Unit 5: DC Circuit (Q No. 5 and Q. No. 6)

1. Explain following types of Electrical Networks.

1) Linear and Non Linear network (2Unilateral and Bilateral network) (3) Active and Passive network (4) Lumped and Distributed Network

Ans: **(1) Linear Network:** A network in which values of the circuit elements (resistance, inductance and capacitance) remain constant, irrespective of change in voltage or current, is known as *'linear network'*. Ohm's law is applicable to such network.

Non Linear Network: On the other hand, if values of the circuit elements change with change in voltage or current, such a network is called *'Non-linear network'*. Ohm's law is not applicable to such a network.

(2) Bilateral Network: If characteristics or behavior of the circuit is independent of direction of current through various elements, such a network is called *'bilateral'*. Network comprised of pure resistance is bilateral one.

Unilateral Network: If characteristic or behavior of the circuit depends on direction of current through one or more elements it is called '*Unilateral Network'*.

A diode allows flow of current only in one direction when it is forward biased, circuit consisting of diode is unilateral one.

(3) Active Network: If electric circuit contains at least single energy source, it is called *'Active network'*. It may be either voltage or current source.

Passive Network: A circuit in absence of an energy source containing only passive elements is called *'Passive network'*.

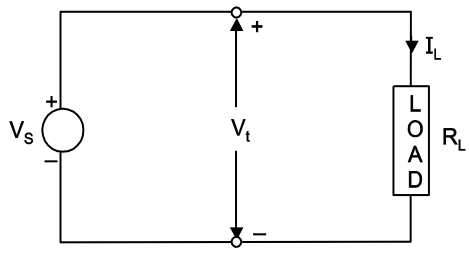
(4)Lumped Network: If all the network elements are physically separable, such a network is called *'lumped network'*. Most of the electrical networks are lumped in nature.

Distributed Network: A network in which elements are not physically separable is known as *'distributed network'*.

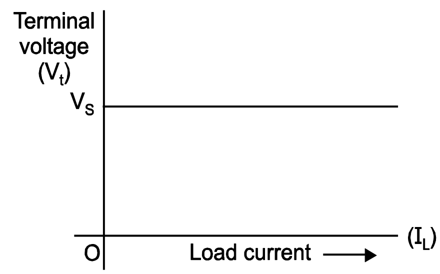
As resistance inductance and capacitance of a transmission line are uniformly distributed over its length, it is a 'distributed network'.

2. Explain concept of ideal and practical voltage and current sources.

Ans: **(1) Ideal Voltage Source:** An ideal voltage source is one which gives constant voltage across its terminals, irrespective of current drawn. Symbol for ideal voltage source and its V-I characteristic are shown in Fig. (a) and (b)



(a) Ideal Voltage Source



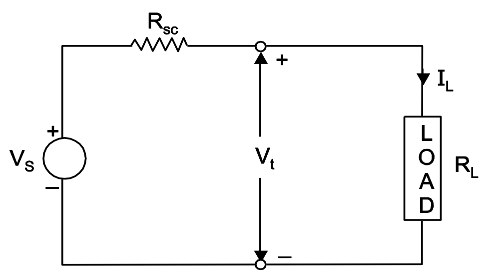
(b) V-I Characteristics

Where,

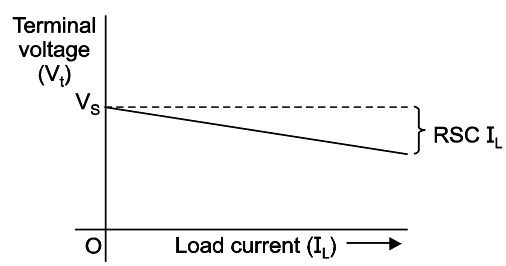
V_t=Terminal voltage, R_L= Load resistance, V_S=Source voltage and I_L=Load current

From V-I characteristic it can be seen that whatever is the value of load current terminal, voltage remains constant.

Practical Voltage Source: In practice, it is not possible because every voltage source has small internal resistance. Symbol of practical voltage source and it's V-I characteristic are shown in Fig. (c) and Fig. (d) respectively.



(C)Practical Voltage Source



(d) V-I Characteristics

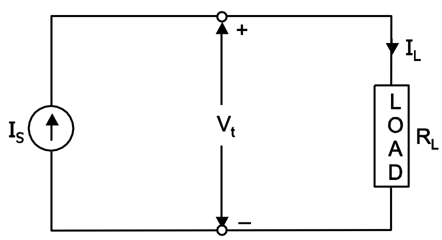
As load current increases ($R_{se} \times I_L$) drop increases and terminal voltage reduces.

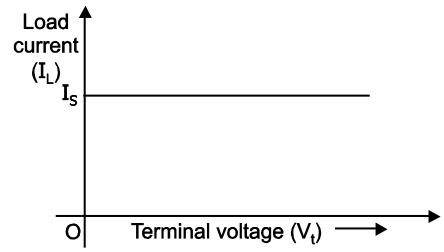
Terminal Voltage, $V_t = V_S - (R_{se} \cdot I_L)$

Thus, ideal voltage source has zero internal resistance, while practical voltage source has small internal resistance.

(2) Ideal Current Source: An ideal current source is one which delivers constant current irrespective of voltage across it's terminals.

Symbol for ideal current source and its V-I characteristics are shown in Fig. (a) and Fig. (b) respectively.

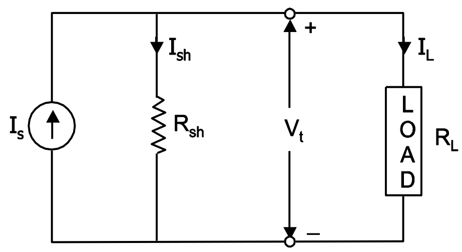


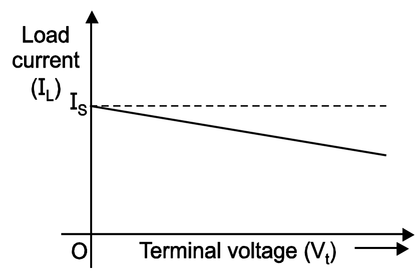


(a)IdealCurrent Source (b) V-I Characteristics

Internal resistance of ideal current source is infinity.

Practical Current Source: The practical current source has high (finite) internal resistance. Hence, with increase in terminal voltage, current delivered by such a source decreases. Symbol for Practical Current Source and it's V-I characteristics are shown in Fig. (c) and Fig. (d) respectively.

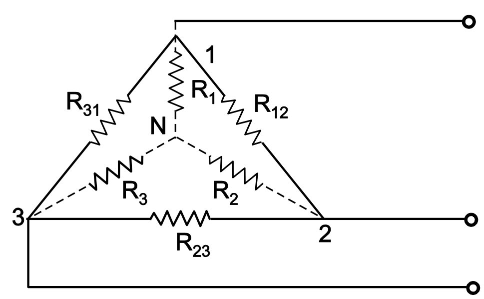




(a)Practical Current Source (b) V-I Characteristics

3. Derive the expression for conversion of a delta connected network into an equivalent star.

Ans: Let us convert this delta connection into an equivalent star connection. Let equivalent star resistances be R_1, R_2 and R_3.



To call these arrangements as equivalent of each other, resistance between two terminals must be same in both types of connections.

Resistance between terminals 1 and 2 will be

For delta connection $= \dfrac{R_{12}.(R_{23}+R_{31})}{R_{12}+R_{23}+R_{31}}$ ... (1)

for star connection $= R_1+R_2$ ... (2)

Equating (1) and (2)

$$\frac{R_{12}\,(R_{23}+R_{31})}{R_{12}+R_{23}+R_{31}} = R_1+R_2 \qquad \ldots(3)$$

Similarly for resistances between terminal 3 and 1, we get

$$\frac{R_{31}.(R_{12}+R_{23})}{R_{12}+R_{23}+R_{31}} = R_1+R_3 \qquad \ldots(4)$$

Also for resistances between terminals 2 and 3 we get,

$$\frac{R_{23}.(R_{31}+R_{12})}{R_{12}+R_{23}+R_{31}} = R_2+R_3 \qquad \ldots(5)$$

Let's find R_1, R_2 and R_3 in terms of R_{12}, R_{23} and R_{31}

Subtracting Equation (4) from equation (5) we get,

$$\frac{R_{23}(R_{31}+R_{12})-R_{31}(R_{12}+R_{23})}{R_{12}+R_{23}+R_{31}} = R_2-R_1 \qquad \ldots(6)$$

Adding equations (3) and (6) we get,

$$\frac{R_{12}(R_{23}+R_{31})+R_{23}(R_{31}+R_{12})-R_{31}(R_{12}+R_{23})}{R_{12}+R_{23}+R_{31}} = 2.R_2$$

$$\therefore \qquad \frac{2.R_{12}.R_{23}}{R_{12}+R_{23}+R_{31}} = 2.R_2$$

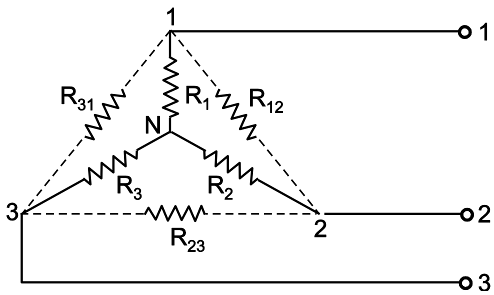
$$\therefore \qquad R_2 = \frac{R_{12}.R_{23}}{R_{12}+R_{23}+R_{31}} \qquad \ldots(7)$$

Similarly, $R_1 = \dfrac{R_{12}.R_{31}}{R_{12}+R_{23}+R_{31}}$ ...(8)

and $R_3 = \dfrac{R_{23}.R_{31}}{R_{12}+R_{23}+R_{31}}$ ... (9)

4. Derive the expression for conversion of a star connected network into an equivalent Delta.

Let 3 resistances R_1, R_2 and R_3 be connected in star as shown in Fig.



Making use of following equations (Delta to star conversion),

$$R_1=\frac{R_{12}.R_{31}}{R_{12}+R_{23}+R_{31}} \qquad \dots(1)$$

$$R_2 = \frac{R_{12}.R_{23}}{R_{12}+R_{23}+R_{31}} \qquad \dots(2)$$

$$R_3= \frac{R_{23}.R_{31}}{R_{12}+R_{23}+R_{31}} \qquad \dots(3)$$

Taking product of equations (1) & (2), (2) & (3) and (3) & (1)

$$R_1.R_2 =\frac{R_{12}^2.R_{31}.R_{23}}{(R_{12}+R_{23}+R_{31})^2} \qquad \dots(4)$$

$$R_2.R_3 = \frac{R_{23}^2.R_{12}.R_{31}}{(R_{12}+R_{23}+R_{31})^2} \qquad \dots(5)$$

$$R_3.R_1= \frac{R_{31}^2.R_{12}.R_{23}}{(R_{12}+R_{23}+R_{31})^2} \qquad \dots(6)$$

Adding equations (4),(5) and (6) we get,

$$R_1.R_2+R_2R_3+R_3R_1=\frac{R_{12}^2R_{31}R_{23}+R_{23}^2R_{12}R_{31}+R_{31}^2R_{12}R_{23}}{(R_{12}+R_{23}+R_{31})^2}$$

$$\therefore \quad R_1R_2+R_2R_3+R_3R_1 =\frac{R_{12}.R_{23}.R_{31}(R_{12}+R_{23}+R_{31})}{(R_{12}+R_{23}+R_{31})^2}$$

$$\therefore \quad R_1R_2+R_2R_3+R_3R_1 =R_{12}.\left(\frac{R_{23}.R_{31}}{R_{12}+R_{23}+R_{31}}\right) \qquad \dots(7)$$

Put $\frac{R_{23}.R_{31}}{R_{12}+R_{23}+R_{31}} = R_3$ in equation (7)

$$\therefore \quad R_1R_2+R_2R_3+R_3R_1 = R_{12}.R_3$$

$$\therefore \quad R_{12} =R_1+R_2+\frac{R_1.R_2}{R_3} \qquad \dots(8)$$

Similarly,

$$R_{23}=R_2+R_3+\frac{R_2.R_3}{R_1} \qquad \dots(9)$$

and

$$R_{31} = R_3 + R_1 + \frac{R_3 . R_1}{R_2} \qquad \dots (10)$$

5. State & explain Kirchhoff's Laws.

Kirchhoff's Current Law (KCL)

Statement: Algebraic sum of currents meeting at any junction point in an electric circuit is always zero. i.e. $\sum I = 0$

In other words, at any junction or node in an electric circuit, sum of incoming currents is equal to sum of outgoing currents.

i.e. at any node, ∑ Incoming Currents= ∑ Outgoing Currents

Explanation: Consider a node as shown in Fig.

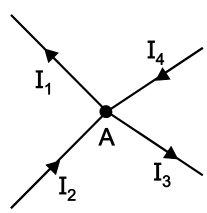


Fig.

Four branches meet at junction or node A. By KCL,

$$I_2 + I_4 = I_1 + I_3$$

where I_2 and I_4 are incoming currents while I_1 and I_3 are outgoing currents.

Kirchhoff's Voltage Law (KVL)

Statement : In any electrical network, algebraic sum of voltage drops across various elements around any closed loop or mesh is equal to algebraic sum of e.m.f.s in that loop.

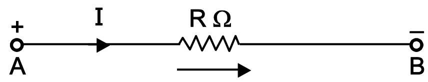
$$\sum IR = \sum E$$

In other words, if we trace any closed path or loop in an electrical network an algebraic sum of branch voltages is always zero.

$$\sum V = 0$$

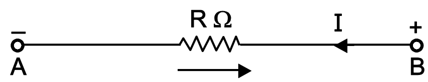
Sign Convention: Direction of current through a circuit element decides polarity of voltage. Current always flows from higher potential to lower potential.

While tracing a closed path, from positively marked terminal of resistor to negatively marked terminal then it indicates potential drop. This is shown in Fig. (a).



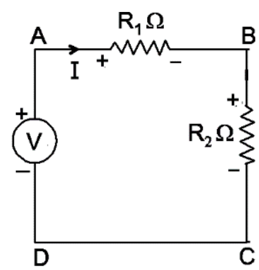
(a) Potential drop

If circuit path is traced from negatively marked terminal to positively marked terminal, then it indicates potential rise. This is shown in Fig. (b)



(b) Potential rise

Consider a network shown below.



Using KVL, $\sum V=0$

$-I R_1 - I R_2 + V=0, \quad V=I(R_1+R_2)$

$I= (V/(R_1+R_2))$

6. State & explain Superposition theorem.

Statement: In any linear, bilateral network containing at least two energy sources, the current flowing through a particular branch is the algebraic sum of the currents flowing through that branch when each source is considered separately and remaining sources are replaced by their respective internal resistances.

Explanation: Consider a network shown in Fig. (a). The current (I) through R_2 is to be estimated using Superposition theorem

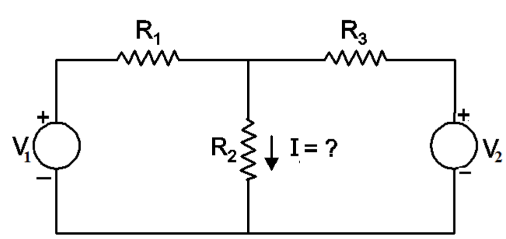


Fig (a)

Consider voltage source V_1 acting alone. Make other voltage source inactive i.e. replace it by it's internal resistance. As internal resistance of an ideal voltage source is zero, it is replaced by short circuit. Circuit will be as shown in Fig. (b).The current through R_2 is I'.

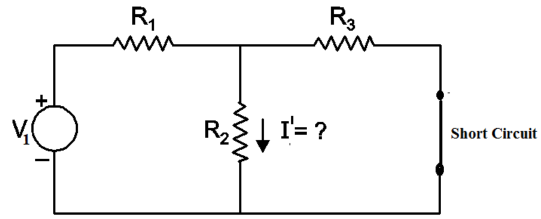


Fig. (b)

Now consider second voltage source V_2 is acting alone. Replace first voltage source V_1 by short circuit. Circuit will be as shown in Fig. (c).The current through R_2 is I''

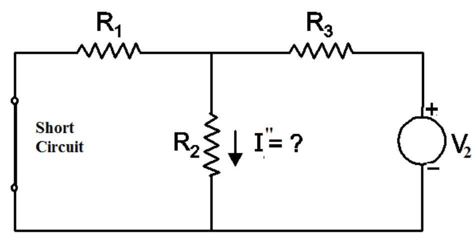


Fig. (c)

I' and I'' can be calculated by using KVL or branch current method.

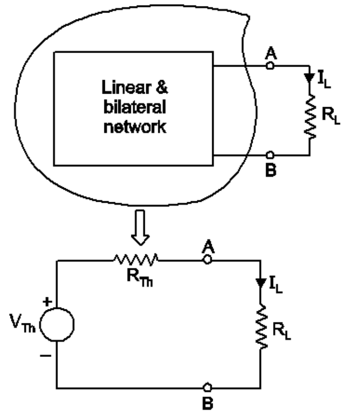
Hence the current flowing through R_2 when both the sources are acting is… **I=I'+I"**

7. State & explain Thevenin's Theorem.

Statement: Any linear, bilateral network containing energy sources and circuit elements can be replaced by an equivalent circuit containing a voltage source V_{Th} and a series resistance R_{Th} or R_{eq} across the terminals under consideration. Value of voltage source V_{Th} is equal to the open-circuit voltage across the terminals under consideration while R_{Th} or R_{eq} is the equivalent resistance measured between the same terminals replacing all the energy sources by their internal resistances.

Explanation:By using this theorem current flowing through any particular circuit element can be calculated.

Consider linear bilateral network as shown in figure. The current (I_L) through branch AB carrying resistance R_L is to be determined using Thevenin's Theorem.



Thevenins Equivalent Circuit

How to apply Thevenin's theorem to calculate I_L?

(i) Remove the circuit element (R_L) under consideration from the network.

(ii) Find the value of open-circuit voltage across those terminals. This is nothing but Thevenin's voltage source (V_{TH}).

(iii) Find the equivalent resistance between the same terminals replacing all energy sources by their internal resistances. Ideal voltage sources are replaced by short circuit, while ideal current sources are replaced by open circuit. This resistance is R_{Th} or R_{eq}.

(iv) Replace the given network across terminals under consideration by Thevenin's equivalent circuit, which is Thevenin's voltage source in series with an equivalent resistance.

(v) Reconnect original element across Thevenin's equivalent circuit and find current through it.

Questions asked in End-Sem Dec 2019 Examination

Q. Define the ideal and practical voltage sources. Draw their V-I characteristics [4M] (Refer Q. No. 02)

Q. Derive the equations to convert Delta connected resistive circuit into equivalent star circuit. [8M] (Refer Q. No. 03)

Q. State and explain KCL & KVL. [4M] (Refer Q. No. 5)

Q. Define: (i) active & passive network (ii) linear & nonlinear network (iii) unilateral & bilateral network. [6M] (Refer Q. No. 01)

BEE, Unit No: 05
DC Circuits
Solved Networks (Numericals)

*Contents:

1. Series-Parallel Combination of resistances & star delta Conversion
2. Kirchhoff's Laws
3. Superposition Theorem
4. Thevenins Theorem

1. Series-Parallel combination of resistances and star Delta Conversion Technique

While dealing with various complex electric networks, it is often required to reduce the network during solution procedure. This task becomes easy, if one knows how to convert given star arrangement of resistances into equivalent delta arrangement and vice-versa.

[Note: Remember that by using star-delta conversion technique, the complex network has to be reduced to a single equivalent resistance between any two asked terminals]

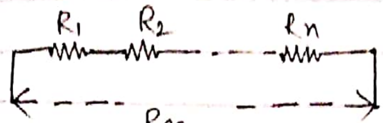
Steps to be followed while solving complex networks using star-delta conversion technique

1. The two terminals between which we have to find equivalent resistance should remain as it, till we get single equivalent resistance between them
2. Observe the network carefully. If any series/parallel arrangement of resistances is observed, find out total series or parallel resistance
3. Again redraw the network and observe it carefully
4. If star arrangement of resistances is observed, convert into delta or vice-versa.
5. Repeat step no. 2 [Step no. 4 may need to be repeated in case of more complex network]
6. Finally reduce the network to get single equivalent resistance between the desired two terminals.

PTO ...

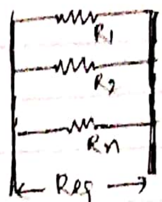
Series- Parallel Combination of resistances

Series Combination



$R_{eq} = R_1 + R_2 + \cdots + R_n$

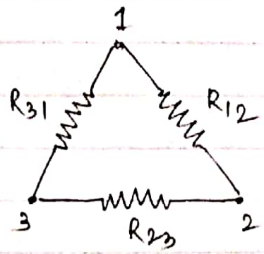
Parallel combination



$$\frac{1}{R_{eq}} = \frac{1}{R_1} + \frac{1}{R_2} + \cdots + \frac{1}{R_n}$$

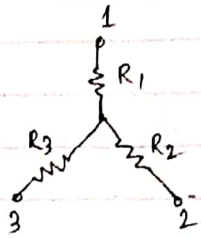
Star - Delta convesion Technique

1. Delta to star



Delta

$\Rightarrow$



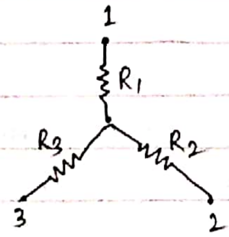
Star

$$R_1 = \frac{R_{12} R_{31}}{R_{12} + R_{23} + R_{31}}$$

$$R_2 = \frac{R_{12} R_{23}}{R_{12} + R_{23} + R_{31}}$$

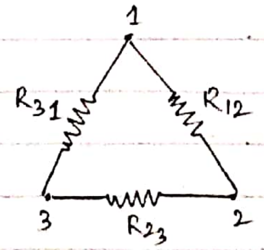
$$R_3 = \frac{R_{23} R_{31}}{R_{12} + R_{23} + R_{31}}$$

2. Star to Delta



Star

$\Rightarrow$



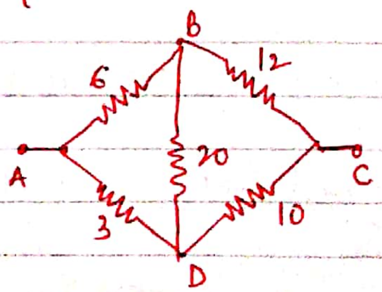
Delta

$$R_{12} = R_1 + R_2 + \frac{R_1 R_2}{R_3}$$

$$R_{23} = R_2 + R_3 + \frac{R_2 R_3}{R_1}$$

$$R_{31} = R_3 + R_1 + \frac{R_3 R_1}{R_2}$$

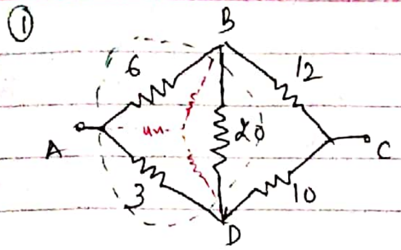
Q1. Find the equivalent resistance between terminals A and C



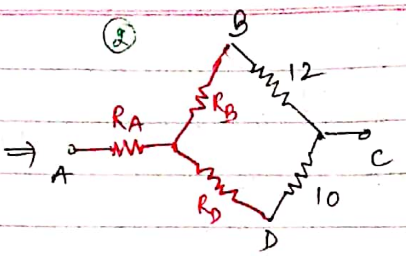
Solution.

converting delta ABD into star

(1)

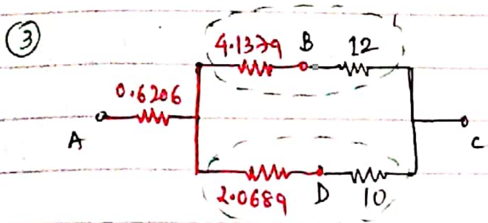


(2)



$$R_A = \frac{6 \times 3}{6+3+20} = 0.6206\,\Omega \qquad R_B = \frac{6 \times 20}{6+3+20} = 4.1379\,\Omega, \qquad R_D = \frac{3 \times 20}{6+3+20} = 2.0689\,\Omega$$

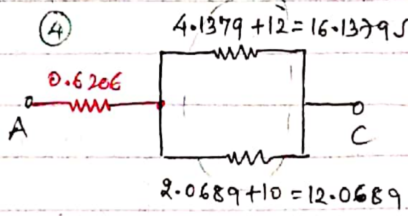
(3)



12 Ω and 4.1379 Ω are in series

2.0689 Ω & 10 Ω are in series

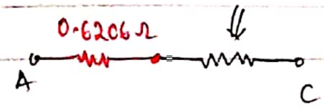
(4)



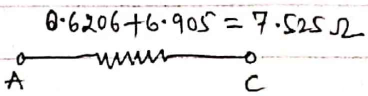
16.1379 Ω and 12.0689 Ω are connected in parallel

(5)

$$\frac{16.1379 \times 12.0689}{16.1379 + 12.0689} = 6.905\,\Omega$$



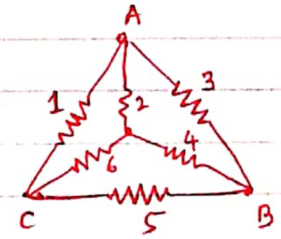
(6)



0.6206 and 6.905 are in series

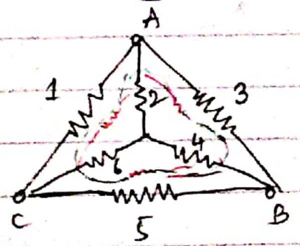
$$\boxed{R_{AC} = 7.525\,\Omega}$$

Q 2 Find the resistance between terminals B and C



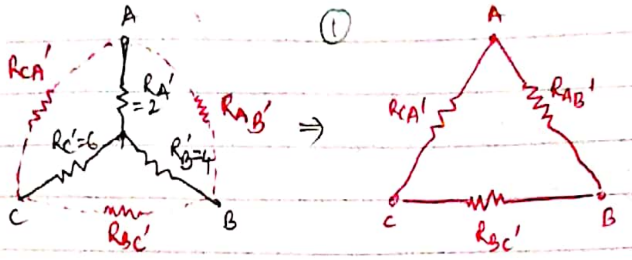
Solution:

Converting inner star into delta



Consider only inner star

①



$$R_{AB}' = R_A' + R_B' + \frac{R_A' R_B'}{R_C'}$$

$$= 2 + 4 + \frac{2 \times 4}{6}$$

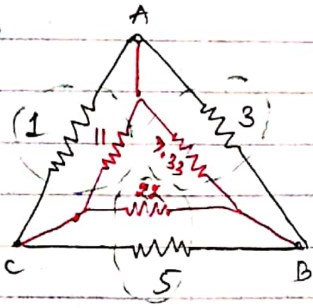
$$= 7.33\,\Omega$$

$$R_{BC}' = R_B' + R_C' + \frac{R_B' R_C'}{R_A} = 4 + 6 + \frac{4 \times 6}{2} = 22\,\Omega$$

$$R_{CA}' = R_C' + R_A' + \frac{R_C' R_A'}{R_B'} = 6 + 2 + \frac{6 \times 2}{4} = 11\,\Omega$$

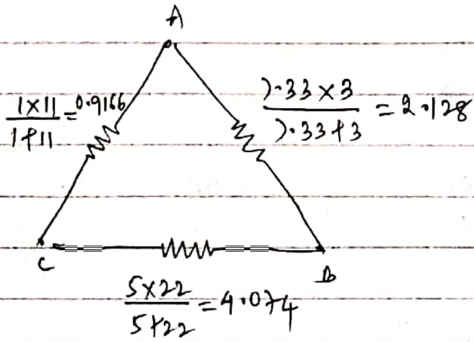
Consider whole network

②

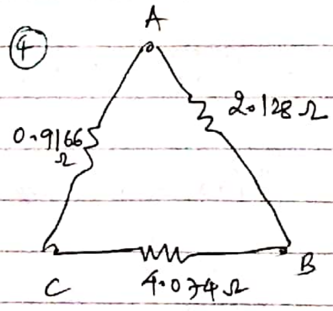


$7.33\,\Omega$ & $3\,\Omega$, $5\,\Omega$ and $22\,\Omega$, $11\,\Omega$ & $1\,\Omega$ are connected in parallel. seperately

③

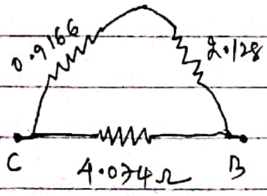


④

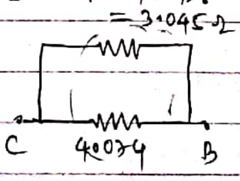


As we have to find equivalent resistance between B & C, resistances 0.9166 and 2.128 Ω ~~are~~ will be in series and this series combination will be in parallel with $4.074\,\Omega$

⑤

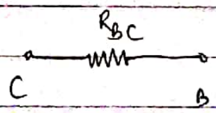


⑥ $0.9166 + 2.128 = 3.045\,\Omega$



3.045 and 4.074 are connected in parallel

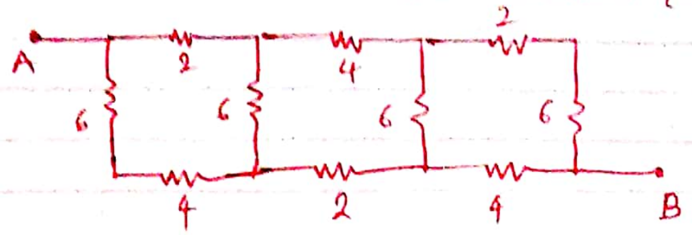
⑦



$$R_{BC} = \frac{3.045 \times 4.074}{3.045 + 4.074} = \underline{\underline{1.742\,\Omega}}$$

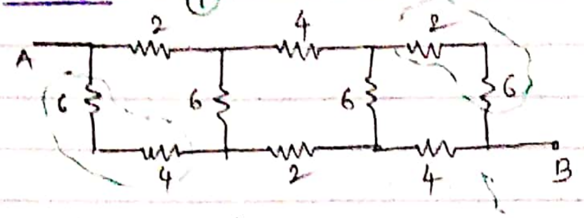
(9)

Q3 Find the resistance between terminals A & B



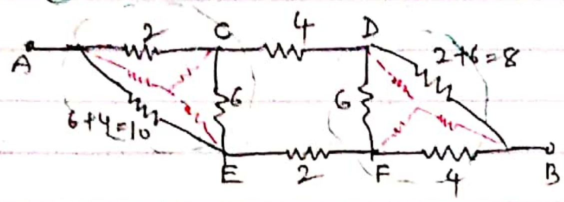
Solution:

(1)



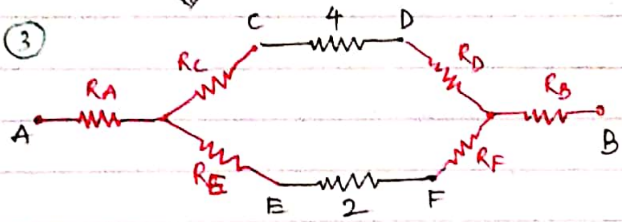
6Ω and 4Ω are in series
2Ω and 6Ω are in series

(2)



converting
delta ACE and delta DFB
into star

(3)



$R_A = \frac{2\times 10}{2+6+10} = 1.11\,\Omega$

$R_D = \frac{6\times 8}{6+8+4} = 2.66\,\Omega$

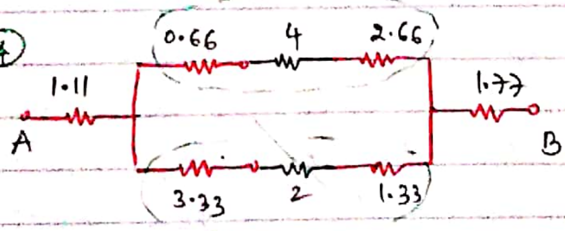
$R_C = \frac{2\times 6}{2+6+10} = 0.66\,\Omega$

$R_F = \frac{4\times 6}{6+8+4} = 1.33\,\Omega$

$R_E = \frac{6\times 10}{2+6+10} = 3.33\,\Omega$

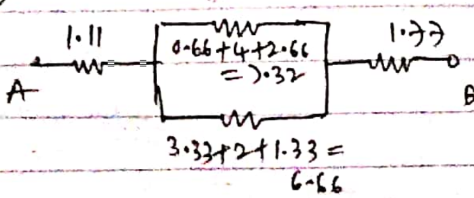
$R_B = \frac{4\times 8}{6+8+4} = 1.77\,\Omega$

(4)

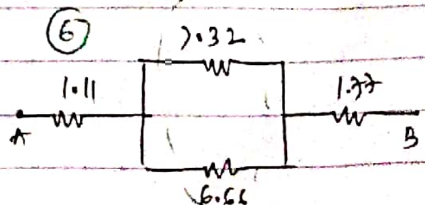


0.66, 4 and 2.66 are in series
3.33, 2 & 1.33 are in series

(5)



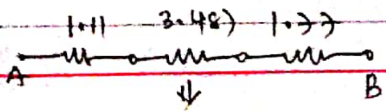
(6)



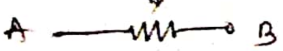
7.32 & 6.66 are in parallel

$\frac{7.32\times 6.66}{7.32+6.66} = 3.487\,\Omega$

(7)



(8)

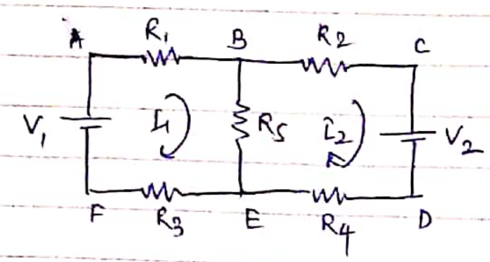


$R_{AB} = \underline{6.367\,\Omega}$

2. Kirchhoff's Laws

Solution of the network using Kirchhofts laws.

Only loop current method has to be used to find out current flowing through a particular resistance.



Consider the network as shown above.

Steps to be followed while getting solution using kirchoffs ~~laws~~ laws.

1. Observe the network carefully.
2. Consider no of loop currents. In above network two individual loops are observed. Hence two loop currents are considered.
3. As generalized method is to be used consider clockwise direction for considered two loop currents. (no. of loop currents = no. of Individual loops)
4. Write voltage equations
5. Solve the equations by using calculator.

Generalized method

The voltage equations can be written as

$$R_{11} I_1 + R_{12} I_2 = V_1 \quad -(i)$$

$$R_{21} I_1 + R_{22} I_2 = V_2 \quad -(ii)$$

where (i) R_{11} & R_{22} are the values obtained by adding all resistances present in loop 1 and loop 2 respectively

(ii) R_{12} and R_{21} are the values of the resistances common to first and second loop, second and first loop respectively.

(iii) V_1 and V_2 are the source values. It will be considered as positive ~~hence~~ when the current approaches negative terminal of battery first. It will be considered negative when the loop current approaches positive terminal first.

Hence the voltage equations will be

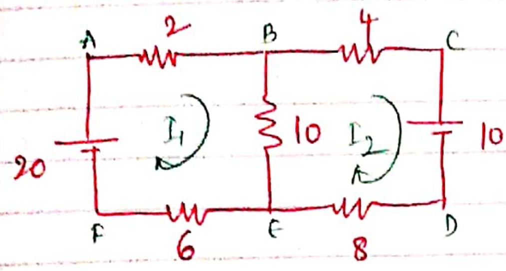
$$(R_1+R_3+R_5) I_1 - R_5 I_2 = V_1 \quad -(i)$$

$$-(R_5) I_1 + (R_2+R_4+R_5) I_2 = -V_2 \quad -(ii)$$

R_{12} and R_{21} will be negative as the currents flowing through them will be in opposite directions.

[Note: Always consider $R_{11}, R_{22}, .. R_{nn}$ as positive and $R_{12}, R_{21}, R_{31}, R_{13}, R_{23}, R_{32}$ as negative]

Q1 find the current flowing through 10Ω resistance.



$I_{10\Omega} = (I_1 - I_2) = ?$

Solution:

Voltage equation for the loop with loop current as I_1

$R_{11} I_1 + R_{12} I_2 = V_1$

$(2+6+10) I_1 - 10 I_2 = 20$

$18 I_1 - 10 I_2 = 20$ — (I)

Voltage equation for the loop with loop current as I_2

$R_{21} I_1 + R_{22} I_2 = V_2$

$-10 I_1 + (4+8+10) I_2 = -10$

$-10 I_1 + 22 I_2 = -10$ — (II)

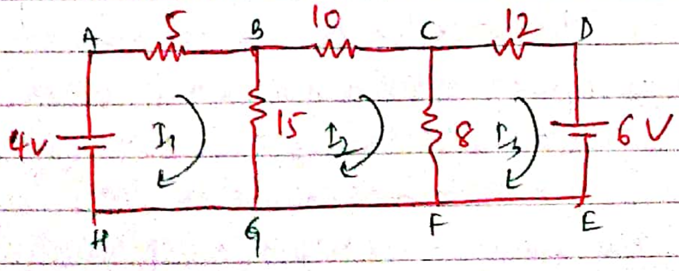
Solve equation (I) and (II) using calculator

$x = I_1 = 1.1486$ Amp $y = I_2 = 0.0675$ Amp

Current through 10Ω

$I_{10\Omega} = (I_1 - I_2) = 1.1486 - 0.0675 = \underline{1.0811 \text{ Amp}}$

Q2 Find Current flowing through 10Ω resistance using loop current method



$I_{10} = I_2 = ?$

Solution

Voltage equation for the loop with loop current as I_1

$R_{11} I_1 + R_{12} I_2 + R_{13} I_3 = V_1$

$(5+15) I_1 - 15 I_2 - 0 = 4$

$$20 I_1 - 15 I_2 - 0 = 4 \quad -(1)$$

Voltage equation for the loop with loop current as I_2

$$R_{21} I_1 + R_{22} I_2 + R_{23} I_3 = V_2$$

$$-15 I_1 + (15+10+8) I_2 - 8 I_3 = 0$$

$$-15 I_1 + 33 I_2 - 8 I_3 = 0 \quad -(II)$$

Voltage equation for the loop with loop current as I_3

$$R_{31} I_1 + R_{32} I_2 + R_{33} I_3 = V_3$$

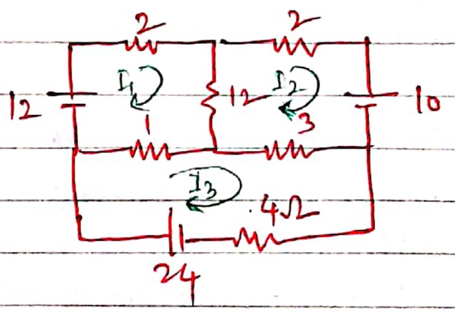
$$-0 I_1 - 8 I_2 + (8+12) I_3 = -6$$

$$0 - 8 I_2 + 20 I_3 = -6 \quad -(III)$$

Solve the equation (I), (II) & (III) and find value of I_2 using calculator directly

$$y = I_2 = 0.0323 \text{ Amp}$$

Q3 find current through 4Ω resistance using loop current method



$$I_{4\Omega} = I_3 = ?$$

Solution:

Voltage equation for the loop with loop current I_1

$$(2+12+1) I_1 - 12 I_2 - 1 I_3 = 12$$

$$15 I_1 - 12 I_2 - I_3 = 12 \quad -(I)$$

Voltage equation for the loop with loop current I_2

$$-12 I_1 + (2+12+3) I_2 - 3 I_3 = -10$$

$$-12 I_1 + 17 I_2 - 3 I_3 = -10 \quad -(II)$$

Voltage equation for the loop with loop current I_3

$$-1 I_1 - 3 I_2 + (1+3+4) I_3 = 24$$

$$-I_1 - 3 I_2 + 8 I_3 = 24 \quad -(III)$$

Solve equations (I) (II) & (III) and find value of I_3 using calculator

$$z = I_3 = 4.11 \text{ Amp}$$

3. Superposition Theorem

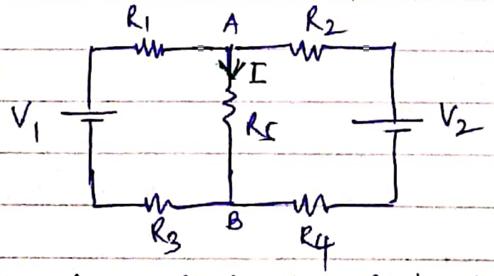
This theorem is used to find current through particular branch in multisource network.

Steps to be followed while solving examples using superposition theorem

1. Assume current flowing through the particular branch (branch through which current has to be calculated) as I Amp.
2. Consider only one source at one time. Replace the remaining sources by their respective internal resistances (Voltage source has to be replaced by zero resistance i.e. short circuit and current source by open circuit (infinite resistance))
3. When first source is considered, assume current through the particular branch as I'
4. When second source is considered, assume current through that particular branch as I''
5. If more than two sources are present in the network, assume current as $I', I'', I''', \ldots$
6. Current I will be algebraic sum of the current $I', I'' \ldots$

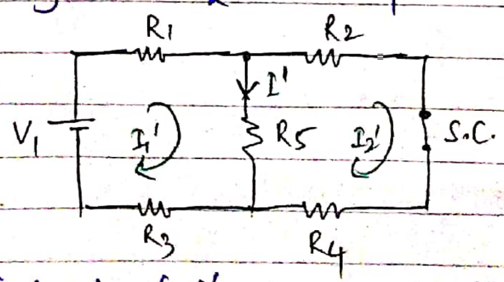
(Note: As per our syllabus, numericals will be asked on the networks with voltage source only)

Consider the network as shown below. [Same network which was considered for kirchhoff's law]



Say the particular branch is, branch AB carrying R_5. Assume current through it as I Amp

Step 1. Consider voltage source V_1 and current through AB is I'.
Voltage source V_2 has to replaced by short circuit.



To find value of I', we can use kirchoff's loop current method.

Current $I' = I_1' - I_2'$

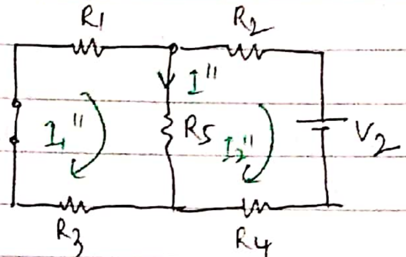
$$(R_1+R_3+R_5)\,I_1' - R_5 I_2' = V_1 \quad \text{— (i)}$$

$$-R_5 I_1' + (R_2+R_4+R_5)\,I_2' = 0 \quad \text{— (ii)}$$

By solving equations (i) & (ii) find I_1' & I_2'

$$I' = I_1' - I_2' \quad \text{— (iii)}$$

Step 2. Consider second source V_2, replace V_1 by short circuit.
Current through AB is I''



Using kirchoffs loop current method, $I'' = I_1'' - I_2''$

$$(R_1+R_3+R_5)\,I_1'' - R_5 I_2'' = 0 \quad \text{— (iv)}$$

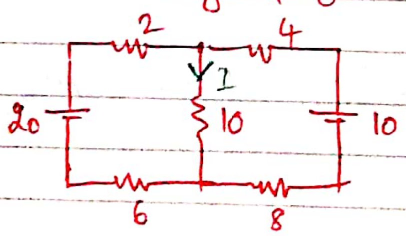
$$-R_5 I_1'' + (R_2+R_4+R_5)\,I_2'' = -V_2 \quad \text{(v)}$$

By solving equation (iv) & (v), find I_1'', I_2''

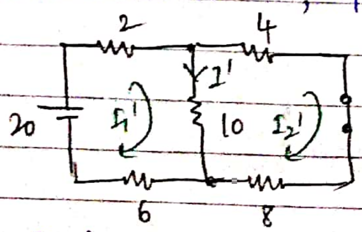
$$I'' = I_1'' - I_2'' \quad \text{--- (vi)}$$

Step 3.
Current through branch AB is
$$I = I' + I''$$

Q.1 Find current flowing through 10Ω using superposition theorem



Solution
Consider 20 Volt source, replace 10 volt source by short circuit



$$I' = I_1' - I_2'$$

Voltage equations:

1st loop
$$(2+10+6)\,I_1' - 10\,I_2' = 20$$

$$18\,I_1' - 10\,I_2' = 20 \quad \text{— (i)}$$

2nd loop $-10I_1' + (4+10+8) I_2' = 0$

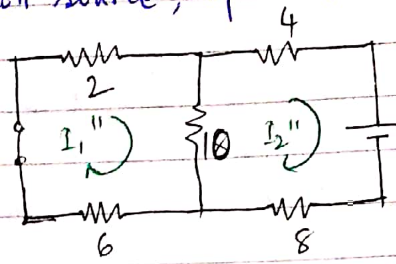
$-10I_1' + 22I_2' = 0$ — (II)

By solving equation (I) and (II)

$I_1' = 1.486$ Amp $\quad I_2' = 0.675$ Amp

$I' = 1.486 - 0.675 = 0.811$ Amp

Consider 10 Volt source, replace 20 Volt Source by short circuit



Voltage equations

1st loop $(2+6+10) I_1'' - 10 I_2'' = 0$

$18 I_1'' - 10 I_2'' = 0$ — (III)

2nd loop $-10I_1'' + (4+10+8) I_2'' = -10$

$-10I_1'' + 22 I_2'' = -10$ (IV)

By solving equations (III) and (IV)

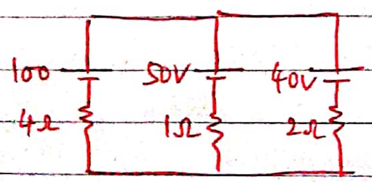
$I_1'' = -0.3378$ Amp $\quad I_2'' = 0.6081$ Amp

$I'' = I_1'' - I_2'' = -0.3378 + 0.6081 = 0.2703$ Amp

Hence the current flowing through 10Ω resistance when both the sources are acting simultaneously

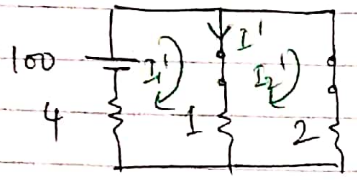
$I = I' + I'' = 0.811 + 0.2703 = \underline{1.0813 \text{ Amp}}$

Q 2. Find current flowing through 1Ω resistance using superposition theorem.



$I_{1\Omega} = ?$

(1) Consider a source of 100 volt is acting alone



$I' = I_1' - I_2'$

Voltage equations

$(4+1) I_1' - 1 I_2' = 100 \Rightarrow 5 I_1' - I_2' = 100$ — (i)

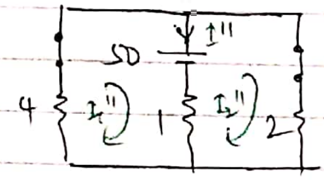
$-1 I_1' + (1+2) I_2' = 0 \Rightarrow -I_1' + 3 I_2' = 0$ — (ii)

Solving equations (i) and (ii)

$I_1' = 21.428$ A $\quad I_2' = 7.142$ Amp

$I' = I_1' - I_2' = 14.286$ Amp

(2) Consider a source of 50 volt acting alone



$I'' = I_1'' - I_2''$

voltage equations

$(4+1) I_1'' - 1 I_2'' = -50 \Rightarrow 5 I_1'' - I_2'' = -50$ — (iii)

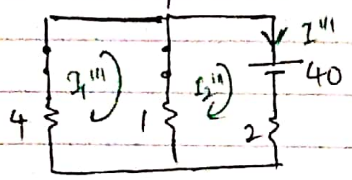
$-1 I_1'' + (1+2) I_2'' = 50 \Rightarrow -I_1'' + 3 I_2'' = 50$ — (iv)

Solving equations (iii) and (iv)

$I_1'' = -7.1428$ A $\quad I_2'' = 14.285$ A

$I'' = I_1'' - I_2'' = -21.427$ Amp

(3) Consider a source of 40 volt acting alone



$I''' = I_1''' - I_2'''$

Voltage equations

$5 I_1''' - I_2''' = 0$ — (v)

$-I_1''' + 3 I_2''' = -40$ — (vi)

Solving equation (v) & (vi)

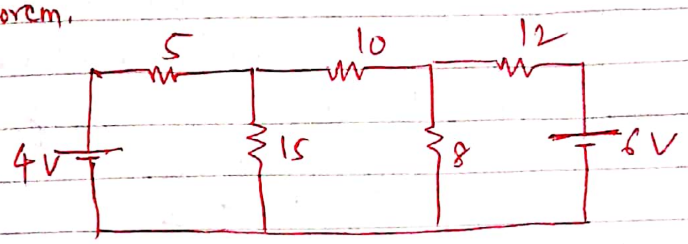
$I_1''' = -2.857$ A $\quad I_2''' = -14.285$ Amp

$I''' = I_1''' - I_2''' = 11.428$ Amp

(4) when all the sources are acting simultaneously

$$I = I' + I'' + I''' = \underline{4.287 \text{ Amp}}$$

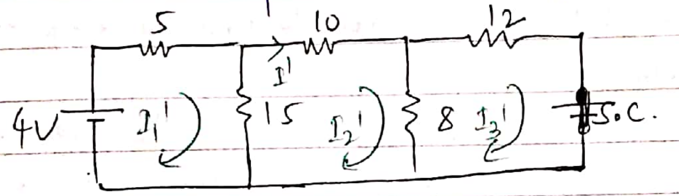
Q3 Find the current flowing through 10Ω resistance using superposition theorem.



$I_{10\Omega} =$

Solution:

(1) Consider a source of 4 volt is acting alone)



$I' = I_2'$

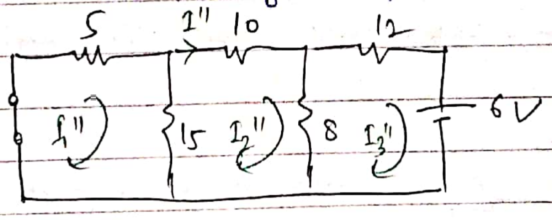
Voltage equations

$$20I_1' - 15I_2' - 0 = 4 \quad \text{—(i)}$$
$$-15I_1' + 33I_2' - 8I_3' = 0 \quad \text{—(ii)}$$
$$-0 - 8I_2' + 20I_3' = 0 \quad \text{—(iii)}$$

By solving equation (i), (ii) & (iii)

$I_1' = 0.3213$ Amp, $\underline{I_2' = 0.1617}$ Amp, $I_3' = 0.0646$ Amp

(ii) Consider 6 Volt source (acting alone)



$I'' = I_2''$

Voltage equations

$$20I_1'' - 15I_2'' - 0 = 0 \quad \text{—(iv)}$$
$$-15I_1'' + 33I_2'' - 8I_3'' = 0 \quad \text{—(v)}$$
$$-0 - 8I_2'' + 20I_3'' = -6 \quad \text{(vi)}$$

By solving equations (iv), (v) & (vi)

$I_1'' = -0.0970$ A, $I_2'' = -0.1293$ A, $I_3'' = -0.3517$ Amp

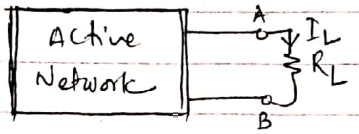
(3) When both the sources are acting simultaneously

$$I = I' + I'' = 0.1617 - 0.1293 = \underline{0.0324 \text{ Amp}}$$

3) Thevenin's Theorem:

This theorem is again used to find the current flowing through a particular resistance.

Consider the network as shown below. The resistance R_L is connected between terminals A & B.



Using thevenin's theorem the current through R_L is given by

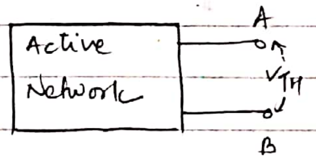
$$I_L = \frac{V_{TH}}{R_{TH} + R_L}$$

where V_{TH} = Thevenin's equivalent voltage, R_{TH} = Thevenins equivalent resistance

* Steps to be followed while solving numericals using thevenin's theorem *

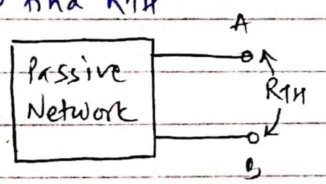
To find current 'I_L', we need to find value of 'V_{TH}' and 'R_{TH}'

1. To find V_{TH}



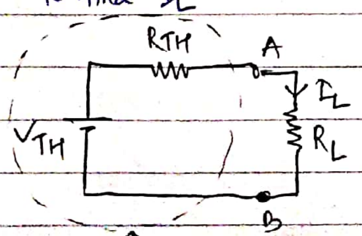
Remove resistance 'R_L' and calculate the voltage between the open circuited terminals A and B. This voltage is called as 'V_{TH}'

2. To find R_{TH}



Remove resistance R_L. Make the network as passive Network. It will be made as passive network by replacing all sources present in the network by their respective internal resistances.
(Simply replace voltage source by short circuit and current source by open circuit)

3. To find I_L



Thevenin's equivalent ckt

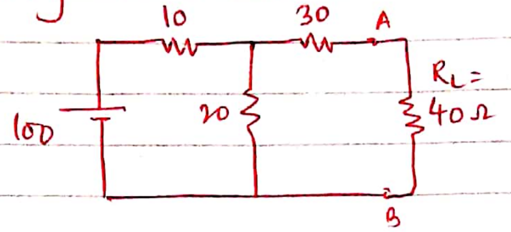
Once we get value of V_{TH} and R_{TH}. Draw thevenin's equivalent circuit and again connect resistance 'R_L' between A and B.

Hence the current through R_L is

$$I_L = \frac{V_{TH}}{R_{TH} + R_L}$$

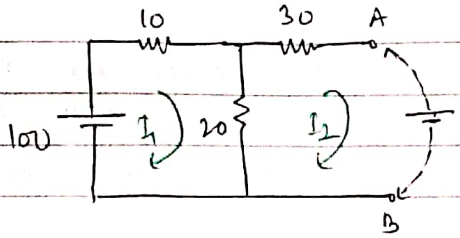
[when V_{TH} will be as per diagram shown above, current flows from A to B But when B is +ve with respect to A, current flows from B to A.]

Q.1 find the current flowing through the resistance of 40Ω using Thevenin's Theorem.



Solution

1. To find V_{TH}



Remove R_L of 40Ω

Assume A is +ve w.r.to B

(V_{TH}: If value of V_{TH} obtained after calculation is +ve, then the polarities assumed are correct otherwise vice-versa means B is +ve w.r.to A)

Using KVL we can find find value of V_{TH}. Here $I_2 = 0$

Voltage equations

$$30 I_1 - 20 I_2 = 100 \quad \& \quad -20 I_1 + 50 I_2 = -V_{TH}$$

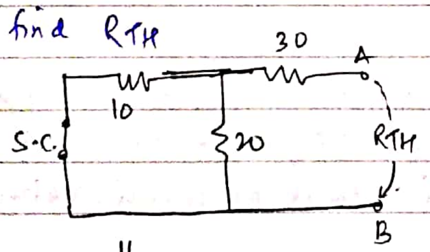
as $I_2 = 0$ $\quad 30 I_1 = 100 \quad$ and $\quad -20 I_1 = -V_{TH}$

OR $V_{TH} = 20 I_1$

$$I_1 = \frac{100}{30} = 3.33 \text{ Amp}$$

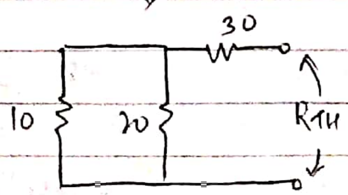
$$V_{TH} = 20 I_1 = 20 \times 3.33 = 66.66 \text{ Volt} \quad (1)$$

2. To find R_{TH}



Voltage source of 100 Volt, need to be replaced by Short circuit.

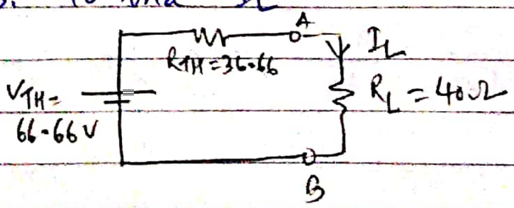
⇓



10Ω & 20Ω are in parallel. And this parallel combination will be in series with 30Ω

$$R_{TH} = \frac{10 \times 20}{10 + 20} + 30 = 36.66\,\Omega$$

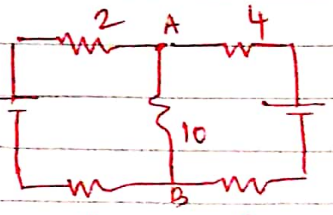
3. To find I_L



$$I_L = \frac{V_{TH}}{R_{TH} + R_L} = \frac{66.66}{36.66 + 40}$$

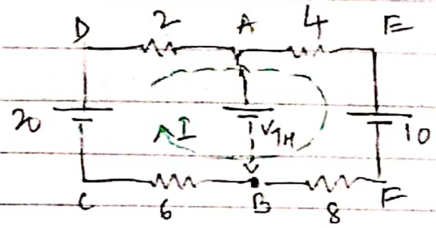
$$I_L = 0.869 \text{ Amp}$$

Q2 find the current flowing through 10Ω using thevenins theorem.



Solution.

1. To find V_{TH}



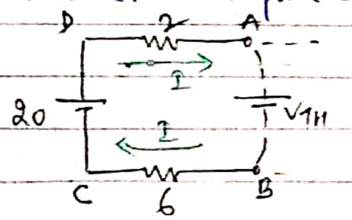
As 10Ω resistance between A and B is removed. Only one loop will be their. Hence current I will flow as shown in figure.

$$(2+4+8+6)I = -10+20 = 10$$

$$20I = 10$$

$$I = 10/20 = 0.5 \text{ Amp}$$

Consider left half part of the circuit. [Right half can be considered]

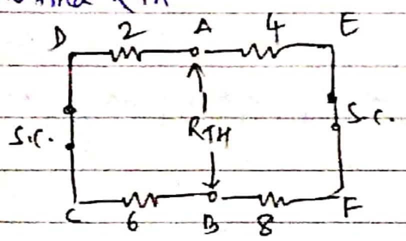


$$(2+6)I = -V_{TH} + 20$$

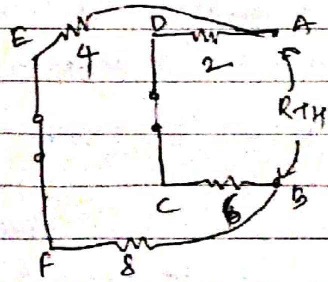
$$V_{TH} = 20 + 8I = 20 + 8(0.5)$$

$$= 20 + 4 = 16 \text{ volt}$$

2. To find R_{TH}



20V and 10V sources are replaced by short circuit as well have make the network as passive network



4 and 8 are in series $4+8 = 12$

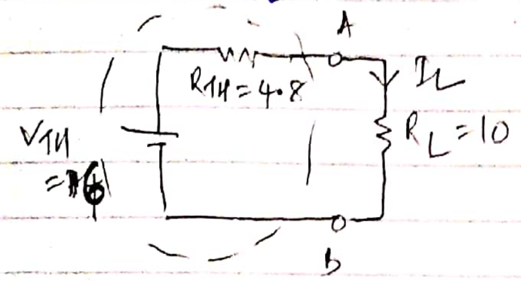
2 and 6 are in series $2+6 = 8$

These series combinations will be parallel to eachother

12Ω is in parallel with 8Ω

$$R_{TH} = \frac{12 \times 8}{12+8} = \frac{96}{20} = 4.8\ \Omega$$

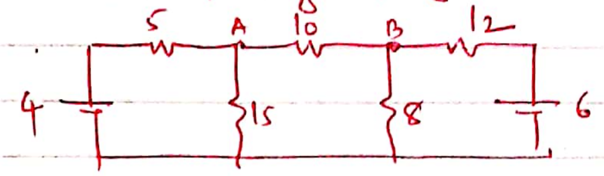
3. To find I_L



Thevenins eq. ckt

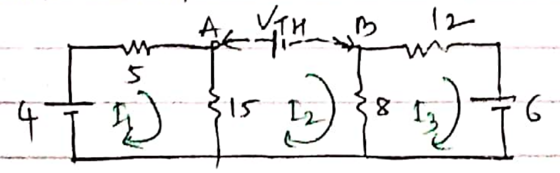
$$I_L = \frac{V_{TH}}{R_{TH}+R_L} = \frac{16}{4.8+10} = \underline{1.081\ Amp}$$

Q3 find the current flowing through $10\,\Omega$ using Thevenins Theorem.



Solution:

1. To find V_{TH}



As $10\,\Omega$ resistance is removed
$I_2 = 0$ Amp

Voltage equations.

$$20I_1 - 15I_2 - 0I_3 = 4 \quad -(I)$$
$$-15I_1 + 23I_2 - 8I_3 = -V_{TH} \quad -(II)$$
$$-0I_1 - 8I_2 + 20I_3 = -6 \quad -(III)$$

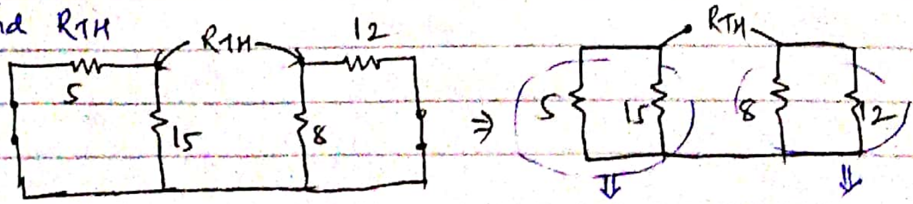
from equation no. (I)
$20I_1 = 4$
$I_1 = 4/20 = 0.2$ Amp

from equation no. (III)
$20I_3 = -6$
$I_3 = -6/20 = -0.3$ Amp

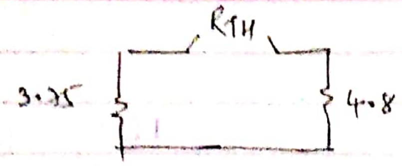
Putting value of I_1 and I_3 in equation no. (II)

$$-15I_1 + 23I_2 - 8I_3 = -V_{TH}$$
$$-15(0.2) + 0 - 8(-0.3) = -V_{TH}$$
$$-3 + 2.4 = -V_{TH}$$
$$\underline{V_{TH}} = 3 - 2.4 = \underline{0.6}\ \text{Volt}$$

2. To find R_{TH}

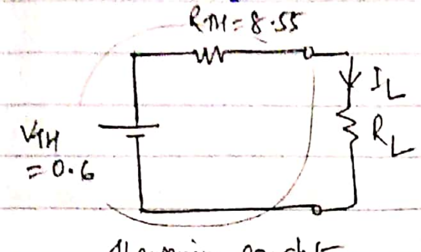


$$\frac{5\times15}{5+15} = 3.75 \qquad \frac{8\times12}{8+12} = \frac{96}{20} = 4.8$$



$R_{TH} = 3.35 + 4.08 = 8.55\ \Omega$

3. To find I_L



Thevenins eq. ckt

$$I_L = \frac{V_{TH}}{R_{TH}+R_L} = \frac{0.6}{8.55+10}$$

$$= 0.0323\ \text{Amp}$$