Chap-6

$~~f(x)~~ = \binom{n}{k} p^k (1-p)^{n-k}$

$P_S(k)$

↓

Occurence of a success

$E[S] = np$

$Var(S) = np(1-p)$

# of trials T upto the first success

$$p_T(t) = (1-p)^{t-1} p \qquad t = 1, 2, \ldots$$

$$E[T] = \frac{1}{p} \qquad var(T) = \frac{1-p}{p^2}$$

$$P(T-n = t \mid T > n) = (1-p)^{t-1} p = P(T=t)$$

$P(\text{Priority}) = p$

Condition for Bernouilli.

→ fixed prob.

→ independent everywhere

→ all events are indepent.

Properties of Bernouilli

→ Memoryless

→ Fresh start

→ Independence

$$\overbrace{B \; B \; B \; \cdots \; B}^{B} \; I$$

$$Z$$

shortcoming
→ discrete Domain

$P(\text{priority}) = p$ $\qquad$ $p(\text{Busy}) = p$

$P(\text{non priority}) = 1-p$ $\qquad$ $p(\text{idle}) = 1-p.$

$$P_T(k) = (p)^{t-1}(1-p) \qquad E[T] = \frac{1}{1-p} \qquad var(T) = \frac{p}{(1-p)^2}$$

$$P_I(k) = (1-p)^{t-1} p$$

Fresh start $(1-p)^2$

Y_k kth success

$$Y_k = T_1 + T_2 + \cdots T_k$$

$$E[Y_k] = E[T_1] + E[T_2] \cdots + E[T_k] = \frac{k}{p}$$

$$var(Y_k) = var(T_1) + var(T_2) \cdots var(T_k)$$
$$= \frac{k(1-p)}{p^2}$$

Pascal PMF:

$$P_{Y_k}(t) = \binom{t-1}{k-1} p^k (1-p)^{t-k}$$

6.5 $P(\text{foul}) = p$
$P(\text{no foul}) = 1-p$

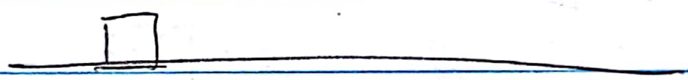
time = min { time for 6th foul, 30 mins }

$\Downarrow$

$\frac{4}{6}$

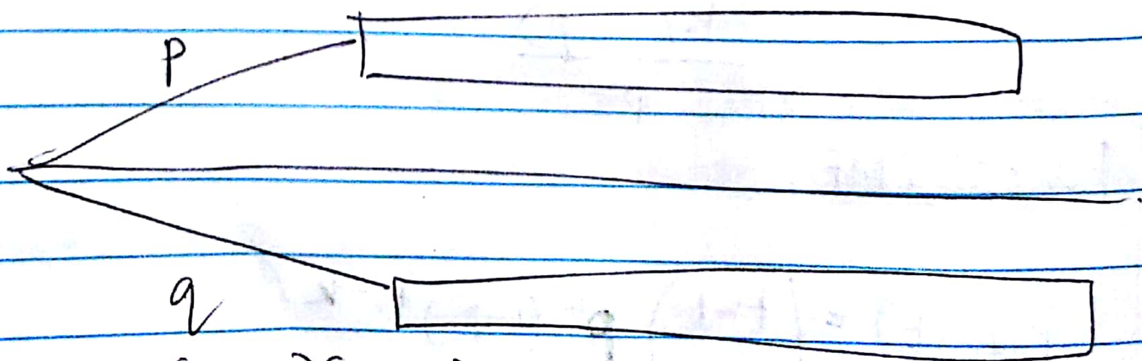
$$\binom{t-1}{5} (1-p)^{t-6} p^6$$

Splitting and Merging



$P(\text{keep ing}) = p$
$P(\text{splitting}) = (1-q)$

P(discarded arrival at each time slot)
$= p(1-q)$



$$1-(1-p)(1-q) = p+q-pq$$

0

Poisson Approximation to the Binomial.

$$\binom{n}{k} p^k (1-p)^{n-k} \qquad \frac{e^{-\lambda}\lambda^k}{k!}$$

The Poisson Process

$$t = \frac{1}{n}$$

$P(K, \tau)$ = P(there are exactly k arrivals during an interval of length τ)

λ = arrival rate or intensity.

$$P(k,\tau) = \begin{cases} 1 - \lambda\tau + o(\tau^2) & k = 0 \\ \lambda\tau + o_1(\tau^2) & k = 1 \\ 0 \quad o_k(\tau) & k > 0 \end{cases}$$

$k > 0 \Rightarrow$ no more than 1 occurrence happens per time stamp.

approximation errors.

Number of arrivals in an Interval

Number of periods = $n = \frac{\tau}{\delta}$

P(success in 1 period) = $p = \lambda\delta$

E[# of arrivals] = $np = \lambda\tau$

$= n\lambda\delta = n\tau$

$$P(k,\tau) = \frac{e^{-\lambda\tau}(\lambda\tau)^k}{k!} \quad k = 0, 1 \ldots$$

$E[N_\tau] = \lambda\tau$ $\quad var(N_\tau) =$

Erlangs PDF.

$$f_Y(y) = \frac{\lambda^k y^{k-1} e^{-\lambda y}}{(k-1)!} \qquad f_T(t) = \lambda e^{-\lambda t} \quad t \geq 0$$

$E[T] = \frac{1}{\lambda}$

$var(T) = \frac{1}{\lambda^2}$

when $k = 1$ $\quad \frac{\lambda\tau}{n} = p$.

	Poisson	Bernoulli
Time of Arrival	Continuous	Discrete
PMF of # of Arrivals	Poisson	Binomial
Interarrival Time CDF	Exponential	Geometric
Arrival Rate	λ/unit time	P/trial

1 Occurrence	Poisson λ	Bernoulli
n Occurrence	Poisson λT	Binomial n
Interarrival Time T_1	Exponential	Geometric
Y_n	$\frac{\lambda^k y^{k-1} e^{-\lambda y}}{(k-1)!}$ Erlang	$\binom{t-1}{k-1} p^k (1-p)^{t-k}$ pascal

8 9

$P(\text{fail}) = \frac{1}{4}$

$\binom{6}{2}\left(\frac{1}{4}\right)^2\left(\frac{3}{4}\right)^4$

$Y_k = T_1 + T_2 + T_3 =$

$E[Y_3] = \frac{3}{1/4} = 12$

3 fails

# of pass = 12 − 3.

$\binom{7}{1}\left(\frac{1}{4}\right)^1\left(\frac{3}{4}\right)^6 \times \frac{1}{4} \times \frac{1}{4}$

$Y_k = T_1 + T_2 \text{ --- --- } T_k$

$Y_k = T_1 \quad T_2 \quad T_3$

$\frac{1}{P} + \frac{1}{P} + \frac{1}{P}$

$\frac{3}{P} \approx \frac{k}{P}$

$= 12$

12 − 3 = 9