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Unit V

Kinematics Of Rectilinear And Curvilinear Motion

ENGINEERING MECHANICS

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UNIT 5

Kinematics of Particle (6 Hr)

Kinematics of linear motion:-

- Basic concepts
- Equation of motion for constant acceleration
- Motion under gravity
- Variable acceleration
- Motion curves

Kinematics of curvilinear motion:-

- Basic Concepts
- Equation of motion in Cartesian coordinates
- Equation of motion in path coordinates
- Equation of motion in polar coordinates
- Motion of projectile.

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Variable acceleration

- $a = dv/dt$
- $v = dx/dt$
- $a = f(t)$ $a = f(v)$ $a = f(x)$
- **At maximum velocity, acceleration should be zero.**
- **At maximum displacement, velocity should be zero.**

Example 1.The motion of particle moving in a straight line is given by the expression

$S = t^3 - 3t^2 + 2t + 5$ where S is the displacement in metres and t is the time in seconds. Determine

(1)Velocity & acceleration after 4 seconds

(2)Maximum or minimum velocity and corresponding displacement

(3) Time at which velocity is zero

- **Solution Key:**

(1) Differentiating S w.r.t. 't' & obtain the equation of velocity [dS / dt = V]

. Differentiating V w.r.t. 't' & obtain the equation of acceleration [dV / dt = a]

V = ? When t = 4sec.

a = ? When t = 4sec.

2) Max . Or min. velocity & corresponding displacement

Condition for maximum or minimum velocity is dv /dt = a = 0

t=? When a(expression) = 0

Vmax./min =?,When t= (above calculated)

- S=?, When t= (above calculated)

(3)t=?,When V (expression)=0

2) Max . or min. velocity & corresponding displacement

Condition for maximum or minimum velocity is

$dv/dt = a = 0$

$a = 6t - 6 = 0$

hence $t = 1$

Max. velocity or min. velocity

$V = 3t^2 - 6t + 2$

$v_{min.}$ or $v_{max} = 3(1)^2 - 6(1) + 2$

$= -1$ m / sec (Ans)

Hence it should be minimum velocity

(or $d^2v / dt =$ is positive hence it is minimum)

Corresponding displacement (t = 1)

$S = t^3 - 3t^2 + 2t + 5$

$= 1^3 - 3(1)^2 + 2(1) + 5$

$= 5$ m(Ans)

3) Time at which velocity is zero

$V = 3t^2 - 6t + 2$

$t = ?$ $v = 0$

$0 = 3t^2 - 6t + 2$

t=1.577 & 0. 423 sec. (Ans)

Example 3.The velocity of a particle moving in a straight line is given by the expression $\mathbf{V= t^3 - t^2 - 2t + 2}$. The particle is found to be at a distance 4 m from station A after 2 seconds . Determine : 1) acceleration & displacement after 4 seconds and 2) maximum or minimum acceleration

Solution Key :-

1. Differentiating V w.r.t. 't' & obtain the equation of acceleration [dV / dt = a]

2. a=?,Put t=4sec in expression of acceleration

3. Obtain the equation of displacement (S), by integrating equation of velocity because dS / dt = V

4. Find out constant of integration (C),Put s=4,t=2sec (Given condition) in expression of S

5. S=?,Put t=4sec in expression of S

6. Differentiating a w.r.t t , equate to zero and find out t [because maximum or minimum acceleration [condition is) da / dt = 0]

7. $a_{max.min}$=?,Put t=(above calculated)

Solution : $V = t^3 - t^2 - 2t + 2$ (Given)

Differentiating V w.r.t. 't' &obtain the equation of acceleration [dV / dt = a]

$a = dv / dt = 3t^2 - 2t - 2$

a = acceleration after 4 seconds

$a = 3t^2 - 2t - 2$

$= 3(4)^2 - 2(4) - 2$

= 38 m/ sec ^{2} (Ans)

Obtain the equation of displacement, by integrating equation of velocity because dS / dt = v

$ds / dt = v = t^3 - t^2 - 2t + 2$

$S = t^4/4 - t^3/3 - t^2 + 2t + C$

Find out constant of integration (C),Put s=4,t=2sec (Given condition) in expression of S

$4 = 2^4/4 - 4^3/3 - 4^2 + 2(4) + C$

$C = 4/3$

$s = t^4/4 - t^3/3 - t^2 + 2t + C$

Displacement after 4 sec.

s = ? when t = 4 sec .

$s = 2^4/4 - 4^3/3 - 4^2 + 2(4) + 4/3$

= 36 m (Ans)

max. or minimum acceleration

$da / dt = 0$

$d / dt \ (3t^2 - 2t - 2) = 0$

i.e . $6t - 2 = 0$ $t = 1/3$ sec.

Since $d^2 a / dt^2$ is positiv quantity , the above condition is for the minimum value

Minimum value of acceleration

$a_{t=1/3} = 3t^2 - 2t - 2$

$= 3(1/3)^2 - 2(1/3) - 2$

= - 2. 333 m/ sec ^{2} (Ans)

Equation of motion under uniform acceleration

S.N.	**Relation**	**Derive**		**Equation**
1	V,u,a,t	a (Rate of change of velocity) $= (V-u)/t$ m/sec^2 $at=V-u$ $V=u+at$		$V=u+at$
2	S,u,a,t	Average Velocity $=(V+u)/2$ Distance,$S=V_{av.} X t$ $=[(V+u)/2]t$ $=[u+at+u)/2]t$ $=[(2u + at)/2)t$ $S =ut + ½ at^2$	**Hint** $V_{av =}(V+u)/2$ $V=u+at$	$S =ut + ½ at^2$
3	u,V,a,s	Distance,$S=V_{av.} X t$ $=[(V+u)/2]t$ $==[(V+u)/2][(V-u)/2]$ $=(V^2-u^2)/2a$ $2as= V^2-u^2$	**Hint** $V_{av =}(V+u)/2$ $a=(V-u)/t$ $at=V-u$ N, $t= (V-u)/2$	$V^2-u^2 = 2as$

MOTION WITH UNIFORM ACCELERATION
(Kinematics)

- If the velocity changes equal in magnitude in equal interval of time is called a body is in uniform acceleration

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Motion with uniform velocity :

In this type ,when motion of a body moving with uniform velocity ,

Use , Distance =Velocity × time

Motion With Uniform Acceleration:

when motion of a body moving with uniform acceleration use following equations

$$V = u + at$$

$$S = ut + \frac{1}{2} at^2$$

$$v^2 - u^2 = 2as$$

Where , u –Initial velocity

V - final velocity

t – time taken for change of velocity from u to v

S – displacement

a – acceleration

Example 1. A motorist is travelling at 80 km/h , when he observes a traffic light 200 m ahead of him turns red .The traffic light is timed to stay red for 10 sec. If the motorist whishes to pass the light without stopping , just as it turns green
The required uniform acceleration of the motor &
the speed of the motor as it passes the light.

Given :

s =200 m

t = 10 sec.

a =?

V= ?

u =80 km/h

=80×1000/ 60 ×60

= 22.22 m/s

Solution Key :

a=? $S = ut + \frac{1}{2} at^2$

V=? $V = u + at$

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$S = ut + \frac{1}{2} at^2$

$200 = 22.22 \times 10 + \frac{1}{2} \times a \times 10^2$

hence**, a = - 0.444 m /s ^{2} (Ans)**

(i.e. it is decelerating)

$V = u + at$

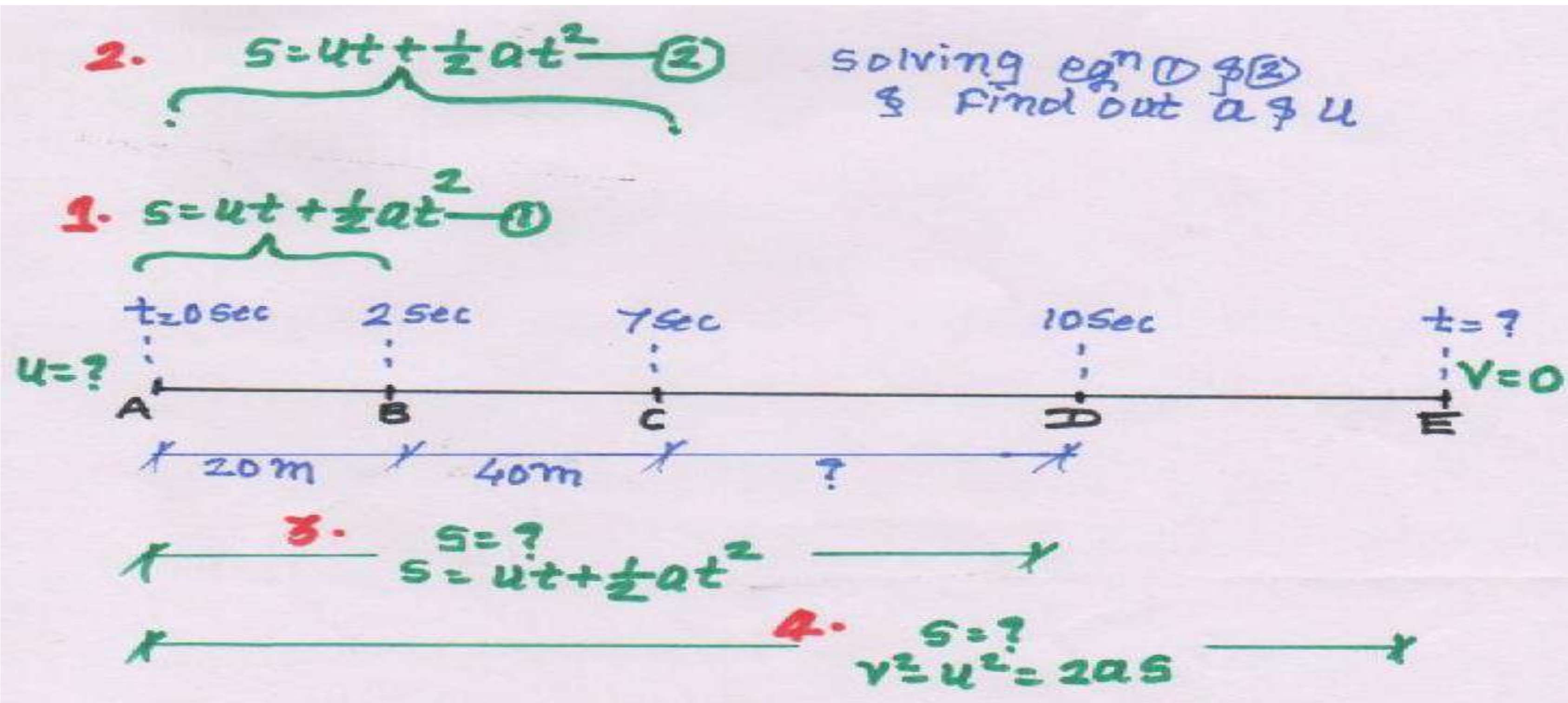
$= 22.22 + (-0.44)\,10$

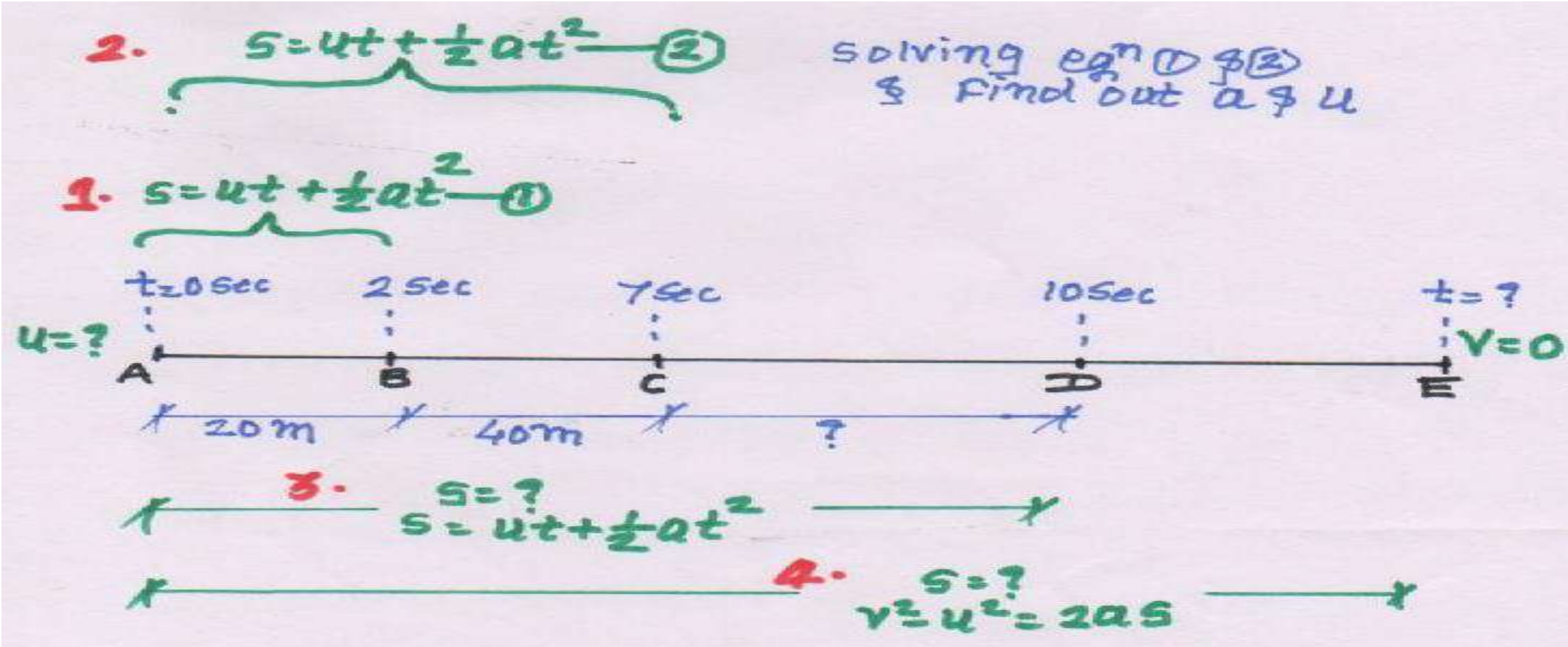
$= 17.78$ m /s

$= 17.78 \times 60 \times 60 / 1000$

= 64 km /h . (Ans)

Example 2.A particle under a constant deaccelaration is moving in a straight line and covers a distance of 20 m in first two seconds & 40 m in the next 5 seconds . Calculate the distance it covers in the subsequent 3 seconds & the total distance covered before it comes to rest





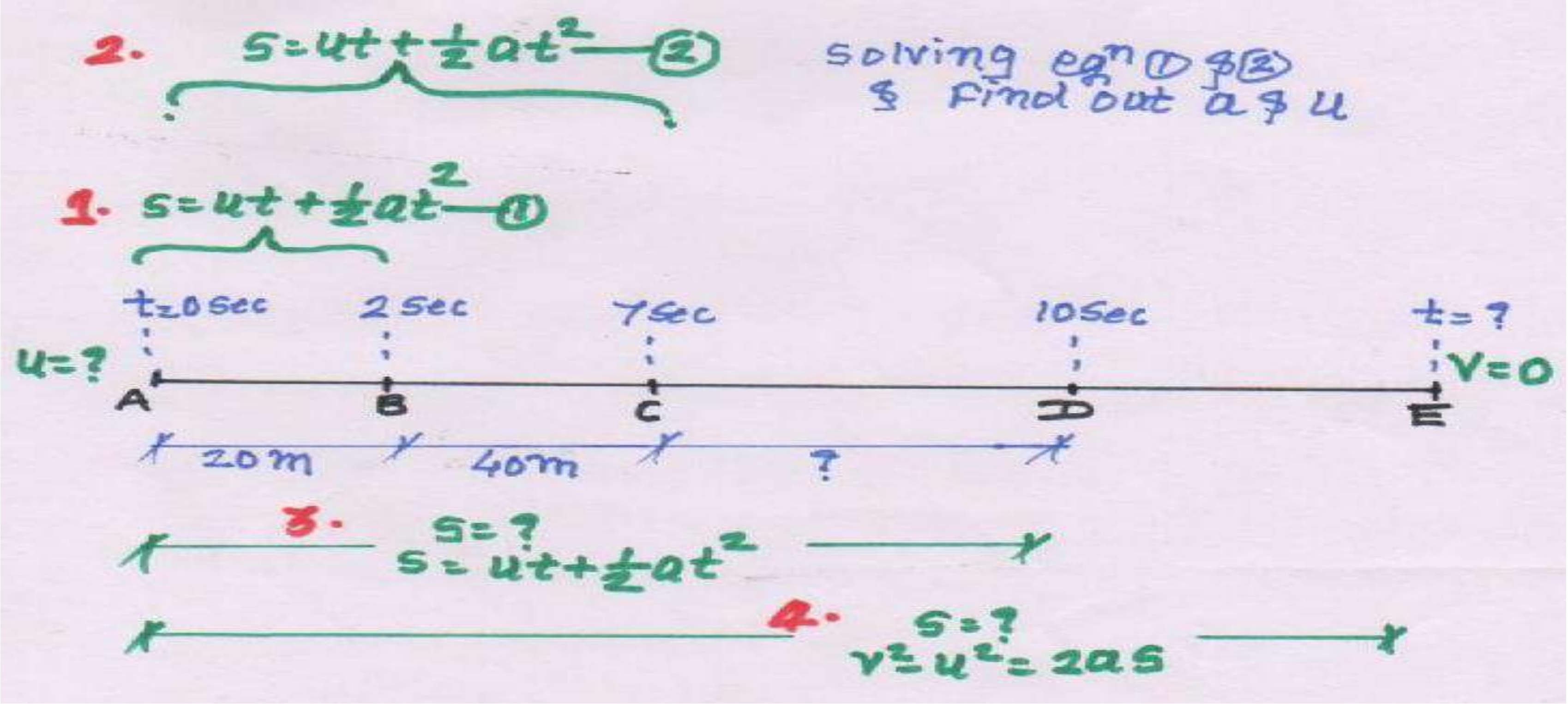
Solution :

Let the particle start from A & come to halt at E as shown in fig.

Let the acceleration of the particle be 'a'

Note that if the particle is having deacceleration the value of 'a' will be negative .

Let the initial velocity be u m/s



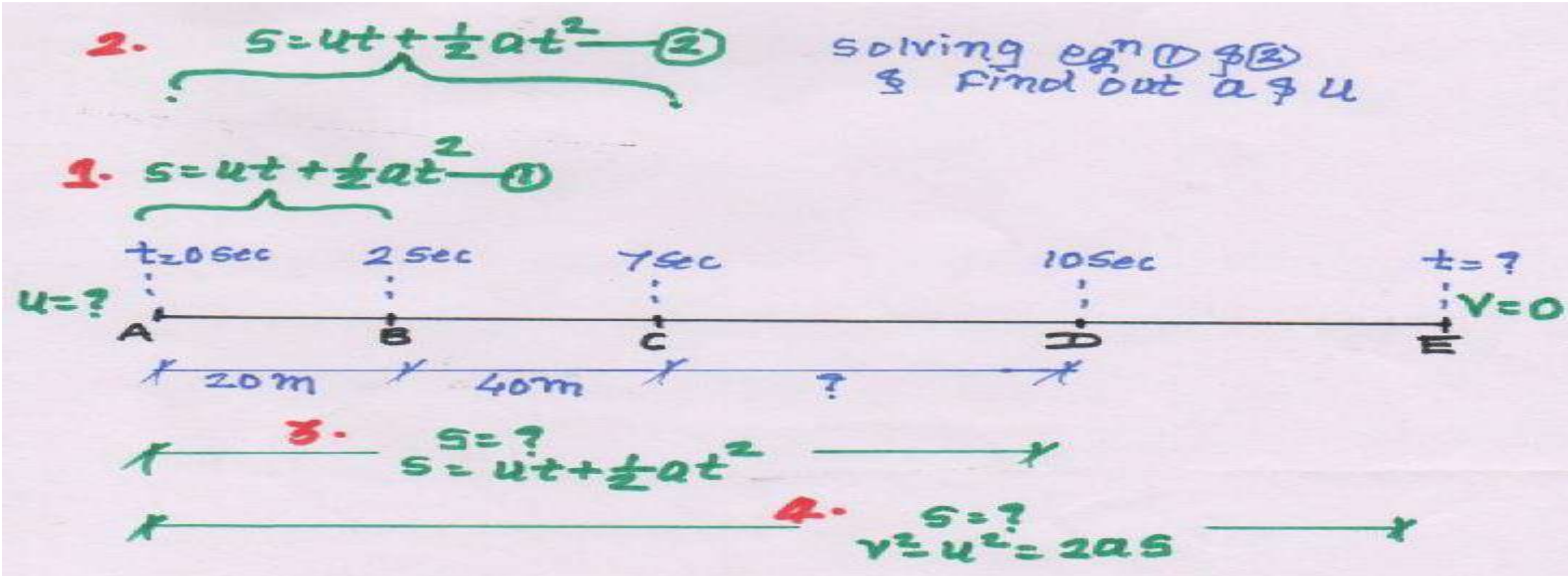
Considering the motion between A & B

$S = ut + \frac{1}{2} at^2$

$20 = u \times 2 + 1/2 \times a \times (2)^2$

$20 = 2u + 2a$

$10 = u + a$ ---------1



Considering the motion between A & C

$S = ut + \frac{1}{2} at^2$

$60 = u \times 7 + 1/2 \times a \times (7)^2$

$60 = 7u + 24.5 a$ ________ 2

Solving equation 1 & 2

$1 \times 7 - 2$

$70 = 7u + 7a$

$60 = 7u + 24.5 a$

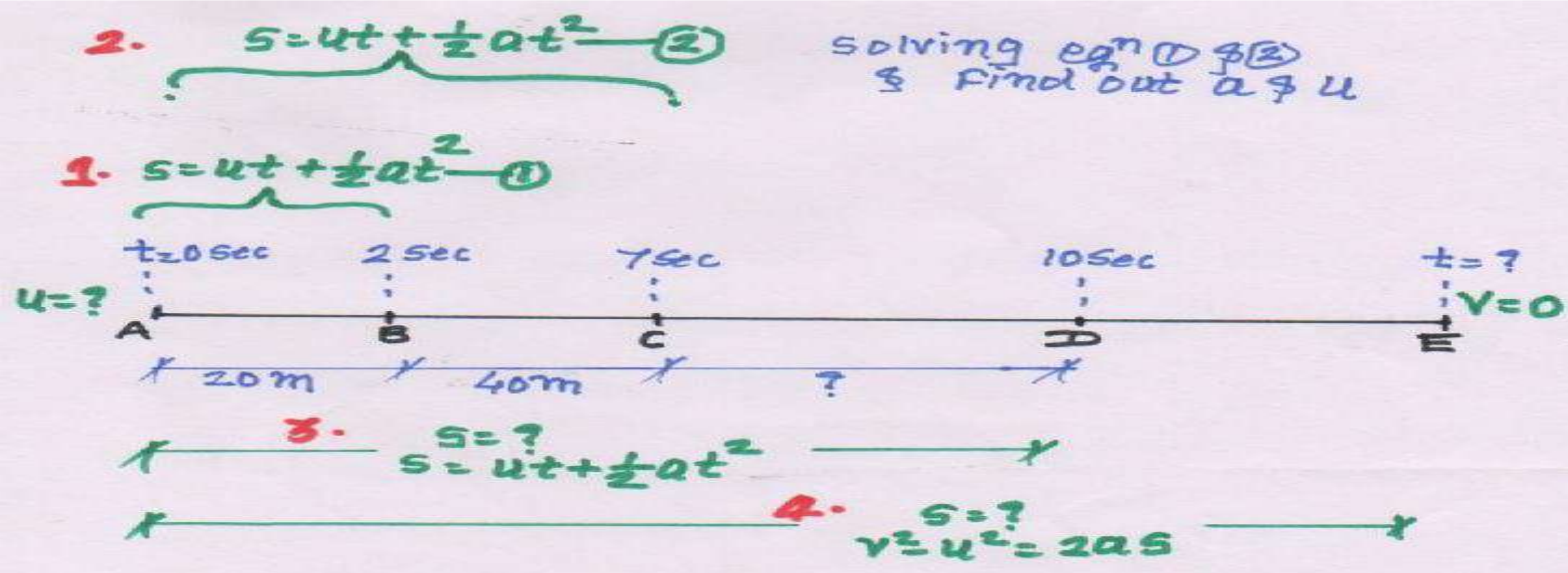
\-

$10 = -17.5 a$

$a = -0.571 \text{ m/sec}^2$

(i.e. body is deacelerating)

$u = 10.571$ m/sec



considering the motion between A & D & find out S = ?

$S = ut + \frac{1}{2} at^2$

$S = 10.57 \times 10 + 1/2 \times (-0.571) \times (10)^2$

$S = 77.16$ m

Distance covered in the interval , 7 seconds to 10 seconds is

C – D = 77.16 – 60 =**<u>17.16 m (Ans)</u>**

Let the particle come to rest at a distance S from the starting point i.e. $A_E = S$

$v^2 - u^2 = 2as$

$0^2 – (10.571)^2 = 2 \times (- 0.571) \times S$

$-111.746 = - 1.142 \times S$

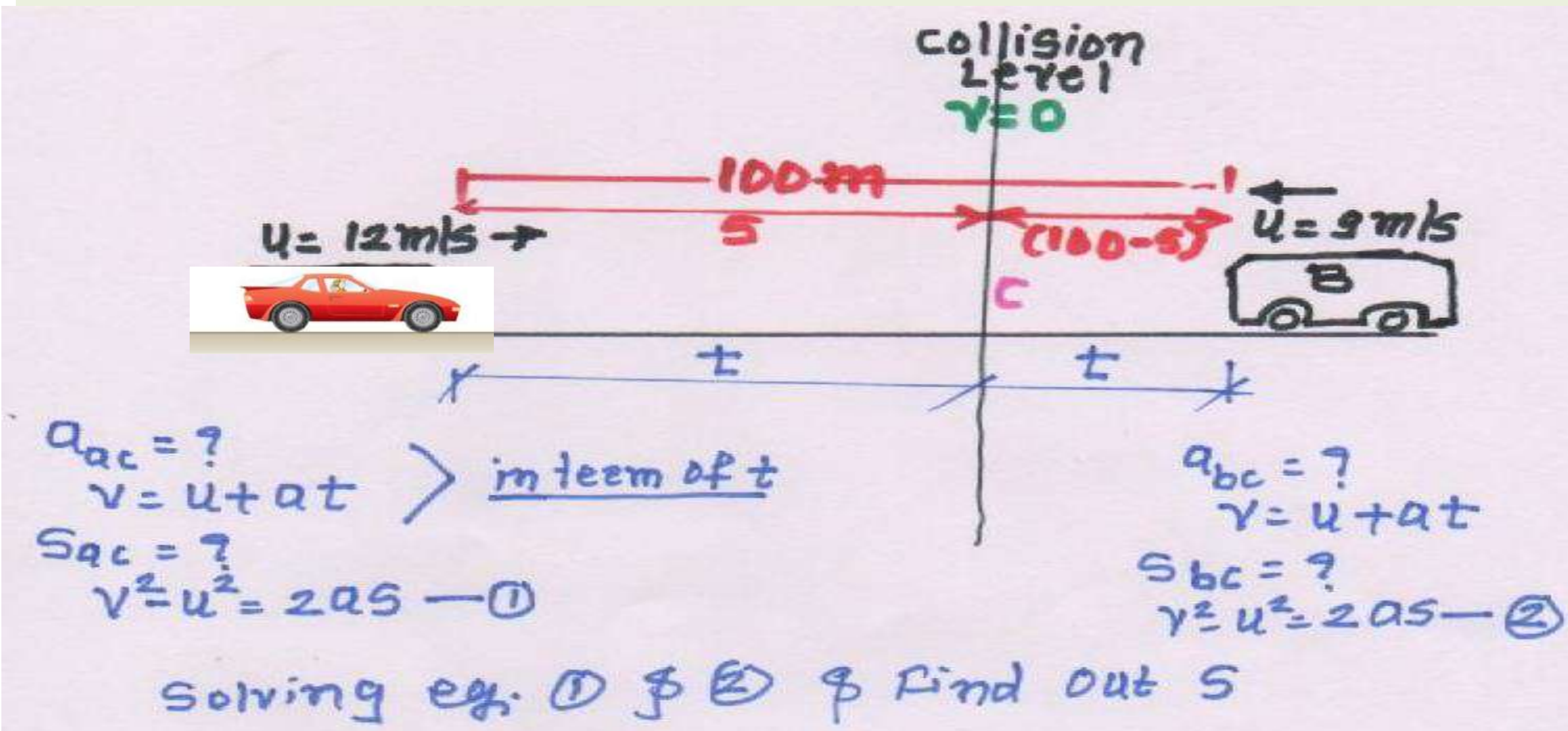
<u>S = 97.85 m (Ans)</u>

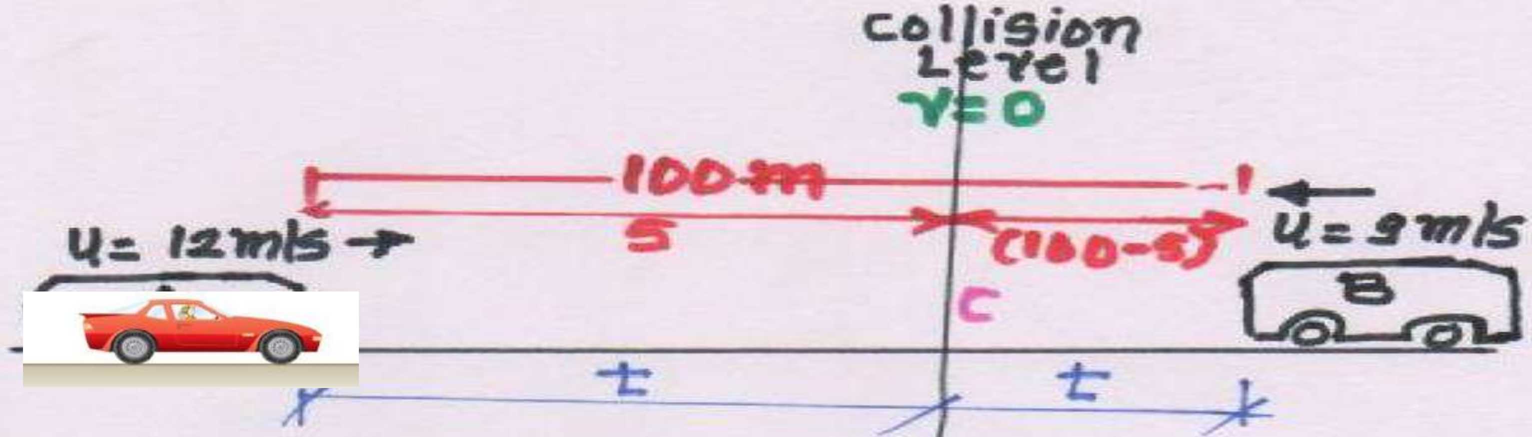
Example 5.Two cars are travelling towards each other on a single lane road at the velocity 12 m/s & 9 m/s respectively . when 100m apart both drivers realize the situation & apply their brakes . They succeed in stopping simultaneously and just short colliding . Determine -

a) time required for cars to stop

b) Deceleration of each car and

c) the distance travelled by each car while slowing down





$a_{ac} = ?$

$v = u + at$ > in term of t

$S_{ac} = ?$

$v^2 - u^2 = 2as$ —①

$a_{bc} = ?$

$v = u + at$

$S_{bc} = ?$

$v^2 - u^2 = 2as$ —②

solving eq. ① & ② & find out S

Solution :

For car A ,

$V = u + at$ (A –C)

$0 = 12 + a (t)$

$at = - 12$

$a_1 = -12 /t$

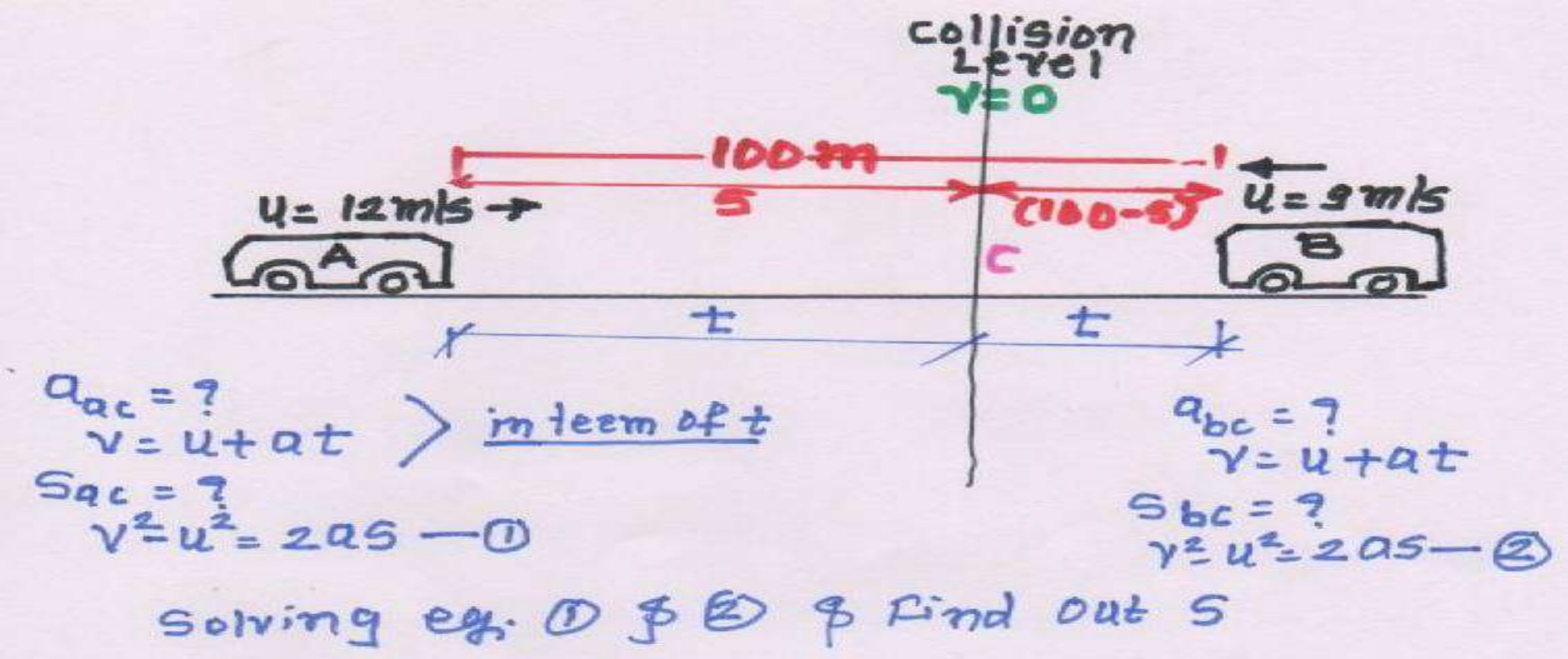
$V^2 - u^2 = 2 as$

$0^2 – (12)^2 = 2 \times (- 12/t) \times S$

$– 12^2 = -24 / t \times S$

$S = 144 \times t / 24$

$= 6 t$ ------------1



For car B , (B – C)

$v = u + at$

$0 = 9 + a_2 (t)$

$a_2 t = -9$

$a_2 = -9/t$

$v^2 - u^2 = 2as$

$0^2 - 9^2 = 2 \times (-9/t) \times (100 - S)$

$-81 = -18/t\,(100 - S)$

$-81t = -1800 + 18S$

$18S = -81t + 1800$

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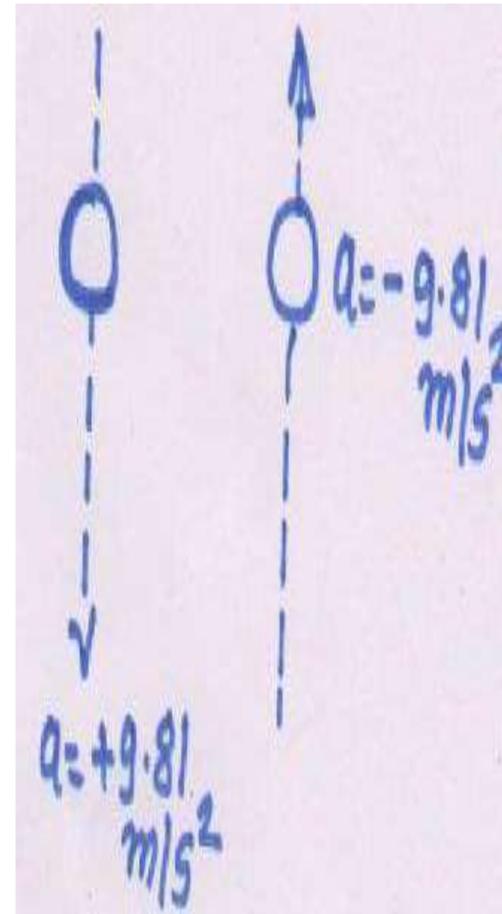
Motion under gravity :

Acceleration due to gravity is constant for all practical purposes when we treat the motion of the bodies near earth's surface.

Its value is found to be 9.81 m / s^2 & is always directed towards centre of the earth i.e . Vertically downwards .

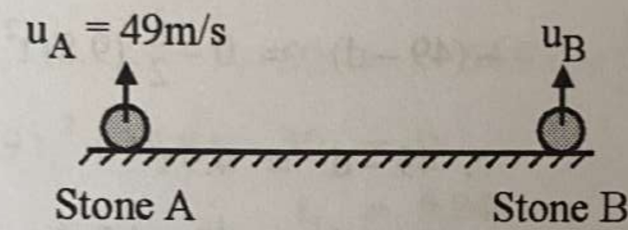
Hence , if vertically downwards motion of a body is considered, the value of acceleration is 9.81 m / S^2 & if vertically upward motion is considered , then

$a = - g = - 9.81 \ m / s^2$



A stone was thrown vertically up from the ground with a velocity of 49 m/s. After 2 sec, another stone was thrown vertically up from the same level. If both stones hit the ground simultaneously, find the velocity with which the 2nd stone was thrown ?

☑ **Soln. :**



A stone was thrown vertically up from the ground with a velocity of 49 m/s. After 2 sec, another stone was thrown vertically up from the same level. If both stones hit the ground simultaneously, find the velocity with which the 2^{nd} stone was thrown ?

Soln. :

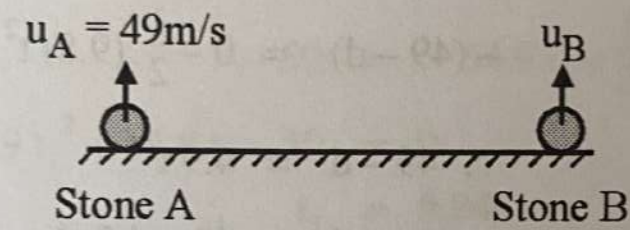


Fig. Ex. 11.8.26

Let 't' be the total time of journey for stone 'A'.

$\therefore$ Total time of journey for stone 'B' must be (t – 2) sec.

For stone A (M.U.G.)

Use, $s = ut - 1/2\ gt^2$

$$0 = +49t - 4.9\,t^2$$

$$\therefore 49\,t = 4.9\,t^2 \qquad \therefore \quad t = \mathbf{10\ sec}$$

[For entire motion s = 0 as point of projection and point of landing are at the same level.]

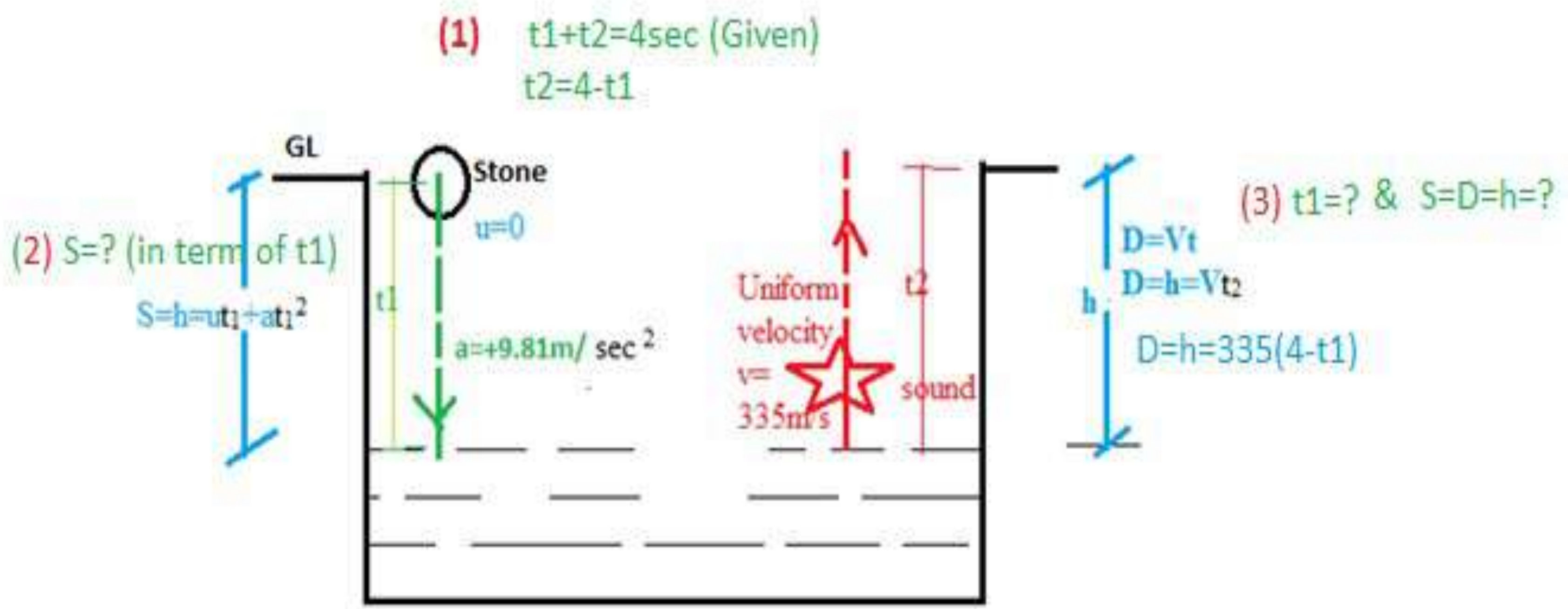
For stone B (M.U.G.)

Use, $s = ut - \frac{1}{2}gt^2$

$$0 = u_B\,(t-2) - \frac{1}{2} \times 9.8\,(t-2)^2;$$

Quiz 1. A stone dropped in to a well is heard to strike the water in 4 seconds. Find depth of well upto water level , assuming the velocity of sound to be 335 m/s.

SOLUTION KEY



Solution : *Let ,S – depth ofwell upt water level*

t_1 – time taken by stone to strike the water

t_2 – time taken by sound to travel distance 's'

$t_1 + t_2 = 4$ sec . (given)

$t_2 = 4 - t_1$

For downward motion of stone (motion under gravity)

$S = ut + ½ \ at^2$

$S = 0 \times t_1 + 1/2 \ \times 9.81 \times t_1^2$

$S = 4.905 \ t_1^{\ 2}$

For upward motion of sound

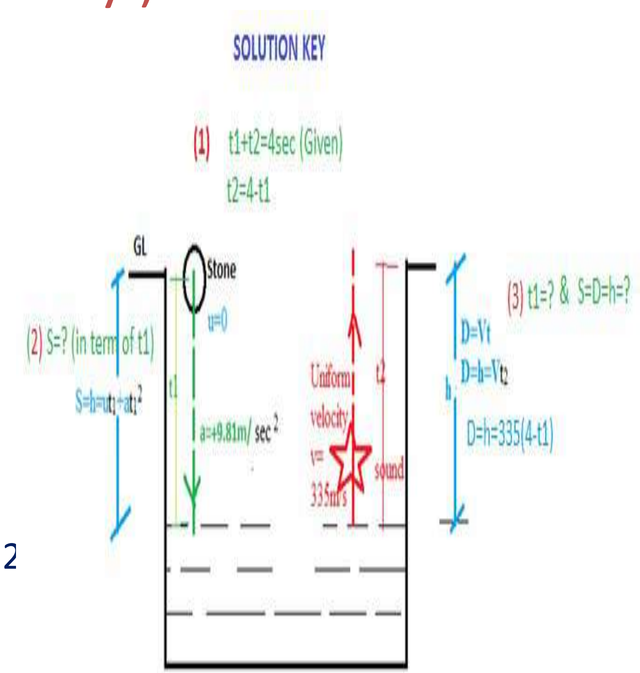
(since sound moves with uniform velocity)

Distance =Velocity × time

$4.905 \ t_1^{\ 2} = 335 \ (4 - t_1)$

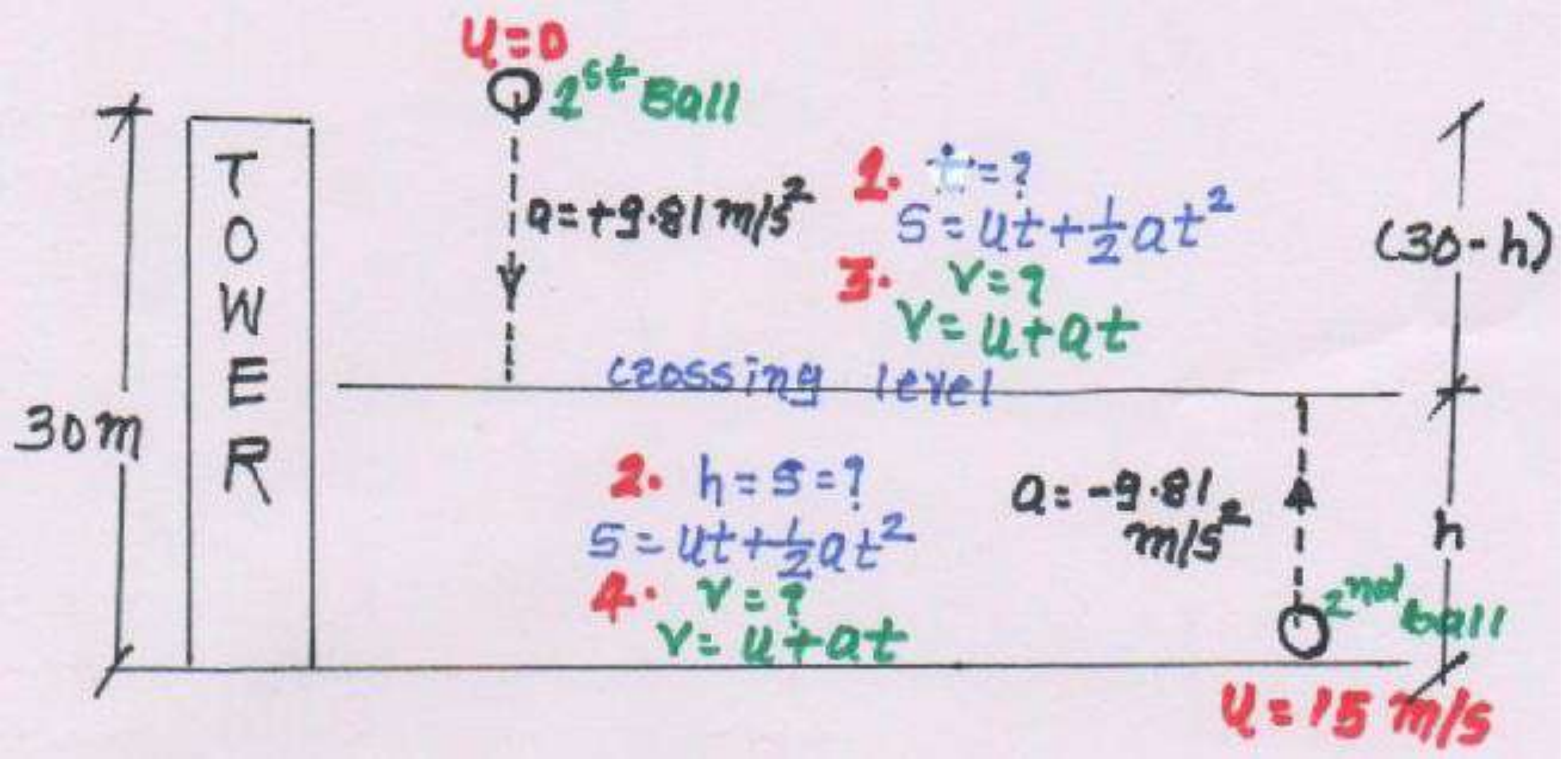
$t_1 = 3.79$ sec

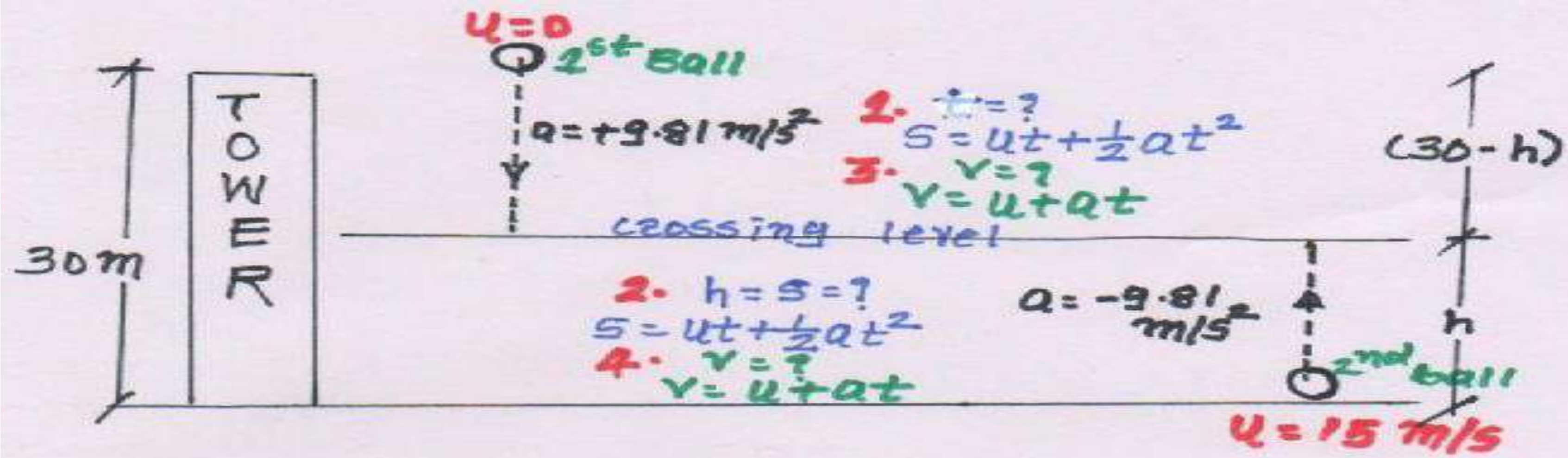
$s=h=ut_1+1/2 \ at_1^2$



<u>**S =70.44 m (Ans)**</u>

Example 4. A ball is dropped from the top of a tower 30 m high . At the same instant a second ball is thrown upward from the ground with an initial velocity of 15 m/sec. when & where do they cross





Solution : Let the two balls cross each other at a height h from the ground after t second

For the motion of first fall

$S = ut + ½\ at^2$

$(30 – h) = 0 \times t + 1/2 \times 9.81 \times t^2$

$30 – h = 4.905\ t^2$

$t^2 = 30 – h / 4.905$

$t = √\ 30 – h / 4.905$

For the motion of second ball

$S = ut + ½\ at^2$

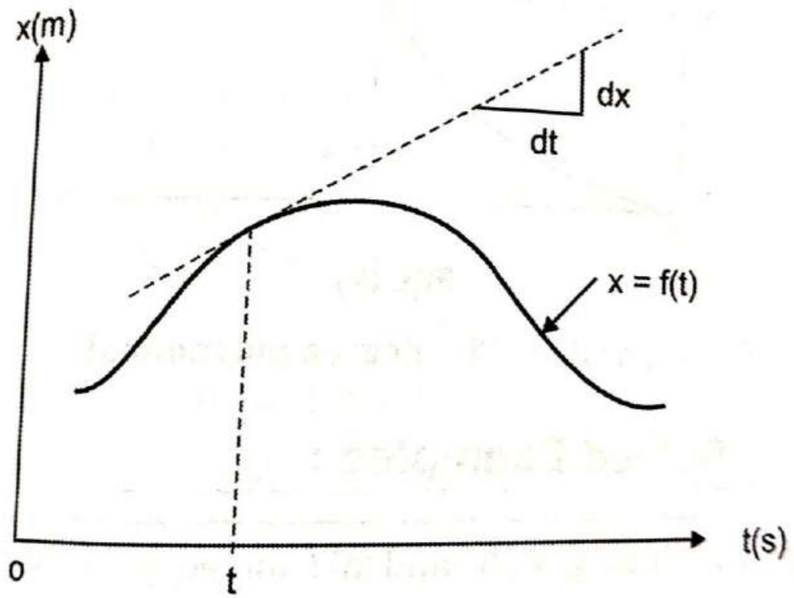
$h = s\ √\ 30 – h / 4.905 + ½ (- 9.81) (30 – h / 4.905)^2$

<u>h = 10.38 m (Ans) , t = 2 sec. (Ans)</u>

MOTION CURVES

- Graphical representation of the displacement , velocity and acceleration with time

S-t Curve



Displacement (S) – time (t) curve is a curve with time abscissa and displacement as ordinate

At any instant of time t_1 , velocity is given by

$V = ds / dt$

Slope of the curve represent velocity

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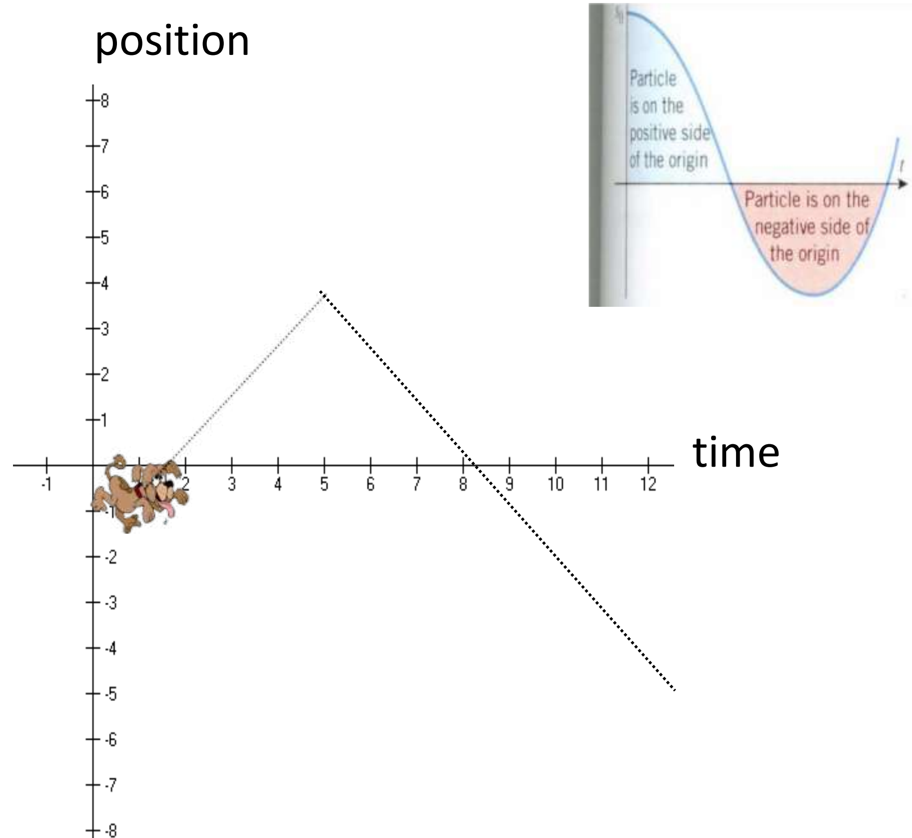
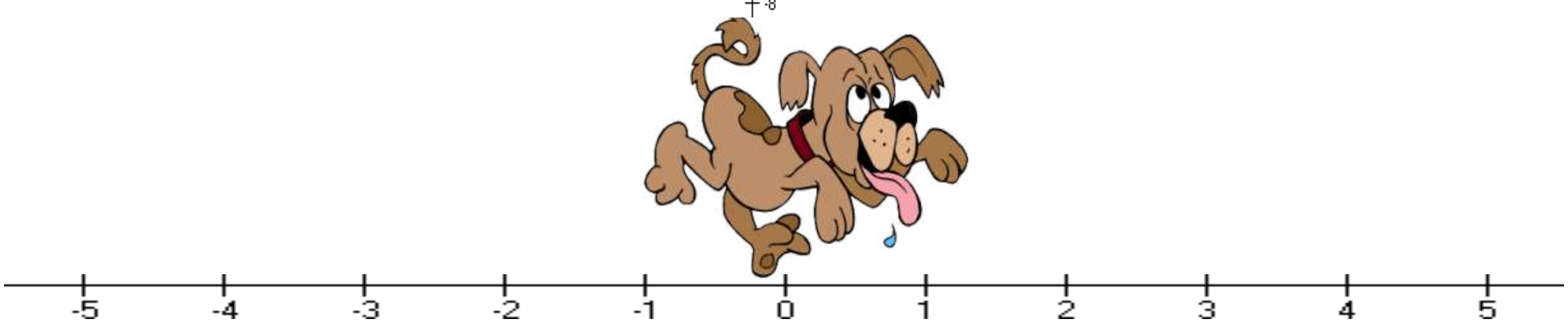
Position Function



- Horizontal axis:
  - time
- Vertical Axis:
  - position on a line

Position function: s(t)
s = position
t = time
s(t)= position changes as time changes



Moving in a direction from the origin

V-t Curve

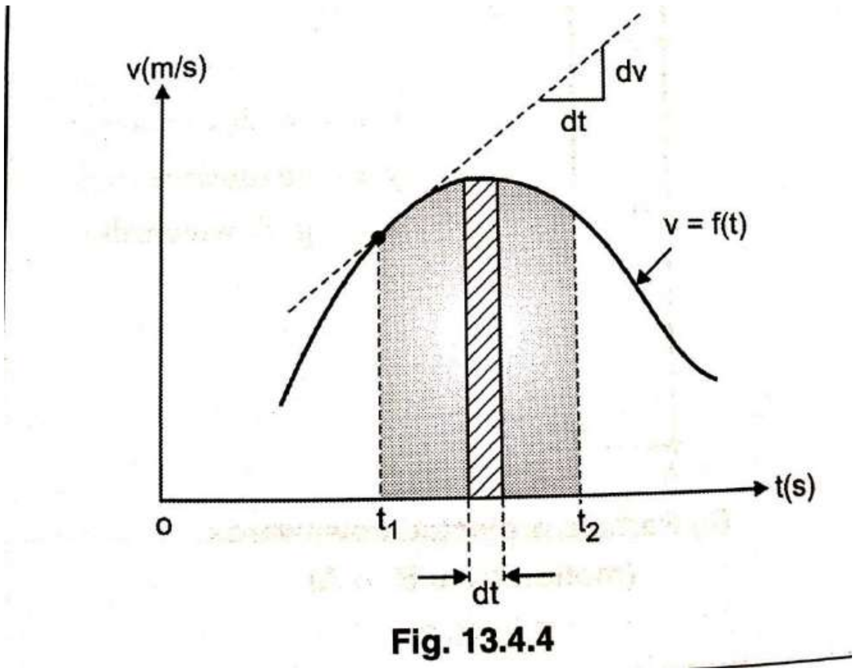


Fig. 13.4.4

a=dv/dt

acceleration at any time is the slope of V-t curve at that time.

v = ds / dt

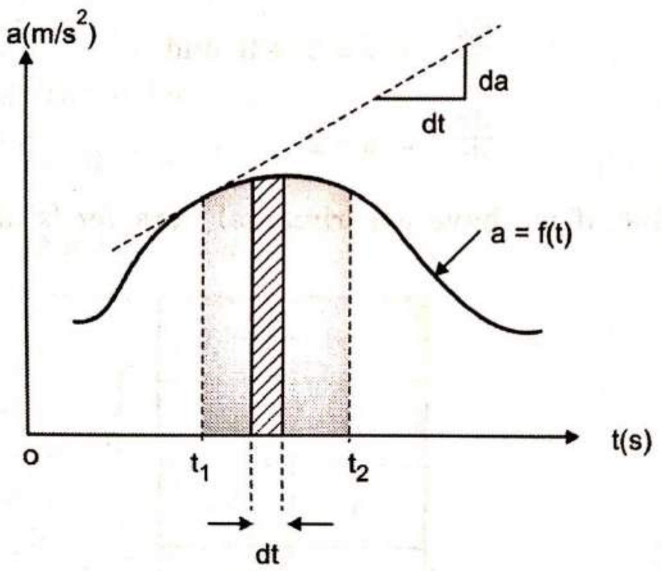
ds = v .dt

s = ∫ v.dt

- **Slope of the curve represent acceleration**

- **Area under the curve represent displacement**

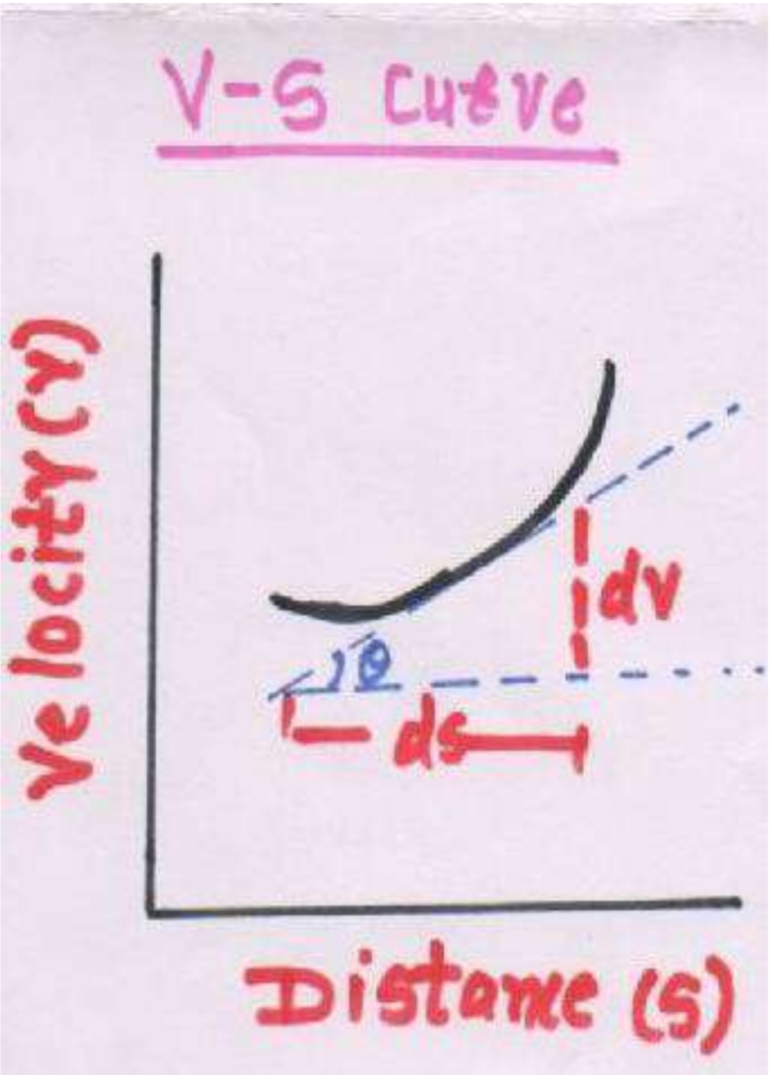
a-t Curve



Area under a-t diagram represents velocity.

$v = \int a.dt$

V-S Curve



$\tan\theta = dv/ds$ ---------1

Multiplying by ds / dt

$\tan\theta \,.\, ds/dt = dv/dS \times ds/dt$

$\tan\theta \,.\, v = dv/dt$

$\tan\theta \,.v = a$

$\tan\theta = a/v$ ------------2

Equate 1 & 2

$dv/ds = a/v$

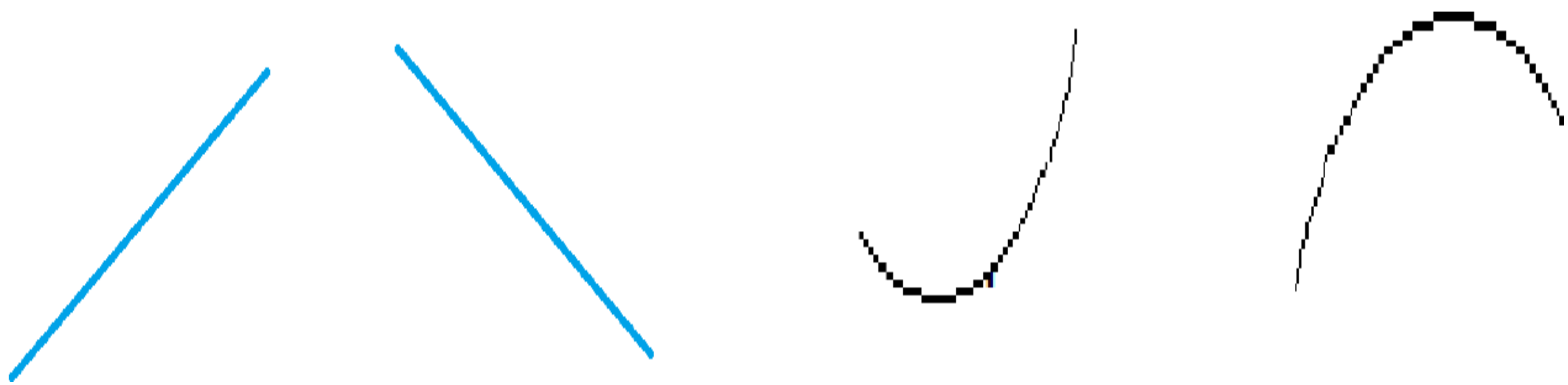
$v.\,dv = a.\,ds$

$\int v.\,dv = \int a.\,ds$

$v^2/2 = (\text{area})_{a\text{-}s}$

Study cases of the curves :

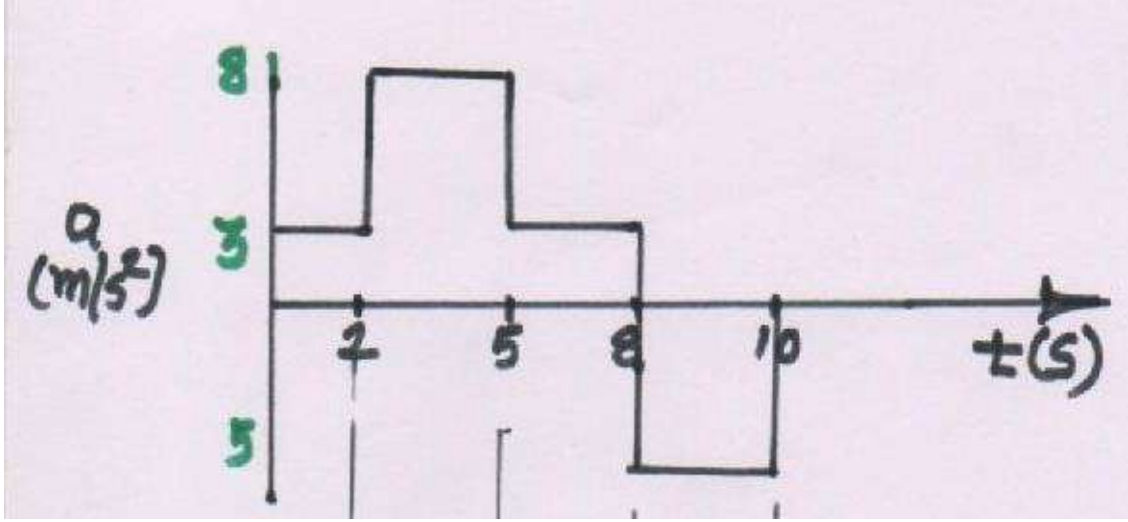
[How to join the points either by straight line of parabola (convex up / down)]



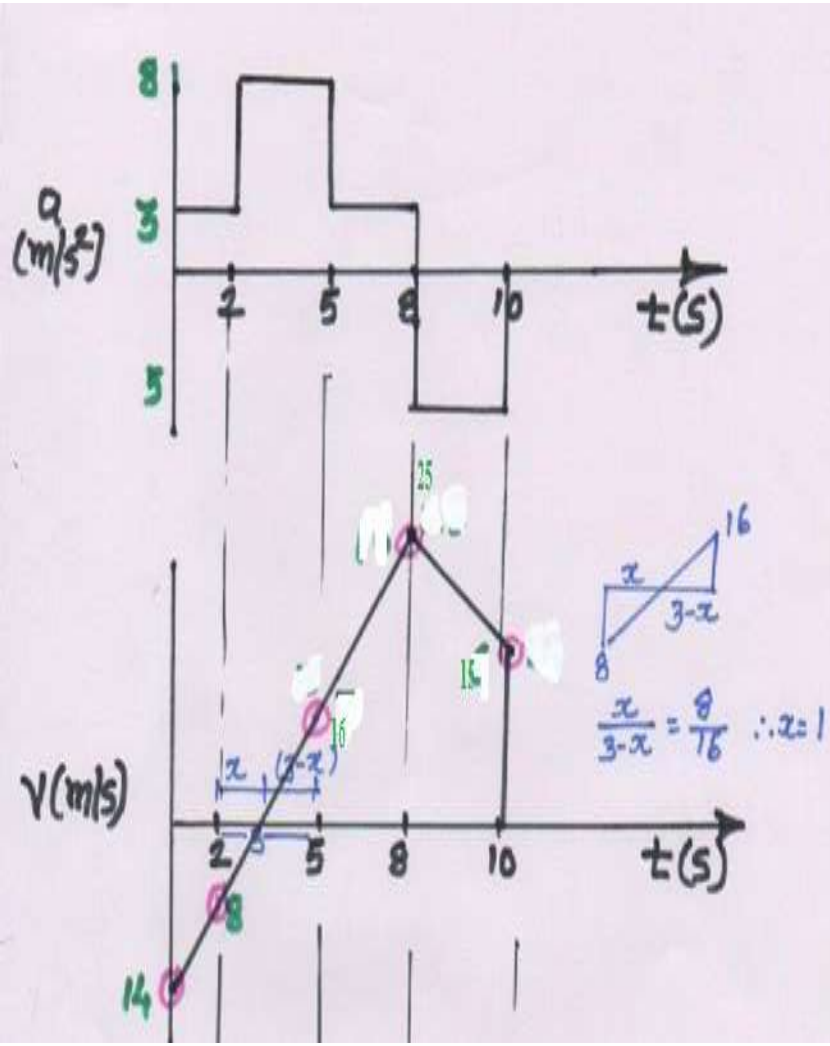
Oblique line having positive slope	**Oblique line having negative slope**	**Concave up**	**Convex up**

S.N.	a – t curve	v – t curve	s – t curve
1	No line When a = 0	Straight line	Oblique line
2	Straight line	Oblique line having Positive slope	Concave up
3	Straight line	Oblique line having Negative slope	Convex up
4	Oblique line having positive slope	Concave up	Convex up
5	Oblique line having Negative slope	Convex up	Concave up

Example 1) A particle moves in a straight line with an acceleration shown in fig. knowing that it starts from origin with initial velocity - 14 m/s .Plot V-t and S-t diagrams and determine velocity of particle , its position and total distance travelled after 10 sec.



Given : Initial velocity = -14 m/s = V_0 = -14 m/s
i.e.velocity at zero time = -14 m/s



V – t diagram calculations

Δv = (area) $_{a-t}$

$V_2 - V_0$ = (area) $_{a-t}$

$V_2 - (-14) = 3 \times 2$

$V_2 = -8$ m/s

$V_5 - V_2$ = (area) $_{a-t}$

$V_5 - (-8) = 8 \times 3$

$V_5 = 16$ m/s

$V_8 - V_5$ = (area) $_{a-t}$

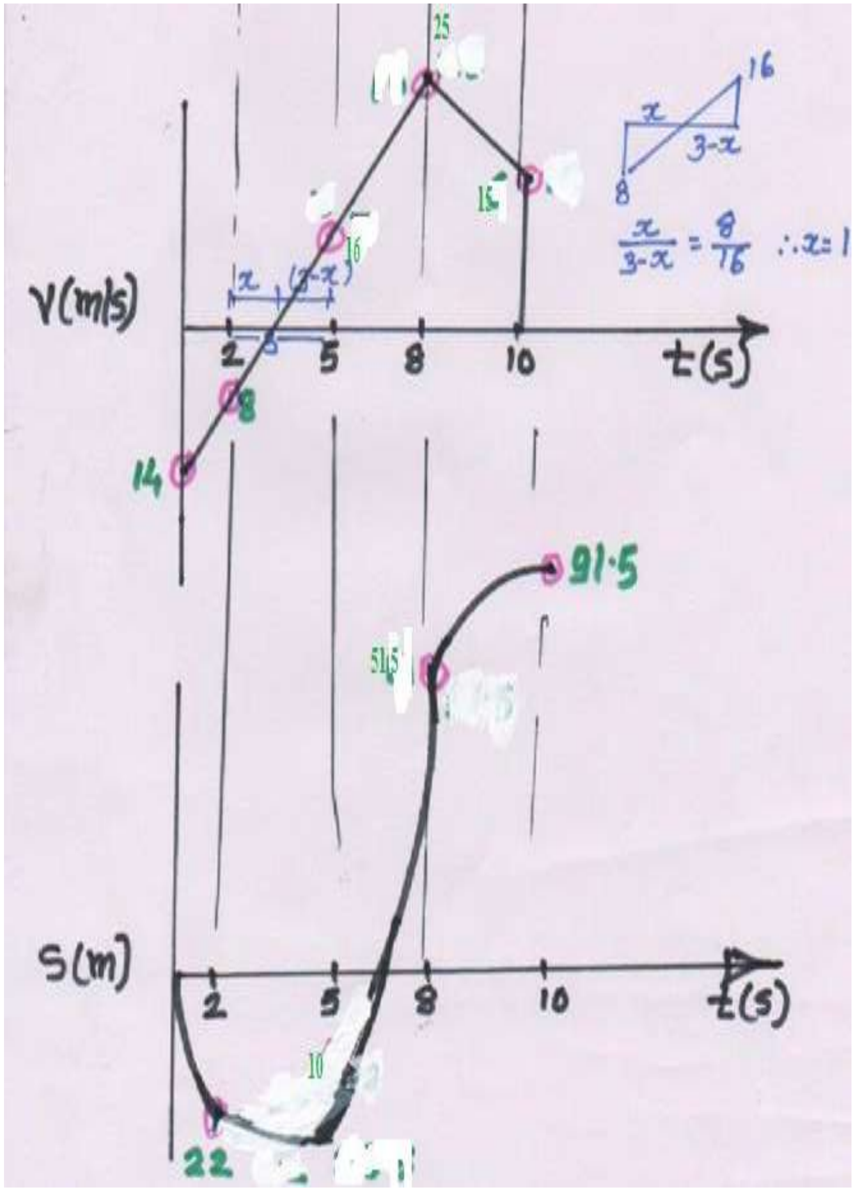
$V_8 - 16 = 3 \times 3$

$V_8 = 25$ m/s

$V_{10} - V_8$ = (area) $_{a-t}$

$V_{10} - 25 = 2(-5)$

$V_{10} = 15$m/s



S – t diagram calculations

$S_0 = 0$

Δs = (area)$_{v\text{-}t}$

($S_2 - S_0$) = (area)$_{v\text{-}t}$

$S_2 - 0 = [\ (-\tfrac{1}{2} \times 2 \times 6) + (-2 \times 8)\ ]$

$S_2 = -22$m

$S_5 - S_2$ = (area)$_{v\text{-}t}$

$S_5 - (-22) = [\ (-\tfrac{1}{2} \times 1 \times 8) + (\tfrac{1}{2} \times 2 \times 16)\ ]$

$S_5 = -10$

$S_8 - S_5$ = (area)$_{v\text{-}t}$

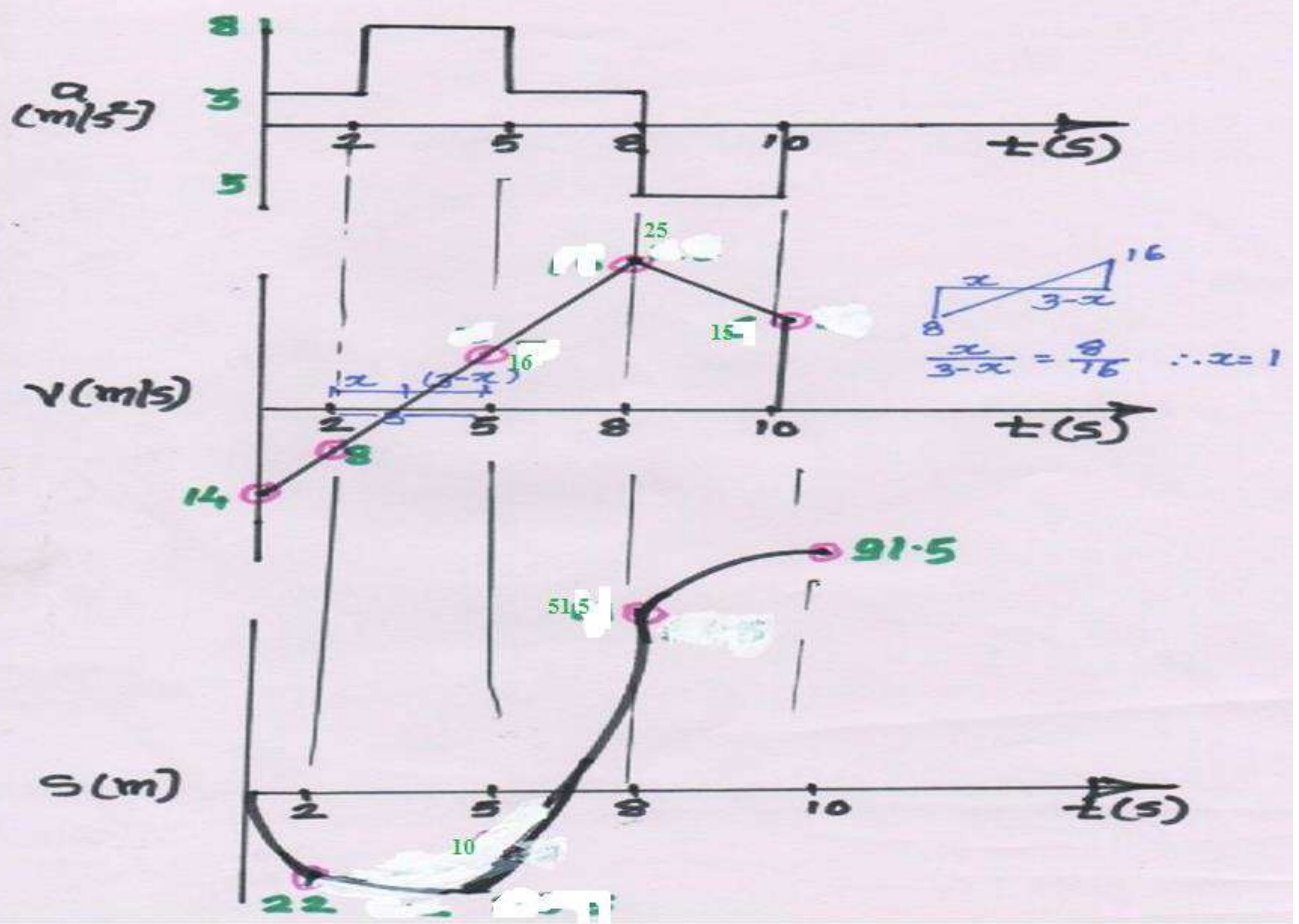
$S_8 - (-10) = [\ (\tfrac{1}{2} \times 3 \times 9) + (3 \times 16)\ ]$

$S_8 = 51.5$m

$S_{10} - S_8$ = (area)$_{v\text{-}t}$

$S_{10} - 51.5 = [\ (\tfrac{1}{2} \times 2 \times 10) + (2 \times 15)\ ]$

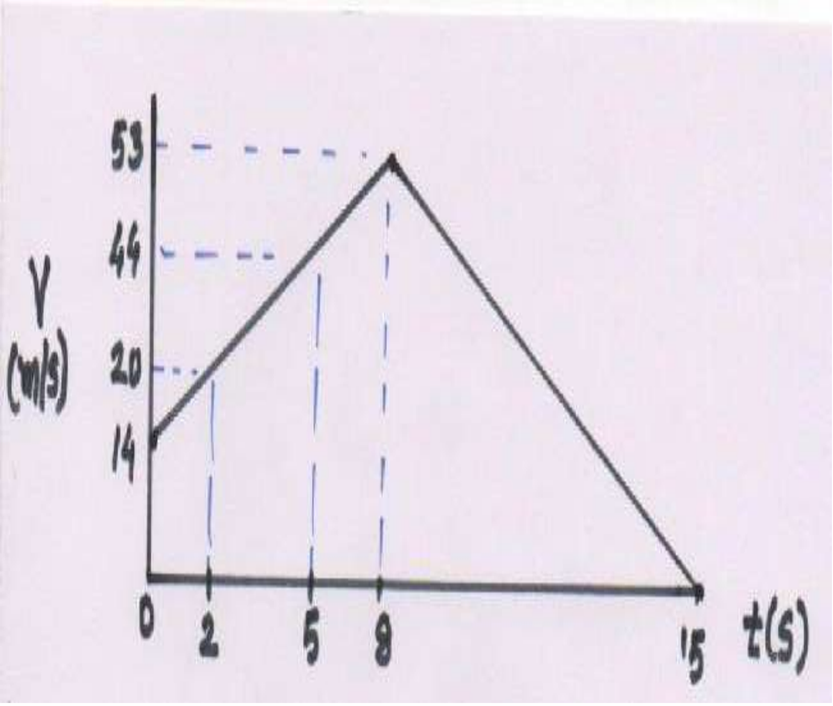
$S_{10} = 91.5$m



a (m/s²)
8
3
5
2
5
8
10
t (s)
25
16
8
x
3-x
x/(3-x) = 8/16 ∴ x = 1
16
15
V (m/s)
x
(3-x)
3
2
5
8
10
t (s)
8
14
91·5
51.5
S (m)
2
5
8
10
t (s)
10
22

Example 2.A particle moves in a straight line with an acceleration shown in fig. knowing that it starts from origin with V_0 = 14 m/s . plot S – t curve for 0 < t < 15 sec. and determine .
a) the max. value of the velocity of the particle , b) the max.

value of its position co-ordinate



Solution : For given V-t diagram

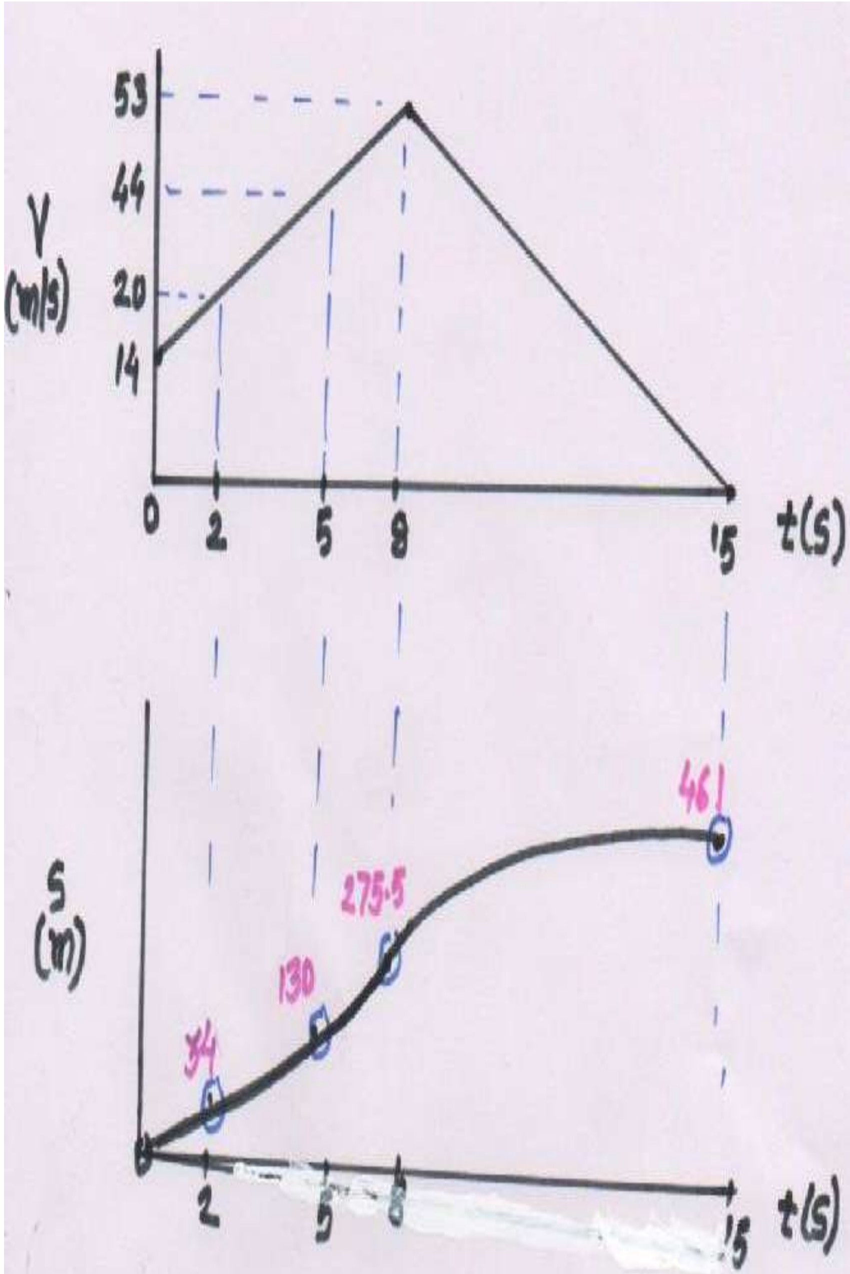
Δv (0 to 2 sec.) = 20 – 14 = 6 m/sec.

Δv (2 to 5 sec.) =44 – 20 = 24 m/sec.

Δv (5 to 8 sec.) = 53 – 44 =9 m/sec.

Δv (8 to15 sec.) =53 – 0 = 53 m/sec.

Max. velocity = 53 m/sec.(Ans)



S – t diagram calculation

$\Delta s = (\text{ area })_{v-t}$ $\qquad S_0 = 0$

$(S_2 - S_0) = (\text{ area })_{v-t}$

$(S_2 - S_0) = [(2 \times 14) + (½ \times 2 \times 6)]$

$S_2 = 34$ m

$S_5 - S_2 = (\text{ area })_{v-t}$

$(S_5 - 34) = [(3 \times 20) + (½ \times 3 \times 24)]$

$S_5 = 130$ m

$S_8 - S_5 = (\text{ area })_{v-t}$

$(S_8 - 130) = [(3 \times 7) + (½ \times 3 \times 9)]$

$S_8 = 275.5$ m

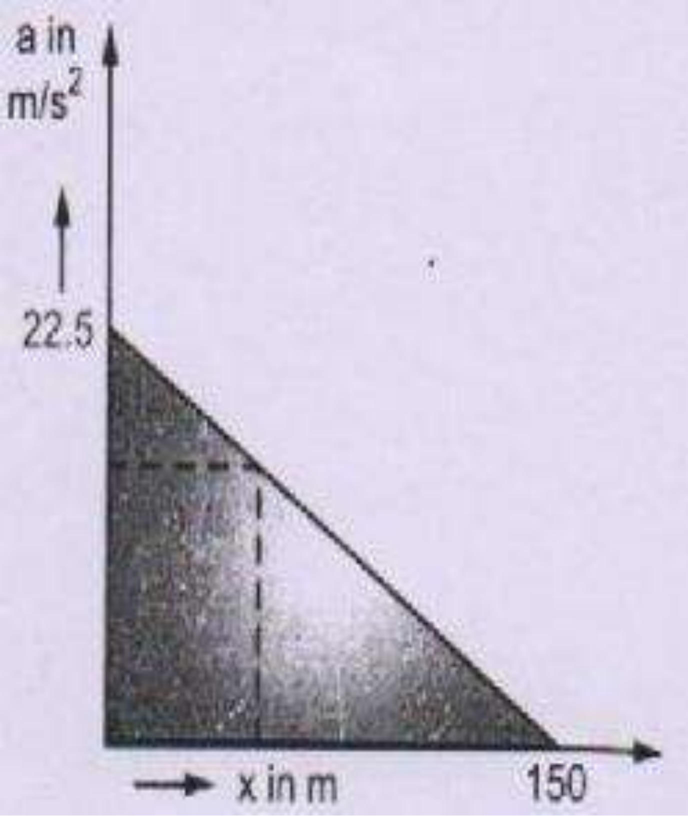
$S_{15} - S_8 = (\text{ area })_{v-t}$

$S_{15} - (275.5) = [(½ \times 7 \times 53)]$

$S_{15} = 461$ m

Max. value of its position co-ordinate = 461 m

Example 3. A jet plane starts from rest at x = 0 and is subjected to the acceleration shown in fig. Determine the speed of the plane when it has travelled 60 m



Given : v = ? when x = 60 m

Key :

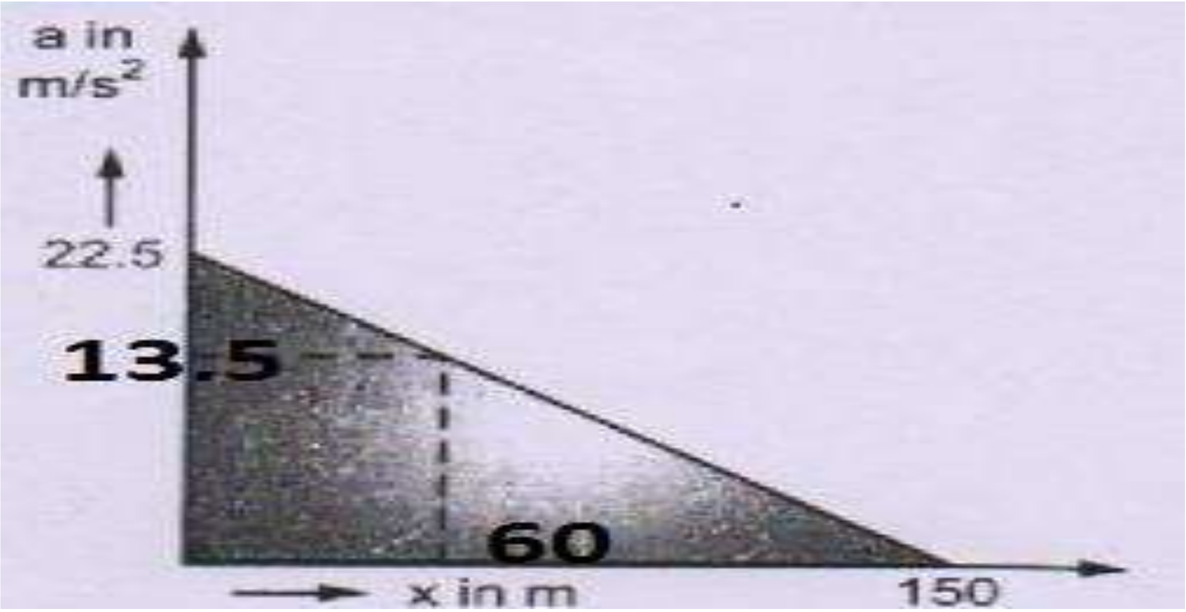
1) a = ? when x = 60m

(by interpolation with the help of a- x curve)

2) v = ? when x = 60

by [$v^2/2 = (\text{area})_{a\text{-}s}$]

S	a
0	22.5
60	?
150	0



1) a = ? when x = 60m

(by interpolation with the help of a- x curve)

$a = 22.5 - 9 = 13.5\ m/s^2$

2) v = ? when x = 60

by **[$v^2/2 = (\text{area})_{a\text{-}s}$]**

$v^2/2 = (\text{area})_{a\text{-}s}$

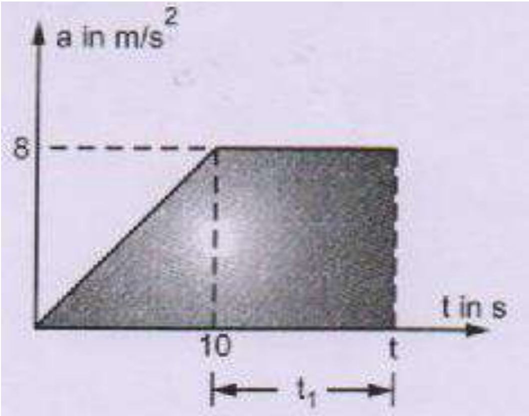
$v^2/2 = [(60 \times 13.5) + (½ \times 60 \times 9)]$

$v^2/2 = 1080$

$v^2 = 2160$

<u>v = 46.48 m/s (Ans)</u>

Example 4. A car starting from rest moves along a straight track with an acceleration shown in fig. Determine time t for the car to reach a speed of 50 m/s and construct the v-t diagram that describe the motion until the time t .



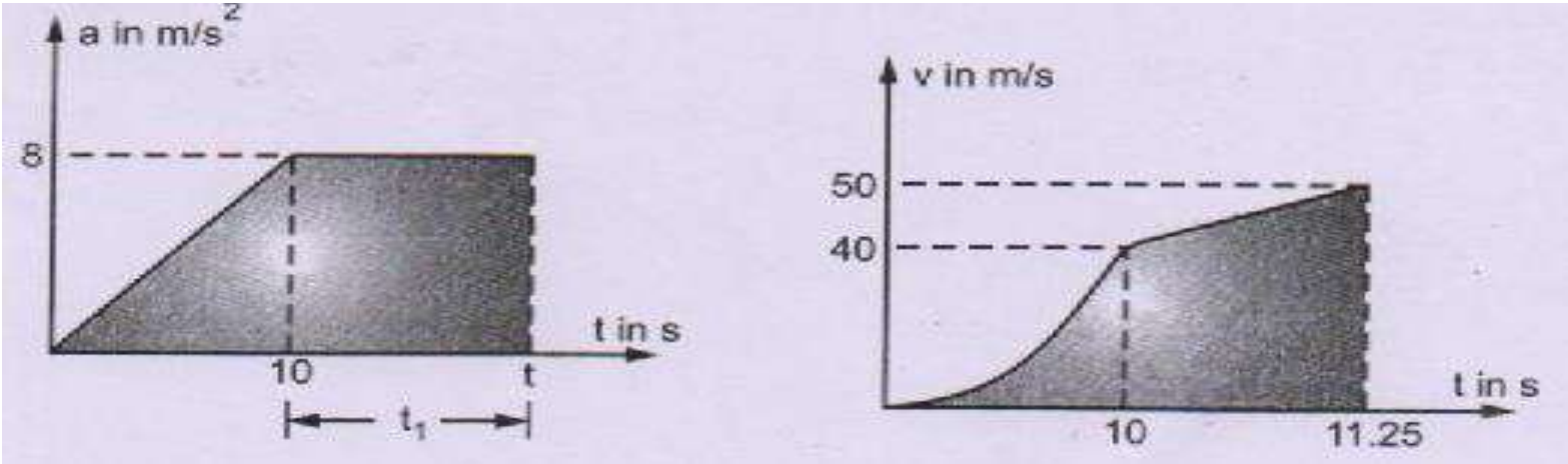
Given & Key

$V_0 = 0$ car starting from rest ,

$V_t = 50$

$t_1 = ?$ KEY $\Delta v = (\text{ area })_{a\text{-}t}$ $V_t - V_0 = (\text{ area })_{a\text{-}t}$

$V_{10} = ?$ $\Delta v = V_{10} - V_0 = (\text{ area })_{a\text{-}t}$



From the property of a – t curve , the area of a- t curve at time t , gives the velocity at that time.

$\Delta v = (\text{ area })_{a\text{-}t}$

$V_t - V_0 = (\text{ area })_{a\text{-}t}$

$50 - 0 = [\,(\tfrac{1}{2} \times 10 \times 8) + (8 \times t_1)\,]$

$50 = 40 + 8\, t_1$

$t_1 = 1.25$ sec.

time for the car to reach a speed of 50 m/s = 10 + 1.25

t = 11.25 sec.

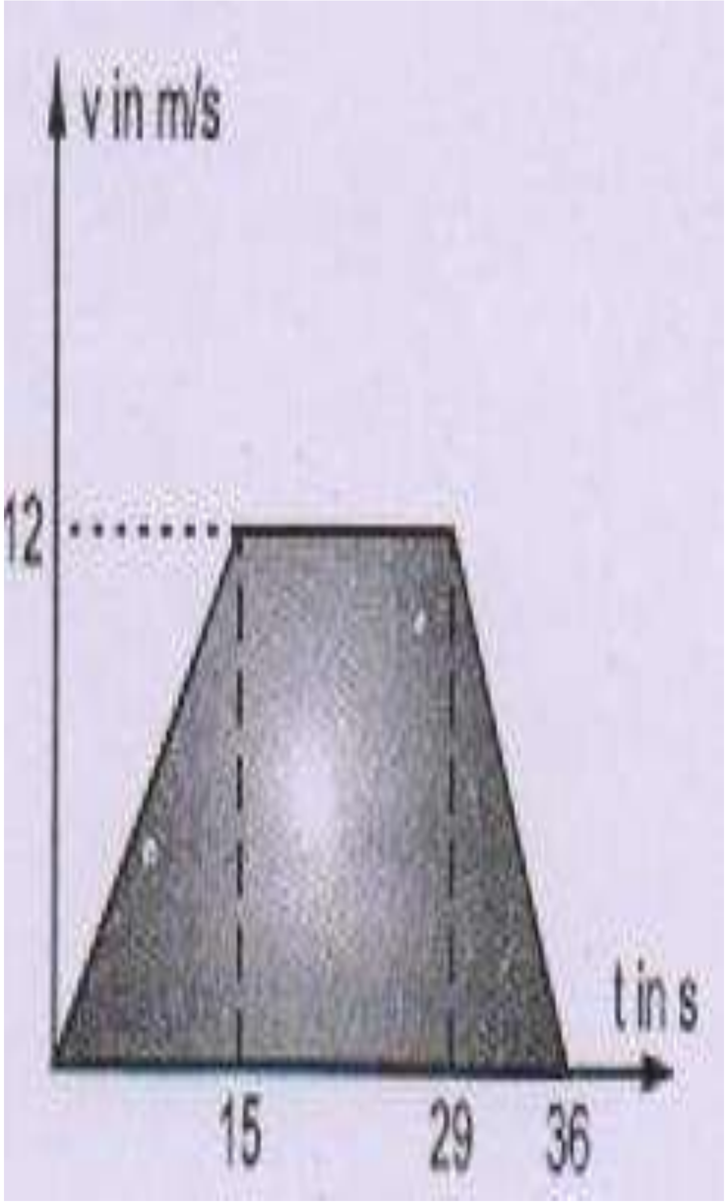
$\Delta v = (\text{ area })_{a\text{-}t}$

$V_{10} - V_0 = [\tfrac{1}{2} \times 10 \times 8\,]$

$V_{10} - 0 = 40$

$V_{10} = 40$ m/s

Example 5.A bus starts from rest at point A and accelerates at the rate of 0.8 m/s till it reaches a speed of 12 m/s . It then proceeds at 12 m/s till the brakes are applied . It comes to rest at point B , 42 m beyond the point where the brakes are applied . Assuming the uniform deacceleration & that the total time of travel from A & B is 36 sec, determine the distance between A & B using V –t diagram .Also draw S –t diagram



Solution :

$\Delta s = (\text{ area })_{v\text{-}t}$

$(S_{15} - S_0) = (\text{ area })_{v\text{-}t}$

$S_{15} - 0 = \frac{1}{2} \times 15 \times 12$

$S_{15} = 90 \text{ m}$

$\Delta s = (\text{ area })_{v\text{-}t}$

$(S_{29} - S_{15}) = (\text{ area })_{v\text{-}t}$

$S_{29} - 90 = (\frac{1}{2} \times 14 \times 12)$

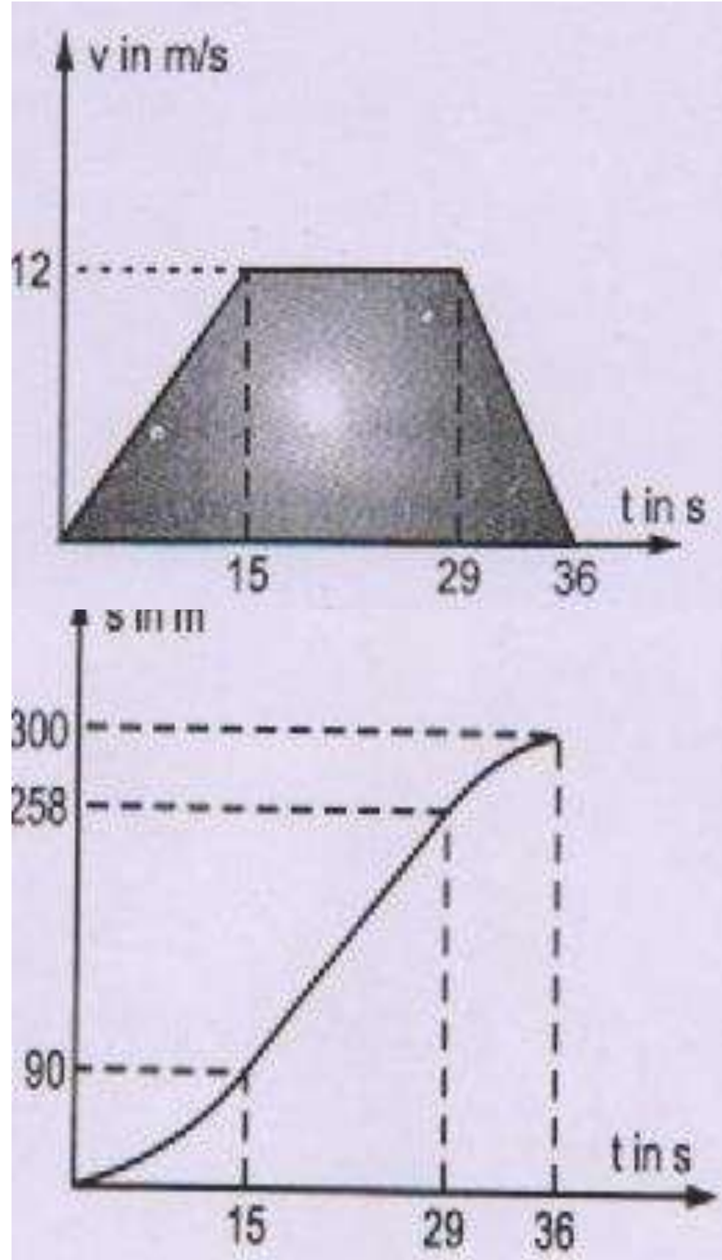
$S_{29} = 258 \text{ m}$

$\Delta S = (\text{ area })_{v\text{-}t}$

$(S_{36} - S_{29}) = (\text{ area })_{v\text{-}t}$

$(S_{36} - 529) = (\frac{1}{2} \times 7 \times 12)$

$S_{36} = 300\text{m}$

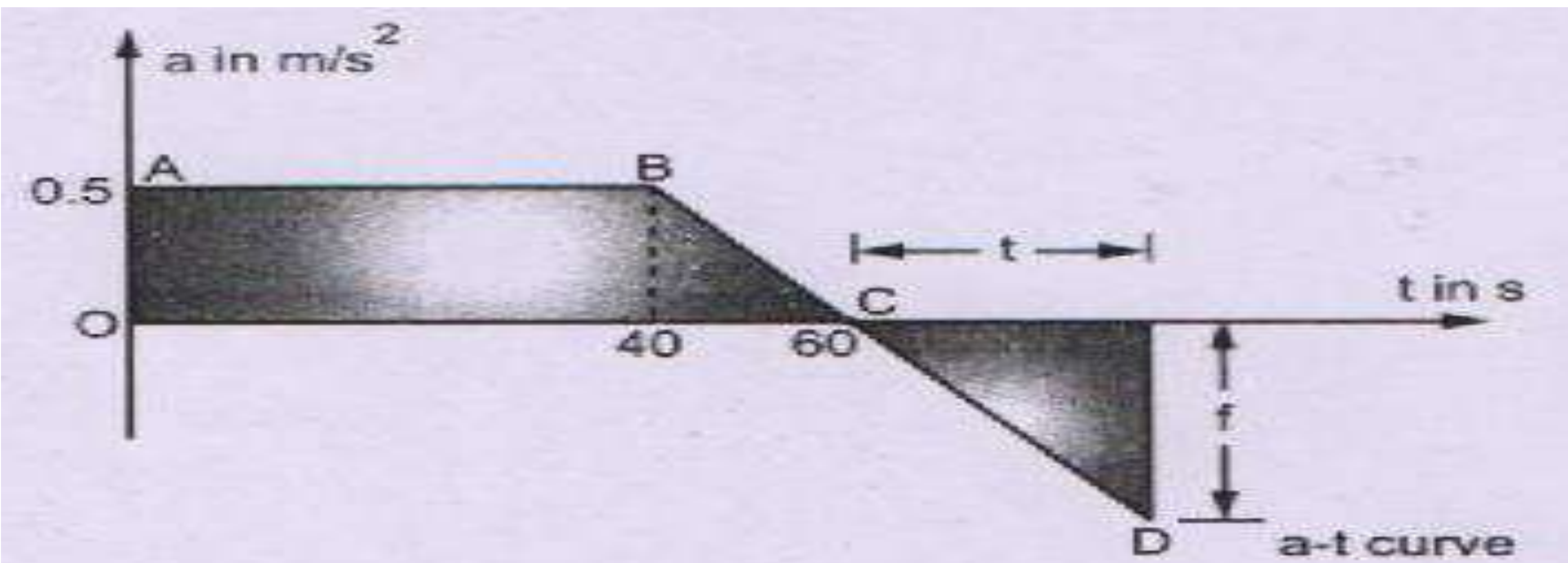


Example 6. Fig . shows a-t diagram for a car which starts from rest & come to halt after time (60 + t) seconds . Find the value of ‘t’ and ‘f’ shown in the diagram find also

velocity at 40 seconds

velocity at 60 seconds

Distance travelled during first 40 seconds



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CURVILINEAR

MOTION

CURVILINEAR MOTION

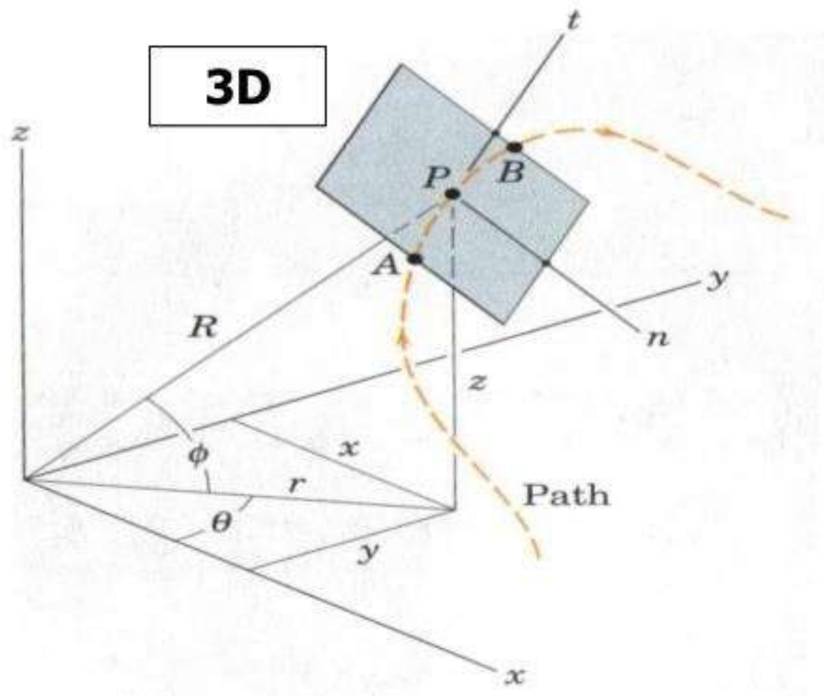
- KINEMATICS
- KINETICS

Topics Covered in Curvilinear Motion

- Plane curvilinear motion
- Coordinates used for describing curvilinear motion
  - Rectangular coords
  - *n-t* coords
  - Polar coords

Plane curvilinear Motion

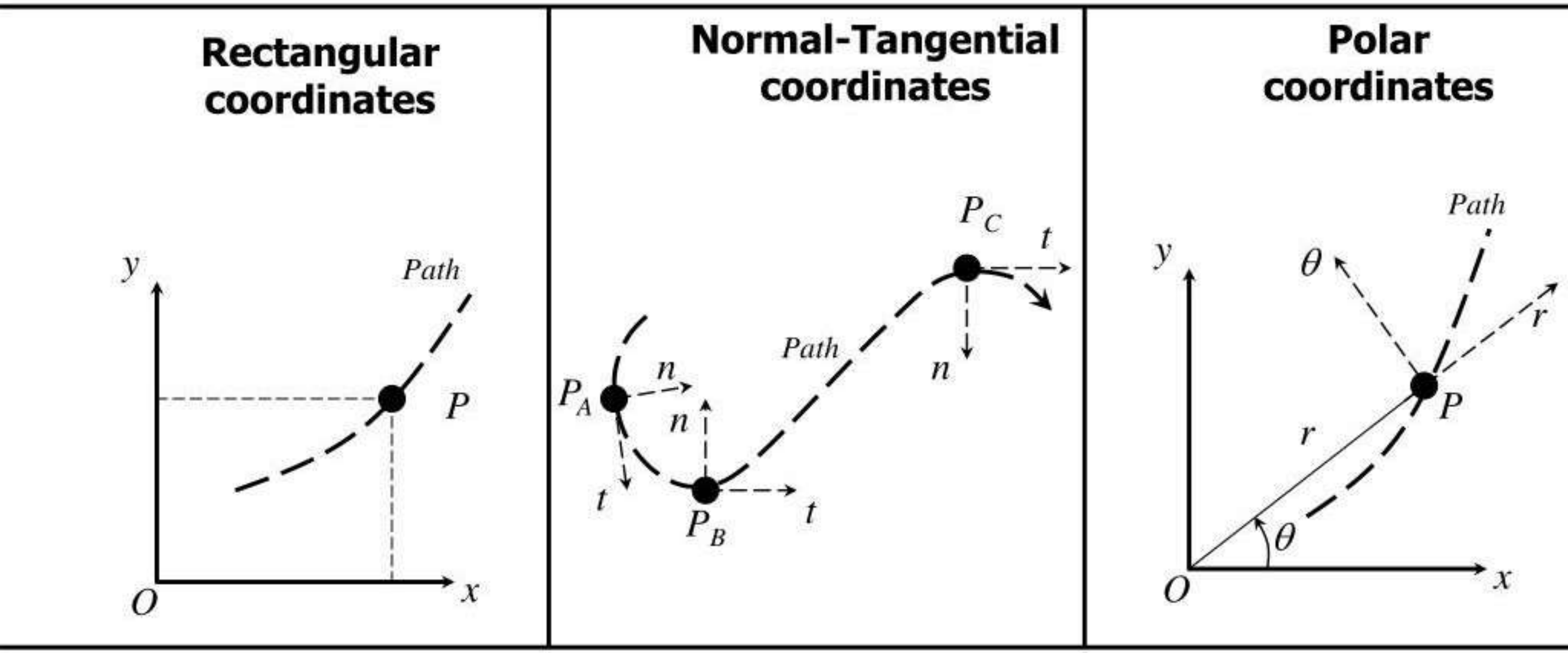
- Studying the motion of a particle along a curved path which lies in a single plane (2D).
- This is a special case of the more general 3D motion.



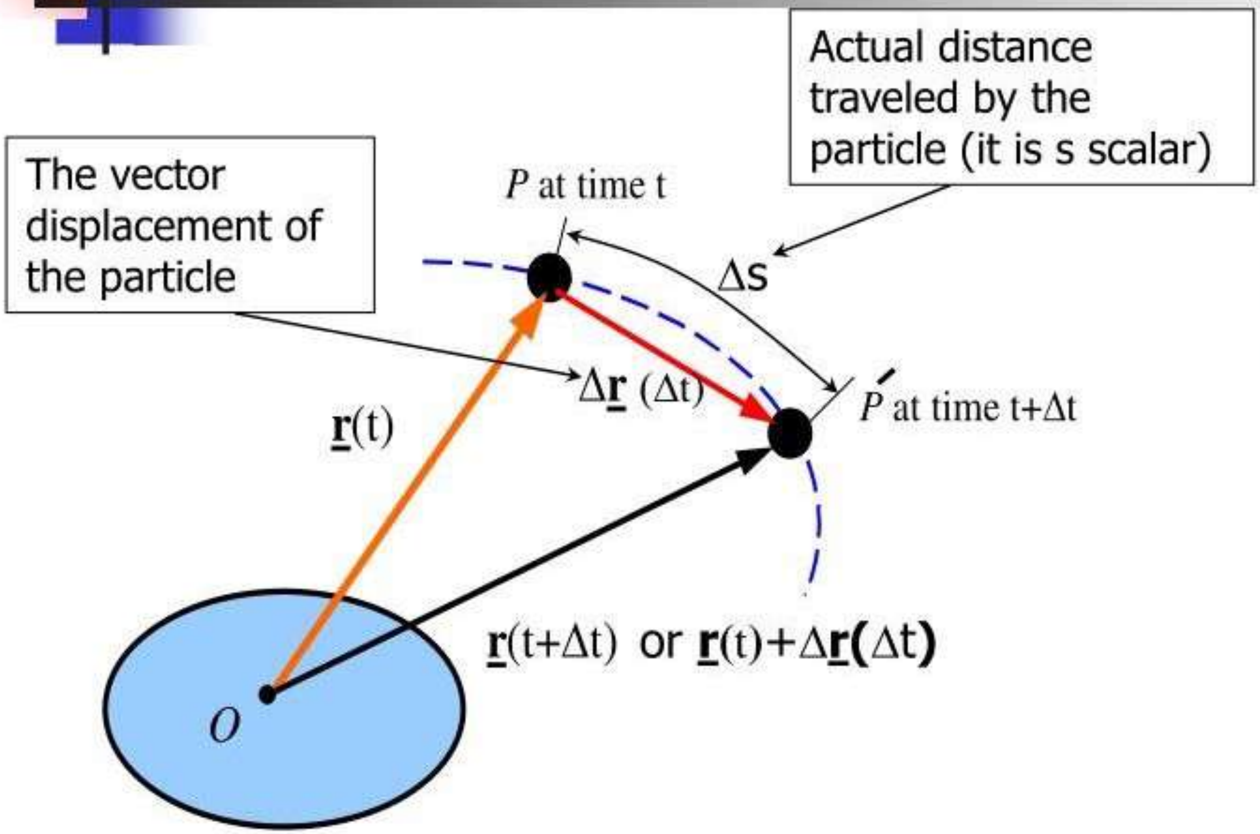
Plane curvilinear Motion – (Cont.)

- If the x-y plane is considered as the plane of motion; from the 3D case, z and φ are both zero, and R becomes as same as r.
- The vast majority of the motion of particles encountered in engineering practice can be represented as plane motion.

Coordinates Used for Describing the Plane Curvilinear Motion



Plane Curvilinear Motion – without Specifying any Coordinates (Displacement)



Note: Since, here, the particle motion is described by two coordinates components, both the magnitude and the direction of the position, the velocity, and the acceleration have to be specified.

Note: If the origin (*O*) is changed to some different location, the position **r**(t) will be changed, but Δ**r**(Δt) will not change.

Plane Curvilinear Motion – without Specifying any Coordinates (Velocity)

- **Average velocity ($\underline{v}_{av}$):**

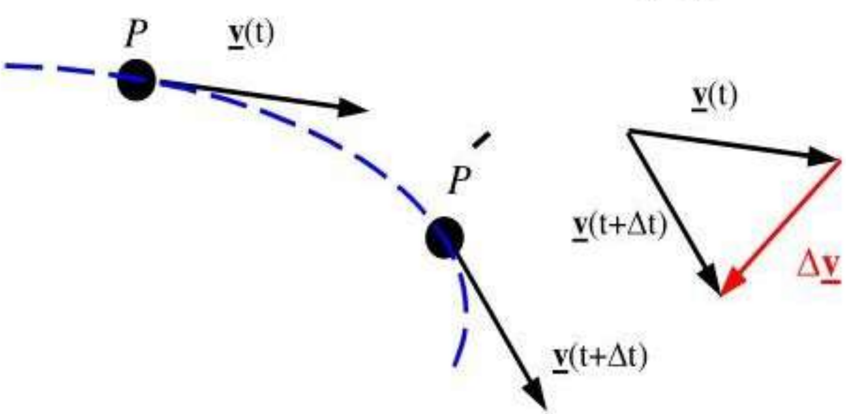
$$\underline{v}_{av} = \frac{\Delta \underline{r}}{\Delta t}$$

Note: $\underline{\mathbf{v}}_{av}$ has the direction of $\Delta\underline{\mathbf{r}}$ and its magnitude equal to the magnitude of $\Delta\underline{\mathbf{r}}$ divided by Δt.

Note: the average speed of the particle is the scalar $\Delta s/\Delta t$. The magnitude of the speed and $\underline{\mathbf{v}}_{av}$ approach one another as Δt approaches zero.

- **Instantaneous velocity ($\underline{v}$):** as Δt approaches zero in the limit,

$$\underline{v} = \lim_{\Delta t \to 0} \frac{\Delta \underline{r}}{\Delta t} \equiv \frac{d\,\underline{r}}{dt} = \dot{\underline{r}}$$



Note: the magnitude of $\underline{v}$ is called the speed, i.e. $v = |\underline{v}| = ds/dt = \dot{s}$.

Note: the velocity vector $\underline{v}$ is always tangent to the path.

Plane Curvilinear Motion – without Specifying any Coordinates (Acceleration)

- **Average Acceleration ($\underline{a}_{av}$):**

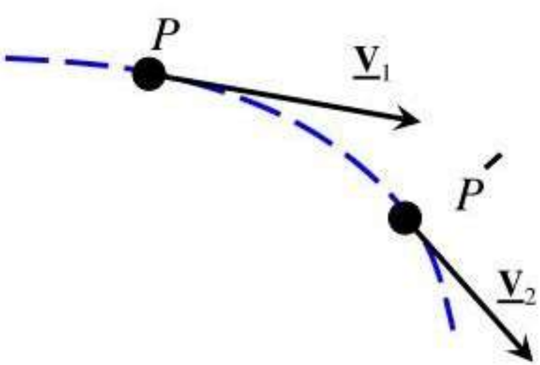
$$\underline{a}_{av} = \frac{\Delta \underline{v}}{\Delta t}$$

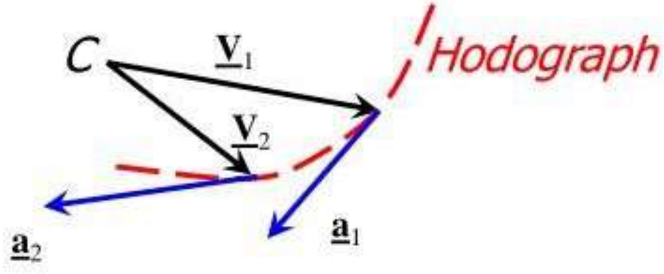
Note: $\underline{\mathbf{a}}_{av}$ has the direction of $\Delta\underline{\mathbf{v}}$ and its magnitude is the magnitude of $\Delta\underline{\mathbf{v}}$ divided by Δt.

- **Instantaneous Acceleration ($\underline{a}$):** as Δt approaches zero in the limit,

$$\underline{a} = \lim_{\Delta t \to 0} \frac{\Delta \underline{v}}{\Delta t} \equiv \frac{d\,\underline{v}}{dt} = \dot{\underline{v}} = \ddot{\underline{r}}$$

Note: in general, the acceleration vector $\underline{a}$ is neither tangent nor normal to the path. However, $\underline{a}$ is tangent to the hodograph.





The description of the Plane Curvilinear Motion in the Rectangular Coordinates (Cartesian Coordinates)

Plane Curvilinear Motion - Rectangular Coordinates

The position vector: $\underline{r} = x\underline{i} + y\underline{j}$

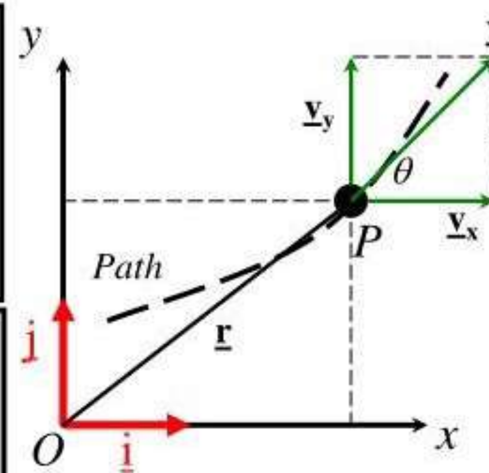
The velocity vector: $\underline{v} = \dot{\underline{r}} = v_x\underline{i} + v_y\underline{j} = \dot{x}\underline{i} + \dot{y}\underline{j}$

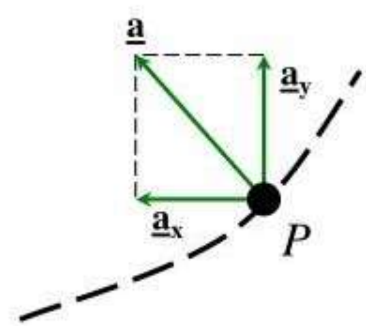
The acceleration vector: $\underline{a} = \dot{\underline{v}} = \ddot{\underline{r}} = a_x\underline{i} + a_y\underline{j} = \ddot{x}\underline{i} + \ddot{y}\underline{j}$

The magnitude of the position vector: $r = \sqrt{x^2 + y^2}$

The magnitude of the velocity vector: $v = \sqrt{{v_x}^2 + {v_y}^2}$

The magnitude of the acceleration vector: $a = \sqrt{{a_x}^2 + {a_y}^2}$





Note: the time derivatives of the unit vectors are zero because their magnitude and direction remain constant.

Note: if the angle θ is measured counterclockwise from the x-axis to $\underline{\mathbf{v}}$ for the configuration of the axes shown, then we can also observe that $dy/dx = \tan\theta = v_y/v_x$.

Plane Curvilinear Motion - Rectangular Coordinates (Cont.)

- The coordinates x and y are known independently as functions of time t; i.e. $x = f_1(t)$ and $y = f_2(t)$. Then for any value of time we can combine them to obtain **r**.
- Similarly, for the velocity **v** and for the acceleration **a**.
- If **a** is given, we integrate to get **v** and integrate again to get **r**.
- The equation of the curved path can be obtained by eliminating the time between $x = f_1(t)$ and $y = f_2(t)$.
- Hence, the rectangular coordinate representation of curvilinear motion is merely the superposition of the components of two simultaneous rectilinear motions in x- and y- directions.

A particle is traversing a curved path of radius 300 m with a speed of 108 kmph and a tangential acceleration 4 m/s^2. Determine total acceleration of the particle.

☑ **Soln. :**

Given data :

$$\text{radius } \rho = 300 \text{ m}$$

$$\text{speed } v = 108 \text{ kmph} = 30 \text{ m/s}$$

$$\text{Tangential acceleration } a_T = 4 \text{ m/s}^2$$

We know that normal acceleration $a_N = \frac{v^2}{\rho} = \frac{(30)^2}{300} = 3 \text{ m/s}^2$

∴ Total acceleration

$$a = \sqrt{a_T^2 + a_N^2} = \sqrt{(4)^2 + (3)^2}$$

$$= 5 \text{ m/s}^2$$ **...Ans.**

$$\tan\theta = \frac{a_N}{a_T} = \frac{3}{4}$$

$$\therefore \theta = 36.87^\circ \text{ with } a_T$$ **...Ans.**

A particle moves along the path $\bar{r} = (8t^2)\,i + (t^3 + 5)j$ m. where t in sec. Determine magnitudes of particle's velocity and acceleration when $t = 3$ sec. Also determine the equation $y = f(x)$ of the path.

A particle moves along the path $\bar{r} = (8t^2)\,i + (t^3 + 5)j$ m. where t in sec. Determine magnitudes of particle's velocity and acceleration when t = 3 sec. Also determine the equation y = f(x) of the path.

☑ **Soln. :**

From position vector, the rectangular position co-ordinates are

$$\bar{r} = (8t^2)\,\hat{i} + (t^3 + 5)\,\hat{j} \text{ m}$$

$$\therefore \quad \frac{d\bar{r}}{dt} = \bar{v} = (16\,t)\,\hat{i} + (3t^2)\,\hat{j} \text{ m/s}$$

$$\frac{d\bar{v}}{dt} = \bar{a} = (16)\,\hat{i} + (6t)\,\hat{j}$$

At t = 3 sec,

$$\bar{v} = 48\,\hat{i} + 27\hat{j}$$

$$|\bar{v}| = \sqrt{(48)^2 + (27)^2} = 55.07 \text{ m/s}$$

$$\bar{a} = 16\,\hat{i} + 18\hat{j}$$

$$|\bar{a}| = \sqrt{(16)^2 + (18)^2} = 24.08 \text{ m/s}^2$$

To find path equation in y = f(x) eliminate 't'

$$x = 8t^2 \qquad \therefore \quad t^2 = x/8$$

$$\therefore \quad t = (x/8)^{1/2}$$

substitute this value in $y = t^3 + 5$

The motion of a particle is defined by $x = (t + 1)^2$ and $y = 4(t + 1)^{-2}$ where x and y are in mt and t in sec. Show that path of particle is part of rectangular hyperbola. Find velocity and acceleration at $t = 0$.

The motion of a particle is defined by $x = (t + 1)^2$ and $y = 4(t + 1)^{-2}$ where x and y are in mt and t in sec. Show that path of particle is part of rectangular hyperbola. Find velocity and acceleration at $t = 0$.

☑ **Soln. :**

Given : $$x = (t+1)^2 \quad \ldots(1)$$

and $$y = 4(t+1)^{-2} = \frac{4}{(t+1)^2} \quad \ldots(2)$$

Now,

To find path equation eliminate

time 't' from Equations (1) and (2)

$$y = \frac{4}{(t+1)^2} = \frac{4}{x}$$

$$\therefore \quad xy = 4$$

This is equation of rectangular hyperbola.

Velocity : $$v_x = \frac{dx}{dt} = 2(t+1)$$

$$v_y = \frac{dy}{dt} = \frac{4(-2)}{(t+1)^3} = \frac{-8}{(t+1)^3}$$

When $t = 0$, $v_x = 2$ m/s $\qquad v_y = -8$ m/s

$$\therefore \text{ velocity } v = \sqrt{v_x^2 + v_y^2} = \sqrt{(2)^2 + (8)^2}$$

$$= \mathbf{8.25 \frac{m}{s}} \quad \ldots\textbf{Ans.}$$

$$\tan\theta = \frac{v_y}{v_x} \Rightarrow \theta = 75.96^\circ \text{ in } 4^{th} \text{ quadrant.}$$

Acceleration

$$a_x = \frac{d}{dt}(v_x) = 2\frac{m}{s^2}$$

$$a_y = \frac{d}{dt}(v_y) = \frac{-24}{(t+1)^4}$$

when $t = 0$, $a_x = 2$ m/s^2, $\qquad a_y = -24$ m/s^2

$$\therefore \quad \text{Total acceleration } a = \sqrt{a_x^2 + a_y^2}$$

$$= 24.083 \text{ m/s}^2 \quad \ldots\text{Ans.}$$

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Kinematics of Particle

Projectile motion

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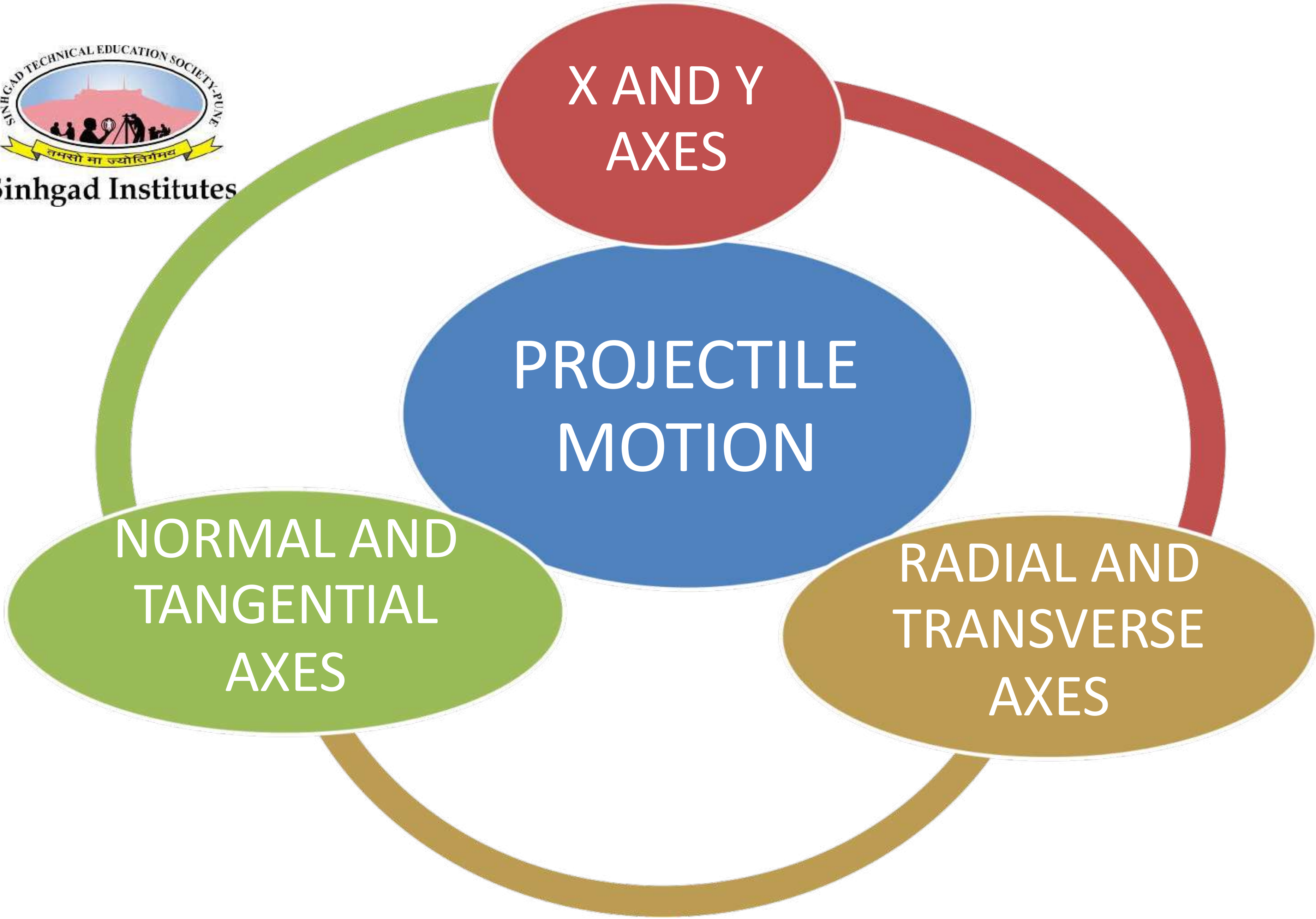
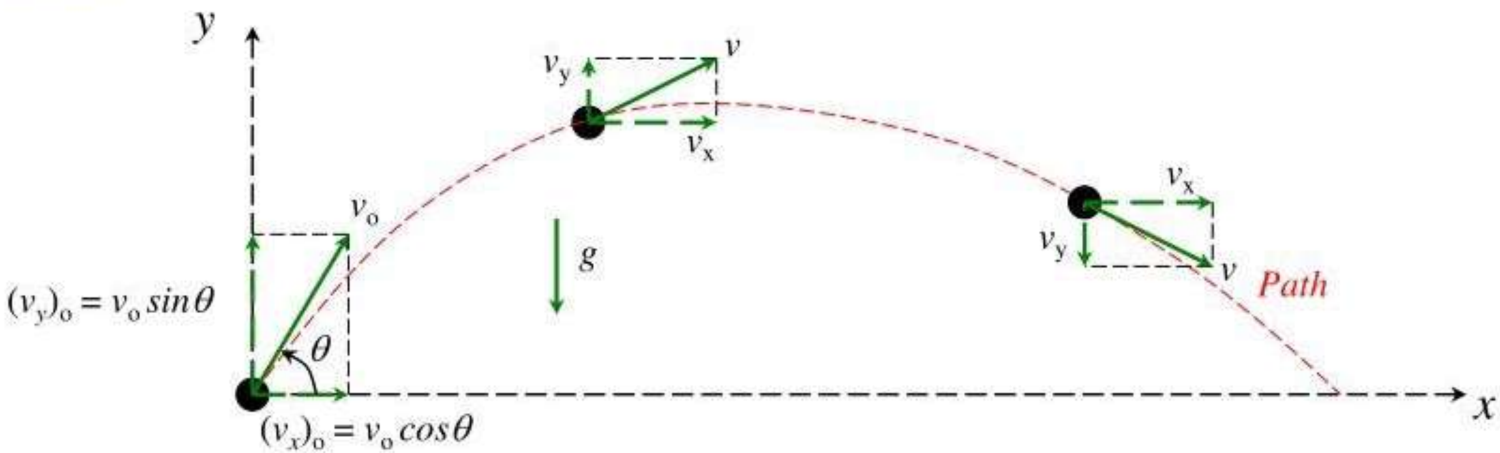


Plane Curvilinear Motion - Rectangular Coordinates (Cont.) – Projectile Motion



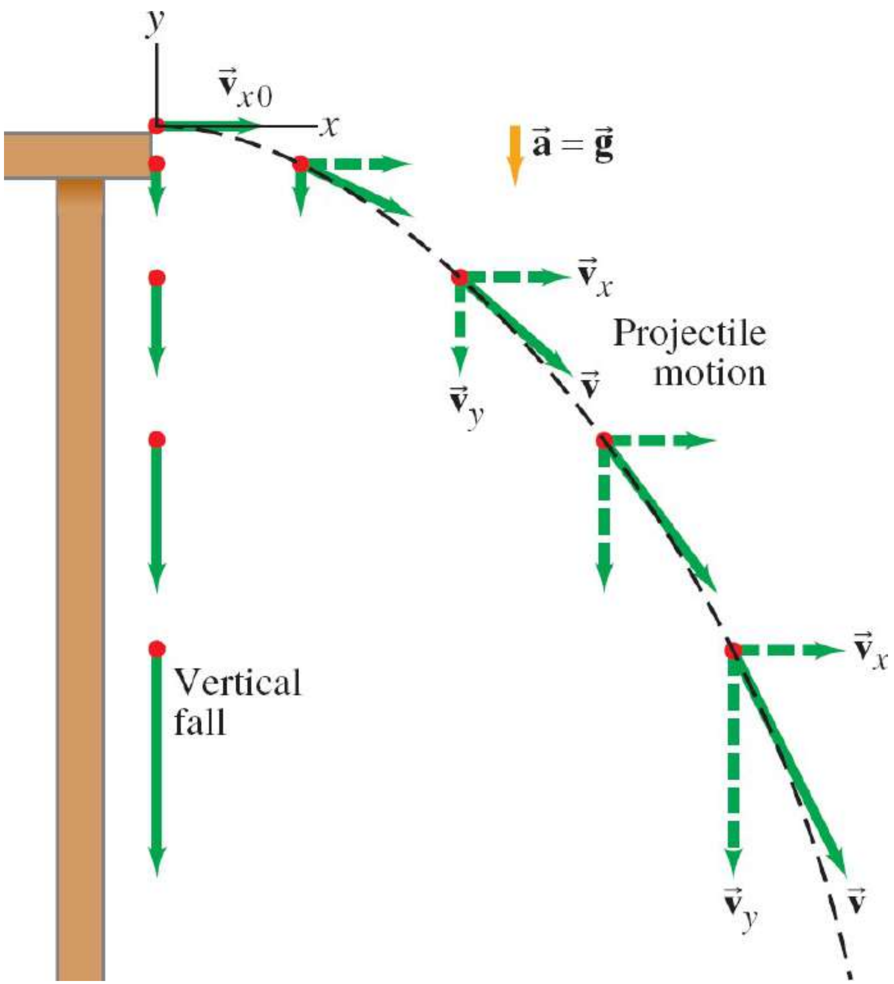
$$a_x = 0 \qquad a_y = -g$$

$$v_x = (v_x)_o \qquad v_y = (v_y)_o - gt$$

$$x = x_o + (v_x)_o t \qquad y = y_o + (v_y)_o t - (1/2)gt^2$$

$$v_y^2 = (v_y)_o^2 - 2g(y - y_o)$$

What is Projectile Motion?



A projectile is an object upon which the only force is gravity.

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Horizontal

Motion of a ball rolling freely along a level surface

Horizontal velocity is ALWAYS constant

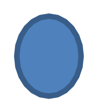
Vertical

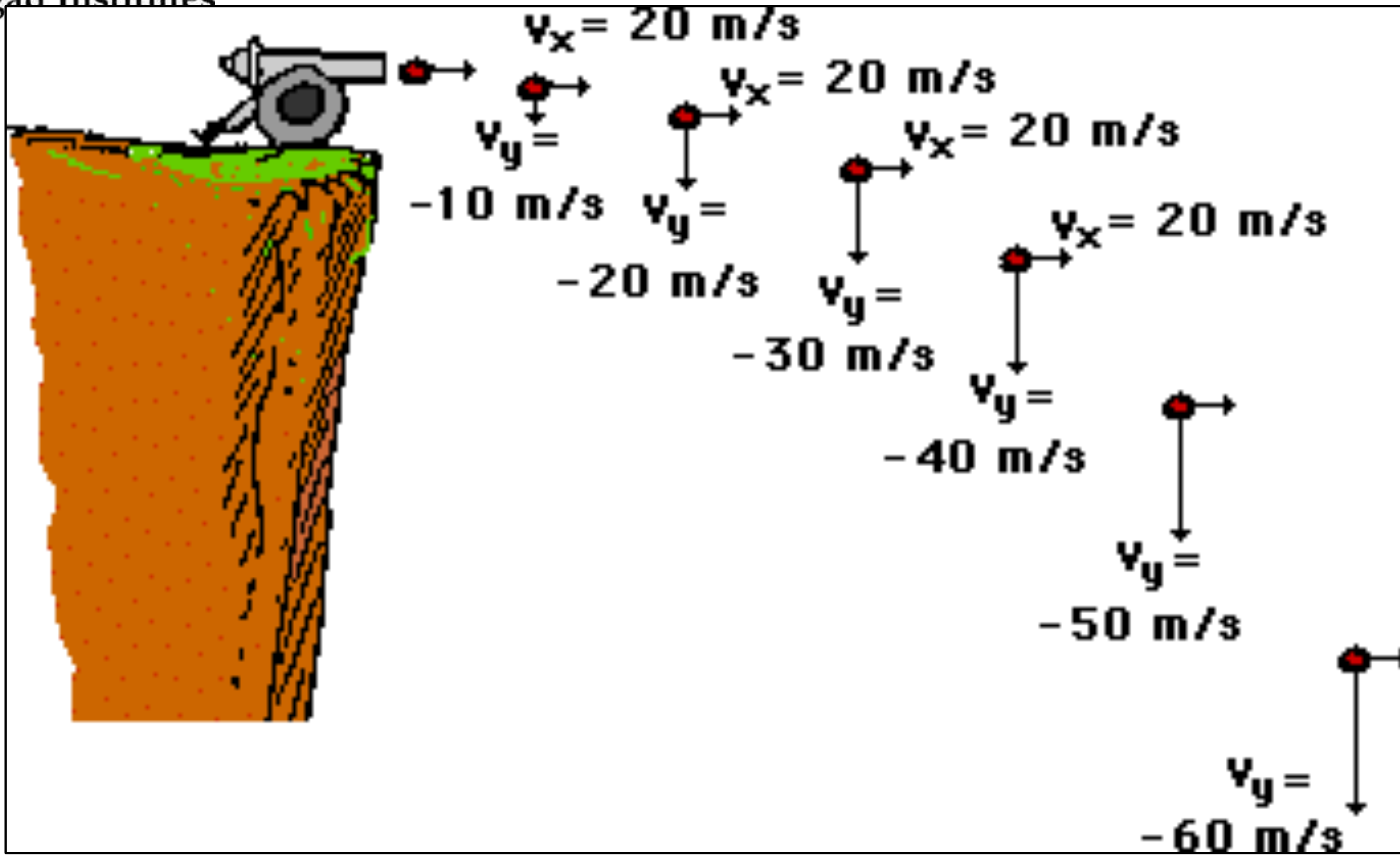
Motion of a freely falling object Force due to gravity

Vertical component of velocity changes with time

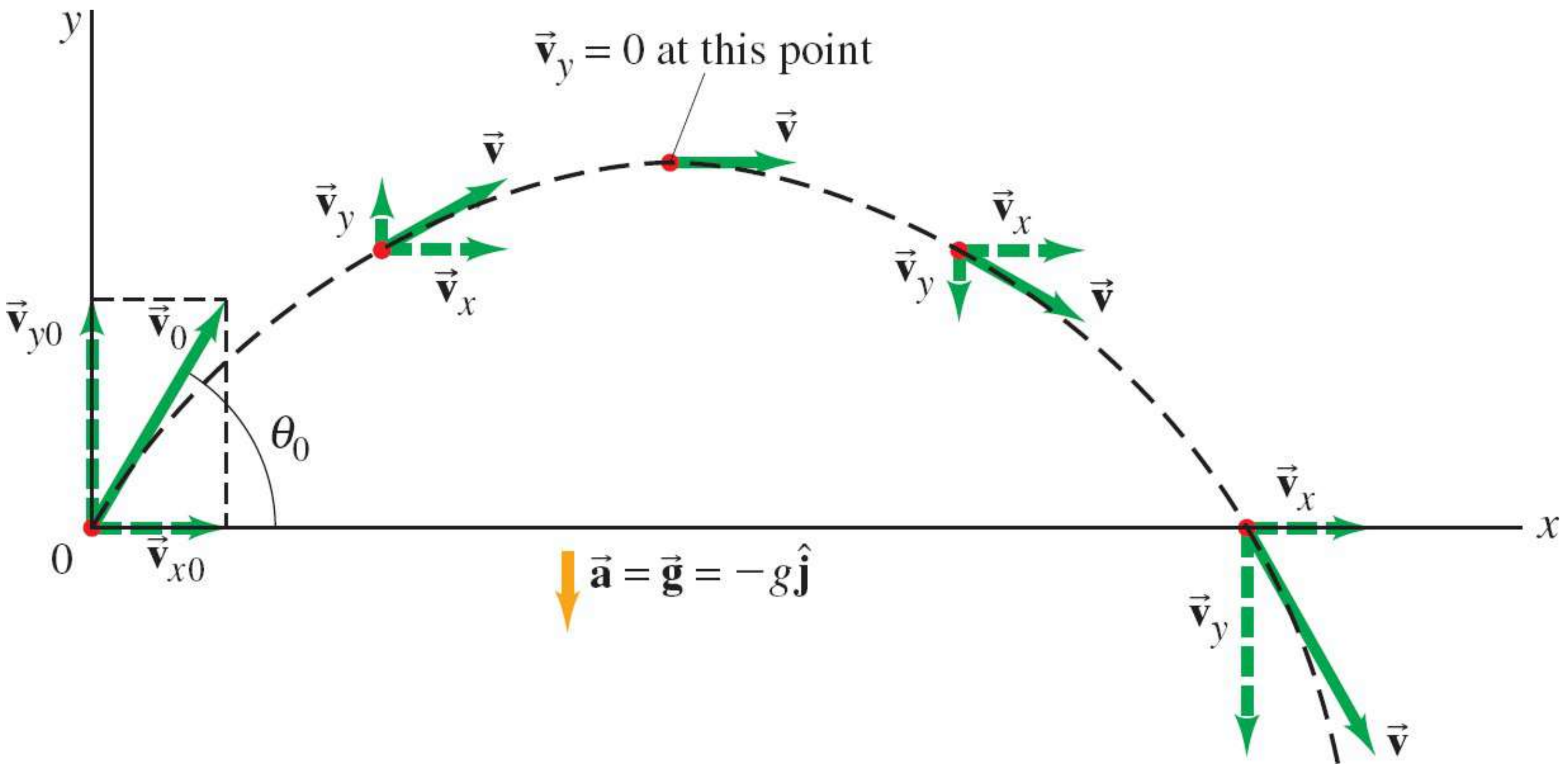
Parabolic

Path traced by an object accelerating only in the vertical direction while moving at constant horizontal velocity

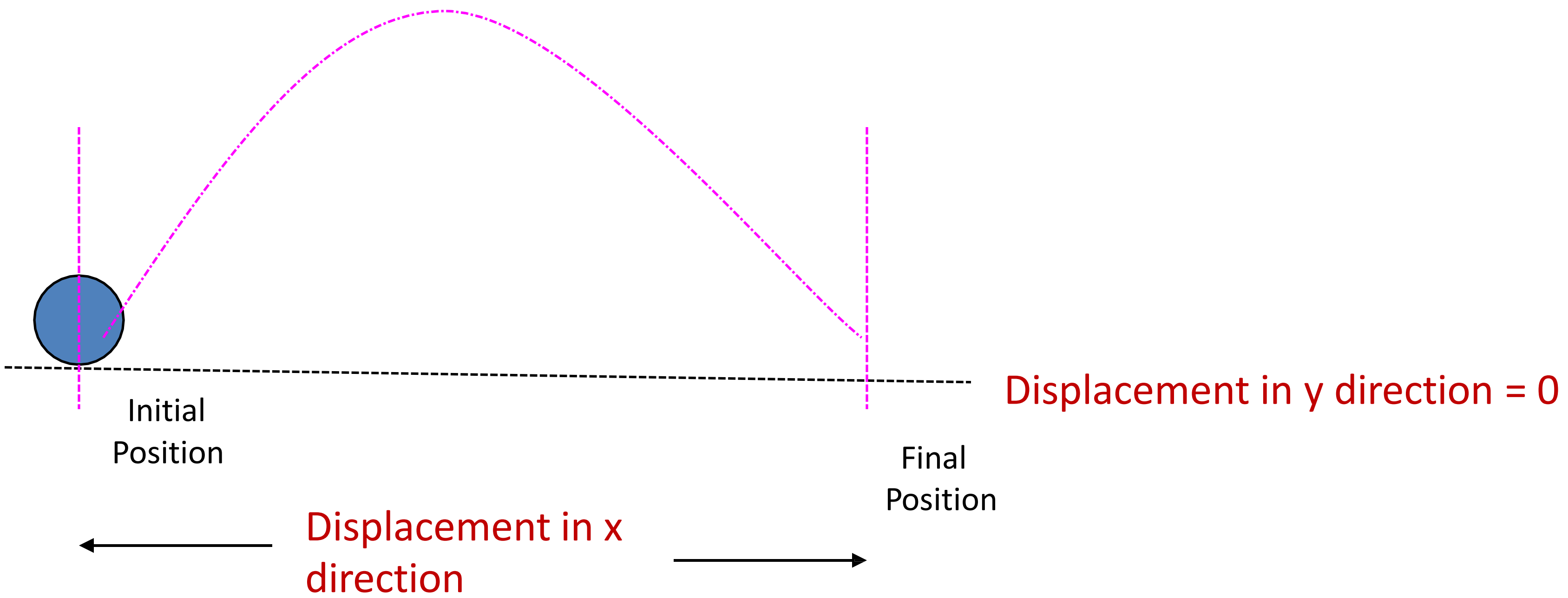



Vx= 20 m/s
Vx= 20 m/s
Vx= 20 m/s
Vx= 20 m/s
Vy= -10 m/s
Vy= -20 m/s
Vy= -30 m/s
Vy= -40 m/s
Vy= -50 m/s
Vy= -60 m/s

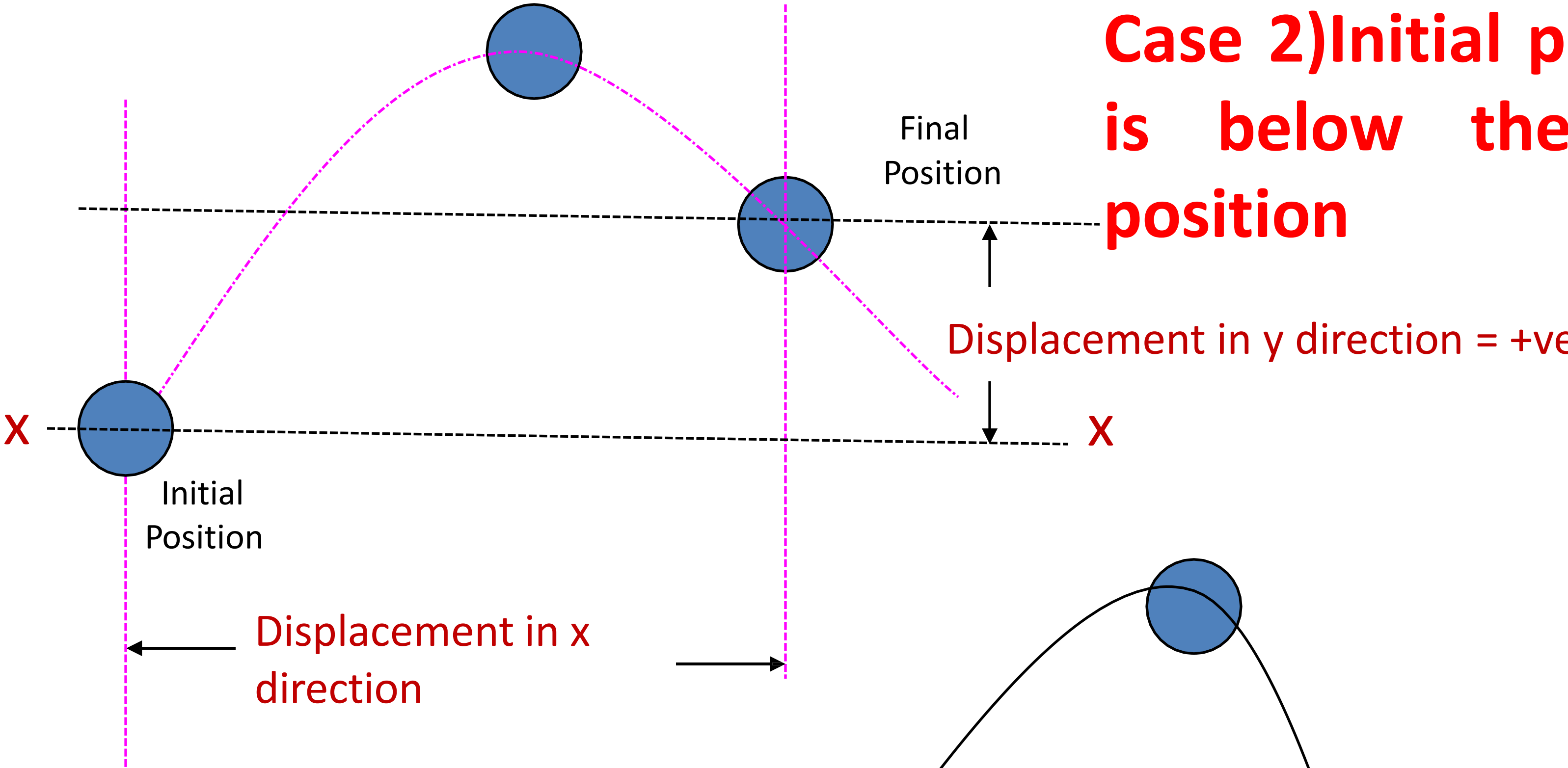
If an object is **launched** at an initial angle of ϑ_0 with the horizontal, the analysis is similar except that the initial velocity has a **vertical** component.



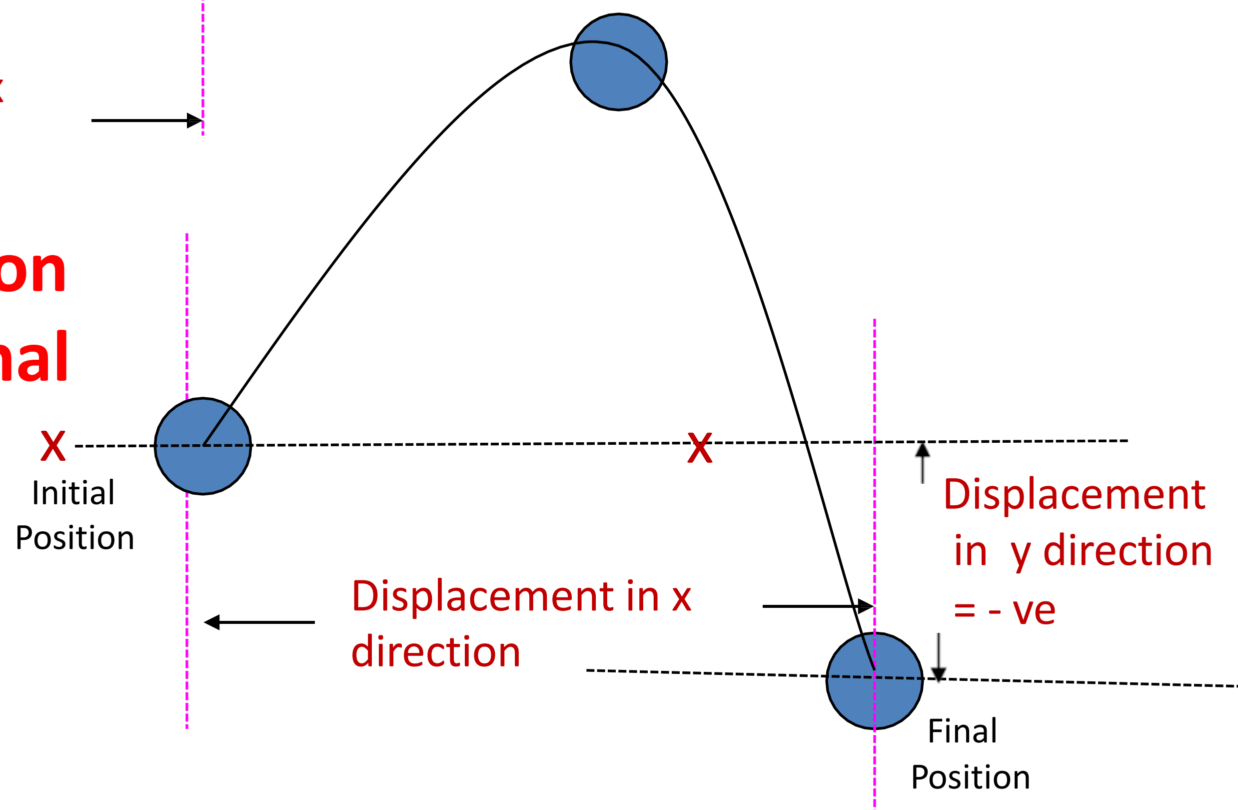
Case 1) Initial and final position are along the same line:



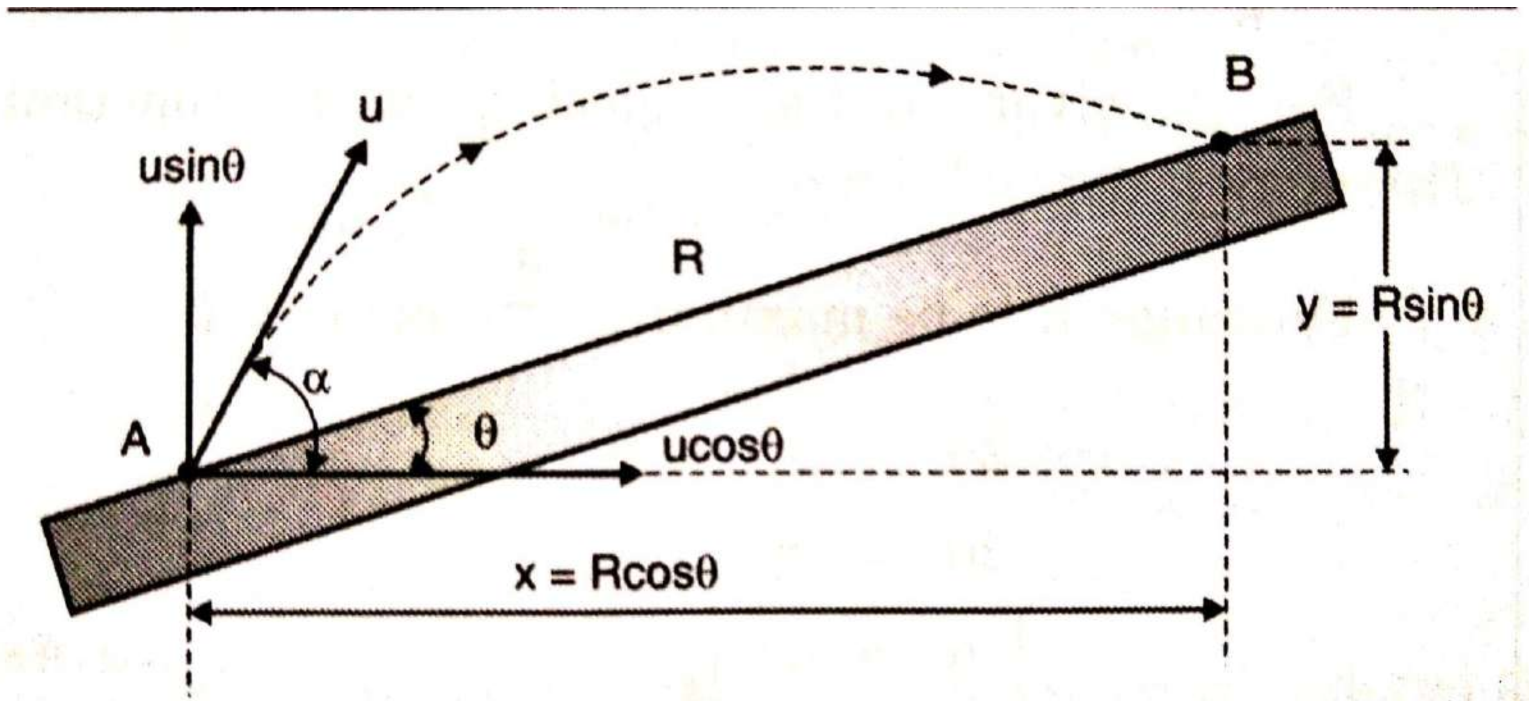
Case 2)Initial position is below the final position



Case 3)Initial position is above the final position



Case 4)Projection on inclined plane

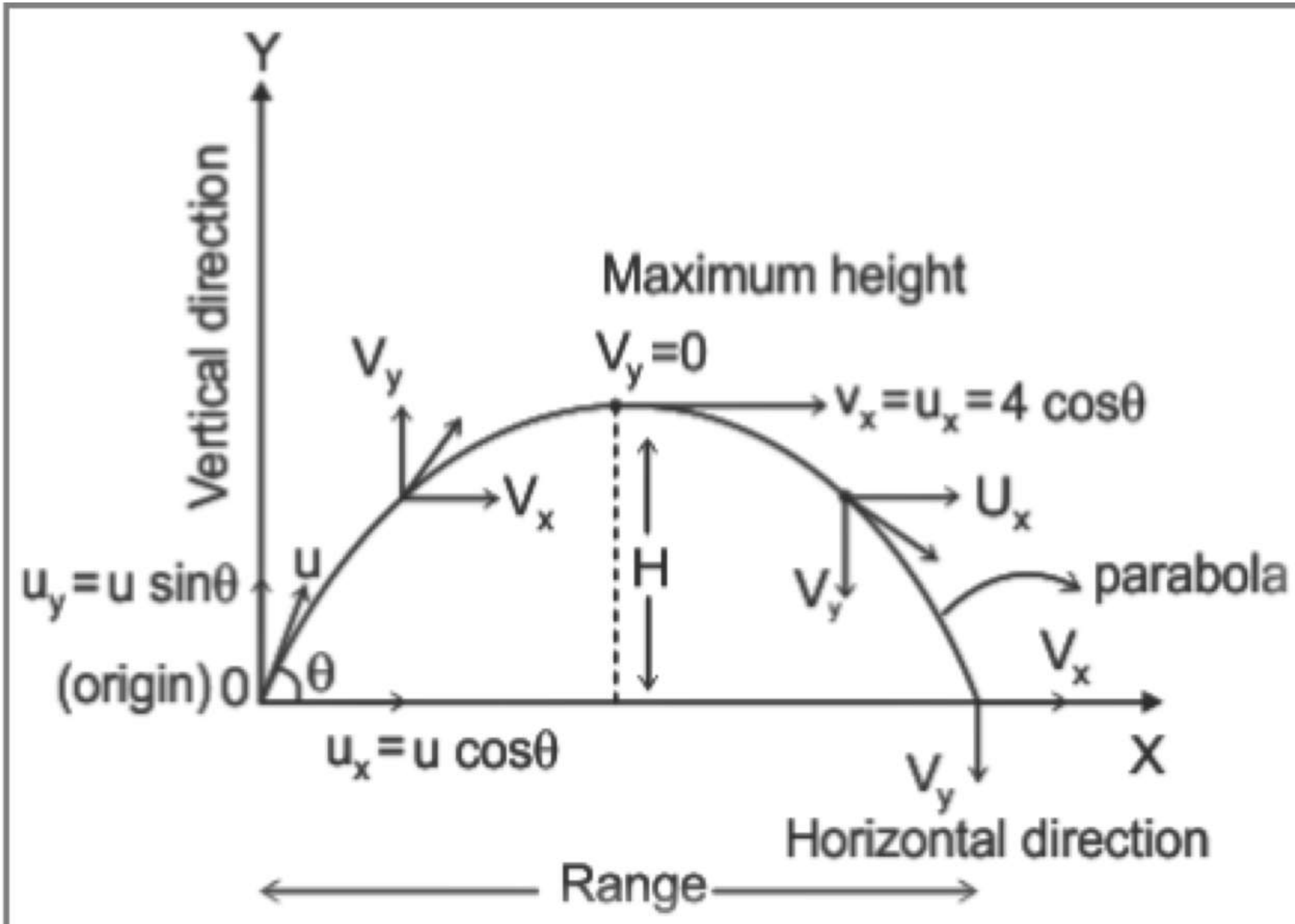


$$\therefore R = \frac{2u^2 \sin(\alpha - \theta)\cos\alpha}{g\cos^2\theta}$$

Range on inclined plane

$$t = \frac{R\cos\theta}{u\cos\alpha}$$

Time of flight



Y
Vertical direction
Maximum height
V_y
$V_y = 0$
$v_x = u_x = 4\cos\theta$
V_x
U_x
H
u
$u_y = u\sin\theta$
V_y
parabola
V_x
(origin) 0
θ
$u_x = u\cos\theta$
X
V_y
Horizontal direction
Range

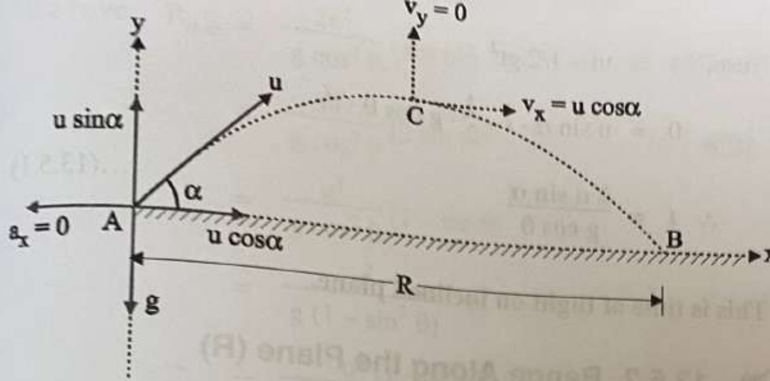


Fig. 13.3.1

13.3.1 Time of Flight

Consider y-motion from A → B (M.U.G.)

$$\text{use} \quad s_y = u_y t - 1/2\, gt^2$$

$$0 = (u \sin \alpha) \cdot t - 1/2\, gt^2$$

$$\therefore \quad t = \frac{2u \sin \alpha}{g} \quad \text{This is time of flight} \quad \ldots(13.3.1)$$

13.3.2 Horizontal Range

Now, consider x-motion from A → B (U.M.)

$$\text{use,} \quad s = \text{vel.} \times \text{time}$$

$$s_x = v_x \cdot t \qquad \therefore \quad R = u \cos \alpha \cdot t$$

Substituting value of

$$t = \frac{2u \sin \alpha}{g}$$

$$\therefore \quad R = u \cos \alpha \cdot \frac{2u \sin \alpha}{g} = \frac{u^2}{g}(2 \sin \alpha \cos \alpha)$$

$$R = \frac{u^2 \cdot \sin 2\alpha}{g} \quad \text{This is horizontal range on H.P.} \ldots(13.3.2)$$

13.3.3 Maximum Range

For the range to be maximum,

$$\frac{d}{d\alpha}(\sin 2\alpha) = 0 \qquad \therefore \quad \cos 2\alpha = 0$$

$$\therefore \quad 2\alpha = 90 \qquad \therefore \quad \alpha = 45°$$

$\therefore$ For maximum range angle of projection should be 45°

$\therefore \quad R_{max} = \frac{u^2}{g}$ **This is maximum range on H.P.** ...(13.3.3)

13.3.4 Maximum Height

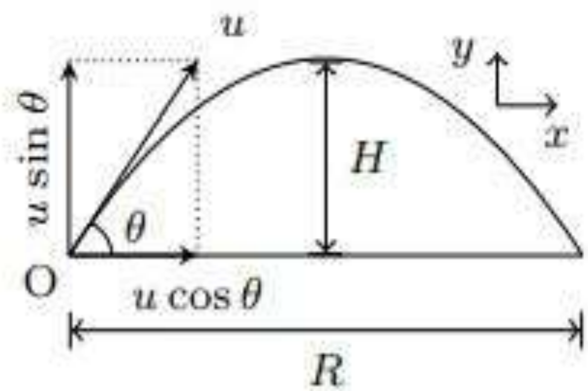
Again consider,

y-motion from A $\rightarrow$ C (M.U.G.)

use, $v_y^2 = u_y^2 - 2\,gs_y \qquad 0 = (u \sin\alpha)^2 - 2\,g\,(H)$

$\therefore \quad H = \frac{u^2 \cdot \sin^2\alpha}{2g}$ **This is maximum height attained.**

Projectile Motion:

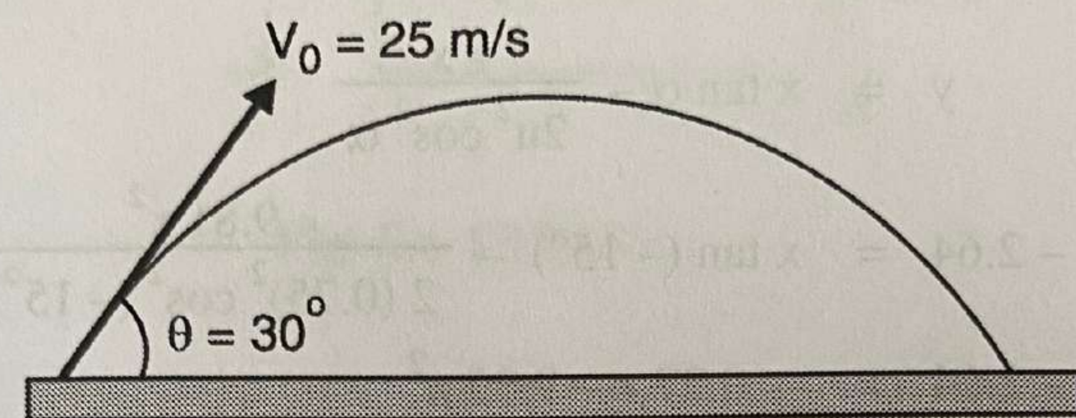


$$x = ut\cos\theta, \quad y = ut\sin\theta - \tfrac{1}{2}gt^2$$

$$y = x\tan\theta - \frac{g}{2u^2\cos^2\theta}x^2$$

$$T = \frac{2u\sin\theta}{g}, \quad R = \frac{u^2\sin 2\theta}{g}, \quad H = \frac{u^2\sin^2\theta}{2g}$$

A projectile is launched With a ,speed of $V_0 = 25$ m/s at an angle of $\theta = 30°$ with horizontal as shown in Fig. Ex. 13.7.22. Determine the maximum distance travelled by projectile along horizontal and vertical direction.



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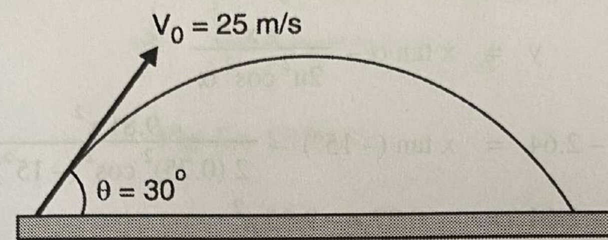


Fig. Ex. 13.7.22

☑ **Soln. :**

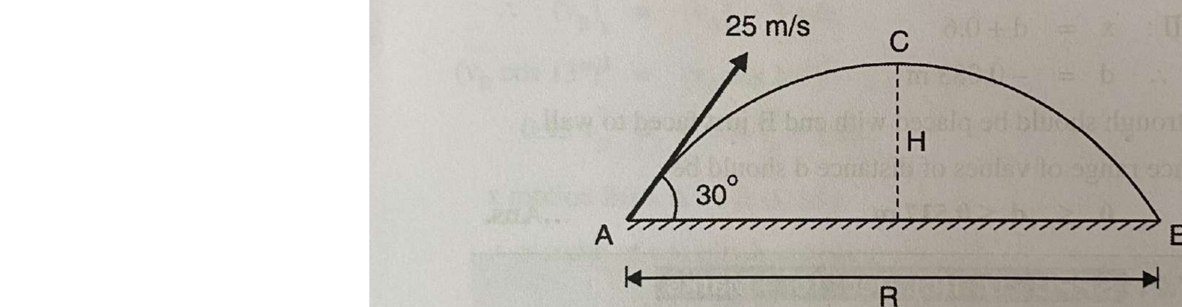


Fig. Ex. 13.7.22(a)

Horizontal range,

$$R = \frac{u^2 \sin 2\alpha}{g} = \frac{(25)^2 \sin(60)}{9.81}$$

$$\mathbf{R = 55.17\ m} \qquad \text{...Ans.(i)}$$

Maximum vertical distance

$$H = \frac{u^2 \sin^2 \alpha}{2g}$$

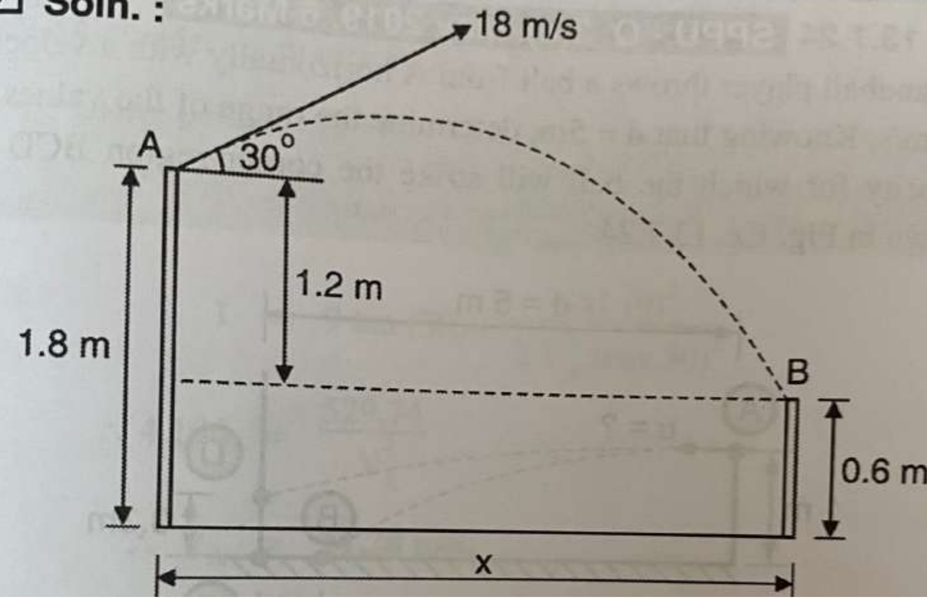
$$= \frac{(25)^2 (\sin 30)^2}{2 \times 9.81}$$

$$\mathbf{H = 7.96\ m} \qquad \text{...Ans.}$$

Q. 1(c), May 14, 4 Marks

A cricket ball shot by a batsman from a height of 1.8 m at an angle of 30° with the horizontal with a velocity of 18 m/s is caught by a fielder at a height of 0.6 m from the ground. Determine the distance between the batsman and fielder.

☑ **Soln. :**



Q. 1(c), May 14, 4 Marks

A cricket ball shot by a batsman from a height of 1.8 m at an angle of 30° with the horizontal with a velocity of 18 m/s is caught by a fielder at a height of 0.6 m from the ground. Determine the distance between the batsman and fielder.

☑ **Soln. :**

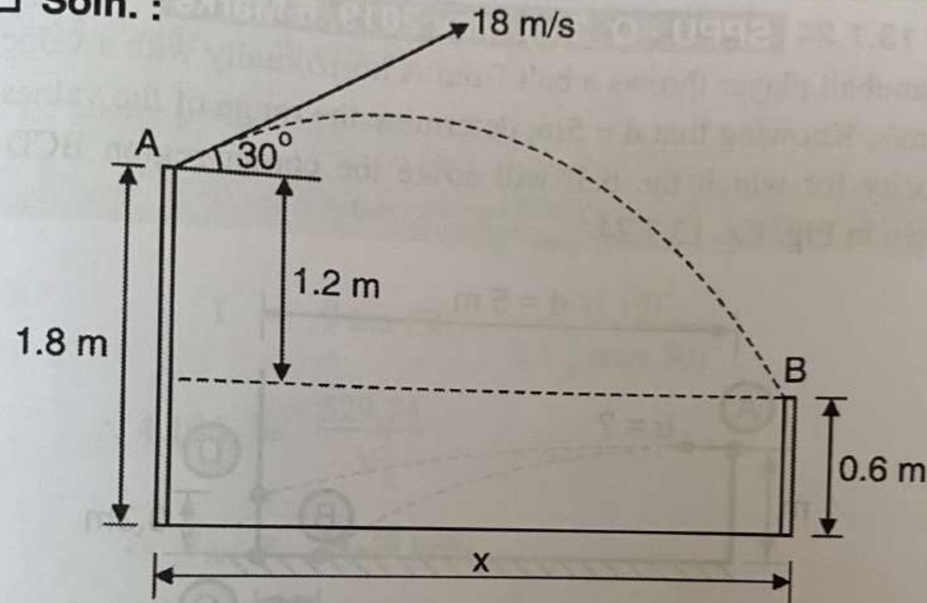


Fig. Ex. 13.7.19

Using,

$$y = x \tan \alpha - \frac{g x^2}{2 u^2 \cos^2 \alpha}$$

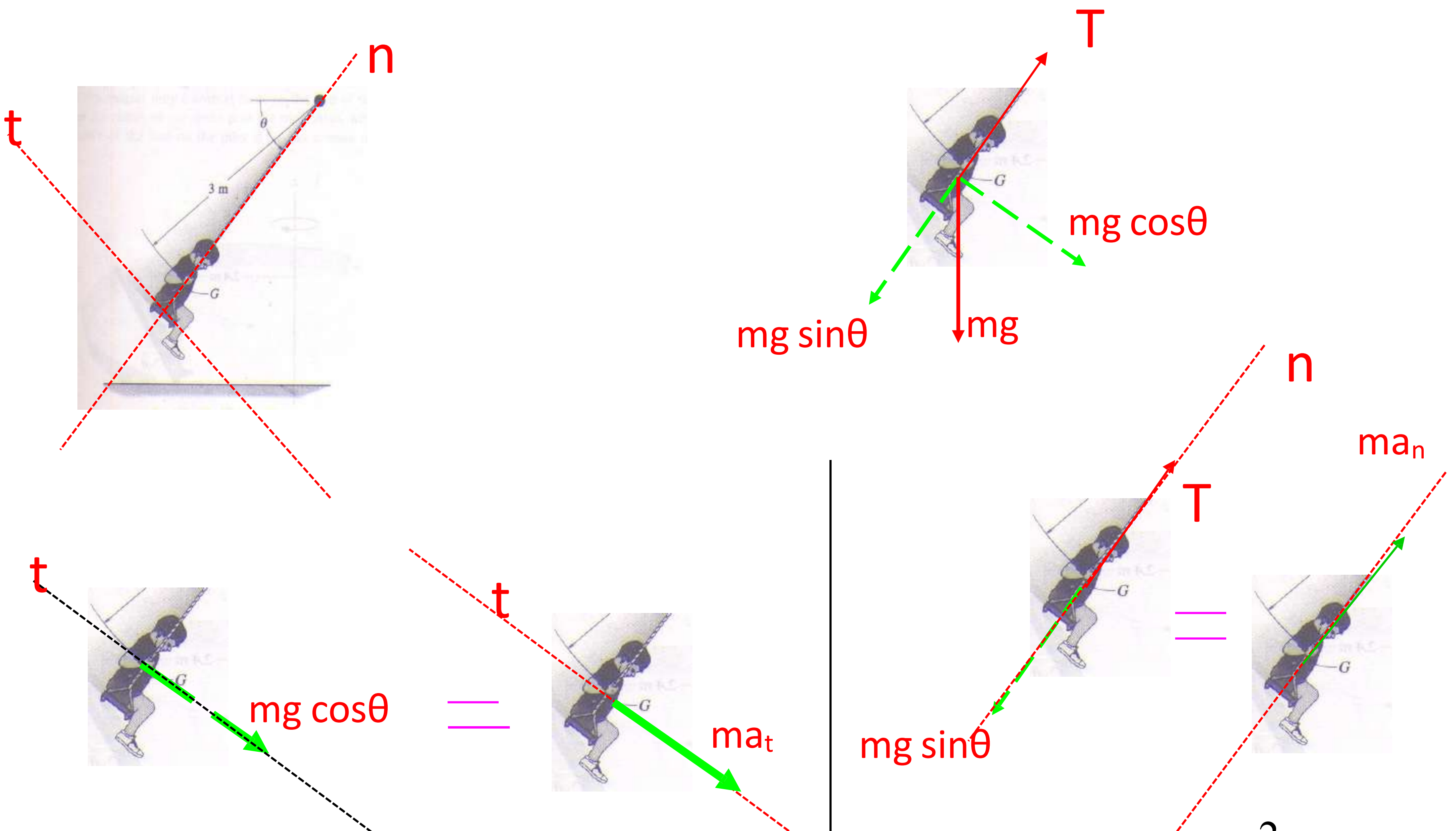
$$-1.2 = x \tan (30°) - \frac{9.81 x^2}{2 (18)^2 \cos^2 30°}$$

$$\therefore \quad -1.2 = 0.577 x - 0.02 x^2$$

$$0.02 x^2 - (0.577)x + 1.2 = 0$$

$$\therefore \quad x = 30.79 \text{ m}$$

6. Newton's second law of motion:



$\sum F_t = ma_t$

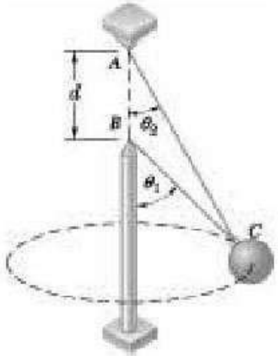
$mg \cos\theta = ma_t$

$a_t = g \cos\theta$

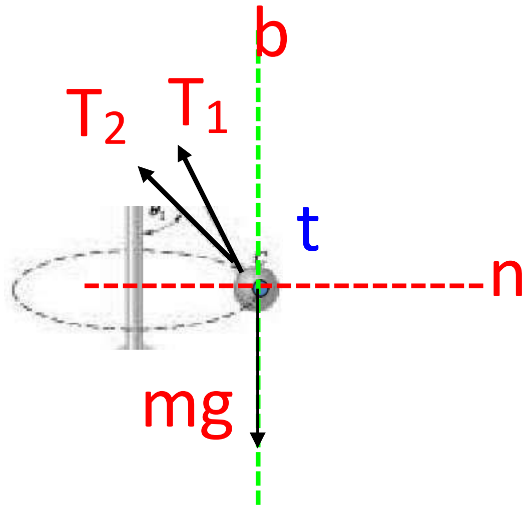
$\sum F_n = ma_n$

$a_n = \frac{v^2}{\rho} = g \cos\theta$

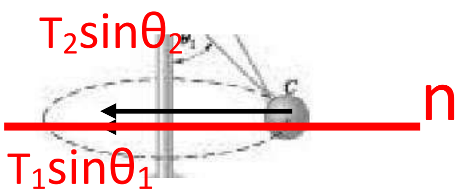
$T - mg \sin\theta = ma_n$



A
d
B
C

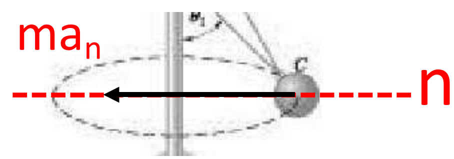


b
T_2
T_1
t
n
mg

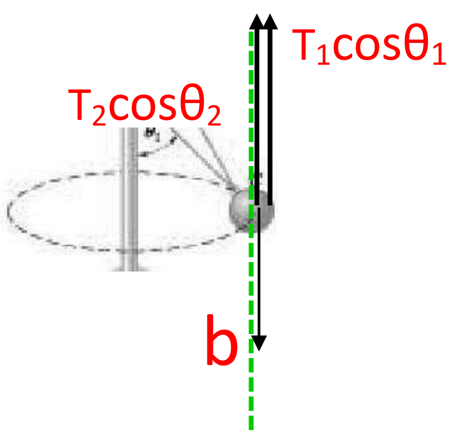


$T_2 \sin\theta_2$
n
$T_1 \sin\theta_1$

=

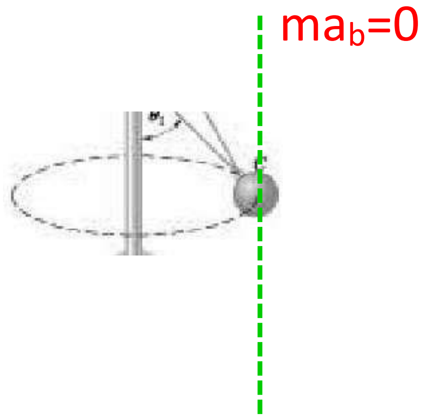


ma_n
n



$T_1 \cos\theta_1$
$T_2 \cos\theta_2$
b

=



$ma_b=0$