EAS 596, Fall 2018, Homework 11
Due Friday 12/7, 9 AM, Box outside Furnas 611

The homework is a bonus assignment. Show all work, including any M-files you have written or adapted. Make sure your work is clear and readable - if the TA cannot read what you've written, he will not grade it. All electronic work (m-files, etc.) **must** be submitted through UBLearns (and submitted by the time class starts on the day it's due). Any handwritten work may be submitted in class. Each problem will be graded according to the following scheme: 2 points if the solution is complete and correct, 1 point if the solution is incorrect or incomplete but was using correct ideas, and 0 points if using incorrect ideas.

1. You are given the following data:

$$x = \begin{bmatrix} 0.1 & 0.2 & 0.4 & 0.6 & 0.9 & 1.3 & 1.5 & 1.7 & 1.8 \end{bmatrix}$$
$$y = \begin{bmatrix} 0.75 & 1.25 & 1.45 & 1.25 & 0.85 & 0.55 & 0.35 & 0.28 & 0.18 \end{bmatrix}$$

   You are tasked with fitting a function of the following form to this data:

$$y = a_0 x e^{a_1 x}$$

   (a) Use a linear least square analysis to fit this data. Hint: Take the natural log of the function.

   (b) Use Newton's method (*i.e.* Newton-Rhapson) to solve the non-linear system of equations. Use an initial guess of $a_0 = 1$ and $a_1 = 1$, and a residual norm tolerance of 10^{-6}.

   (c) Use the Gauss-Newton method to obtain the minimum residual fit of the function to the data. Use an initial guess of $a_0 = 1$ and $a_1 = 1$, and a residual norm tolerance of 10^{-6}.

   (d) In all three cases, plot the data and the curve fit to the data. Compare the solutions. Which is better?

2. Runge's function is given by:

$$f(x) = \frac{1}{1 + 25x^2}$$

   Generate 3 sets of points from [-1,1] containing: 5 points, 11 points, 21 points.

(a) Compute the Lagrange interpolant for each set and plot the interpolating polynomial and the actual function on the same plot.

(b) Construct fits using linear splines for each set and plot the interpolating function and the actual function on the same plot. Use the MATLAB function *interp1* to construct the spline.

(c) Construct fits using cubic splines (not-a-knot end conditions) for each set and plot the interpolating function and the actual function on the same plot. Use the MATLAB function *interp1* to construct the spline.

3. The arclength of the curve $y = \log(x)$, $1 < x < 2$, is given by

$$\int_1^2 \sqrt{1 + x^{-2}}$$

Estimate the value of this integral using

(a) Composite Trapezoidal rule to within an estimated error of 0.1%, using an appropriate error estimate.

(b) Composite Simpson's rule to within an estimated error of 0.1%, using an appropriate error estimate.

(c) Composite 2-point Gaussian Quadrature to within an estimated error of 0.1%, using an appropriate error estimate.

(d) How many intervals were required for each of the two methods?