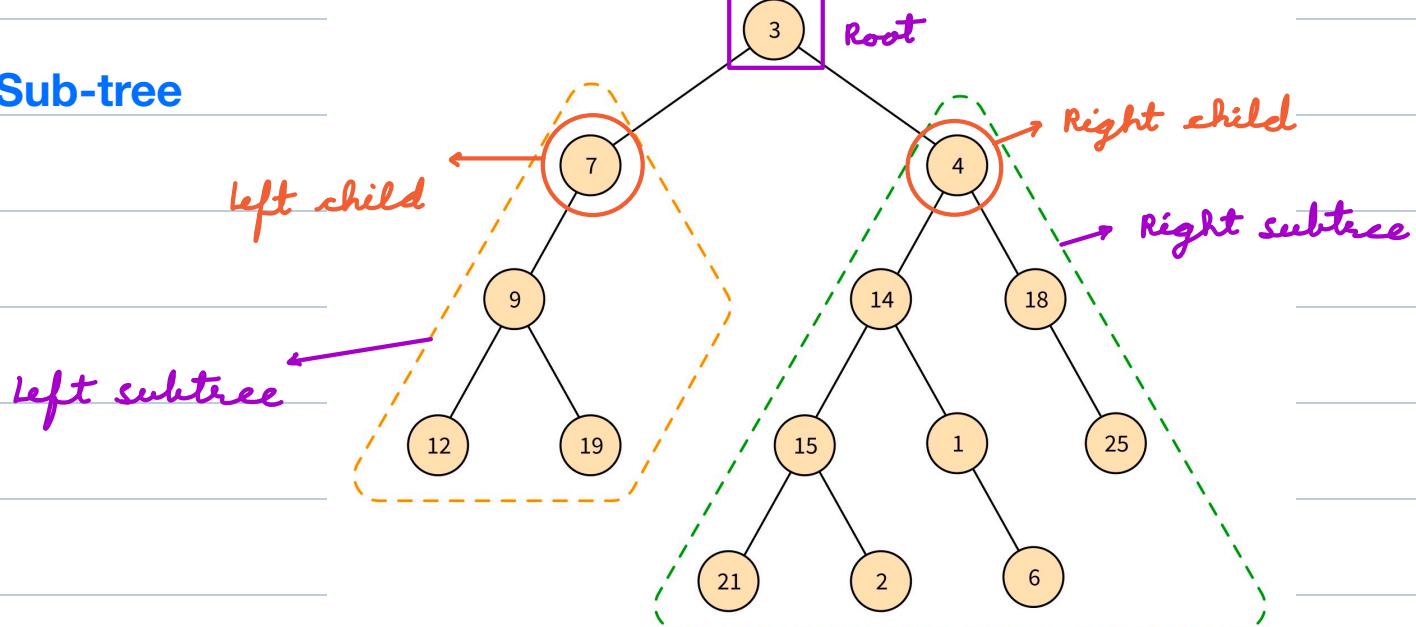
class TreeNode {

    int val;

    TreeNode left, right;

}

Sub-tree



Preorder

N L R

Levelorder → Next class

Inorder

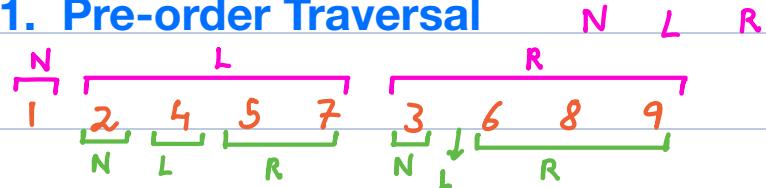
L N R

Postorder

L R N

# Binary Tree Traversal

## 1. Pre-order Traversal



```
void preorder(root) {
```

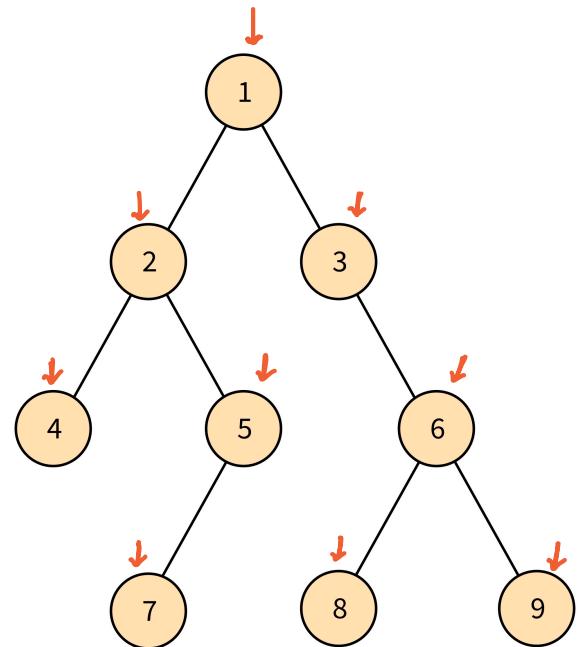
```
    if (root == null) return
```

```
    print (root.val)           // Node
```

```
    preorder (root.left)      // Left
```

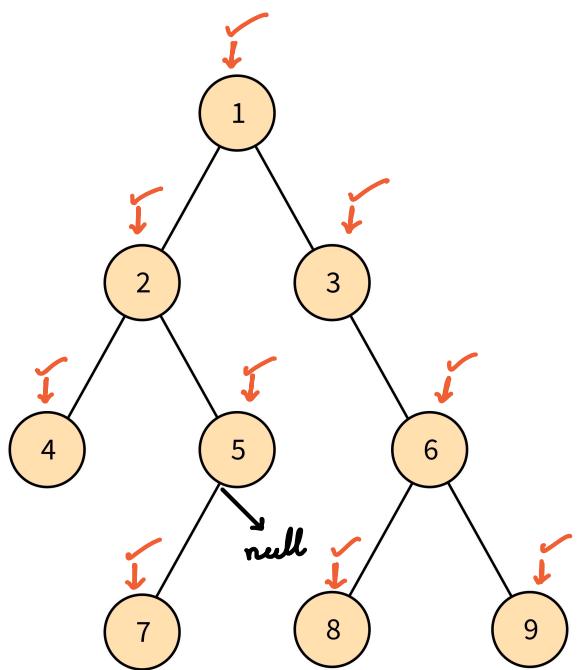
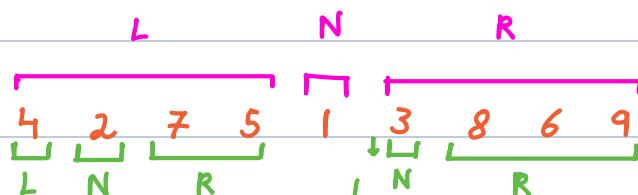
```
    preorder (root.right)     // Right
```

TC = O(N)   SC = O(H)



## 2. In-order Traversal

$L \ N \ R$



```

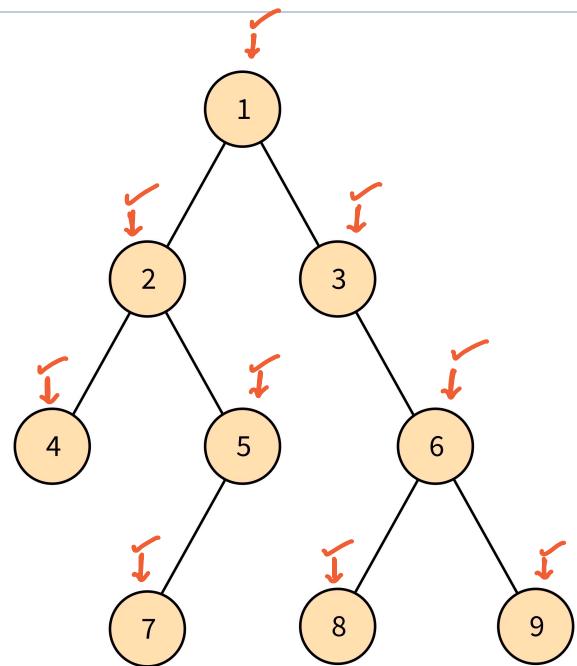
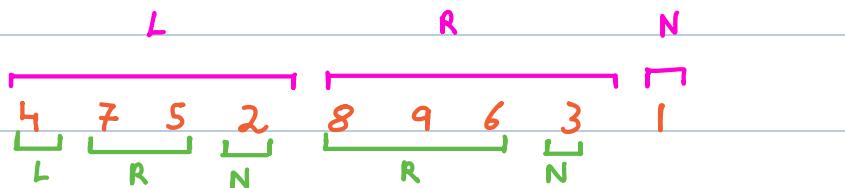
void inorder (root) {
    if (root == null) return
    inorder (root.left) // left
    print (root.val)      // Node
    inorder (root.right) // Right
}

```

$TC = \underline{O(N)}$     $SC = \underline{O(H)}$

## 2. Post-order Traversal

L R N



```

void postorder (root) {
    if (root == null) return
    postorder (root.left) // left
    postorder (root.right) // Right
    print (root.val)      // Node
}

```

$TC = \underline{O(N)}$     $SC = \underline{O(H)}$

# Iterative In-order Traversal

L N R

DS to use → Stack

1) st.push(cur)

cur = cur.left

2) cur = st.pop()

print(cur.val)

cur = root

cur = cur.right

→ null

o/p → 4 2 7 5 1 3 10 8 6 9

→ null → 8 → null → null → 5 → null  
 → 3 → null → 6 → 8 → 10 → null → 10 → null → 8  
 → null → 6 → null → null

cur = root

while (cur != null || !st.isEmpty()) {

if (cur != null) { st.push(cur)

cur = cur.left }

else { cur = st.pop()

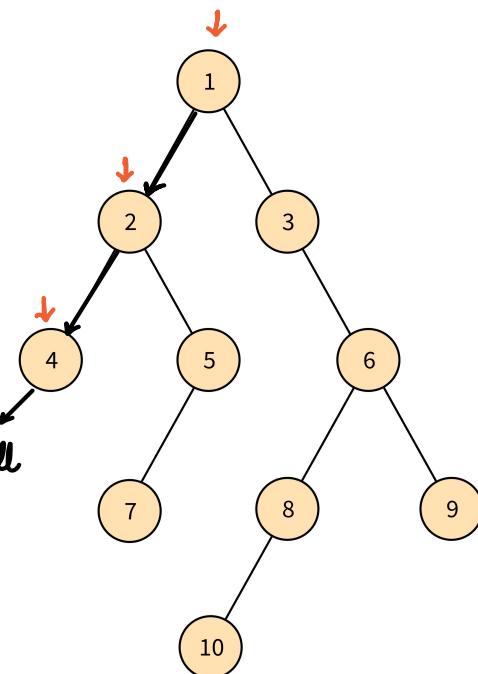
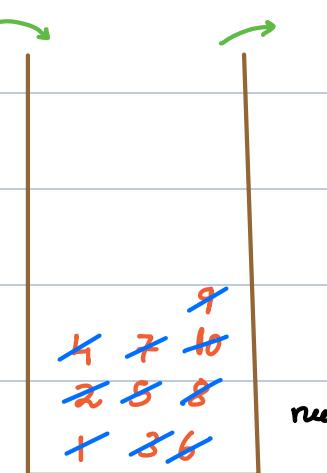
print(cur.val)

cur = cur.right }

}

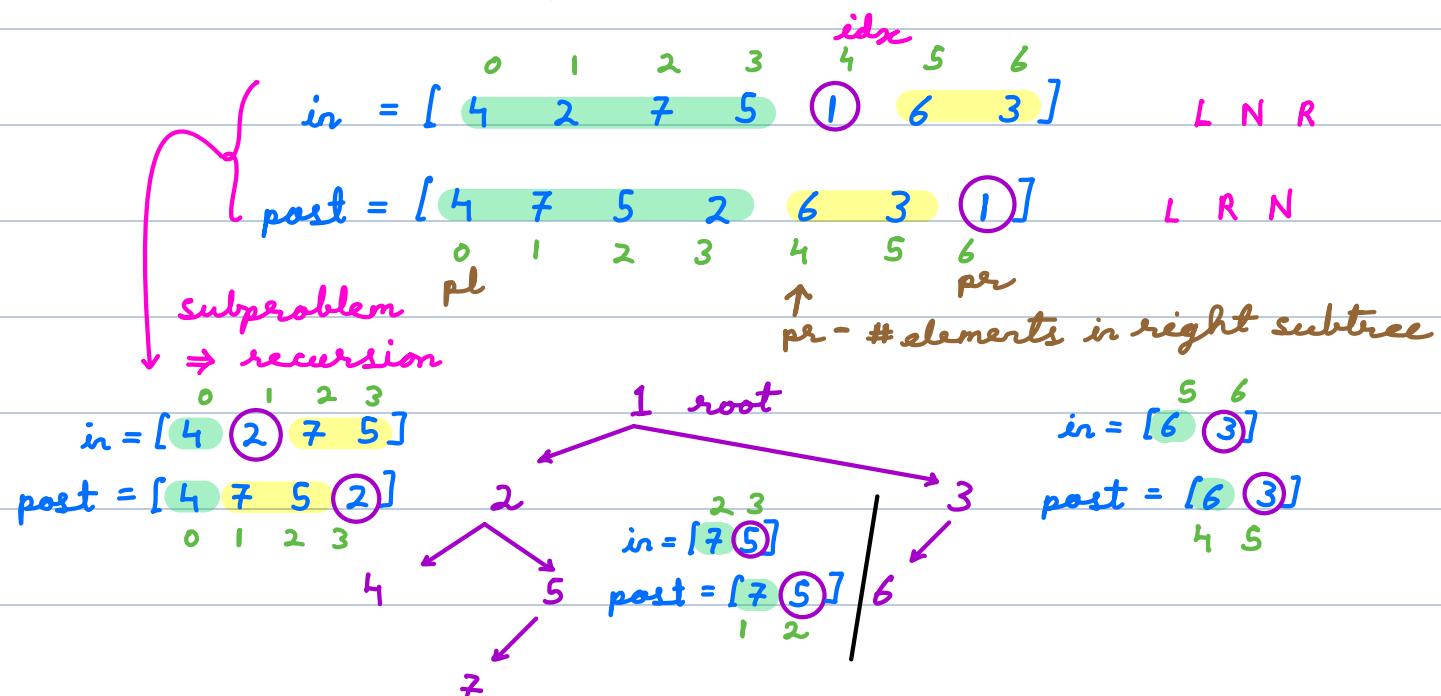
TC = O(N) SC = O(H)

H.W → Try iterative code for preorder & postorder. ✓



< Question > : Construct the binary tree from in-order and post-order.

Given two arrays in[ ], post[ ]. (unique elements)



TreeNode build ( in[], il, ir, post[], pl, pr) {

    if ( ir < il ) return null // empty array

    root = new TreeNode ( post[pr] )

    idx = index ( in, root.val ) // iterate  
 $\leftarrow A[i], i \rightarrow \text{hashmap} \checkmark$

    // in  $\rightarrow$  left    l = il    r = idx - 1

    //      right    l = idx + 1    r = ir

    crt\_r = ir - idx      // R - L + 1    [L R]

    // post  $\rightarrow$  left    l = pl      r = pr - crt\_r - 1

    right    l = pr - crt\_r    r = pr - 1

    root.left = build ( in, il, idx - 1, post, pl, crt\_r - 1 )

    root.right = build ( in, idx + 1, ir, post, crt\_r, pr - 1 )



*return root*

*}*

$$TC = \underline{O(N)}$$

$$SC = \underline{O(N)}$$

# Trees 2

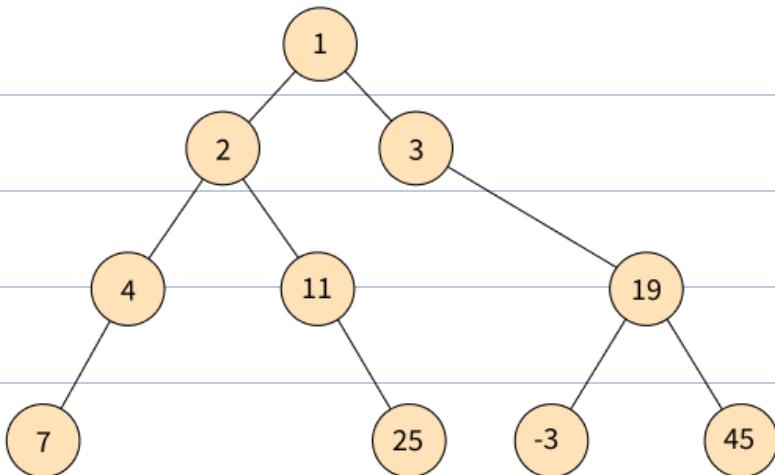
## TABLE OF CONTENTS

1. Level Order Traversal
2. Right view of Binary Tree
3. Left view of Binary Tree
4. Vertical Order Traversal
5. Top - View of Binary Tree
6. Bottom view of Binary Tree Idea
7. Types of Binary Tree
8. Balanced Binary Tree



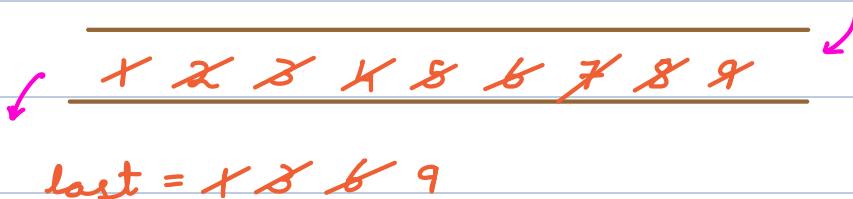
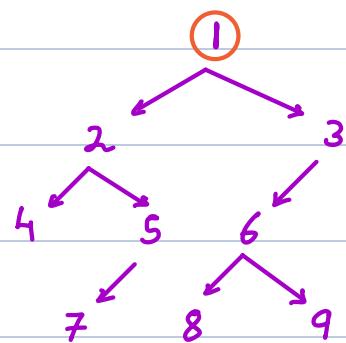
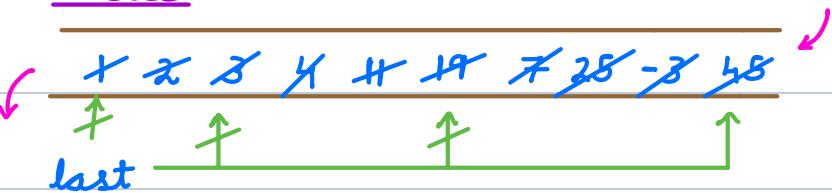
# Level Order Traversal

L0  
L1  
L2  
L3



o/p → 1 ✓  
2 ✓ 3 ✓  
4 ✓ 11 ✓ 19 ✓  
7 ✓ 25 ✓ -3 ✓ 45 ✓

queue (FIFO)



last = x x x

x = x x x x x x x x

o/p → 1  
2  
3  
4 5 6  
7 8 9



$q.\text{enqueue}(\text{root})$

$\text{last} = \text{root}$

while ( $! q.\text{isEmpty}()$ ) {  $\leftarrow$

$x = q.\text{dequeue}()$

$\text{print}(x.\text{val})$

$\text{if } (x.\text{left} \neq \text{null}) \quad q.\text{enqueue}(x.\text{left})$

$\text{if } (x.\text{right} \neq \text{null}) \quad q.\text{enqueue}(x.\text{right})$

$\text{if } (x == \text{last} \ \& \ ! q.\text{isEmpty}()) \{$

$\text{print}("\n")$

$\text{last} = q.\text{rear}()$

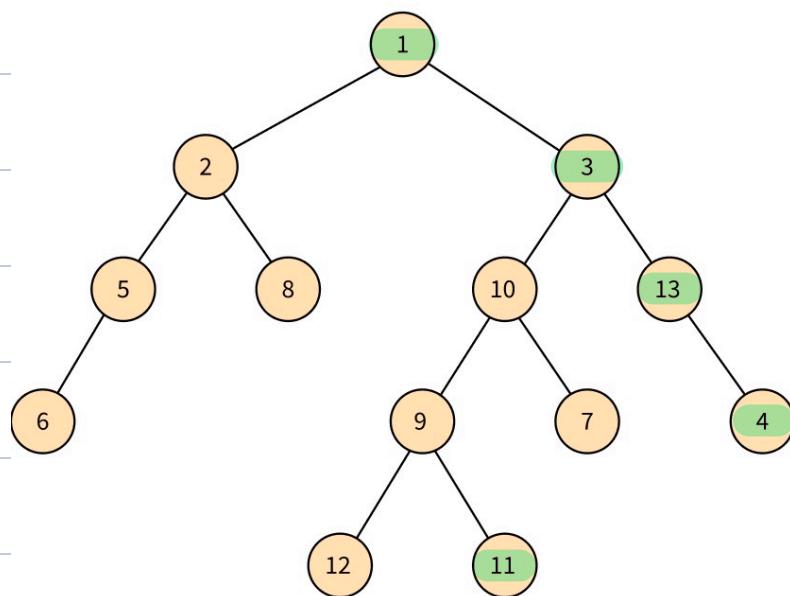
}

}

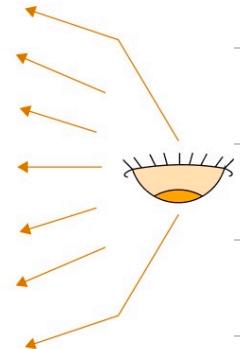
$TC = \underline{O(N)}$

$SC = \underline{O(N)}$

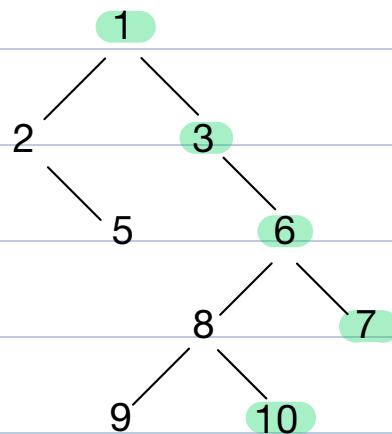
**< Question > :** Find right view of binary tree.



*o/p → 1 3 13 4 11*



**Quiz :**



*o/p → 1 3 6 7 10*

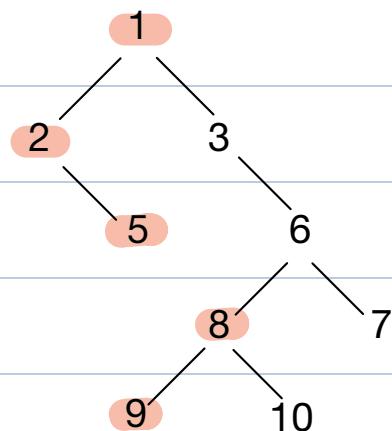
*Sol → Print last node*

*of the level. ✓*

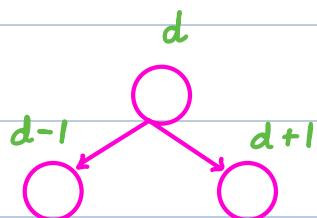
**< Question > :** Find left view of binary tree.

*o/p → 1 2 5 8 9*

*H.W ✓*



# Vertical Order Traversal of Binary Tree



Print each vertical line top to bottom.  
If there is overlap print the node coming from left side first.

o/p  $\rightarrow$  6

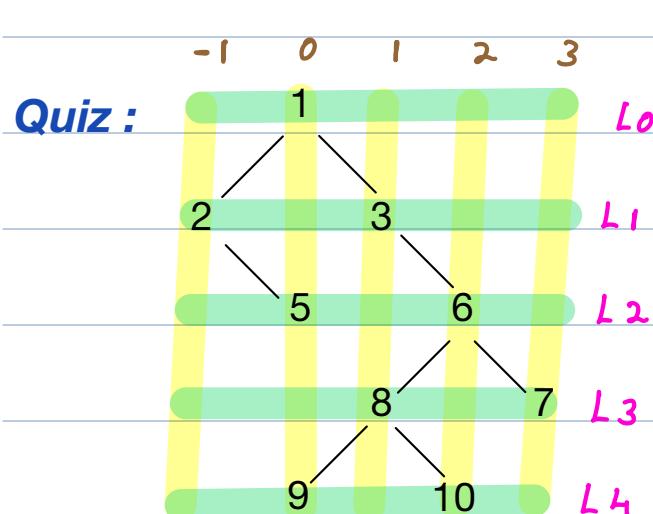
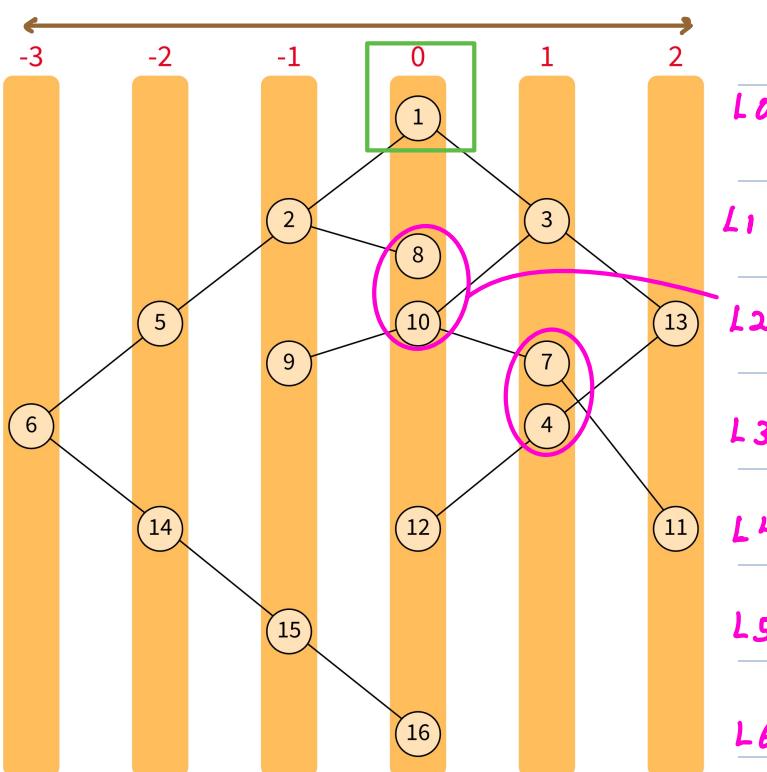
5 14

2 9 15

1 8 10 12 16

3 7 4

13 11



o/p  $\rightarrow$  2

1 5 9

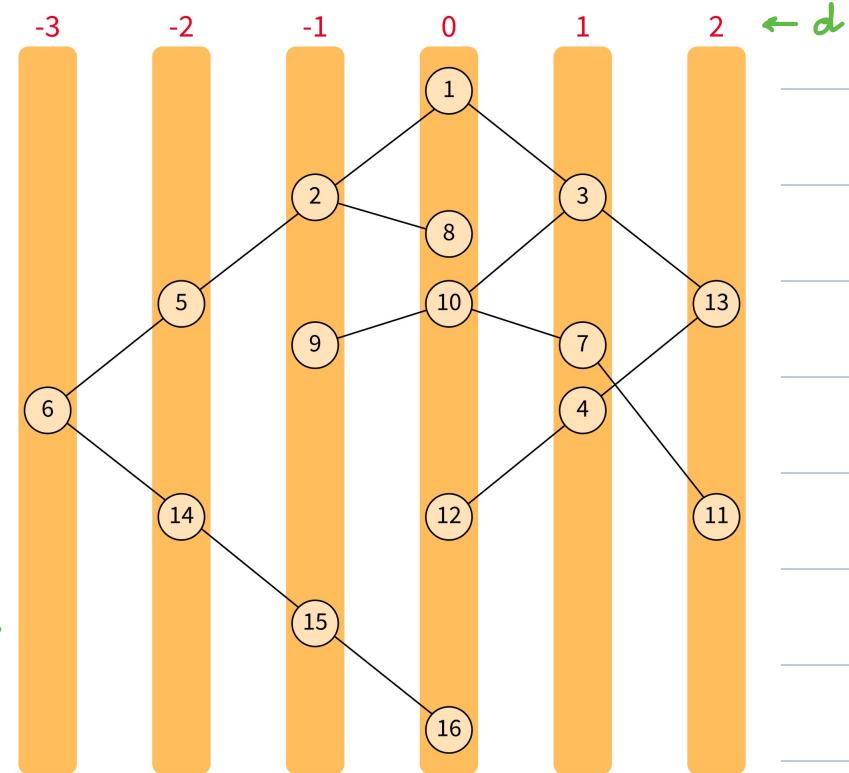
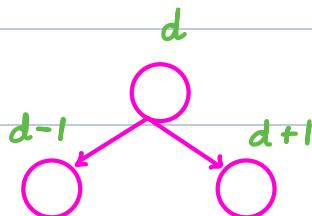
3 8

6 10

7

Print top to bottom

⇒ Level order traversal



forall nodes, store distance

⇒ Pair  $\langle \text{Node}, d \rangle$  /

forall  $i$ ,  $d[i] \rightarrow \text{distance of } i$  ✓

forall level, store list of

nodes ⇒ HashMap < Level, list of Nodes >

queue

→ ~~1 2 3 8 4 6 8 7 9 10~~

$d = \begin{bmatrix} 0 & -1 & 1 & 0 & 0 & 2 & 3 & 1 & 0 & 2 \end{bmatrix}$

Nodes → 1 to N

HashMap < dist, list of Nodes >

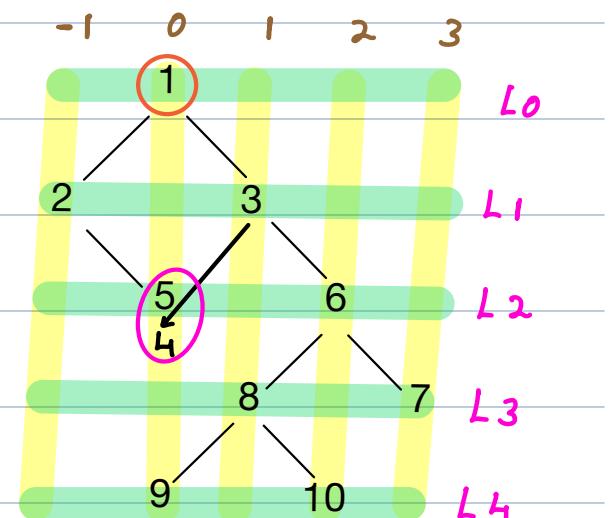
0 → {1, 5, 4, 9}

-1 → {2}

1 → {3, 8}

2 → {6, 10}

3 → {7}





$\text{min\_d} \rightarrow \text{min horizontal distance} \quad // -1$

$\text{max\_d} \rightarrow \text{max horizontal distance} \quad // 3$

for  $d \rightarrow \text{min\_d to max\_d}$  {

    for  $x$  in  $\text{hn.get}(d)$  { // for each loop

        print( $x$ )

}

    print("ln")

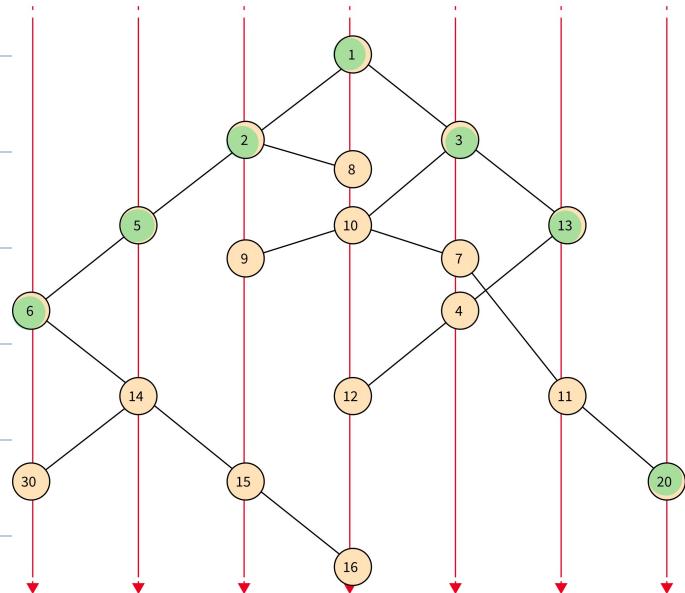
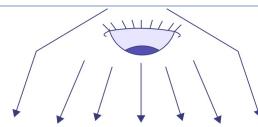
}

$TC = \underline{O(N)}$

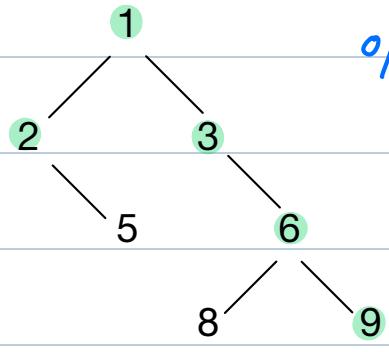
$SC = O(N + N + N) = \underline{O(N)}$

---

**< Question > :** Find top view of binary tree.



**Quiz :**



*o/p → 2 1 3 6 9*

*sol → Print first element*

*of every vertical line ⇒*

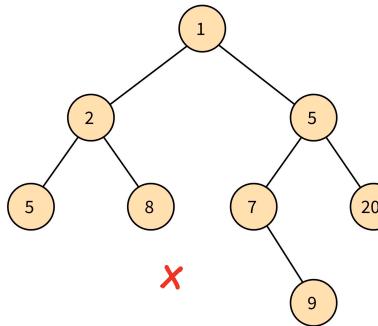
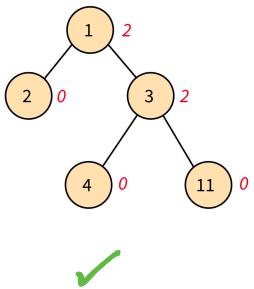
*first node & list in tn. ✓*

*H.W → Bottom view .*

# Types of Binary Tree [ Structure ]

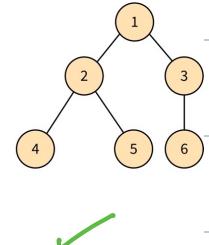
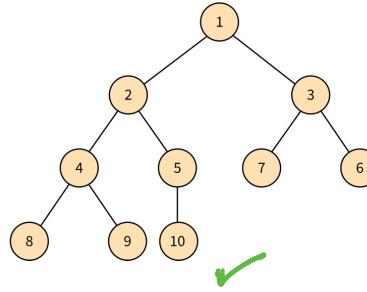
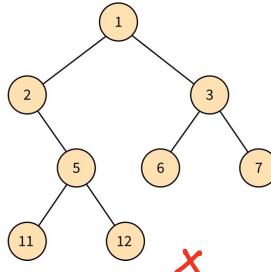
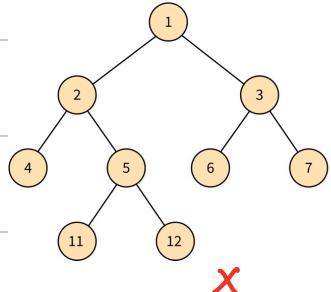
## 1. Proper/ Full Binary Tree

*Every node has either 0 or 2 children.*



## 2. Complete Binary Tree (C.B.T)

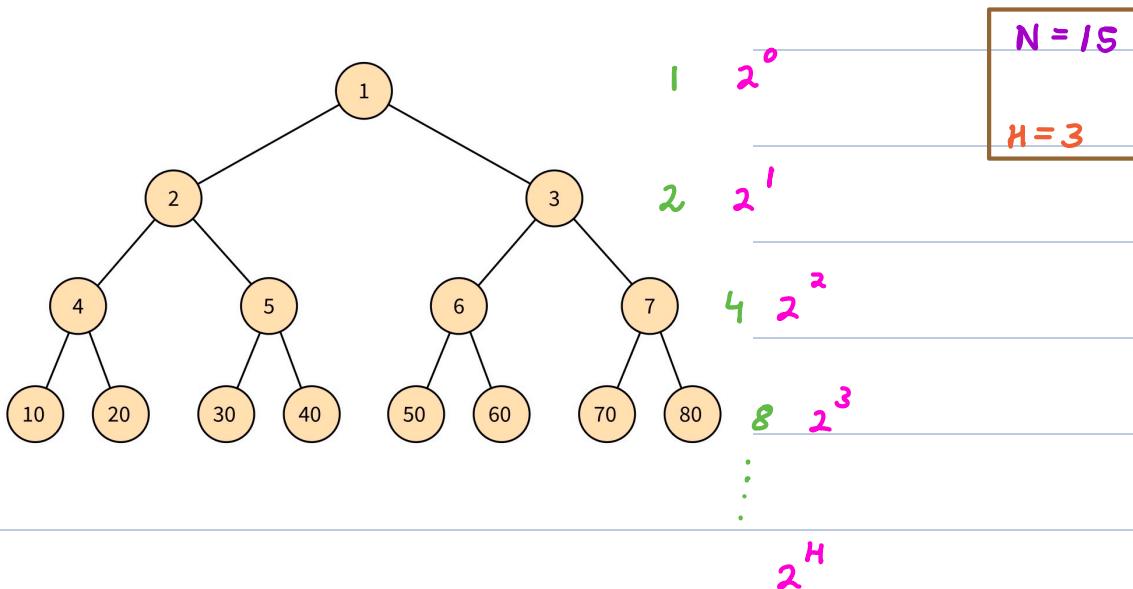
*All levels must be completely filled except possibly the last level  
and the last level must be filled from left to right.*





## 3. Perfect Binary Tree

All levels are completely filled.



Q → Given a perfect binary tree with  $N$  nodes, find height of the tree.

$$1 + 2 + 4 + \dots + 2^H = N$$
$$= \frac{1(2^{H+1} - 1)}{2 - 1} = 2^{H+1} - 1 = N$$

$$\Rightarrow 2^{H+1} = N + 1$$

$$\Rightarrow H + 1 = \log_2(N + 1)$$

$$\Rightarrow H = \log_2(N + 1) - 1 \rightarrow \underline{\underline{O(\log(N))}}$$

## Balanced Binary Tree

For all nodes

$$\left| \begin{array}{c} \text{ht. of} & \text{ht. of} \\ \text{left} & - & \text{right} \\ \text{child} & & \text{child} \end{array} \right| \leq 1$$

$$|1| - (-1)| = \underline{2} \neq 1$$

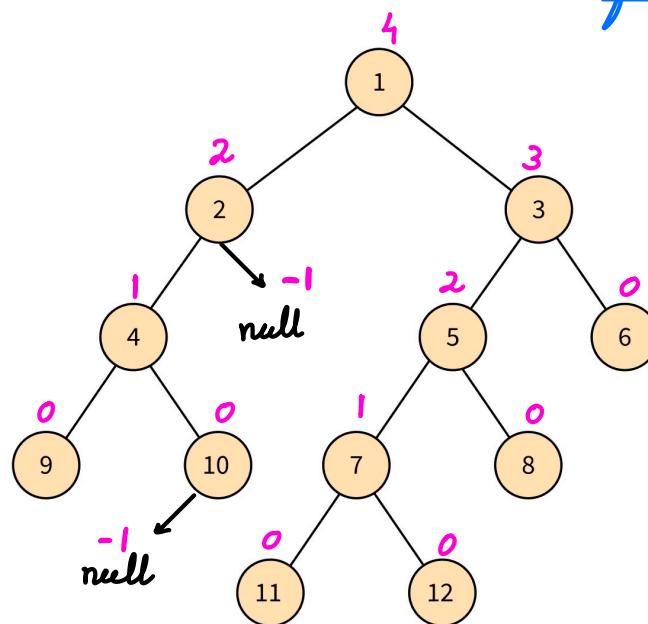
< Question > : Given a binary tree, check if it is balanced or not.

Ans = false

Height  $\rightarrow$  Distance of node to farthest leaf.

left Right Node

isB = true



int height (root) {

if (root == null) return -1

L = height (root.left)

R = height (root.right)

if (abs (L - R) > 1) isB = false

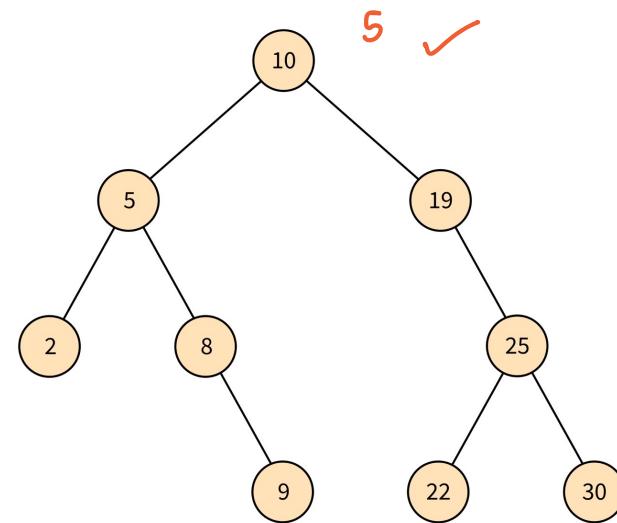
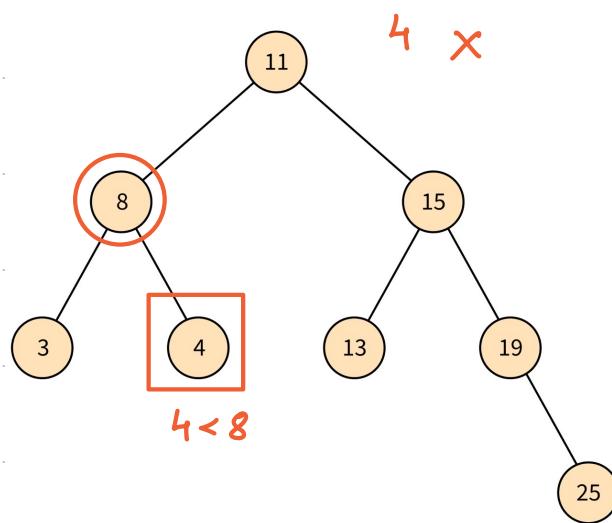
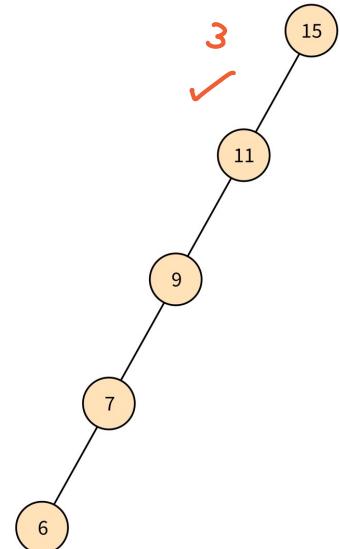
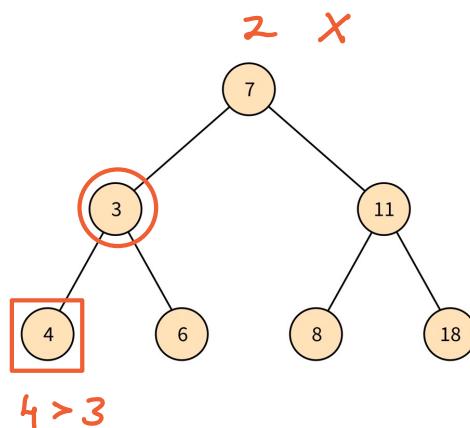
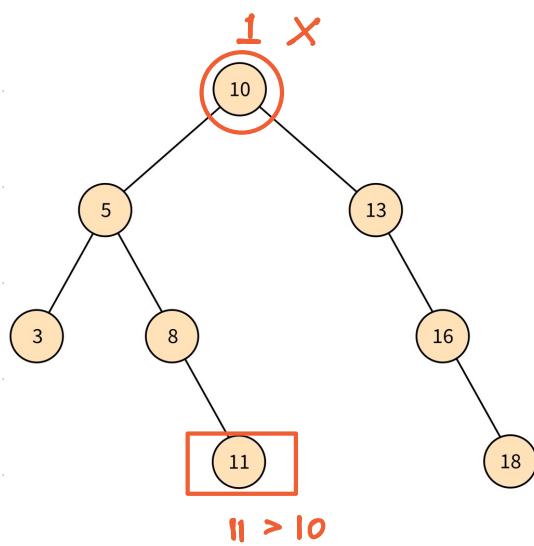
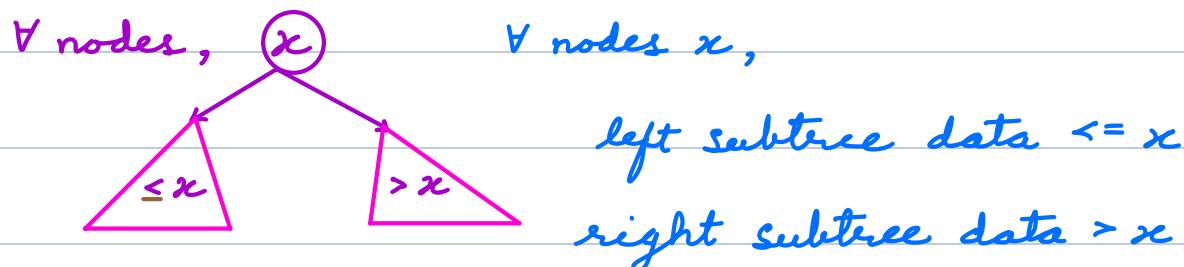
return max (L, R) + 1

}

TC = O(N)

SC = O(N)

# Binary Search Tree [ BST ]



# Scenario

Flipkart stores and manages the history of all the orders that were processed and stores them efficiently such that whenever some information is required regarding any order, it can be fetched easily.

## Problem

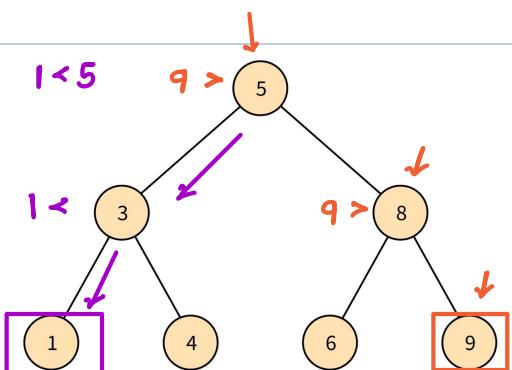
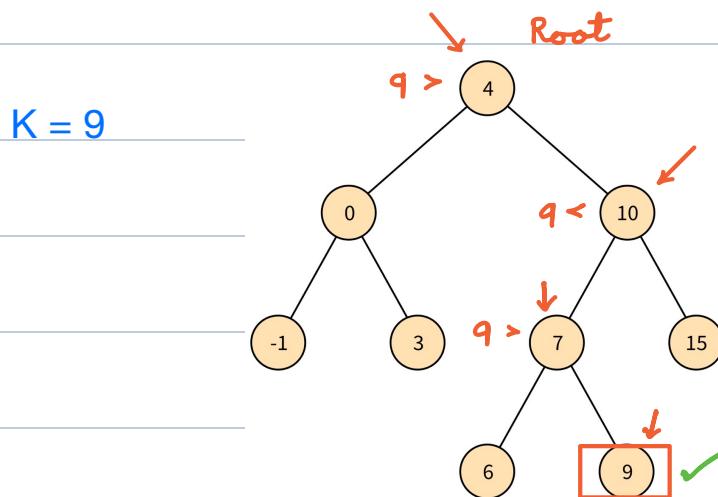
Develop a system for Flipkart that efficiently stores and manages the history of all processed orders. The system should allow easy access to information regarding any specific order when needed.

## Requirements

There are two types of requirements :

- **Add Order :** Insert a new record.
- **Find Order Time :** The system should be able to quickly retrieve the time of placing the order, given the unique order ID.

< Question > : Search an element K in Binary Search Tree.



$temp = root$

```
while (temp != null) {  
    if (temp.val == K) return temp  
    if (temp.val > K) temp = temp.left  
    else temp = temp.right  
}
```

return null

$TC = \underline{O(H)}$

$SC = \underline{O(1)}$

# Insertion in B.S.T

insert (7)

insert (2)

insert (7)

Temp = root

while (temp != null) {

    if (K <= temp) {

        if (temp.left == null) {

            temp.left = new TreeNode (K)

            break

    }

    temp = temp.left

}

    if (temp.right == null) {

        temp.right = new TreeNode (K)

        break

    }

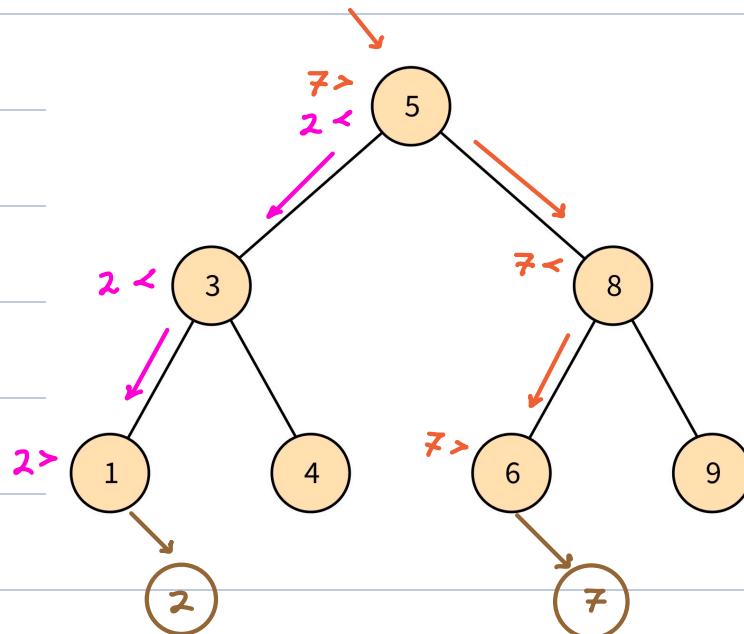
    temp = temp.right

}

}

$TC = O(H)$

$SC = O(1)$



## Find the Smallest Element in B.S.T

temp = root

while (temp.left != null) {

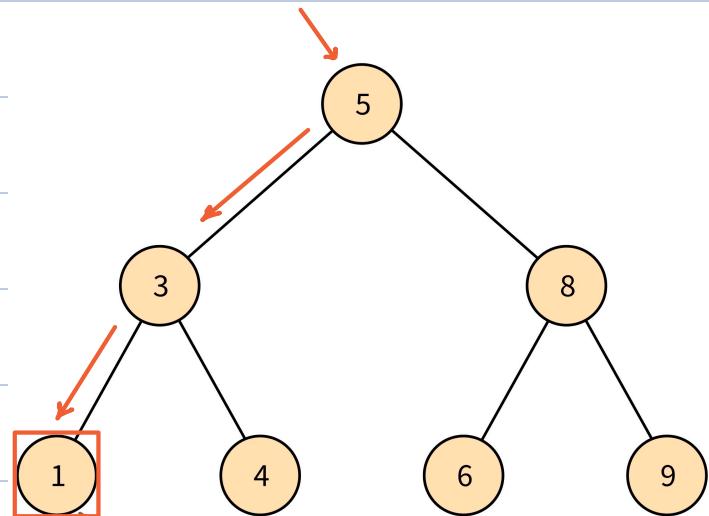
    temp = temp.left

}

return temp.

TC = O(H)

SC = O(1)



**< Question > :** How to find the largest element in Binary Search Tree.

temp = root

while (temp.right != null) {

    temp = temp.right

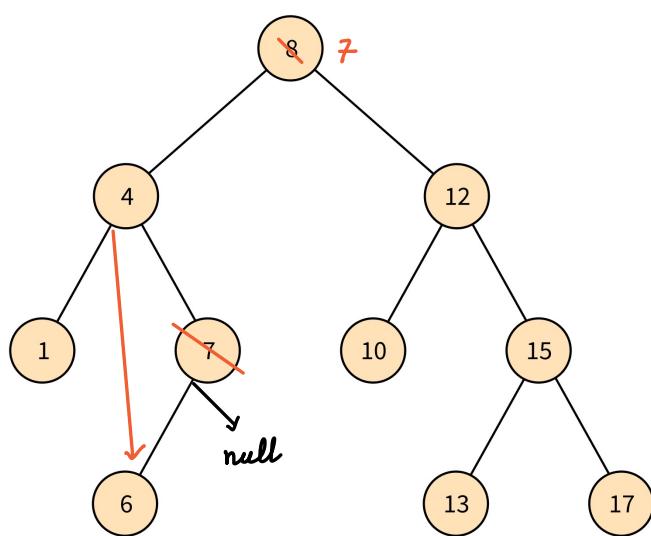
}

return temp

TC = O(H)

SC = O(1)

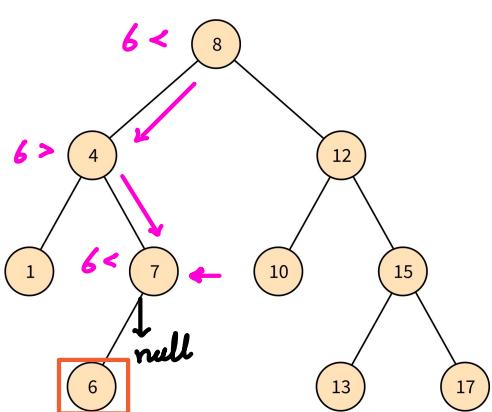
# Deletion in B.S.T



delete (6) ✓  
 delete (7) ✓  
 delete (8) ✓

1. Leaf node ✓
2. Node with single child ✓
3. Node with both the children ✓

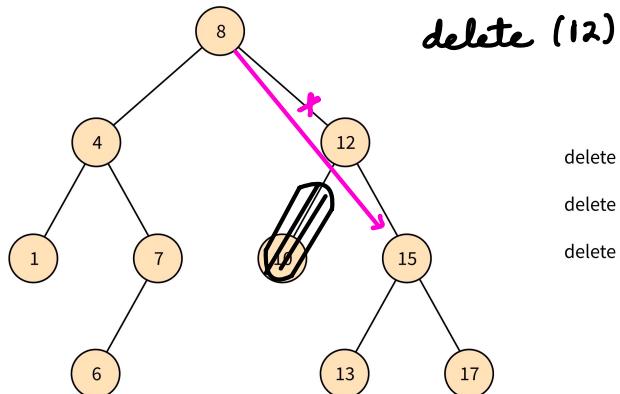
## Leaf Node



→ Travel till parent of K.

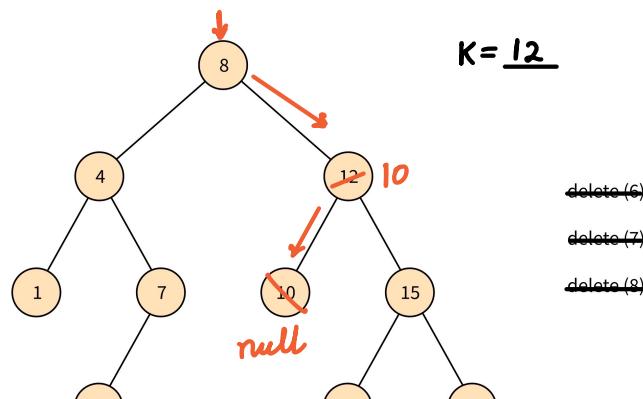
→ Update the child to null.

delete (6) ✓  
 delete (7)  
 delete (8)

Node with single child

delete (6)  
delete (7)  
delete (8)

- 1) Travel till parent of K.
- 2) Update link to K with link to child of K.

Node with both children

- 1) Travel till node K.
- 2) Find largest node in left subtree & delete it.
- 3) Replace K with largest node in left subtree.

root = delete (root, K)

int  
TreeNode delete (root, K) {  
 if (root == null) return null;  
 if (K < root.data)  
 root.left = delete (root.left, K) ✓;  
 else if (K > root.data)  
 root.right = delete (root.right, K);  
 else {  
 if (root.left == null || root.right == null)  
 return root.right == null ? root.left : root.right;  
 else {  
 TreeNode temp = root.right;  
 while (temp.left != null)  
 temp = temp.left;  
 root.data = temp.data;  
 root.right = delete (root.right, temp.data);  
 }  
 }  
 return root;  
}

```

    L   root.right = delete (root.right, K) ✓
else { // K == root.data
    {
        if (root.left == null &&
            root.right == null) return null
    {
        if (root.left == null) return root.right
        if (root.right == null) return root.left
    }
}

```

```

temp = root.left
while (temp.right != null)
    |
    |   temp = temp.right
    |
root.data = temp.data
root.left = delete (root.left, temp.data)
return root
}

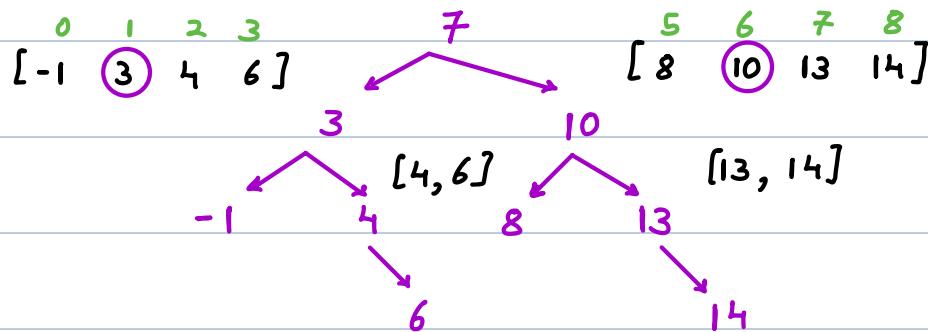
```

$$TC = O(H) \quad SC = O(H)$$

H.W  $\rightarrow$  Read about Red-Black Tree } Balanced  
& AVL Tree } BST.

## Construct B.S.T from sorted array

arr  $\rightarrow$  [-1, 3, 4, 6, 7, 8, 10, 13, 14]  
 0 1 2 3 4 5 6 7 8



TreeNode build (A[], L, R) {

if (L > R) return null

$m = (L + R) / 2$

root = new TreeNode(A[m])

root.left = build (A, L, m-1)

root.right = build (A, m+1, R)

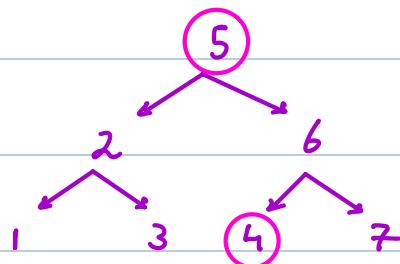
return root

}

TC = O(N)

SC = O(log<sub>2</sub>(N))

## Check if given Binary Tree is a B.S.T

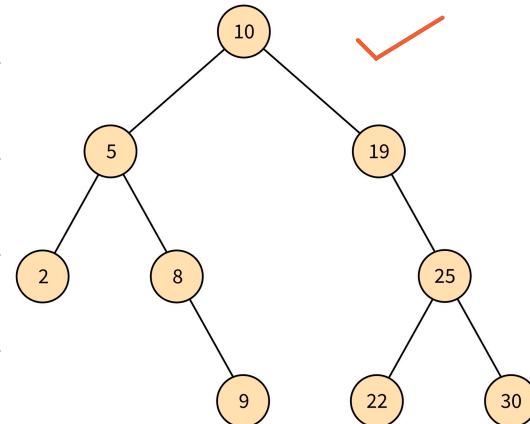
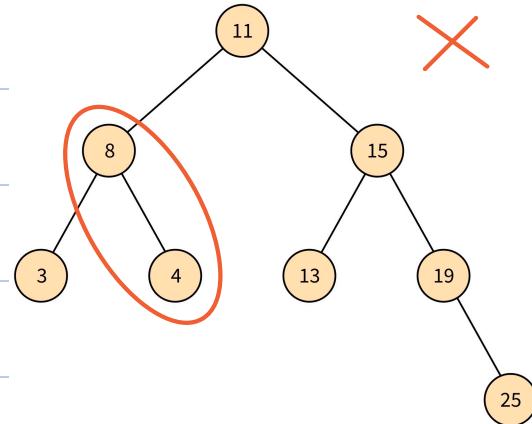


Not a BST

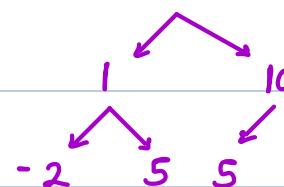
Sol 1 → **left    Node    Right**

Inorder traversal is sorted.

Unique data ✓



→  $in = -2 \ 1 \ 5 \ 5 \ 5 \ 10$

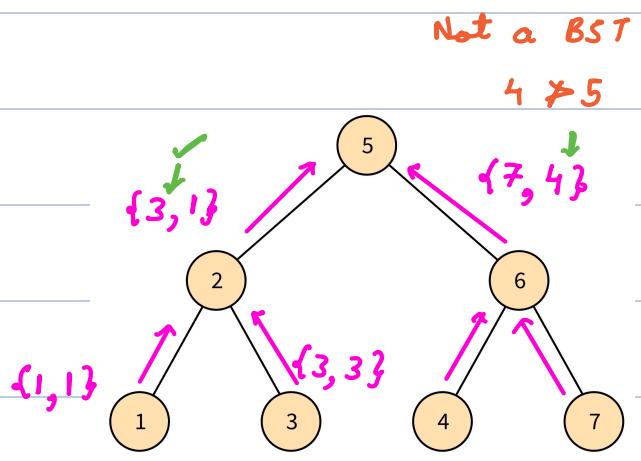


Sol 2 →

forall nodes  $x$ ,

left subtree data  $\leq x$

right subtree data  $> x$



largest in left subtree  $\leq x$

{max, min}

& smallest in right subtree  $> x$  ✓

## Trees - 4

### TABLE OF CONTENTS

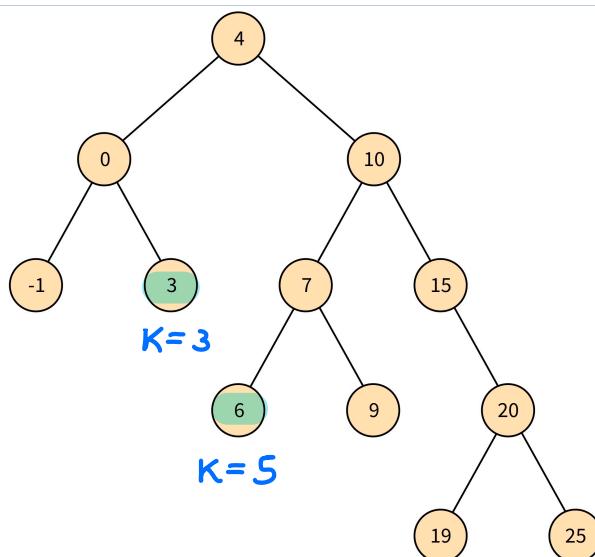
1. K<sup>th</sup> Smallest Element in B.S.T
2. Morris Traversal
3. Node to Root Path in a given tree
4. L.C.A of Binary Tree
5. L.C.A of Binary Search Tree



Notes

**< Question > :** Find Kth smallest element in B.S.T

K = 3



Nodes, x



Sol → Inorder → Left Node Right  
 $K^{th}$  element in inorder traversal.

TC = O(N)

SC = O(H)

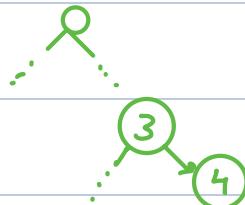
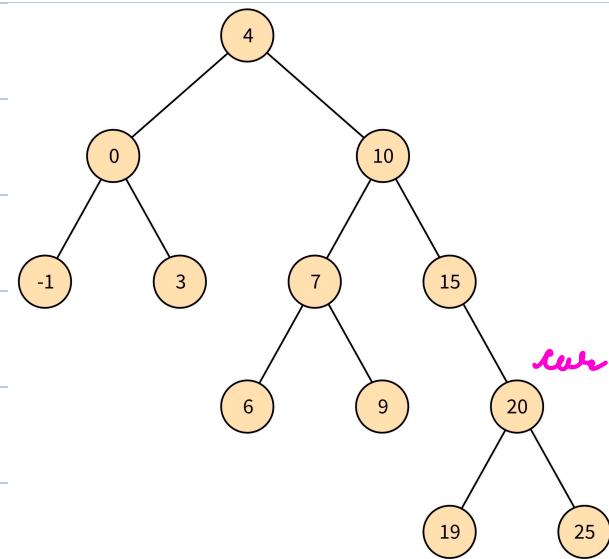
O(1)

# Morris Traversal

Left Node Right

# In-Order Traversal of a Binary Tree

Expected S.C  $\rightarrow O(1)$



$o/p \rightarrow -1 \ 0 \ 3 \ 4 \ 6 \ 7 \ 9 \ 10 \ 15 \dots$

cur = root

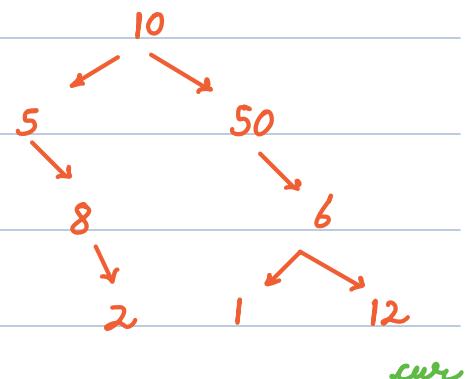
while ( cur != null ) {

if (cur.left == null) {

print (cur.val)

cur = cur. right

```
    } } else {
```



`p = cur.left`

o/p → 5 8 2 10 50 16 12

while ( p.right != null && p.right != cur )

$p = p.\text{right}$

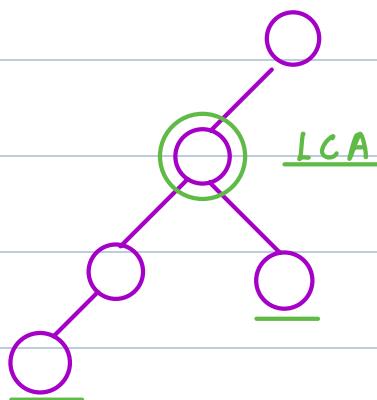
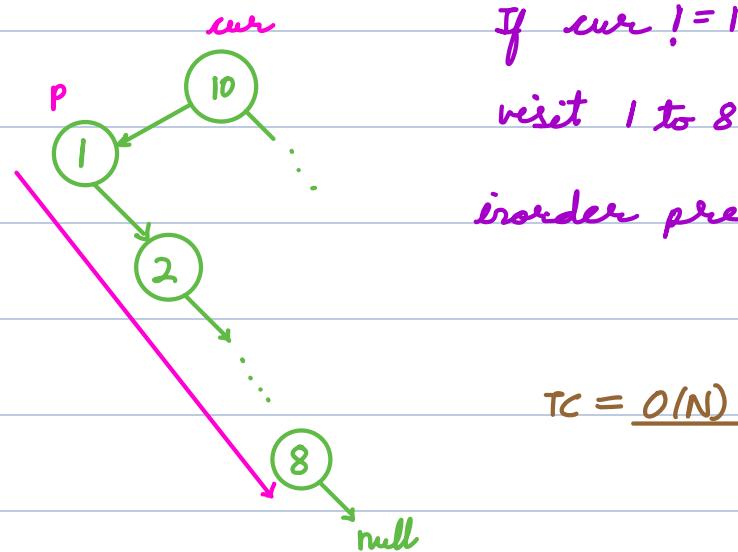
```
if ( p->right == null ) { // First time
```

p. right = cur

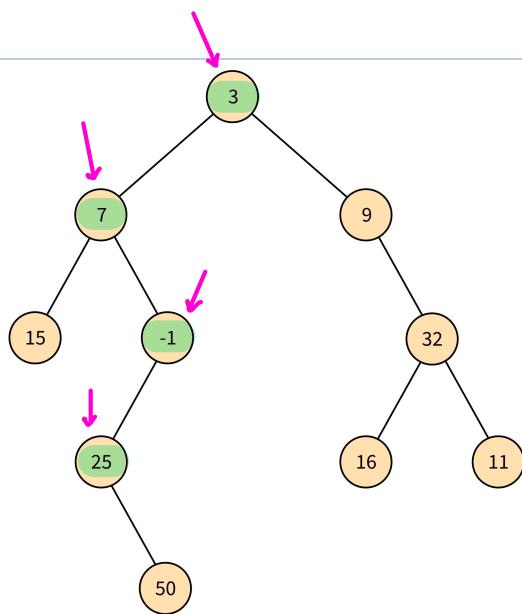
```

    cur = cur.left
}
else {           // Second time
    print (cur.val)
    cur = cur.right
    p.right = null
}
}

```



## Path from Root to Node



$K = 25$

void travel (root, found) {  
if (root == null || found)  
return

a. add (root)

if (root.val == K) {  
found = true  
return

}

travel (root.left, found)

travel (root.right, found)

a. removeLast()

}

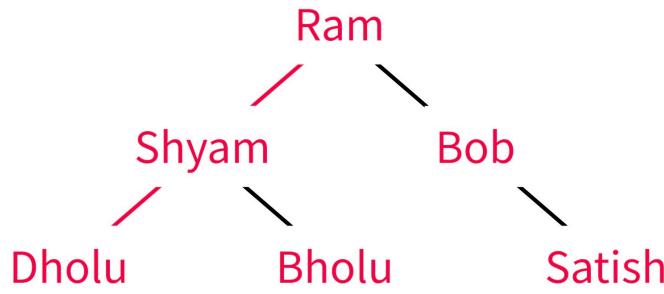
TC = O(N)

SC = O(H)

# Problem Statement

It is said that we all humans are related through some common ancestor at some point of time. Assume that a person can have 0, 1 or 2 children only.

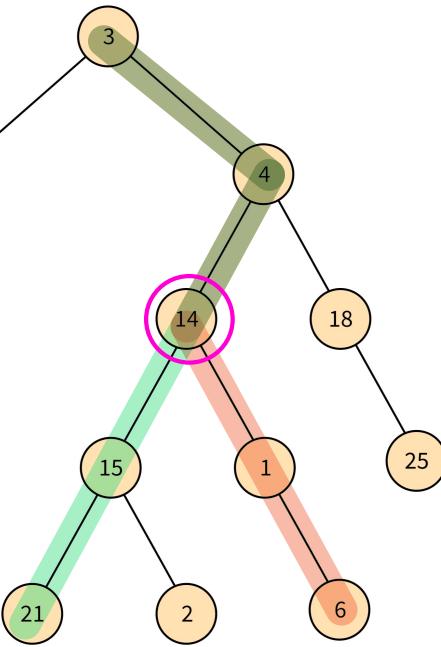
Given the Binary tree A representing the family tree, discover the earliest common family member who connects two given people B and C in a family tree.



- Find LCA of **Dholu** and **Bob**
- Answer = **Ram**

# Lowest Common Ancestor

Flipkart	Acolite	Amazon	Microsoft	OYO Rooms	Snapdeal	MakeMyTrip
PayU	Times Internet	Cisco	PayPal	Expedia	Twitter	American Express



L.C.A(21, 6)  $\rightarrow$  14

L.C.A(2, 6)  $\rightarrow$  14

L.C.A(21, 4)  $\rightarrow$  4

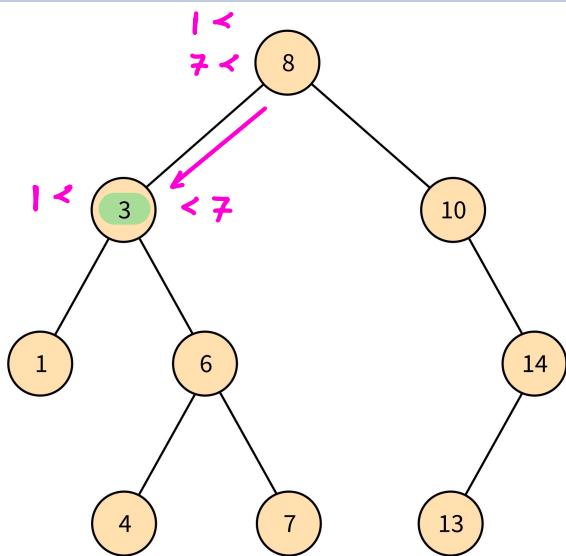
L.C.A(12, 6)  $\rightarrow$  3

LCA( $x, y$ )

Sol → 1) Find root to node path for  $x$  &  $y$ . ✓  
2) Ans = last common node.

$$TC = O(N) \quad SC = O(H)$$

# L.C.A in B.S.T



Above sol  $\rightarrow$   $TC = O(H)$   $SC = O(H)$   
 $O(1)$

$LCA(1, 7) = 3$

$cur = root$

```
while (cur != null) {
    if (x < cur.val && y < cur.val)
        cur = cur.left
    else if (x > cur.val && y > cur.val)
        cur = cur.right
    else
        return cur
}
```

$TC = O(H)$   $SC = O(1)$

## Trees - 5

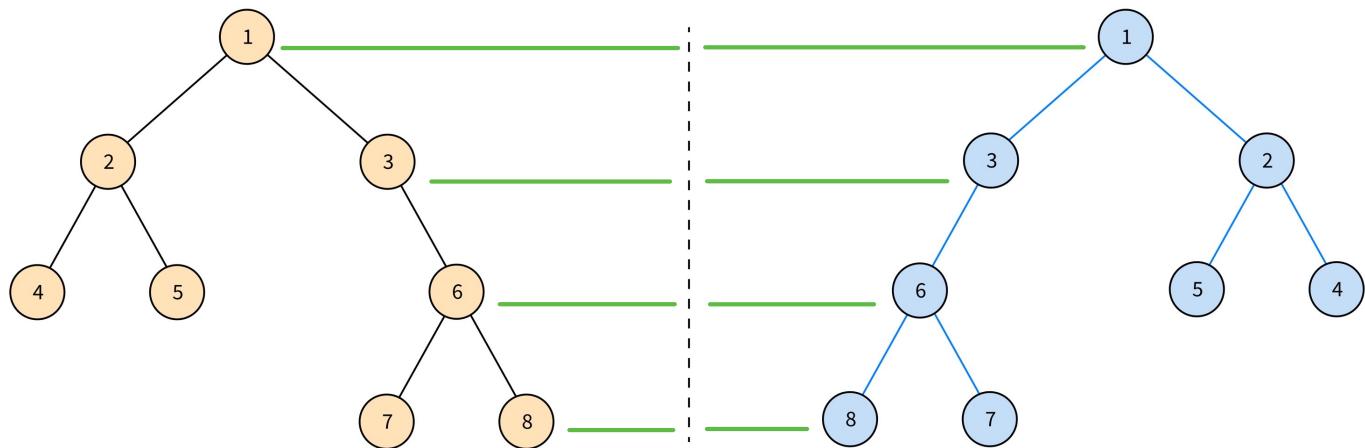
### TABLE OF CONTENTS

1. Invert Binary Tree
2. Equal Tree Partition
3. Next Pointer in Binary Tree
4. Root to Leaf Path Sum = K
5. Diameter of a Binary Tree



Notes

# Invert a Binary Tree



Is the below inversion correct for given binary tree ?



No

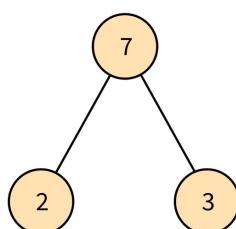
```

void invert (root) {
    if (root == null)
        return
    temp = root.left
    root.left = root.right
    root.right = temp
    invert (root.left)    // Left
    invert (root.right)  // Right
}
  
```

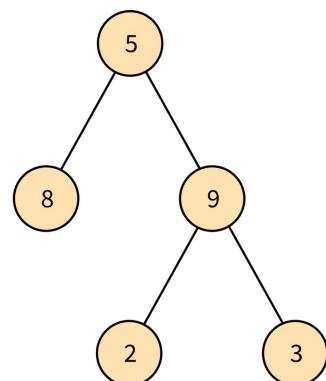
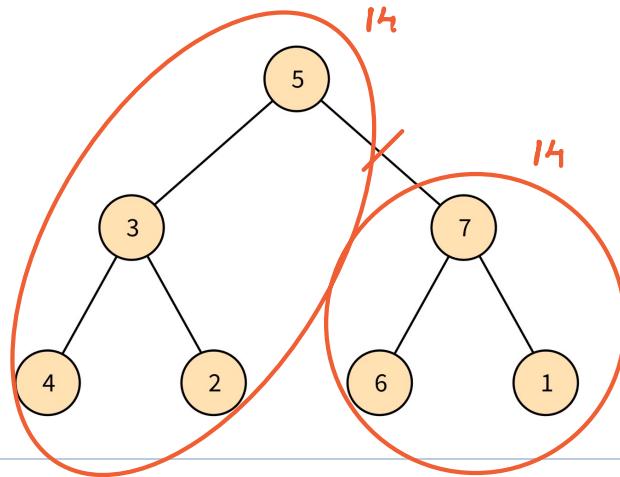
$$TC = \underline{O(N)} \quad SC = \underline{O(H)}$$

# Equal tree Partition

- Check if it is possible to remove an edge from the given binary tree such that sum of resultant two trees is equal. *← half of total.*



False



False

True

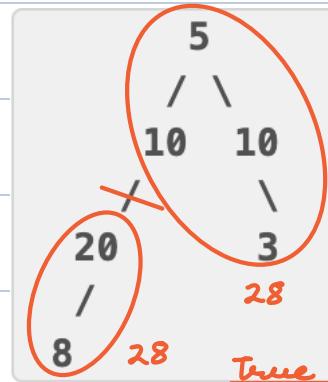
`sum = getSum(root)`

`if (sum % 2 == 1) return false`

`ans = false`

`checkSum(root, sum/2)`

`return ans`



`int getSum(root) {`

`if (root == null) return 0`

`return root.val + getSum(root.left) +`

`getSum(root.right)`

`}`

```

int checkSum(root, halfSum) {
    if (root == null) return 0
    s = root.val + checkSum(root.left, halfSum) +
        checkSum(root.right, halfSum)
    if (s == halfSum) ans = true
    return s
}

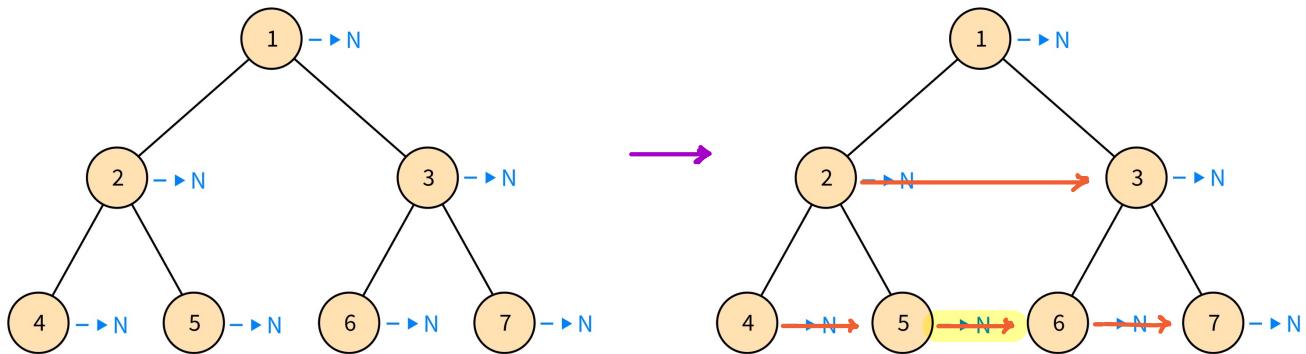
```

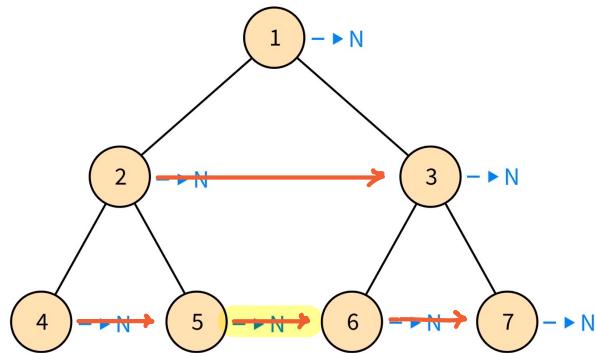
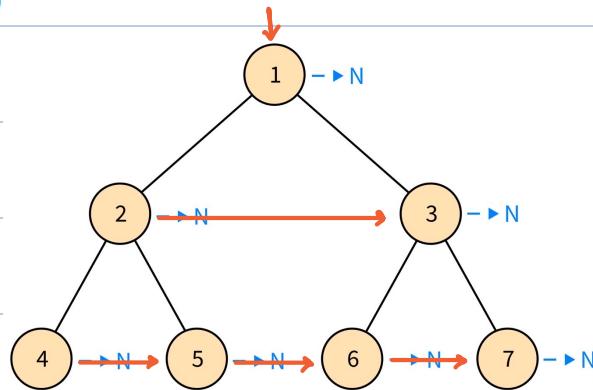
$$TC = O(N + N) = \underline{O(N)}$$

$$SC = \underline{O(H)}$$

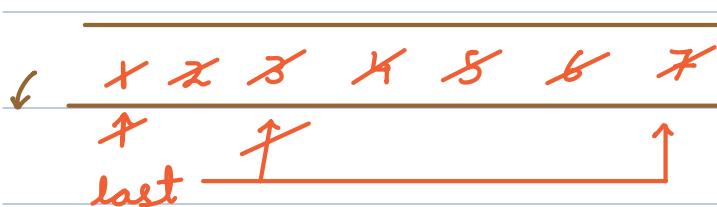
## Next Pointer in Binary Tree

- Given a perfect tree initially with all next pointers set to `nullptr`, modify the tree in-place to connect each node's next pointer to the next node in the same level from left to right, following an in-order traversal.





Level Order Traversal → Queue



if (root == null) return

// Queue → q

q.enqueue(root)

last = root

while (!q.isEmpty()) {

    cur = q.dequeue()

    if (cur.left != null) q.enqueue(cur.left)

    if (cur.right != null) q.enqueue(cur.right)

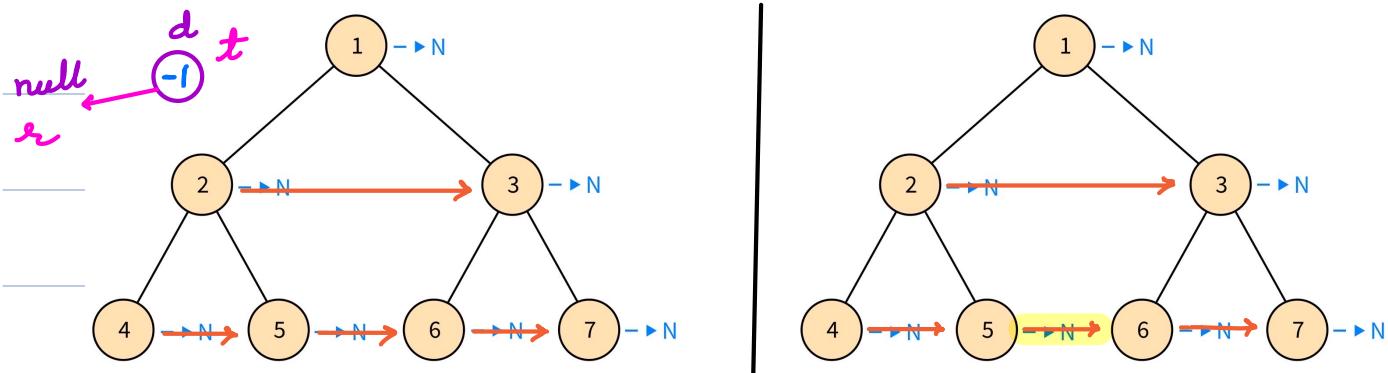
    if (cur != last) cur.next = q.front()

    else last = q.rear()

}

TC = O(N)

SC = O(N)



```
if (root == null) return
dummy = new TreeNode (-1)
```

```
temp = dummy
```

```
while (root != null) {
```

```
if (root.left != null) {
```

```
temp.next = root.left
```

```
temp = temp.next
```

```
} if (root.right != null) {
```

```
temp.next = root.right
```

```
temp = temp.next
```

```
} root = root.next ← ✓
```

```
if (root == null) {
```

```
root = dummy.next
```

```
dummy.next = null
```

```
temp = dummy
```

```
}
```

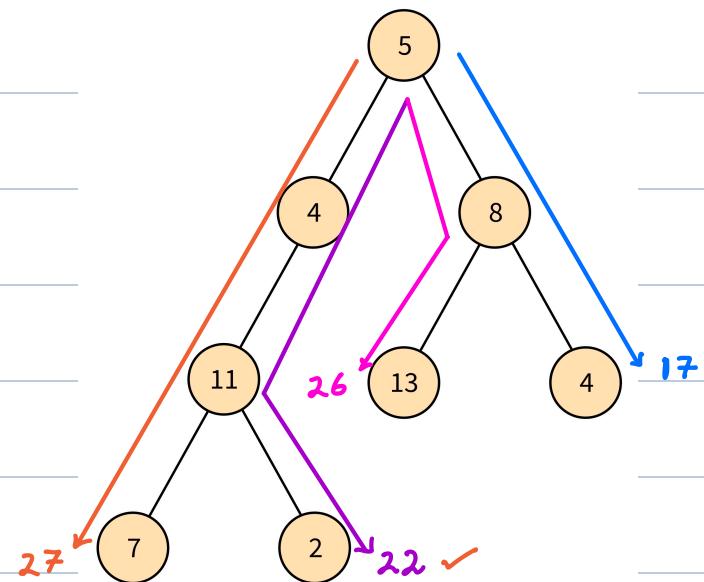
TC = O(N)

SC = O(1)

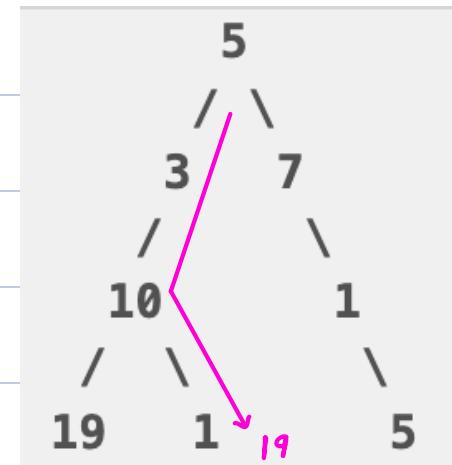
< Question > : Check if root to leaf path sum equals to K.

a.  $K = 22 \rightarrow \text{true}$

b.  $K = 19 \rightarrow \text{false}$



```
boolean check (root, K) {  
    if (root == null) return false  
    if (root.left == null &&  
        root.right == null)  
        return (root.val == K)  
  
    return check (root.left, K-root.val) ||  
           check (root.right, K-root.val)  
}
```

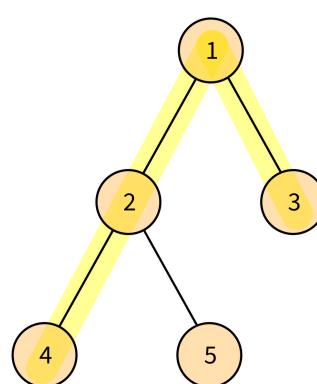


$TC = \underline{O(N)}$     $SC = \underline{O(H)}$

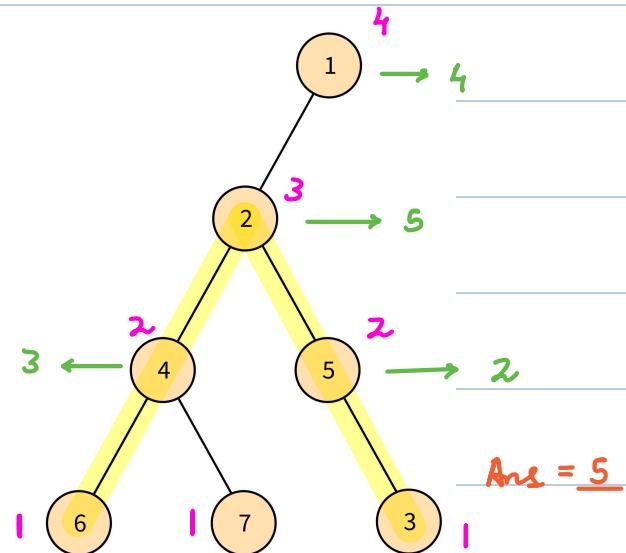
# Diameter of Binary Tree

- **Definition of Diameter :**

The diameter of a binary tree is defined as the **number of nodes** along the longest path between any two leaf nodes in the tree. This path may or may not pass through the root.



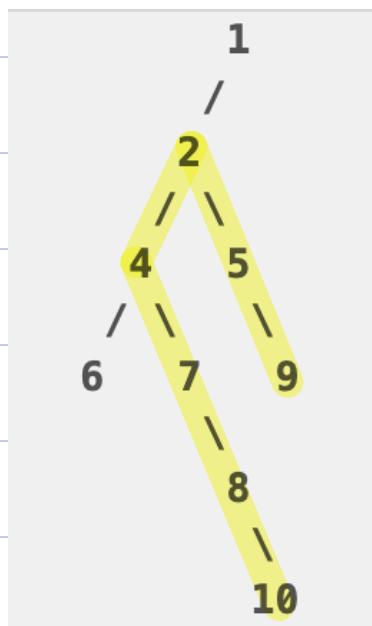
Ans = 4



Ans = 5

Hint → use height of tree .

height (leaf) = 1



height (node) = max (left, right) + 1

diameter (node) = height (left) +

Ans = 7

height (right) + 1