EAS 596, Fall 2018, Homework 7

Due Weds. 10/17, **5 PM**, Box outside Furnas 611

Work all problems. Only MATLAB code will be accepted. Show all work, including any M-files you have written or adapted. Make sure your work is clear and readable - if the TA cannot read what you've written, he will not grade it. All electronic work (m-files, etc.) **must** be submitted through UBLearns and submitted by the due time. Any handwritten work may be submitted by the due time. Each problem will be graded according to the following scheme: 2 points if the solution is complete and correct, 1 point if the solution is incorrect or incomplete but was using correct ideas, and 0 points if using incorrect ideas.

1. (a) Determine the eigenvalues and eigenvectors of the $n \times n$ identity matrix.

   (b) Let λ and $\mathbf{x}$ be the eigenvalue and eigenvector of $\mathbf{A}$: $\mathbf{Ax} = \lambda\mathbf{x}$. Show that the eigenvalue of $\mathbf{A} + \mathbf{I}$ is $\lambda + 1$ while the eigenvector is unchanged.

2. Let $\mathbf{P} = \begin{bmatrix} 0.6 & 0.2 \\ 0.4 & 0.8 \end{bmatrix}$ be the transition matrix for a Markov-Chain process. Using the eigendecomposition of $\mathbf{P}$, determine the limit of $\mathbf{P}^n$ as $n \to \infty$. You *must* compute the eigendecomposition by hand, but you can check it using a computer. You do not need to include any code you use to check the eigendecomposition.

3. Let $\mathbf{A}$ be diagonalizable with one eigenvalue of $\lambda = 2$ and eigenvector of $\mathbf{x}_1 = [1;0]^T$ and another eigenvalue of $\lambda_2 = 5$ and $\mathbf{x}_2 = [1;1]^T$. Determine $\mathbf{A}$.

4. (a) Determine all eigenvalue and eigenvectors of the matrix $\mathbf{A} = \begin{bmatrix} 5 & -4 & 0 \\ 1 & 0 & 2 \\ 0 & 2 & 5 \end{bmatrix}$.

   (b) Is it possible diagonalize matrix $\mathbf{A}$? Why or why not?