Sinhgad Institutes

SINHGAD COLLEGE OF ENGINEERING PUNE.

Engineering mechanics

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BE Civil, M.Tech Construction Technology & Managment,
(Ph.D Construction Materials)

Syllabus

Unit I - Resolution and Composition of Forces (07Hrs)
Principle of statics, Force system, Resolution and composition of forces, Resultant of concurrent forces. Moment of a force, Varignon's theorem, resultant of parallel force system, Couple, Equivalent force couple system, Resultant of parallel general force system.

Unit II - Distributed Forces and Friction (06Hrs)
Moment of area, Centroid of plane lamina and wire bends, Moment of Inertia. Friction- Laws of friction, application of friction on inclined planes Wedges and ladders friction Application to flat belt.

Unit III	**Equilibrium**	**(06Hrs)**
Free body diagram Equilibrium of concurrent, parallel forces in a plane Equilibrium of general forces in a plane Equilibrium of three forces in a plane, Types of beams, simple and compound beams, Type of supports and reaction, Forces in space, Resultant of concurrent and parallel forces in a space, Equilibrium of concurrent and parallel forces in a space.		
Unit IV	**Analysis of Structures**	**(06 Hrs)**
Two force member, Analysis of plane trusses by Method of joints Analysis of plane trusses by method of section, Analysis of plane frames, Cables subjected to point load multi force member.		
Unit V	**Kinematics of Particle**	**(06 Hrs)**
Kinematics of linear motion- Basic concepts Equation of motion for constant acceleration Motion under gravity, Variable acceleration motion curves. Kinematics of curvilinear motion- Basic Concepts Equation of motion in Cartesian coordinates Equation of motion in path coordinates Equation of motion in polar coordinates Motion of projectile.		
Unit VI	**Kinetics of Particle**	**(06Hrs)**
Kinetics- Newton's Second Law of motion Application of Newton's Second Law. Work, power, energy, conservative and non-conservative forces Conservation of energy for motion of particle, Impulse, Momentum, Direct central impact. Coefficient of restitution, Impulse Momentum principle of particle.		

Books and other Resources

Text Books: 1. Vector Mechanics for Engineers, by F. P. Beer and E. R. Johnson, McGraw-Hill Publication 2. Engineering Mechanics by R. C. Hibbeler, Pearson Education
Reference Books: 1. Engineering Mechanics by S. P. Timoshenko and D. H. Young, McGraw- Hill publication 2. Engineering Mechanics by J. L. Meriam and Craige, John Willey 3. Engineering Mechanics by F L Singer, Harper and Rowe publication 4. Engineering Mechanics by A. P. Boresi and R. J. Schmidt, Brooks/Cole Publication

Examination Scheme

In semester exam : 30 marks **Total marks : 125**

End Semester exam : 70 marks **Credits : 4**

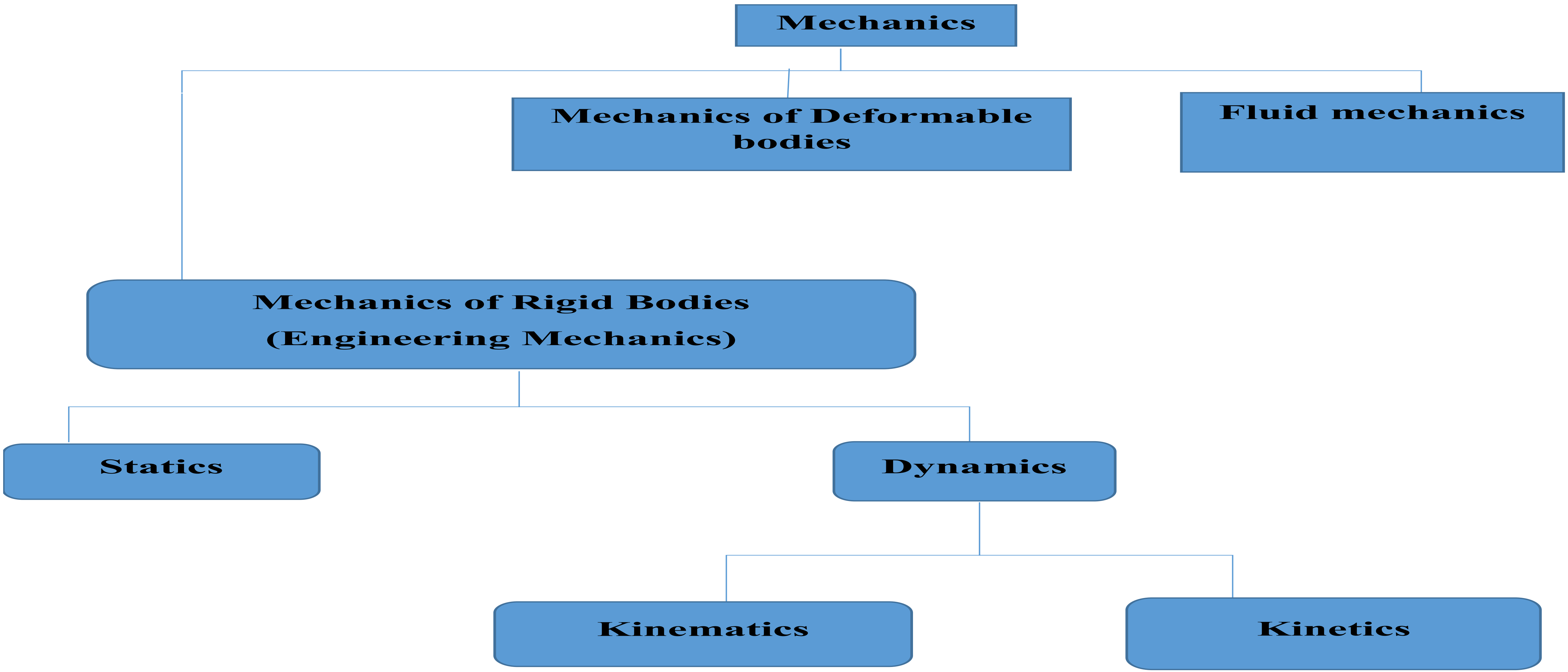
PR : 25

Unit Number	Phase I ISE Marks Weightage	Phase II ESE Marks Weightage
1	15	-
2	15	-
3		18
4		17
5		18
6		17

Unit-1 Resolution and Composition of forces

Principle of Statics, Force system, Resolution and Composition of forces , Resultant of Concurrent forces. Moment of force, Varignons theorem, resultant of parallel force system , Couple, Equivalent force-couple system, Resultant of parallel general force system.

Mechanics : It is the branch of engineering science which deals with study of forces on body when body is at rest or it is in motion.



Engineering Mechanics

What is Engineering Mechanics?

Engineering Mechanics is that branch of *applied science*, which deals with the state of rest or motion of bodies under the action of forces.

- <u>**Categories of Mechanics:**</u>
  - **Rigid bodies**
    - ***Statics***
    - ***Dynamics***
  - **Deformable bodies**
  - **Fluids**

- ***Statics*** **deals with the forces and their effects, while acting upon the bodies at rest or move with a constant velocity**,

- ***Dynamics*** **is concerned with the accelerated motion of bodies.**

➢Kinematics : It is the study of forces on bodies in motion without considering cause of motion (forces).

➢Kinetics : It is the study of forces on bodies in motion with considering cause of motion (forces)

Basic Concepts and Definitions

- ***Space*** **- associated with the notion of the position of a point P given in terms of three coordinates measured from a reference point or origin.**
- ***Time*** **- is the measure of the succession of events**

\- ***Mass*** **is a measure of the inertia of a body the quantity of matter in a body.**

is the property of every body by which it experiences mutualattraction to other bodies.

- *Force* - represents the action of one body on another.

The force is characterized characterized by its point of application, magnitude, and direction, i.e., a force is a vector quantity. In Newtonian Mechanics, *space, time, and mass* are *absolute* concepts, independent of each other. Force is not independent of the other three

Weight refers to the gravitational attraction of the earth on a body or quantity of mass. Its magnitude depends upon the elevation at which the mass is located

• Weight of a body is the gravitational force acting on it.

Concentrated Force -A **concentrated force represents the effect of a loading, which is assumed to act at a point on a body .**

Particle: **is a body of negligible dimensions**
Rigid body: **A rigid body can be considered as a combination of a large number of particles in which all the particles remain at a fixed distance from one another both before and after applying a load.**

Scalar is the measurement of a medium strictly in magnitude.

Vector is a measurement that refers to both the magnitude of the medium as well as the direction of the movement the medium has taken.

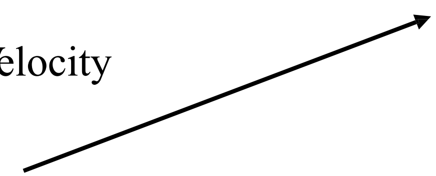
Scalar Quantities

Length, area, volume, speed, mass, density, pressure, temperature, energy, entropy, work, power

Vector Quantities

Displacement, velocity, acceleration, momentum, force, lift, drag, thrust, weight, torque, acceleration due to gravity.

Velocity



The Basic Units of Mechanics Cont.

System of Units

Name	Length	Time	Mass	Force
International System of Units (SI)	meter (m)	second (s)	kilogram (kg)	newton* (N) $\left(\frac{kg \cdot m}{s^2}\right)$
U.S. Customary (FPS)	foot (ft)	second (s)	slug* $\left(\frac{lb \cdot s^2}{ft}\right)$	pound (lb)

*Derived unit.

CHARACTERISTICS OF A FORCE

A Force has following basic characteristics

i) Magnitude

ii) Direction

iii) Point of application

iv) Line of action

Force is represented as a vector .i.e an arrow with its magnitude.

e.g. for the force shown in Fig. 2.1, magnitude of force is 4KN, direction is 40° with the horizontal in fourth quadrant, point of application is C and line of action is AB.

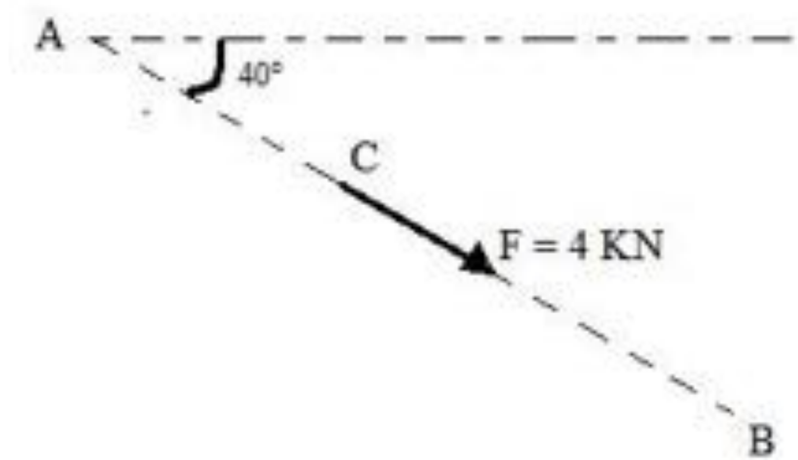


Fig. Characteristics of a force

Smaller magnitudes of forces are measured in newton (N) and larger in kilonewton (KN).

SYSTEMS OF FORCES

When a mechanics problem or system has more than one force acting, it is known as a ‘force system’ or ‘system of force’.

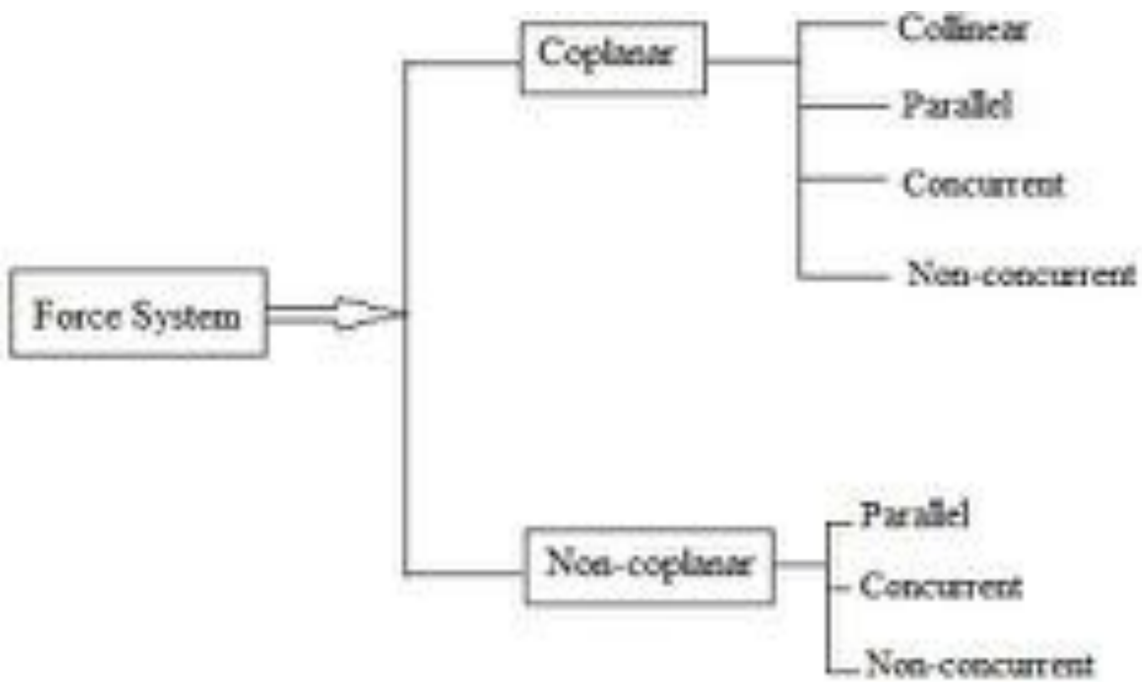


Fig. Force System

Coplanar Force System

When the lines of action of a set of forces lie in a single plane is called coplanar force system.

Non-Coplanar Force System

When the line of action of all the forces do not lie in one plane, is called Non-coplanar force system

Collinear Force System

When the lines of action of all the forces of a system act along the same line, this force system is called collinear force system.

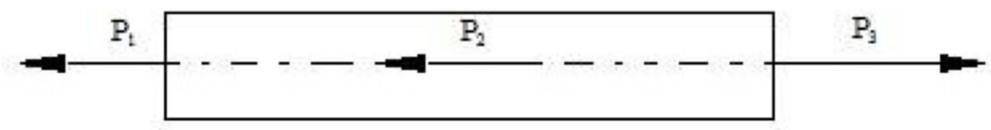


Fig. Force System

Parallel Forces

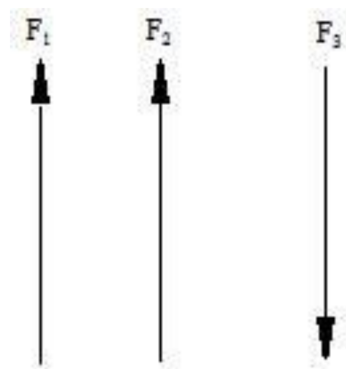


Fig. Force System

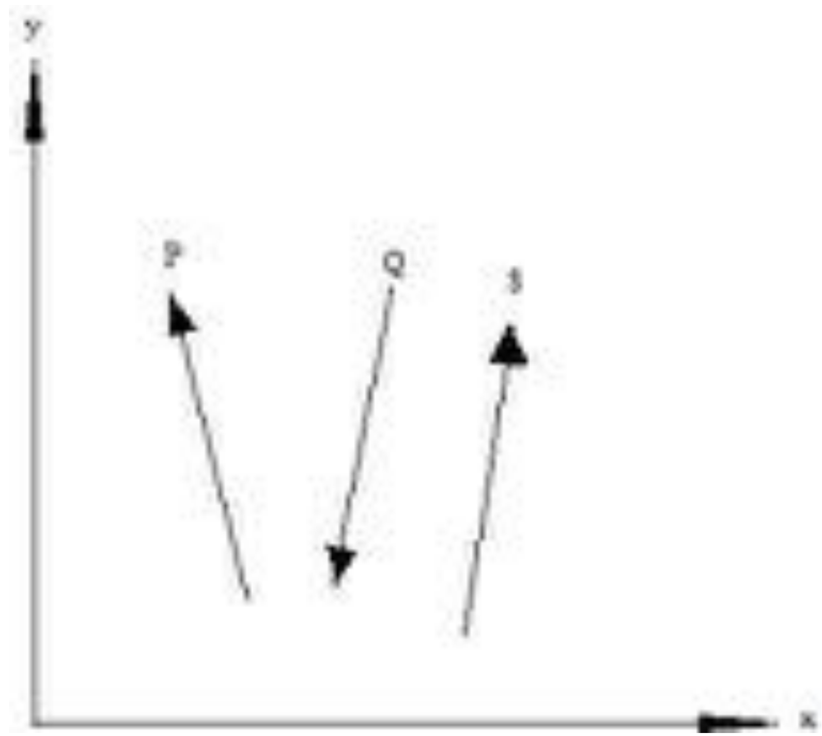


Fig. Force System

Concurrent Force System

The forces when extended pass through a single point and the point is called point of concurrency. The lines of actions of all forces meet at the point of concurrency. Concurrent forces may or may not be coplanar.

Non-concurrent Force System

When the forces of a system do not meet at a common point of concurrency, this type of force system is called non-concurrent force system. Parallel forces are the example of this type of force system. Non-concurrent forces may be coplanar or non-coplanar.

PRINCIPLE OF SUPERPOSITION OF FORCES

- This principle states that the combined effect of force system acting on a particle or a rigid body is the sum of effects of individual forces.
- Consider two forces P and Q acting at A on a boat as shown in Fig.3.1. Let R be the resultant of these two forces P and Q. According to Newton's second law of motion, the boat will move in the direction of resultant force R with acceleration proportional to R. The same motion can be obtained when P and Q are applied simultaneously.

EQUILIBRIUM OF FORCES

- Equilibrium is defined as the condition of a body, which is subjected to a force system whose resultant force is equal to zero.
- It means the effect of the given force system is zero and the particle or rigid body is said to be in equilibrium.
- For example, a particle subjected to two forces will be in equilibrium when the two forces are equal in magnitude, opposite in direction and act along the same line of action as shown in Figure.
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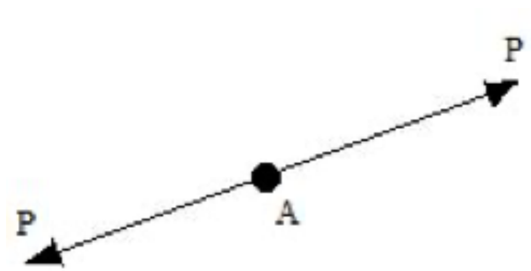


Equilibrium of forces

RESOLUTION OF A FORCE INTO COMPONENTS

- A given force F can be resolved into (or replaced by) two forces, which together produces the same effects that of force F. These forces are called the components of the force F.
- This process of replacing a force into its components is known as resolution of a force into components.
- A force can be resolved into two components, which are either perpendicular to each other or inclined to each other.
- If the two components are perpendicular to one another, then they are known as rectangular components and when the components are inclined to each other, they are called as inclined components.
- The resolution of force into components is illustrated as follows.

Resolution of a Force into Rectangular Components

Consider a force F acting on a particle O inclined at an angle Θ as shown in Fig.4.1(a).
Let x and y axes can be the two axes passing through O perpendicular to each other. These two axes are called rectangular axes or coordinate axes. They may be horizontal and vertical or inclined as shown in Fig. 4.1(b)

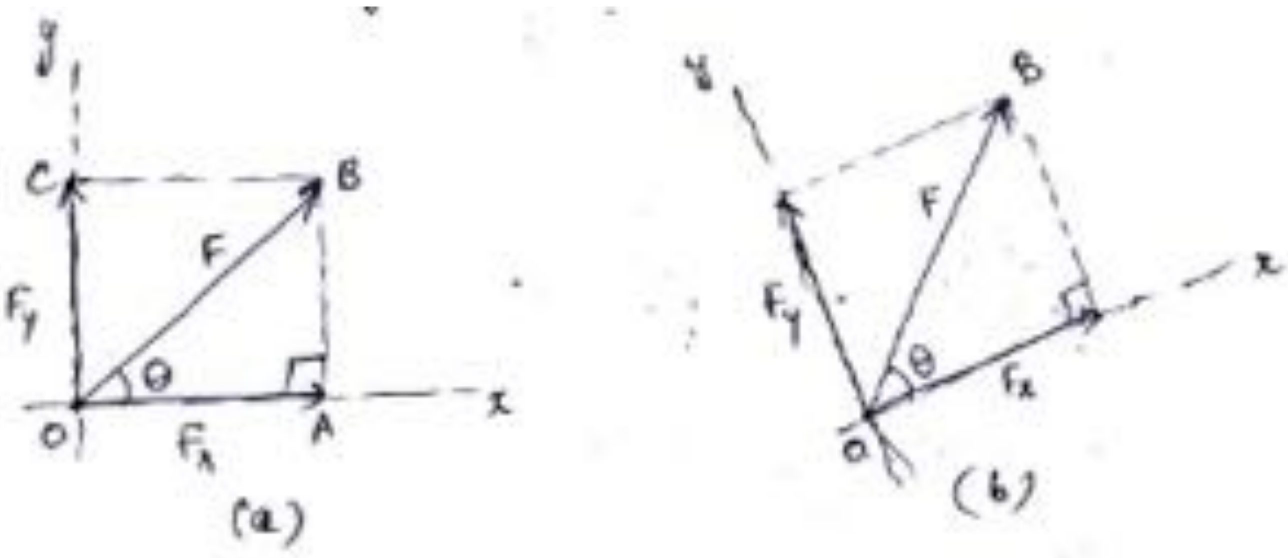


Fig. Resolution of force into rectangular components

The force F can now be resolved into two components F_x and F_y along the x and y axes and hence, the components are called rectangular components. Further, the polygon constructed with these two components as adjacent sides will form a rectangle OABC and, therefore, the components are known as rectangular components.

Therefore,
OA = OB × cos θ
Or

$$F_x = \mathrm{OA} = F\cos\theta \qquad \text{(a)}$$

Therefore,
AB = OB × sin θ

$$F_y = \mathrm{OC} = \mathrm{AB} = F\sin\theta \qquad \text{(b)}$$

Therefore, the two rectangular components of the force F are:
$F_x = F\cos\theta$ and $F_y = F\sin\theta$

- The conventional coordinate directions are used for the sign conventions of the components of the force.
- That is, the components along the coordinate directions are considered as positive components and the one in the opposite direction as negative components.
- The sign conventions shown in Fig.4.2 are used in general.

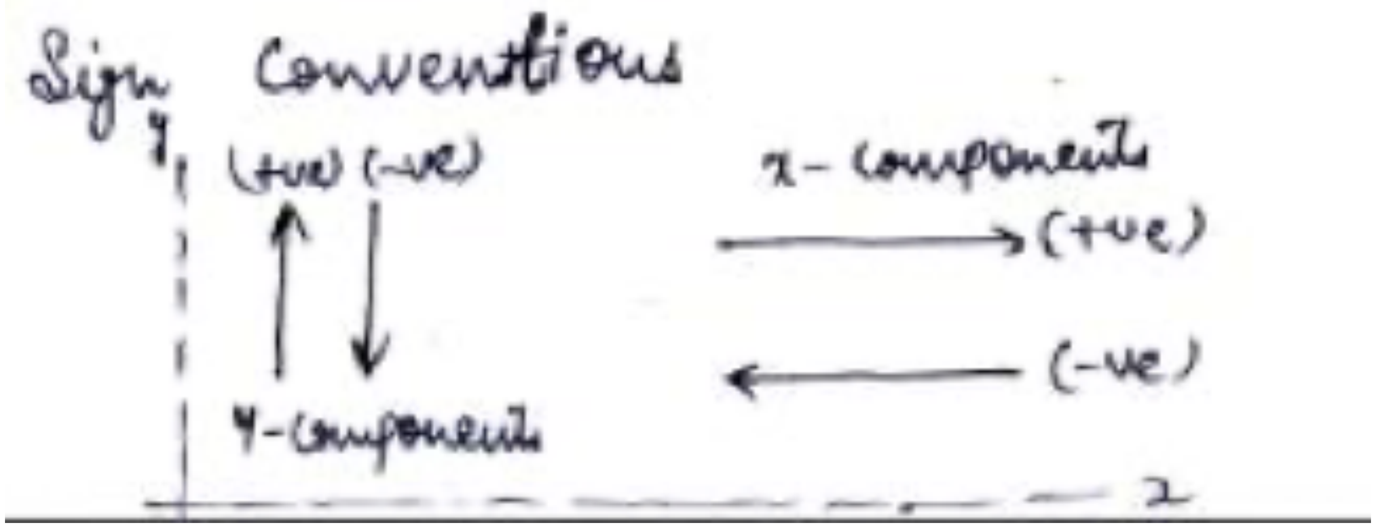
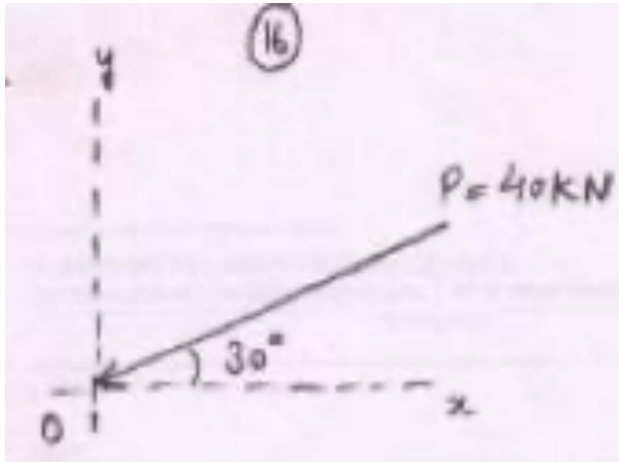
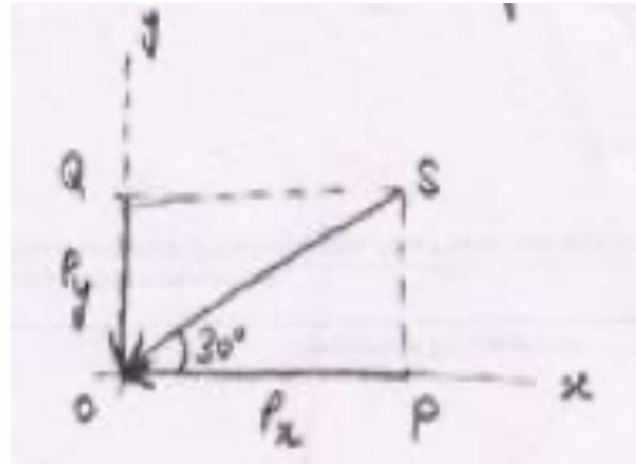


Fig. Sign conventions

Example : Determine the components of force $P = 40$ kN along x and y as shown in Figure.





Solution: Plot a rectangle OPSQ taking the force P (that is OS) as the diagonal as illustrated in Fig.4.4, the two components P_x and P_y can be obtained.

Consider the right angle triangle OPQ in which

$\cos 30^\circ = {{OP} \over {OS}}$

OrOP = OS cos 30°

Therefore,

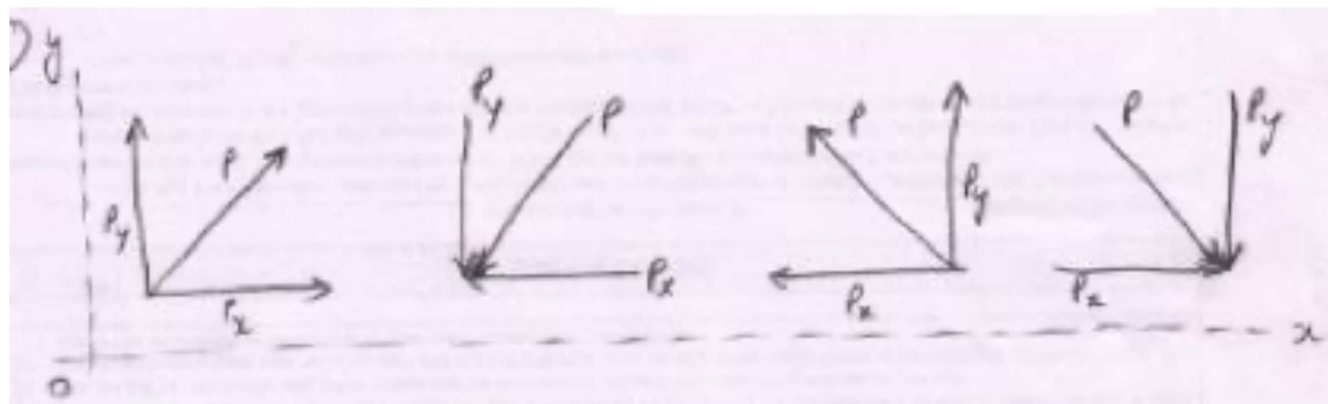
$P_x = P \cos 30^\circ = 40 \cos 30^\circ = 34.64$ KN (←)

Hence,

OS = OQ sin 30°

$P_y = P \sin 30^\circ = 40 \sin 30^\circ = 20$ KN (↓)

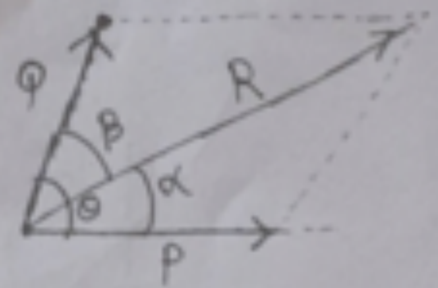
Note: The directions of P_x and P_y are obtained based on the direction of P as shown follows:



* Composition of forces :- It is the process of finding resultant.

Resultant :- A single force which has same effect as that of number of forces.

Parallelogram law of forces :-



$\theta = \alpha + \beta$

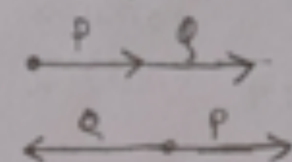
If two forces are represented in magnitude and direction by two adjecent sides of parallelogram, then their resultant is represented in magnitude and direction by diagonal of parallelogram passing through the intersection of two forces.

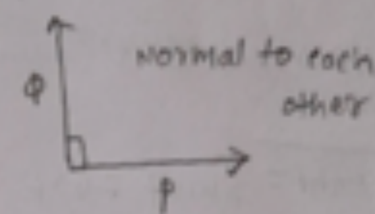
$$R = \sqrt{P^2 + Q^2 + 2PQ\cos\theta} \qquad \tan\alpha = \frac{Q\sin\theta}{P + Q\cos\theta} \qquad \tan\beta = \frac{P\sin\theta}{Q + P\cos\theta}$$

$R \to R_{max}$ $\quad \theta = 0^\circ$ $\quad R_{max} = P + Q$

$R \to R_{min}$ $\quad \theta = 180^\circ$ $\quad R_{min} = P - Q$

$\theta = 90^\circ$ $\quad R = \sqrt{P^2 + Q^2}$





(P.8) If two forces of magnitude P and 2P acts on body, then their a) maximum resultant b) minimum resultant will be?

Max resultant (R_{max}) $= P + 2P = \underline{\underline{3P}}$, Min resultant ($R_{min}$) $= 2P - P = \underline{\underline{P}}$

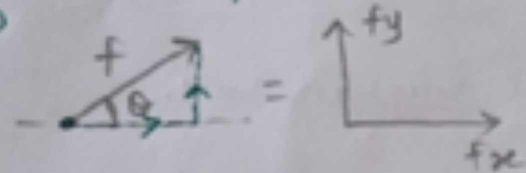
(P.9) If two forces of magnitude 7N and 8N acts at 60° then their resultant will be?

$P = 7N$, $Q = 8N$, $\theta = 60^\circ$ $\quad R = \sqrt{P^2 + Q^2 + 2PQ\cos\theta} = \underline{\underline{13N}}$.

* **Resolution :-** It is the process of spliting given force into two or more components without changing its effect.

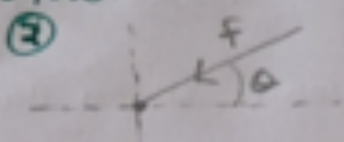
a) **Orthogonal resolution / Perpendicular resolution**
Components are at 90° to each other.

(1)

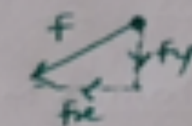


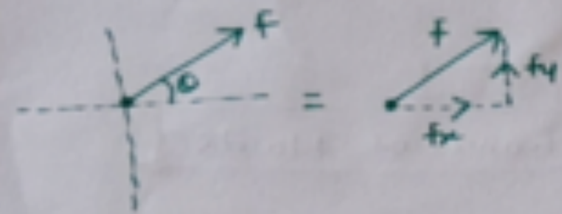
$f_x = f\cos\theta$
$f_y = f\sin\theta$

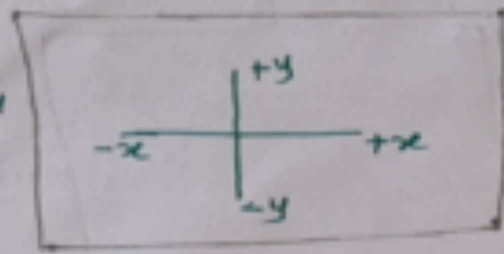
(2)

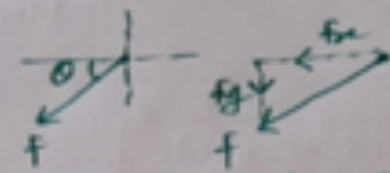


$f_x = -f\cos\theta$
$f_y = -f\sin\theta$



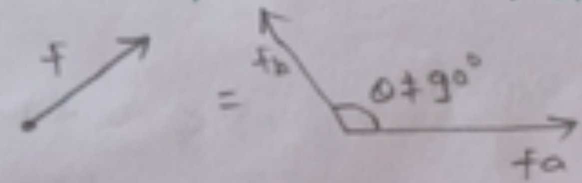


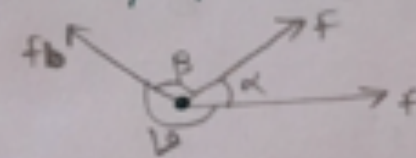




$f_x = -f\cos\theta$
$f_y = -f\sin\theta$

b) **Non-orthogonal resolution / Non-perpendicular resolution**
components are not at 90°

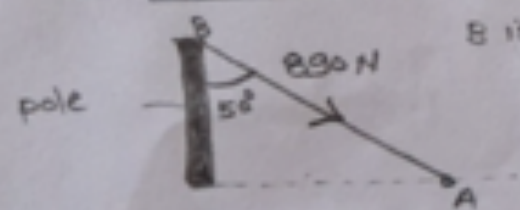




$$\frac{f_a}{\sin\beta} = \frac{f}{\sin\gamma} = \frac{f_b}{\sin\alpha}$$

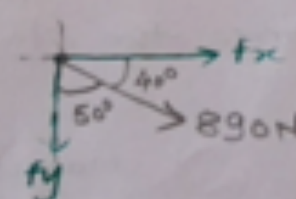
Lamis Theorem :- If body is in the equilibrium under action of three forces then each force is proportional to sin of angle between remaining two force.

P.1 A telephone pole is supported by wire which exert pull of 890N on the top of the pole. If angle between pole and the wire is 50°. What is the horizontal and vertical components of force.



B is top of pole

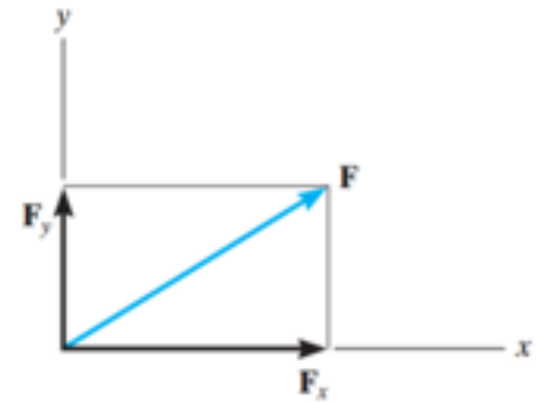
FBD of B



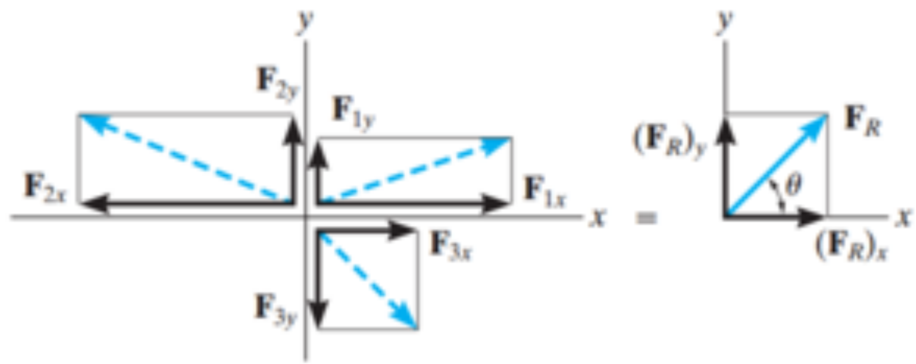
$f_x = 890\cos 40 = 681.8\text{N}$
$f_y = 890\sin 40 = 572.1\text{N}$

2.2. Rectangular Components : Two Dimensions

Vectors F_x and F_y are rectangular components of F.



The resultant force is determined from the algebraic sum of its components.



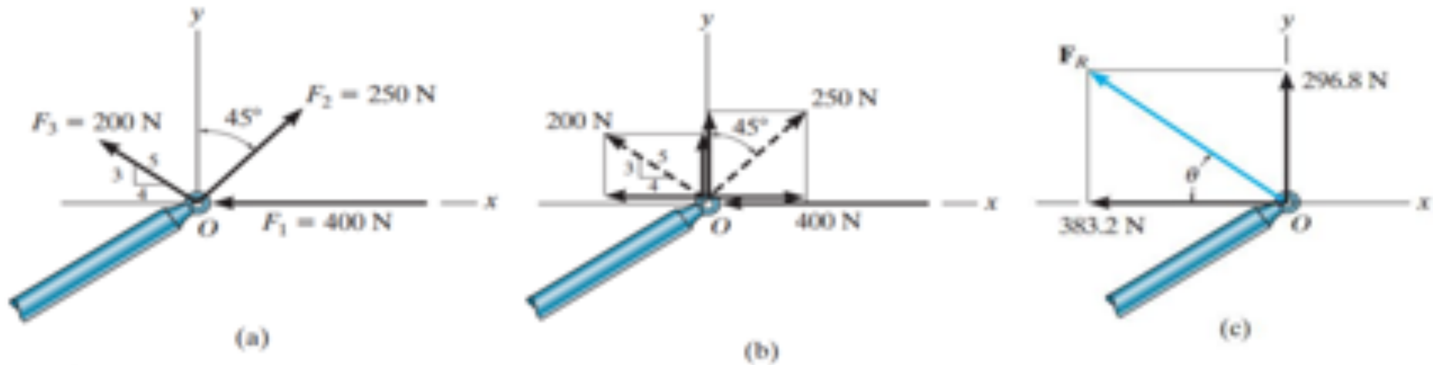
$$(F_R)_x = \Sigma F_x$$

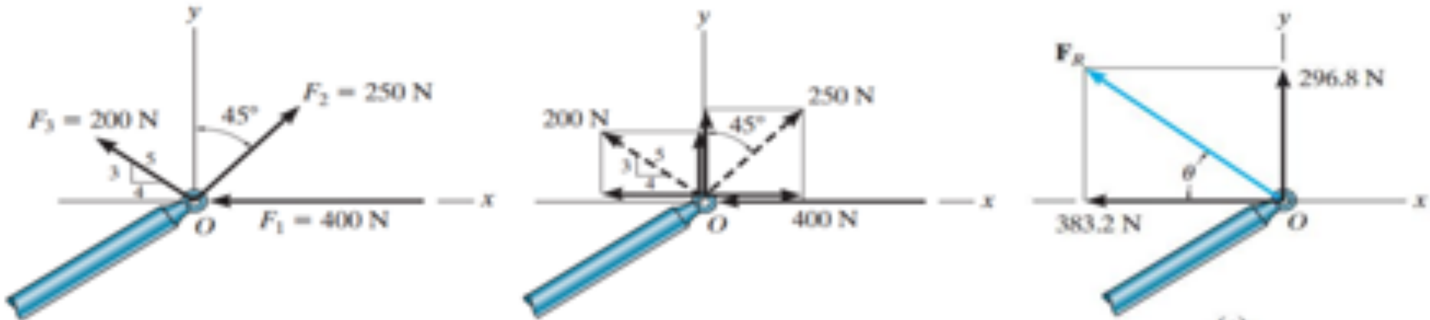
$$(F_R)_y = \Sigma F_y$$

$$F_R = \sqrt{(F_R)_x^2 + (F_R)_y^2}$$

$$\theta = \tan^{-1}\left|\frac{(F_R)_y}{(F_R)_x}\right|$$

Example: The end of the boom O in Figure (a) below is subjected to three concurrent and coplanar forces. Determine the ***magnitude*** and ***direction*** of the resultant force.





(a) (b) (c)

Solution:

Each force is resolved into its x and y components, Figure (b), Summing the x-components and y-components:

$$\xrightarrow{+} (F_R)_x = \Sigma F_x; \qquad (F_R)_x = -400 \text{ N} + 250 \sin 45^\circ \text{ N} - 200\left(\tfrac{4}{5}\right) \text{ N}$$
$$= -383.2 \text{ N} = 383.2 \text{ N} \leftarrow$$

$$+\uparrow (F_R)_y = \Sigma F_y; \qquad (F_R)_y = 250 \cos 45^\circ \text{ N} + 200\left(\tfrac{3}{5}\right) \text{ N}$$
$$= 296.8 \text{ N}\uparrow$$

The resultant force, shown in Figure c, has a magnitude of:

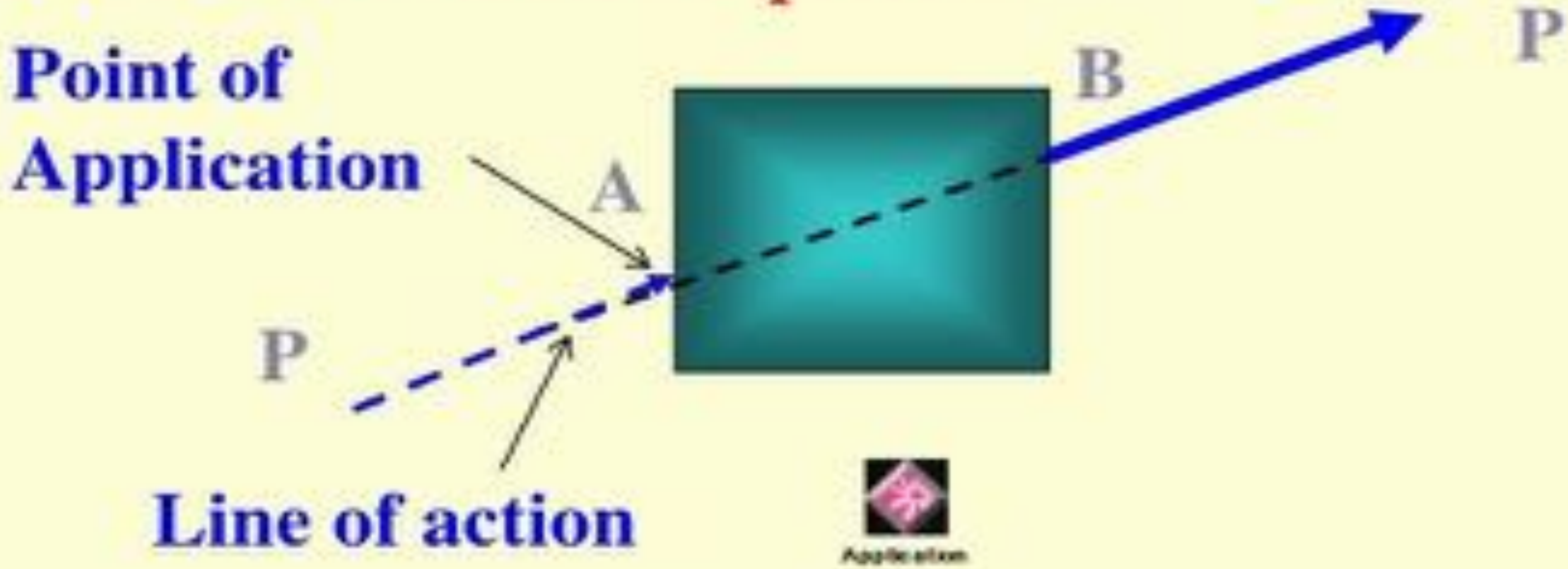
$$F_R = \sqrt{(-383.2 \text{ N})^2 + (296.8 \text{ N})^2}$$
$$= 485 \text{ N}$$

The direction angle θ is:

$$\theta = \tan^{-1}\left(\frac{296.8}{383.2}\right) = 37.8^\circ$$

Principle of Transmissibility

If we move *(TRANSMIT)* the Force P from point A to point B which lies on the Line of Action of Force P, the **same effect would be expected.**



Principle of transmissibility states that a force acting on a rigid body at different points along the force's line of action will produce the same effect on the body.

zero and the particle or rigid body is said to be in equilibrium.

For example, a particle subjected to two forces will be in equilibrium when the two forces are equal in magnitude, opposite in direction and act along the same line of action as shown in Figure.

Equations of equilibrium for a concurrent, coplanar force system

The resultant of a concurrent, coplanar force system is a single force through the point of concurrence. When the resultant force is zero, the body on which the force system acts in equilibrium.

Consider the force system as shown in figure:

If the sum of the *x* components of the forces of the system is equal to zero, the resultant can act only along the *y* axis.

If in addition, the sum of the *y* components of the forces of the system is equal to zero, the resultant must be zero. Consequently, one complete set of equations of equilibrium for a concurrent, coplanar force system is

$$\sum F_x = 0, \quad \sum F_y = 0 \qquad (1)$$

Again, if the sum of the *x* components of the forces of the system is equal to zero, the resultant can be only a force along the *y* axis and if the sum of the moments of the forces of the system with respect to an axis through A is equal to zero where A is any point not on the *y* axis is not zero. Thus, another set of equations which assure equilibrium for this system is

$$\sum F_x = 0, \quad \sum M_A = 0 \qquad (2)$$

Where A is not on the *y* axis.

In a similar manner, a third set of independent equations can be shown to be

$$\sum M_A = 0, \quad \sum M_B = 0 \qquad (3)$$

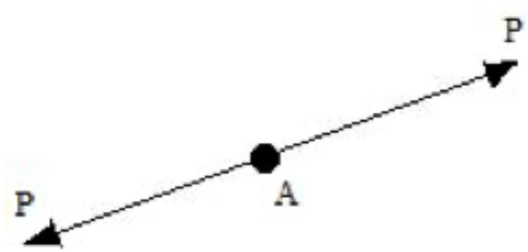


Fig. Equilibrium of forces

- Where line AB does not pass through the point of concurrence of the forces of the system. There are only two independent equations of equilibrium for a concurrent, coplanar force system. When a force system of this type contains not more than two unknowns (two magnitudes, one magnitude and one slope, or two slopes), they can be determined directly from the equations of equilibrium.
- When a concurrent, coplanar force system contains more than two unknowns, they cannot all be determined from the equations of equilibrium alone, and the force system is said to be statically indeterminate.
- For a collinear force system, Eq.(1) reduces to one equation,
- $\sum F_x = 0$
- Where the *x* axis is parallel to the forces. Likewise, Eq.(2) can be reduced to the equation
- $\sum M_A = 0$

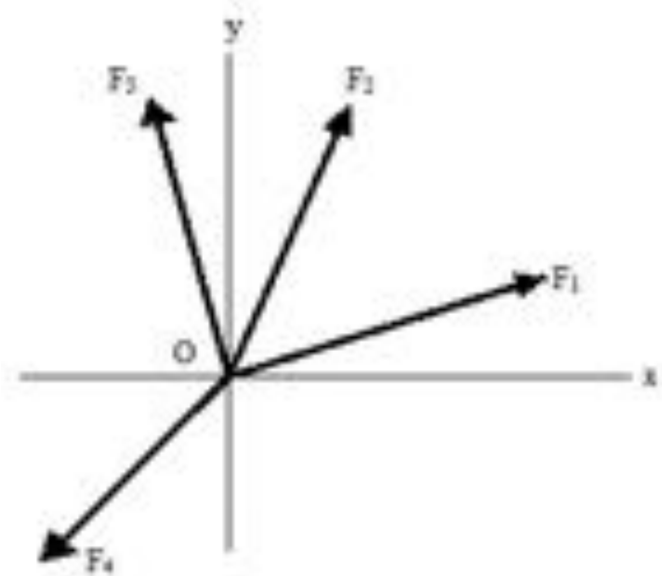


Fig. Equilibrium of concurrent and coplanar Force system

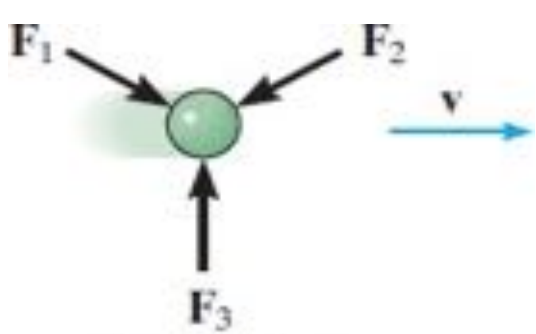
Newton’s Three Laws of Motion. Engineering mechanics is formulated on the basis of Newton’s three laws of motion, the validity of which is based on experimental observation. These laws apply to the motion of a particle as measured from a nonaccelerating reference frame. They may be briefly stated as follows.

First Law. A particle originally at rest, or moving in a straight line with constant velocity, tends to remain in this state provided the particle is not subjected to an unbalanced force. Fig. a

Second Law. A particle acted upon by an *unbalanced force* **F** experiences an acceleration **a** that has the same direction as the force and a magnitude that is directly proportional to the force. Fig. b

If **F** is applied to a particle of mass *m*, this law may be expressed mathematically as

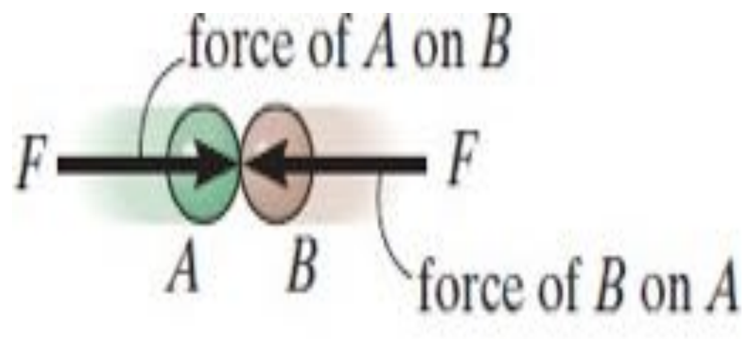
$$\mathbf{F} = m\mathbf{a}$$



Equilibrium
(a)

Accelerated motion
(b)

Third Law. The mutual forces of action and reaction between two particles are equal, opposite, and collinear. Fig. c



Action – reaction

(c)

Newton's Law of Gravitational Attraction. This law states that a body attracts every other body in the universe with a force which is directly proportional to the product of their masses but also inversely proportional to the square of the distance between their centers. Mathematically ,

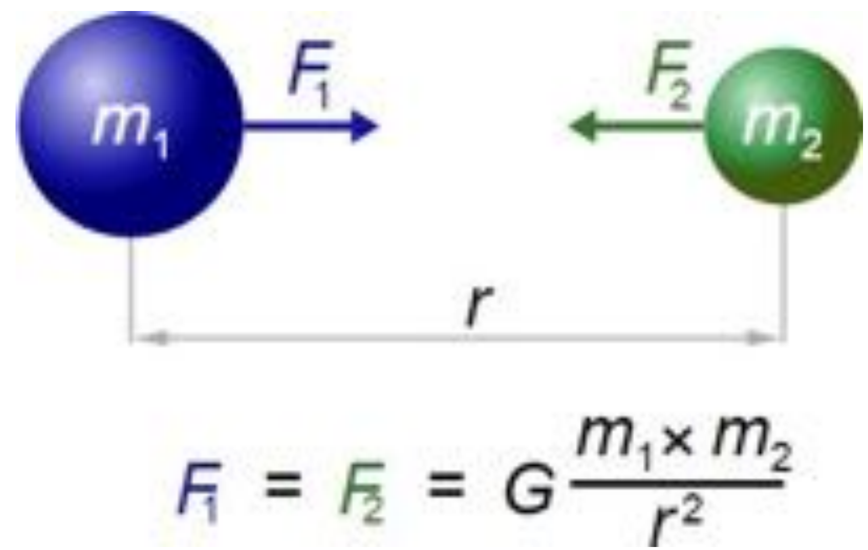
$$F \propto m_1 m_2$$

$$F \propto \frac{1}{r^2}$$

Combining above two equations,

$$F \propto \frac{m_1 m_2}{r^2}$$

$$F = G \frac{m_1 m_2}{r^2}$$



where:

- F is the force between the masses;
- G is the gravitational constant (6.674×10^{-11} N · (m/kg)^2)
- m_1 is the first mass;
- m_2 is the second mass;
- r is the distance between the centers of the masses.

Fundamental Principles

Newton's Law of Gravitation: **Newton postulated a law governing the gravitational attraction between any two particles. This law is known as Newton's Law of Gravitation and is expressed mathematically by the equation:**

$$F = G\frac{m_1 m_2}{r^2}$$

Weight: **The magnitude *w* of the weight of a particle of mass *m* is**
$w = m$ **g**

Gravitation Problems Answers

1. Find the gravitational force of attraction between two elephants, one of mass 1.00×10^3 kg and the other of mass 8.00×10^2 kg, when they are separated by 5.0 m.

$m_1 = 1000$ kg $\quad m_2 = 800$ kg $\quad r = 5.0$ m

$G = 6.67 \times 10^{-11}$ N m^2/kg^2 $\quad F_g = ?$

Gravitation Problems Answers

1. Find the gravitational force of attraction between two elephants, one of mass 1.00×10^3 kg and the other of mass 8.00×10^2 kg, when they are separated by 5.0 m.

$m_1 = 1000\ \text{kg} \qquad m_2 = 800\ \text{kg} \qquad r = 5.0\ \text{m}$

$G = 6.67 \times 10^{-11}\ \text{N m}^2/\text{kg}^2 \qquad F_g = ?$

$$F_g = \frac{G\, m_1\, m_2}{r^2}$$

$$= \frac{(6.67 \times 10^{-11}\ \text{N}\ \cancel{\text{m}^2}/\cancel{\text{kg}^2})(1000\ \cancel{\text{kg}})(800\ \cancel{\text{kg}})}{(5.0\ \cancel{\text{m}})^2}$$

$$\boxed{F_g = 2.1 \times 10^{-6}\ \text{N}}$$

Introduction

- A force will cause motion along its direction.

- A force may also cause rotation about a fixed point some distance away.

- This rotation or turning effect of a force is called MOMENT.

Example: what is the moment of the 40^{KN} about point A?

Solution:

The perpendicular distance from *A* to the line of action of the force is

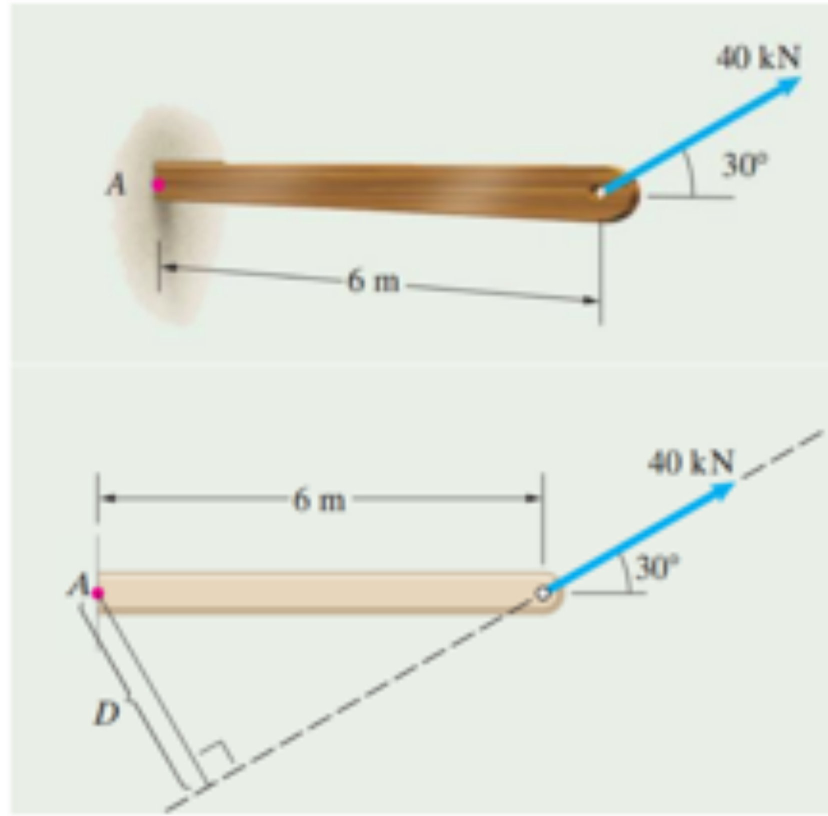
$D = (6\text{ m})\sin 30° = 3\text{ m}.$

Therefore the magnitude of the moment is

$(3\text{ m})(40\text{ kN}) = 120\text{ kN-m}.$

The direction of the moment is counterclockwise, so

$M_A = 120\text{ kN-m}.$



Practice Problem Resolve the 40-kN force into horizontal and vertical components and calculate the sum of the moments of the components about *A*.

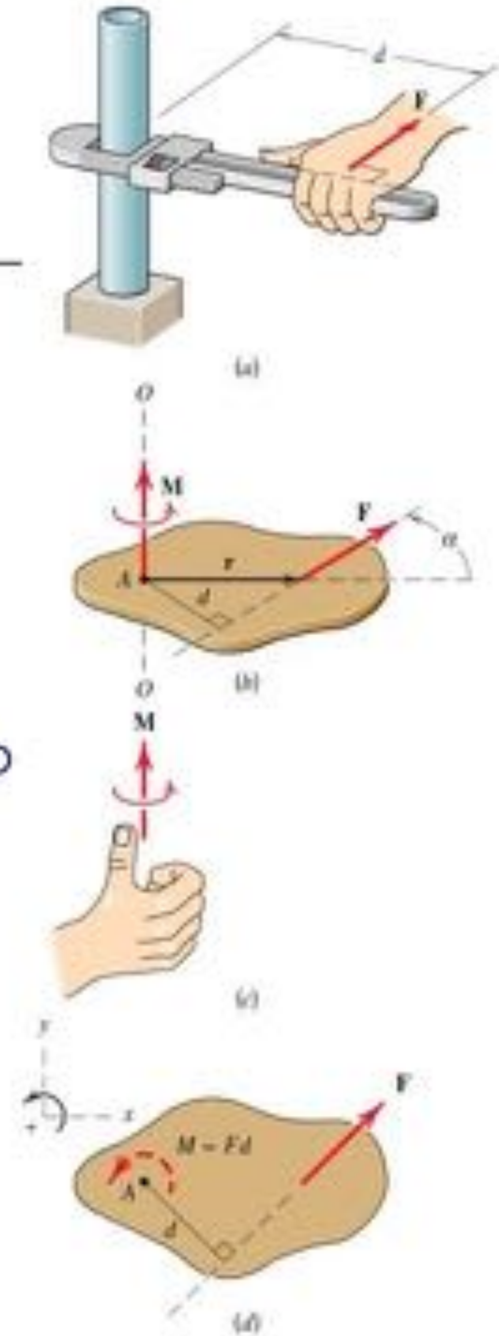
Moment

- A vector approach for moment calculations is proper for 3D problems.
- Moment of **F** about point A maybe represented by the cross-product

$$\boxed{\mathbf{M} = \mathbf{r} \times \mathbf{F}}$$

where **r** = a position vector from point A to **any** point on the line of action of **F**

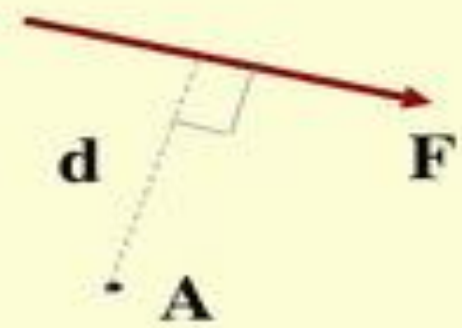
$$\boxed{M = Fr \sin \alpha = Fd}$$



Moment of A Force

- The moment *M* of a force *F* about a fixed point *A* is defined as the *product of the magnitude of force* ***F*** *and the perpendicular distance* ***d*** *from point A to the line of action of force F.*

$$M_A = F \times d$$



Where *force F* is in newtons, N

And *distance d* is in meters, m

Thus *moment M_A* is in newton-meter, Nm.

Moment Sign Convention

Anti-clockwise : + VE
(Counter-clockwise)

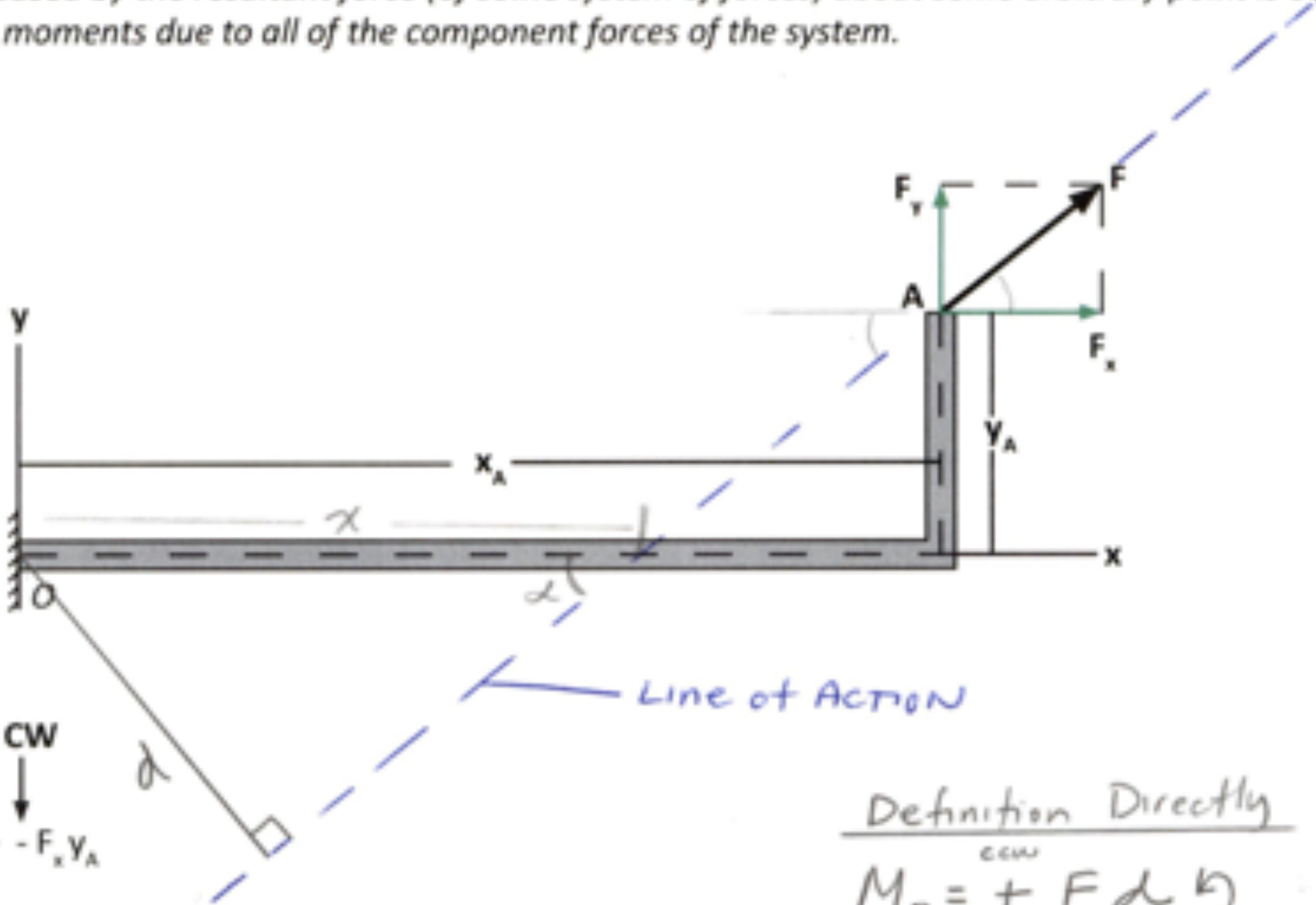
Clockwise : - VE

Varignon's Theorem

- Varignon's Theorem states that " *the moment of a force about any point* is *equal to the sum of the moments of its components about the same point*".
- To calculate the moment of any force with a slope or at an angle to the x or y-axis, resolve the force into the F_x and the F_y components, and calculate the sum of the moment of these two force components about the same point.

Varignon's Theorem

The moment caused by the resultant force (of some system of forces) about some arbitrary point is equal to the sum of the moments due to all of the component forces of the system.



CCW CW

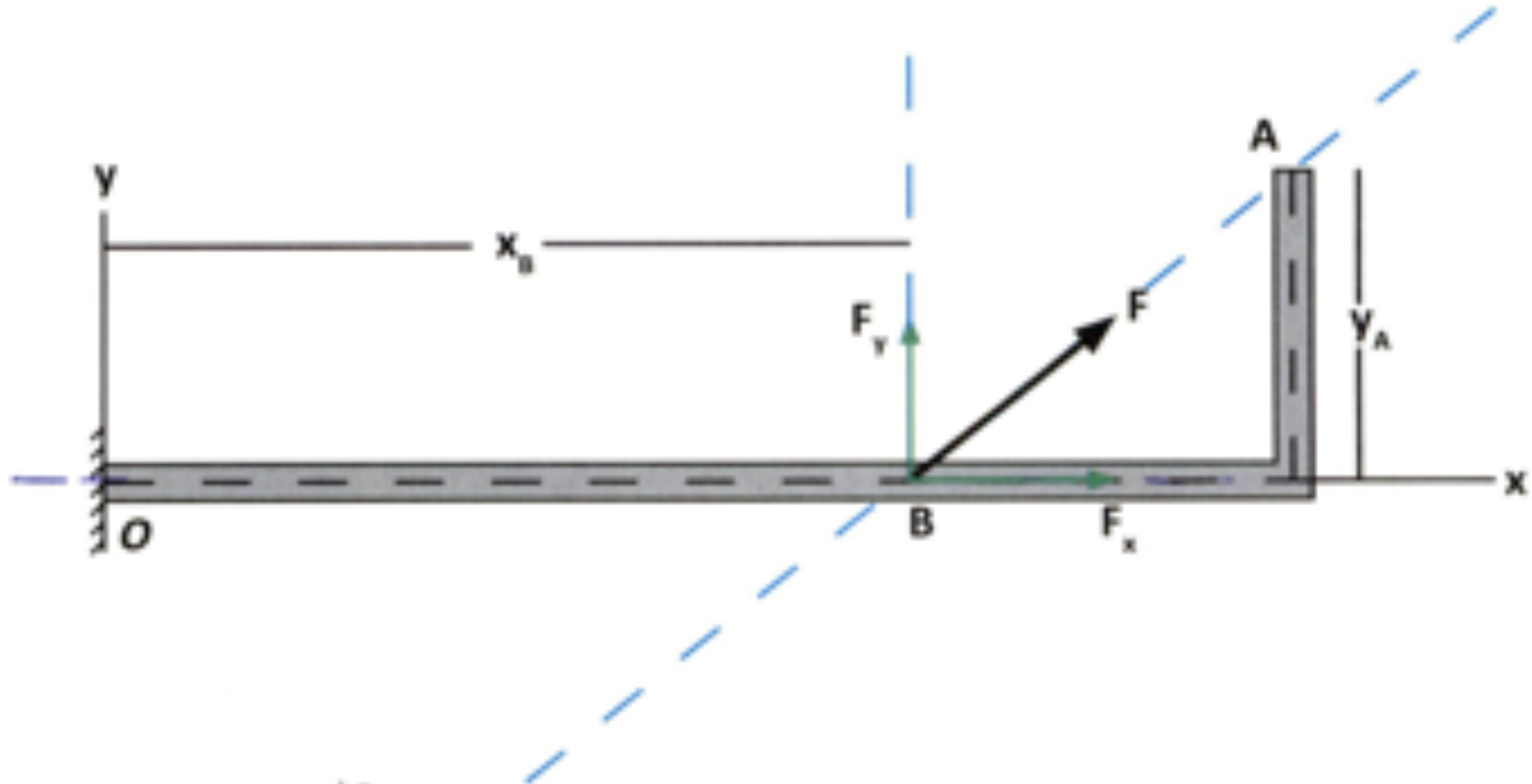
$$M_O = +F_y\, x_A + -F_x\, y_A$$

Definition Directly

$$M_O \overset{ccw}{=} + F d \circlearrowleft$$

Principle of Transmissibility
Moment arm is independent of the point of application of a force.

As long as the magnitude, direction, and the line of action of a force are defined, the moment of the force about a given point may be determined by placing the force at any point along its line of action.

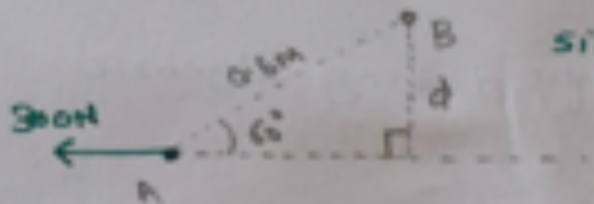


CCW

$M_O = +F_y\, x_B + F_x \times 0 = F_y\, x_B$

Since the line of action of the component F_x passes through the moment center O, it produces no moment about O.

(P22) A member AB of 600mm is inclined at 60° to the horizontal. A force of 300N acts towards left horizontally at point 'A'. The moment at point B will be ______



$\sin 60 = \frac{d}{0.6}$

$d = 0.52\,m$

$M_B = F \times d$

$= 300 \times 0.52$

$M_B = 155.88\ N\cdot m$ (↻)

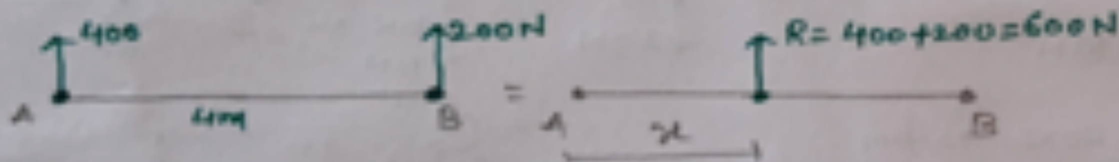
* Varignon's theorem :-

The algebric sum of moments of all forces about any point is equal to moment of resultant about same point.

$$\Sigma M_A = M_{R_A}$$

- It is used to find position (location) of resultant.
- It is also called as 'principle of moments'.

(P23) Two like parallel forces P = 400N and Q = 200N acting at ends of rod 4m length, distance of resultant from 400N force will be ______



From Varignons theorem

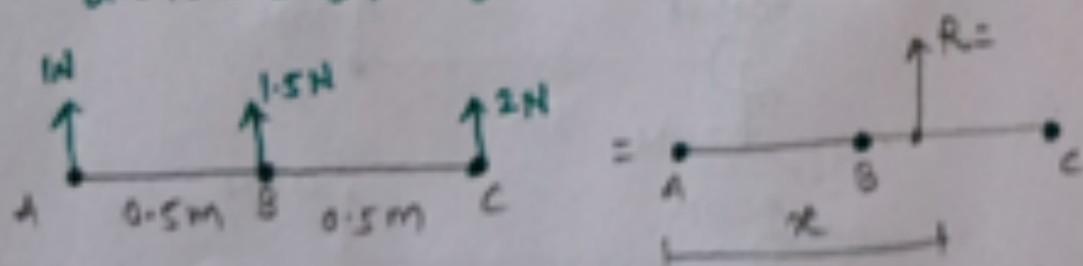
$\Sigma M_A = M_{R_A}$

$400 \times 0 + 200 \times 4 = 600 \times x$

$\underline{\underline{x = 1.33\,m}}$ from A

from B = 2.67m

(P24) If three like parallel forces 1N, 1.5N and 2N acts at distance of 0.5 m each find distance of resultant from 1N force.



$R = 1+1.5+2 = 4.5N$

from Varignons theorem

$\Sigma M_A = M_{RA}$

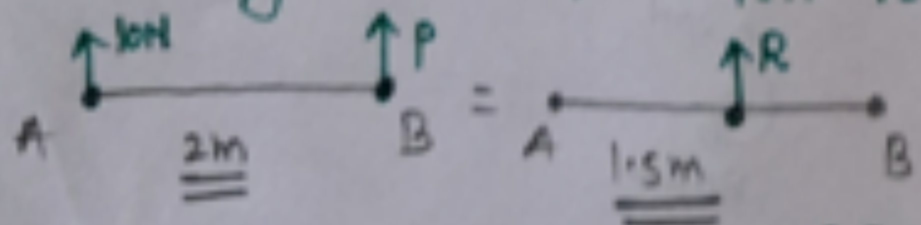
$1.5\times0.5+2\times1 = 4.5\times x$

$x = \underline{0.61\,m}$ from A

$= 0.11\,m$ from B

$= 0.39\,m$ from C

(P25) Two like parallel of 10N and PN acts at ends of rod 2m. If the resultant is acting at 1.5m from 10N force, value of P will be ——



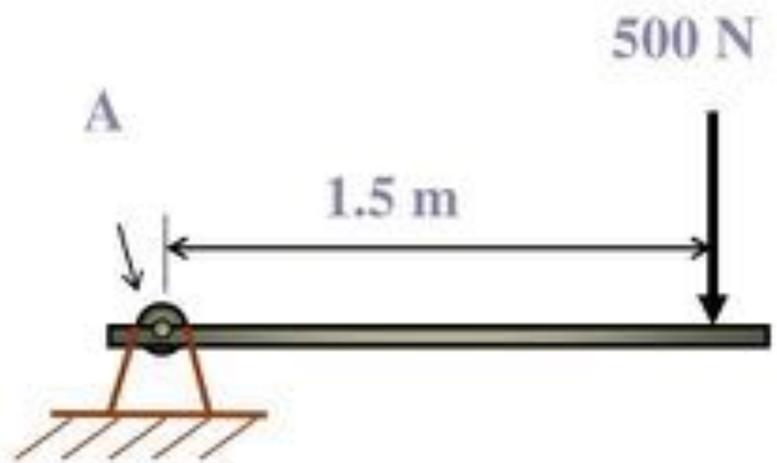
According to Varignons theorem

$\Sigma M_A = M_{RA}$

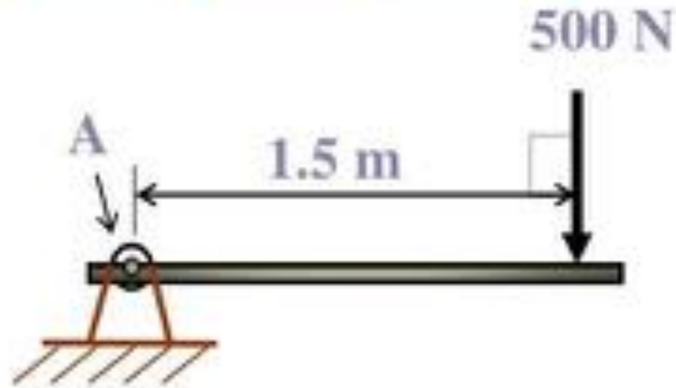
$2P = 1\cdot$

$\underline{P = 30N}$

- Calculate the moment of the 500 N force about the point A as shown in the diagram.



Example 3.1
Solution



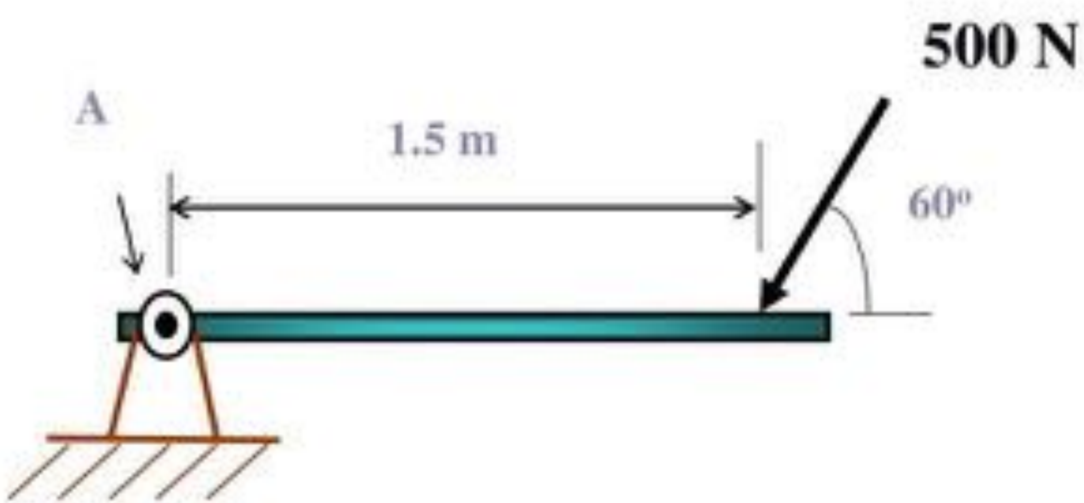
Since the perpendicular distance from the force to the axis point A is 1.5 m, from

$$M_A = F \times d$$

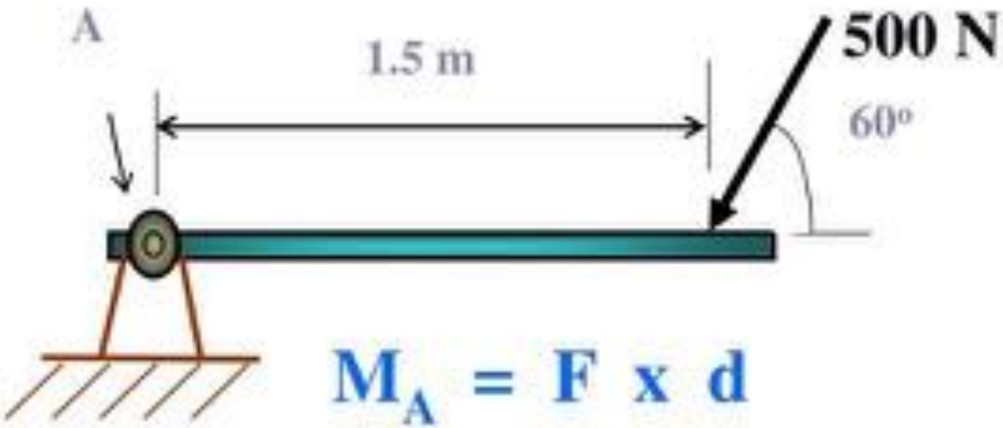
$$\begin{aligned} M_A &= -500 \times 1.5 \\ &= -750 \text{ Nm} \\ &= 750 \text{ Nm} \end{aligned}$$

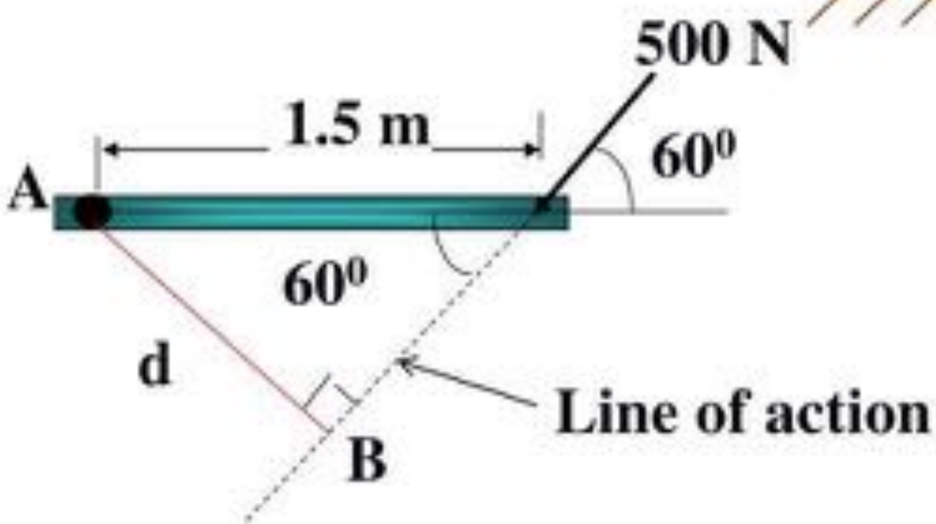
Example 3.2

- Calculate the moment about point A caused by the 500 N force as shown in the diagram.



Example 3.2
Solution:





$$M_A = F \times d$$

$$= -500 \times AB$$

$$= -500 \times 1.5 \sin 60$$

$$= -649.5 \text{ Nm}$$

$$= 649.5 \text{ Nm}$$

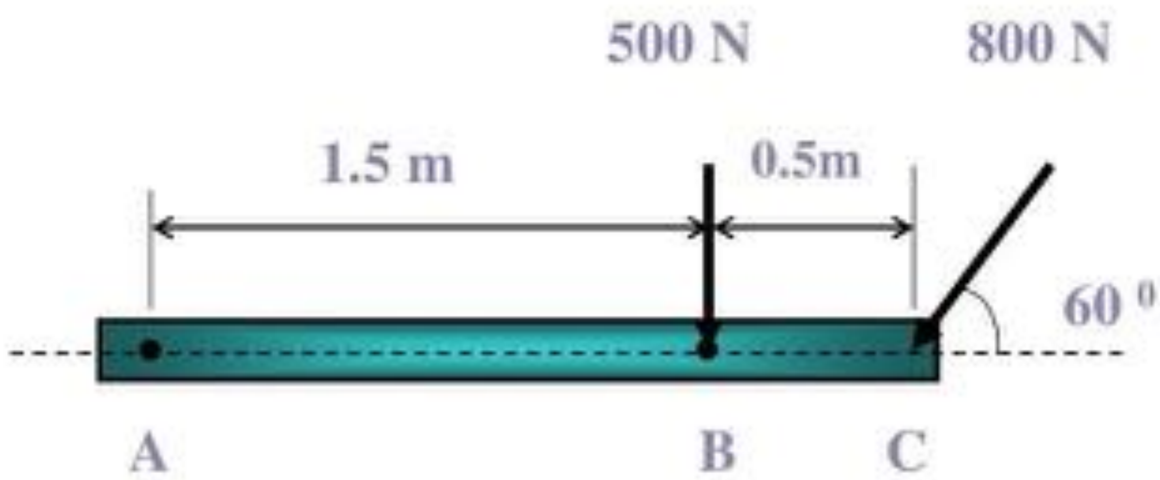
Addition of Moments of Coplanar Forces

- *Coplanar Forces* refers to forces acting on the *Same Plane* (i.e. *2-D*).
- For a system of several coplanar forces, the combined turning effect of the forces can be determined by *adding algebraically* the moment caused by each individual force, taking into account their sense of rotation,

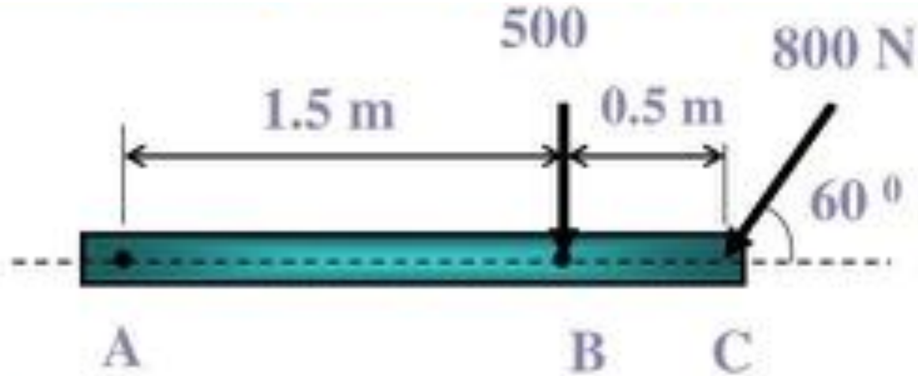
  i.e +ve for anti-clockwise and

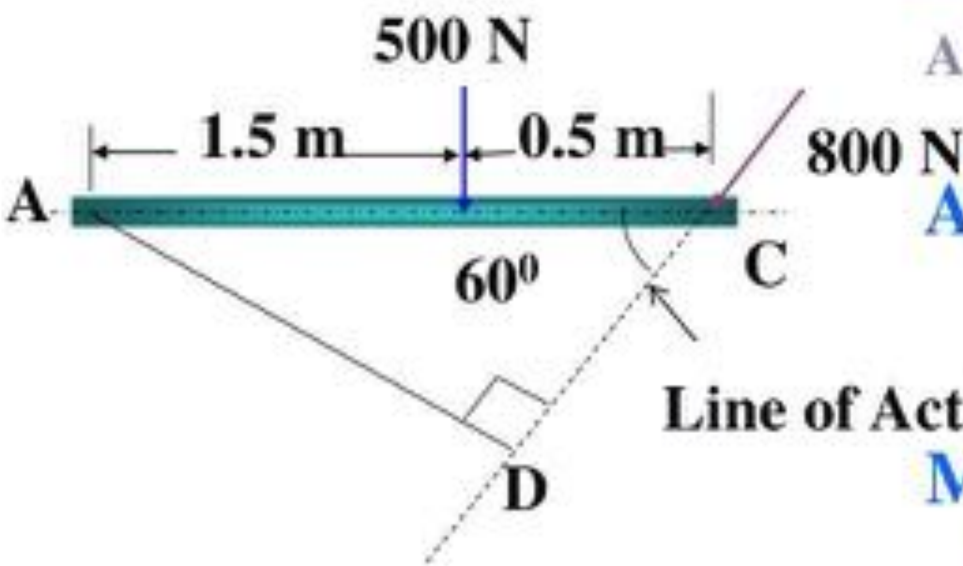
  –ve for clockwise rotation.

Determine the resulting moment about point A of the system of forces on bar ABC as shown in the diagram.



Example 3.3
Solution:

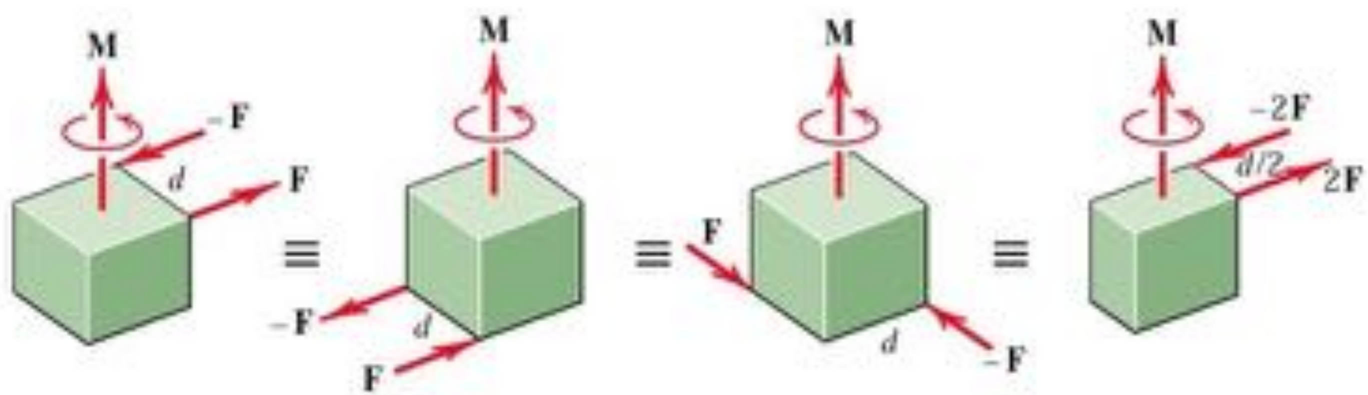




$$AD = (1.5 + 0.5) \sin 60$$
$$= 2.0 \sin 60$$
$$= 1.732 \text{ m}$$
$$M_A = (-500 \times 1.5) + (-800 \times 1.732)$$
$$= -750 - 1385.6$$
$$= -2135.6 \text{ Nm}$$
$$= 2135.6 \text{ Nm}$$

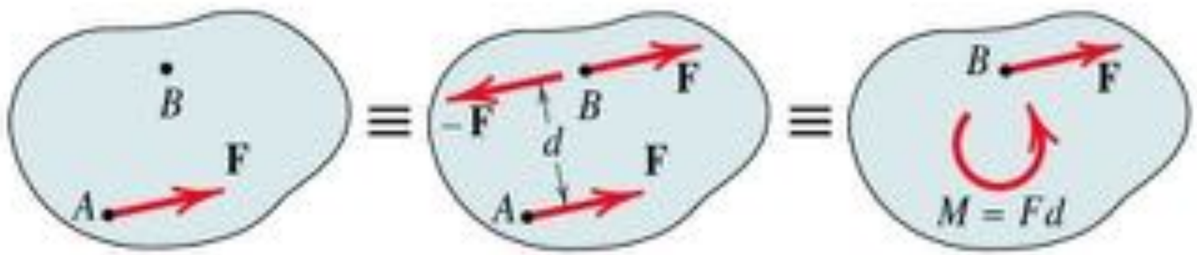
Couple

- Equivalent couples
  - Change of values *F* and *d*
  - Force in different directions but parallel plane
  - Product *Fd* remains the same

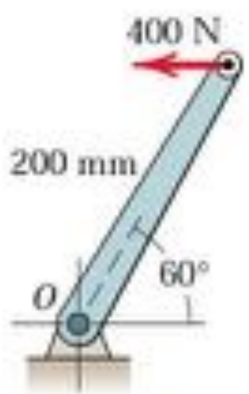


Force-Couple Systems

- Replacement of a force by a force and a couple
- Force **F** is replaced by a parallel force **F** and a counterclockwise couple *Fd*

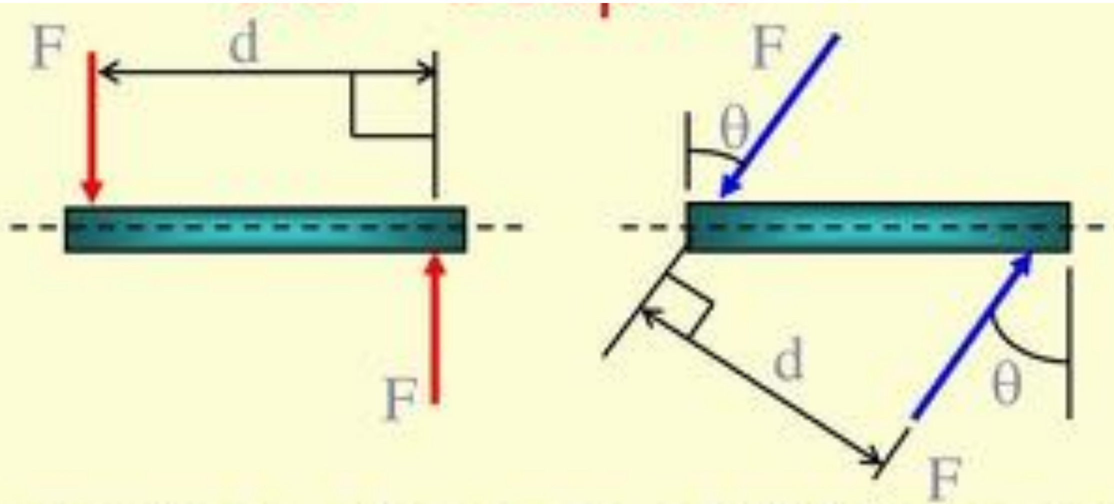


<u>Example</u> Replace the force by an equivalent system at point O



Also, reverse the problem by the replacement of a force and a couple by a single force

COUPLE



A couple consists of a pair of 2 forces which has the following properties :-

- *Equal magnitude and opposite in direction*
- *Act along parallel lines of action*
- *Separated by a perpendicular distance d.*

What a couple does?

A couple causes a body to *rotate only* without translational motion since the two forces '*cancels*' out each other giving zero resultant.

A couple acting in a system of forces will only contribute to the resulting moment but not to the resulting force.

Couple: Example

Moment required to turn the shaft connected at center of the wheel = 12 Nm

Case I: Couple Moment produced by 40 N forces = 12 Nm

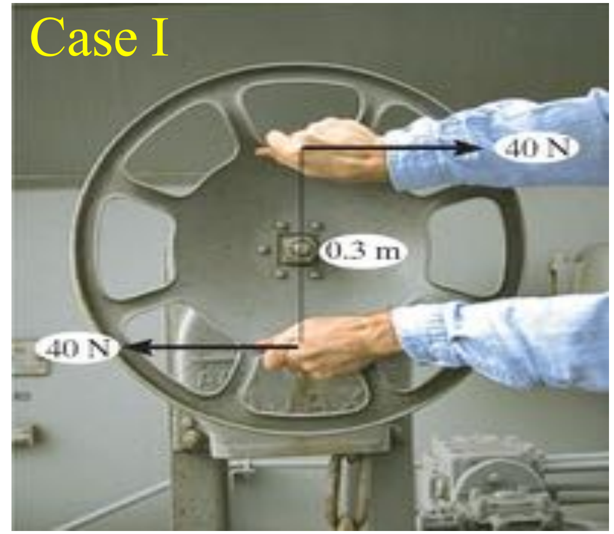
Case II: Couple Moment produced by 30 N forces = 12 Nm

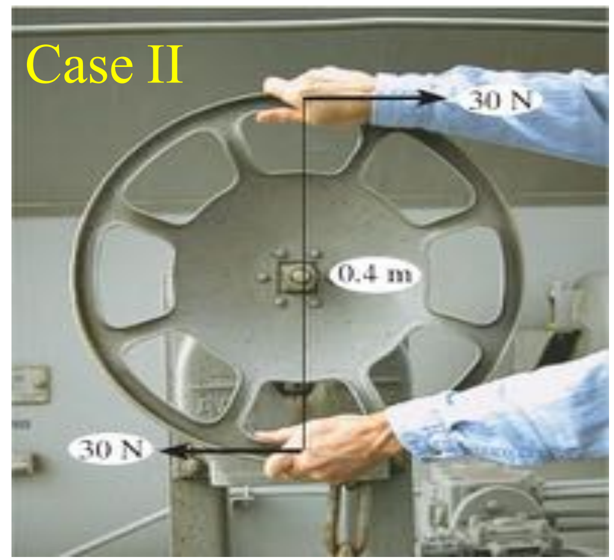
If only one hand is used?

Force required for case I is 80N Force required for case II is 60N

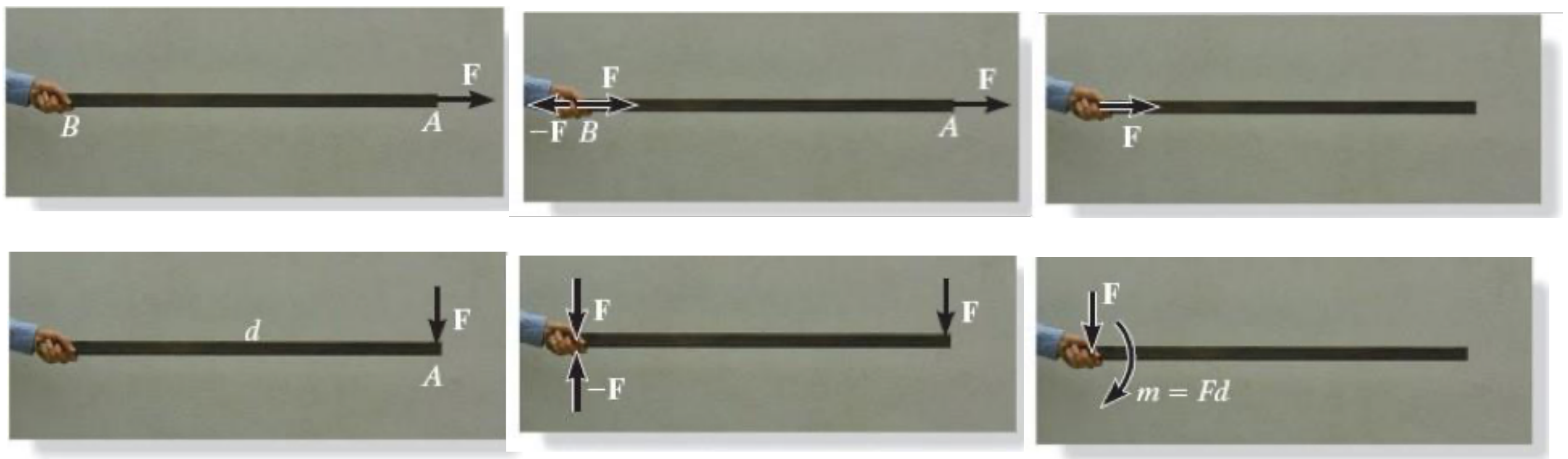
What if the shaft is not connected at the center of the wheel?

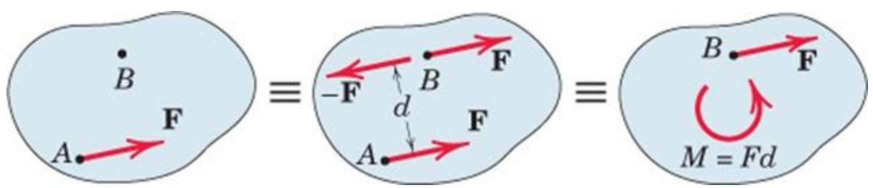
Is it a Free Vector?





Equivalent System



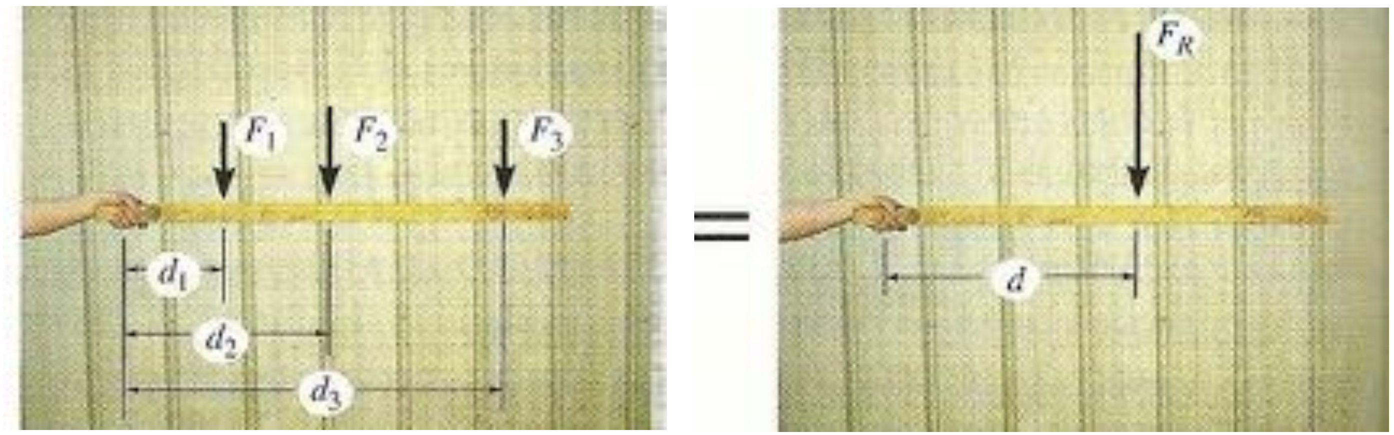


At support O

$$W_r = W_1 + W_2$$

$$M_o = W_1 d_1 + W_2 d_2$$

Equivalent System



$F_R = F_1 + F_2 + F_3$

What is the value of d?

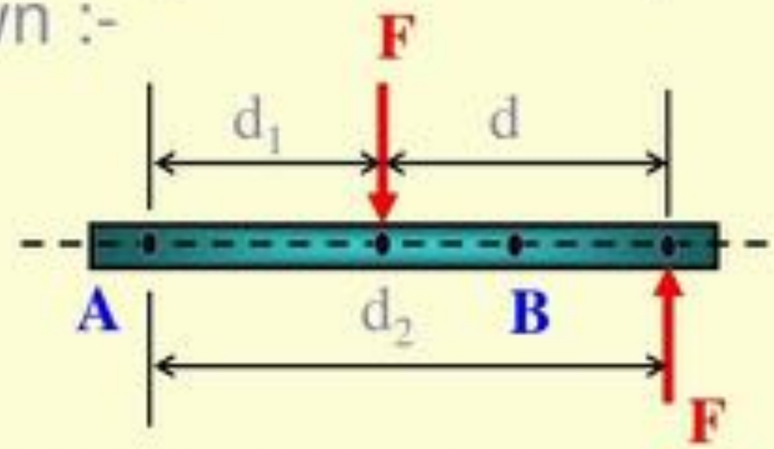
Moment of the Resultant force about the grip must be equal to the moment of the forces about the grip

$$\boxed{\mathbf{F_R d = F_1 d_1 + F_2 d_2 + F_3 d_3}}$$

Equilibrium Conditions

Magnitude of a Couple

Consider a light bar acted upon by a couple as shown :-



The moment of the couple about A is

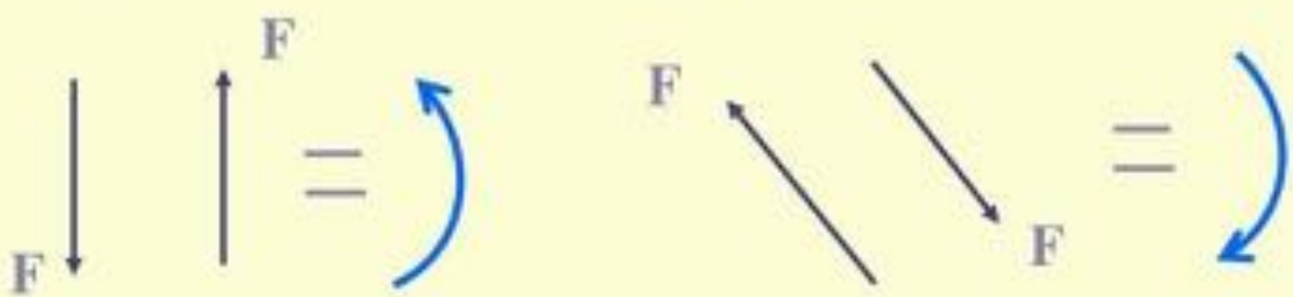
$$M_A = +(F \times d_2) - (F \times d_1)$$
$$M_A = F \times (d_2 - d_1)$$
$$M_A = F \times d$$

What is the total moment of the couple about point B ?

From above, we see that :

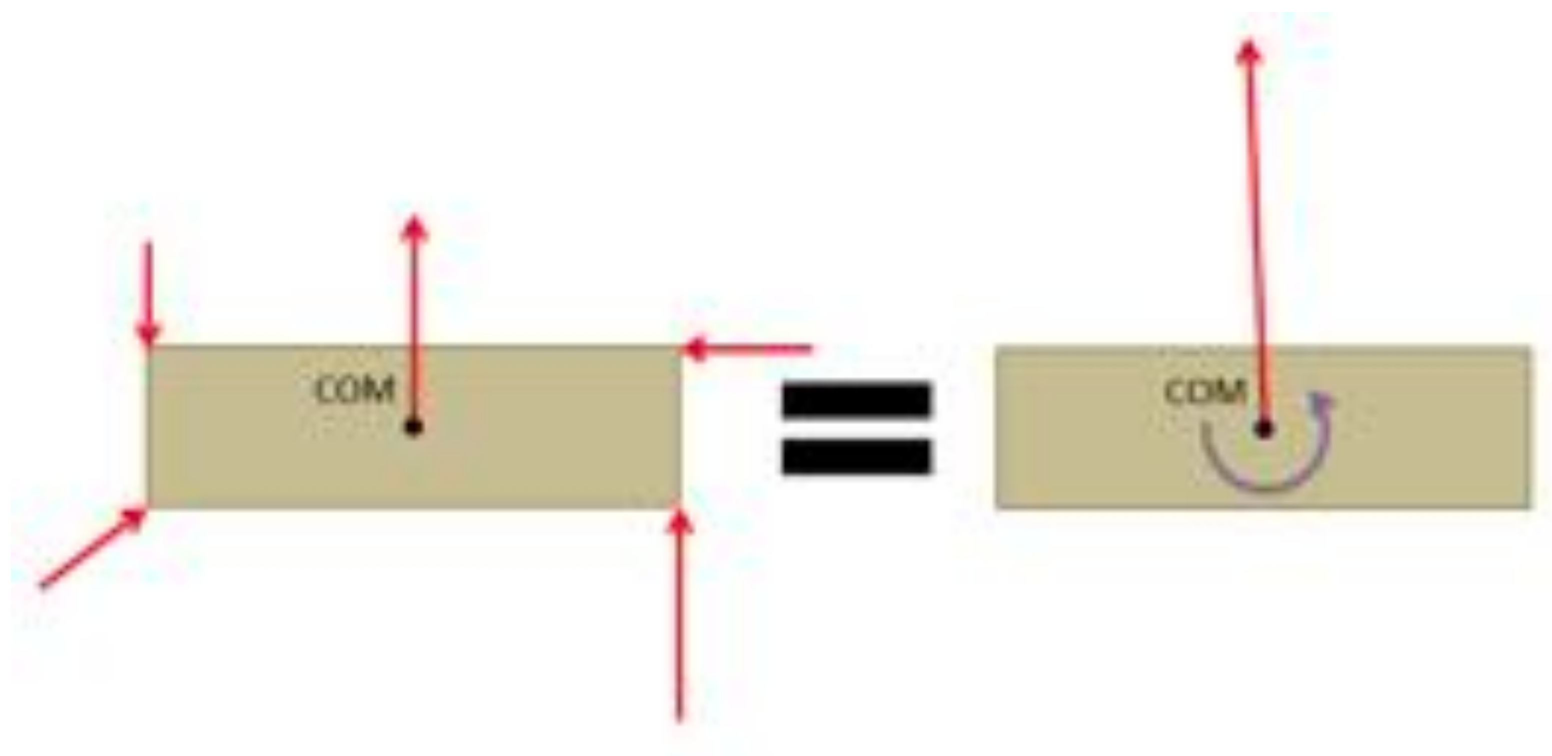
The couple's moment about *any pivot point* is equal to *F x d*

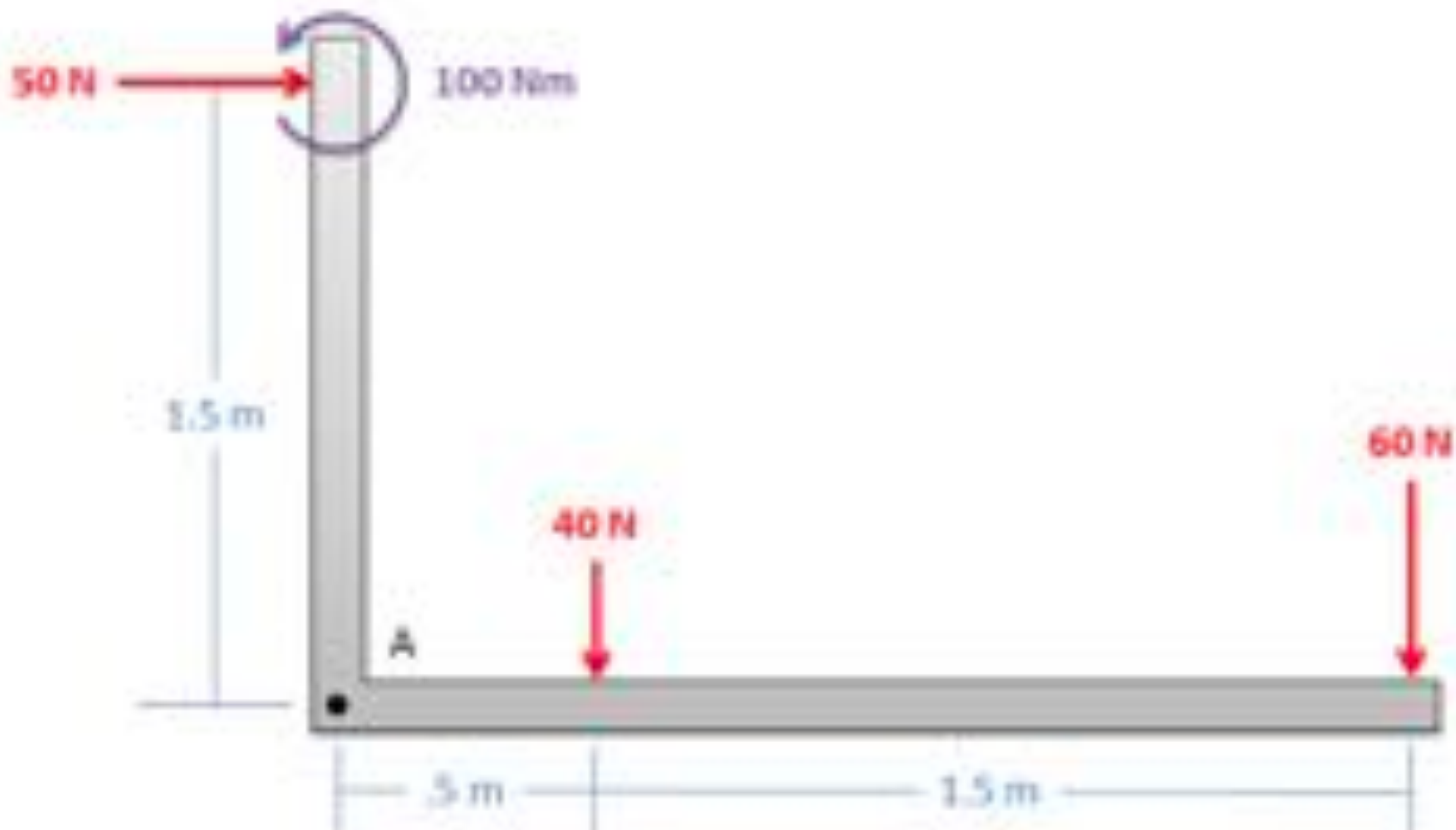
A couple has the same moment about all points on a body.



Equivalent Force Couple System

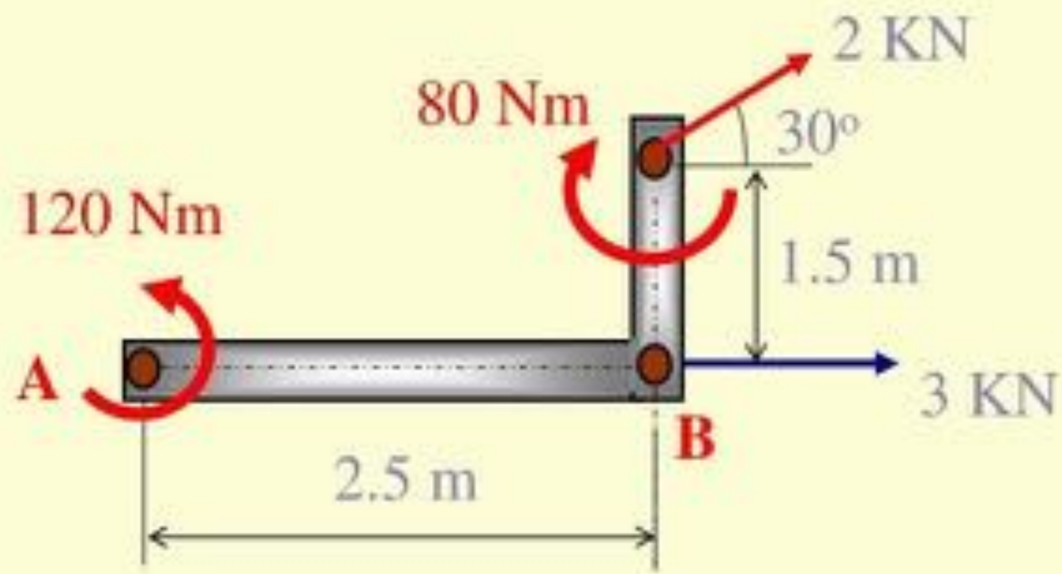
Every set of forces and moments has an **equivalent force couple system**. This is a single force and pure moment (couple) acting at a single point that is **statically equivalent** to the original set of forces and moments.



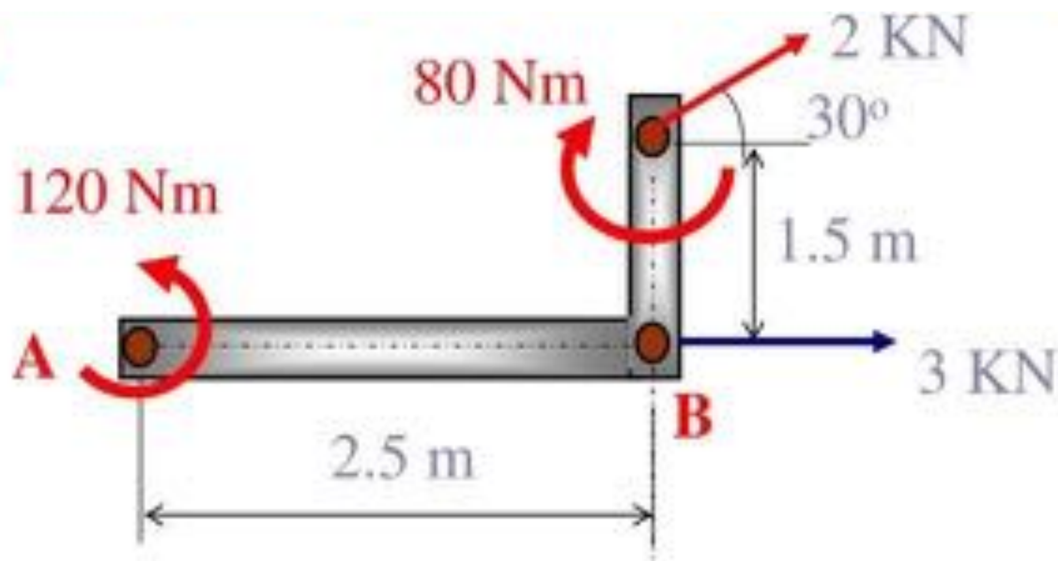

50 N
100 Nm
1.5 m
60 N
40 N
A
.5 m
1.5 m

Example 3.6

A light bracket ABC is subjected to two forces and two couples as shown. Determine the moment at (a) *point A* and (b) *point B*.



Example 3.6
Solution:



a) $\Sigma M_A = (2000 \cos 60 \times 2.5) - (2000 \sin 60 \times 1.5) + (3000 \times 0)$
$+ 120 - 80$
$= 2500 - 2598 + 0 + 120 - 80$
$= -58 \text{ Nm} = 58 \text{ Nm}$

(b) $\Sigma M_B = (2000 \cos 60 \times 0) - (2000 \sin 60 \times 1.5) + (3000 \times 0)$
$+ 120 - 80$
$= -2558 \text{ Nm}$
$= 2558 \text{ Nm}$

EQUIVALENT FORCE-COUPLE SYSTEM

A force at any given point on a rigid body can always be replaced by another force of same direction but acting at different point along with an associated couple.

Let P be a force acting on a rectangular plane at point A as shown in Fig. (a). Introducing two collinear forces of magnitude P acting opposite to one another at point B and parallel to the one acting at A as indicated in fig. 12(b), the condition remain same as in Fig. (a) itself. According to the principle of superposition, the systems shown in Fig. (a) and (b) are statically equivalent. Subsequently, the force P acting at A and the one acting at B opposite to that at A can be combined together to form a couple, the moment of which is $M = Pa$ acting in counter-clockwise direction [see fig. (c)]. This couple can be applied at any point on the plate and is shown in fig. (a) is statically equivalent to the force-couple system shown in Figures. This indicates that any given force can be reduced into an equivalent force-couple system and vice versa.

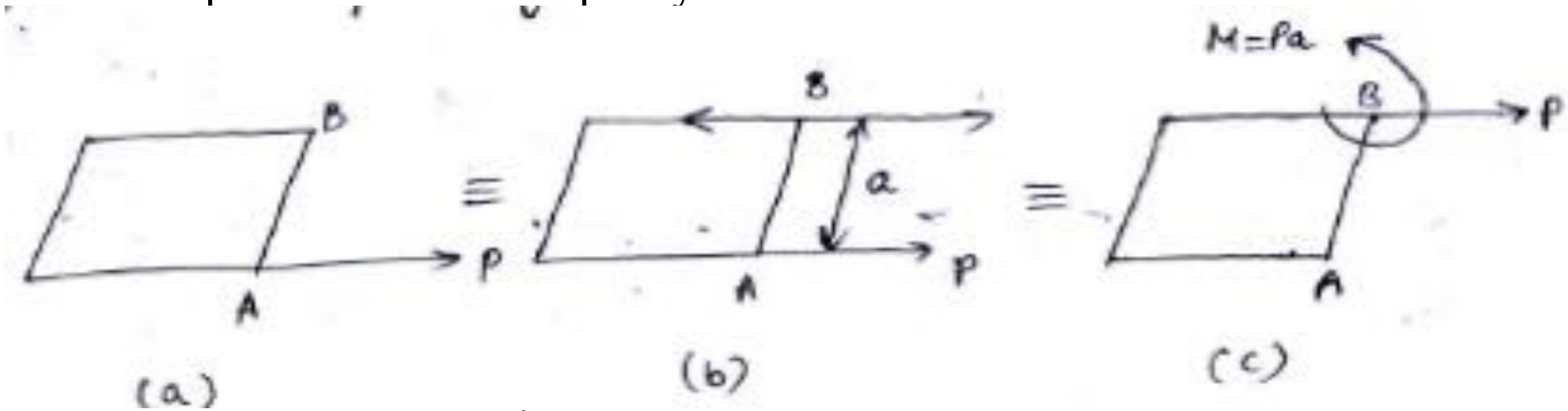


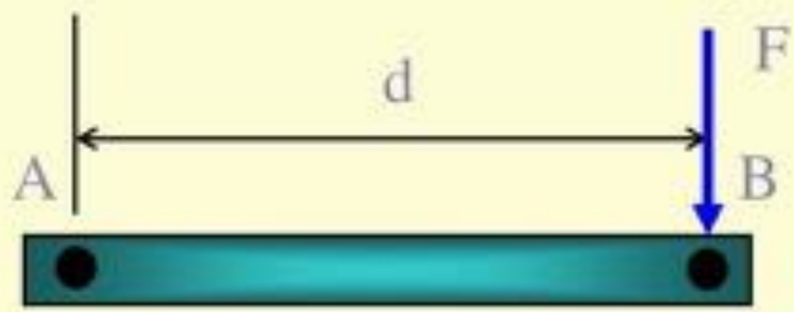
Fig. Force-couple system

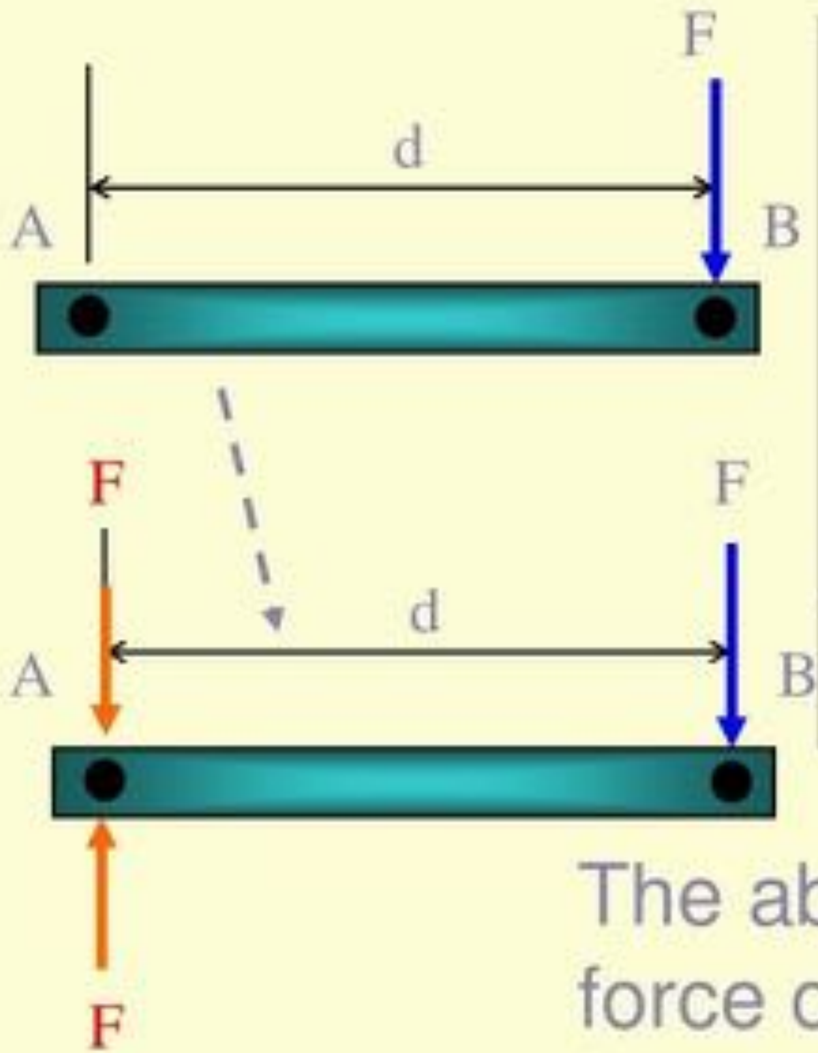
3.7 Force Couple Equivalent

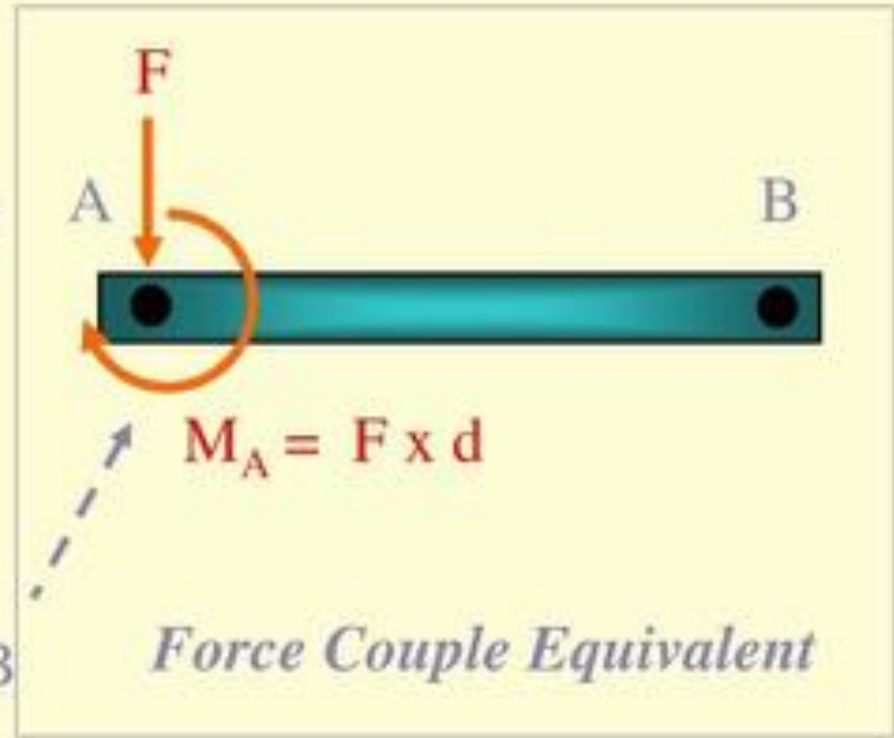
The *Force-Couple Equivalent* concept will enable us *to transfer a force to another location outside its line of action.*

Consider a force F acting at a point B on a rigid body as shown in diagram (a) below.

How do we transfer the force F from point B to point A?



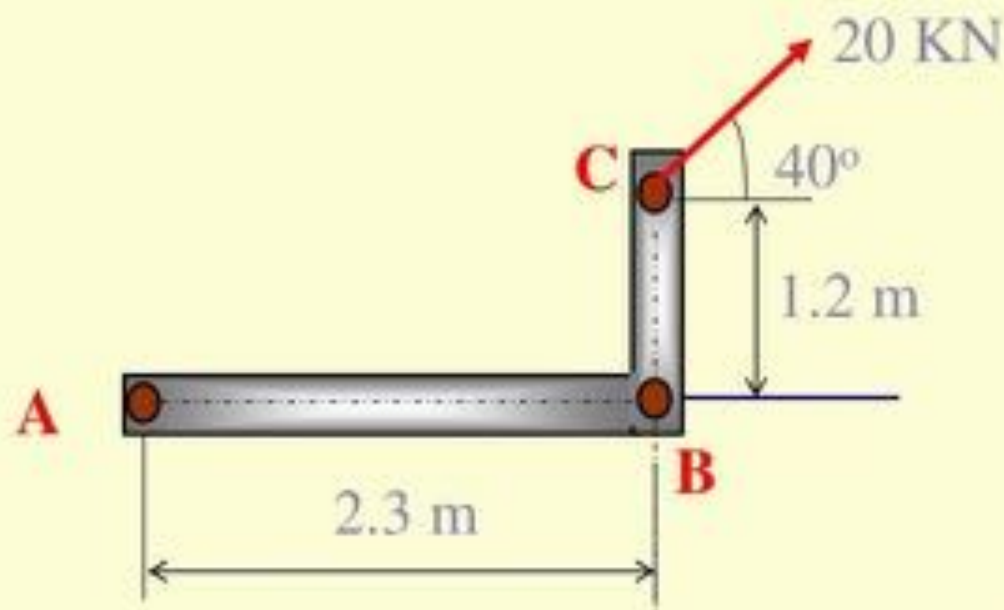




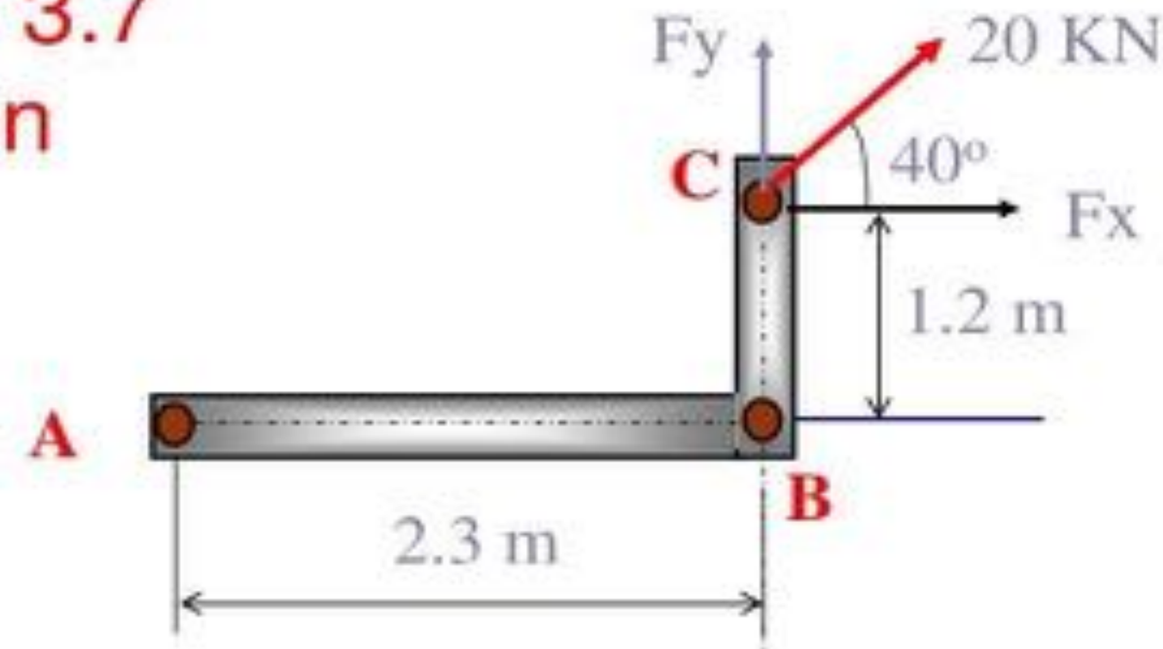
The above shows how a force can be replaced by a force-couple equivalent.

Example 3.7 (Single Force System)

Determine the force-couple equivalent at point A for the single force of 20 kN acting at point C on the bracket ABC.

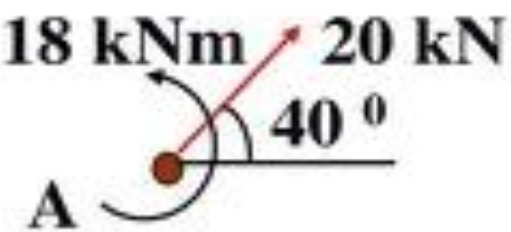


Example 3.7 solution



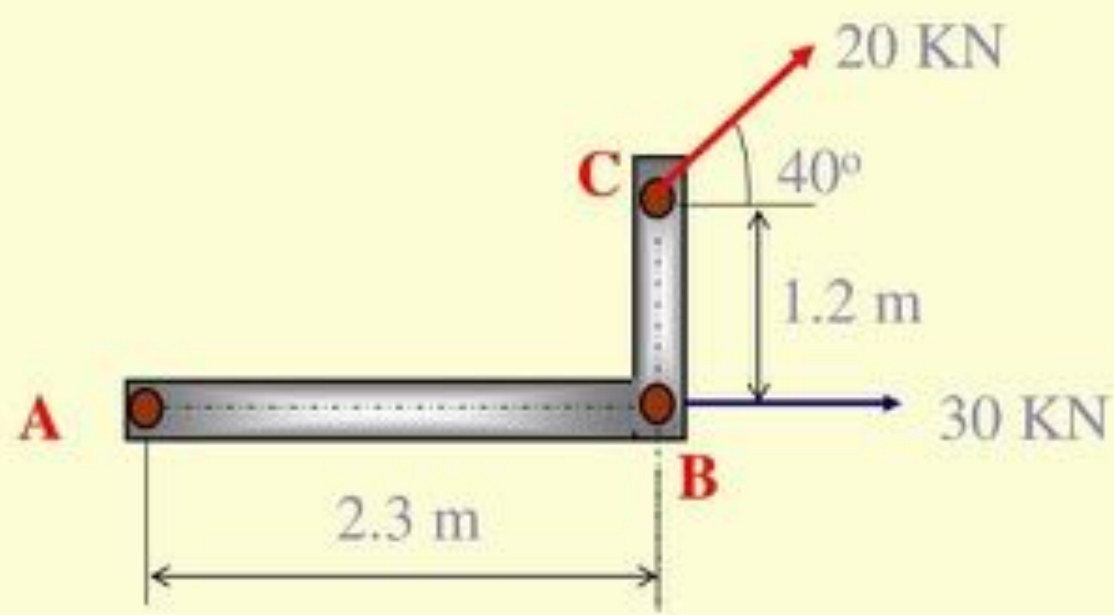
$$\Sigma M_A = (20 \sin 40 \times 2.3) - (20 \cos 40 \times 1.2)$$
$$= 11.18 \text{ kNm}$$

Answer:

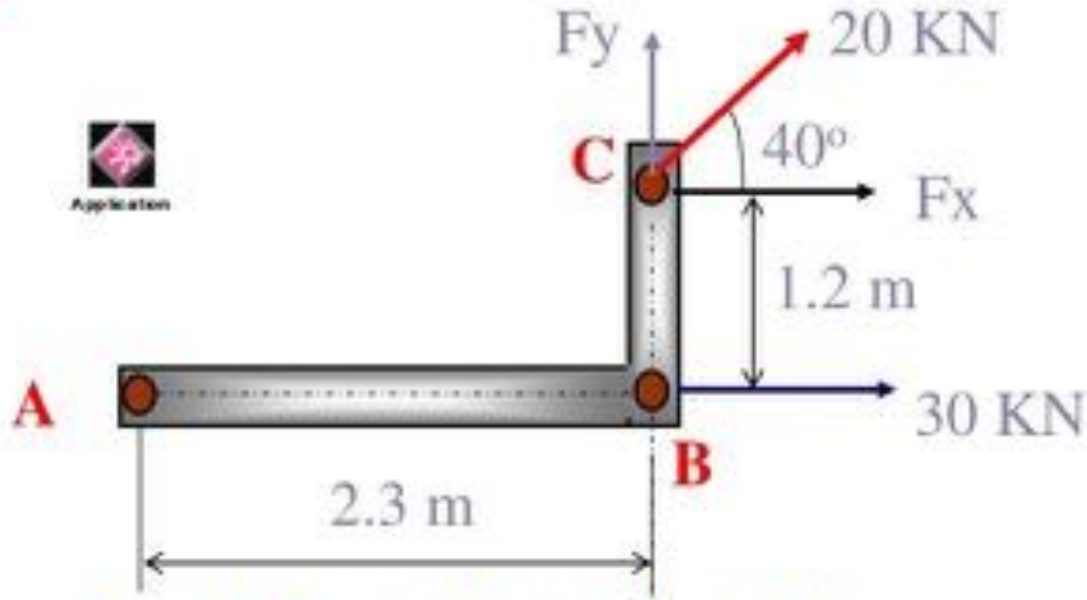


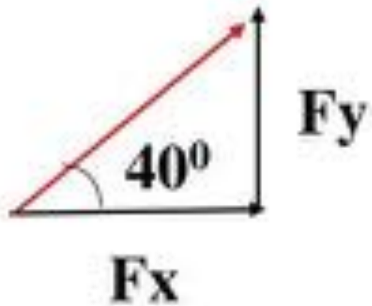
Example 3.8 (Multiple force system)

Another 30 kN horizontal force is added to Example 3.7 at point B. Determine the force-couple equivalent at point A.



Example 3.8
Solution





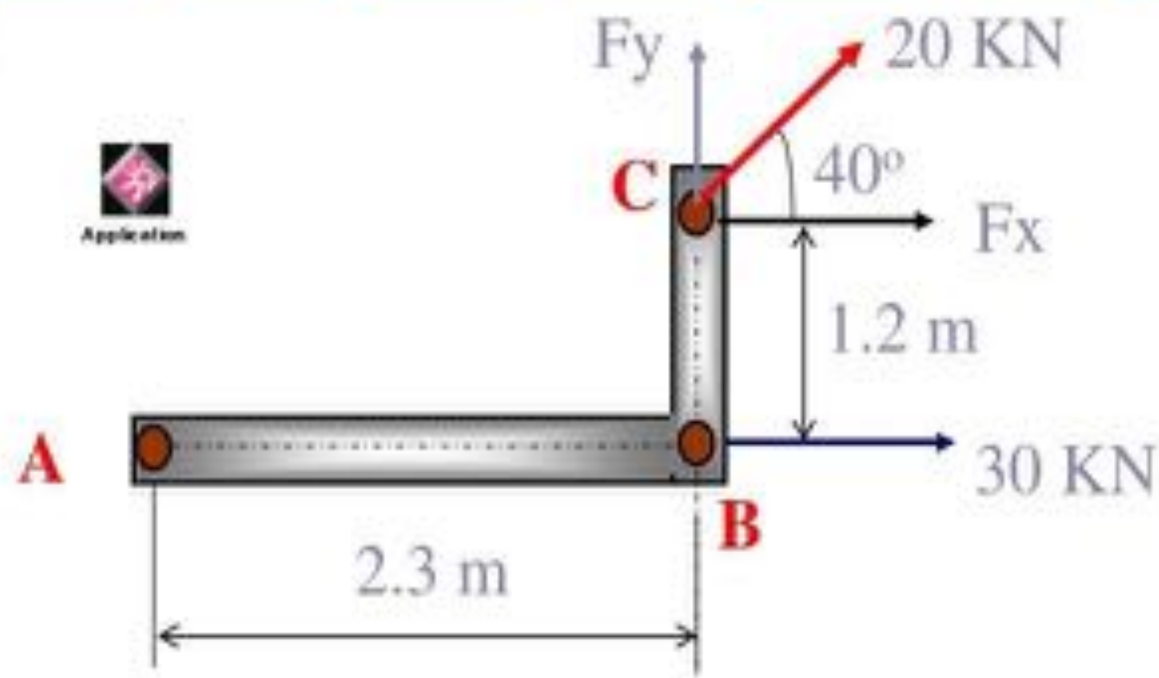
$$R_x = \Sigma F_x = 20 \cos 40 + 30 \longrightarrow$$
$$= 45.32 \text{ kN}$$

$$R_y = \Sigma F_y = 20 \sin 40 \quad \uparrow$$
$$= 12.86 \text{ kN}$$

Therefore $R = \sqrt{(45.32^2 + 12.86^2)}$
$$= 47.11 \text{ kN}$$

And $\tan\theta = \dfrac{Ry}{Rx} = \dfrac{12.86}{45.32} = 15.84^0$

Example 3.8
Solution



$$\Sigma M_A = (20 \sin 40 \times 2.3) - (20 \cos 40 \times 1.2) + (30 \times 0)$$
$$= 11.18 \text{ kNm}$$

Answer: