Sinhgad Institutes

Engineering Mechanics
Unit 4

Syllabus

- **Unit IV Analysis of Structures (06 Hrs)**

A) Two force member, Analysis of plane trusses by Method of joints, Analysis of plane trusses by method of section, Analysis of plane frames, Cables subjected to point load multi force member.

Sub-topics of Unit 4

Applications of General Force System

1. Plane Trusses

    a) Method of Joints

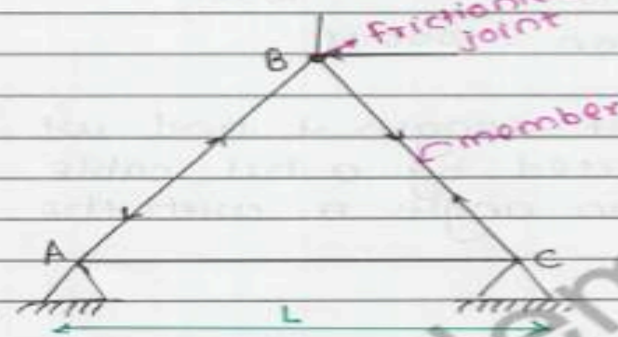
    b) Method of sections

2. Plane Frames

3. Cables subjected to point loads.

* Equ^m of truss :-

Def^n: • Truss is a rigid structure composed of a straight slender member and connected by means of frictionless pins at a jt to form a pattern of Δ.



* Assumptions:

• All the members are straight and having negligible c/s

• All the members are connected at the jt through pin connections which are frictionless.

• The external force must be applied at the jts

• Couple moment which produces bending do not act on member of the truss

• All members have only axial force members (no transverse force)

A truss may be defined as a system of uniform members joined together at their ends by riveting etc. The truss is formed from *two-force members, i.e., members of truss are straight members* with end point connections, And loads are applied only at joints.

Classification

Types of Trusses: Trusses may be of following types:

a) **Plane Truss or Space Truss:** In plane truss all the members lie in the same plane and the forces act along the plane of the truss, e.g. Bridge Trusses and roof trusses. In Space truss all the members do not lie in the same plane, e.g. suspension tower, Tripod etc.

b) **Statically Determinate and Statically Indeterminate Truss:** The force analysis can be done by equation of static in the case of S.D.F. Only static equations are not sufficient there is need of considering their deformation also in case of SIF.

c) **Perfect or Rigid Truss:** This is non-collapsible when external supports are removed. A perfect truss is the one, which contains such number of members as are just sufficient to prevent distortion of its shape when loaded externally. A perfect truss should satisfy the equation, $m = 2j - 3$, where m = no. of members, j = no. of joints.

Plane Trusses - Sub- Unit 1

Plane Truss: Structure made of two force members connected by pins at the ends.

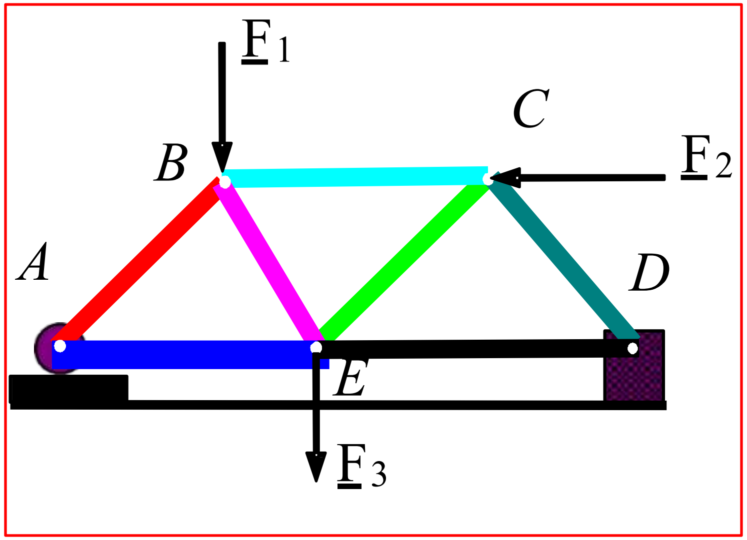


Fig.5 Plane Truss

Simple Truss: If $m = 2j - 3$

where, m = no. of members,

j = no. of joints.

Deficient Truss : if $m < 2j - 3$

Redundant Truss: if $m > 2j - 3$

Nature of Force in the member

Fig.6a Compressive Force

Fig.6b Tensile Force

(i) **Deficient or Collapsible Truss:** A truss that contains less members than required to be just rigid and is collapsible, is known as Deficient truss. It cannot resist distortion/ retain its shape under external load. $m < 2j - 3$.

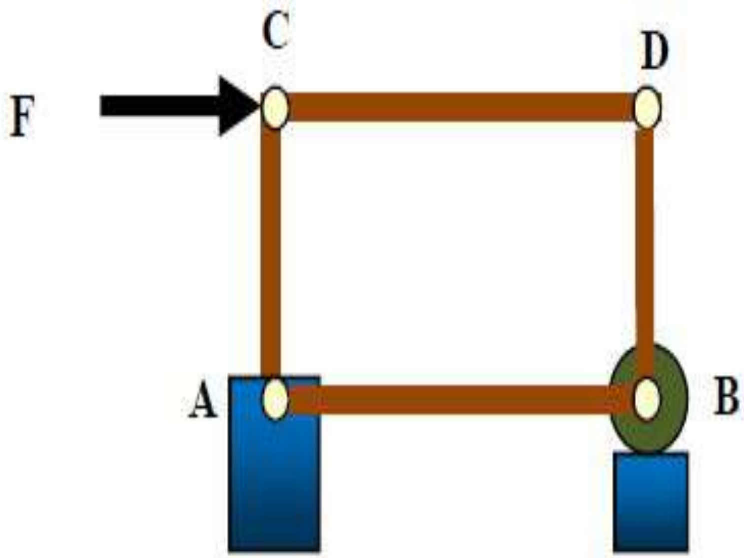


Figure: Deficient / Collapsible Truss

(ii) Redundant or Over Rigid Truss: A truss that contains more members than required to be just rigid, is known as redundant or over rigid truss. $m > 2j - 3$, since number of unknowns $(m+3)$ are more than number of equations $(2j)$ hence such a truss is statically indeterminate.

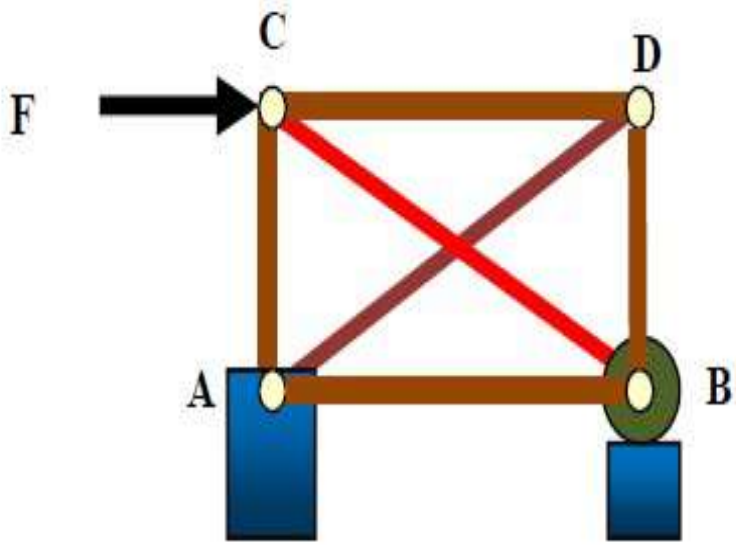


Figure: Redundant / over rigid Truss

Basic Assumptions of Truss Analysis:

Following assumptions are made while making such analysis:

1. All members of truss are pin joined. The forces are transmitted from one member to another through smooth pins (no friction).
2. The truss is a perfect one and statically determinate.
3. All the members are straight, rigid, slender, and uniform in cross section and lie in same plane.
4. The external loads and reactions are acting at Joints only.
5. The self-weight of members is neglected.
6. Every joint is treated separately as a free body in equilibrium, i.e., the sum of all the vertical forces as well as the horizontal forces acting on the joint is equated to zero.

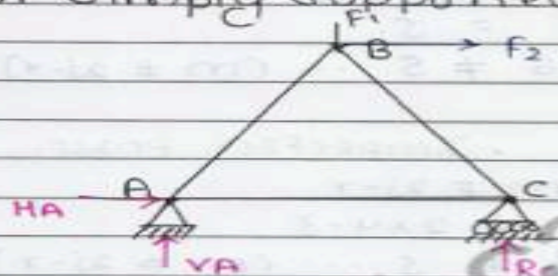
$\Sigma Fx = 0$ and $\Sigma FY = 0$

- To check the stability of truss use the following relation.

$$\boxed{m = 2j - r}$$

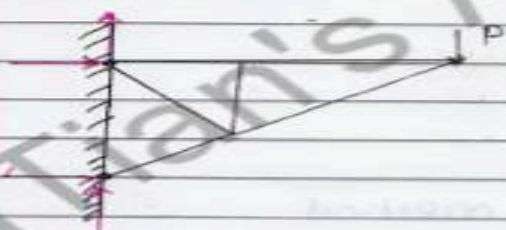
where, m = no of members.
j = no of joints.
r = external support reactions.

★ For simply supported truss.



$m = 2j - 3$

★ For cantilever truss:



$m = 2j - 4$

★ Classification of truss:

Truss. $(m = 2j - r)$

- perfect truss $(m = 2j - r)$ (stable)
- Imperfect truss (unstable) $(m \neq 2j - r)$
  - Redundant truss $(m > 2j - r)$
  - Deficient truss $(m < 2j - r)$

* Trusses :-

- Frames
  - Rigid jointed
    - 2D plane
      - 2 equilibrium
      - $\Sigma F_x = 0$
      - $\Sigma F_y = 0$
      - $\Sigma M = 0$
    - 3D space
      - 6 equilibrium
      - $\Sigma F_x = 0$
      - $\Sigma F_y = 0$
      - $\Sigma F_z = 0$
      - $\Sigma M_x = 0$
      - $\Sigma M_y = 0$
      - $\Sigma M_z = 0$
  - Pin-jointed (Truss)
    - 2D Truss
      - 2D plane
      - 2 equilibrium equation
      - $\Sigma F_x = 0$
      - $\Sigma F_y = 0$
      - It is coplaner concurrent forces system
      - Ex. Roof truss, Bridge truss
    - 3D space
      - 3 eqm eqn.
      - $\Sigma F_x = 0$
      - $\Sigma F_y = 0$
      - ~~ΣM~~
      - $\Sigma F_z = 0$
      - It is non-coplaner concurrent force system
      - Ex: Observation Tower, Transmission tower.

Note:- Pin jointed frame are

1) Madeup of slender member
2) Pin connected @ ends
3) capable of taking load @ joints.

Uses:- supporting sloping roofs, bridge decks, arms of machineries (boom of crane)

* Classification of truss based upon static indeterminacy

$Ds = R - E$

$Ds = (m+3) - 2j$

m = member

3 = reactions i.e. no. of eqm condn

j = joints

condition			Indeterminate truss
+ve	$R > E$ $(m+3) > 2j$	—	Redundant (over stable)
−ve	$R < E$ $(m+3) < 2j$	—	Deficient truss (unstable)
zero	$R = E$ $(m+3) = 2j$ economical	—	Perfect truss (Determinate) Actual truss provided.

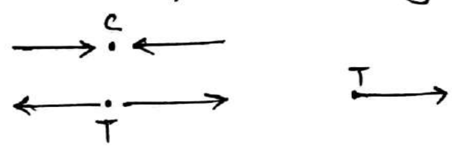
* **Assumptions :—**

1) Truss is always loaded at joints
2) Truss member always carry axial loads

* **Assumptions in truss analysis :—**

1) Ends are pinned connected
2) Load acts as jointed only ie, member carry axial load only
3) Self wt of member is neglected
4) Member have uniform cross-section throughout

Note:- In case of truss analysis we mark force on joints



* Method of Analysis :—

1) Method of joints
2) Method of section
3) Method of tension coeff.
4) Graphical method.

Ex:— Williot - Mohr's method — useful for finding displacement of trusses.

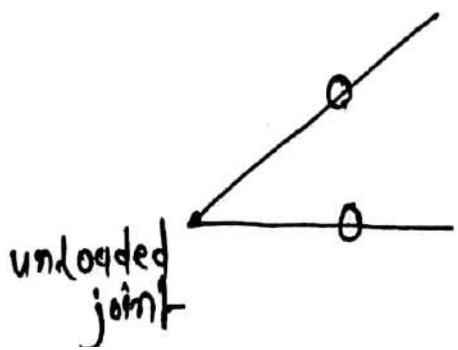
* Method of joints :—

- At every joint forces in member constitute a system of concurrent forces
- Truss always concurrent - coplaner force system.

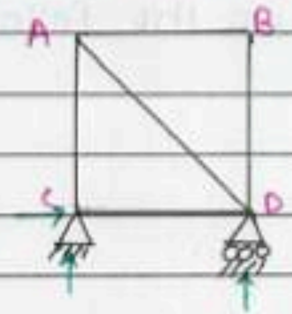
  Coplaner - 2 (F_x, F_y) , concurrent = 3
- Usually we start analysis from joint having min^m no. of unknown
- Equilibrium cod^n findout $\Sigma F_x, \Sigma F_y, \Sigma M$ ie, supports reactions of overall equilibrium of frame.

* Observation of zero force member :-

1) IF two non-colinear member meet @ an unloaded joint then both member are zero force members.



1)



• Perfect truss

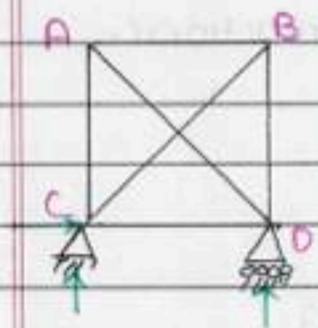
$R.H.S = 2j - r$

$= 2 \times 4 - 3$

$= 5$

$\therefore L.H.S = R.H.S \cdots (m = 2j - r)$

2)



• Imperfect truss.

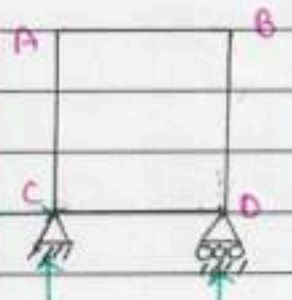
$m \neq 2j - r$

$= 2 \times 4 - 3$

$= 5.$

$6 \neq 5 \cdots [m \neq 2j - r]$

3)



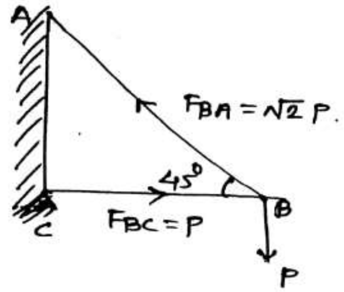
• Imperfect truss.

$m \neq 2j - r.$

$\neq 2 \times 4 - 3$

$4 \neq 5 \cdots (m \neq 2j - r]$

Que 1>



$\rightarrow$ $\Sigma F_x = 0$

$F_{BC} = F_{BA} \cos 45^0$

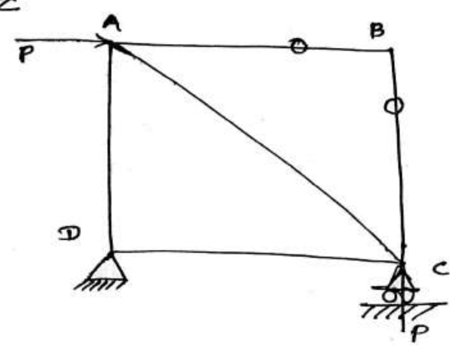
$\boxed{F_{BC} = P}$

$\Sigma F_y = 0$

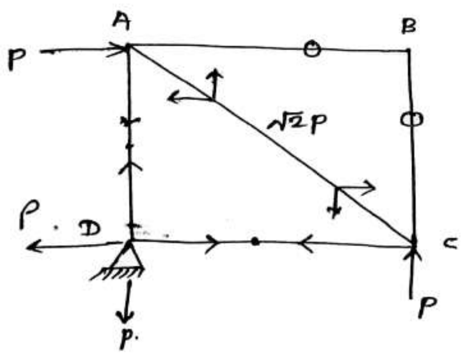
$P = F_{BA} \sin 45$

$F_{BA} = \cancel{\frac{P}{\sqrt{2}}}\ \sqrt{2}P$.

Que 2>



$\rightarrow$



Tension member	compression member
AD	AC
DC	

Que: for the given truss there are how many zero force membes

Ans – Two

Analysis of Simple Trusses

It is the process of finding;

a) Support reactions, and

b) Induced member forces.

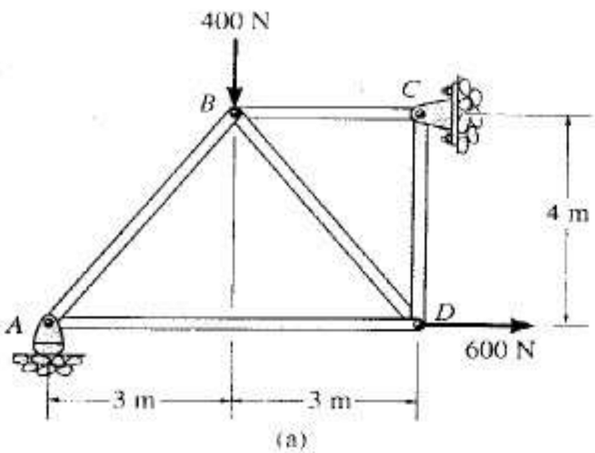
   i. Method of Joints

   ii. Method of Sections

Method of joints: The following procedure is used for analysis of trusses

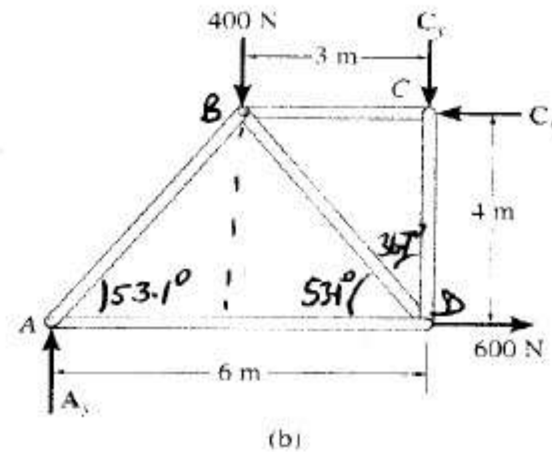
i. Check that truss is a perfect truss ($m = 2j - 3$)
ii. Consider the free body diagram of entire truss and compute the support reactions using the equations of equilibrium ($R_x = 0$, $R_y = 0$, $M_A = 0$). Determination of support reaction may not be necessary in case of cantilever type of truss.
iii. Assume and mark directions of the axial forces in the members away from the joint on the diagram.
iv. Consider equilibrium of each joint independently and calculate magnitude of axial forces in members. Conditions of equilibrium are $Rx = 0$, $R_y = 0$. Hence at a time only two unknown forces can be determined. Therefore start from a joint at which not more than 2 unknown forces appear.
v. If the magnitude of the force comes out to be negative, the nature of force in that member is compressive and if it is positive than nature of force in that member is tensile.
vi. If the force is pushing the joint, it is compressive and if it is pulling the joint, it is of tensile nature.

Example 1: Determine the force in each member of the truss shown. Indicate whether the mem
are in tension or compression.



(a)

Space diagram



(b)

Free-body Diagram.

External Forces Determination

$\sum$ Fx = 0 i.e. 600 N - Cx = 0 , Cx = 600 N

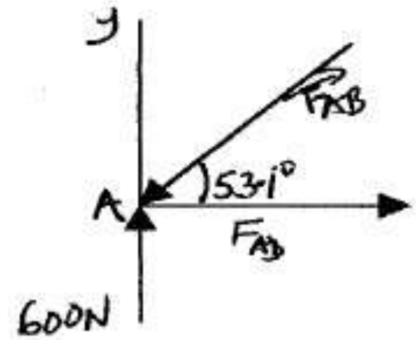
$\sum$ Mc = 0 i.e. - Ay (6 m) + 400 N (3 m) + 600 N (4 m) = 0

6 Ay = 1200 + 2400 = 3600 N, Ay = 600 N

$\sum$ Fy = 0 i.e. 600 N - 400 N - Cy = 0 , Cy = 200 N

Choose a joint where there are no more than two unknowns. Start from A or

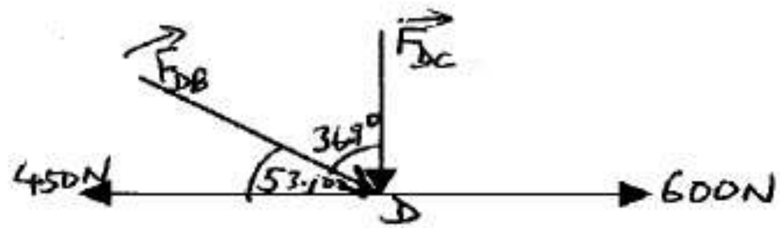
Joint A



$\sum F_y = 0$ i.e. $600\ N = F_{AB} \sin 53.1°$; **$F_{AB}$ = 750 N (C)**

$\sum F_x = 0$ i.e. $F_{AD} = F_{AB} \cos 53.1° = 750 \cos 53.1° =$ **450 N (T)**

Joint D



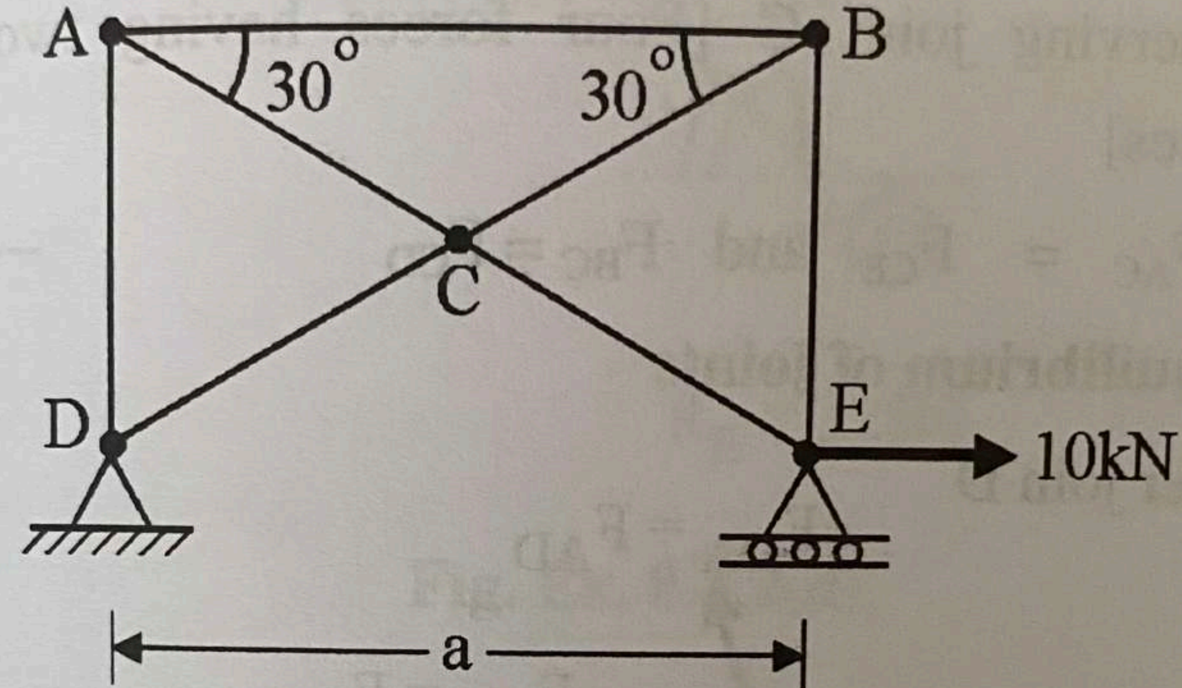
$\sum F_x = 0$ i.e. $-450 + F_{DB} \cos 53.1° + 600 = 0$

$0.6\ F_{DB} = -150\ N$ i.e. $F_{DB} = -250\ N$

The negative sign indicates that F_{DB} acts in the opposite sense to that shown:

Hence **F_{DB} = 250 N (T)**

Find the forces in all the members of the truss shown in Fig. Ex. 8.3.1.



TYPE I : EXAMPLES BASED ON METHOD OF JOINTS

Ex. 8.3.1

Find the forces in all the members of the truss shown in Fig. Ex. 8.3.1.

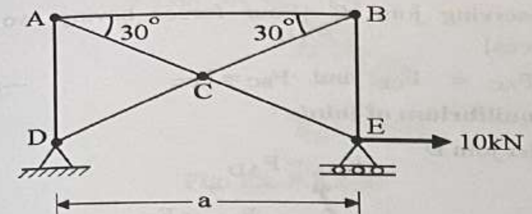


Fig. Ex. 8.3.1

☑ **Soln. :**

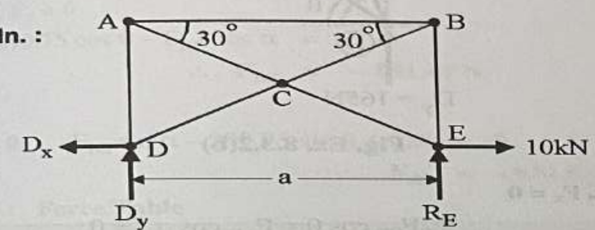


Fig. Ex. 8.3.1(a)

Step 1 : Stability check

Number of joints $J = 5$

Number of members $n = 7, \quad R = 3$

$2j - R = 2(5) - 3 = 7 = n$

$\therefore \; n = 2j - R$ so truss is perfect

Step 2 : F.B.D. of truss and apply conditions of equilibrium.

$\Sigma F_x = 0 \qquad -D_x + 10 = 0$

$\therefore \; D_x = 10 \text{ kN} \leftarrow$ ...(1)

$\Sigma F_y = 0 \qquad D_y + R_E = 0$

$\Sigma M_D = 0 \qquad R_E \cdot a = 0$

$\therefore \; R_E = 0$

$\therefore \; D_y = 0$

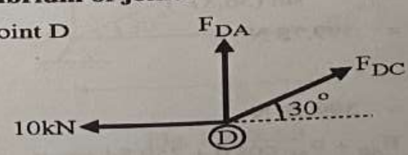
Step 3 : Forces by observation

By observing joint C [Four forces having two pairs of collinear forces] --- result (1)

$\therefore \; F_{AC} = F_{CE}$ and $F_{CD} = F_{BC}$

Step 4 : Equilibrium of joints

a) Consider joint D



Apply, $\Sigma F_x = 0$

$F_{DC} \cos 30° - 10 = 0$

$\therefore \; \mathbf{F_{DC} = 11.55 \text{ kN}}$

$\therefore$ by result (1) $\mathbf{F_{BC} = 11.55 \text{ kN}}$

$\Sigma F_y = 0 \qquad F_{DC} \cdot \sin 30° + F_{DA} = 0$

$11.55 \sin 30° + F_{DA} = 0$

$\therefore \; \mathbf{F_{DA} = -5.775 \text{ kN}}$

b) Consider joint A

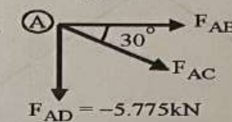


Fig. Ex. 8.3.1(c)

Apply, $\Sigma F_y = 0$

$-F_{AC} \cdot \sin 30° + 5.775 = 0$

$\therefore \; \mathbf{F_{AC} = 11.55 \text{ kN}}$

$\therefore$ by result (1) $\mathbf{F_{CE} = 11.55 \text{kN}}$

$\Sigma F_x = 0 \qquad F_{AB} + F_{AC} \cdot \cos 30° = 0$

$\therefore \; F_{AB} + 11.55 \cos 30° = 0$

$\therefore \; \mathbf{F_{AB} = -10 \text{ kN}}$

c) Consider joint B

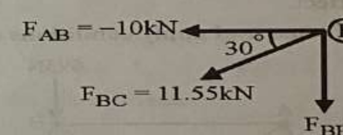


Fig. Ex. 8.3.1(d)

Apply, $\Sigma F_y = 0$

$-F_{BE} - 11.55 \sin 30° = 0$

$\therefore \; \mathbf{F_{BE} = -5.775 \text{ kN}}$

Step 5 : Force table

Sr. No.	Member	Magnitude (kN)	Nature
1	AB	10	C
2	AD	5.775	C
3	AC	11.55	T
4	BE	5.775	C
5	BC	11.55	T
6	CD	11.55	T
7	CE	11.55	T

Method of Sections:

In this method, the equilibrium of a portion of truss is considered which is obtained by cutting the truss by some imaginary section

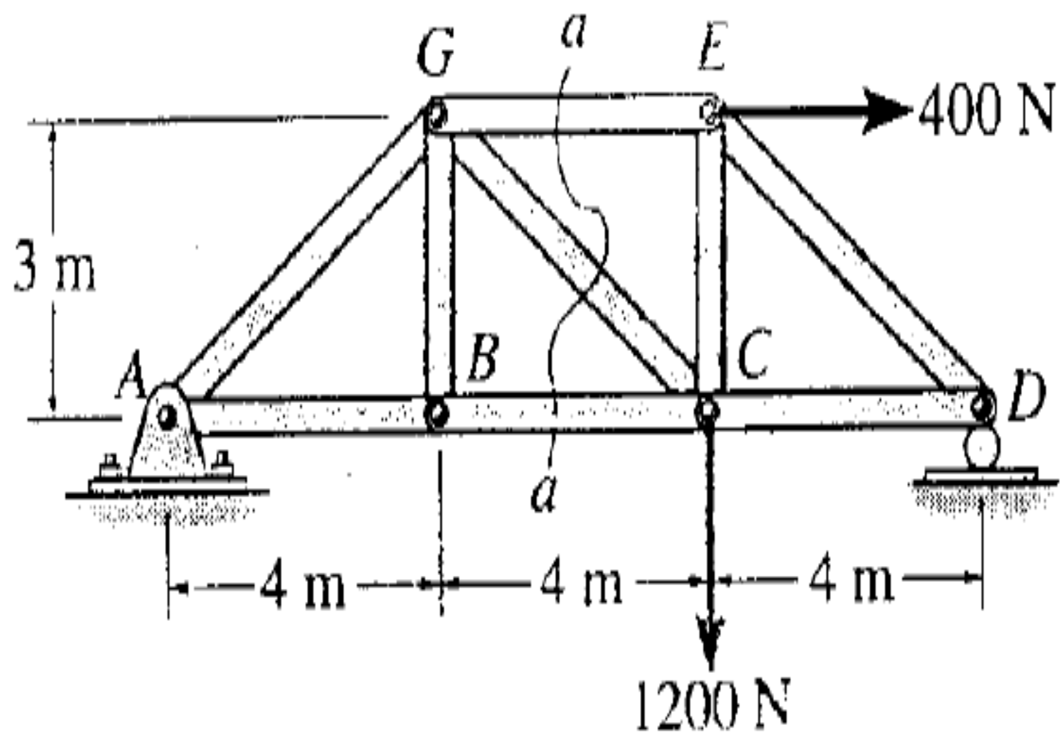
Points to be considered:

- The section should pass through the members and not through joints.
- A section should divide the truss into two clearly separate and unconnected portions.
- A section should cut only three members since only three unknowns can be determined. (In special cases, a section may cut more than three members.)
- When using moment equations, the moment can be taken about any convenient point, which may or may not lie on the section under consideration.

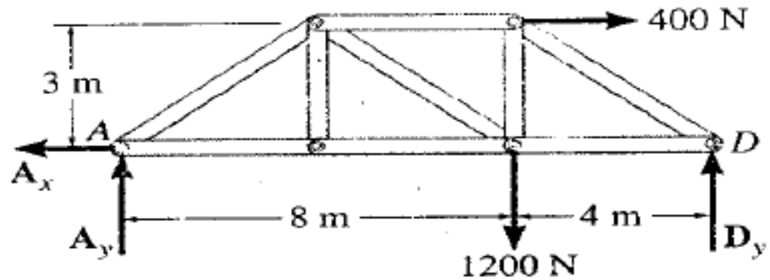
The following steps are carried out to find forces in members through method of sections.

i. Check that truss is a perfect truss ($m = 2j - 3$)
ii. Consider the entire truss as a free body and determine reactions at the supports.
iii. Cut the truss into two separate portions by passing an imaginary section through those members in which forces are to be determined. (More than one such sections are possible)
iv. The internal forces in these members become external forces acting on the two portions of the truss.
v. Assume and mark directions of the axial forces in the cut members away from the joint on the diagram.
vi. Consider equilibrium of one portion of the truss and calculate magnitude of three unknown axial forces in members using equations of equilibrium $Rx = 0$, $R_y = 0$, $M_A = 0$.

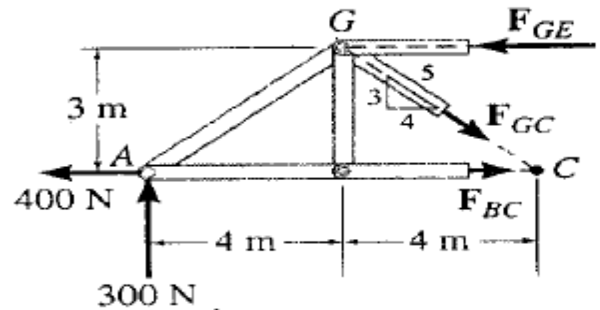
Example 2: Determine the force in members GE, GC and BC of the truss shown in the Figure. Indicate whether the members are in tension or compression.



(a)



(b)



(c)

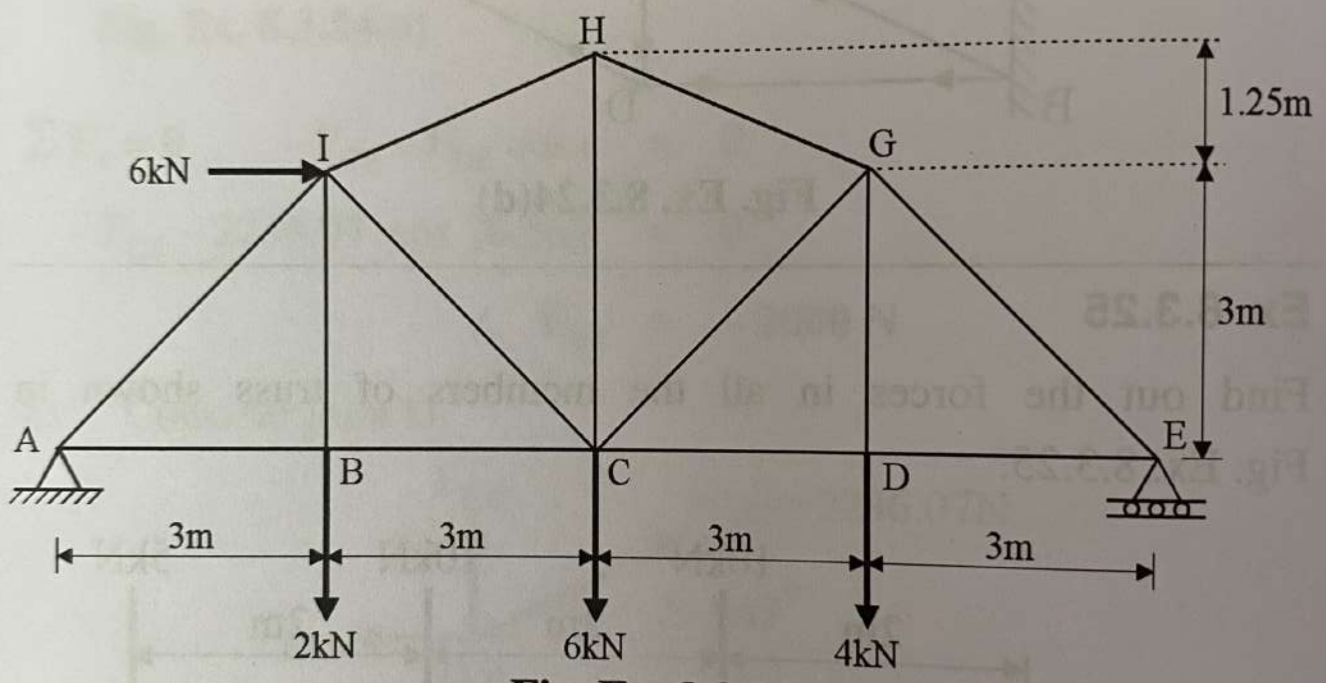
External Forces Determination

$\sum$ Fx = 0 i.e. $\mathbf{A_x}$ **= 400 N**

$\sum$ M_A = 0 ; i.e. - 1200 x 8 - 400 x 3 + 12 D_y = 0; $\mathbf{D_y}$ **= 900 N**

$\sum$ Fy = 0 i.e. A_y - 1200 + 900 = 0; $\mathbf{A_y}$ **= 300 N**

Determine the forces in members GH, CG and CD.



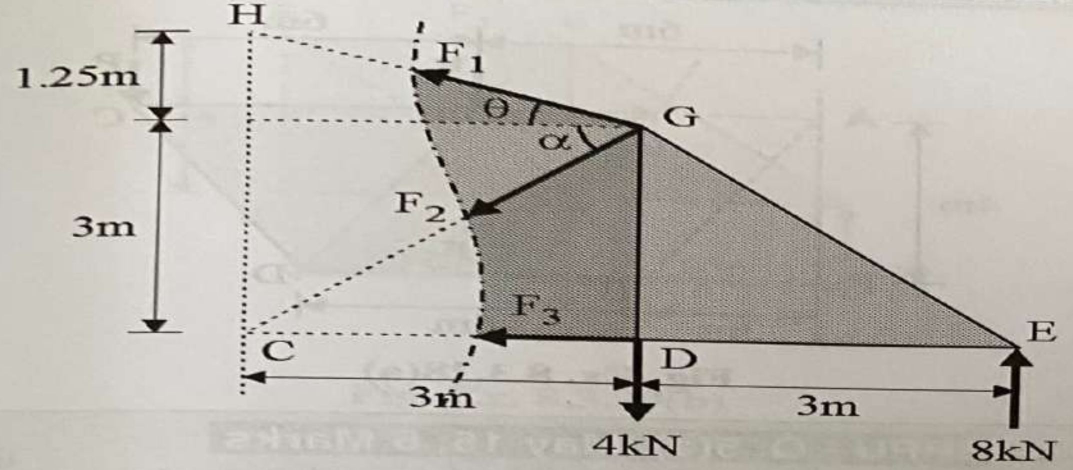


Fig. Ex. 8.3.26(b)

Step 4 : Conditions of equilibrium $\Sigma M = 0$

1) $\Sigma M_G = 0$ [As two forces F_1 and F_2 are passing through point G, we can find F_3 easily]

$$(-F_3 \times 3) + (8 \times 3) = 0$$

$$\therefore \quad F_3 = 8 \text{ kN}$$

2) $\Sigma M_C = 0 \quad (-4 \times 3) + (8 \times 6) + (F_1 \cos \theta \times 4.25) = 0$

[Resolving F_1 at point H]

$$\therefore \quad F_1 = -9.176 \text{ kN}$$

3) $\Sigma M_D = 0$

$$(8 \times 3) + (F_2 \cos \alpha \times 3) + (F_1 \cos \theta \times 3) = 0$$

[Resolving F_1 at G and F_2 at G]

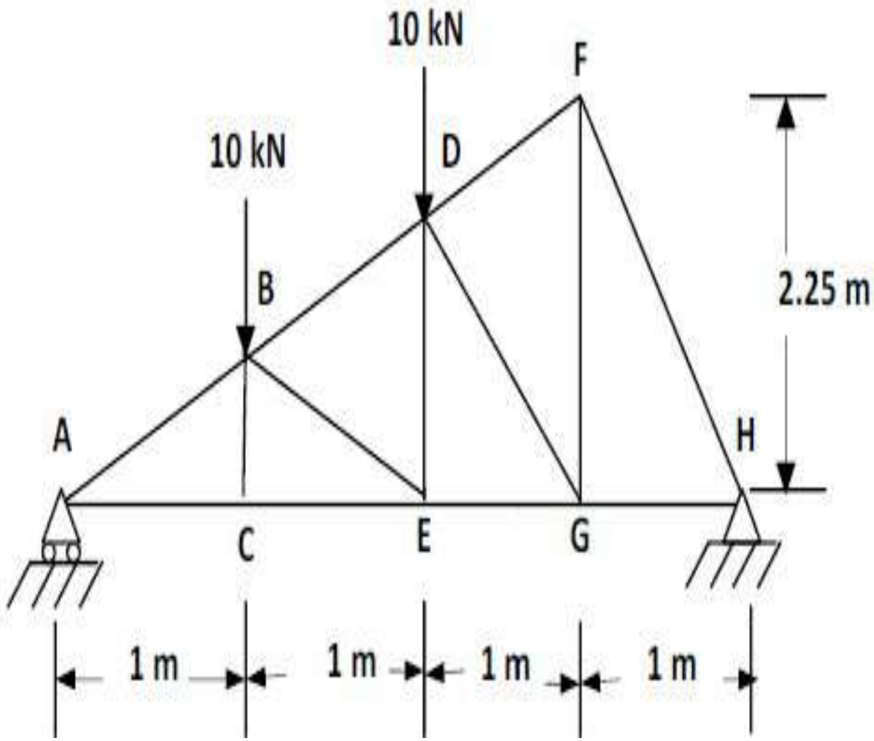
$$\therefore \quad 24 + F_2 \cos 45° \times 3 - 9.176 \cos 22.62° \times 3 = 0$$

$$\therefore \quad F_2 = 0.664 \text{ kN}$$

Step 5 : Force Table

Sr. No.	Member	Magnitude (kN)	Nature
1	GH	9.176	C
2	CG	0.664	T
3	CD	8	T

Example 3: For the simply supported truss shown in figure, find the force in members BD, DE, EG and CE using the method of sections. **(UPTU 2003, Jan'2011)**



These reactions will be calculated with the equations of equilibrium as under:

$\sum Fx = 0$, so $R_{HX} = 0$ ------ (1)

(as there was no horizontal or inclined force was acting on the truss, it was oblivious that horizontal component of reaction at hinged support is ZERO)

$\sum Fy = 0$, so $R_A + R_{HY} = 10 + 10 = 20$ ------ (2)

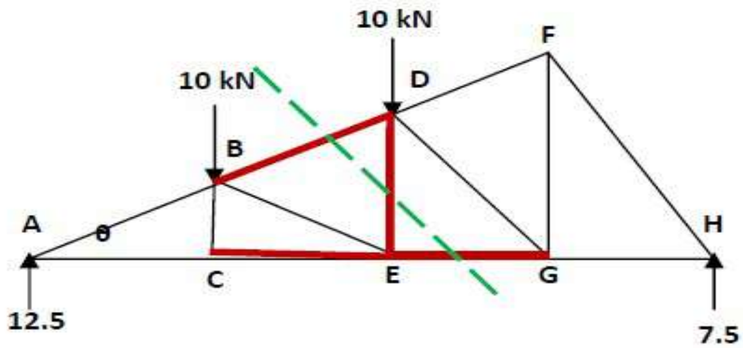
$\sum M_A = 0$, so $4\ R_{HY} = 10 \text{ x } 1 + 10 \text{ x } 2 = 30$

$R_{HY} = 30 / 4 = 7.5$ ------ (3)

$\sum M_H = 0$, so $4\ R_A = 10 \text{ x} 3 + 10 \text{ x } 2 = 50$

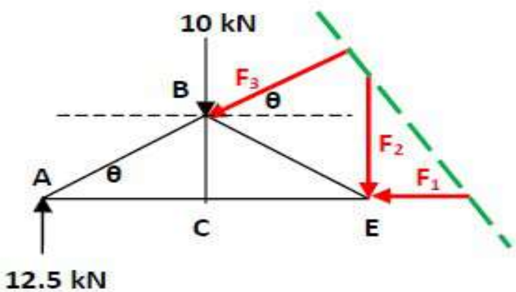
$R_A = 50 / 4 = 12.5$ ------ (4)

We can crosscheck our answer with the help of equation (2).



As we can consider LHS or RHS, LHS is selected and the forces in member BD, DE and EG are considered as Compressive.

$$\theta = \tan^{-1}(2.25/3) \quad = \quad 36.9^0$$



We can find F_1, F_2 and F_3 by three equations of equilibrium or by moment equation only.

$\sum Fx = 0$

$F_3 \cos 36.9 + F1 = 0 \quad (1)$

$\sum F_Y = 0$

$F_3 \sin 36.9 + F_2 + 10 = 12.5$

$F_3 \sin 36.9 + F_2 = 2.5 \quad (2)$

$\sum M_A = 0$

$10 \times 1 + F_2 \times 2 = 0$

$F_2 \times 2 = -10 / 2$ $= -5$ kN (Tensile Force) (Member DE)

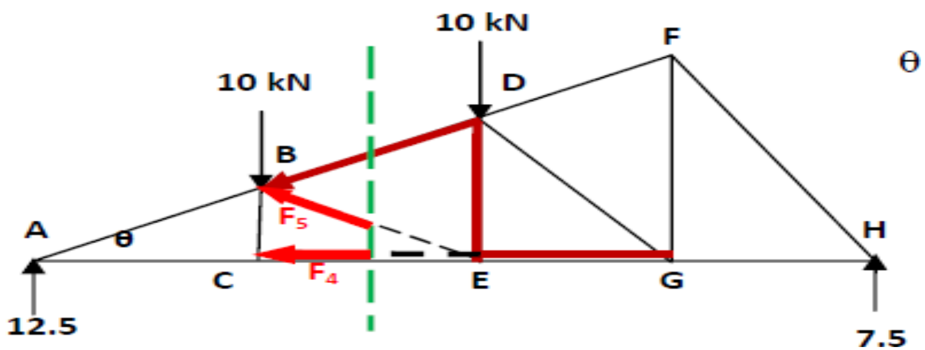
$F_3 \sin 36.9 + (-5) = 2.5$ (2)

So $F_3 = 7.5 / \sin 36.9 =$ 12.5 kN (Compressive) (Member BD)

$12.5 \cos 36.9 + F1 = 0$ (1)

$F_1 = - 12.5 \cos 36.9 = - 10$ kN (Tensile) (Member EG)

To find the force in the section will be cut as under:



Let's assume the force in cut members CE and BE are F4 and F5 respectively (both Compressive). We have already calculated force in member BD (12.5 kN, Comp.)

$\sum M_B = 0$

$12.5 \times 1 + F_4 \times 0.75 = 0$

By similarity of triangles FG / AG = BC / AC

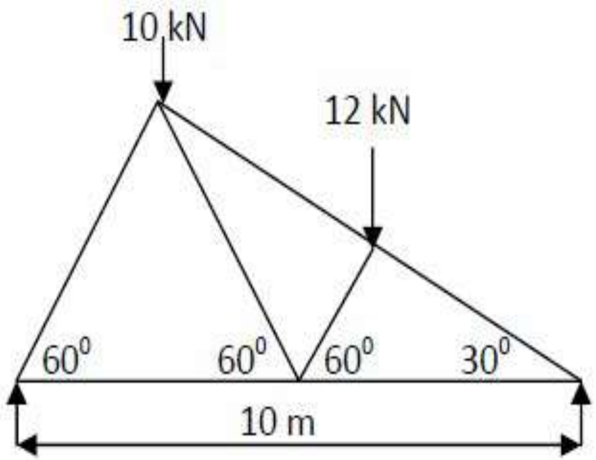
$BC = (2.25/3) \times 1 \quad = \quad 0.75$

$F_4 = \quad (-12.5 \times 1)/0.75 = -16.67$ kN (Tensile) in Member CE.

The final answer is tabulated as under:

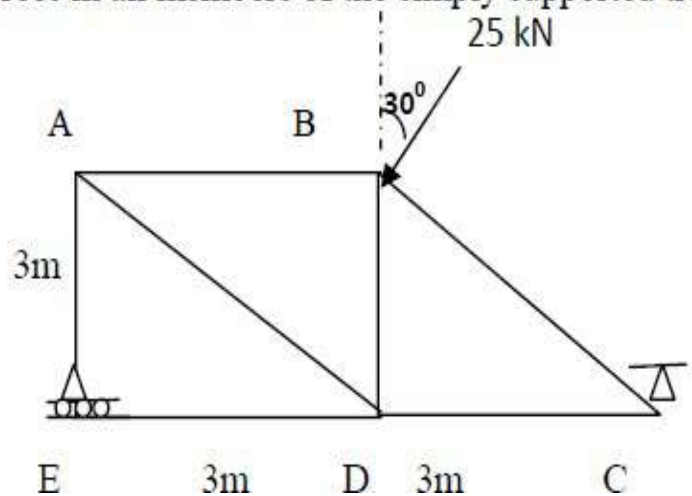
Sr. No.	Member	Force	Magnitude (kN)	Nature
1	EG	F_1	**10**	Tensile
2	DE	F_2	**5**	Tensile
3	BD	F_3	**12.5**	Compressive
4	CE	F_4	**16.67**	Tensile

c) Find the axial forces in all members of following truss.

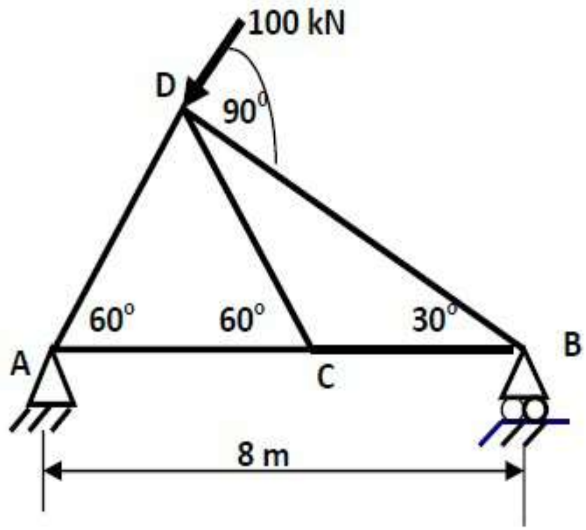


OR

Find the axial forces in all members of the simply supported truss as shown in below fig.

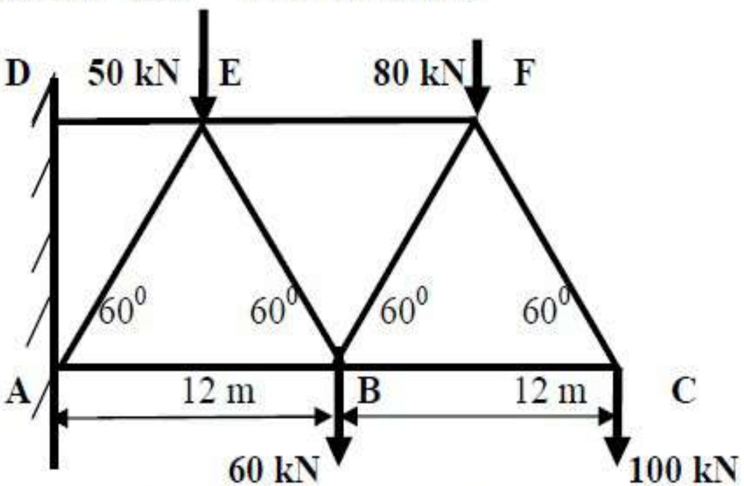


5. Determine the forces in all the members of the Truss by **Joint Method** as shown in figure.

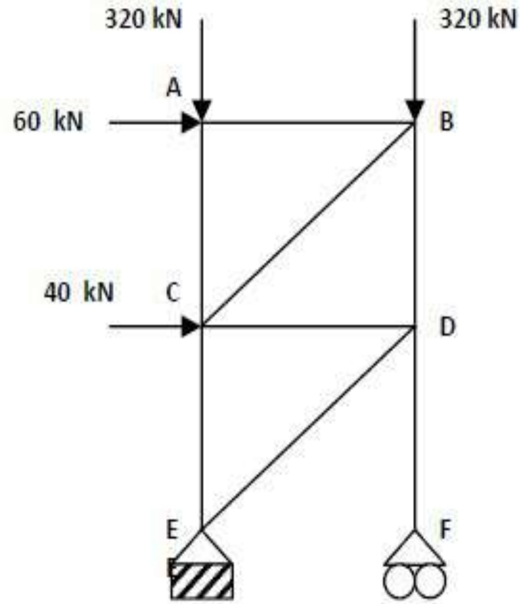


6. Determine the forces in the members of the truss by **Methods of Section**, shown in figure.

- FE, AE, AB and DE, DB and BC



7. Find the forces of the members of following Truss.

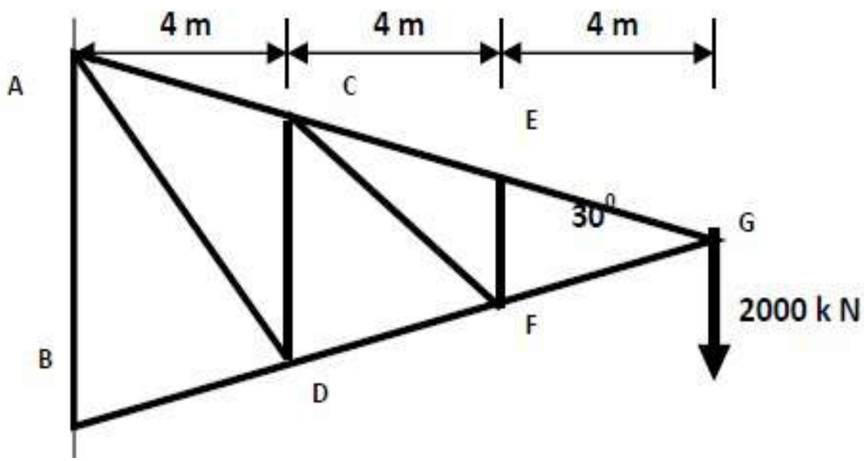


Given

AB = 3 m

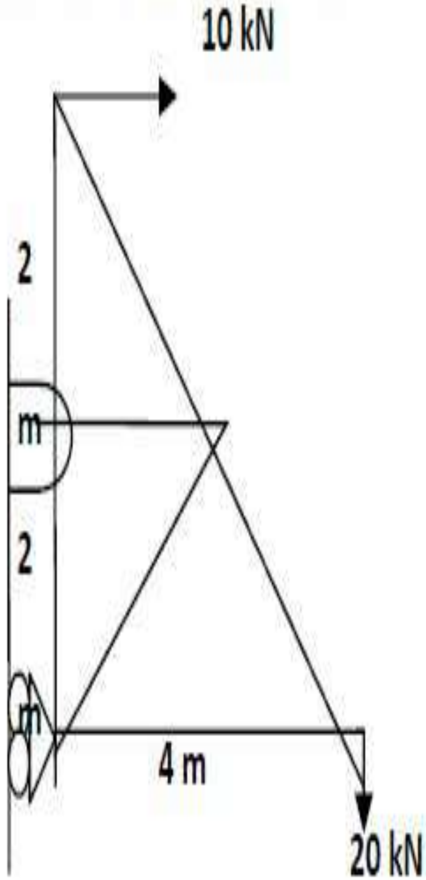
AC = CE = 4 m

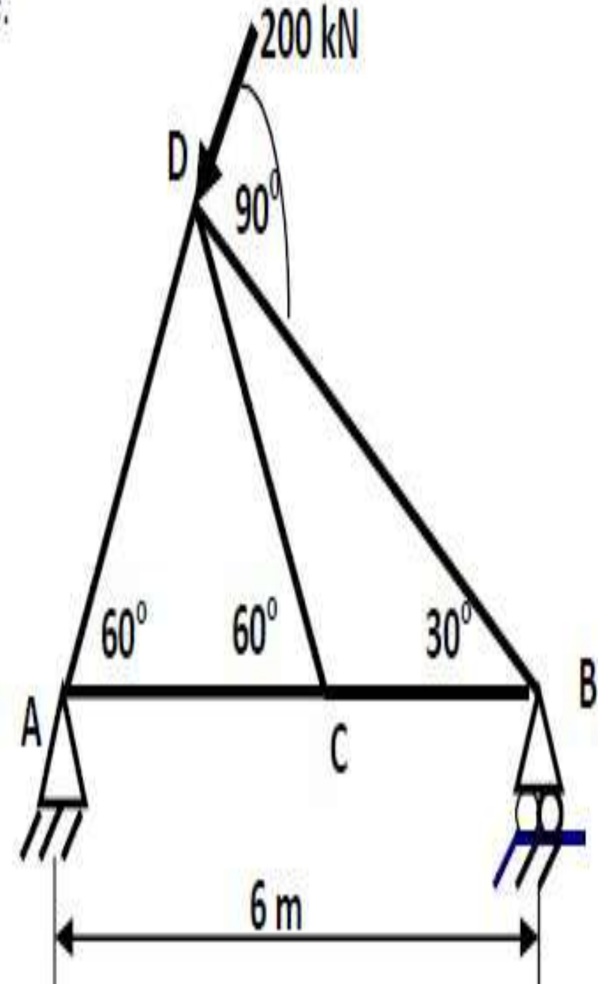
8.



5

a) Find the forces in the members of the Truss shown in the left figure:





(b) Determine the forces in **DB, DC and AC** members of the Truss by **Section Method** as shown in right figure.

FRAMES

FRAMES

- Structures having two force & multi force members
- Pin Connected

Types of Problem

- ❖ To find Support Reactions
- ❖ To find Pin Reactions
- ❖ Force in Two-Force Member

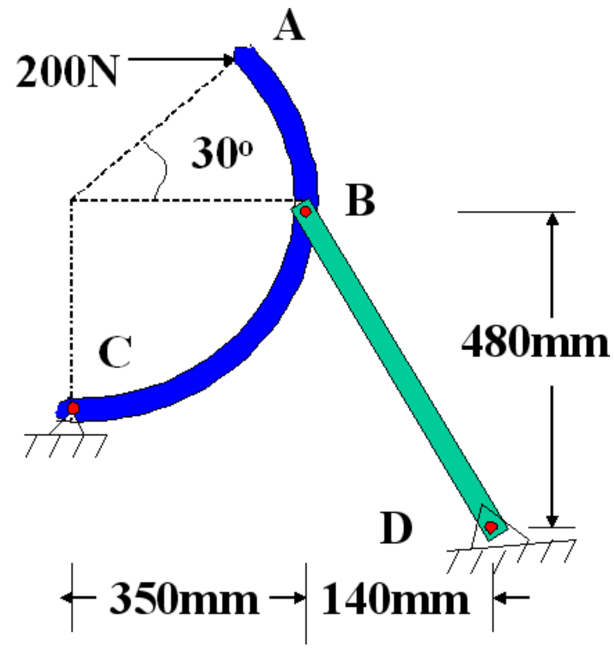


Fig. 16. Frame

STEPS

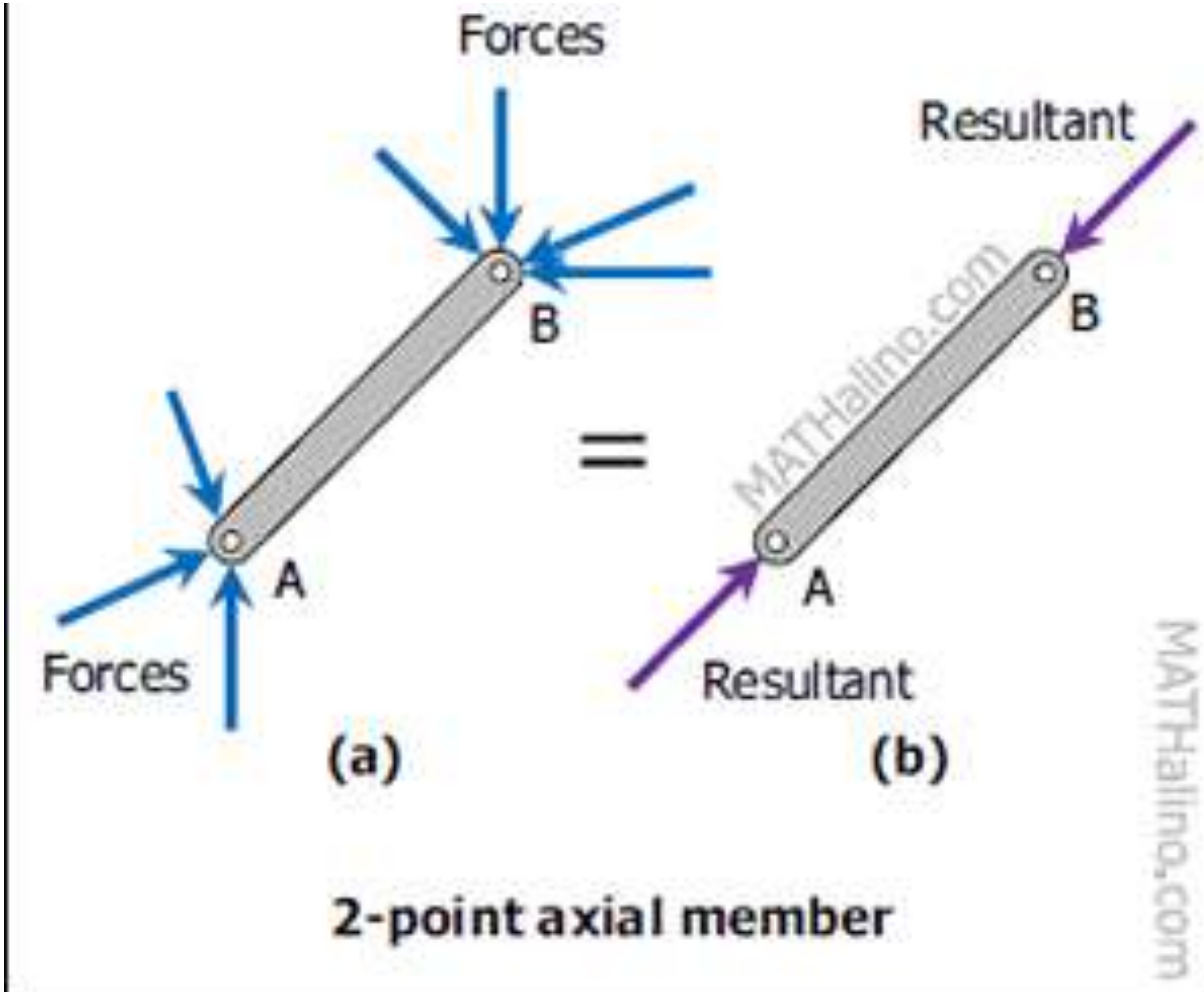
i. *FBD* of Each member

ii. $\sum F_x = 0;$

iii. $\sum F_y = 0;$

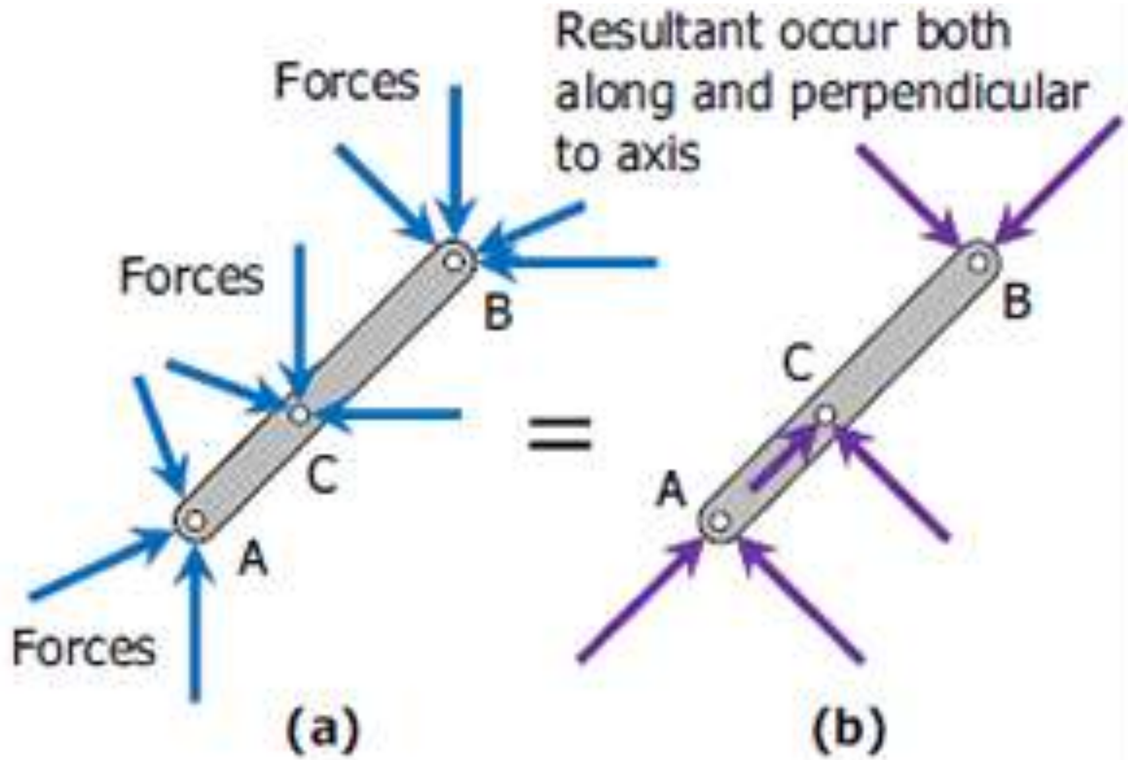
iv. $\sum M = 0$

Frames



Forces
B
A
Forces
(a)
=
Resultant
B
A
Resultant
(b)
MATHalino.com
MATHalino.com
2-point axial member

Frames



3-point non-axial member

9.6 Difference between a Truss and a Frame

Sr. No.	Truss	Frame
1.	All members are two force members	At least one member must be a multi-force member.
2.	All joints are pin joints.	Members are connected by internal hinge.
3.	All members are straight	A bent member can be used.
4.	Loads are acting only on the joints.	A member can be loaded.
5.	Used to support inclined roof covering or in construction of a bridge and tower.	Used to support or to lift loads.

9.5 Key Tips in F.B.D. of Entire Frame

a) By inspection always identify the two force member.
Note that the direction of the line joining the two points of force application, and not the shape of the member, determines the direction, of the forces acting on a two force-member (see Fig. 9.5.1(a))

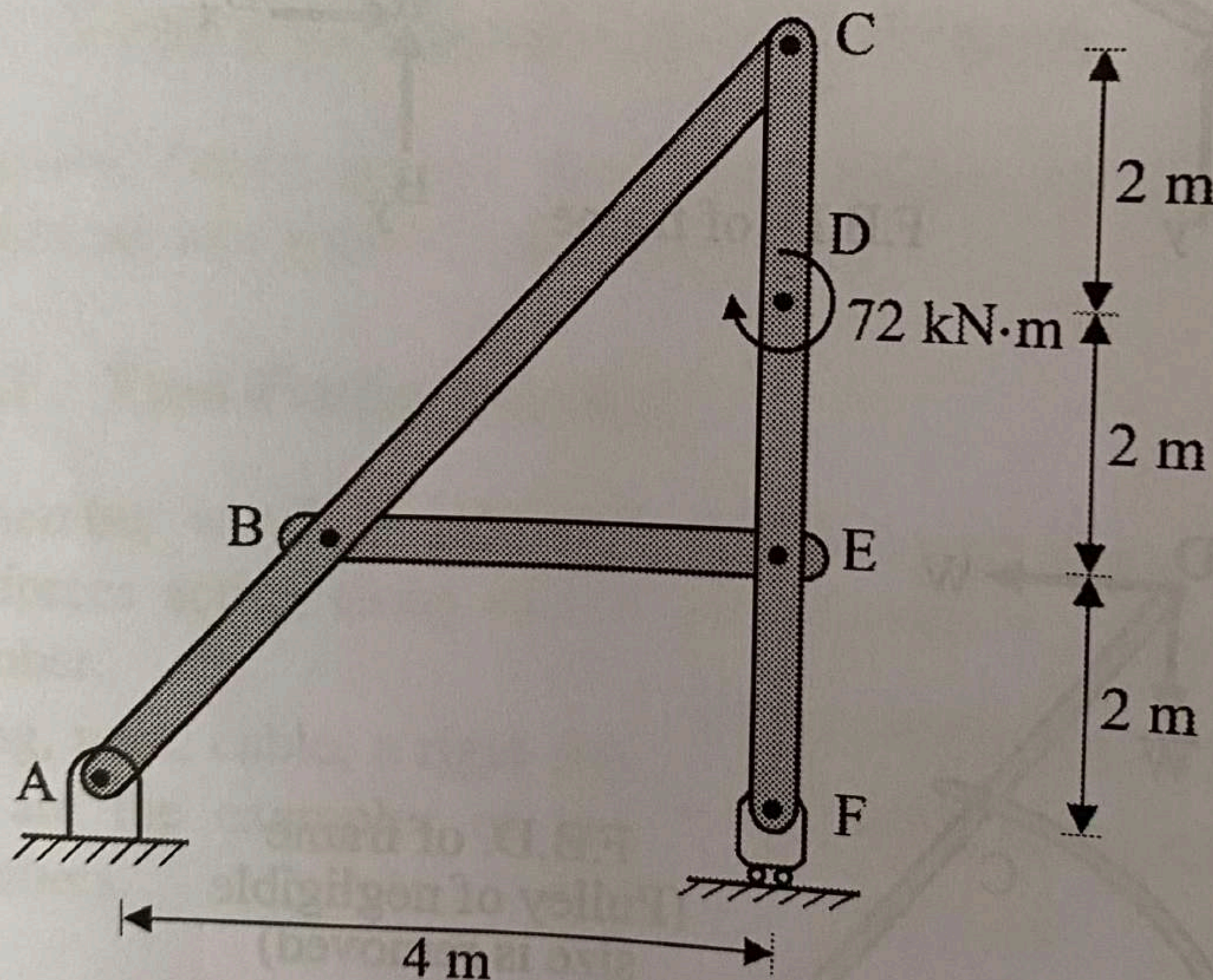
b) If a string attached with a load at one end is passing over a smooth pulley and **connected with any member of the frame**, the tension is considered as **internal** and do not consider this tension while drawing F.B.D. of entire frame (See Fig. 9.5.1(a))

c) If a string attached with a load at one end is passing over a smooth pulley and **connected with any external surface or plane**, the tension is considered as **external** and consider this tension while drawing F.B.D. of entire frame
(see Fig. 9.5.1(b) and (c)).

d) If size of pulley is not mentioned, remove the pulley and transfer the forces to centre of pulley. (See Fig. 9.5.1(c)).

e) The tensions or compressions which were internal initially become external after dismembering the frame.

For given frame, determine the components of forces acting on members CDEF and ABC.



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Ex. 9.6.1

For given frame, determine the components of forces acting on members CDEF and ABC.

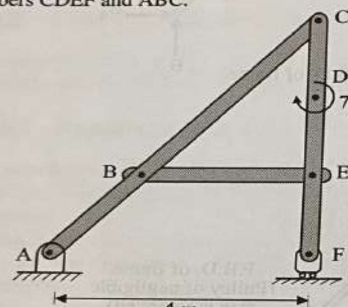


Fig. Ex. 9.6.1

☑ **Soln. :**

Step 1 : F.B.D. of entire frame : (Refer Fig. Ex. 9.6.1(a)).

Consider F.B.D. of entire frame, by assuming reaction components at A and F as shown in Fig. Ex. 9.6.1(a)

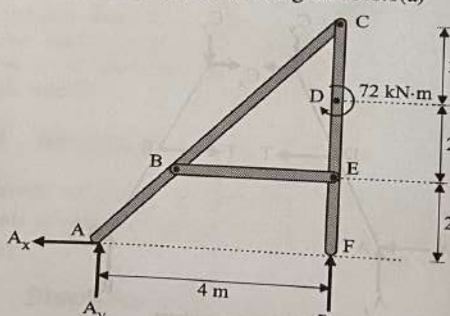


Fig. Ex. 9.6.1(a)

Note : By inspection it can be seen that BE is a two force member subjected to tensile or compressive force. Since BE is connected between two members of frame, its force is internal.

Step 2 : Apply 3 conditions of equilibrium to entire frame

$\Sigma F_x = 0 \qquad -A_x = 0 \qquad \therefore \; A_x = 0$

$\Sigma F_y = 0 \qquad A_y + R_F = 0 \qquad \therefore \; A_y = -R_F \qquad ...(1)$

$\Sigma M_A = 0 \qquad (R_F \times 4) - 72 = 0 \qquad R_F = 18 \text{ kN} \uparrow$

Substituting in Equation (1)

$A_y = -18$ kN (Negative sign indicates assumed direction of A_y is wrong)

$\therefore \; A_y = 18 \text{ kN} \downarrow$

Step 3 : F.B.D. of members

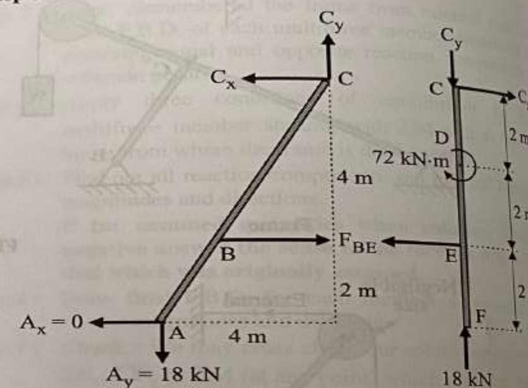


Fig. Ex. 9.6.1(b)

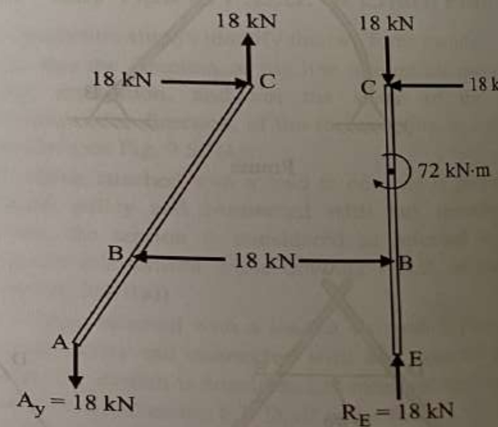


Fig. Ex. 9.6.1(c) (Final F.B.D)

Dismembered the frame and draw F.B.D. of members by assuming reaction components as equal and opposite.

Step 4 : Conditions of equilibrium

As only three unknowns, C_x, C_y and F_{BE} are involved apply three conditions of equilibrium to **any one** member.

Member ABC

$\Sigma M_C = 0$ $F_{BE} \times 4 + 18 \times 4 = 0$

$\therefore$ $F_{BE} = -18$ kN (Negative sign indicates assumed nature of force in member BE is wrong)

$\therefore$ $F_{BE} = 18$ **kN (compressive)**

$\Sigma F_x = 0$ $-C_x + F_{BE} = 0$

$\therefore$ $C_x = F_{BE}$ $\therefore$ $C_x = -18$ **kN**

$\Sigma F_y = 0$ $C_y - 18 = 0$ $\therefore$ $C_y = 18$ **kN**

Check : One can find unknowns by considering member CDEF as follows :

$\Sigma M_C = 0$ $-72 - F_{BE}(4) = 0$ $\therefore$ $F_{BE} = -18$ kN

$\therefore$ $F_{BE} = 18$ kN (compressive)

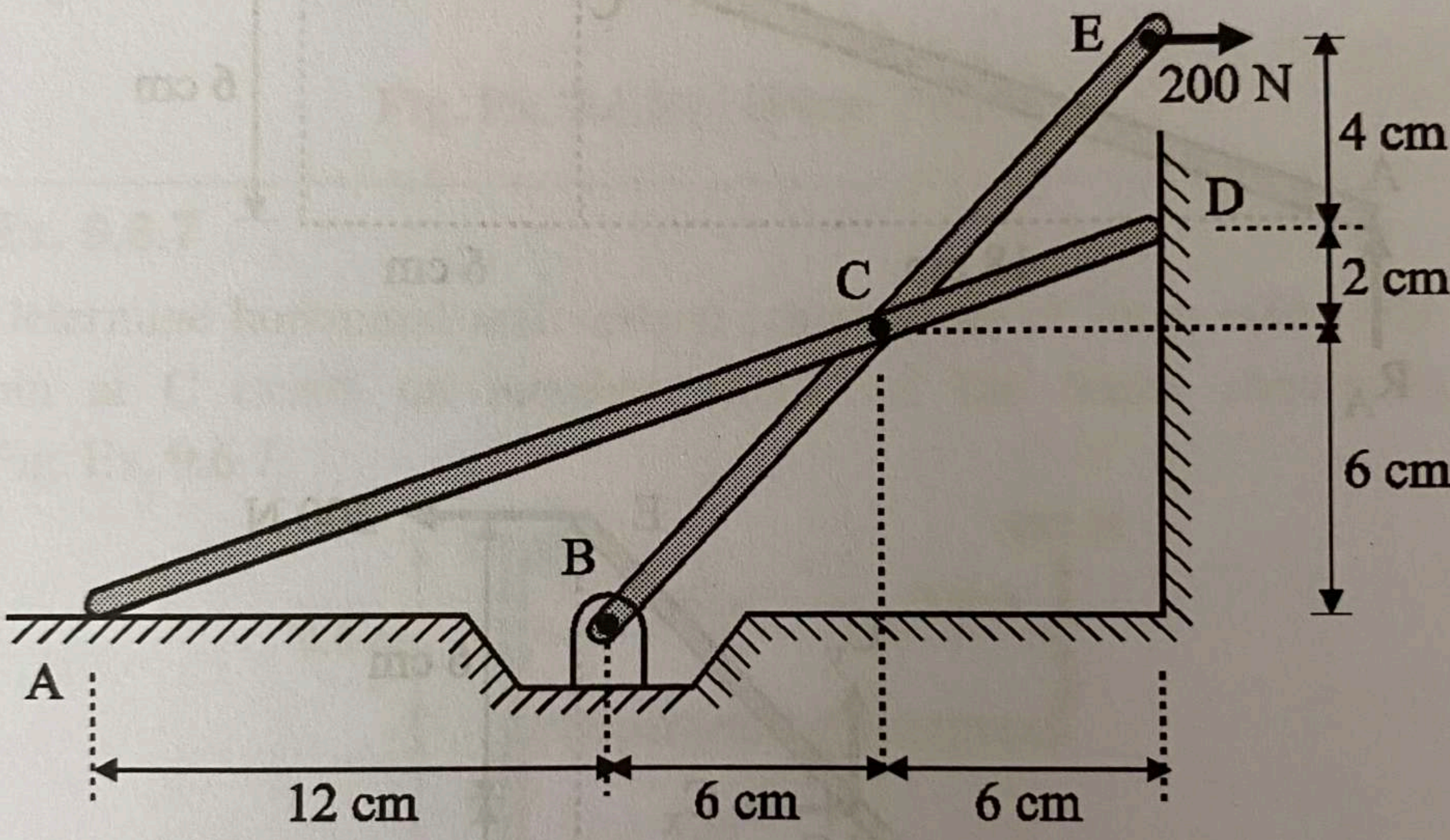
$\Sigma F_x = 0$ $C_x - F_{BE} = 0$ $\therefore$ $C_x = -18$ **kN**

$\Sigma F_y = 0$ $-C_y + 18 = 0$ $\therefore$ $C_y = 18$ **kN**

Step 5 : Final F.B.D. of members

Draw final F.B.D. of members showing magnitudes and actual directions of forces.

If surfaces at A and D are smooth, determine components of all forces exerted on member ACD and BCE. Refer Fig. Ex. 9.6.5.



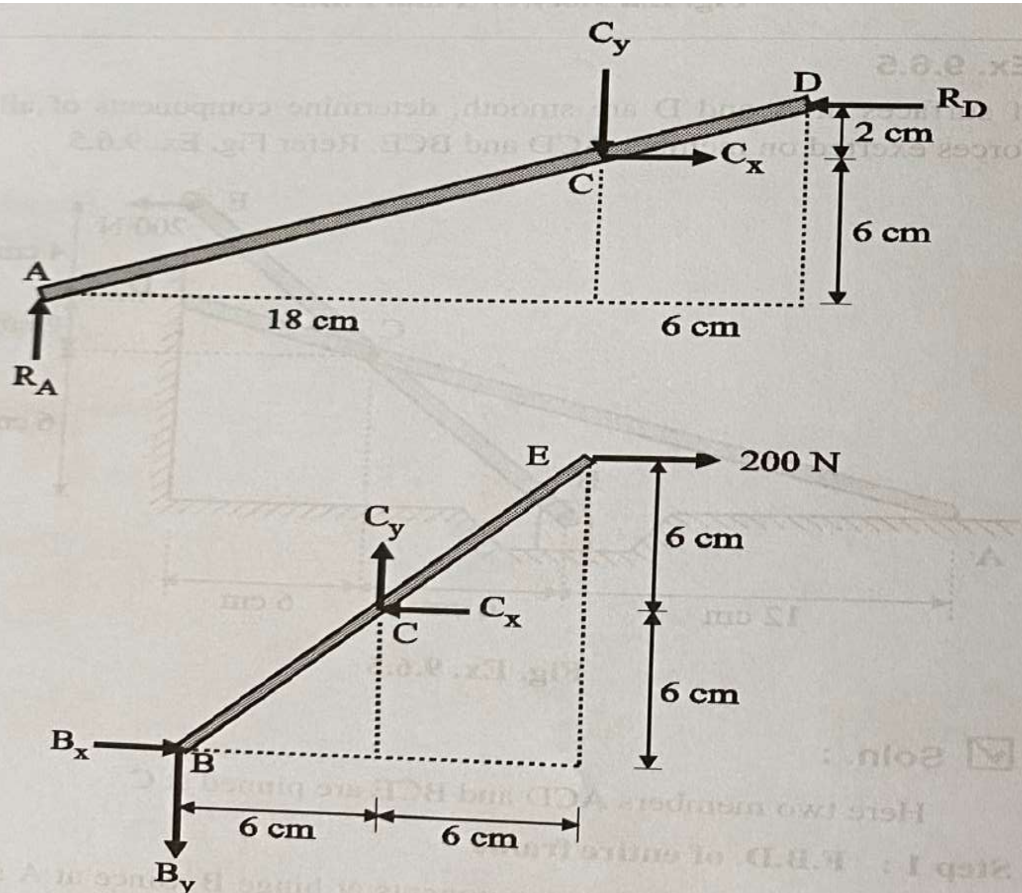


Fig. Ex. 9.6.5(b)

Step 4 : Equations of equilibrium

Member ACD

$\Sigma M_C = 0$ $\qquad -R_A \times 18 + R_D\,(2) = 0$

$$\therefore \quad R_D = 9\,R_A \qquad \ldots(4)$$

Substituting this value in Equation (3)

$$-12\,R_A + 8\,(9\,R_A) = 2400$$

$$\therefore \quad R_A = 40\text{ N}\uparrow$$

$\therefore$ By Equation (4) $\quad R_D = 360\text{ N}\leftarrow$

$\Sigma F_x = 0 \qquad C_x - R_D = 0$

$$\therefore \quad C_x = 360\text{ N}$$

$\Sigma F_y = 0 \qquad R_A - C_y = 0$

$$\therefore \quad C_y = 40\text{ N}$$

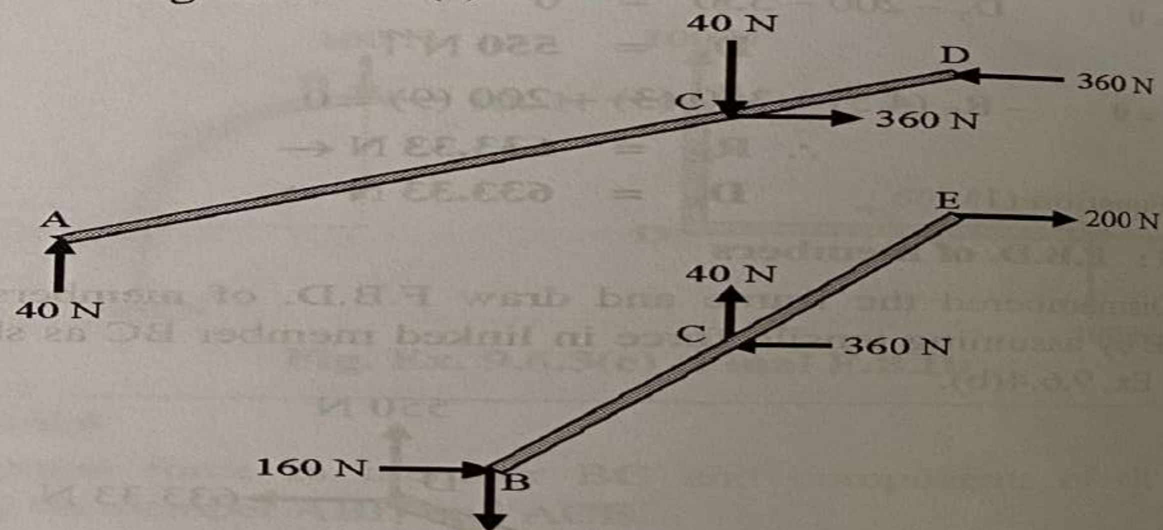
Now, substituting value of R_D in Equation (1)

We get $\quad B_x = 360 - 200 \qquad B_x = 160\text{ N}\rightarrow$

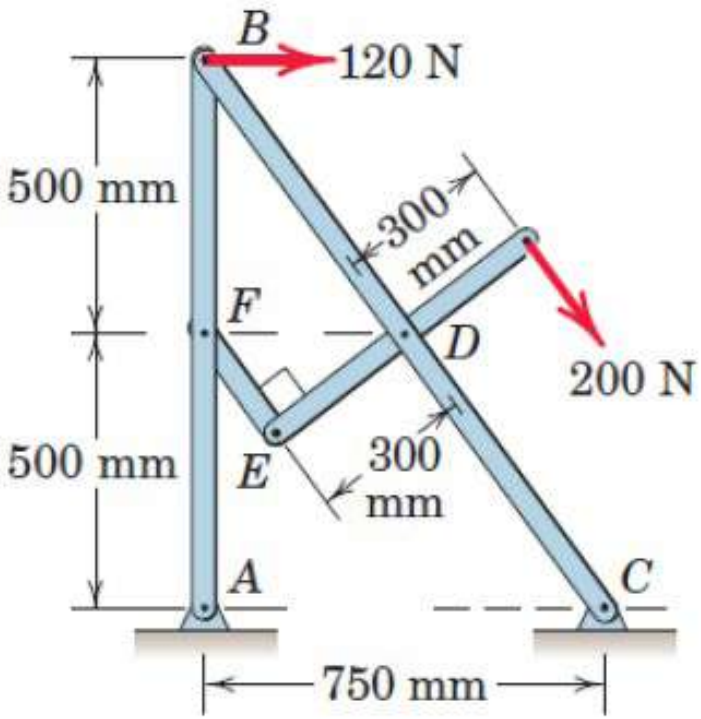
By Equation (2)

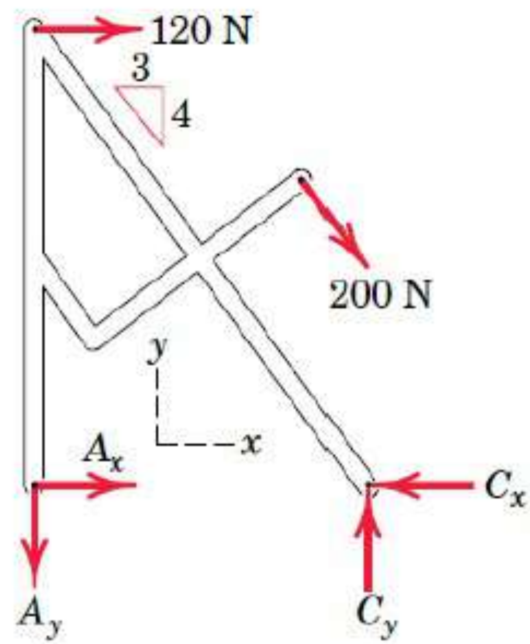
$B_y = R_A \qquad \therefore \quad B_y = 40\text{ N}\downarrow$

Step 5 : Final F.B.D. of members as shown in Fig. Ex. 9.6.5(c).



Neglect the weight of the frame and compute the forces acting on all of its members





$[\Sigma M_C = 0]$	$200(0.3) + 120(0.1) - 0.750A_y = 0$	$A_y = 240\text{ N}$
$[\Sigma F_y = 0]$	$C_y - 200(4/5) - 240 = 0$	$C_y = 400\text{ N}$

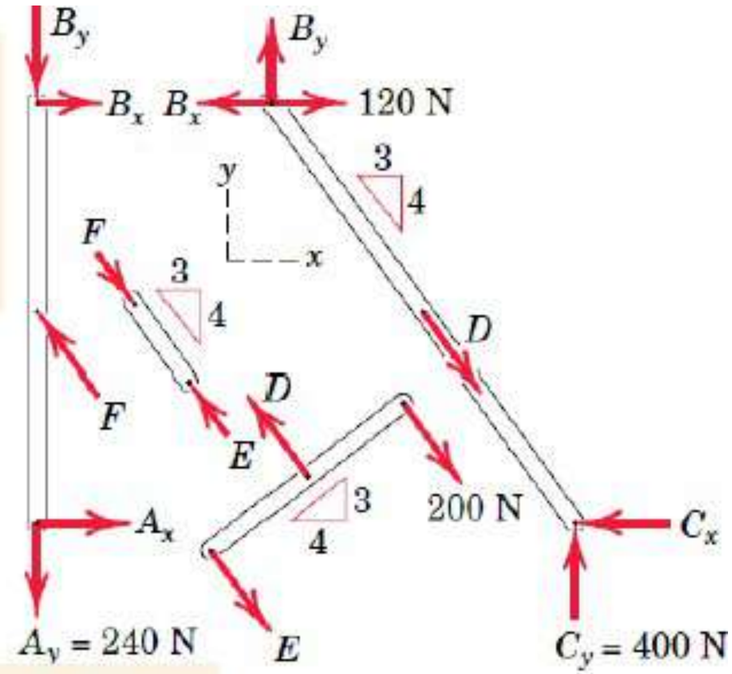
Member ED. The two unknowns are easily obtained by

$[\Sigma M_D = 0] \qquad 200(0.3) - 0.3E = 0 \qquad E = 200 \text{ N}$

$[\Sigma F = 0] \qquad D - 200 - 200 = 0 \qquad D = 400 \text{ N}$

Member EF.

Clearly *F* is equal and opposite to *E* with the magnitude of 200 N.



Member AB. Since F is now known, we solve for B_x, A_x, and B_y from

$[\Sigma M_A = 0] \qquad 200(3/5)(0.5) - B_x(1.0) = 0 \qquad B_x = 60 \text{ N}$

$[\Sigma F_x = 0] \qquad A_x + 60 - 200(3/5) = 0 \qquad A_x = 60 \text{ N}$

$[\Sigma F_y = 0] \qquad 200(4/5) - 240 - B_y = 0 \qquad B_y = -80 \text{ N}$

The minus sign shows that we assigned B_y in the wrong direction.

10

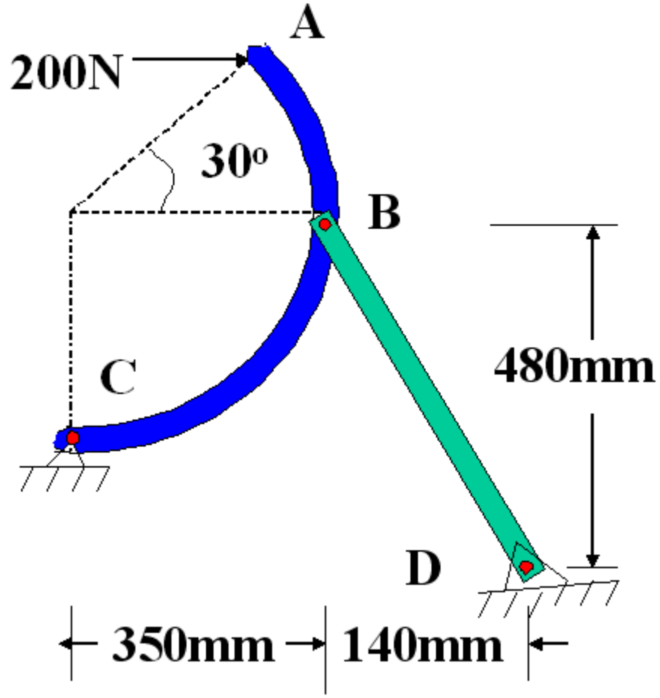
Member BC. The results for B_x, B_y, and D are now transferred to BC, and the remaining unknown C_x is found from

$$[\Sigma F_x = 0] \qquad 120 + 400(3/5) - 60 - C_x = 0 \qquad C_x = 300 \text{ N} \qquad \textit{Ans.}$$

We may apply the remaining two equilibrium equations as a check. Thus,

$$[\Sigma F_y = 0] \qquad 400 + (-80) - 400(4/5) = 0$$

$$[\Sigma M_C = 0] \qquad (120 - 60)(1.0) + (-80)(0.75) = 0$$



Example:

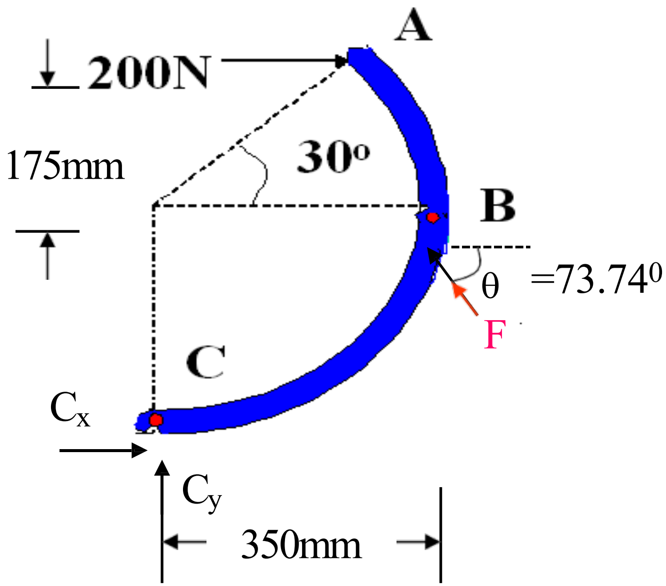


Fig 17 FBD of ABC

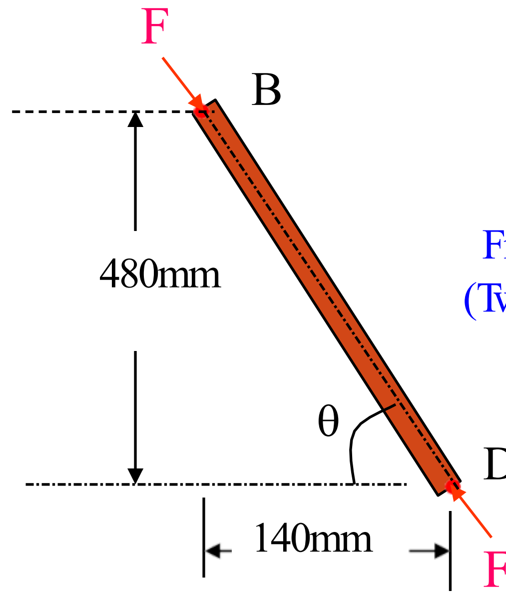


Fig 18 FBD of BD (Two Force Member)

From FBD of ABC

$$\sum M_C = 0; \quad \text{i.e.,} F\cos 73.74 \text{ x } 350 + F\sin 73.74 \text{ x } 350 - 200 \text{ x } 175 = 0$$

$$\therefore F = 80.65\text{N} \quad (\nwarrow)$$

$$\sum F_x = 0; \text{ i.e., } C_x + 200 - F\cos 73.74 = 0; \quad \Rightarrow A_x = 177.42\text{N}(\rightarrow)$$

and, $$\sum F_y = 0; \; C_y + F\sin 73.74 = 0; \quad \Rightarrow C_y = -77.42\text{N}(\downarrow)$$

ANALYSIS OF FRAME [Cont…]

Ex:2.

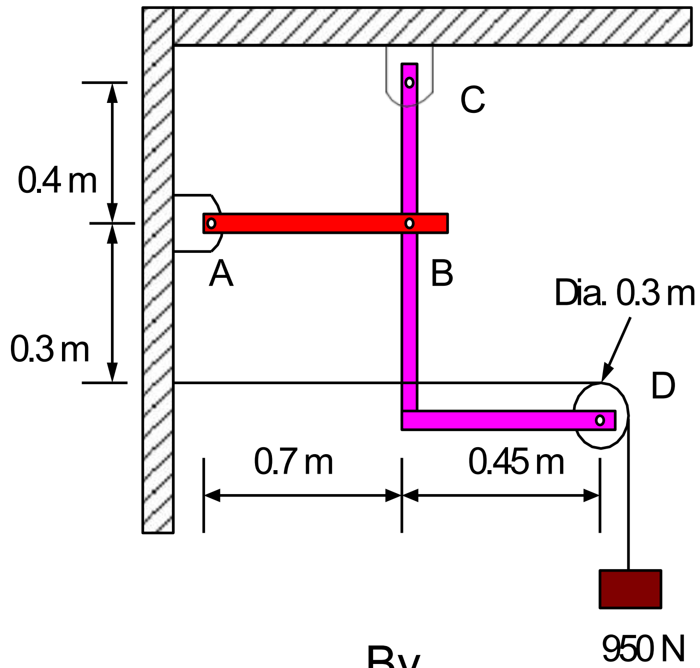


Fig 19 Frame

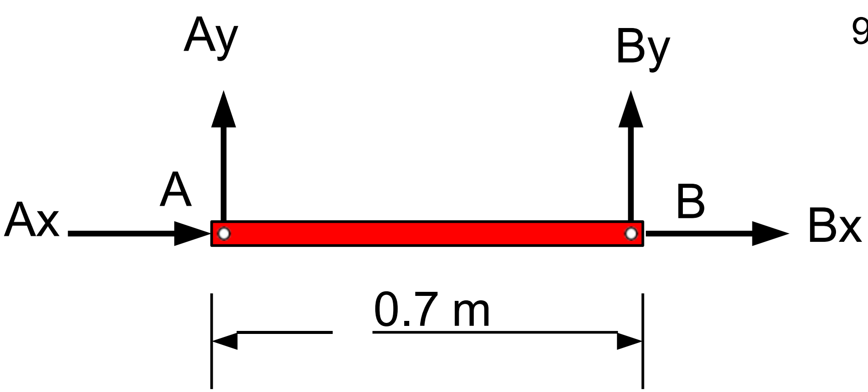


Fig 20(a) FBD of AB

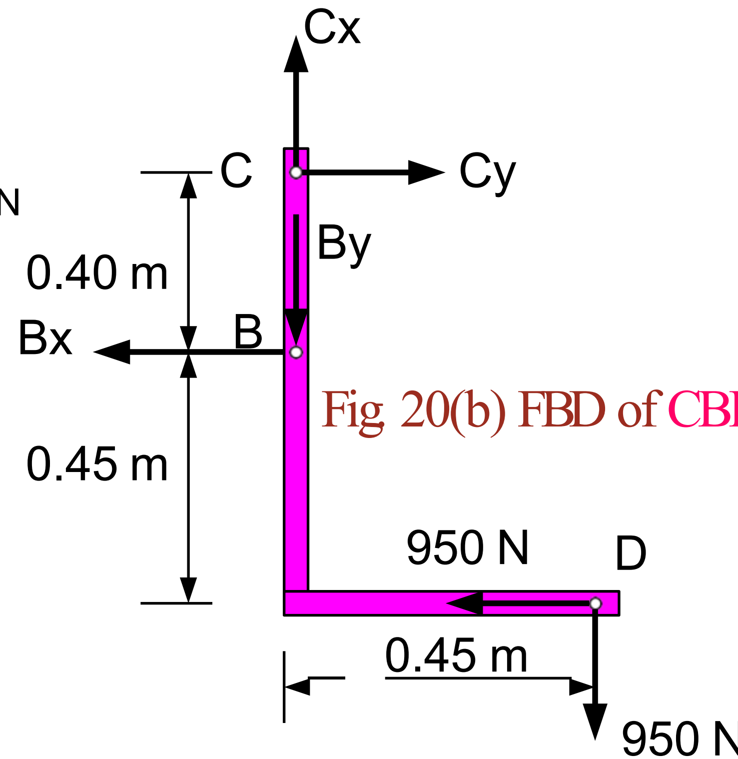


Fig 20(b) FBD of CBD

Cables Subjected to Point Loads

Cables:

- Cables are flexible structures
- Hinged supports
- Carry only tensile forces.

Types of problems on cables

- To find support reactions,
- To find sag of loaded Joint,
- To find tension in different parts of cable,
- To find max. tension in the cable.

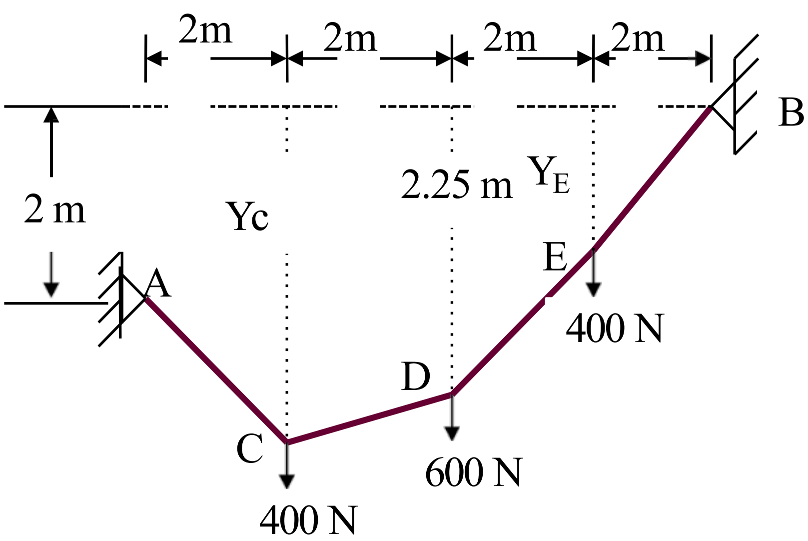


Fig 21 Cable

Assumptions for Cable

1. Cable is perfectly flexible and inextensible.
2. Cable has constant length before and after the load is applied.
3. The geometry of cable remains fixed after loads have been applied.
4. The weight of cable is comparatively small, so neglected.
5. The tensile force in cable is directed along cable.

Important tips

a) During analysis of cable, a question arises in our mind that how many parts of cable we should consider ?

For better solution observe number of unknown forces. If number of unknown forces are ‘n’ then consider (n – 3) parts of cable for finding the unknown quantities.

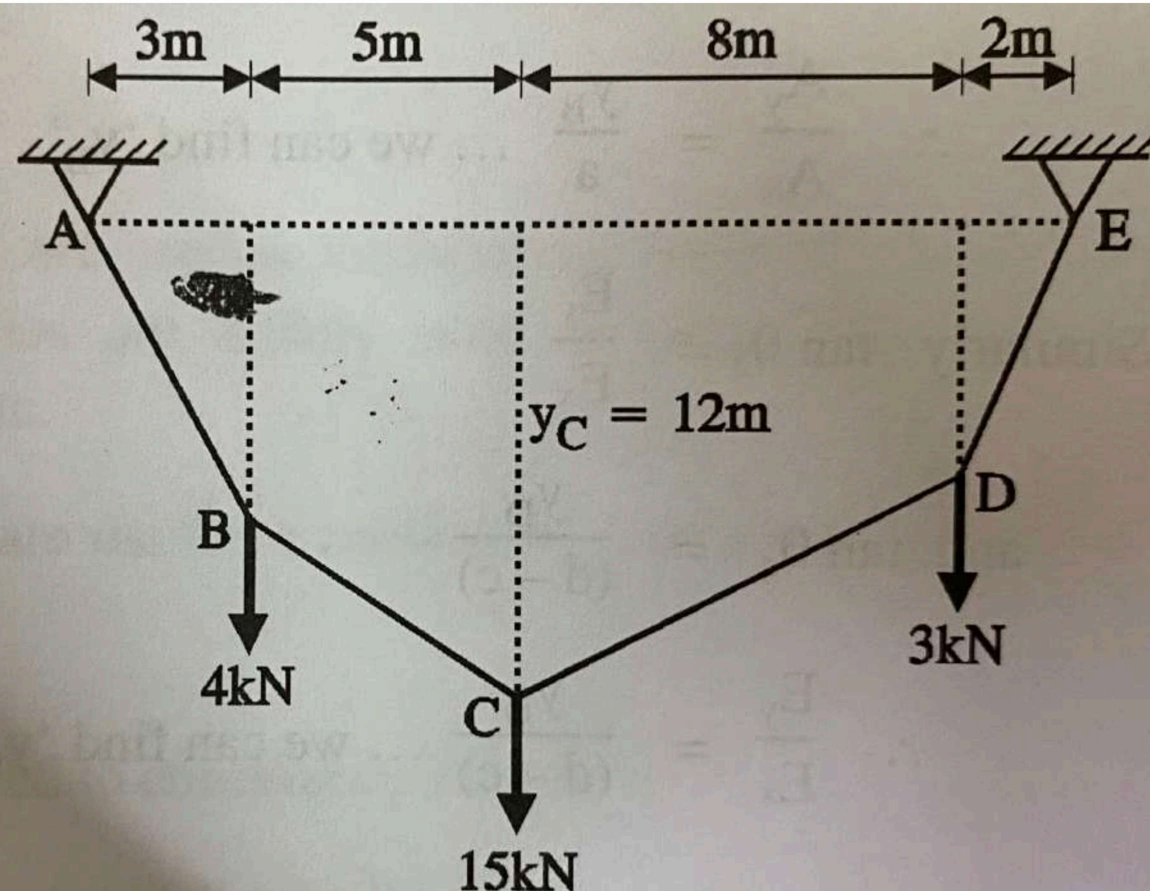
b) While selecting number of parts care should be taken that parts must be starting from **“same support”** upto the points where sags of points are known.

c) **Remember**

Although the tensions in different portions of the cable are different but their horizontal components are equal.

Ex. 10.2.1

Determine the tension in each segment of the cable shown in Fig. Ex. 10.2.1.



Step I : F.B.D. of entire cable system ABCDE

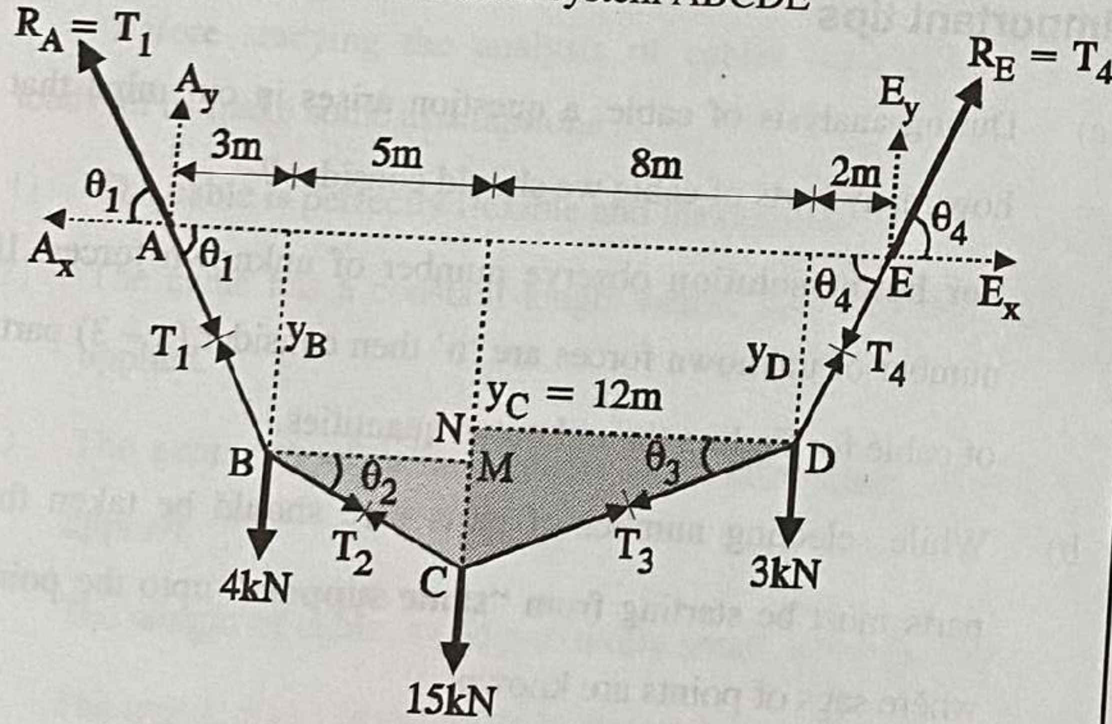


Fig. Ex. 10.2.1(a)

ep 2 : Apply conditions of equilibrium for entire cable ABCDE

$\Sigma F_x = 0$ $\quad A_x = E_x$ ...(1)

$\Sigma F_y = 0$ $\quad A_y + E_y - 4 - 15 - 3 = 0$

$\therefore \quad A_y + E_y = 22 \text{ (kN)}$ ...(2)

$\Sigma M_A = 0$ $\quad (E_y \times 18) - (4 \times 3) - (15 \times 8) - (3 \times 16) = 0$

$\therefore \quad E_y = 10 \text{ kN} \uparrow$

Substituting in Equation (2) $\quad \therefore \quad A_y = 12 \text{ kN} \uparrow$

TRICK BUT NOT A RULE ! : For entire cable moment is taken at point A, so consider part from opposite support E.

Step 3 : Consider equilibrium of part EDC, (as sag of C is known)

Use $\Sigma M_C = 0 \qquad (E_y \times 10) - (E_x \times 12) - (3 \times 8) = 0$

$$\therefore \quad (10 \times 10) - (3 \times 8) = E_x (12)$$

$$\therefore E_x = \mathbf{6.33 \text{ kN}} \rightarrow$$

$\therefore$ by Equation (1) $\qquad A_x = \mathbf{6.33 \text{ kN}} \leftarrow$

Step 4 : Tensions in segment AB, BC, CD and DE

a) $\quad T_1 = T_{AB} = R_A = \sqrt{A_x^2 + A_y^2} = \sqrt{(6.33)^2 + (12)^2}$

$\quad \mathbf{T_1 = T_{AB} = 13.57 \text{ kN}}$ ...**Ans.**

b) $\quad T_4 = T_{DE} = R_E = \sqrt{E_x^2 + E_y^2} = \sqrt{(6.33)^2 + (10)^2}$

$\quad = \mathbf{11.835 \text{ kN}}$ ...**Ans.**

c) To find T_{BC} and T_{CD}, apply Lami's theorem at joint C.

F.B.D. of joint C.

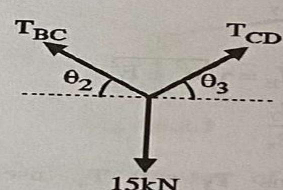


Fig. Ex. 10.2.1(b)

Procedure to find out angles θ_2 and θ_3.

$$\tan \theta_1 = \frac{A_y}{A_x} = \frac{y_B}{3}$$

$$\frac{12}{6.33} = \frac{y_B}{3}$$

$$\therefore \quad y_B = 5.69 \text{ m}$$

$$\tan \theta_2 = \frac{CM}{BM} = \frac{12 - y_B}{5} = \frac{12 - 5.69}{5}$$

$$\therefore \quad \theta_2 = \mathbf{51.61^\circ}$$

$$\tan \theta_4 = \frac{E_y}{E_x} = \frac{y_D}{2}$$

$$\frac{10}{6.33} = \frac{y_D}{2}$$

$$\therefore \quad y_D = 3.14 \text{ m}$$

$$\tan \theta_3 = \frac{CN}{ND} = \frac{12 - y_D}{8} = \frac{12 - 3.14}{8}$$

$$\therefore \quad \theta_3 = \mathbf{47.92^\circ}$$

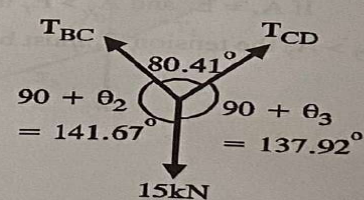


Fig. Ex. 10.2.1(c)

$\therefore$ By la...

$$\frac{T_{BC}}{\sin 137.92^\circ} = \frac{15}{\sin 80.41^\circ} = \sin 14$$

$\therefore \quad T_{BC} = \mathbf{10.194 \text{ kN}}$

$\quad T_{CD} = \mathbf{9.434 \text{ kN}}$

Ex. 10.2.2

Knowing that the maximum tension ... determine (a) sag y_B (b) sag y_C.

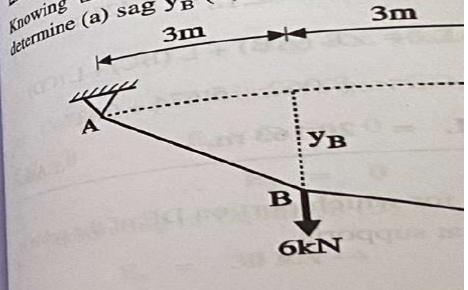


Fig. Ex. 1

☑ **Soln. :**

Concept : Here total unknown ... and D_y) so four equ... analysis.

Hence use 3 conditions of eq... not possible to consider any part... unknown. Therefore use maximum...

Step 1 : F.B.D. of entire cable s...

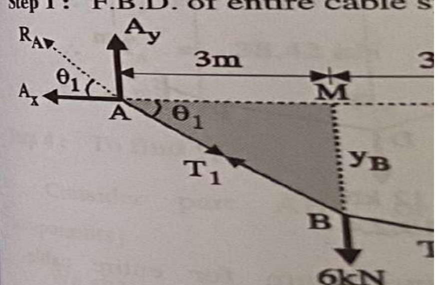


Fig. E

Step 2 : Apply conditions of ...

1) $\Sigma F_x = 0$

2) $\Sigma F_y = 0 \quad A_y + D_y - 6 - 1$...

3) ΣM...

CABLES [Cont……]

1. To find Support Reactions

- Draw FBD of whole cable
- Apply Equations of Equilibrium

$$\sum F_x = 0, \quad (1), \quad \sum F_Y = 0, \quad (2)$$

$$\sum M_A = 0 \quad (3)$$

- Cut a cable at a joint (say D) whose sag is known
- Consider FBD of left or right part of cable.
- Applying, $\sum M_D = 0;$ (4)

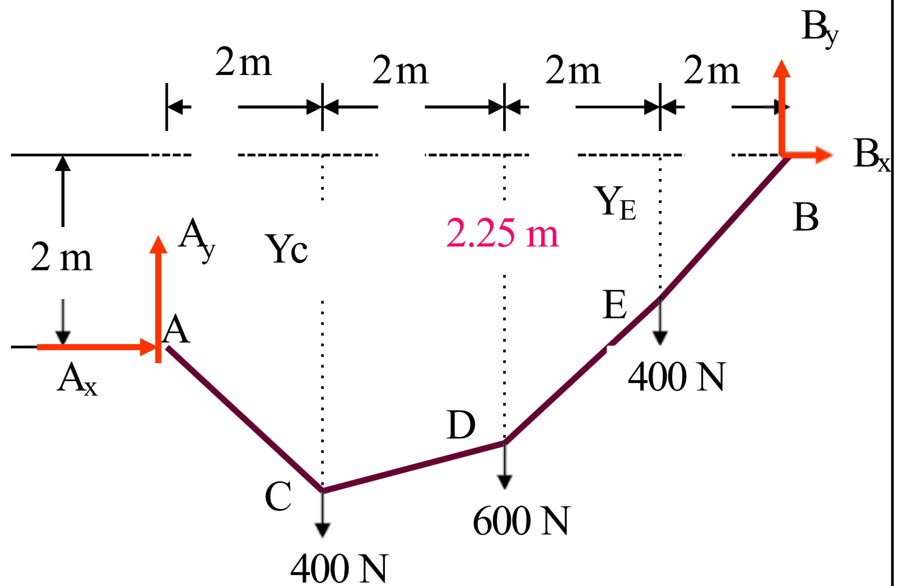


Fig22 FBD of Whole Cable

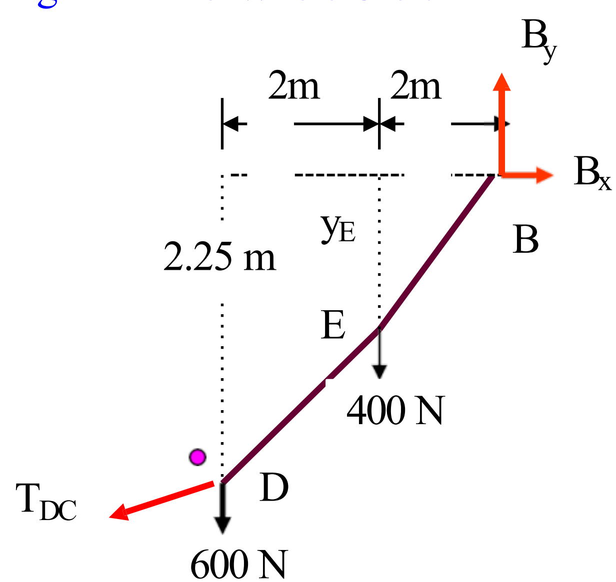


Fig23 FBD of Part DEB

CABLES [Cont……..]

To Find Sag of Joints

- Cut cable at joint whose sag is required.
- Consider FBD of part cable
- Apply $\sum M_C = 0;$

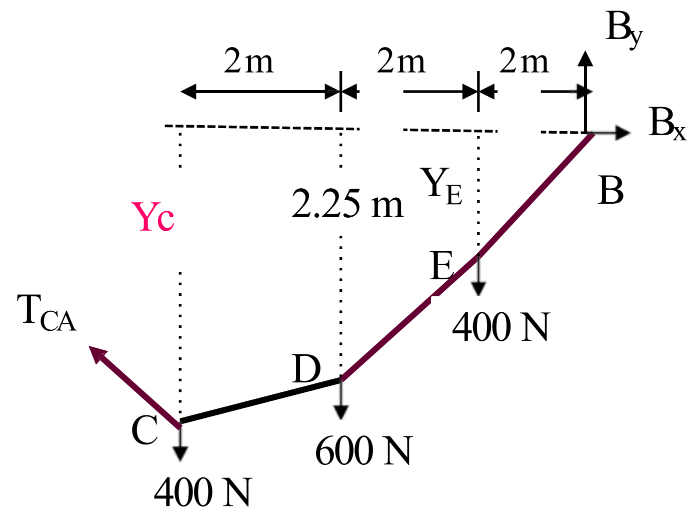


Fig 24 FBD of part CDEB

To Find Max Tension

- Determine Slope of each part i.e., θ_1, θ_2, θ_3 & θ_4
- Part of cable carries maximum tension whose slope is maximum

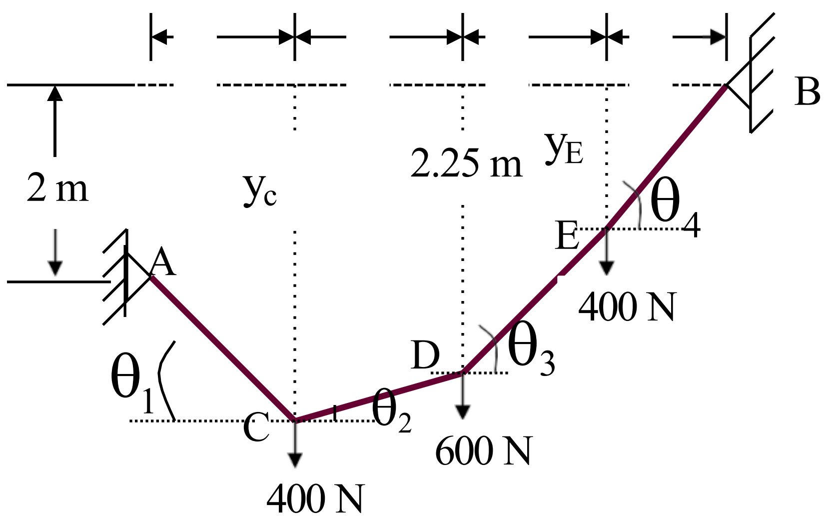


Fig 25 FBD of part CDEB