Unit - II

Relations & Functions

Q. Define ordered pair?

* Ordered Pair:

(a,b) $(1,2)$ $(3,5)$

Let A & B two sets, $a \in A$ & $b \in B$ or $1 \in A$ & $2 \in B$ or $3 \in A$ & $5 \in B$ then (a,b) denotes ordered pair & a is known as first co-ordinate of ordered pair & b is known as second co-ordinate of ordered pair

* Cartesian Product

Let A & B be the two sets then $A \times B$ called Cartesian product of A & B

$A \times B$ is a set of all distinct ordered pair, in which first coordinate of ordered pair from set A & second coordinate is from set B

$A = \{1, 2, 3\}$

$B = \{a, b\}$

$A \times B = \{(1,a), (1,b), (2,a), (2,b), (3,a), (3,b)\}$

if A contains 3 elements & B contains 2 elements then $A \times B$ contains $3 \times 2 = 6$ elements.

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Q. Explain complement of relation?

* Complement of relation

Let S be a set of family members

$S = \{ f, m, b1, b2, s1 \}$

then $5 \times 5 = 25$

$$S \times S = \begin{cases} (f, f) \ (f, m) \ (f, b1), (f, b2), f(s_1) \\ (m, f), (m, m) \\ \vdots \end{cases}$$

R be a relation on the set S. R be ~~further~~ father relation

f is father of b1
f is father of b2
f is father of s1

$$R = \{ (f, b1), (f, b2), f(s_1) \}$$

$$R \subseteq S \times S.$$

Ref. book - Discrete mathamatical structures

Author - J. P. Trembley

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* complement of a relation

The complement of a relation R denoted by $\bar{R}$ is defined as

$$\bar{R} = \{ (a,b) \mid (a,b) \in R \}$$

example

let $A = \{1,2,3,4\}$ & $B(a,b,c)$

Let $R = \{ (1,a), (1,b), (2,c), (3,a), (4,b) \}$

&

$S = \{ (1,b), (1,c), (2,a) (3,b), (4,b) \}$

Find $\bar{R}$ & $\bar{S}$

ii] verify demorgan's law for R & S

$A \times B = \{ (1,a), (1,b), (1,c) \ldots$

$\bar{R} = \{ (1,c), (2,a), (2,b), (3,b), (3,c) (4,a), (4,b), (4,c) \}$

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$\overline{S} = \{(1,a), (2,b), (2,c), (3,a),$
$(3,c), (4,a), (4,c)\}$

Demorgen's laws states that

$\overline{A \cup B} = \overline{A} \cap \overline{B}$

$\overline{A \cap B} = \overline{A} \cup \overline{B}$

$R \cup S$

$\overline{R \cup S} = (A \times B) - (R \cup S)$

$= \{(2,b), (3,c), (4,a), (4,c)\}$

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* Converse of a relation;

Let R be a relation from A to B then converse of R, denoted by R^c

R^c is relation from B to A

$$R^c = \{(b,a) / (a,b) \in R\}$$

$$R^c \subseteq B \times A$$

Q. Let $A = \{1,2,3,4\}$

$B = \{a,b,c\}$

$R = \{(1,a), (3,a), (3,c)\}$

i] $R^c = \{(a,1), (a,3) (c,3)\}$

$B \times A = \{(a,1), (a,2), (a,3), (a,4), (b,1) (b,2), (b,3)\}$

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Q. Explain representation of Relation?

* Representation of Relations :-

i] Relation as a Matrix :-

let $P = \{a_1, a_2, \ldots a_m\}$ &
$Q = \{b_1, b_2, \ldots b_n\}$

are finite sets, Containing m & n no of elements respectively

R is a relation from P to Q

The relation R can be represented by $m \times n$ matrix $M = [M_{ij}]$ defined as

$$M_{ij} = 0 \text{ if } (a_i, b_j) \notin R$$
$$= 1 \text{ if } (a_i, b_j) \in R$$

Q. $P = \{1, 2, 3, 4\}$, $Q = \{a, b, c, d\}$

$R = \{(1,a), (1,b), (1,c), (2,b), (2,c), (2,d)\}$

Find the relation matrix

$$M_R = \begin{array}{c} \\ 1 \\ 2 \\ 3 \\ 4 \end{array} \begin{array}{c} \begin{array}{cccc} a & b & c & d \end{array} \\ \begin{bmatrix} 1 & 1 & 1 & 0 \\ 0 & 1 & 1 & 1 \\ 0 & 0 & 0 & 0 \\ 0 & 0 & 0 & 0 \end{bmatrix} \end{array}$$

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Q. consider $A = \{1, 2, 3, 4\}$
$B = \{1, 4, 6, 8, 9\}$

aRb if & only if iff $b = a^2$ then find relation matrix M_R

$$A \times B = \begin{Bmatrix} (1,1) & (1,4) & (1,6) & (1,8) & (1,9) \\ (2,1) & (2,4) & (2,6) & (2,8), & (2,9) \\ (3,1) & (3,4) & (3,6) & (3,8) & (3,9) \\ (4,1) & (4,4) & (4,6) & (4,8) & (4,9) \end{Bmatrix}$$

$$R = \{(1,1)\ (2,4)\ (3,9)\}$$

$$M_R = \begin{array}{c} \\ 1 \\ 2 \\ 3 \\ 4 \end{array} \begin{array}{c} \begin{array}{ccccc} 1 & 4 & 6 & 8 & 9 \end{array} \\ \begin{bmatrix} 1 & 0 & 0 & 0 & 0 \\ 0 & 1 & 0 & 0 & 0 \\ 0 & 0 & 0 & 0 & 1 \\ 0 & 0 & 0 & 0 & 0 \end{bmatrix} \end{array}$$

[Ref. Book. Discrete mathematics
Author - S.K. Chakroborty
page No. 25]

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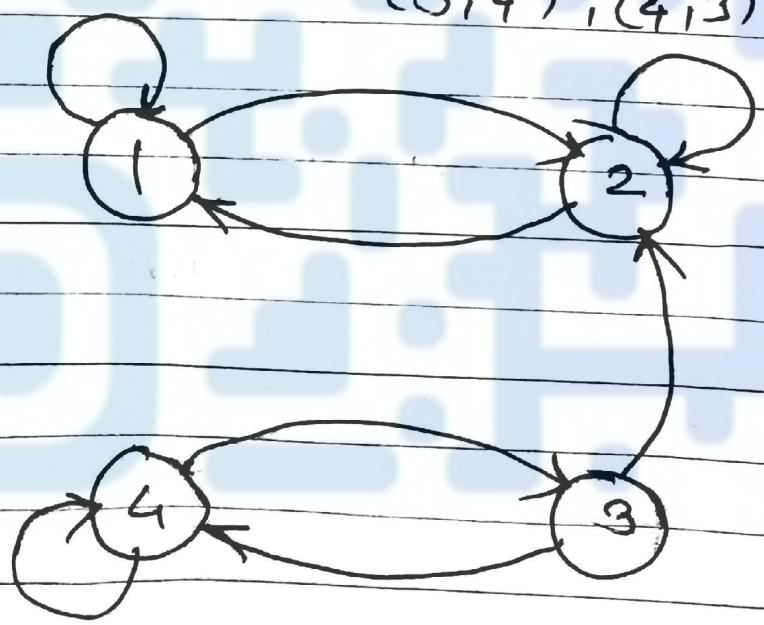
* Relation as a directed Graph

If P is a finite set & R is a relation on P, R can be represented as a directed graph as follows

Draw a small circle for each element of P & label the circle with corresponding element of P. These circles are called vertices.

Draw a directed line from vertex a_i to vertex a_j if $a_i R a_j$
These directed lines are called edges.

eg. $P = \{1, 2, 3, 4\}$
$R = \{(1,1), (1,2), (2,1), (2,2), (3,2), (3,4), (4,3), (4,4)\}$



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~~Pro~~

Q. Explain Closure Properties of relations?

→ 1] Reflexive closure:

Consider the relation R on a set A. Relation R_F is called reflexive closure of R if R_F is smallest relation containing R, having the reflexive property i.e. $R_F = R \cup \Delta$

where Δ is diagonal relations.

Q. Consider the relation R on $A = \{7, 8, 9\}$ defined by

$$R = \{(7, 8), (7, 9), (8, 8), (9, 7)\}$$

Find the reflexive closure of R

$$\Delta = \{(7, 7), (8, 8), (9, 9)\}$$

$$R_F = R \cup \Delta = \{(7, 7), (7, 8), (7, 9), (8, 8) (9, 7), (9, 9)\}$$

[Ref book: Discrete Mathematics.
Author: S.K. Chakraborthy

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2] Symmetric closure:-

consider a relation R on set A. Relation Rs is called the symmetric closure of R is Rs is a smallest relation containing R, having the symmetric property

i.e. $R_s = R \cup R^{-1}$

Q. Consider relation R on A $\{4, 5, 6, 7\}$ defind by $R = \{(4,5), (5,5), (5,6), (6,7), (7,4), (7,7)\}$

Find the symmetric closure of R

$R = \{(4,5), (5,5), (5,6), (6,7), (7,4), (7,7)\}$

$R^{-1} = \{(5,4), (5,5), (6,5), (7,6), (4,7), (7,7)\}$

$R_s = R \cup R^{-1}$

$= \{(4,5), (5,5), (5,6), (6,7), (7,4), (7,7), (5,4), (6,5), (7,6), (4,7)\}$

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3] Transitive closure :-

consider the relation R on a set A.
The transitive closure R on a relation R is the smallest transitive relation containing R

Let $A = \{a, b, c, d\}$

$R = \{(a,b), (a,c), (b,a), (b,c), (c,d), (d,a)\}$

calculate R^*

$$R^* = R \cup R^2 \cup R^3 \cup \dots \cup R^n$$

n is set of element

$R^2 = R.R$ $\qquad$ $R^3 = R.R.R$.

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Q. Explain Warshall's algorithm?

→ Finding the transitive closure of a relation by computing various power of R or products of the relation matrix M_R, is quite impractical for large set. of relation.

– Warshalls algorithm offers an alternative but efficient method for computing the transitive closure of R

Q. Use Warshall's algorithm to find transitive closure of R where

$$M_R = \begin{bmatrix} 1 & 0 & 1 \\ 0 & 1 & 0 \\ 1 & 1 & 0 \end{bmatrix} \quad \& \quad A = \{1, 2, 3\}$$

→ W_0, W_1, W_2

Step I = Here $n = |A| = 3$ to find warshalls set W_0, W_1, W_2, W_3

$$W_0 = M_R = \begin{array}{c} \\ 1 \\ 2 \\ 3 \end{array} \begin{array}{c} \begin{array}{ccc} 1 & 2 & 3 \end{array} \\ \begin{bmatrix} 1 & 0 & 1 \\ 0 & 1 & 0 \\ 1 & 1 & 0 \end{bmatrix} \end{array}$$

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step 2

How to find ω_1 from ω_0 we have row number 1 & column 1

column consider as P_{ij}

Row consider as q_{ij}

$$P = (1,1) \quad \overset{i,\ j}{(3,1)}$$

$$q = (1,1) \quad (1,3)$$

Therefore add

$(P \cdot q) = (1,1), (3,1), (1,3), (3,3)$ in ω_1

$$W_1 = \begin{array}{c|ccc} & 1 & 2 & 3 \\ \hline 1 & 1 & 0 & 1 \\ 2 & 0 & 1 & 0 \\ 3 & 1 & 1 & 1 \end{array}$$

step 3

Now to find ω_2 from ω_1 we have row number 2 & column 2

column as a p & row as q

$$P = (2,2) \cdot (3,2)$$

$$q = (2,2)$$

Therefore add

$$P, q = (2,2), (3,2)$$

$$\omega_2 = \begin{array}{c|ccc} & 1 & 2 & 3 \\ \hline 1 & 1 & 0 & 1 \\ 2 & 0 & 1 & 0 \\ 3 & 1 & 1 & 1 \end{array}$$

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step 4

Now to find W_3 from W_2 we consider row 3 & column 3 for row q & column p

$p = (1,3), (3,3)$

$q = (3,1), (3,2), (3,3)$

Therefore add

$(1,1), (1,2), (1,3), (3,1) (3,2), (3,3)$

$$W_3 = \begin{array}{c} \\ 1 \\ 2 \\ 3 \end{array} \begin{array}{c} \begin{array}{ccc} 1 & 2 & 3 \end{array} \\ \begin{bmatrix} 1 & 1 & 1 \\ 0 & 1 & 0 \\ 1 & 1 & 1 \end{bmatrix} \end{array}$$

$R^* = W_3$ is the transitive closure of R

$R^* = \{(1,1)\ (1,2), (1,3), (2,2), (3,1), (3,2), (3,3)\}$

Ref. Book. Discrete Mathematics
Author :- S.K. Chakraborthy
page No. 36

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Q. Explain Equivalance relation?

→ Equivalance relation

Let R be a relation in set A, then R is a equivalance relation on A iff.

i] R is reflexive relation

i.e $\forall a \in A \Rightarrow aRa$

or $(a, a) \in R$

ii] R is symmetric relation

i.e $(a, b) \in R \Rightarrow (b, a) \in R$

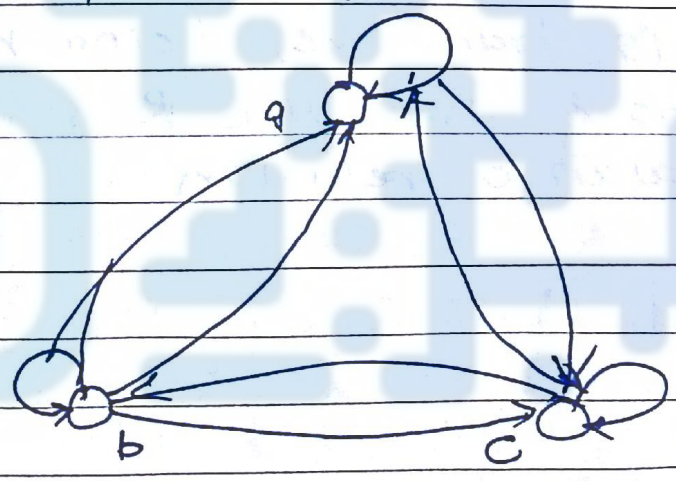
$aRb \Rightarrow bRa$

iii] R is transitive relation

i.e. aRb and $bRc \Rightarrow aRc$

or $(a, b) \in R$ and $(b, c) \in R \Rightarrow (a, c) \in R$

* Diagraph of equivalance relation



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eg. Let I be the set of all integers, let R be a relation of I defined as aRb iff a−b is divisible by 5 or 5 | a−b

i.e. $R = \{(a,b);\ 5 | a-b\}$

i] $\forall a \in I$, $5 | a-a$

as zero is divisible by 5

$\Rightarrow (a,a) \in R$ or aRa

hence R is reflexive

ii] If $a, b \in I$

& $5 | a-b$

then 5 also divides $b-a$

i.e $5 | b-a$

$\Rightarrow$ bRa or $(b,a) \in R$

hence R is symmetric

iii] if $a, b, c \in I$

& $5 | a-b$ also $5 | b-c$

then $5 | a-b+b-c$, i.e $5 | a-c$

hence if aRb, bRc $\Rightarrow$ aRc

hence R is transitive, being reflexive, symmetric & transitive R is an equivalance relation.

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Q. Explain Partial order relation.

* Partial order relations

A relation R on a set A is called a partial order relation iff R is

i] Reflexive relation

$aRa \ \forall \ a \in A$

i.e $(a,a) \in R$

ii] Antisymmetric relation

if aRb & bRa then $a=b$

$(a,b) \in R \quad (b,a) \in R \Rightarrow (a=b)$

where $a,b \in A$

iii] Transitive relation

$aRb, bRc \Rightarrow aRc$

$(a,b) \in R, \ (b,c) \in R \Rightarrow (a,c) \in R$

where $a,b,c \in A$

iv] Symmetric $(a,b) \in R \Rightarrow (b,a) \in R$

$aRb \Rightarrow bRa$

Q. Let $A = \{1,2,3,4\}$ &

$R = \{(1,1), (1,3), (2,2), (2,4), (3,1), (3,3), (4,2), (4,4)\}$

Show that R is partial order relation.

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→ Reflexive

$aRa \ \forall \ a \in A$

$= \{(1,1), (2,2), (3,3), (4,4)\} \in R$

$\therefore$ relation R is reflexive

Antisymmetric

If aRb, bRa then $a = b$

$(1,3) \in R, \ (3,1) \in R$

$(2,4) \in R, \ (4,1) \in R$

$(3,1) \in R, \ (1,3) \in R$

$(4,2) \in R, \ (2,R) \in R$

$\therefore$ R is antisymmetric

Transitive

$aRb, bRc \Rightarrow aRc$

$aRc = \{(1,3), (1,1), (1,3), (2,4), (2,1)$

$(2,4) (3,1), (3,3), (3,1), (4,2), (4,4)\}$

$aRc \in R$

$\therefore$ R is transitive.

$\therefore$ Given relation is partial order relation.

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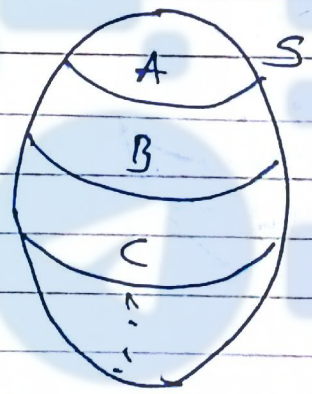
* Partitions :-

Let S be a non empty set. A set $A = \{A, B, C, \ldots\}$ of non empty subsets of S will be called portion of S if

i] $A \cup B \cup C \cup \ldots = S$, i.e the set S is the union of the sets of P and

ii] The intersection of every pair of distinct subsets of $S \in P$ is the null set, i.e. if A and $B \in P$ then either $A = B$ or $A \cap B = \phi$



eg. $S = \{1, 2, 3, 4, 5, 6, 7, 8, 9, 10\}$ & its subsets

$B_1 = \{1, 3\}$, $B_2 = \{7, 8, 10\}$,

$B_3 = \{2, 5, 6\}$, $B_4 = \{4, 9\}$

the set $P = \{B_1, B_2, B_3, B_4\}$ is such that

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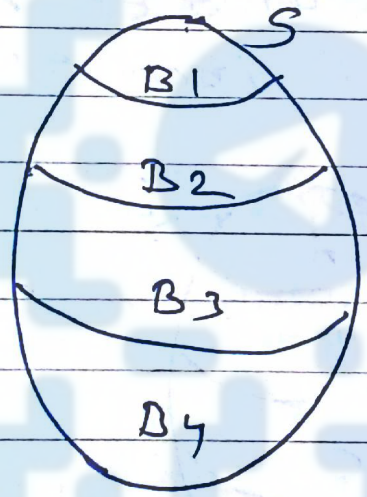
i] B_1, B_2, B_3, B_4 are all non empty subsets of S

ii] $B_1 \cup B_2 \cup B_3 \cup B_4 = S$ and

iii] For any sets B_i, B_j either $B_i = B_j$ or $B_i \cap B_j = \phi$

Hence the set $\{B_1, B_2, B_3, B_4\}$ is a partition of S



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Q. What is Hasse diagram? Explain in detail.

* Hasse diagrams :-

Poset can be represented by diagraphs. A simplest way of representing poset is Hasse diagram

method of find Hasse diagram from diagraph

1. omitt loops as relation is reflexive on poset

2. All arrows are appear on the edges are omitted

3. Eliminate all edges that are implied by transitive relation

eg. if aRb, bRc, then aRc so eliminate (a,c) edge.

4. An arc pointing upward is drawn from a to b if $a \neq b$ and aRb.

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eg. Let R be the relation on the set A

$A = \{5, 6, 8, 10, 28, 36, 48\}$

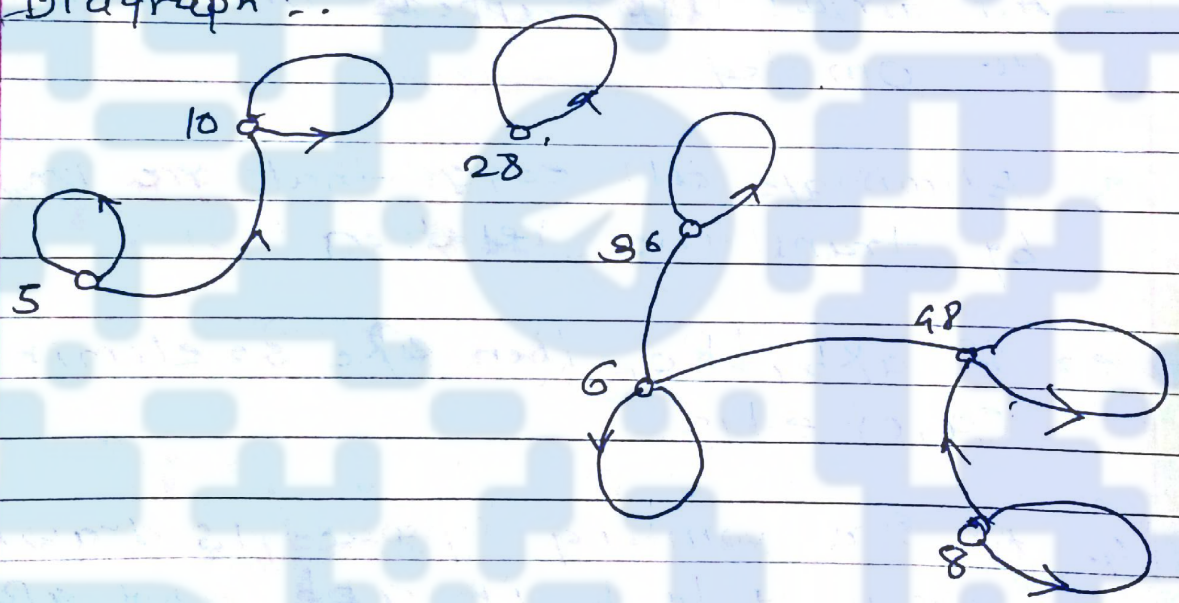
Let $R = \{(a,b) \mid a \text{ is a divisior of } b\}$

Draw the Hasse diagram. Compare with diagraph. Determine wheather R is equivalance relation.

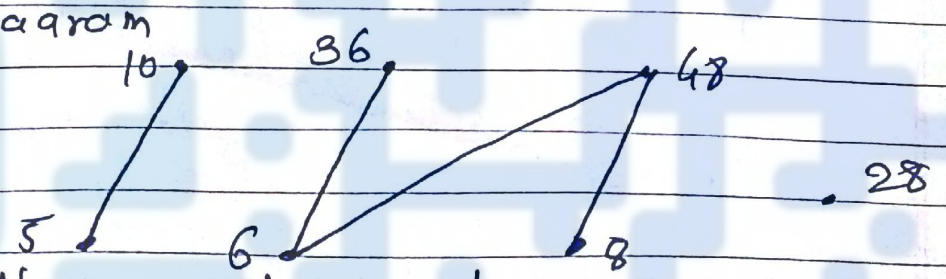
→ $A = \{5, 6, 8, 10, 28, 36, 48\}$

$R = \{(5,5), (6,6), (8,8), (10,10), (28,28), (36,36), (48,48), (5,10), (6,36), (6,48), (8,48)\}$

Diagraph :-



Hasse diagram



R is reflexive relation, but R is not symmetric relation, Hence R is not equivalance relation

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Q. Define Lattice?

* Lattice:-

A lattice is a poset $(A, \leq)$ in which every subset $\{a, b\}$ of A, has a least upper bound & a greatest lower bound.

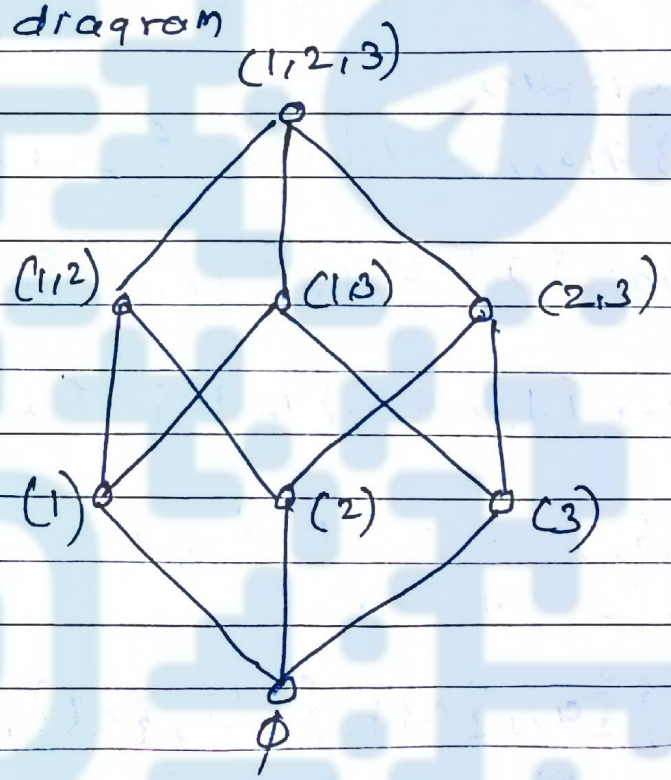
Ex.

$A = \{1, 2, 3\}$

$P(A) = \{\{\phi\}, \{1\}, \{2\}, \{3\}, \{1,2\}, \{1,3\}, \{2,3\}, \{1,2,3\}\}$

$(P(A), \subseteq)$ is a poset

Hasse diagram



Here every pair of elements has a lub and a glb hence $(P(A), \subseteq)$ is a lattice.

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Q. Define chain & Antichain.

* chains and Antichains

Let $(A, \leq)$ be a poset, A subset of A is known as chain if every pair of elements in the subset are related. The number of elements in a chain is called the length of the chain.

A subset of A is known as antichain if no two distict elements in a subset are related.

ex: $S = \{a, b, c\}$ poset $(P(S), \subseteq)$

Hasse diagram

$\{\phi, \{a\}, \{a,b\}, \{a,b,c\}\}, \{\phi, \{a\}, \{a,c\}, \{a,b\}\}$

$\{\phi, \{b\}, \{a,b\}, \{a,b,c\}\}, \{\phi, \{b\}, \{b,c\}, \{a,b,c\}\}$

$\{\phi, \{c\}, \{a,c\}, \{a,b,c\}\}, \{\phi, \{c\}\}, \{b,c\}, \{a,b,c\}\}$

$\{\phi, \{a\}, \{a,b\}\}, \{a, \{a,c\}, \{a,b,c\}\}$

are ex. of chains

$\{\{a\}, \{b\}\}, \{\{a\}, \{c\}\}, \{\{b\}, \{c\}\}, \{\{a,b\}, \{a,c\}\}$

$\{\{b,c\}\}, \{a,c\}\}, \{\{a,b\}, \{b,c\}\}$

are ex. of antichains.

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* Functions

~~te~~ Let A and B be two non-empty set. Then a function or mapping f rom the set A to the set B is a ~~tule~~ rule which assians to each element $a \in A$ to unique element $b \in B$

f maps element a to element b and that f maps set A to set B

The notation denote that f maps a to b is

$$f(a) = b \text{ or } (a, b) \in f$$

* Representation by diagram :-

Let the interiors of the two closed area represent the set A & set B

Let $a_1, a_2, a_3, \ldots$ be the element of A & $b_1, b_2, b_3 \ldots$ be the elements of B the mapping or function $f: A \to B$ is represented by means of arc of lines joining the points representing the elements of A to the element B

1. Every $a \in A$ is joined to some $b \in B$
2. Two or more points in A may be joined to the same point in B (as, a_1, a_2 are joined to b_2 in the fig.)
3. For mapping, two or more points of B cannot be joined to the same point in A

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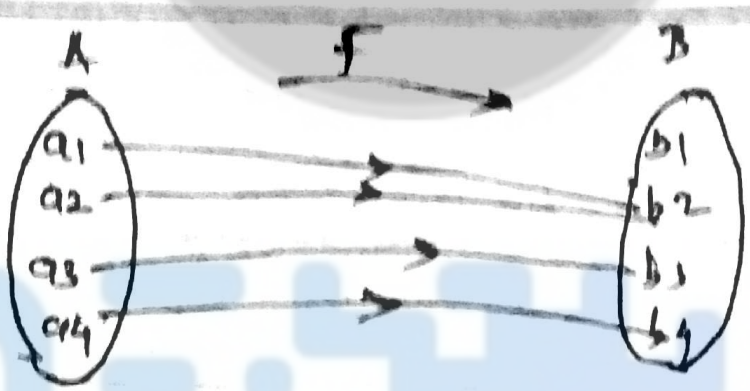
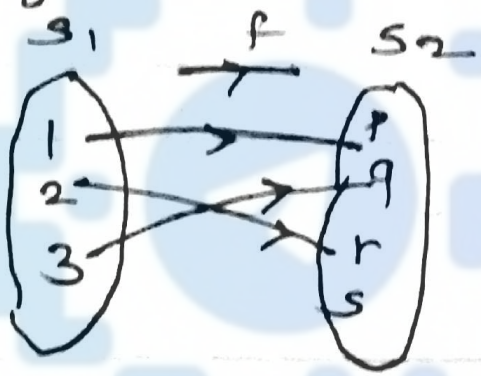
Eg.



1. Let $S_1 = \{1, 2, 3\}$
$S_2 = \{p, q, r, s\}$

for below fig.



$f : S_1 \rightarrow S_2$ is a function It can also be denoted by $S_1 \xrightarrow{f} S_2$

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Q. Explain Types of functions

→

1. One to one mapping (injective function)

A function $f : A \to B$ is said to be one to one mapping or one to one correspondance or <u>injective function</u> or univalent fun^n

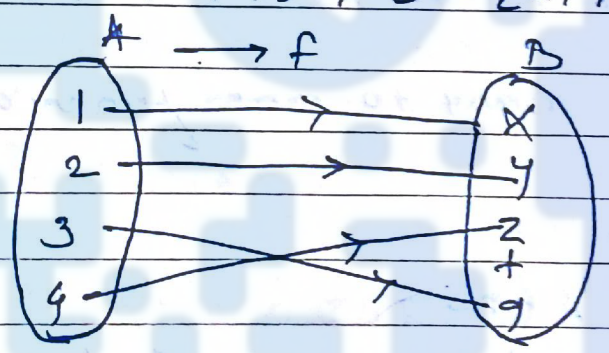
If $\forall a_1, a_2 \in A$

$$f(a_1) = f(a_2) \Rightarrow a_1 = a_2$$

or if $a_1 \neq a_2 \Rightarrow f(a_1) \neq f(a_2)$

Thus under an injective function on element of B can have at the most one element of A as preimage.

eg. $A = \{1, 2, 3, 4\}$, $B = \{x, y, z, t, q\}$



$f : A \to B$ is one to one mapping

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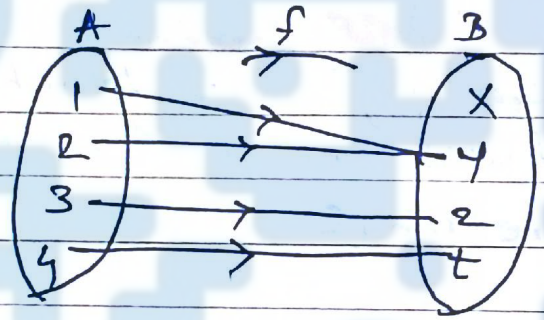
2] Many to one Mapping

Let $f: A \to B$
then f is many to one function if $a_1 \neq a_2$ but $f(a_1) = f(a_2)$ i.e f is many to one function if two (or more than two) distict element of A have the same f-image in B

eg. $A = \{1, 2, 3, 4\}$, $B = \{x, y, z, t\}$



$1 \neq 2$ but $f(1) = f(2)$

So f is many to one function

3] Into Mapping

If $f: A \to B$
then f is into mapping from $A \to B$ if $\exists$ at least one element in B which is not f-image of any element of A. In this case range of f is proper subset of co-domain set i.e. $f(A) \subset B$

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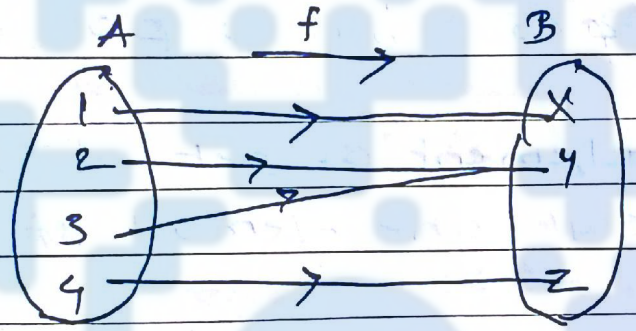
4) Onto mapping (surjective function)

→ A function $f: A \to B$ is said to be onto mapping if each element of B is the f-image of at least one element of set A

In this case range of f is co-domain set B i.e $f(A) = B$

eg. $A = \{1, 2, 3, 4\}$, $B = \{x, y, z\}$



Every element of B is f-image of at least one element of A hence f is 'onto' function.

Hence range of f = co-domain set B

onto mapping is also known as Surjection function if f is an 'onto' function from A to B. then

$f: A \xrightarrow{\text{onto}} B$.

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6) Many to one mapping

If $f: A \to B$ is many one mapping & also f is onto mapping then f is said to be many one onto mapping

hence f is many one onto mapping if

1] for some $a_1 \neq a_2 \Rightarrow f(a_1) = f(a_2)$

or

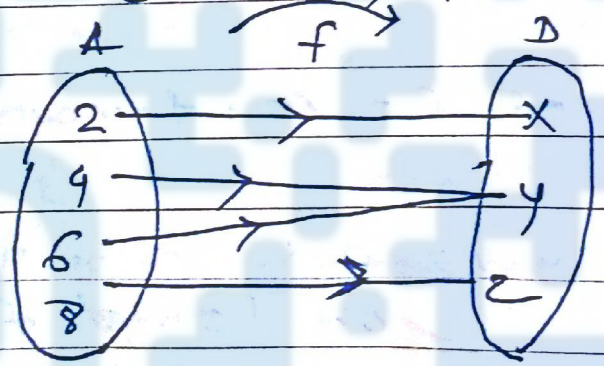
$f(a_1) = f(a_2)$ but $a_1 \neq a_2$

2) Every element of set B is f-image of at least one element of set A

i.e. range of f = codomain set B

or $f(A) = B$

Eg. $A = \{2, 4, 6, 8\}$, $B = \{x, y, z\}$



f is many one mapping, f is also mapping so f is many one ~~onto~~ mapping

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* One one onto mapping (Bijection)

A mapping $f: A \to B$ which is one one as well as onto is called one one onto mapping or bijection

Hence f is one one onto if

i] $f(a_1) = f(a_2) \Rightarrow a_1 = a_2$

(or $a_1 \neq a_2 \Rightarrow f(a_1) \neq f(a_2)$)

ii] Every element of set B is f- image of at least one element of set A.

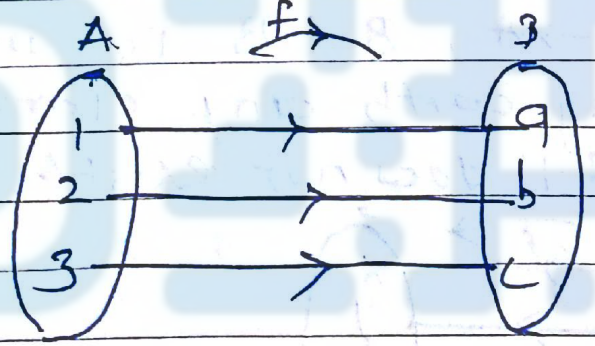
i.e. range of f = co-domain set B

or $f(A) = B$

or $\forall b \in B \; \exists \; a \in A$

$f(a) = b$

Eg $A = \{1, 2, 3\}$, $B = \{a, b, c\}$



f is one one & onto function

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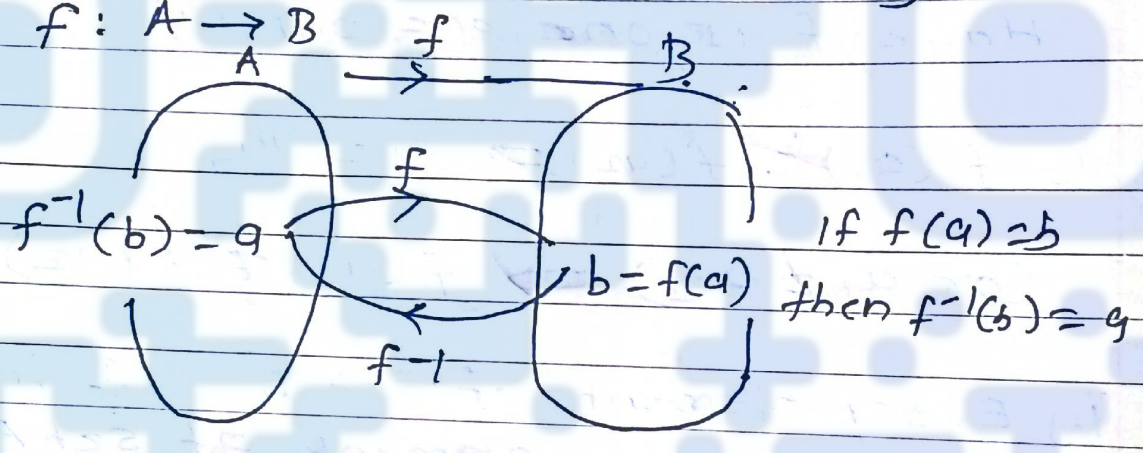
Q. What is Inverse function

→ * Inverse function

If $f: A \to B$ is an one one onto (bijective) mapping, then the mapping $f^{-1}: B \to A$, which associates to element $b \in B$, to the unique element $a \in A$ such that $f(a) = b$ is called the inverse of mapping $f: A \to B$

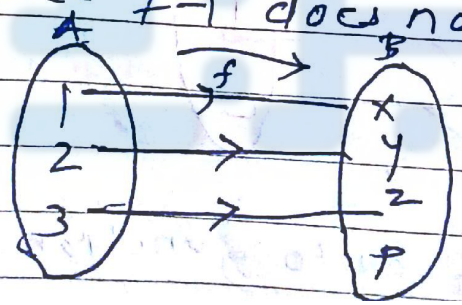


1] If $f: A \to B$ is one one onto mapping then only f^{-1} exists & f^{-1} is also one one onto mapping

2] In case of one one into, many one into and many one into, inverse does not exist as in case of-

i] For $f^{-1}: B \to A$, $B \in B$ is not corresponding with any element of A, hence f^{-1} does not exist.

i]



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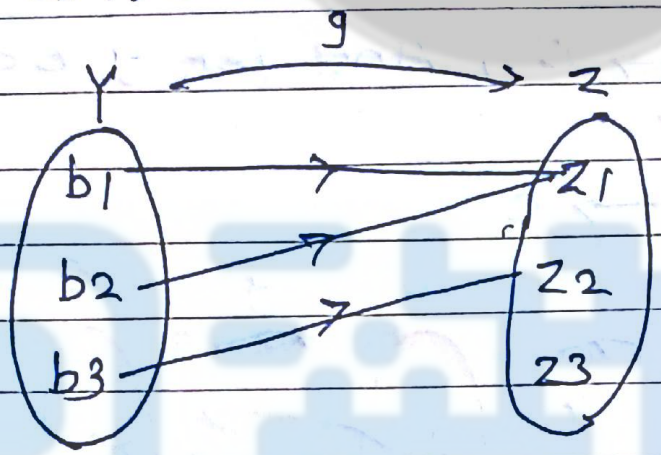
ii] For $g^{-1}: Z \to Y$, $z_3 \in Z$ is not corresponding with any element of Y & z_1 is having two different images

ii]



hence g^{-1} does not exist

iii] for h^{-1}, $B_1 \to A_1$, $B_3 \in B_1$ is having two different images, hence h^{-1} does not exist.

Hence f^{-1} exists only in case of f is one one & onto mapping,

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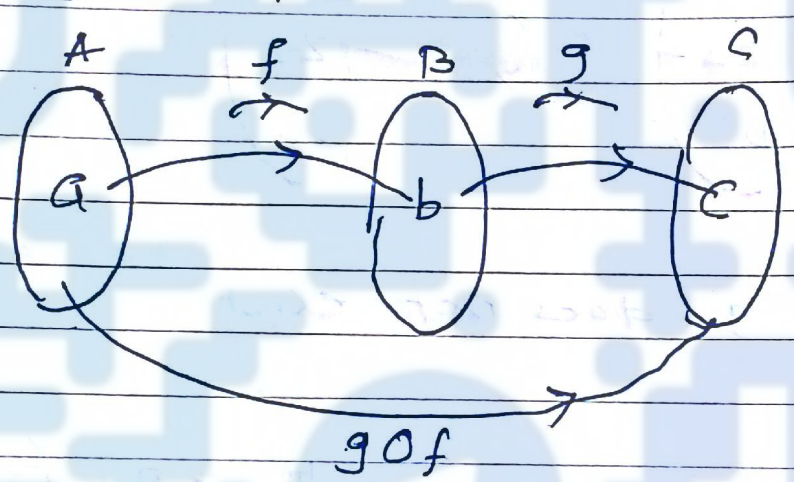
3- Explain composite of functions.

Let A, B, C be three sets & let f be a mapping from A to B
i.e $f : A \to B$, And let g be a mapping from B to C
i.e. $g : B \to C$



then gof, composite of f & g is a function from A to C

$$gof(a) = g[f(a)]$$
$$= g[b]$$
$$= c, \ \forall a \in A$$

Domain of gof is set A & Co-domain of gof is set B

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Q. Explain pigeonhole Principle with example.

→ * The Pigeonhole Principle

Pigeon hole principle says that if there are 'many' pigeons & 'a few' pigeon holes, then there must be some pigeon holes occupied by two or more pigeon

Let A & B be finite sets & $|A| > |B|$ then for any function f from A to B, $\exists a_1, a_2 \in A$ such that $f(a_1) = f(a_2)$

* The extended pigeon hole principle

If there are m pigeon holes & more than 3m pigeons then 4 or more pigeons will occupy at least one of the pigeon hole. In general if the number of pigeons is much larger than the number of pigeon holes, the pigeon hole principle can be extended as,

Theorem :-

If n pigeons are assign to m pigeon holes, then one of the pigeon hole must be occupied by, at least $\left\lceil \frac{n-1}{m} \right\rceil + 1$ pigeons

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Recurrences Relations

consider the following instruction for generating a sequence

1. Start with 2
2. Give any term, add 5 to get the next term

If we list the terms of the seq in queue we obtain

2, 7, 12, 17, 22

The 1st term is 2 because of instruc^n 1.
The 2nd term is 7 because instruction 2 says to add 5 to 2 to get the next term 7.
The 3rd term is 12 because instruc^n 2 says to add 5 to 7 to get next term 12. by following instructions 1 and 2, we can compute any term in the sequence.

If we denote the sequence (1) as $a_1, a_2 \ldots$ we may rephrase instruc^n 1 as

$$a_1 = 2$$

and we may rephrase instruc^n 2 as

$$a_n = a_{n-1} + 5; \quad a \geq 2$$

Taking $n = 2$ in eq^n (3)

$$a_2 = 2 + 5 = 7$$

taking $n = 3$ eq^n (3)

$$a_3 = a_2 + 5$$
$$= 7 + 5 = 12$$

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Eq^n (2) & (3) are equivalent to instr^n 1 & (2)

eq^n (3) is an example of <u>recurrence relation</u>

A recurrence relation defines a sequence by giving the nth value in terms of certain of its predecessors. In order for a recurrence relation such as (3) to define a sequence, a "start up" value or values, such as (2), must be given.

These startup values are known as initial condition

Definition :-

A recurrence relation for the sequence $a_0, a_1 \ldots$ is an equation that relates a_n to certain of its predecessors $a_0, a_1, \ldots a_{n-1}$

A difference eq^n is a specific type of recurrence relation.

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Q. Explain Job scheduling problem.

* Job Scheduling Problem

Scheduling problem arise in many domains. In the manufacturing domain task often referred to as a jobs, correspond to parts or batches of parts that need to be processed on a set of machines.

Scheduling problem arise in schools, where tasks are classes & resources can be teachers, classrooms & students, other examples include transportation related problems, computer scheduling problem etc.

Job shop scheduling is an optimization problem in computer science in which ideal jobs are assigned to resources at particular times

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