EAS 596, Fall 2018, Homework 2
Due Weds. 9/12, **5 PM**, Box outside Furnas 611

Work all problems. Show all work, including any M-files you have written or adapted. Make sure your work is clear and readable - if the TA cannot read what you've written, he will not grade it. All electronic work (m-files, etc.) **must** be submitted through UBLearns (and submitted by the time class starts on the day it's due). Any handwritten work may be submitted in class. Each problem will be graded according to the following scheme: 2 points if the solution is complete and correct, 1 point if the solution is incorrect or incomplete but was using correct ideas, and 0 points if using incorrect ideas.

1. Let $\mathcal{V}$ be a vector space. For every vector $\boldsymbol{v}$ in $\mathcal{V}$ and every real number a, we have

   (a) $a\mathbf{0} = \mathbf{0}$

   (b) $0\boldsymbol{v} = \mathbf{0}$

   (c) $(-1)\boldsymbol{v} = -\boldsymbol{v}$

   (d) If $a\boldsymbol{v} = \mathbf{0}$ then either $a = 0$ or $\boldsymbol{v} = \mathbf{0}$

   Parts (a) and (d) were proven in class. In this homework, prove parts (b) and (c).

2. Prove that the set $\mathcal{P}_3$ of all third order polynomials with real coefficients forms a vector space under the usual operations of polynomial addition and scalar multiplication.

3. Which of the following subsets of $\mathbb{R}^3$ are subspaces?

   (a) All vectors of the form $[a, b, a]$.

   (b) All linear combinations of $[1, 4, 0]$ and $[2, 2, 2]$

   (c) All vectors of the form $[a, b, c]$ where $a \leq b \leq c$.

4. What is the smallest set that will span all 3×3 upper triangular matrices?

5. Let the following vectors be defined.

$$\boldsymbol{a} = \begin{bmatrix} 4 \\ 1 \\ 5 \end{bmatrix}, \quad \boldsymbol{b} = \begin{bmatrix} 3 \\ 4 \\ 5 \\ -5 \\ 4 \end{bmatrix}, \quad \boldsymbol{c} = \begin{bmatrix} -4 \\ 6 \\ -13 \\ 8 \\ -19 \end{bmatrix}, \quad \boldsymbol{d} = \begin{bmatrix} -8 \\ -7 \\ -5 \end{bmatrix}, \quad \boldsymbol{e} = \begin{bmatrix} 16 \\ 9 \\ 15 \end{bmatrix}$$

Answer the following questions regarding these vectors:

(a) $\text{span}(\{\boldsymbol{d}, \boldsymbol{e}\})$ is a subset of what vector space?

(b) Are the vectors $\boldsymbol{b}$ and $\boldsymbol{c}$ sufficient to span all of $\mathbb{R}^5$?

(c) Are the vectors $\boldsymbol{a}$, $\boldsymbol{d}$, $\boldsymbol{e}$ sufficient to span all of $\mathbb{R}^3$?

(d) Write three vectors which span a subspace of $\mathbb{R}^4$