Abhishek Kumar

EAS 596 Take Home Part.

Prof. David Salac.

1.b

From the question we have

$$y' = \begin{cases} y\left(-2t + \frac{1}{t}\right) & , \ t \neq 0 \\ 1 & , \ t = 0 \end{cases}$$

For $t \neq 0$, we have

$$y' = y\left(-2t + \frac{1}{t}\right)$$

$$\Rightarrow \frac{y'}{y} = \left(-2t + \frac{1}{t}\right)$$

$$\Rightarrow \int \frac{y'}{y} = \int \left(-2t + \frac{1}{t}\right) dt$$

$$\Rightarrow \ln y = -t^2 + \ln t + C$$

$$\Rightarrow \ln y - \ln t + t^2 = C$$

$$\Rightarrow \ln y/t = -t^2 + c$$

$$\Rightarrow y = t e^{-t^2 + c}$$

Again, on differentiating both sides, we get

$$\frac{dy}{dt} = t\left((-2t)\, e^{-t^2+c}\right) + e^{-t^2+c}$$

$$\dot{y} = (1-2t^2)e^{-t^2+c}$$

Putting initial condition $\dot{y}(0) = 1$ and solving

$$\dot{y}(0) = 1 \Rightarrow (1-0)(e^{-0+c}) = 1$$

$$\Rightarrow e^{c} = 1 \Rightarrow c = 0$$

$\therefore$ We have

$$\dot{y} = (1-2t^2)e^{-t^2}$$

and

$y = t e^{-t^2}$ (last page)

1.(c) Using $y = te^{-t^2}$ from part (b), we get

$$y = te^{-t^2}$$

$$y(1) = 1 \cdot e^{-1} \quad \cancel{= 0.09}$$

$$= 0.3678$$

2.(a).

The position of point p is given by

$$x = L_1 \cos\theta_1 + L_2 \cos\theta_2$$
$$h = L_1 \sin\theta_1 + L_2 \sin\theta_2$$

As θ_1, L_1, L_2 and h are given, the only unknown we have is θ_2 and x. To convert it into a root problem let's form a system of equations and get $f(x, \theta_2) = 0$

The system of non-linear equations we get are:

$$h = L_1 \sin\theta_1 + L_2 \sin\theta_2$$

$$\sin\theta_2 = \frac{h - L_1 \sin\theta_1}{L_2}$$

$$\therefore \quad \theta_2 = \sin^{-1}\left(\frac{h - L_1 \sin\theta_1}{L_2}\right)$$

and the other equation is:

$$x = L_1 \cos\theta_1 + L_2 \cos\theta_2$$

Now our next problem is :

$$L_1 \cos\theta_1 + L_2 \cos\theta_2 - x = 0$$

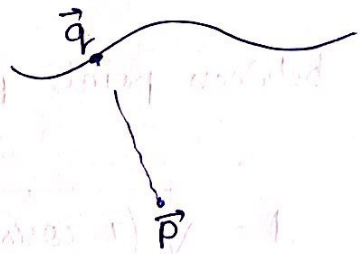
$$\sin^{-1}\left(\frac{h - L_1 \sin\theta_1}{L_2}\right) - \theta_2 = 0$$

After solving these equations, we'll get θ_2 & x,

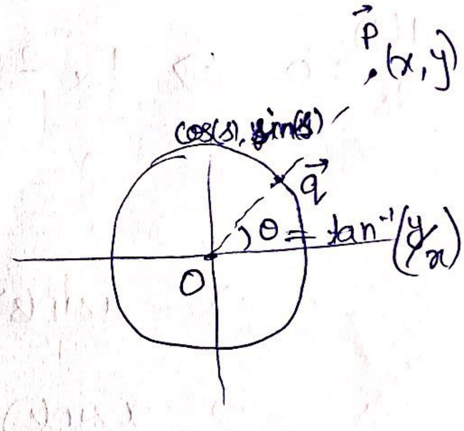
$$x = L_1 \cos\theta_1 + L_2 \cos\left(\sin^{-1}\left(\frac{h - L_1 \sin\theta_1}{L_2}\right)\right)$$

Q3.

(a) The function which is at a minimum when $\vec{q}$ is the closest to point $\vec{p}$ is $|\vec{q} - \vec{p}|$



(b) We know that point $\vec{q}$ lies on the circumference of the circle with radius 1 so its magnitude will be 1.



Also, its very easy to infer from the diagram that point $\vec{p}$, $\vec{q}$ and center O of the circle will be ~~collline~~ co-linear, hence the angle that point $\vec{q}$ makes with x-axis can be given as

$\theta = \tan^{-1}\left(\frac{y}{x}\right)$, (x,y) is point $\vec{P}$.

Therefore point $\vec{q}$ can be given as:

$$(1 \times \cos\theta,\ 1 \times \sin\theta) \equiv \left(\frac{x}{\sqrt{x^2+y^2}},\ \frac{y}{\sqrt{x^2+y^2}}\right)$$

(c) To show the result of (b), we can write the distance between points $\vec{p}$ & $\vec{q}$ as:

$$d = \sqrt{(x-\cos(\delta))^2 + (y-\sin(\delta))^2}$$

To minimize d, we set:

$$\frac{d(d)}{d\delta} = 0 \Rightarrow \frac{1}{2} \times \frac{2\left((x-\cos(\delta)).\sin(\delta) + 2(y-\sin(\delta))(-\cos(\delta))\right)}{\sqrt{(x-\cos(\delta))^2 + (y-\sin(\delta))^2}} = 0$$

$$\Rightarrow x\sin(\delta) - \sin(\delta)\cos(\delta) - y\cos(\delta) + \sin(\delta)\cos(\delta) = 0$$

$$\Rightarrow x\sin(\delta) = y\cos(\delta)$$

$$\Rightarrow \tan(\delta) = y/x$$

or $\delta = \tan^{-1}(y/x)$

Here we see that point $\vec{q}$ on the interface is at minimum distance from point $\vec{p}$ when $\delta = \tan^{-1}(y/x)$ which is consistent with the angle θ we had assumed in part (b).

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