EAS595 HW6

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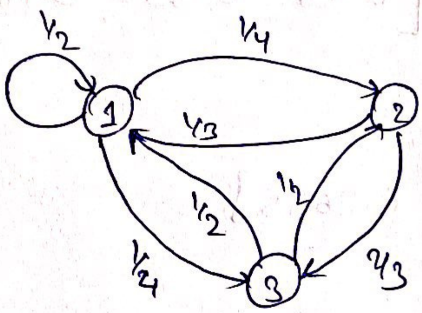
Extra Prob 1.

$$A = \begin{bmatrix} 0.25 \\ 0.25 \\ 0.5 \end{bmatrix} \qquad P = \begin{bmatrix} 1/2 & 1/4 & 1/4 \\ 1/3 & 0 & 2/3 \\ 1/2 & 1/2 & 0 \end{bmatrix}$$

State transition diagram.



(a) $P(X1=3, X2=2, X3=1) = P(X1=3) \cdot P(3\to2) \cdot P(2\to1)$

$$P(X1=3) = \begin{bmatrix} 0.25 & 0.25 & 0.5 \end{bmatrix} \begin{bmatrix} 1/2 & 1/4 & 1/4 \\ 1/3 & 0 & 2/3 \\ 1/2 & 1/2 & 0 \end{bmatrix} = \begin{bmatrix} 0.458 & 0.3125 & 0.229 \end{bmatrix}$$

$\therefore P(X1=3) = 0.229$

$= P(X1=3, X2=2, X3=1) = 0.229 \times 0.25 \times \frac{1}{3} = 0.038$

$P(X3=2)$

$$X3 = AP^3 = \begin{bmatrix} 0.25 & 0.25 & 0.5 \end{bmatrix} \begin{bmatrix} 1/2 & 1/4 & 1/4 \\ 1/3 & 0 & 2/3 \\ 1/2 & 1/2 & 0 \end{bmatrix}^3$$

$$= \begin{bmatrix} 0.25 & 0.25 & 0.5 \end{bmatrix} \begin{bmatrix} 0.458 & 0.26 & 0.28 \\ 0.43 & 0.167 & 0.40 \\ 0.48 & 0.33 & 0.187 \end{bmatrix}$$

$$= \begin{bmatrix} 0.462 & 0.272 & 0.2635 \end{bmatrix}$$

$\therefore P(X3=2) = 0.272$

At steady state

$AP = A$, let $A = [a \; b \; c]$

$$\Rightarrow [a \; b \; c] \begin{bmatrix} 1/2 & 1/4 & 1/4 \\ 1/3 & 0 & 2/3 \\ 1/2 & 1/2 & 0 \end{bmatrix} = [a \; b \; c]$$

$\Rightarrow \frac{a}{2} + \frac{b}{3} + \frac{c}{2} = a \quad \Rightarrow 3a + 2b + 3c = 6a \quad \Rightarrow 3a = 2b + 3c$ (1)

$\Rightarrow \frac{a}{4} + \frac{2b}{3} = b \quad \Rightarrow 3a + 8c = 12b$ — (2)

$\Rightarrow \frac{a}{4} + \frac{c}{2}$

$$\Rightarrow \frac{a}{4} + \frac{c}{2} = b \quad \Rightarrow a + 2c = 4b \quad - (2)$$

$$\Rightarrow \frac{a}{4} + \frac{2b}{3} = c \Rightarrow 3a + 8b = 12c \quad - (3)$$

from (1) & (3)

$$(2b + 3c) + 8b = 12c$$

$$\Rightarrow 10b = 9c \quad \Rightarrow b = \frac{9}{10}c$$

$$a + 2c = \frac{4 \times 9}{10}c \Rightarrow a + 2c = 3.6c \Rightarrow a = 1.6c$$

We know that

$$a + b + c = 1$$

$$\Rightarrow 1.6c + 0.9c + c = 1 \quad \Rightarrow 3.5c = 1 \quad \Rightarrow c = \frac{1}{3.5}$$

$\therefore$ Steady state prob = $[0.457 \quad 0.257 \quad 0.286]$

Extra Problem 2.

The mean 1st passage times t_i to reach state 1 starting from i, are given by

$$t_1 = 0, \quad t_i = 1 + \sum_{j \neq 1}^{3} P_{ij} t_j \qquad \text{for all } j \neq 1$$

$$\therefore t_2 = 1 + \cancel{\tfrac{1}{3} t_1} + \tfrac{2}{3} t_3 \Rightarrow t_2 = 1 + \tfrac{2}{3} t_3$$

$$t_3 = 1 + \cancel{\tfrac{1}{2} t_1} + \tfrac{1}{2} t_3 \Rightarrow t_3 = 2, \; t_2 = \tfrac{7}{3}$$

$E[R \mid X_0 = 1]$ is the mean return time to state 1.

It is given by

$$t_1^* = 1 + P_{11} t_1 + P_{12} t_2 + P_{13} t_3$$

$$= 1 + \tfrac{1}{2} \times 0 + \tfrac{1}{4} \times \tfrac{7}{3} + \tfrac{1}{4} \times 2 = 1 + \tfrac{7}{6} + \tfrac{1}{2} = \tfrac{16}{6} = \tfrac{8}{3}$$