EAS 596, Fall 2018, Homework 3

Due Weds. 9/19, **5 PM**, Box outside Furnas 611

Work all problems. Show all work, including any M-files you have written or adapted. Make sure your work is clear and readable - if the TA cannot read what you've written, he will not grade it. All electronic work (m-files, etc.) **must** be submitted through UBLearns (and submitted by the time class starts on the day it's due). Any handwritten work may be submitted in class. Each problem will be graded according to the following scheme: 2 points if the solution is complete and correct, 1 point if the solution is incorrect or incomplete but was using correct ideas, and 0 points if using incorrect ideas.

1. Let the following matrix and vectors be defined.

$$A = \begin{bmatrix} 1 & 2 \\ -2 & 1 \end{bmatrix}, \quad x = \begin{bmatrix} x_1 \\ x_2 \end{bmatrix}, \quad b = \begin{bmatrix} b_1 \\ b_2 \end{bmatrix}$$

   (a) Write the matrix-vector system $Ax = b$ as a system of two linear equations of x_1 and x_2.

   (b) Assuming that $b_1 = 1$ and $b_2 = 0$, solve this linear system for x_1 and x_2. Write this as a column vector.

   (c) Assuming that $b_1 = 0$ and $b_2 = 1$, solve this linear system for x_1 and x_2. Write this as a column vector.

   (d) Form a 2x2 matrix where the first column of the matrix is the answer vector from part (b) and the second column is the answer vector from part (c). Show that this matrix is the inverse of matrix A.

2. Consider the graphs shown in Figure 1.

   (a) For each of the graphs, give the corresponding adjacency matrix.

   (b) Which of these matrices are symmetric?

   (c) For Graph G_1, how many paths of length 3 are there between points P_1 and P_3?

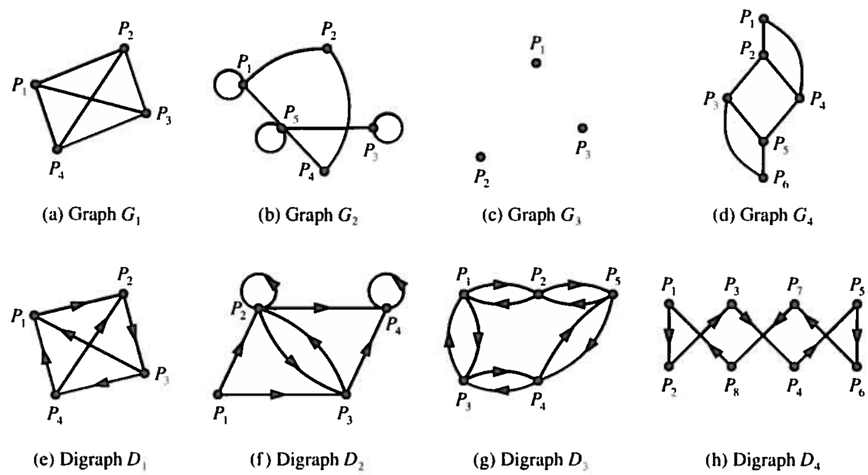

Figure 1: Graphs for Problem 2.

3. Which of the following matrices could be the adjacency matrix for a simple graph or digraph? Draw the corresponding graph and/or digraph when appropriate.

(a) $\mathbf{A} = \begin{bmatrix} -1 & 4 \\ 0 & 1 \\ 6 & 0 \end{bmatrix}$

(b) $\mathbf{B} = \begin{bmatrix} 2 & 0 \\ 0 & -1 \end{bmatrix}$

(c) $\mathbf{C} = \begin{bmatrix} 6 & 0 & 0 & 0 \\ 0 & 6 & 0 & 0 \\ 0 & 0 & 6 & 0 \\ 0 & 0 & 0 & 6 \end{bmatrix}$

(d) $\mathbf{D} = \begin{bmatrix} -1 \\ 4 \\ 2 \end{bmatrix}$

(e) $\mathbf{E} = \begin{bmatrix} 0 & 0 & 0 & 6 \\ 0 & 0 & 6 & 0 \\ 0 & -6 & 0 & 0 \\ -6 & 0 & 0 & 0 \end{bmatrix}$

(f) $\mathbf{F} = \begin{bmatrix} 1 & 0 & 1 & 0 & 1 \\ 0 & 1 & 0 & 0 & 1 \\ 1 & 0 & 0 & 1 & 1 \\ 0 & 0 & 1 & 0 & 0 \\ 1 & 1 & 1 & 0 & 1 \end{bmatrix}$

(g) $\mathbf{G} = \begin{bmatrix} 1 & 1 & 1 \\ 0 & 1 & 1 \\ 0 & 0 & 1 \end{bmatrix}$

(h) $\mathbf{H} = \begin{bmatrix} 0 & 0 & 0 \\ 1 & 0 & 0 \\ 1 & 1 & 0 \end{bmatrix}$

(i) $\mathbf{I} = \begin{bmatrix} 1 & 0 & 0 \\ 0 & 1 & 0 \\ 0 & 0 & 1 \end{bmatrix}$

(j) $\mathbf{J} = \begin{bmatrix} 1 & 2 & 3 & 4 \\ -2 & 1 & 5 & 6 \\ -3 & -5 & 1 & 7 \\ -4 & -6 & -7 & 1 \end{bmatrix}$

(k) $\mathbf{K} = \begin{bmatrix} 0 & 1 \\ 1 & 0 \end{bmatrix}$

(l) $\mathbf{L} = \begin{bmatrix} 0 & 1 & 0 & 0 \\ 1 & 0 & 1 & 1 \\ 0 & 1 & 1 & 1 \\ 0 & 1 & 1 & 0 \end{bmatrix}$

(m) $\mathbf{M} = \begin{bmatrix} -2 & 0 & 0 \\ 4 & 0 & 0 \\ -1 & 2 & 3 \end{bmatrix}$

4. Suppose that each of the following represents the transition matrix $\mathbf{M}$ and the initial probability vector $\mathbf{p}$ for a Markov chain. Find the probability vectors $\mathbf{p}_1$ and $\mathbf{p}_2$.

   (a) $\mathbf{M} = \begin{bmatrix} \frac{1}{4} & \frac{1}{3} \\ \frac{3}{4} & \frac{2}{3} \end{bmatrix}$, $\mathbf{p} = \begin{bmatrix} \frac{2}{3} \\ \frac{1}{3} \end{bmatrix}$

   (b) $\mathbf{M} = \begin{bmatrix} \frac{1}{2} & \frac{1}{3} & 0 \\ 0 & \frac{2}{3} & \frac{1}{2} \\ \frac{1}{2} & 0 & \frac{1}{2} \end{bmatrix}$, $\mathbf{p} = \begin{bmatrix} \frac{1}{3} \\ \frac{1}{6} \\ \frac{1}{2} \end{bmatrix}$

   (c) $\mathbf{M} = \begin{bmatrix} \frac{1}{4} & \frac{1}{3} & \frac{1}{2} \\ \frac{1}{2} & \frac{1}{3} & \frac{1}{6} \\ \frac{1}{4} & \frac{1}{3} & \frac{1}{3} \end{bmatrix}$, $\mathbf{p} = \begin{bmatrix} \frac{1}{4} \\ \frac{1}{2} \\ \frac{1}{4} \end{bmatrix}$

5. Suppose that citizens in a certain community tend to switch their votes among political parties, as shown in the following transition matrix. The ordering is Party A, Party B, Party C, and Nonvoting.

$$\begin{bmatrix} 0.7 & 0.2 & 0.2 & 0.1 \\ 0.1 & 0.6 & 0.1 & 0.1 \\ 0.1 & 0.2 & 0.6 & 0.1 \\ 0.1 & 0 & 0.1 & 0.7 \end{bmatrix}$$

   (a) Suppose that in the last election 30% of the citizens voted for Party A, 15% voted for Part B, and 45% voted for Party C. What is the likely outcome of the next election? What is the likely outcome of the election after that?

   (b) If current trends continue, what percentage of the citizens will vote for Party A one century from now? Party C?