HW 4

CAS 595

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① $n = 40$, $\mu = 97$, $\sigma = 5$

Let W be the total estimated weight of the 40 boxes

$\therefore$ Using CLT, we need

$P(W < 3900)$

$$\therefore \quad z = \frac{3900 - 40\times 97}{5\sqrt{40}} = \frac{3900 - 3880}{10\sqrt{10}} = \frac{2}{\sqrt{10}} = 0.6325$$

$\therefore \phi(z) = 0.7357$

$\therefore P(W < 3900) = 0.7357$

② $\mu = 2.4$, $\sigma = 2$, $n = 100$

$\therefore P(t_{100} < 200)$

$$z_1 = \frac{200 - 100\times 2.4}{2\times\sqrt{100}} = \frac{200 - 240}{20} = -2$$

$\phi(z_1) = 0.0228$

$\therefore P(t_{100} < 200) = 0.0228$

$\therefore P(t_{100} > 200) = 0.9772$

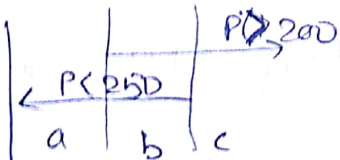
Now, calculate

$P(d_{100} < 250)$

$$z_2 = \frac{250 - 100 \times 2.4}{2\sqrt{100}} = \frac{250 - 240}{20} = 0.5$$

$\therefore \phi(z_2) = 0.6915$

$P(\cancel{S} d_{100}$



$\therefore P(200 < d_{100} < 250) = 0.9772 + 0.6915 - 1$
$= 0.6687$

$\therefore a + 2b + c = P(A) + P(B)$
$1 + b = P(A) + P(B)$

Now, let's calculate

$P(d_{100} > 400)$

$$\therefore z_3 = \frac{400 - 100 \times 2.4}{2\sqrt{100}} = \frac{160}{20} = 8 \simeq 0$$

$\therefore$ The expected value of my profit would be

$P(d_{100} < 200) \times 10{,}000 + P(200 < d_{100} < 250) \times 6000 - P(d_{100} > 400) \times 4000$

$\Rightarrow 0.0228 \times 10000 + 0.6687 \times 6000 - 0 \times 4000$

$\Rightarrow 228 + 4012.2 = \$4240.2$

(3) (a) $\mu = 0.8$, $\sigma = 0.16$, $n = 100$

$\therefore P(79 < a < 81) = P(a > 79) + P(a < 81) - 1$

<u>$P(a > 79)$</u>,

$z_1 = \frac{79 - 100 \times 0.8}{0.16\sqrt{100}} = \frac{79 - 80}{1.6} = -\frac{\cancel{10}}{\cancel{16}}\ \frac{5}{8}$

$\therefore \phi(z_1) = p(a < 79)$

$\therefore P(a > 79) = 1 - \phi(z) = 1 - 0.2676 = 0.7324$

<u>$P(a < 81)$</u>

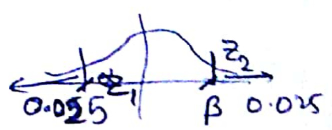
$z_2 = \frac{81 - 100 \times 0.8}{0.16\sqrt{100}} = \frac{81 - 80}{1.6} = \frac{1}{1.6} = \frac{5}{8}$

$\therefore \phi(z_2) = P(a < 81) = 0.7324$

$\therefore P(79 < a < 81) = 0.7324 + 0.7324 - 1 = 0.4648$

(b) We need to calculate α & β, such that

~~$P(2.5 < a < 97.5)$~~ $P(\alpha < a < \beta) = 0.95$



$\therefore \phi(z) = 0.025$

$\therefore z_1 = \frac{\alpha - 100 \times 0.8}{0.16 \times \sqrt{100}} = \cancel{0.025}\ 1.96$

$\Rightarrow$ ~~$\frac{\alpha - 80}{1.6} = 0.025$~~

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$\therefore$ ~~$x = 0.025 \times 1.6 + 80 = .0400$~~

$\Rightarrow \dfrac{x - 80}{1.6} = 1.96$

$\Rightarrow x = 1.96 \times 1.6 + 80 = 80 + 3.136 = 83.136$

Similarly, due to symmetry, we'll have

$80 - 3.136 = 76.864$

$\therefore \alpha = 76.864$ & $\beta = 83.136$

(4) weekly consumption $\mu = 50000$, $\sigma = 10000$, starting supply $= 74000$

Weekly delivery $= 47000$

(a) $P(G < 20000)$

$\therefore z = \dfrac{20000 - (74000 + [47000 - 50000] \times 11)}{10000 \times \sqrt{11}}$

$= \dfrac{20000 - (74000 - 33000)}{10000\sqrt{11}} = \dfrac{20000 - 41000}{10000\sqrt{11}}$

$= \dfrac{-2.1}{\sqrt{11}} = -0.63317$

$\therefore P(G < 20000) = 0.2643$

4b. $p(G < 20000) = 0.005$

$\therefore \phi(z) = 0.005$

$\therefore z = -2.57$

Let x be the weekly supply.

$$\therefore Z = \frac{20000 - (74000 + 11(x - 50000))}{\sqrt{11} \times 10000} = -2.57$$

$$\therefore -54000 - 11x + 550000 = -25700 \times \sqrt{11}$$

$$\Rightarrow x = \frac{550000 + 25700 \times \sqrt{11} - 54000}{11}$$

$\therefore x = 52839.7506$

$\therefore$

⑤ Let her current fortune be Y_0 at the start.

So, she invests $Y_0/2$ into stocks at day 1

Her expected fortune at the end of day 1 is :

$$Y_0/2 \times 1.7 \times \frac{1}{2} + \frac{Y_0}{2} \times 0.5 \times \frac{1}{2}$$

$$= \frac{Y_0}{2}\left(\frac{1.7+0.5}{2}\right) = \frac{Y_0}{2}\left(\frac{2.2}{2}\right) = 0.55 Y_0$$

$\therefore$ Her expected Net fortune after day 1 is

$$Y_1 = 0.55 Y_0 + Y_0/2 = 1.05 Y_0$$

So, on day 2, she invests $Y_1/2$ and at the end of the day her expected fortune is $1.05 Y_1 = (1.05)^2 Y_0$

Like wise, after n days her expected fortune will be $(1.05)^n Y_0$

$\therefore$ As $n \to \infty$, her fortune will tend towards infinite, a very large sum of money.

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