EAS 596, Fall 2018, Homework 6
Due Weds. 10/10, **5 PM**, Box outside Furnas 611

Work all problems. Only MATLAB code will be accepted. Show all work, including any M-files you have written or adapted. Make sure your work is clear and readable - if the TA cannot read what you've written, he will not grade it. All electronic work (m-files, etc.) **must** be submitted through UBLearns and submitted by the due time. Any handwritten work may be submitted by the due time. Each problem will be graded according to the following scheme: 2 points if the solution is complete and correct, 1 point if the solution is incorrect or incomplete but was using correct ideas, and 0 points if using incorrect ideas.

1. Write MATLAB functions to compute a QR decomposition of a matrix A using 1. Classical Gram-Schmidt, 2. Modified Gram-Schmidt, 3. Householder Reflectors. Use each of your three MATLAB functions to compute a QR decomposition of the matrix

$$A = \begin{bmatrix} 1 & 2 & 3 \\ 4 & 5 & 6 \\ 7 & 8 & 7 \\ 4 & 2 & 3 \\ 4 & 2 & 2 \end{bmatrix}$$

   Compare your results.

2. This problem will study the loss of orthogonality that can occur using the Gram-Schmidt procedure for ill-conditioned matrices.

   (a) Write a MATLAB script that will compute the QR decomposition of the Hilbert matrix from sizes 2 all the way through 10 (use the `hilb` command in MATLAB) using your classical Gram-Schmidt, modified Gram-Schmidt, and Householder QR functions.

   (b) Compare your resulting Q matrices with the output from MATLAB's `qr` function. In particular, compute $\|Q^T Q - I\|$ for each method and comment on your results.

3. Consider the data in following table:

$$x = \begin{bmatrix} 0 & 1 & 2 & 3 & 4 & 5 & 6 \end{bmatrix}$$
$$y = \begin{bmatrix} -0.02 & 1.1 & 1.98 & 3.05 & 3.95 & 5.1 & 6.02 \end{bmatrix}$$

(a) Find a least-squares solution to fitting the data using a linear function $f(x) = a_0 + a_1 x$. Plot the data and the resulting regression line.

(b) Now fit the data using a quadratic polynomial $f(x) = a_0 + a_1 x + a_2 x^2$. Plot the data and the resulting regression line.

(c) Which of the two functions do you think is the more appropriate fit of the data?

4. Let $m = 50$ and $n = 12$. Let t be the m-vector corresponding to linearly spaced grid points from 0 to 1. Let A be the $m \times n$ matrix associated with least squares fitting of an $n-1$ order polynomial. Take b to be the vector of points $\cos(4t)$ evaluated on the grid. Compute the solution to the least squares problem using:

   (a) the formation and solution of the normal equations

   (b) QR factorization using your modified Gram-Schmidt function

   (c) QR factorization using your householder reflector function

   (d) QR factorization using MATLAB's `qr` function

   (e) MATLAB's `A\b` (this is based on QR factorization)

   Comment on each of the solutions and the differences you observe.