UNIT 3

Syllabus

- **Unit III Equilibrium (06Hrs)**

A)Free body diagram, Equilibrium of concurrent, parallel forces in a plane, Equilibrium of general forces in a plane, Equilibrium of three forces in a plane, Types of beams, simple and compound beams, Type of supports and reaction,

B)Forces in space, Resultant of concurrent and parallel forces in a space, Equilibrium of concurrent and parallel forces in a space.

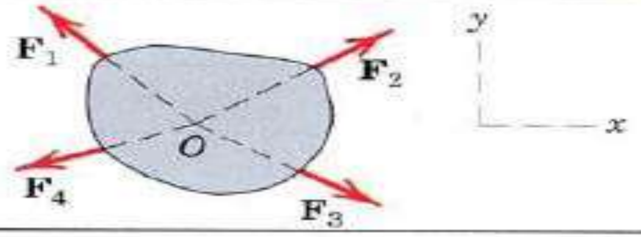
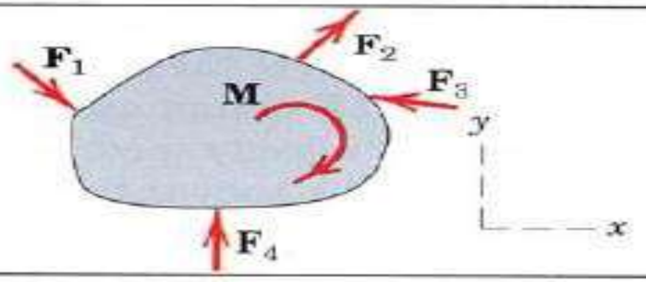
Equilibrium

- Equilibrium of single force doesn't exist.
- If two forces are in equilibrium then they are equal, opposite and collinear.
- Equilibriumunder three forces –
- Can be solved by Lami's theorem or $\sum Fx = 0$, $\Sigma\ Fy = 0$
- Equilibrium under four or more forces
- 1) $\sum Fx = 0$
- 2) $\Sigma\ Fy = 0$ 3)$\Sigma\ M = 0$

Conditions of Equilibrium

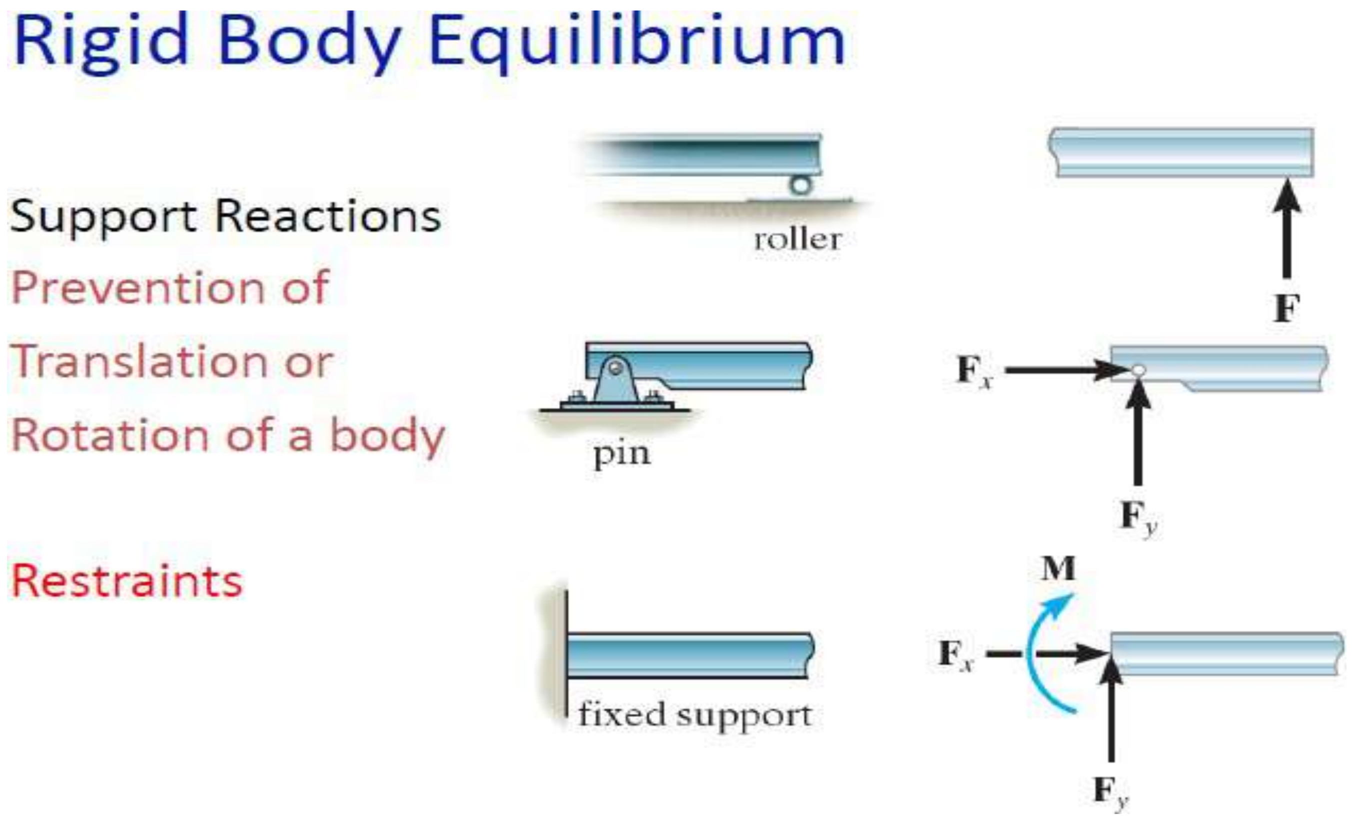
Rigid Body Equilibrium

Categories in 2-D

CATEGORIES OF EQUILIBRIUM IN TWO DIMENSIONS		
Force System	Free-Body Diagram	Independent Equations
1. Collinear		$\Sigma F_x = 0$
2. Concurrent at a point		$\Sigma F_x = 0$ $\Sigma F_y = 0$
3. Parallel		$\Sigma F_x = 0$ $\Sigma M_z = 0$
4. General		$\Sigma F_x = 0$ $\Sigma M_z = 0$ $\Sigma F_y = 0$

Free Body diagram (FBD)

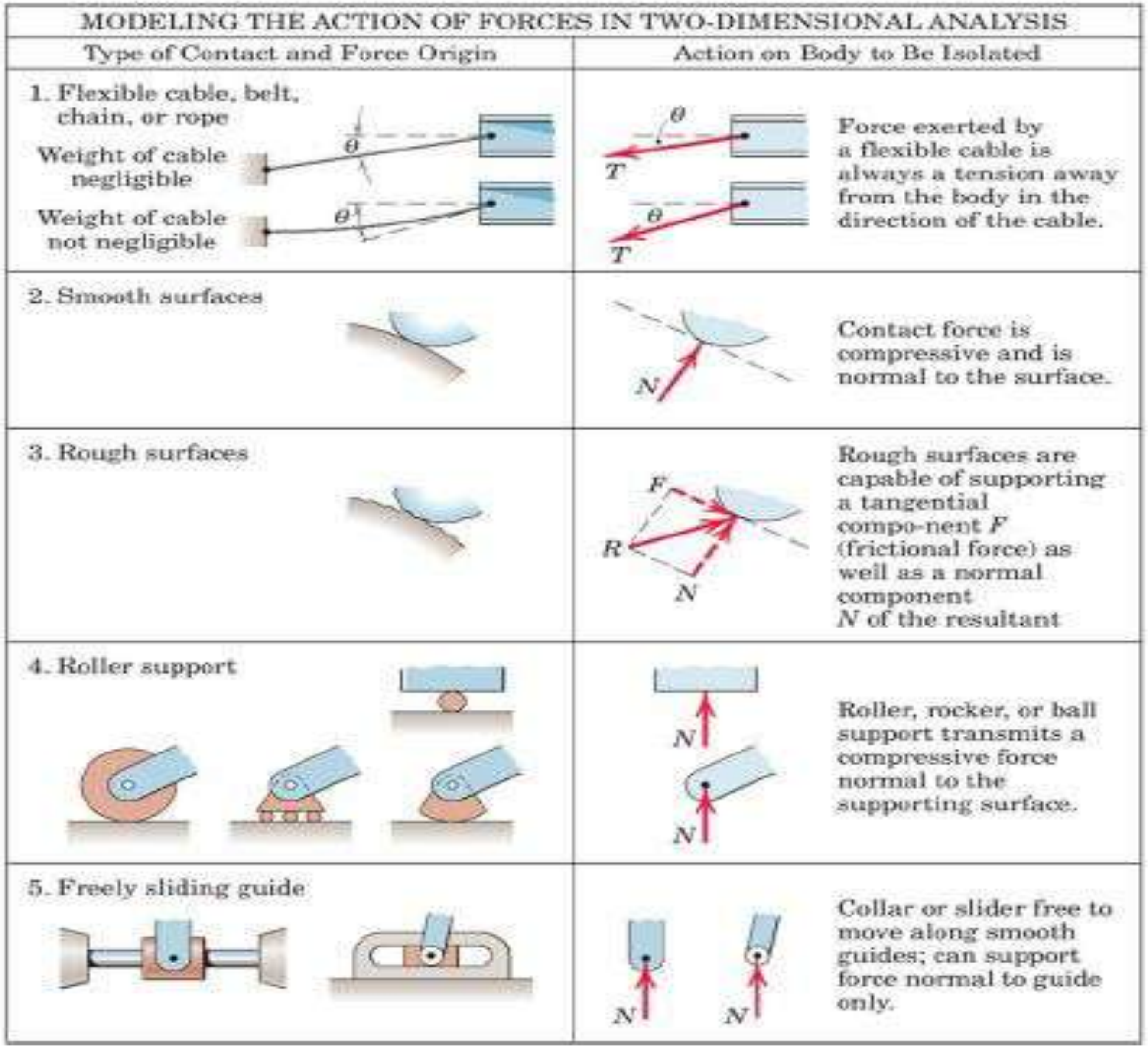
- An isolated body separated from all other connected bodies or surfaces is free body diagram.



Free Body diagram

Rigid Body Equilibrium

Various Supports
2-D Force
Systems

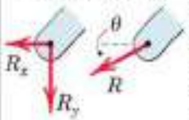
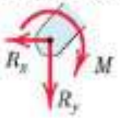
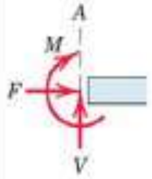
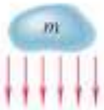
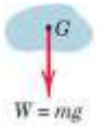
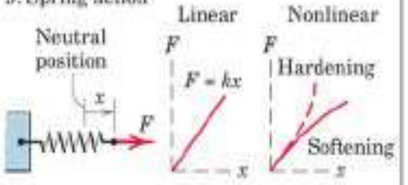
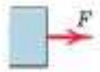


MODELING THE ACTION OF FORCES IN TWO-DIMENSIONAL ANALYSIS	
Type of Contact and Force Origin	Action on Body to Be Isolated
1. Flexible cable, belt, chain, or rope Weight of cable negligible θ Weight of cable not negligible θ	θ T θ T Force exerted by a flexible cable is always a tension away from the body in the direction of the cable.
2. Smooth surfaces	N Contact force is compressive and is normal to the surface.
3. Rough surfaces	F R N Rough surfaces are capable of supporting a tangential compo-nent F (frictional force) as well as a normal component N of the resultant
4. Roller support	N N Roller, rocker, or ball support transmits a compressive force normal to the supporting surface.
5. Freely sliding guide	N N Collar or slider free to move along smooth guides; can support force normal to guide only.

Free Body diagram

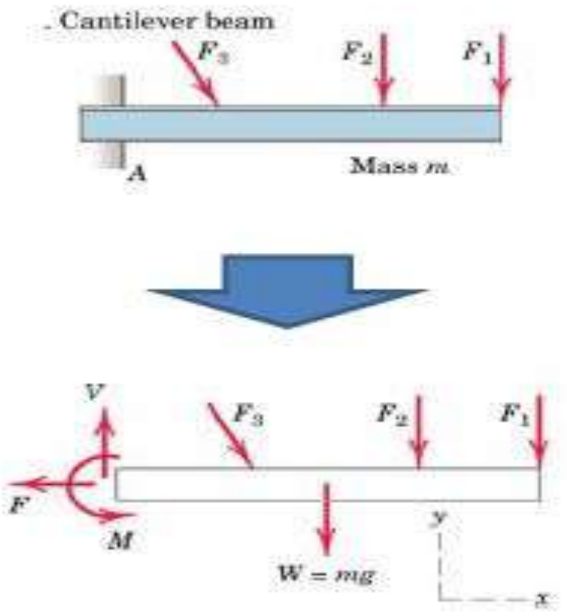
Rigid Body Equilibrium

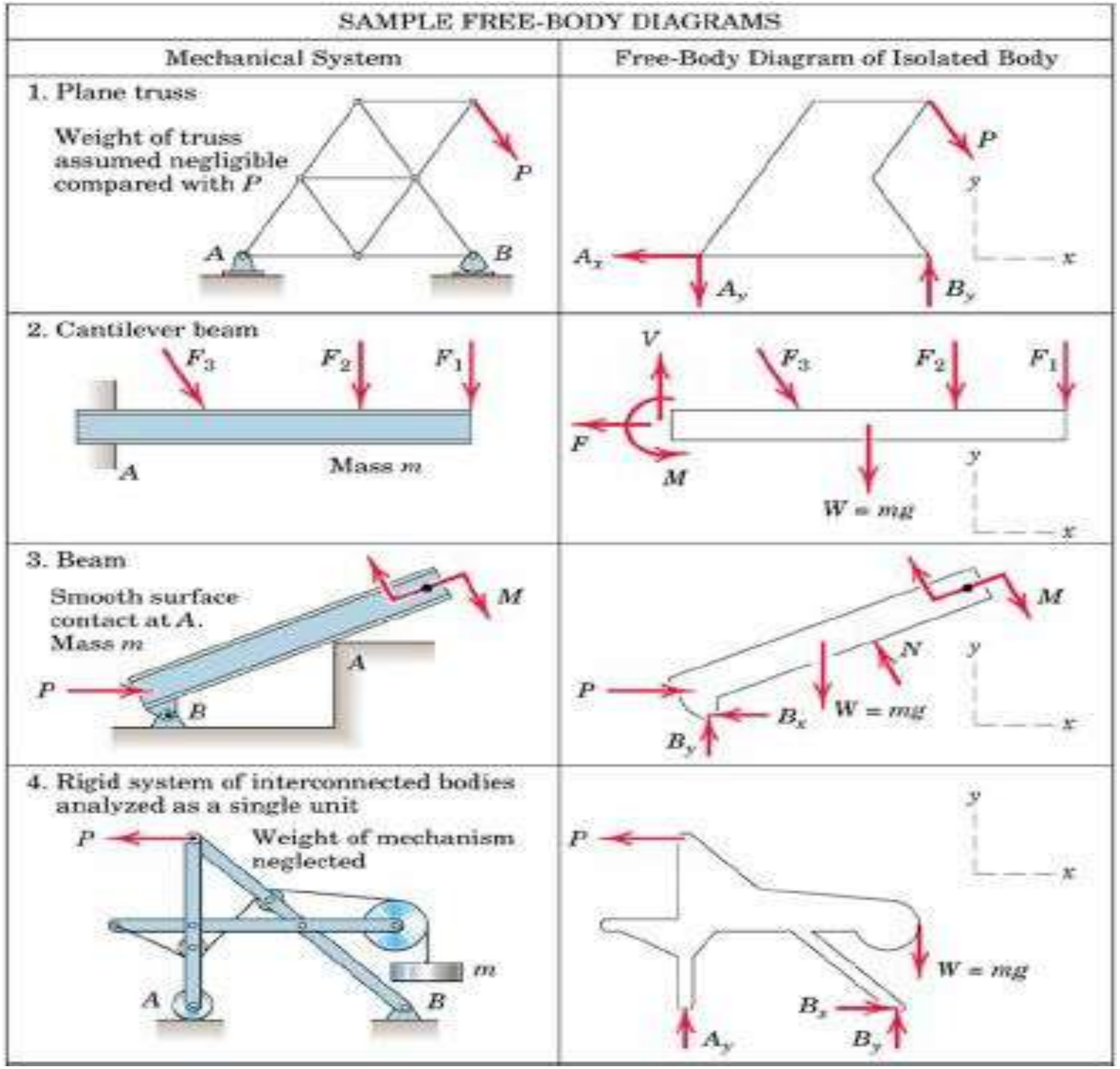
Various Supports 2-D Force Systems

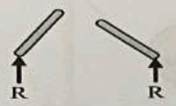
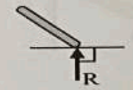
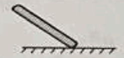
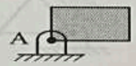
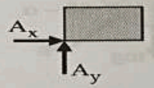
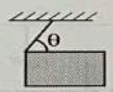
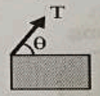
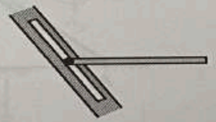
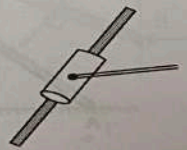
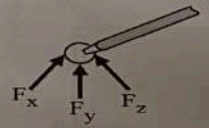
MODELING THE ACTION OF FORCES IN TWO-DIMENSIONAL ANALYSIS (*cont.*)		
Type of Contact and Force Origin	**Action on Body to Be Isolated**	
6. Pin connection 	Pin free to turn  Pin not free to turn 	A freely hinged pin connection is capable of supporting a force in any direction in the plane normal to the pin axis. We may either show two components R_x and R_y or a magnitude R and direction θ. A pin not free to turn also supports a couple M.
7. Built-in or fixed support 		A built-in or fixed support is capable of supporting an axial force F, a transverse force V (shear force), and a couple M (bending moment) to prevent rotation.
8. Gravitational attraction 		The resultant of gravitational attraction on all elements of a body of mass m is the weight $W = mg$ and acts toward the center of the earth through the center mass G.
9. Spring action 		Spring force is tensile if spring is stretched and compressive if compressed. For a linearly elastic spring the stiffness k is the force required to deform the spring a unit distance.

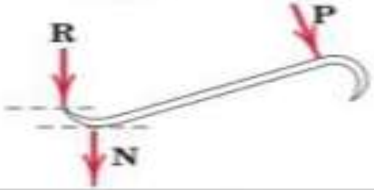
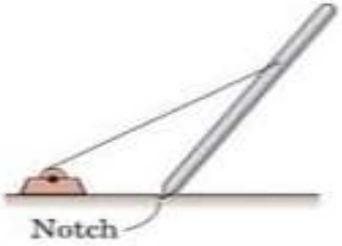
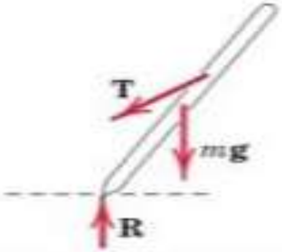
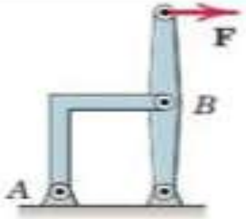
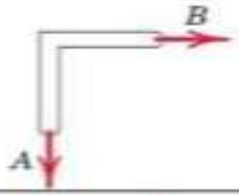
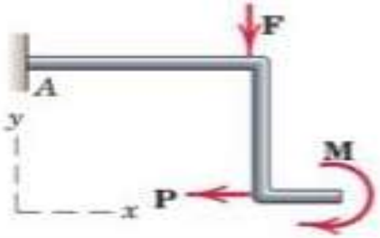
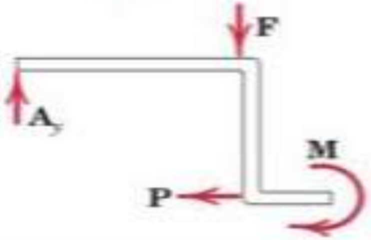
Free Body diagram

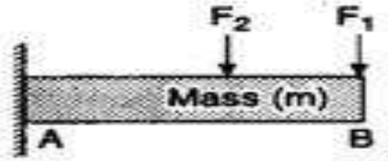
Free body diagram



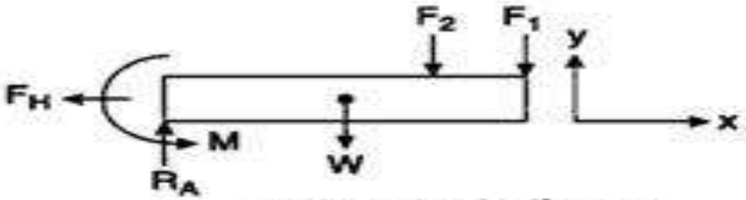


Sr. No.	Support / Connection	Sketch	Reaction	Specification	No. of unknowns
1.	Rollers			Known reaction which is ⊥r to plane of roller	One
2.	Smooth surface			Reaction is ⊥r to the surface	One
3.	Rough surface			Two reaction components with unknown directions	Two
4.	Smooth pin or Hinge			Two reaction components with unknown directions	Two
5.	Flexible cord, rope or cable of negligible weight.			One axial force acting away from body (Tension)	One
6.	Fixed support			Two reaction components and one moment with all components unknown in directions.	Three
7.	A smooth pin in a slot			Reaction with known line of action which is always ⊥r to slot in which pin is sliding.	One
8.	A sliding collar			Reaction is perpendicular to the rod along which collar is sliding without friction	One
9.	Ball and socket joint			Three reaction components in unknown directions.	Three
10.	A short link			Force with known line of action	One

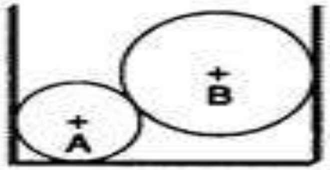
	Body	Wrong or Incomplete FBD
1. Lawn roller of mass m being pushed up incline θ.		
2. Prybar lifting body A having smooth horizontal surface. Bar rests on horizontal rough surface.		
3. Uniform pole of mass m being hoisted into position by winch. Horizontal supporting surface notched to prevent slipping of pole.		
4. Supporting angle bracket for frame; pin joints.		
5. Bent rod welded to support at A and subjected to two forces and couple.		



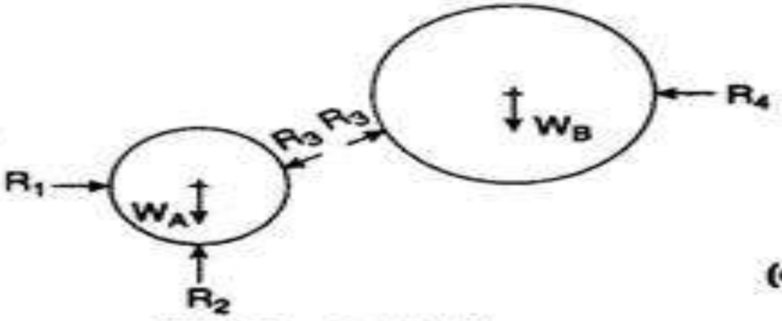
a(i) Cantilever beam



a(ii) Free body diagram



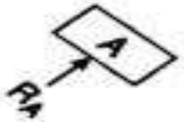
b(i) Two spheres in equilibrium



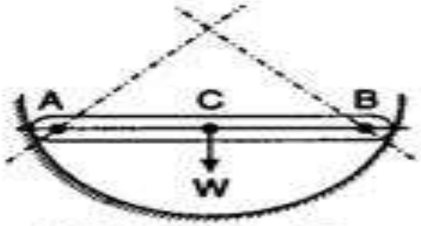
b(ii) Free body diagram

(c) Ball resting on a surface

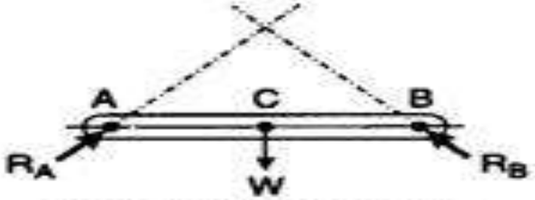
d(i) Frictionless surface



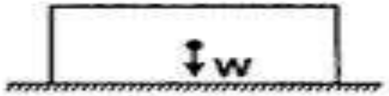
d(ii) Free body diagram



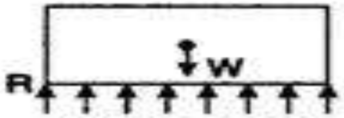
e(i) A bar placed in a hemispherical cup



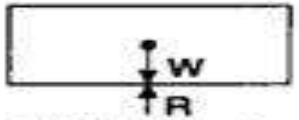
e(ii) Free body diagram



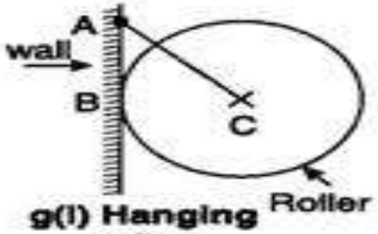
f(i) Block on surface



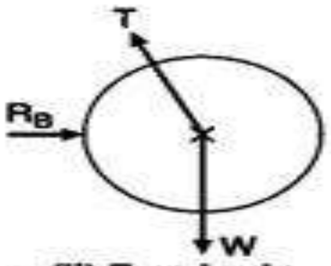
f(ii) Free body diagram



f(iii) Free body diagram

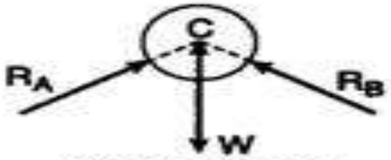


g(i) Hanging roller

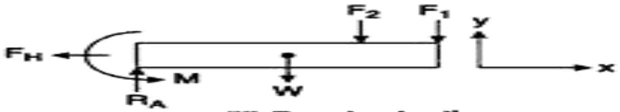
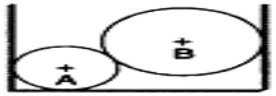
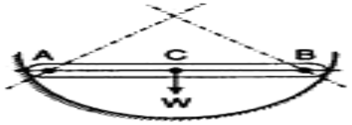
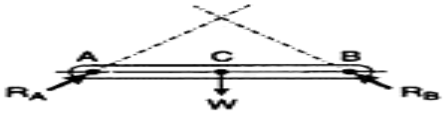
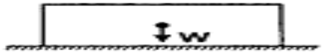
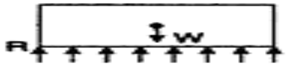
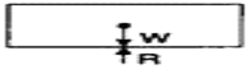
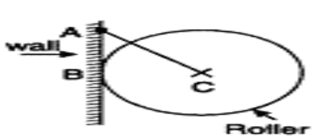
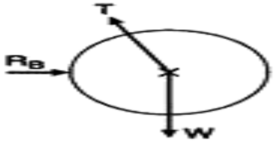


g(ii) Free body diagram

h(i) Sphere in groove



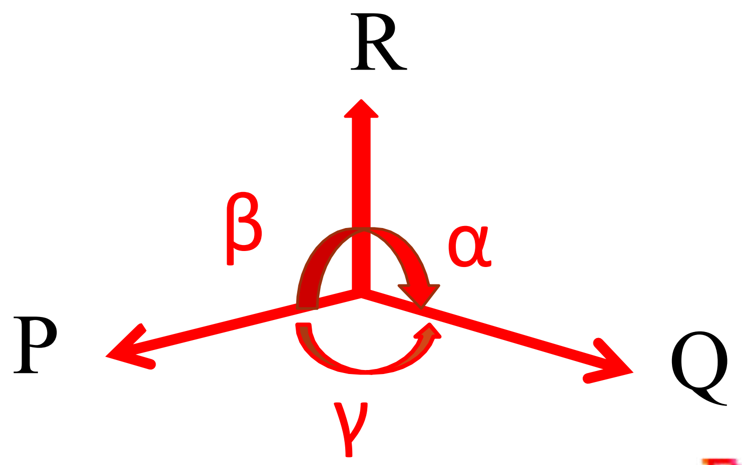
h(ii) Free body diagram

Problem	Free Body Diagram
 a(i) Cantilever beam	 a(ii) Free body diagram
 b(i) Two spheres in equilibrium	 b(ii) Free body diagram
 (c) Ball resting on a surface	 c (ii) Free body diagram
 d(i) Frictionless surface	 d(ii) Free body diagram
 e(i) A bar placed in a hemispherical cup	 e(ii) Free body diagram
 f(i) Block on surface	 f(ii) Free body diagram  f(iii) Free body diagram
 g(i) Hanging roller	 g(ii) Free body diagram
 h(i) Sphere in groove	 h(ii) Free body diagram

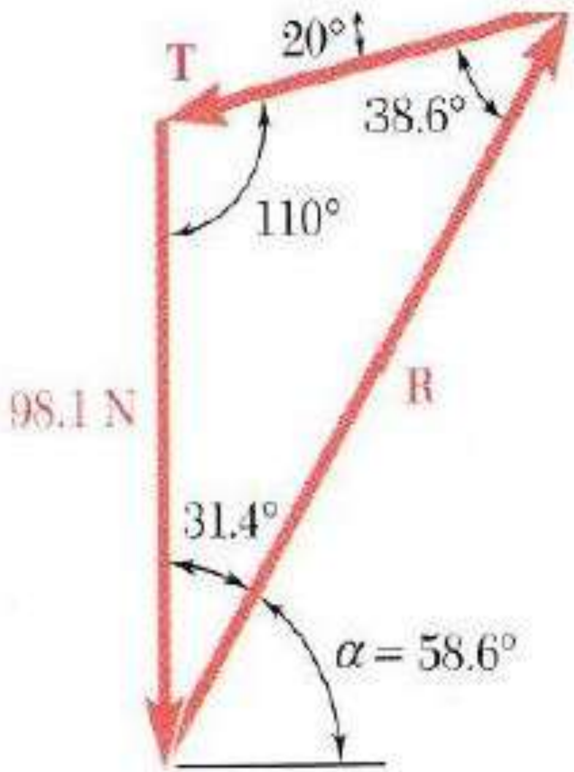
Equilibrium

- Lami's Theorem (applicable for three concurrent coplanar forces)

If body is in equilibrium under three concurrent coplanar forces then each force is proportional to sine of angle between remaining two forces.



$$\frac{P}{Sin\alpha} = \frac{Q}{sin\beta} = \frac{R}{sin\gamma}$$

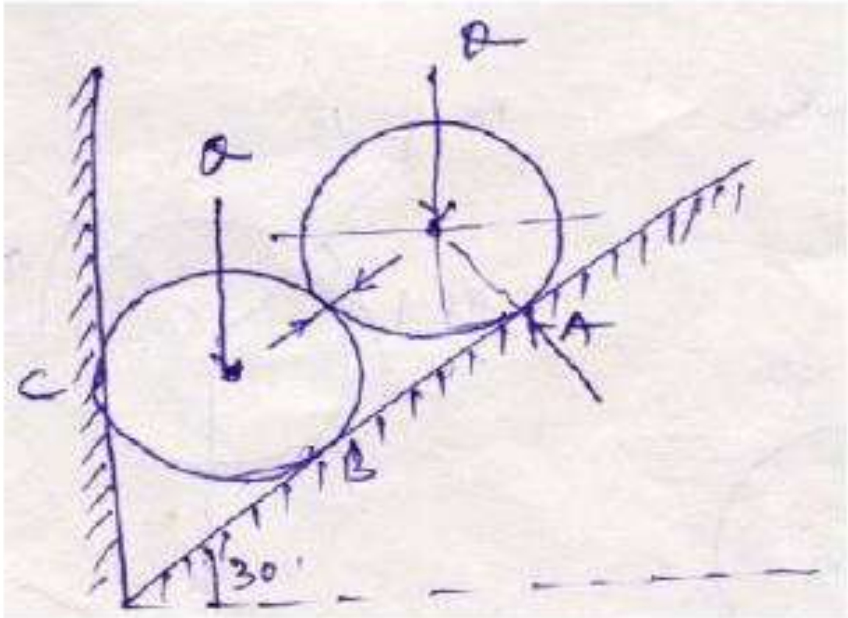


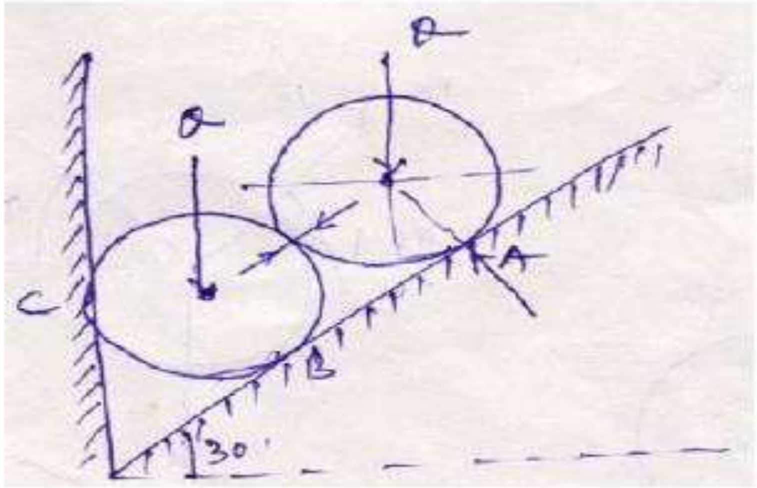
- Determine the magnitude of the reaction force R.

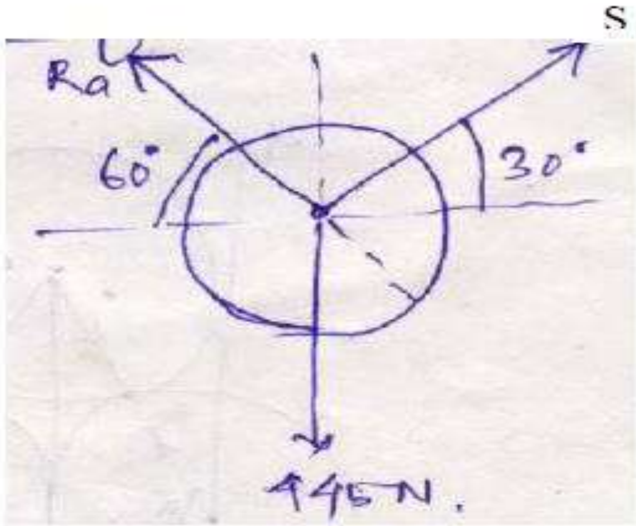
$$\frac{T}{\sin 31.4^\circ} = \frac{R}{\sin 110^\circ} = \frac{98.1\,\text{N}}{\sin 38.6^\circ}$$

$$\boxed{\begin{aligned} T &= 81.9\,\text{N} \\ R &= 147.8\,\text{N} \end{aligned}}$$

Problem: Two identical rollers each of weight Q = 445 N are supported by an inclined plane and a vertical wall as shown in the figure. Assuming smooth surfaces, find the reactions induced at the points of support A, B and C.







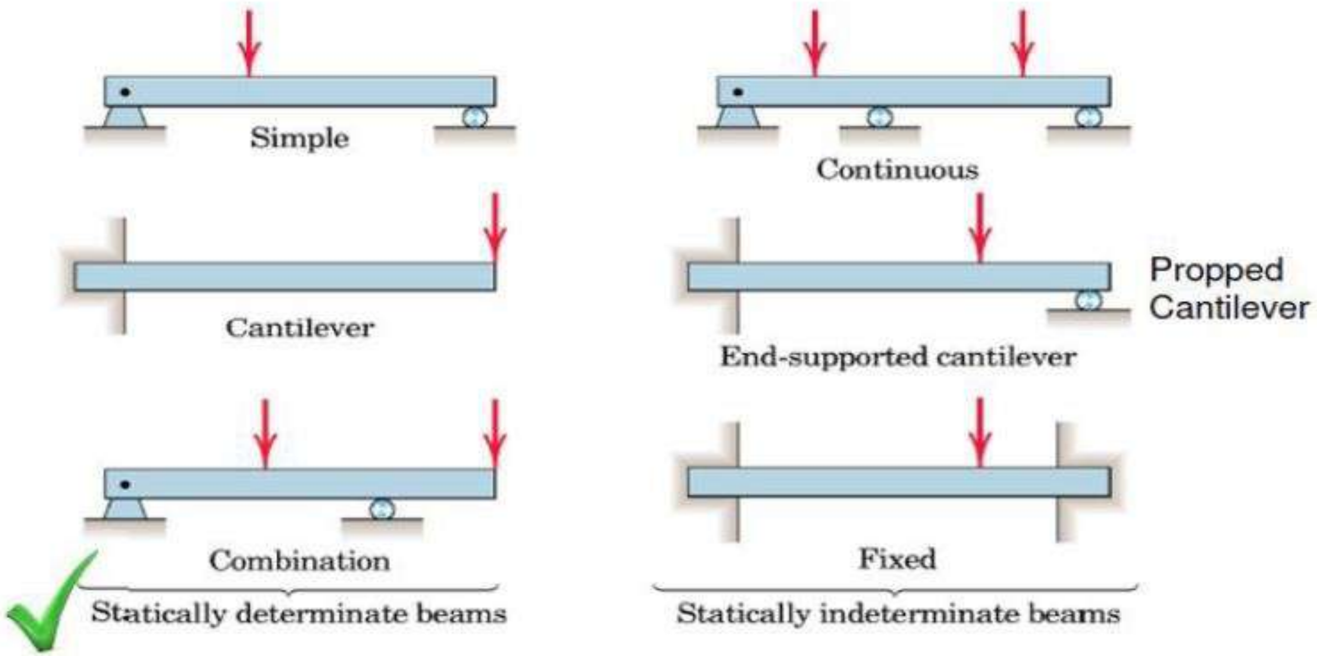
$$\frac{R_a}{\sin 120} = \frac{S}{\sin 150} = \frac{445}{\sin 90}$$

$$\Rightarrow R_a = 385.38N$$
$$\Rightarrow S = 222.5N$$

Equilibrium of Beams

Sub Unit -2

- Beams are structural elements with various engineering applications like roofs, bridges, mechanical assemblies
  - Based on support conditions



Types of loading

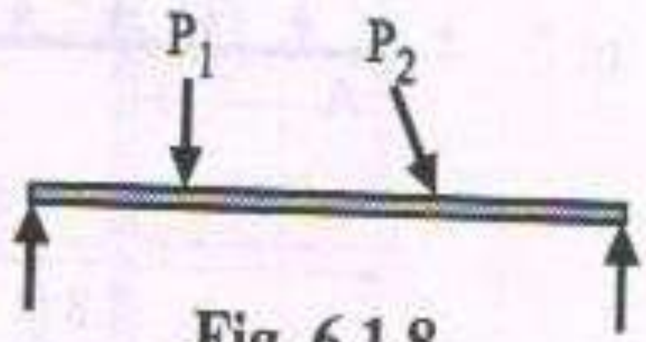
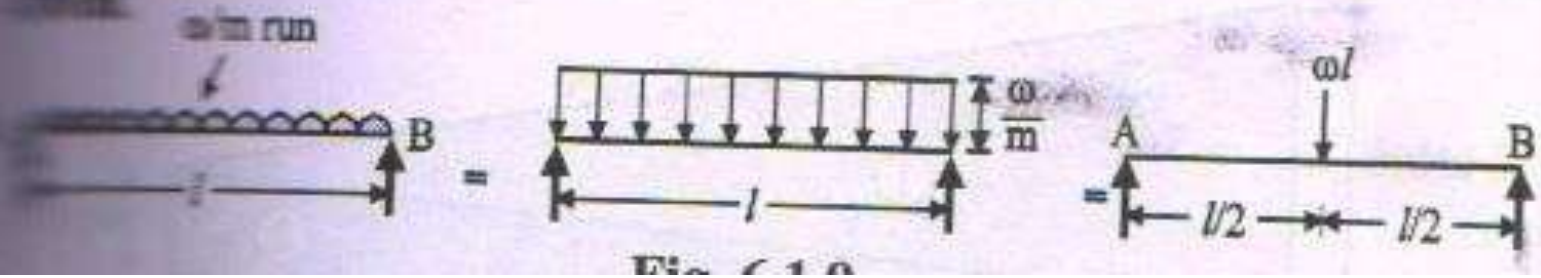


Fig. 6.1.8

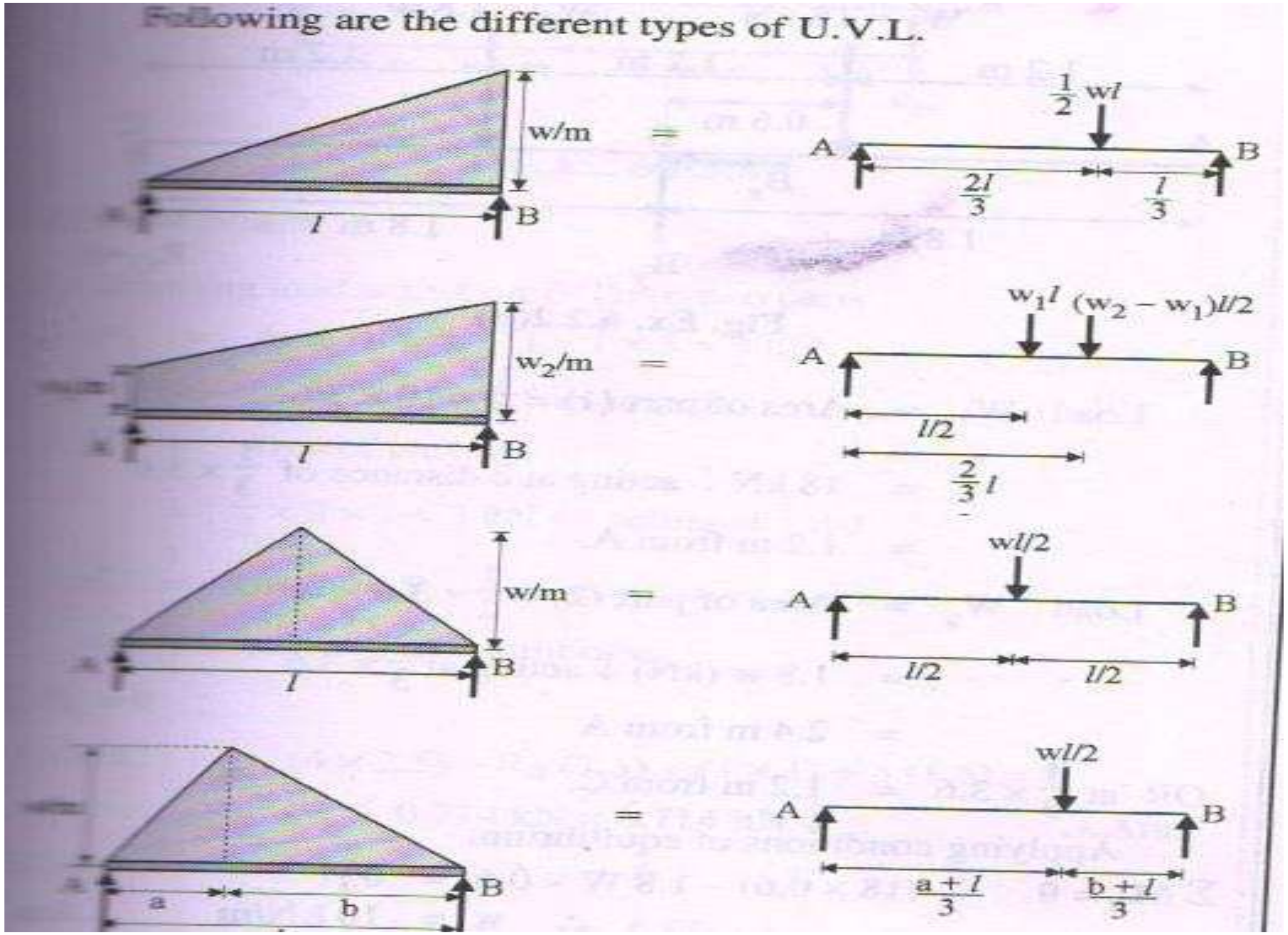
Point load

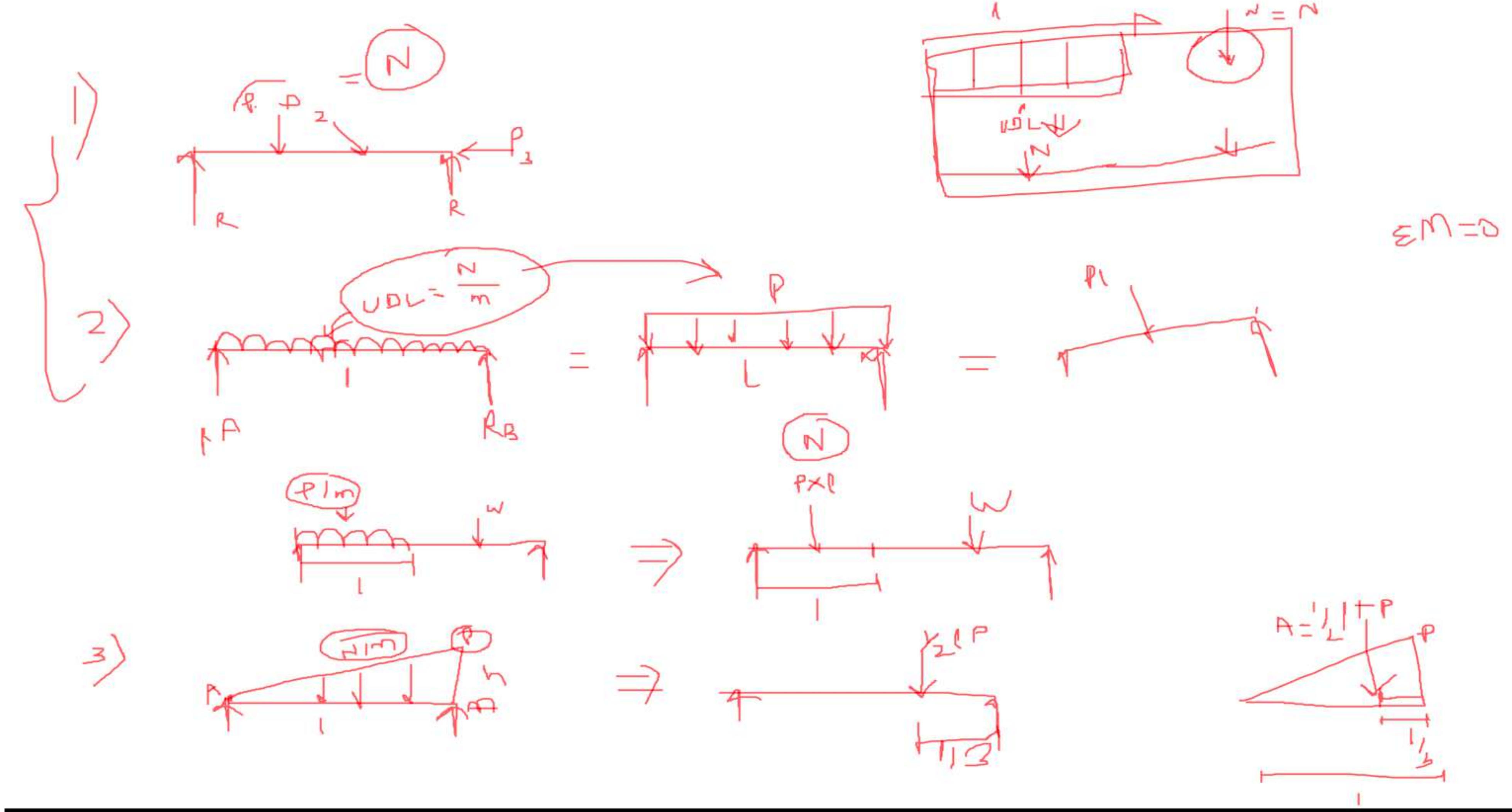
(2) Uniformly Distributed Load (U.D.L.)

A load which is spread over the beam or part of beam uniformly is called U.D.L. It is always represented in **N/m or kN/m**.



Types of loading





1)
N
P
2
R
R
UDL
N
2)
UDL = N/m
l
RA
RB
P
L
Pl
ΣM = 0
N
P/m
w
P×l
W
l
3)
N/m
P
h
A
B
l
1/2 l P
l/3
A = 1/2 l × P
P
l

$\Sigma M_A = 0,$

$W_{AB}\ (150 \sin \theta) - W_{BC}\ [100 \cos \theta - 300 \sin \theta] = 0$

$(AL\rho)_{AB} \cdot 150 \sin \theta = (AL\rho)_{BC}\ [100 \cos \theta - 300 \sin \theta]$

But $A_{AB} = A_{BC}$ (C.S. area is same)

and $\rho_{AB} = \rho_{BC}$ (Material is same, so density is same)

$\therefore 300\ [150 \sin \theta] = 200[100 \cos \theta - 300 \sin \theta]$

$\therefore 1050 \sin \theta = 200 \cos \theta$

$\therefore \tan \theta = \frac{200}{1050}$

$\theta = 10.78°$ ...Ans.

TYPE III : EXAMPLES BASED ON EQUILIBRIUM OF GENERAL FORCES IN PLANE

Part A : Equilibrium of bars/bracket

Ex. 5.10.30

A uniform bar AB of length L and weight W lies in a vertical plane with its ends resting on two smooth surfaces on OA and OB. Find angle θ for equilibrium of bar.

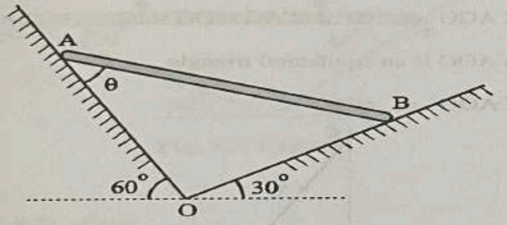


Fig. Ex. 5.10.30

Soln. :

Step 1 : Observe the number of forces acting

Here bar is subjected to three forces viz.

(1) Normal reaction at A, R_A.

(2) Normal reaction at B, R_B.

(3) Self weight of bar W.

Step 2 : Draw F.B.D. of bar

Since bar is **in equilibrium under the action of three non-parallel forces,** they must be concurrent (at point D as shown in Fig. Ex. 5.10.30(a)).

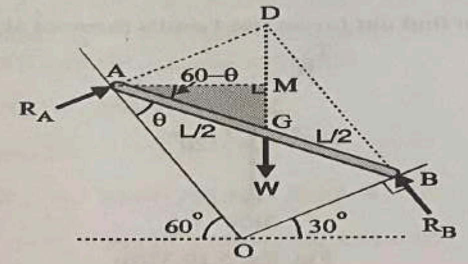


Fig. Ex. 5.10.30(a)

Step 3 : Conditions of equilibrium

$\Sigma M_A = 0 \quad R_B\ (AD) - W\ (AM) = 0$

$\therefore R_B \cdot L \sin \theta = W \cdot \frac{L}{2} \cos (60° - \theta)$

$R_B \sin \theta = \frac{W}{2} \cos (60° - \theta)$ ...(1)

Geometry

In ⊿ AOB, $OB = L \sin \theta,$

$\therefore AD = L \sin \theta$

In ⊿ AMG, $AM = \frac{L}{2} \cdot \cos (60° - \theta)$

Now, to find relation between the forces either apply conditions $\Sigma F_x = 0$ and $\Sigma F_y = 0$ or use Lami's theorem at D.

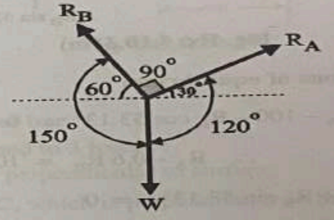


Fig. Ex. 5.10.30(b)

Using Lami's theorem at D.

$$\therefore \frac{R_A}{\sin 150°} = \frac{R_B}{\sin 120°} = \frac{W}{\sin 90°}$$

$$\therefore R_B = \frac{W \sin 120°}{\sin 90°} \qquad R_B = 0.87\ W$$

Substituting this value in Equation (1),

$$0.87\ W \sin \theta = \frac{W}{2} \cos (60° - \theta)$$

$$\therefore 1.74 \sin \theta = \cos 60° \cdot \cos \theta + \sin 60° \cdot \sin \theta$$

$$0.866 \sin \theta = 0.5 \cos \theta$$

$$\therefore \frac{\sin \theta}{\cos \theta} = \frac{0.5}{0.866} \qquad \therefore \theta = 30°$$ ...A

Ex. 5.10.31

A bar 12 m long and of negligible weight is acted upon by f as shown in Fig. Ex. 5.10.31. Determine angle θ for equilibri bar.

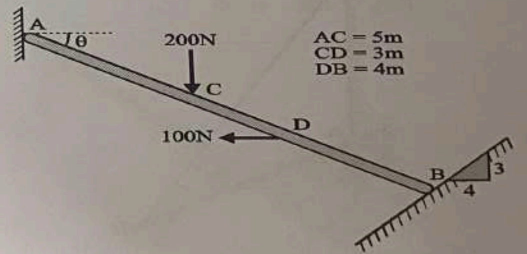
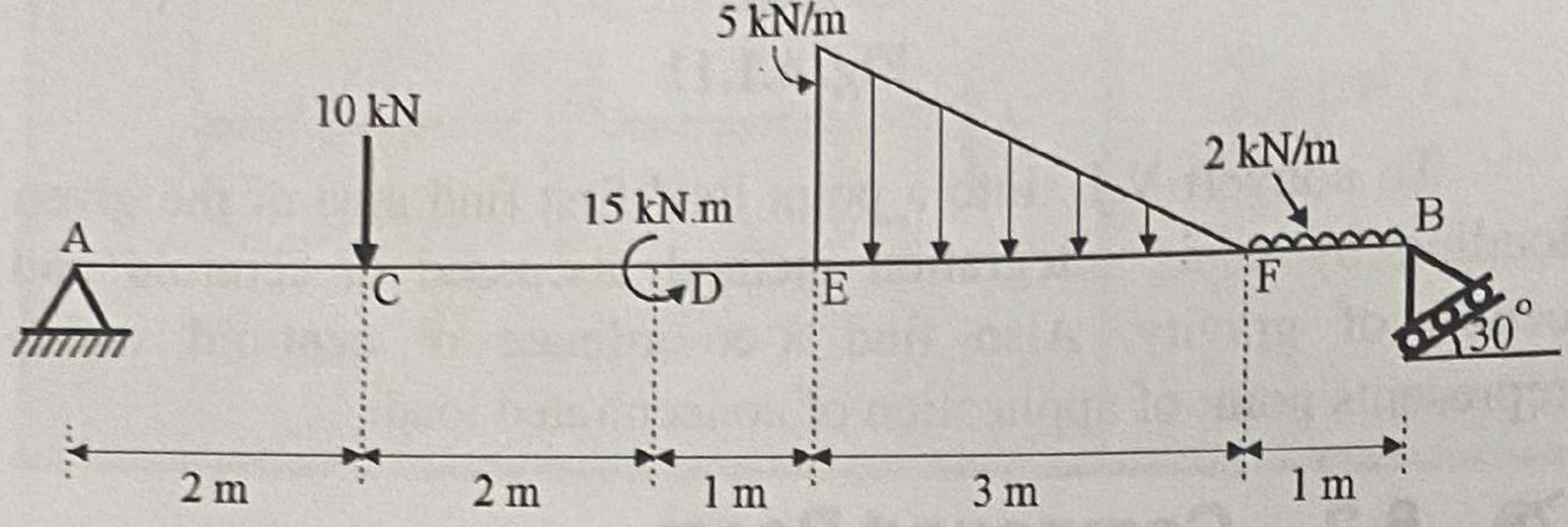


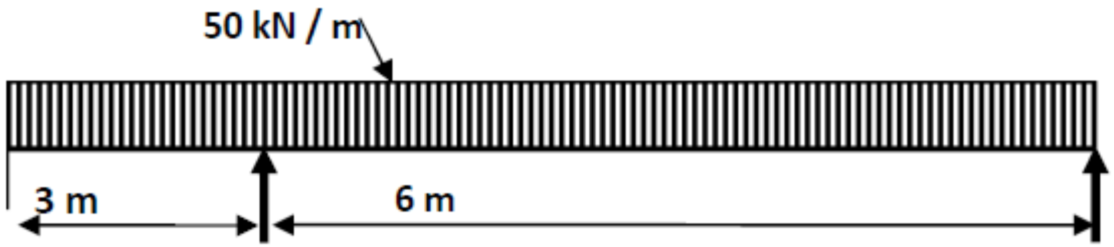
Fig. Ex. 5.10.31

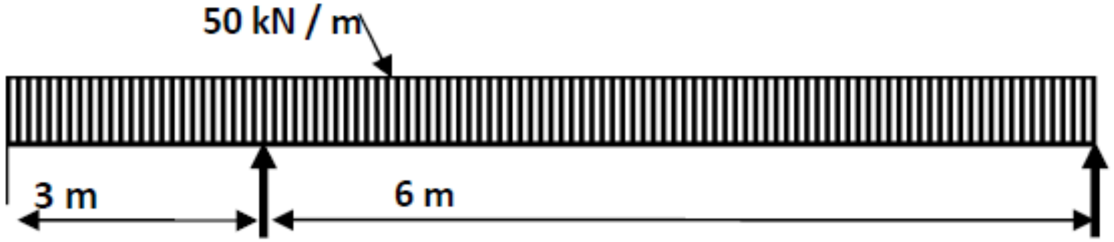
Find reactions at the supports for given beam as shown in Fig. Ex. 6.2.1.

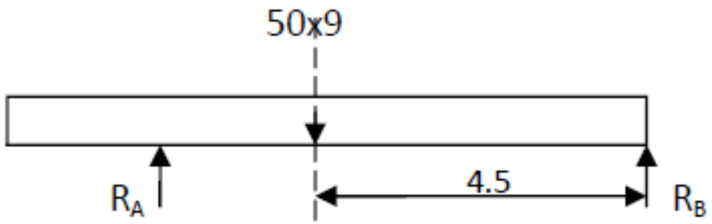


Problems

Find reactions at support for the following beam .



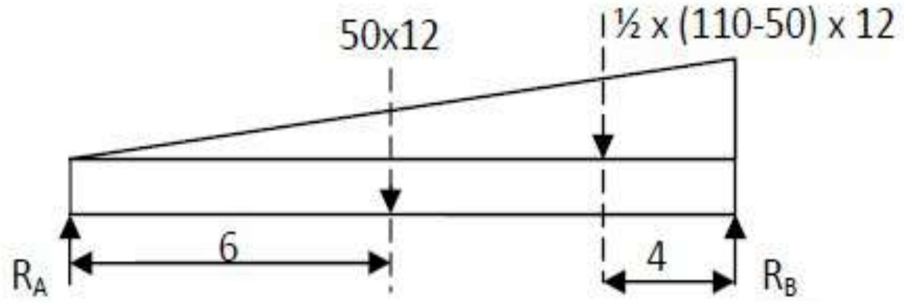


FBD	
$\sum F_x$	Not applicable
$\sum F_y$	$R_A + R_B = 50x9 + 450$ ------ (1)
$\sum M_A$	R_B x $6 = 450$ x 1.5; $R_B = 112.5$ kN and from (1) so $R_A = 337.5$ kN

Problems

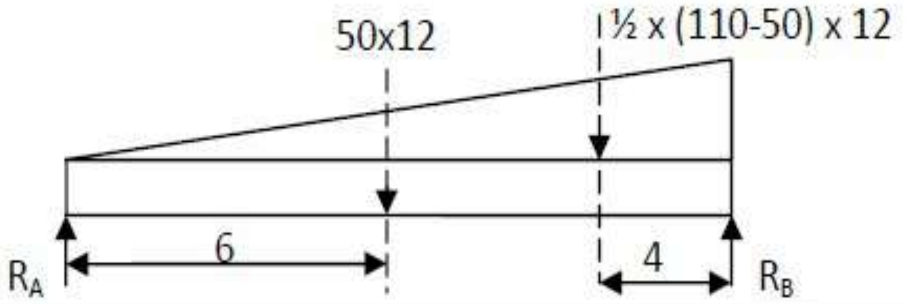
A simply supported beam of span 12 m is loaded with a uniformly varying load of 50 kN / m at left end and 110 kN / m at right end. Find reactions at support.

FBD



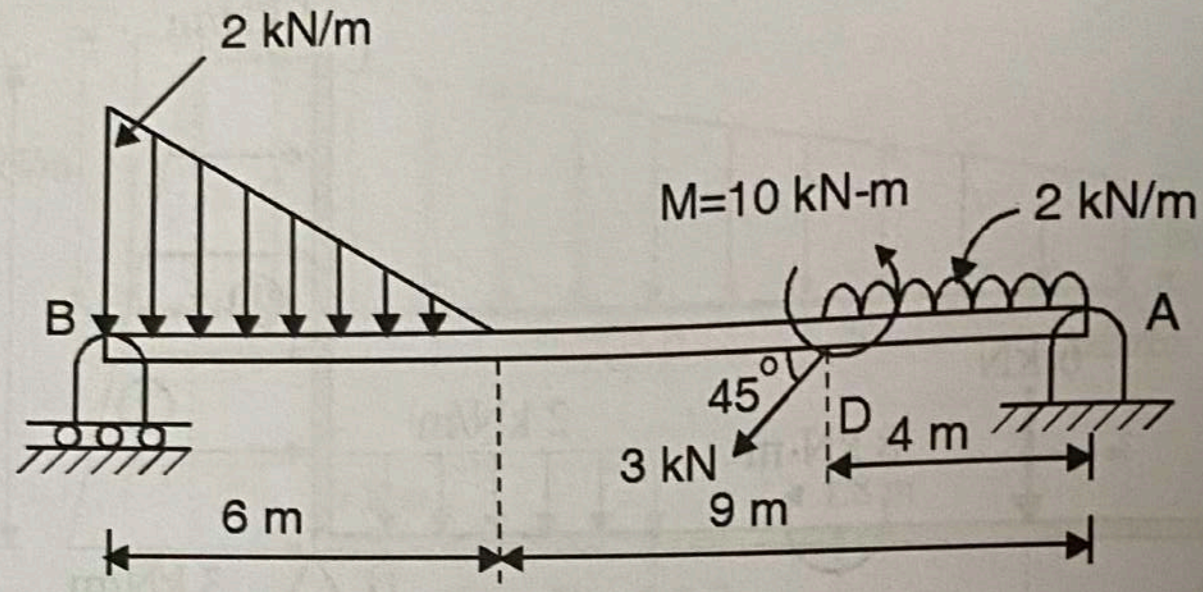
Problems

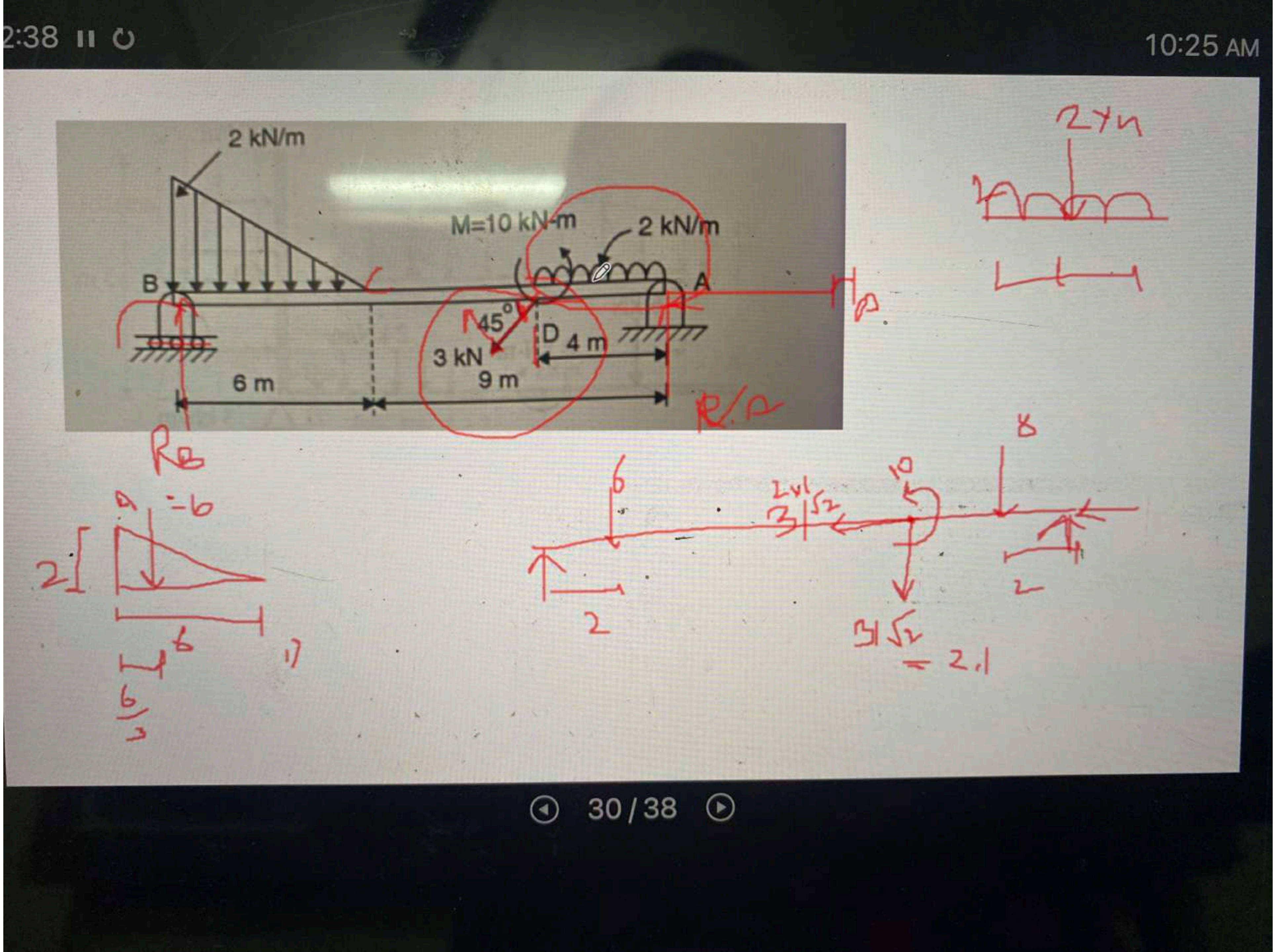
A simply supported beam of span 12 m is loaded with a uniformly varying load of 50 kN / m at left end and 110 kN / m at right end. Find reactions at support.

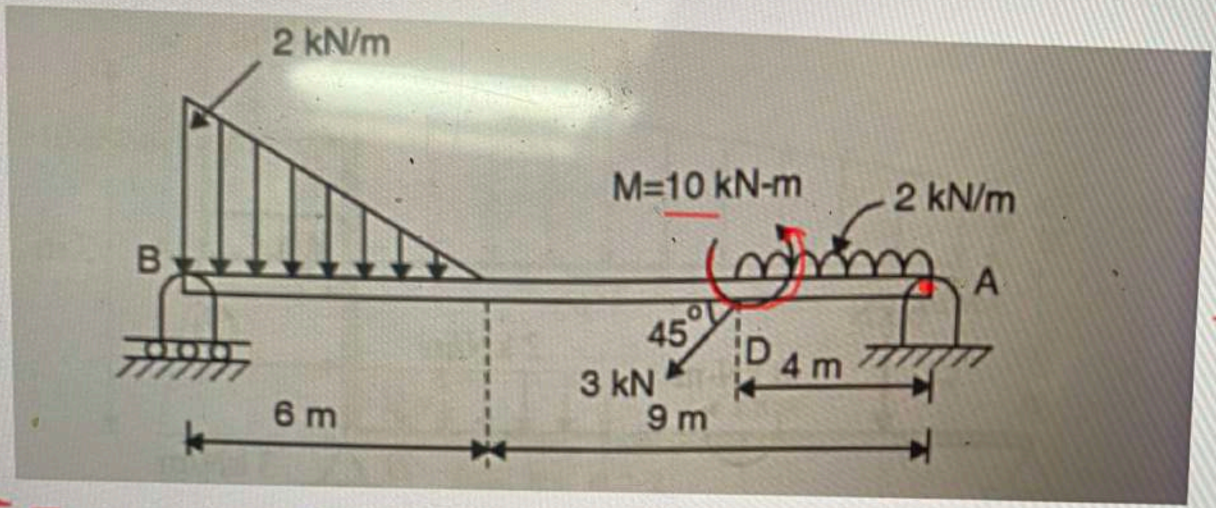
FBD	
$\sum F_x$	Not applicable
$\sum F_y$	$R_A + R_B = 50x12 + ½ (60x12) = 960$ ------ (1)
$\sum M_A$	R_B x 12 = 600 x 6 + 360 x 8; R_B = 540 kN and from (1) so R_A = 420 kN
For checking the answer $\sum M_B$	R_A x 12 = 600 x 6 + 360 x 4 R_A = 420 kN
Answer	R_A= 420 kN ; R_B = 540 kN

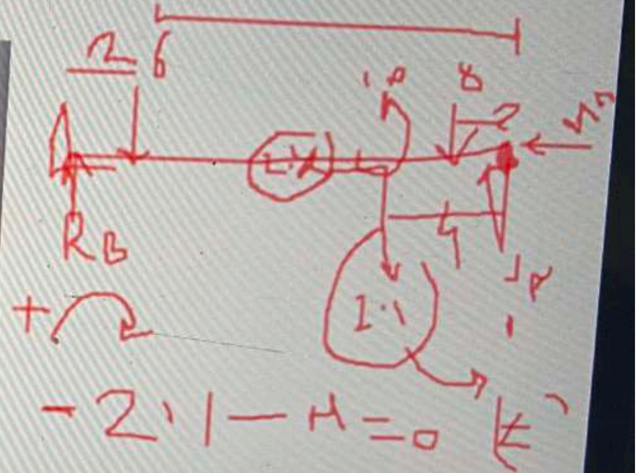
Ex. 6.2.7

Find the reactions at the supports of the beam AB loaded as shown in the Fig. Ex. 6.2.7.




2:38
10:25 AM
2 kN/m
M=10 kN-m
2 kN/m
B
A
45°
3 kN
D
4 m
6 m
9 m
30 / 38





$$-2.1 - H = 0$$

$$H = -2.1 \leftarrow$$

$$= 2.1 \rightarrow$$

$$\Sigma F_y = 0 = R_B - 6 - 2.1 - 8 + R_{VA}$$

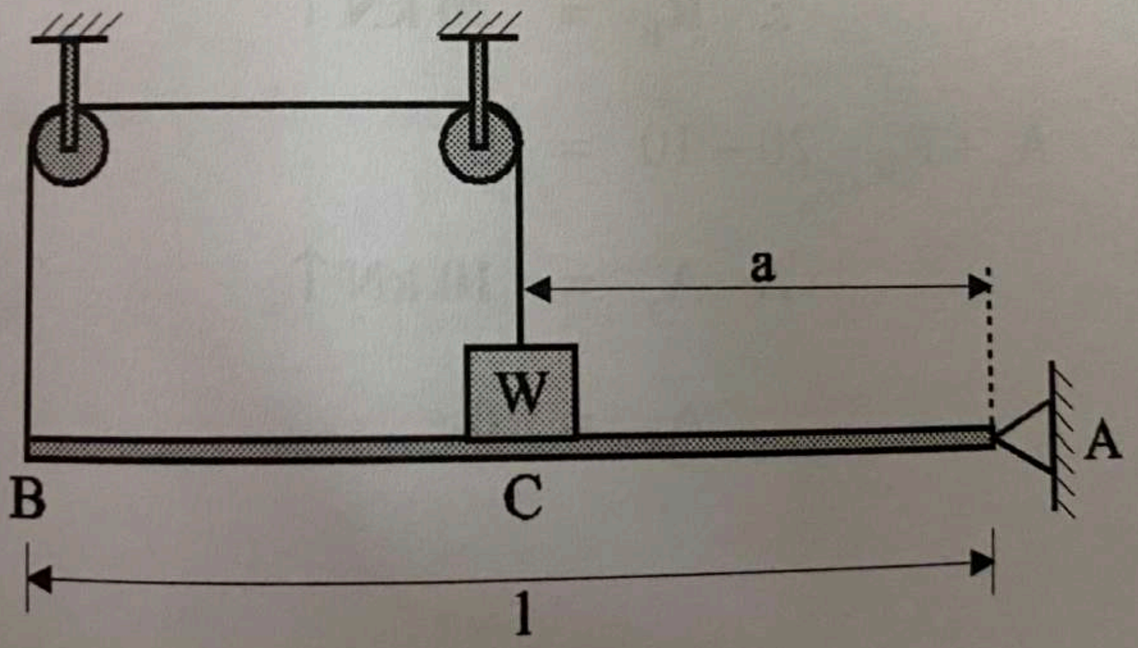
$$\boxed{R_B + V_A = 6 + 2.1 + 8 = 16.1}$$

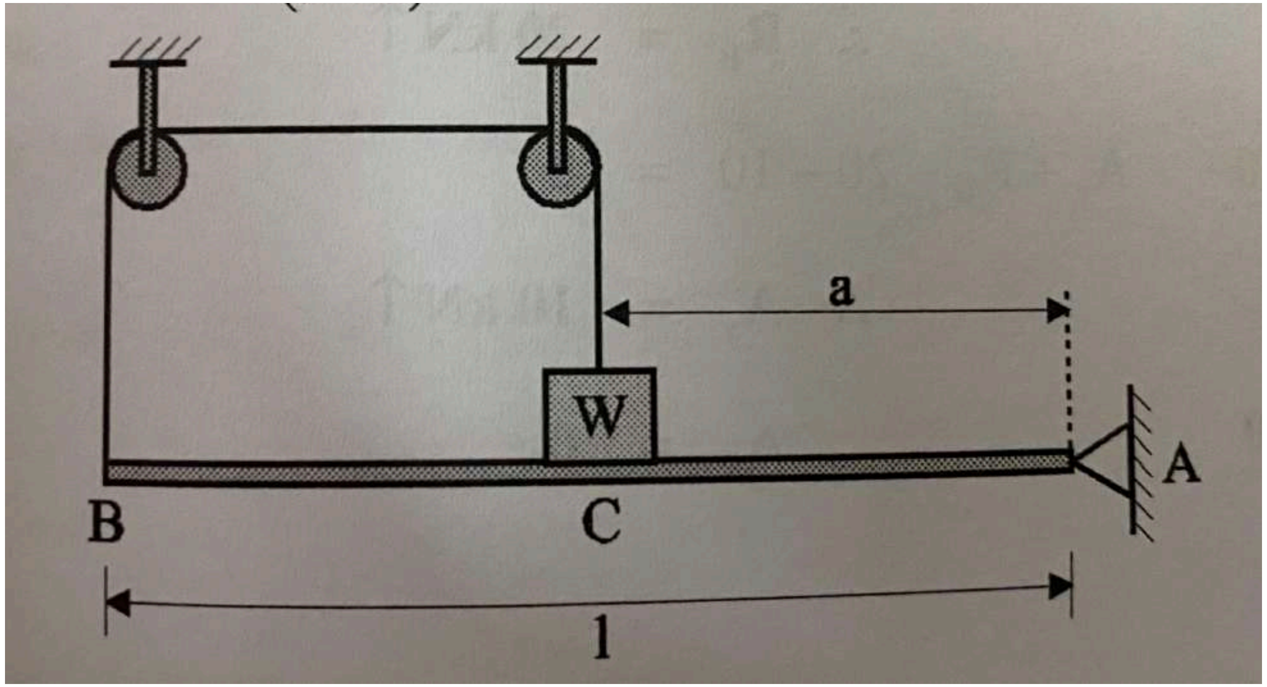
$$\Sigma M_A = 0 = R_B\, 15 - \underbrace{13 \times 6}_{78} - \underbrace{2.1 \times 4}_{8.4} - \underbrace{8 \times 2}_{16} - 10$$

$$\boxed{R_B = 7.49 \text{ kN}}$$

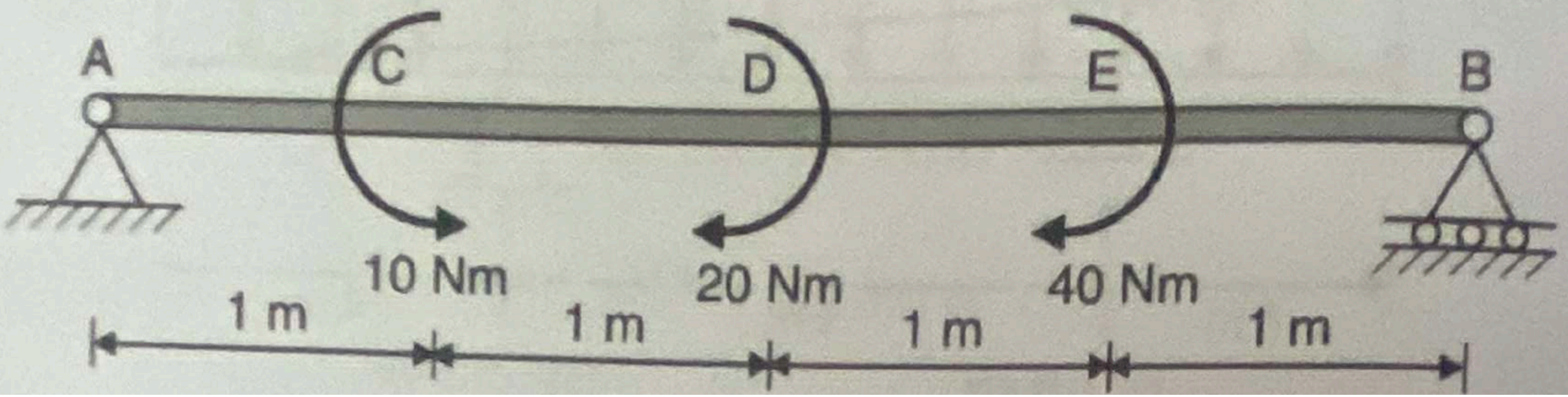
$$V_A = 16.1 - 7.49 = 8.6$$

A weight W rests on beam AB of negligible weight. The string connecting the load is passing over a smooth pulley. Show that reaction at A is $\dfrac{W(l-a)}{(l+a)}$. (Refer Fig. Ex. 6.2.9).

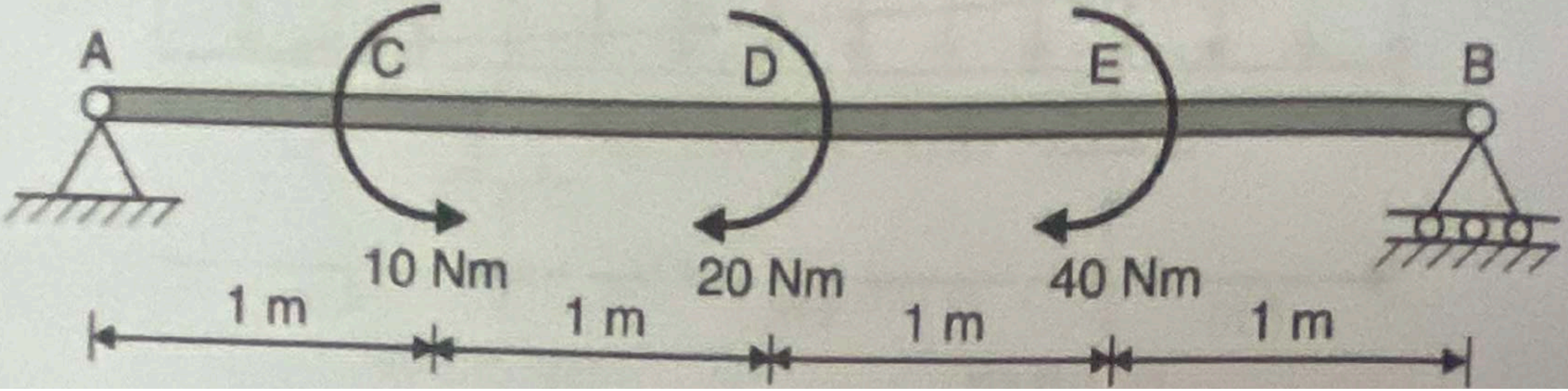



a
W
A
B
C
l

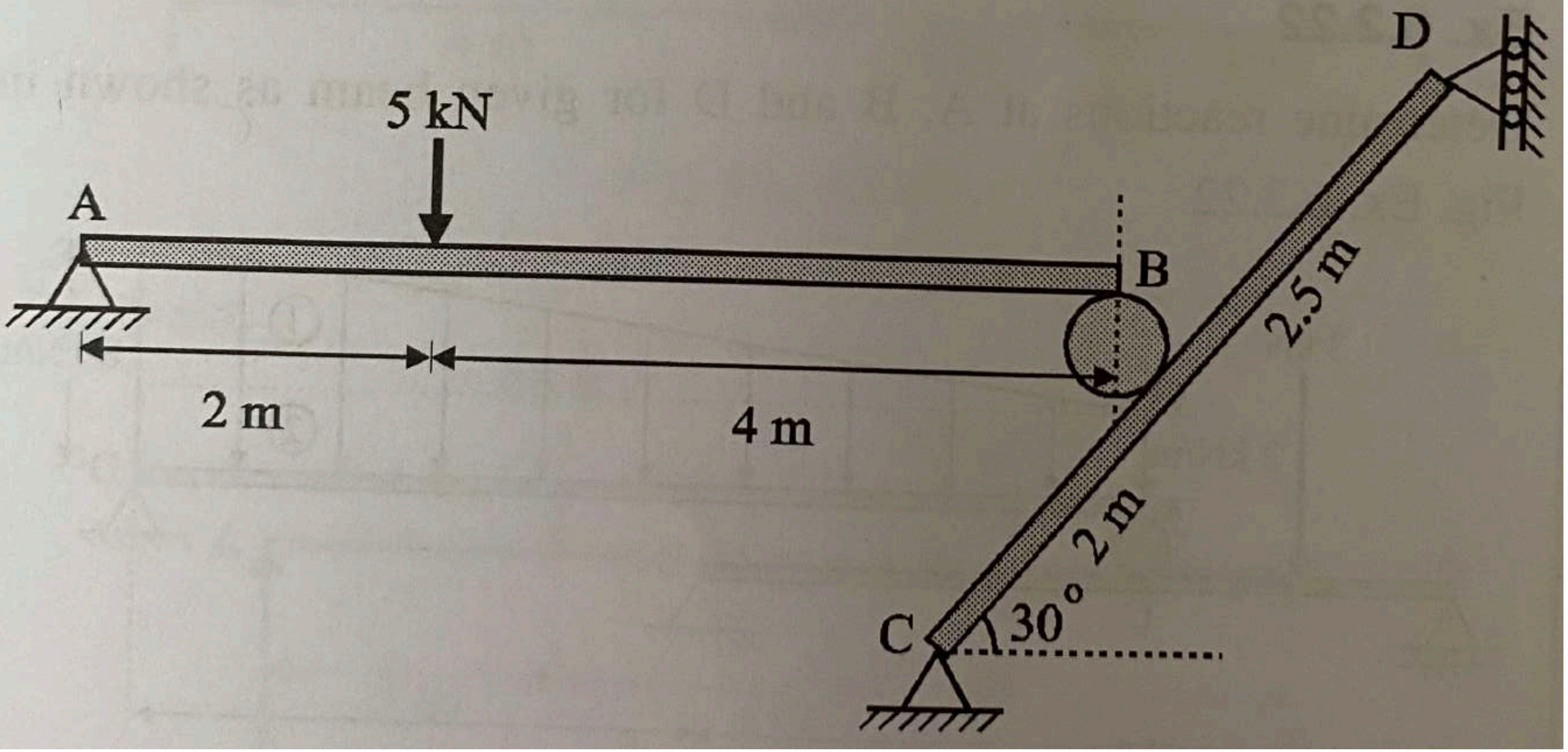
Determine reaction at A and B for the beam loaded and supported as shown in Fig. Ex. 6.2.18. Moments are act at point C, D and E.



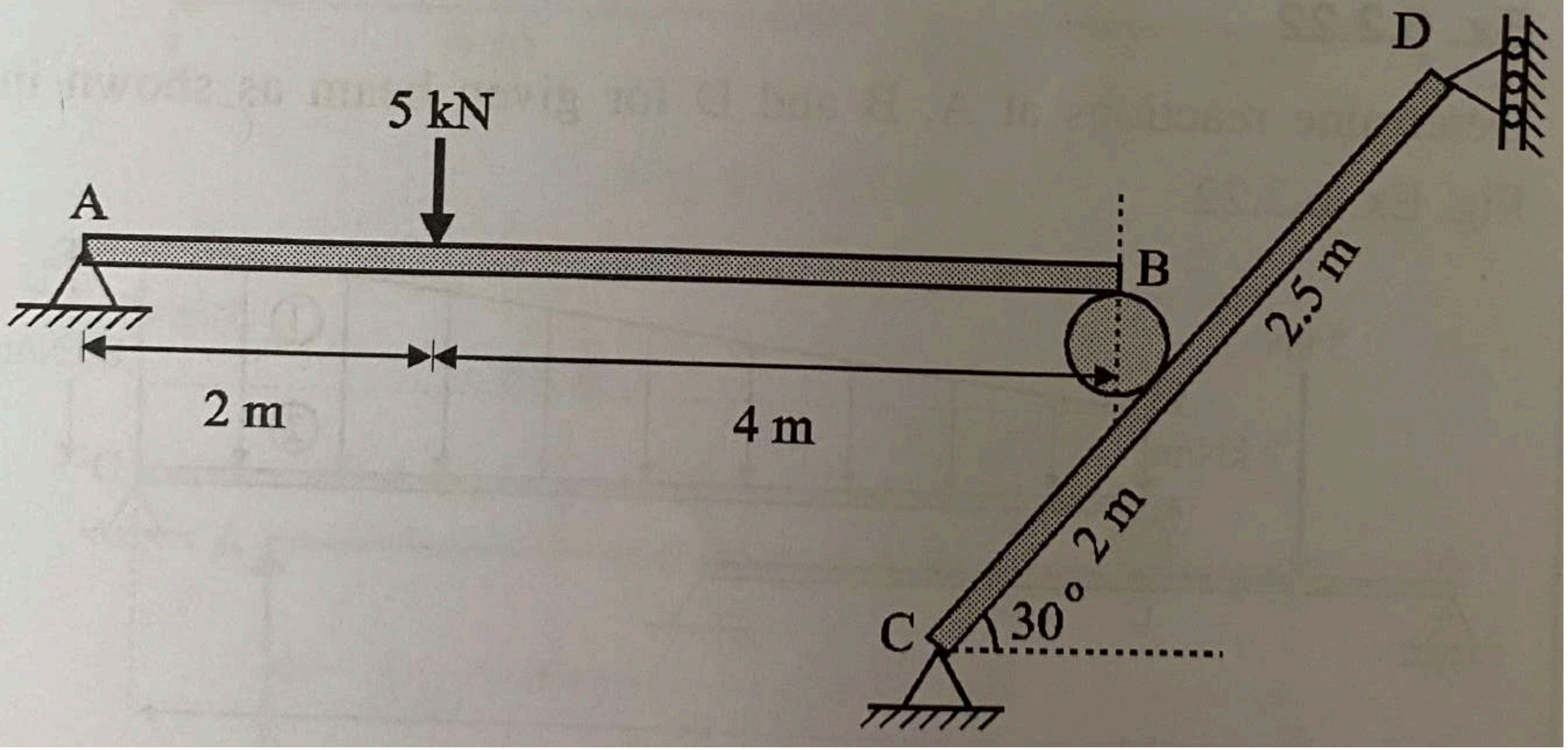
Determine reaction at A and B for the beam loaded and supported as shown in Fig. Ex. 6.2.18. Moments are act at point C, D and E.



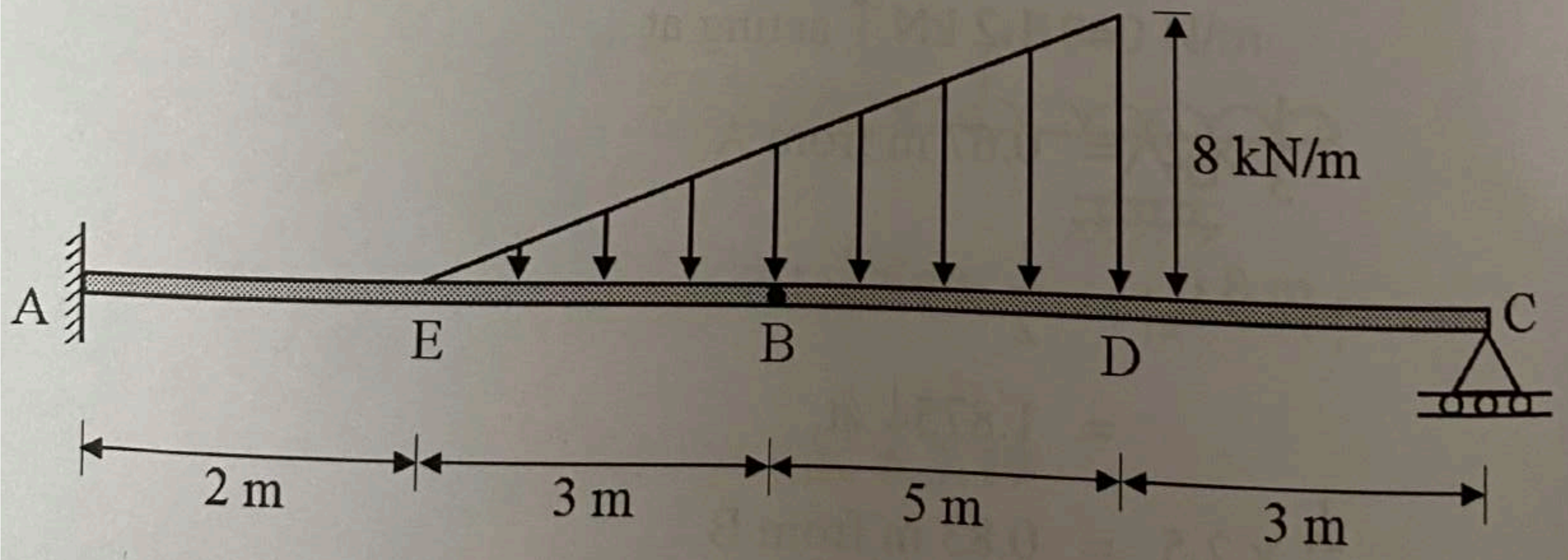
For compound beam determine reactions at supports. Refer Fig. Ex. 6.2.23.



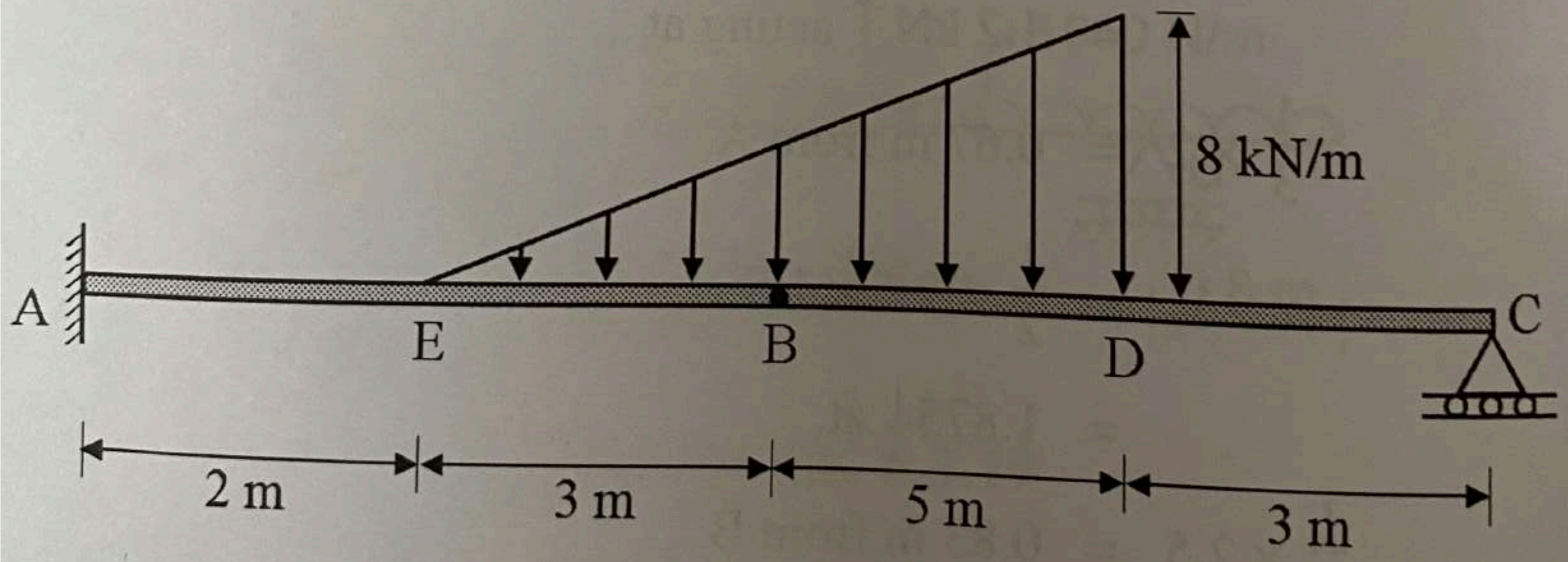
For compound beam determine reactions at supports. Refer Fig. Ex. 6.2.23.



Find reactions at A, B and C for given compound beam as shown in Fig. Ex. 6.2.24.



Find reactions at A, B and C for given compound beam as shown in Fig. Ex. 6.2.24.



Representation of Force In three-dimensional space

Force vector **F** in three-dimensional space is shown in Fig. 2.11 using right hand coordinate system. The force F is written as

$$\mathbf{F} = F^x\mathbf{i} + F^y\mathbf{j} + F^z\mathbf{k} \tag{2.21}$$

where F^x, F^y, F^z are rectangular components of the force F and **i**, **j**, **k** are unit vectors along x, y, z axes respectively.

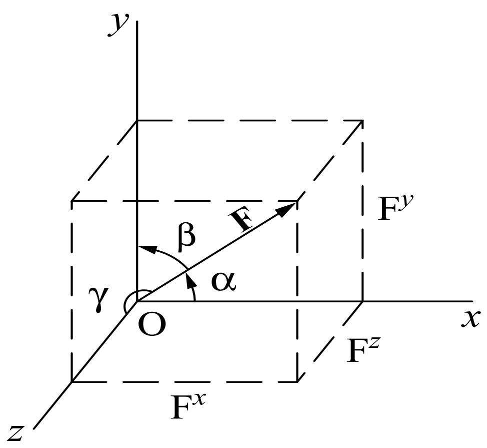


Fig. 2.11 Three-dimensional force.

α is the angle between force vector F and x-axis.

β is the angle between force vector F and y-axis.

γ is the angle between force vector F and z-axis.

$$F^x = F\cos\alpha, \quad \cos\alpha = \frac{x}{L} = l$$

$$F^y = F\cos\beta, \quad \cos\beta = \frac{y}{L} = m$$

$$F^z = F\cos\gamma, \quad \cos\gamma = \frac{z}{L} = n \tag{2.22}$$

$$F = \sqrt{\left(F^x\right)^2 + \left(F^y\right)^2 + \left(F^z\right)^2} \tag{2.23}$$

Fig. 2.11 Three-dimensional force.

$$F = \sqrt{\left(F^x\right)^2 + \left(F^y\right)^2 + \left(F^z\right)^2} \tag{2.23}$$

If **F** is represented by length L to some scale and similarly F^x by x, F^y by y and F^z by z and l, m, n are called direction cosines.

$$l^2 + m^2 + n^2 = 1 \tag{2.24}$$

$$\frac{F^x}{x} = \frac{F^y}{y} = \frac{F^z}{z} = \frac{F}{L} \tag{2.25}$$

$\frac{F}{L}$ is known as force multiplier.

Force is also written as

$$\mathbf{F} = F(l\mathbf{i} + m\mathbf{j} + n\mathbf{k}) \tag{2.26}$$

Let **n** be the unit vector in the direction force

$$\mathbf{n_F} = l\mathbf{i} + m\mathbf{j} + n\mathbf{k} \tag{2.27}$$

$$\mathbf{F} = F\mathbf{n_F} \tag{2.28}$$

Resultant force F given the rectangular components F^x, F^y, F^z:

(i) Let us take F^y and F^z and find the resultant of these components using parallelogram law in *YOZ* plane. (Fig. 2.12)
(ii) Adding F^x component to R vectorially the resultant force F is obtained (Fig. 2.13).

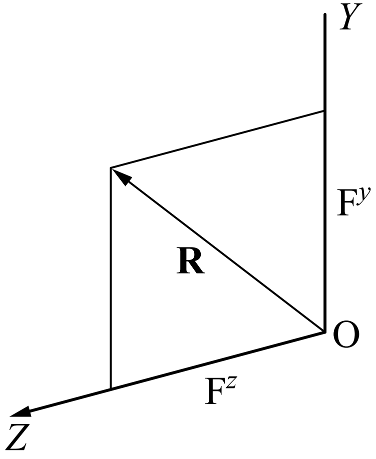


Fig. 2.12 Resultant in *YOZ* plane.

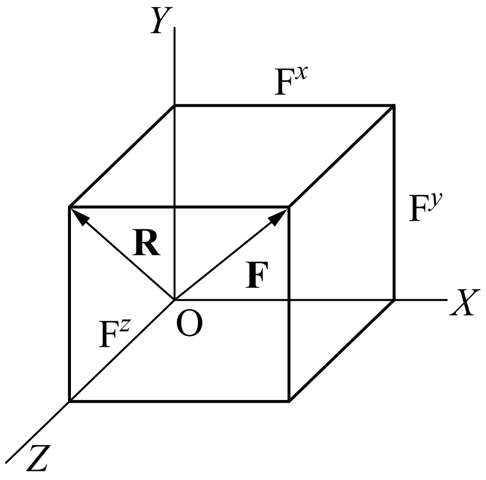


Fig. 2.13 Resultant in three-dimensional space.

2.8. RESULTANT OF COPLANAR CONCURRENT FORCES

$$\mathbf{R} = \mathbf{F}_1 + \mathbf{F}_2 + \mathbf{F}_3 + \mathbf{F}_4 \tag{2.29}$$

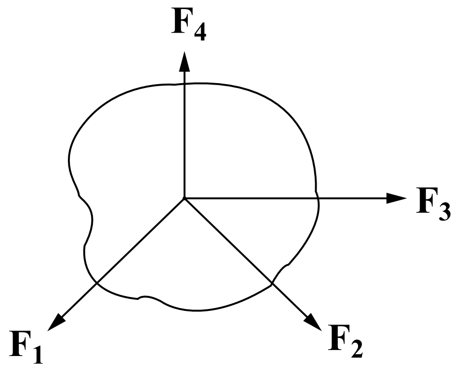


Fig. 2.14 Coplanar concurrent forces.

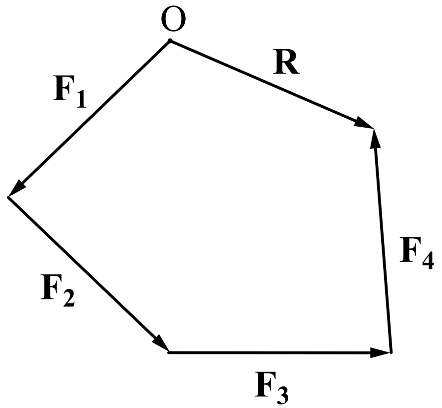


Fig. 2.15 Polygon of forces.

Resultant of forces with be obtained by adding the vectors graphically and the closing side of the polygon will be the resultant R.

2.9. RESULTANT OF NON-CONCURRENT COPLANAR FORCES

In general a system of forces acting in a plane on a body may be non-concurrent. The system of forces acting on the body must be reduced to a simple force system which does not alter the external effect of the original force system on the body. The equivalent of the forces acting on the body is the resultant force F acting at an arbitrary point and a couple. The choice of this arbitrary point will depend upon the convenience for the particular problem under consideration.

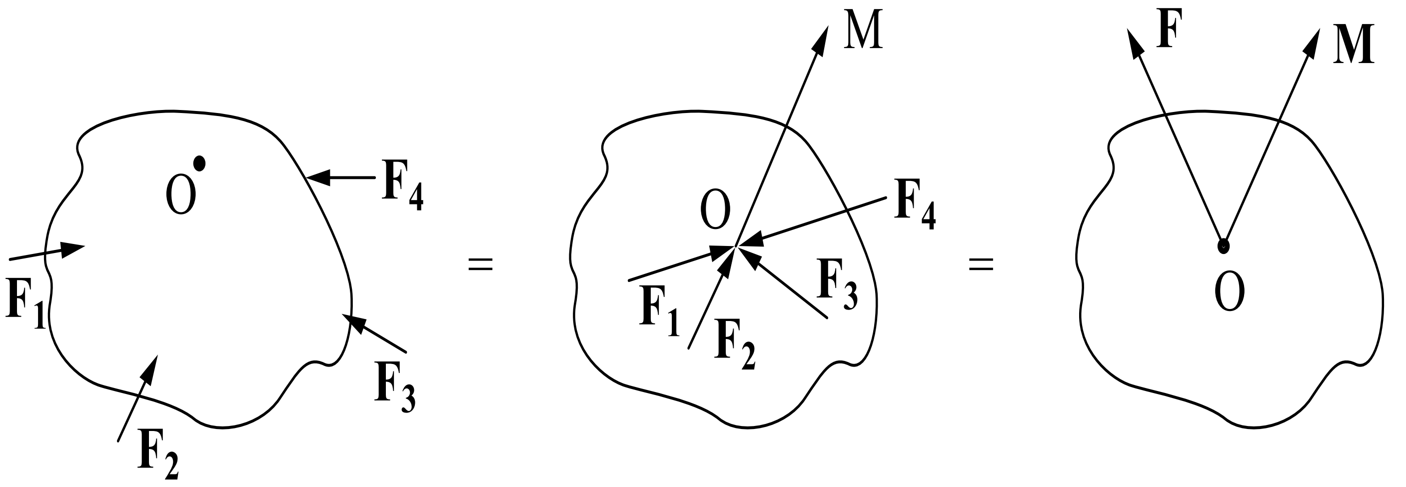
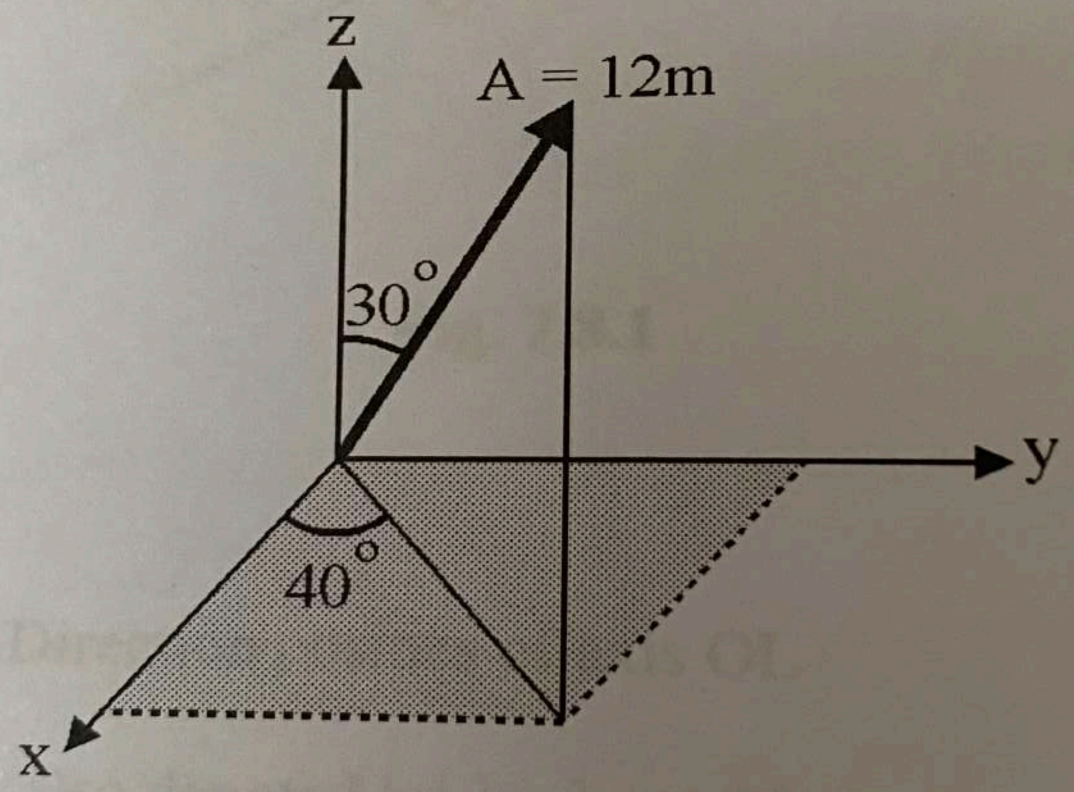


Fig. 2.16 Force system.

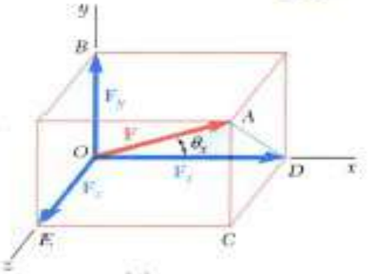
Let $\mathbf{F_1}$, $\mathbf{F_2}$, $\mathbf{F_3}$, $\mathbf{F_4}$ are the forces acting on the body. The system of forces are moved to an arbitrary point O so that it can be reduced to a resultant force **F** and corresponding resultant couple **M**.

Determine rectangular representation of position vector A = 12m. and find angles between A and each of the positive co-ordinates axes. Refer Fig. Ex. 7.10.3.

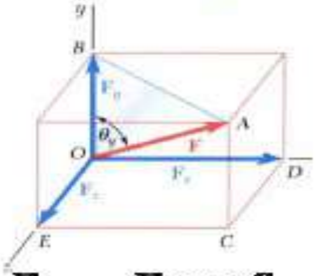


Forces in space Sub-topic-3

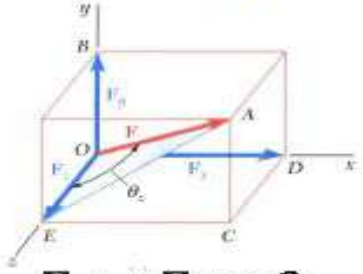
Rectangular Components in Space



$F_x = F\cos\theta_x$



$F_y = F\cos\theta_y$



$F_z = F\cos\theta_z$

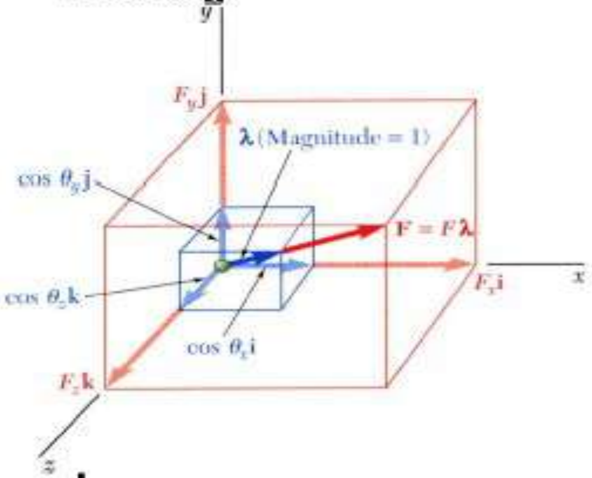
$$\boldsymbol{F} = F_x\boldsymbol{i} + F_y\boldsymbol{j} + F_z\boldsymbol{k}$$

$$\boldsymbol{F} = F\cos\theta_x\boldsymbol{i} + F\cos\theta_y\boldsymbol{j} + F\cos\theta_z\boldsymbol{k}$$

$$\boldsymbol{F} = F(\cos\theta_x\boldsymbol{i} + \cos\theta_y\boldsymbol{j} + \cos\theta_z\boldsymbol{k})$$

$$\boldsymbol{F} = F\boldsymbol{\lambda}$$

Where $\boldsymbol{\lambda} = \cos\theta_x\boldsymbol{i} + \cos\theta_y\boldsymbol{j} + \cos\theta_z\boldsymbol{k}$



$\boldsymbol{\lambda}$ is a unit vector along the line of action of $\boldsymbol{F}$ and $\cos\theta_x$, $\cos\theta_y$ and $\cos\theta_z$ are the direction cosine for $\boldsymbol{F}$

Moment of Force

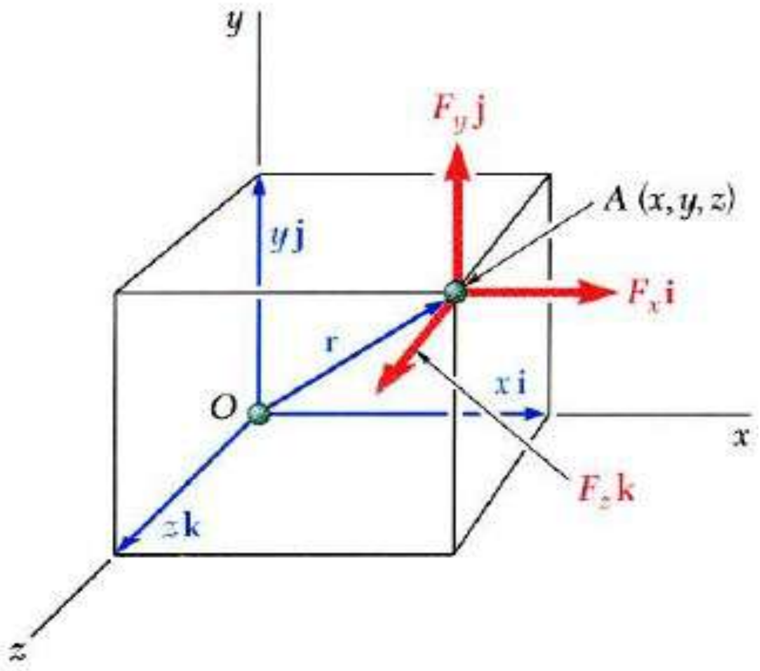
Rectangular Components of a Moment

The moment of $\boldsymbol{F}$ about O,

$$\boldsymbol{M_o} = \boldsymbol{r} \times \boldsymbol{F}$$

$$\boldsymbol{F} = F_x\boldsymbol{i} + F_y\boldsymbol{j} + F_z\boldsymbol{k}$$

$$\boldsymbol{r} = x\boldsymbol{i} + y\boldsymbol{j} + z\boldsymbol{k}$$



$$\boldsymbol{M_o} = M_x\boldsymbol{i} + M_y\boldsymbol{j} + M_z\boldsymbol{k}$$

$$= \begin{vmatrix} \boldsymbol{i} & \boldsymbol{j} & \boldsymbol{k} \\ x & y & z \\ F_x & F_y & F_z \end{vmatrix}$$

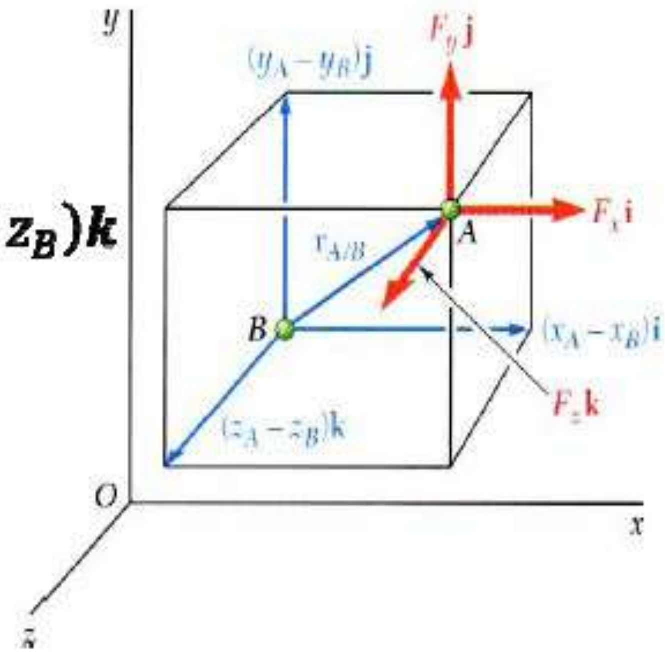
$$= (yF_z - zF_y)\boldsymbol{i} + (zF_x - xF_z)\boldsymbol{j} + (xF_y - yF_x)\boldsymbol{k}$$

The moment of $\boldsymbol{F}$ about B,

$$\boldsymbol{M_B} = \boldsymbol{r_{AB}} \times \boldsymbol{F}$$

$$\boldsymbol{r_{AB}} = (x_A - x_B)\boldsymbol{i} + (y_A - y_B)\boldsymbol{j} + (z_A - z_B)\boldsymbol{k}$$

$$\boldsymbol{F} = F_x\boldsymbol{i} + F_y\boldsymbol{j} + F_z\boldsymbol{k}$$



$$\boldsymbol{M_B} = M_x\boldsymbol{i} + M_y\boldsymbol{j} + M_z\boldsymbol{k}$$

$$= \begin{vmatrix} \boldsymbol{i} & \boldsymbol{j} & \boldsymbol{k} \\ x_A - x_B & y_A - y_B & z_A - z_B \\ F_x & F_y & F_z \end{vmatrix}$$

- Moment of force
- R= F1+F2+F3+……Fn;
- ΣFx=Rx;   ΣFy=Ry;   ΣFz=Rz
- R= ΣFx i+ΣFy j+ΣFz k
- Moment of force about any line AB

$$F = F_x i + F_y j + F_z k$$

$$e_{AB} = e_x i + e_y j + e_z k$$

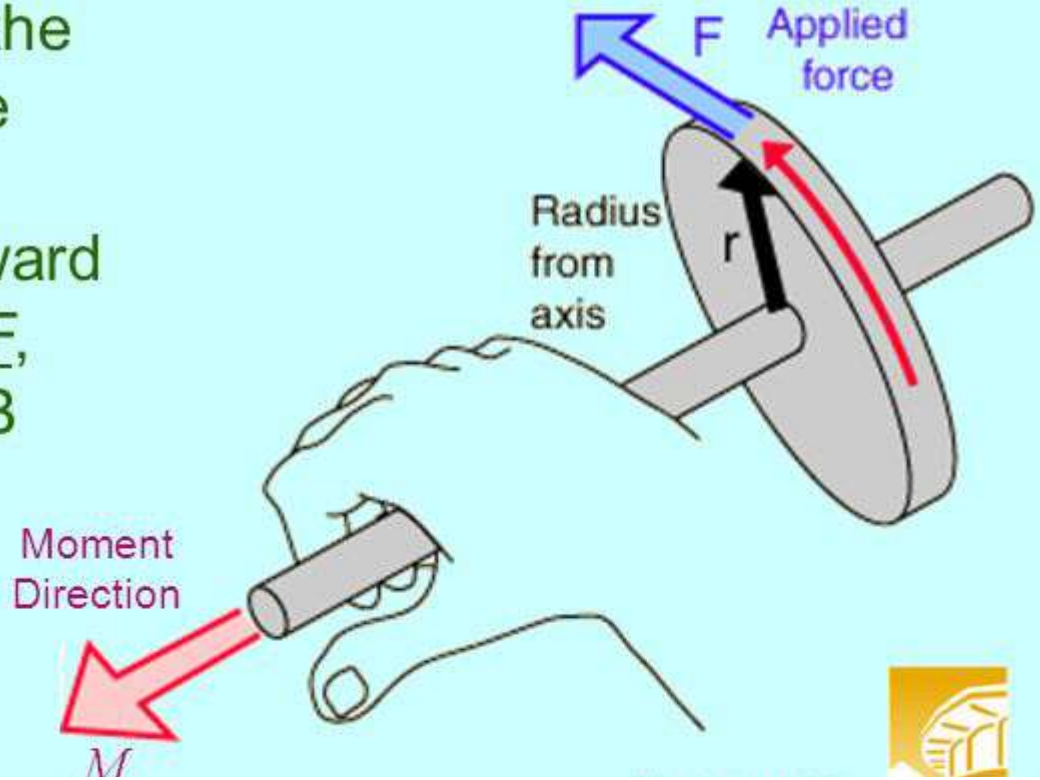
$$r = r_x i + r_y j + r_z k$$

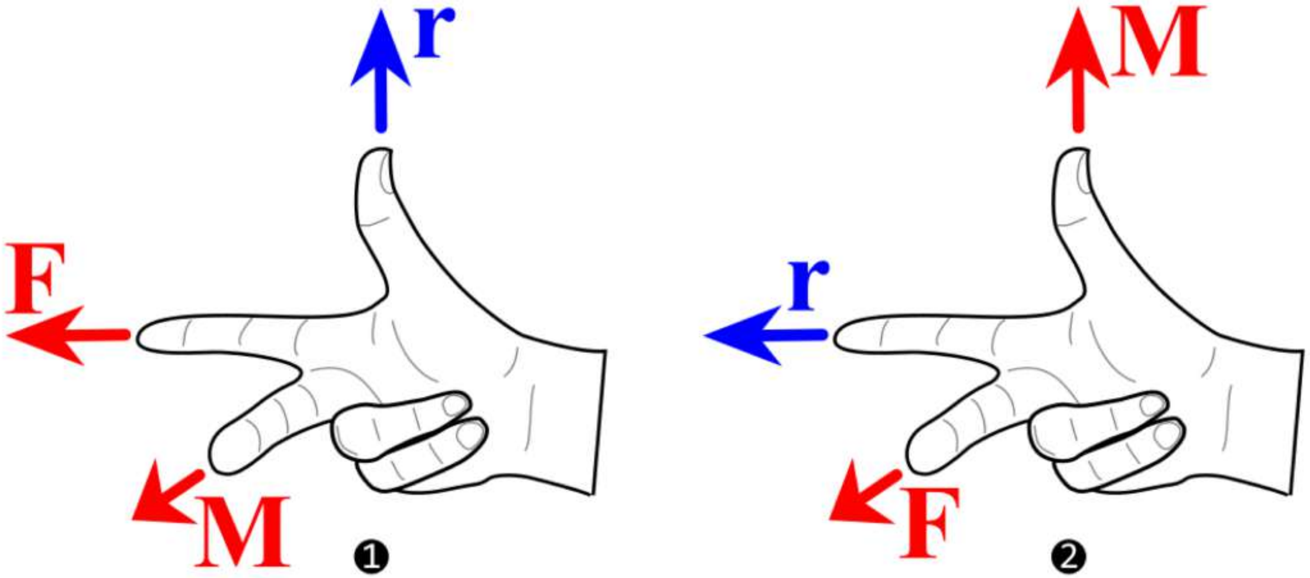
$$M_{AB} = \begin{vmatrix} e_x & e_y & e_z \\ r_x & r_y & r_z \\ f_x & f_y & f_z \end{vmatrix}$$

$\underline{M} = \underline{r} \times \underline{F}$

2. The sense of the moment may be determined by the right-hand rule

- If the fingers of the RIGHT hand are curled from the direction of $\underline{r}$ toward the direction of $\underline{F}$, then the THUMB points in the direction of the Moment




r
F
M
❶
M
r
F
❷

By curling the fingers of right hand about the moment axis in the sense of rotation, the extended thumb represents the direction of moment vector (M_O).

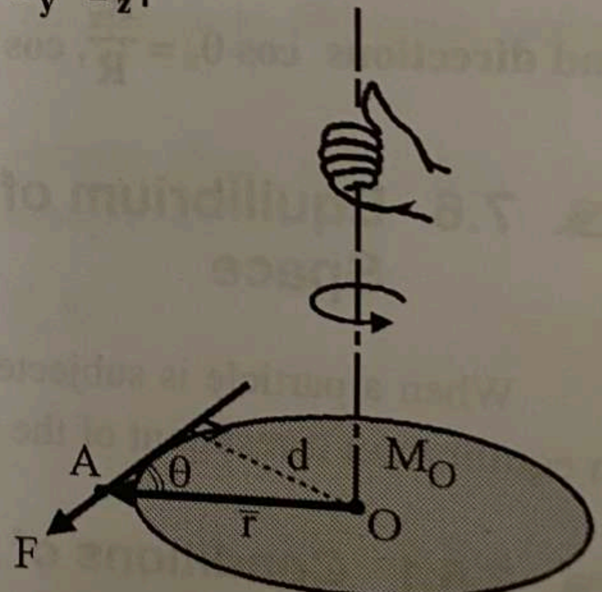
If θ be the angle between lines of action of the position vector r and force F, the magnitude of moment about point O is

$$\mathbf{M_O = rF \cdot \sin\theta.}$$

The moment of a force can be expressed in following form

$$\mathbf{M_O} = \mathbf{r \times F} = \begin{vmatrix} \mathbf{i} & \mathbf{j} & \mathbf{k} \\ \mathbf{r_x} & \mathbf{r_y} & \mathbf{r_z} \\ \mathbf{F_x} & \mathbf{F_y} & \mathbf{F_z} \end{vmatrix} \qquad \text{...(7.7.1)}$$

Where r_x, r_y and r_z are the components of position vector drawn from point O to any point on line of action of the force F_x, F_y, F_z represent components of force.



3.6. MOMENT OF A FORCE ABOUT A POINT

Let us now consider a force **F** acting on a rigid body (Fig. 3.12*a*). As we know, the force **F** is represented by a vector which defines its magnitude and direction. However, the effect of the force on the rigid body depends also upon its point of application A. The position of A can be conveniently defined by the vector **r** which joins the fixed reference point O with A; this vector is known as the *position vector* of A.† The position vector **r** and the force **F** define the plane shown in Fig. 3.12*a*.

We will define the *moment of* **F** *about* O as the vector product of **r** and **F**:

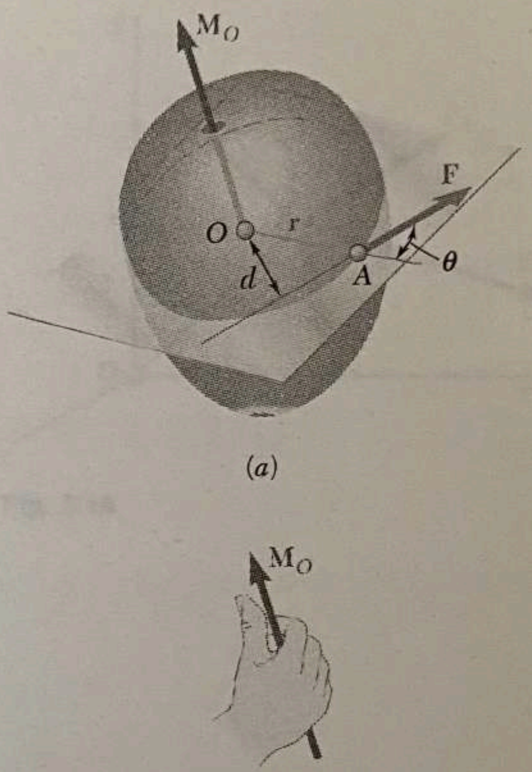
$$\mathbf{M}_O = \mathbf{r} \times \mathbf{F} \tag{3.11}$$

According to the definition of the vector product given in Sec. 3.4, the moment $\mathbf{M}_O$ must be perpendicular to the plane containing O and the force **F**. The sense of $\mathbf{M}_O$ is defined by the sense of the rotation which will bring the vector **r** in line with the vector **F**; this rotation will be observed as *counterclockwise* by an observer located at the tip of $\mathbf{M}_O$. Another way of defining the sense of $\mathbf{M}_O$ is furnished by a variation of the right-hand rule: Close your right hand and hold it so that your fingers are curled in the sense of the rotation that **F** would impart to the rigid body about a fixed axis directed along the line of action of $\mathbf{M}_O$; your thumb will indicate the sense of the moment $\mathbf{M}_O$ (Fig. 3.12*b*).

Finally, denoting by θ the angle between the lines of action of the position vector **r** and the force **F**, we find that the magnitude of the moment of **F** about O is

$$M_O = rF \sin\theta = Fd \tag{3.12}$$

where d represents the perpendicular distance from O to the line of action of **F**. Since the tendency of a force **F** to make a rigid body



(*a*)

(*b*)

Fig. 3.12

3.8. RECTANGULAR COMPONENTS OF THE MOMENT OF A FORCE

In general, the determination of the moment of a force in space will be considerably simplified if the force and the position vector of its point of application are resolved into rectangular x, y, and z components. Consider, for example, the moment $\mathbf{M}_O$ about O of a force $\mathbf{F}$ whose components are F_x, F_y, and F_z and which is applied at a point A of coordinates x, y, and z (Fig. 3.15). Observing that the components of the position vector $\mathbf{r}$ are respectively equal to the coordinates x, y, and z of the point A, we write

$$\mathbf{r} = x\mathbf{i} + y\mathbf{j} + z\mathbf{k} \tag{3.15}$$
$$\mathbf{F} = F_x\mathbf{i} + F_y\mathbf{j} + F_z\mathbf{k} \tag{3.16}$$

Substituting for $\mathbf{r}$ and $\mathbf{F}$ from (3.15) and (3.16) into

$$\mathbf{M}_O = \mathbf{r} \times \mathbf{F} \tag{3.11}$$

and recalling the results obtained in Sec. 3.5, we write the moment $\mathbf{M}_O$ of $\mathbf{F}$ about O in the form

$$\mathbf{M}_O = M_x\mathbf{i} + M_y\mathbf{j} + M_z\mathbf{k} \tag{3.17}$$

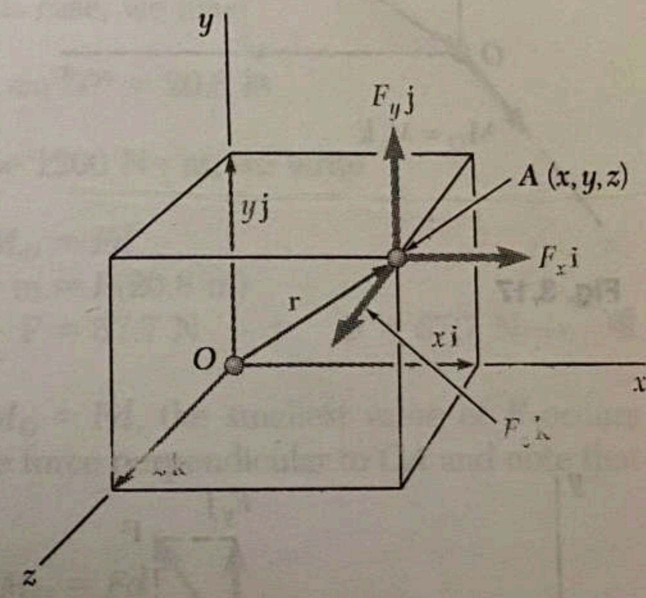


Fig. 3.15

Moment of a force about origin

Consider a force $\bar{F} = F_x\, i + F_y\, j + F_z\, k$ passing through points $P\,(x_1, y_1, z_1)$ and $Q\,(x_2, y_2, z_2)$.

$$\therefore \textbf{Moment about origin } M_O = \begin{vmatrix} i & j & k \\ x_1 & y_1 & z_1 \\ F_x & F_y & F_z \end{vmatrix}$$

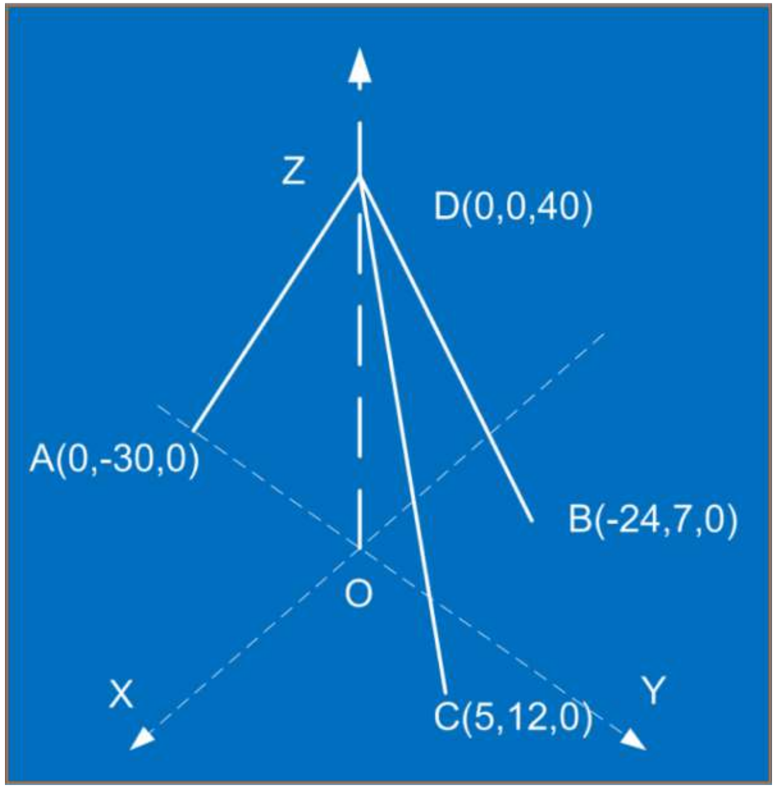
$$\textbf{OR } M_O = \begin{vmatrix} i & j & k \\ x_2 & y_2 & z_2 \\ F_x & F_y & F_z \end{vmatrix}$$

The moment of this force about a point C (x_3, y_3, z_3) is

$$\mathbf{M_C} = \begin{vmatrix} \mathbf{i} & \mathbf{j} & \mathbf{k} \\ (x_1 - x_3) & (y_1 - y_3) & (z_1 - z_3) \\ F_x & F_y & F_z \end{vmatrix}$$

$$\text{OR } \mathbf{M_C} = \begin{vmatrix} \mathbf{i} & \mathbf{j} & \mathbf{k} \\ (x_2 - x_3) & (y_2 - y_3) & (z_2 - z_3) \\ F_x & F_y & F_z \end{vmatrix}$$

- Three cables DA,DB,DC support a vertical mast OD with ball and socket joint at ‘O’ as shown n the figure. Tension in cable DA=100kN.Find the tension in the other cables and the force in the mast



Unknowns are $T_{DB,}$ T_{DC} and force in mast.

T_{DA+} $T_{DB+}$$T_{DC+}$P=0

ΣFx=0

ΣFy=0

ΣFz=0

Solving unknowns can be found out

Unit vector along DA=0i+30j-40k/50

= 0i+0.6j-0.8k

Force along DA=T_{DA}=100(0i+0.6j-0.8k) T_{DA}=0i+60j-80k

Similarly T_{DB}=-22.3872i+6.5296j-37.312k

T_{DC}=22.35i+53.636j-178.788k

R= T_{DA+} T_{DB+} T_{DC}

R=-0.034i+0.166j-296.1k

Parallel Force
System in equilibrium

The direction of the forces can be taken as one of the three coordinate axes.

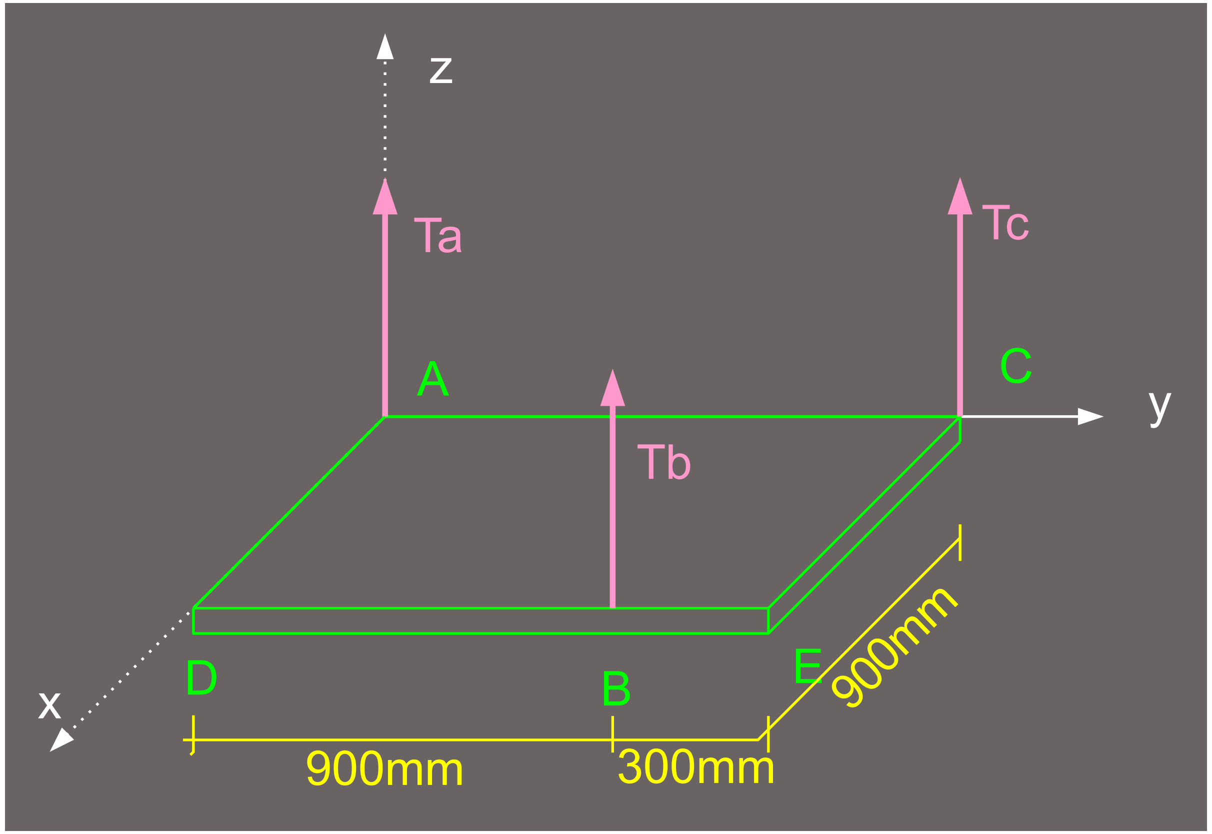
Say z-axis

ΣF has no components in other axes x & y

Also moment around an axis parallel to itself is zero; ΣM has no component along z axis

Hence $\Sigma F = 0$, $\Sigma M = 0$ give three scalar equations $\Sigma F_z = 0$, $\Sigma M_x = 0$, $\Sigma M_y = 0$ to evaluate up to three unknowns.

A homogenous metal plate of 805.205 N weight is held in position by three wires. Determine tensions Ta, Tb, Tc in the wires.



$\Sigma Fz=0$, $\Sigma Mx=0$,$\Sigma My=0$ to evaluate up to three unknowns Ta,Tb,Tc.

$\Sigma My=0$

900xTb=805.205x450

$\Sigma Mx=0$

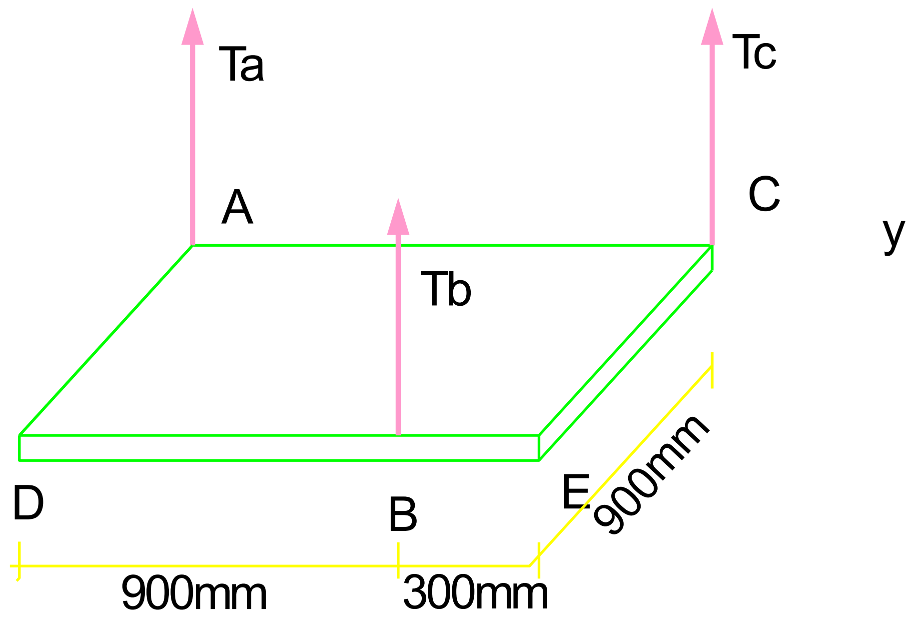
900xTb+1200xTc=805.205x600

$\Sigma Fz=0$

Ta+Tb+Tc=805.206

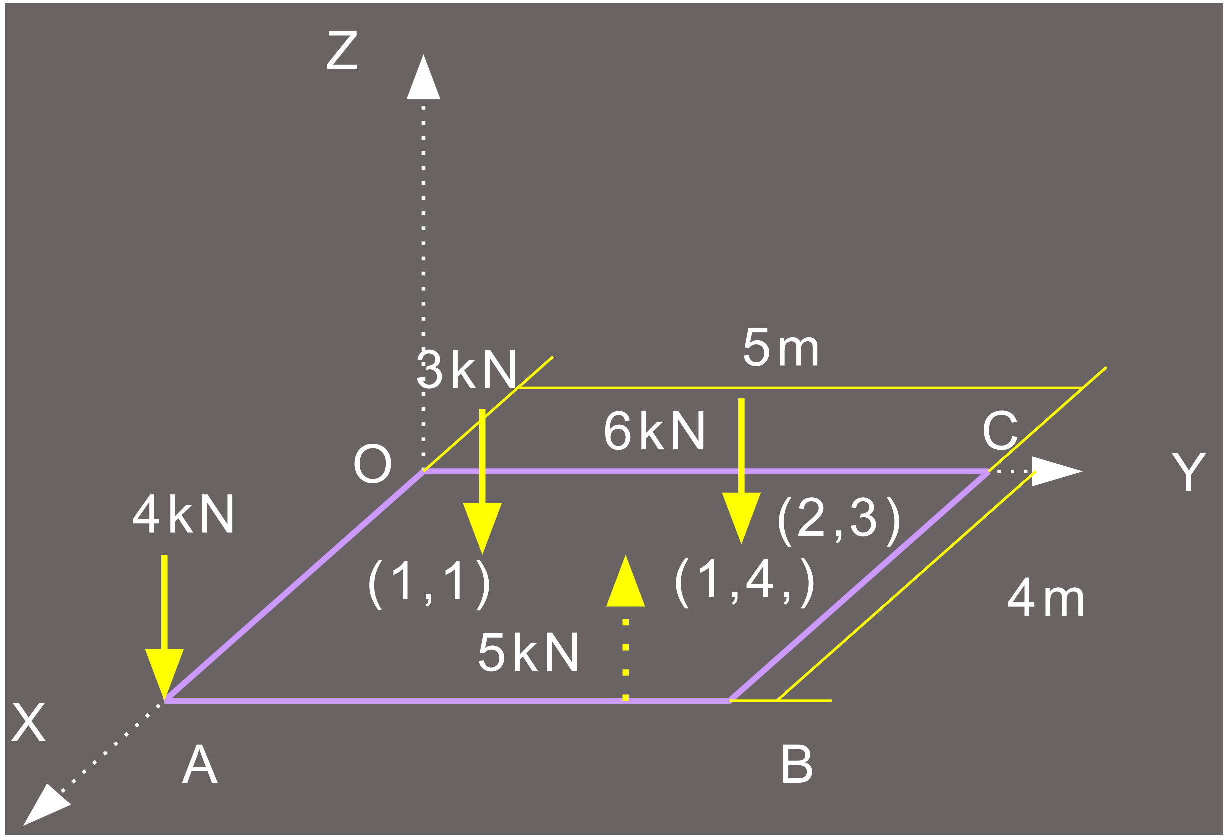
Check: $\Sigma M@DE=0$

(Ta+Tc)x900=805.205x450



Problems

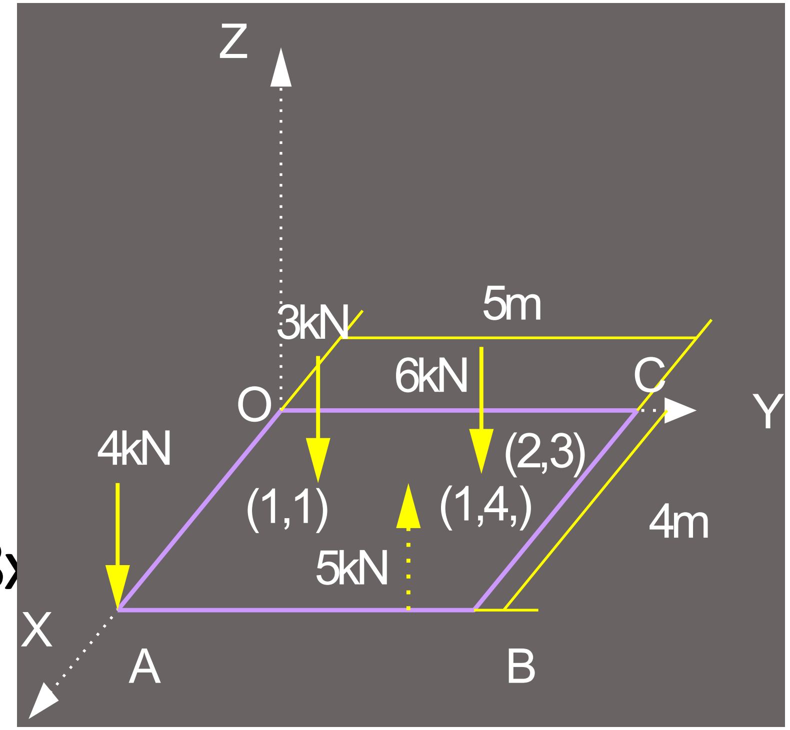
Determine Resultant and its position for given force system.



$R = \Sigma F = 5k - 4k - 3k - 6k = -8k$

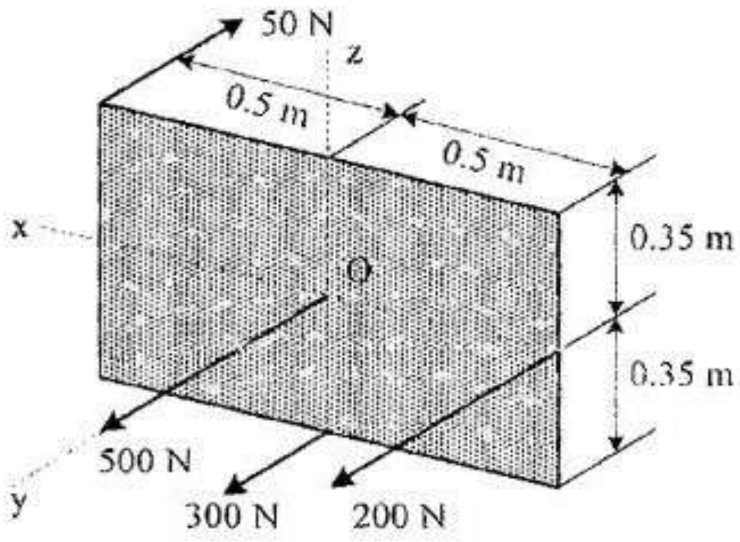
We assume the resultant to act at (x, y)

- $\Sigma Mx = MR$
- $-(3x1)+(5x4)-(6X3) = -(8xy)$
- $Y = 0.125m$
- $\Sigma My = MR$
- $(4x4)+(3X1)-(5X1)+(6X2) = 8x$
- $X = 3.25m$

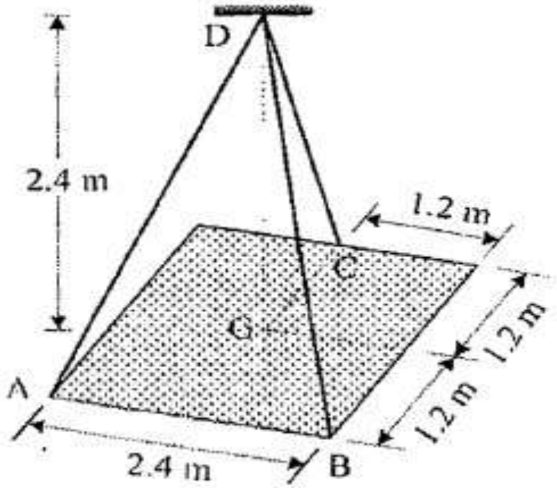


University Question paper problems

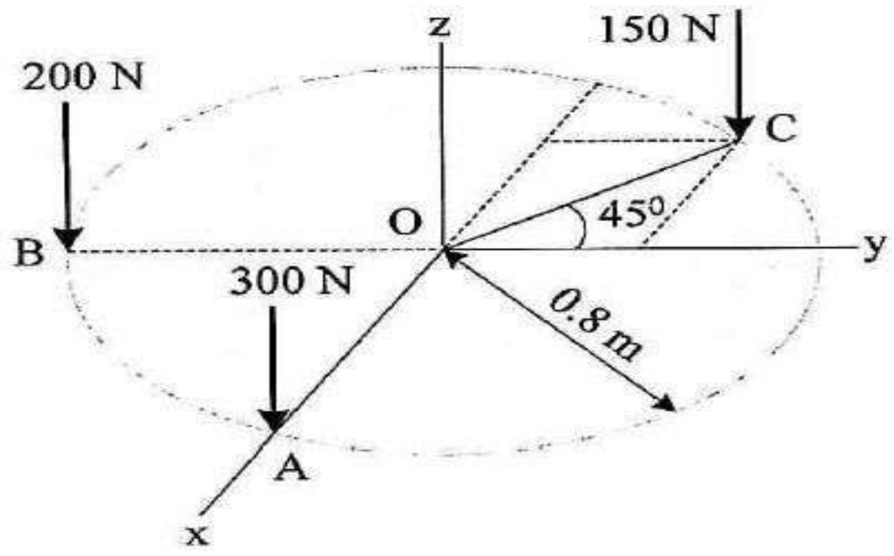
Determine the resultant of the parallel force system which act on the plate as shown in Fig. 3(*a*) : [6]



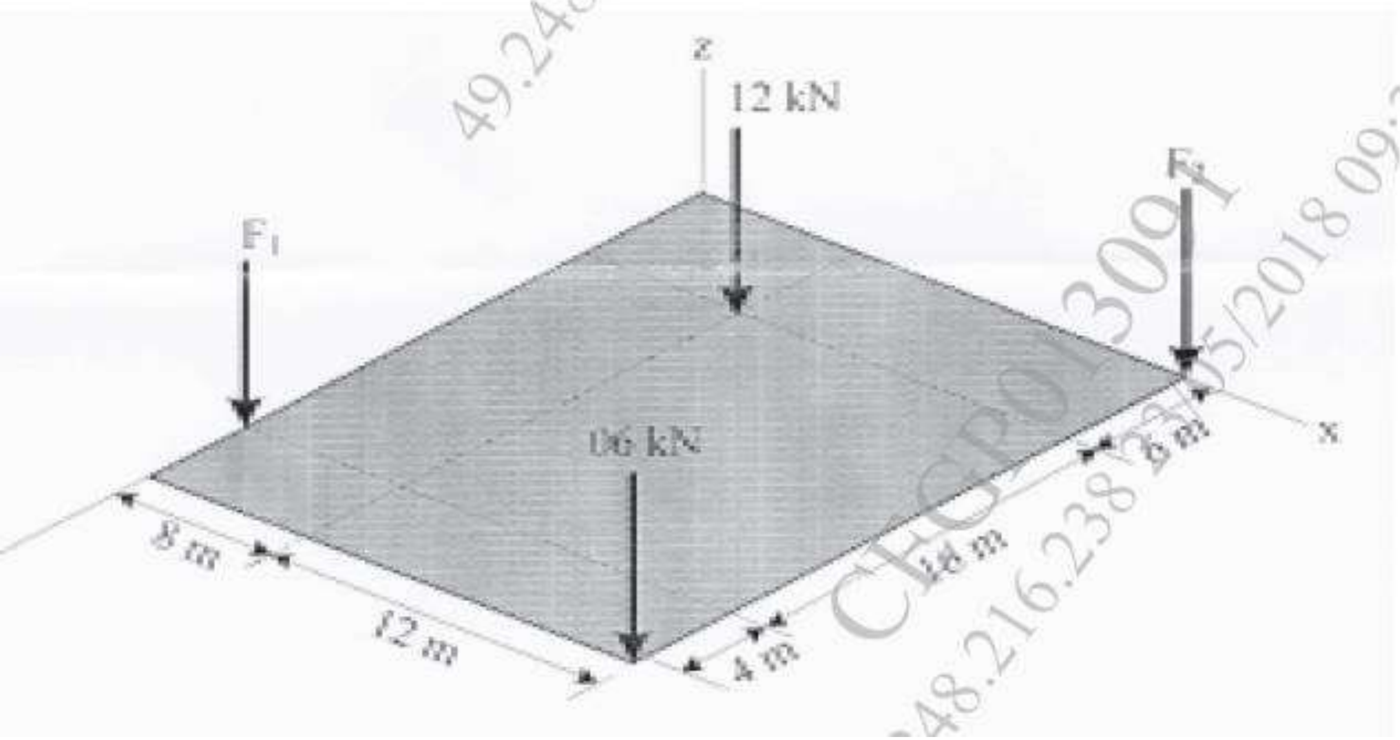
The square steel plate has a mass of 1500 kg with mass center at its center G. Calculate the tension in each of the three cables with which the plate is lifted while remaining horizontal. Refer Fig. 4 (a). [6]



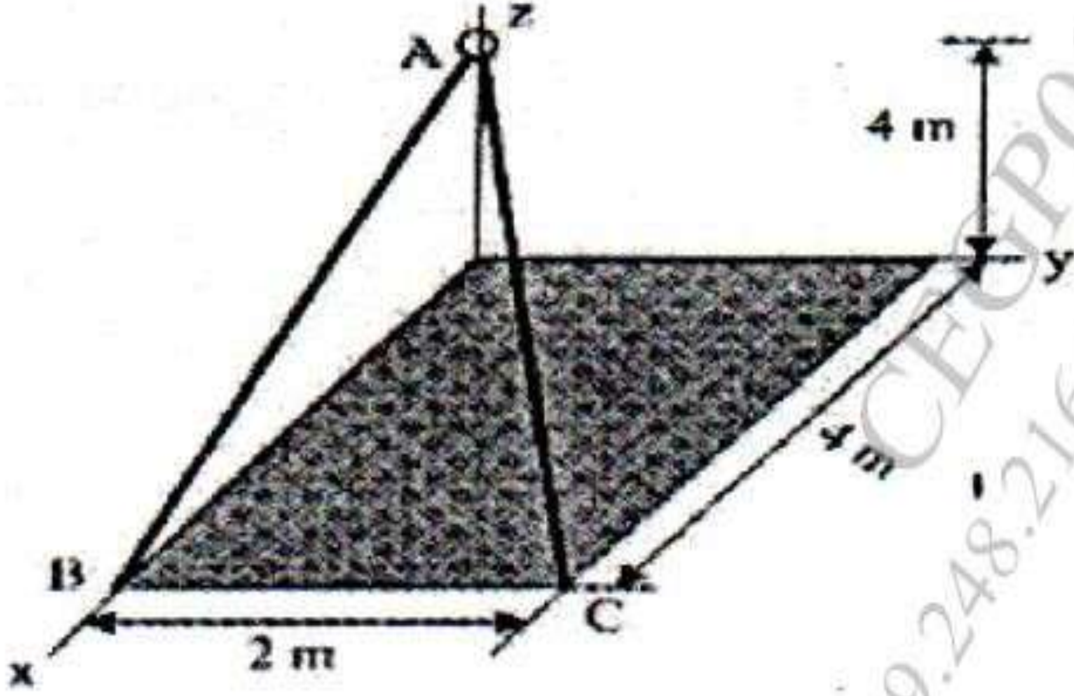
Three parallel bolting forces act on the rim of the circular cover plate as shown in Fig. 3 (*b*). Determine the magnitude, nature and point of application of the resultant force with respect to origin O. [6]



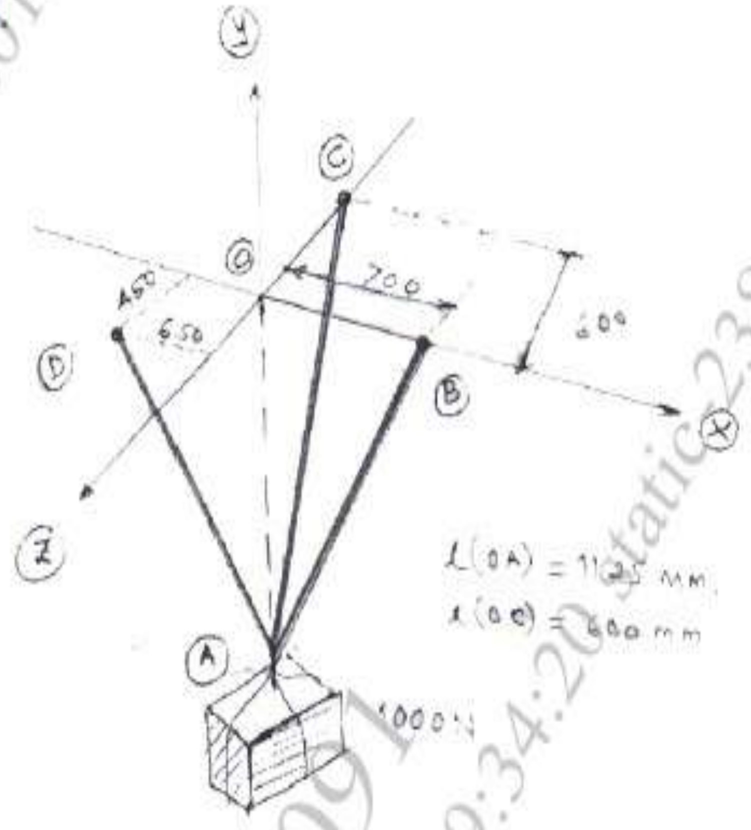
The building slab is subjected to four parallel column loading shown in Figure. Determine F1 and F2 if the resultant force acts through point (12m, 10m). **[6]**



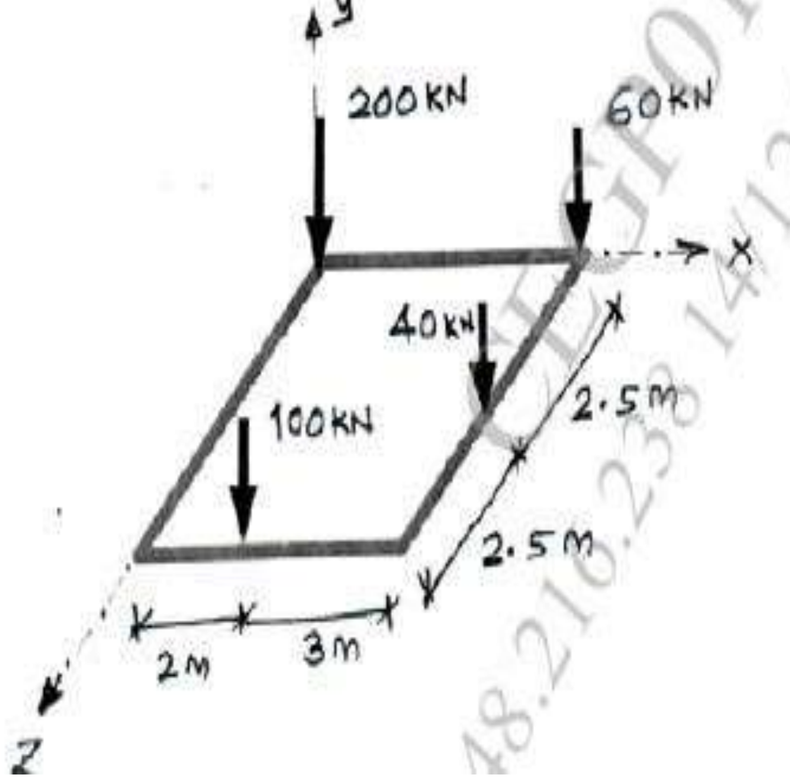
The cables exert forces $F_{AB} = 100$ N and $F_{AC} = 120$ N on the ring at A as shown in Fig. Determine the magnitude of the resultant force acting at A.[6]



Three cables are used to support a container as shown in the **Fig. 5b.** Determine the tension in the cables AB, AC and AD if the weight of the container is 1000N. **[7]**

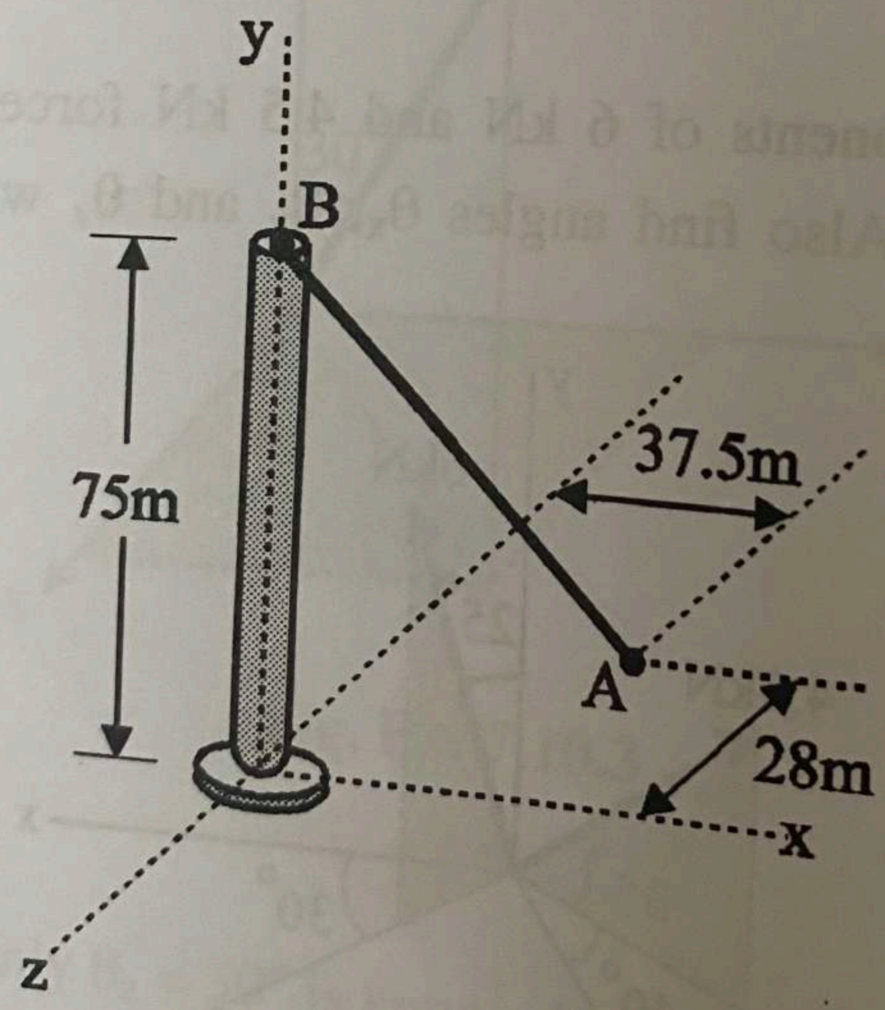


The square mat foundations supports four columns as shown in the **Fig.6b.** Determine the magnitude and position of the resultant force w.r. to origin 'O'. [7]



A wire is connected by a bolt at A. If tension in wire is 3 kN, determine (a) Components of force acting at A.

Refer Fig. Ex. 7.10.7. The angles θ_x, θ_y and θ_z.



[To find position vector r from A to B follow the path starting from A, ends at B by going along each axis.
(Here A → P → O → B)]

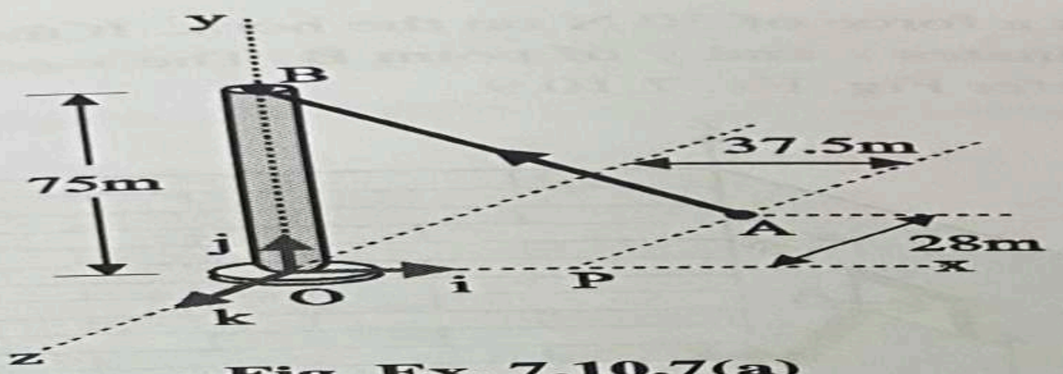


Fig. Ex. 7.10.7(a)

$$\therefore \quad \bar{T}_{AB} = T_{AB} \cdot \bar{e}_{AB}$$

$$= 3\left[\frac{28\,k - 37.5\,i + 75\,j}{\sqrt{(28)^2 + (-37.5)^2 + (75)^2}}\right]$$

$$= \frac{3}{88.403}\,[-37.5\,i + 75\,j + 28\,k]$$

$$\bar{T}_{AB} = -1.272\,i + 2.545\,j + 0.95\,k$$

$\therefore$ x component $F_x = -1.272$ kN ...A

y component $F_y = 2.545$ kN ...A

z component $F_z = 0.95$ kN ...A

The angles, cos

$$\theta_x = \frac{F_x}{F} = \frac{-1.272}{3}$$

$$\therefore \quad \theta_x = 115.087^\circ$$

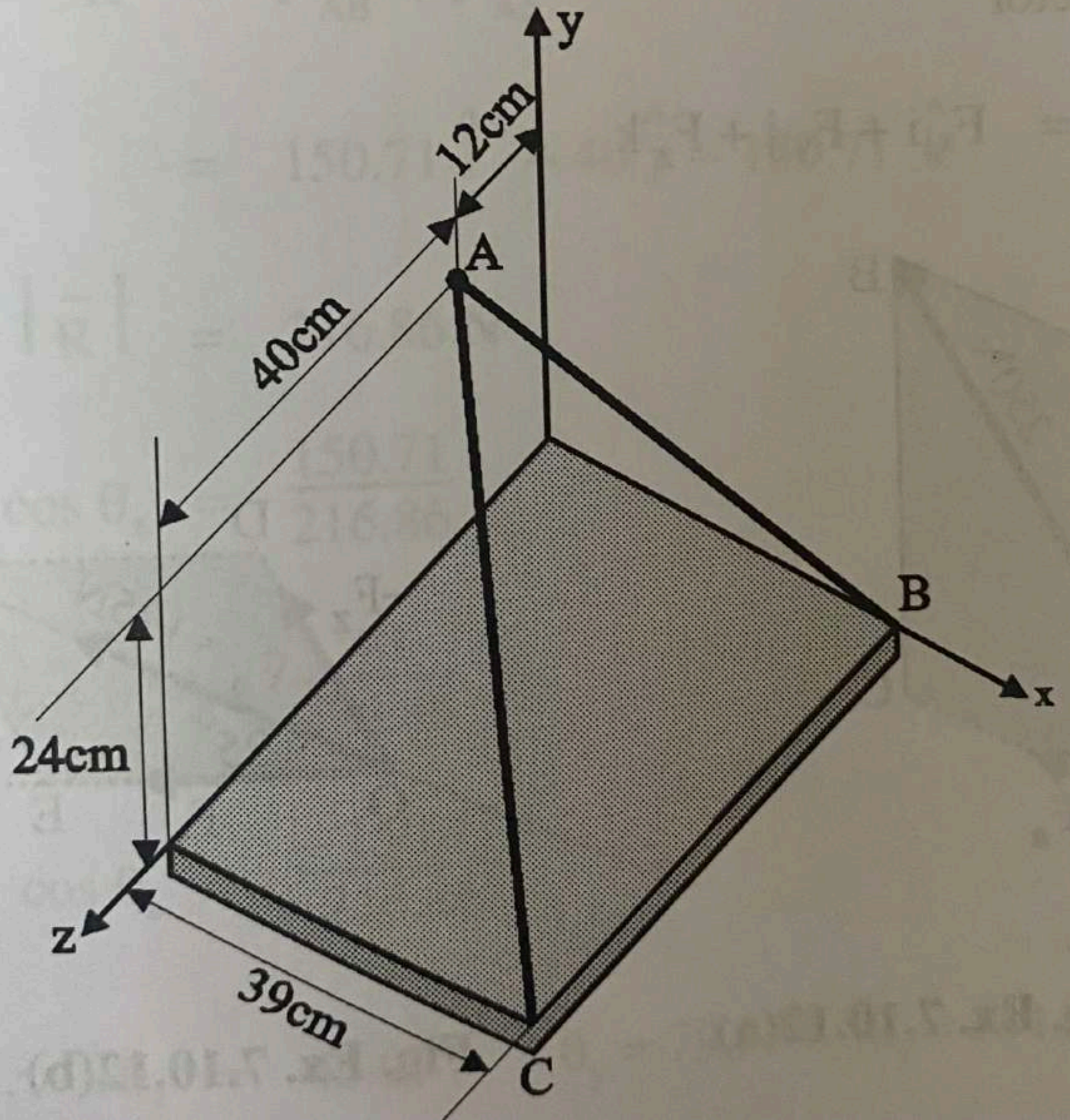
$$\cos\theta_y = \frac{F_y}{F} = \frac{2.545}{3}$$

$$\therefore \quad \theta_y = 31.97^\circ$$

$$\cos\theta_z = \frac{F_z}{F} = \frac{0.95}{3}$$

$$\therefore \quad \theta_z = 71.54^\circ$$

If tension in cable AB is 275 N and in cable AC is 450 N, determine magnitude and direction of resultant of forces acting at A by two cables. Refer Fig. Ex. 7.10.10.



Here, Resultant of two forces acting at A is required, so forces are acting from A to B and A to C as shown in Fig. Ex. 7.10.10(a).

Force vectors

1. $$\bar{T}_{AB} = T_{AB} \cdot \bar{e}_{AB}$$

$$= 275 \left[\frac{-12\,k - 24\,j + 39\,i}{\sqrt{(-12)^2 + (-24)^2 + (39)^2}} \right]$$

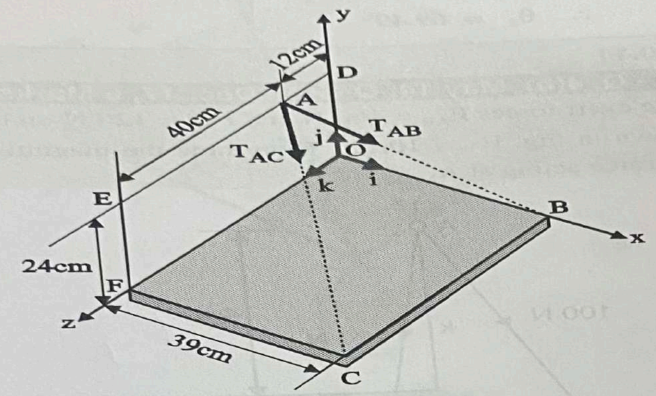


Fig. Ex. 7.10.10(a)

Follow the path A → D → O → B to reach from A to B

$$A \rightarrow D = -12\,k, \quad D \rightarrow O = -24\,j, \quad O \rightarrow B = +39\,i$$

arranging, $$\bar{T}_{AB} = \frac{275}{47.34}\,[39\,i - 24\,j - 12\,k]$$

$$\bar{T}_{AB} = 226.55\,i - 139.42\,j - 69.71\,k$$

2. $$\bar{T}_{AC} = T_{AC} \cdot \bar{e}_{AC}$$

$$= 450 \left[\frac{40\,k - 24\,j + 39\,i}{\sqrt{(40)^2 + (-24)^2 + (39)^2}} \right]$$

Follow the path A → E → F → C to reach from A to C

$$A \rightarrow E = 40\,k, \quad E \rightarrow F = -24\,j, \quad F \rightarrow C = +39\,i$$

Arranging, $$\bar{T}_{AC} = \frac{450}{60.8}\,[39\,i - 24\,j + 40\,k]$$

$$\bar{T}_{AC} = 288.65\,i - 177.63\,j + 296.05\,k$$

Resultant $$\bar{R} = \bar{T}_{AB} + \bar{T}_{AC}$$

$$= [(226.55 + 288.65)\,i + (-139.42 - 177.63)\,j + (-69.71 + 296.05)\,k\,]$$

$$\bar{R} = (515.2)\,i + (-317.05)\,j + (226.34)\,k$$

∴ Magnitude of resultant

$$R = \sqrt{(R_x)^2 + (R_y)^2 + (R_z)^2}$$

$$= \sqrt{(515.2)^2 + (317.05)^2 + (226.34)^2}$$

$$R = 645.89\ N$$

...

1. $\bar{T}_{AB} = T_{AB} \cdot \bar{e}_{AB}$

$$= 275\left[\frac{-12\,k - 24\,j + 39\,i}{\sqrt{(-12)^2 + (-24)^2 + (39)^2}}\right]$$

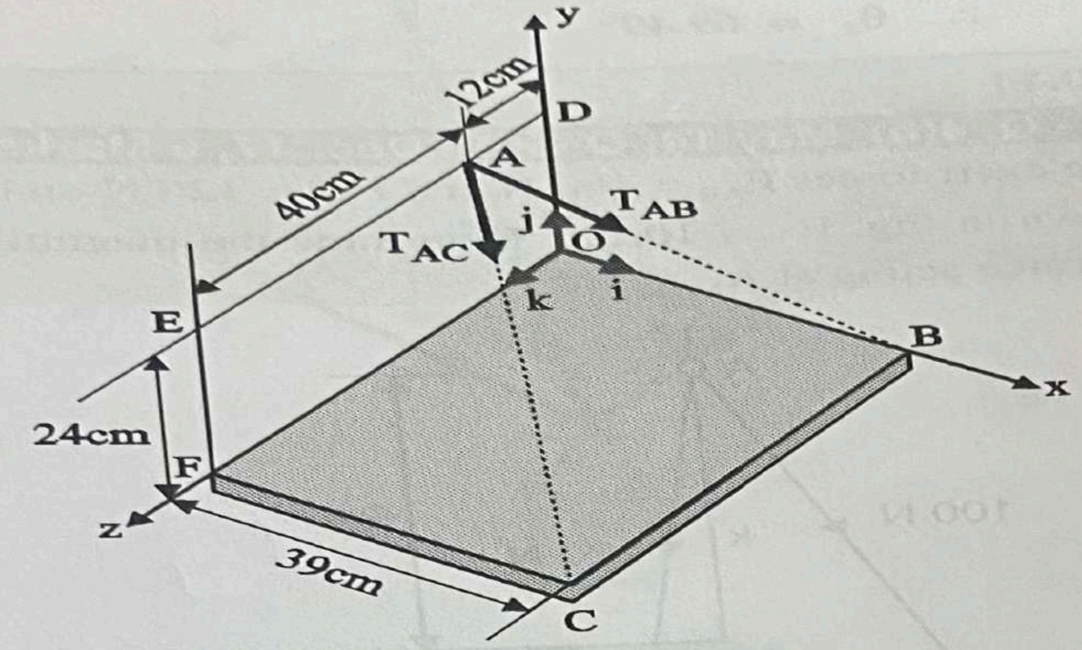


Fig. Ex. 7.10.10(a)

Follow the path A → D → O → B to reach from A to B

A → D = – 12 k, D → O = – 24 j, O → B = + 39 i

arranging, $\bar{T}_{AB} = \dfrac{275}{47.34}\,[39\,i - 24\,j - 12\,k]$

$$\bar{\mathbf{T}}_{\mathbf{AB}} = \mathbf{226.55\,i - 139.42\,j - 69.71\,k}$$

2. $\bar{T}_{AC} = T_{AC} \cdot \bar{e}_{AC}$

$$= 450\left[\frac{40\,k - 24\,j + 39\,i}{\sqrt{(40)^2 + (-24)^2 + (39)^2}}\right]$$

at

Follow the path A → E → F → C to reach from A to C

$$A \rightarrow E = 40\ k,\quad E \rightarrow F = -24\ j,\quad F \rightarrow C = +39\ i$$

Arranging,

$$\bar{T}_{AC} = \frac{450}{60.8}\,[39\ i - 24\ j + 40\ k]$$

$$\bar{T}_{AC} = 288.65\ i - 177.63\ j + 296.05\ k$$

Resultant

$$\bar{R} = \bar{T}_{AB} + \bar{T}_{AC}$$

$$= [(226.55 + 288.65)\ i + (-139.42 - 177.63)\ j + (-69.71 + 296.05)\ k\ ]$$

$$\bar{R} = (515.2)\ i + (-317.05)\ j + (226.34)\ k$$

∴ Magnitude of resultant

$$R = \sqrt{(R_x)^2 + (R_y)^2 + (R_z)^2}$$

$$= \sqrt{(515.2)^2 + (317.05)^2 + (226.34)^2}$$

...

$$R = 645.89\ N$$

$$\text{Directions,} \quad \cos\theta_x = \frac{R_x}{R} = \frac{515.2}{645.89}$$

$$\therefore \quad \theta_x = \mathbf{37.09^\circ}$$

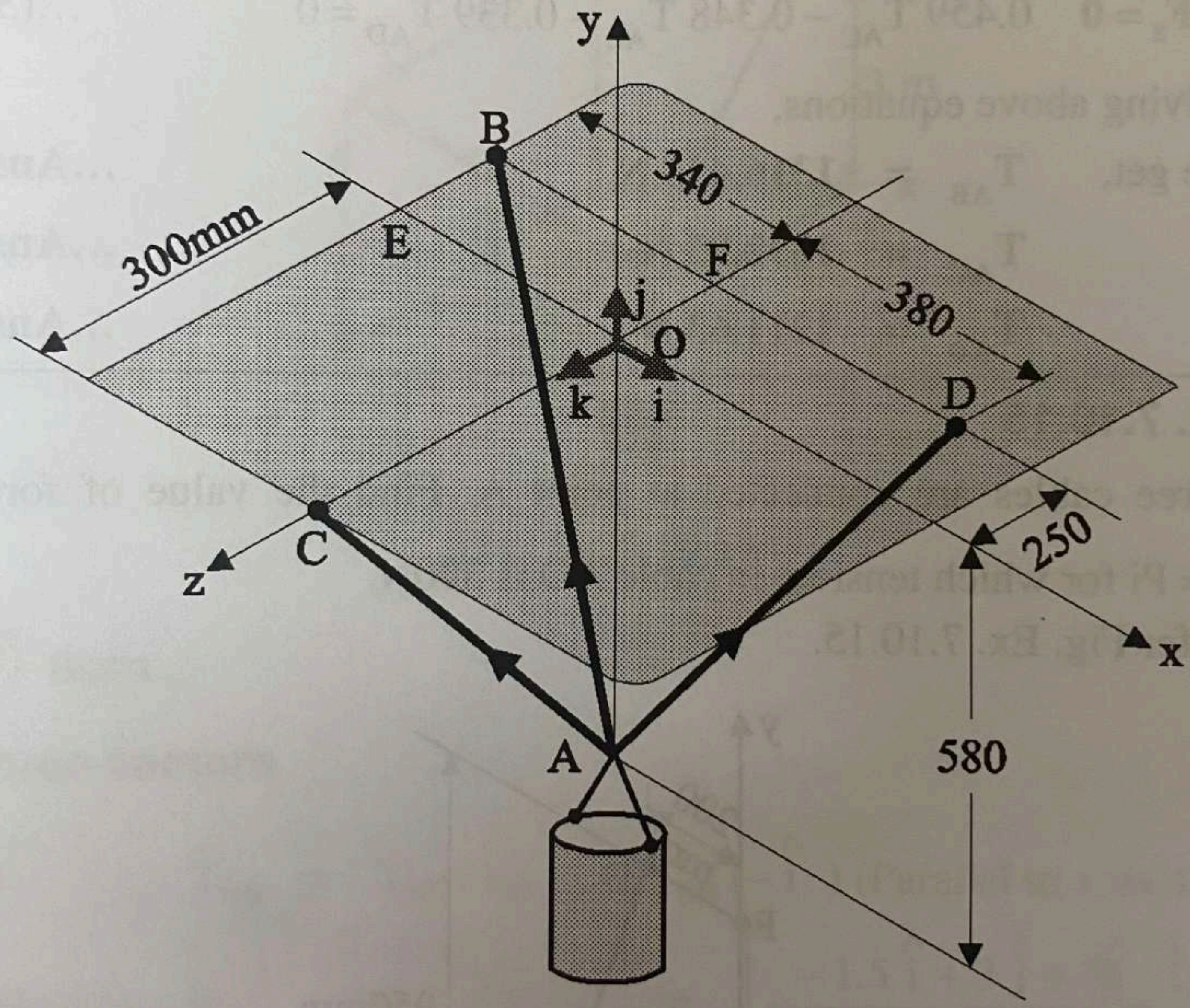
$$\cos\theta_y = \frac{R_y}{R} = \frac{-317.05}{645.89}$$

$$\therefore \quad \theta_y = \mathbf{119.4^\circ}$$

$$\cos\theta_z = \frac{R_z}{R} = \frac{226.34}{645.89}$$

$$\therefore \quad \theta_z = \mathbf{69.49^\circ}$$

Determine the tensions in cables AB, AC and AD. If weight of cylinder is 160 kg. Refer Fig. Ex. 7.10.14.



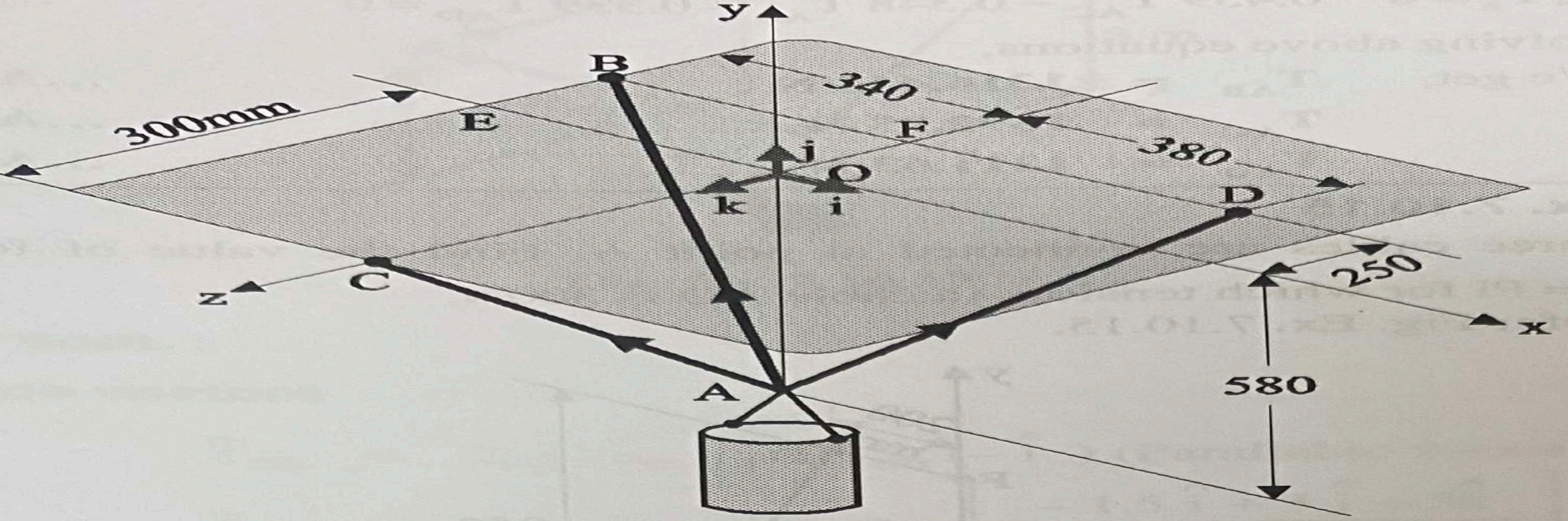


Fig. Ex. 7.10.14

☑ **Soln. :**

The concurrent forces acting at A are, $\bar{T}_{AC}$, $\bar{T}_{AB}$, $\bar{T}_{AD}$ and $\overline{W}$

Now force vectors are

1. $$\bar{T}_{AC} = T_{AC} \cdot \left[\frac{580\,j + 300\,k}{\sqrt{(580)^2 + (300)^2}}\right]$$

Follow the path A → O → C to reach from A to C

A → O = 580 j, O → C = 300 k

$$\bar{T}_{AC} = T_{AC}\,[0.888\,j + 0.459\,k)$$

2. $$\bar{T}_{AB} = T_{AB} \cdot \bar{e}_{AB}$$

$$= T_{AB}\left[\frac{580\,j - 340\,i - 250\,k}{\sqrt{(580)^2 + (-340)^2 + (-250)^2}}\right]$$

2. $\bar{T}_{AB} = T_{AB} \cdot \bar{e}_{AB}$

$$= T_{AB}\left[\frac{580\,j - 340\,i - 250\,k}{\sqrt{(580)^2 + (-340)^2 + (-250)^2}}\right]$$

Follow the path A $\rightarrow$ O $\rightarrow$ E $\rightarrow$ B to reach from A to B

A $\rightarrow$ O = 580 j, O $\rightarrow$ E = – 340 i, E $\rightarrow$ B = – 250 k

$$\bar{T}_{AB} = T_{AB}\,[-0.474\,i + 0.808\,j - 0.348\,k]$$

3. $\bar{T}_{AD} = T_{AD} \cdot \bar{e}_{AD} = T_{AD}\left[\frac{580\,j - 250\,k + 380\,i}{\sqrt{(580)^2 + (-250)^2 + (380)^2}}\right]$

Follow the path A $\rightarrow$ 0 $\rightarrow$ F $\rightarrow$ D

A $\rightarrow$ 0 = 580 j, 0 $\rightarrow$ F = – 250 k

F $\rightarrow$ D = 380 i

$$\bar{T}_{AD} = T_{AD}\,[0.515\,i + 0.786\,j - 0.339\,k]$$

4. $\overline{W} = -W \cdot j = -160 \times 9.81\, j$

$\overline{W} = \mathbf{-1569.6\, j\ (N)}$

Now, apply conditions of equilibrium,

$\sum \mathbf{F_x = 0} \quad -0.474\, T_{AB} + 0.515\, T_{AD} = 0$

$\sum \mathbf{F_y = 0} \quad 0.888\, T_{AC} - 0.808\, T_{AB} + 0.786\, T_{AD} - 1569.6 = 0$

$\sum \mathbf{F_z = 0} \quad 0.459\, T_{AC} - 0.348\, T_{AB} - 0.339\, T_{AD} = 0$

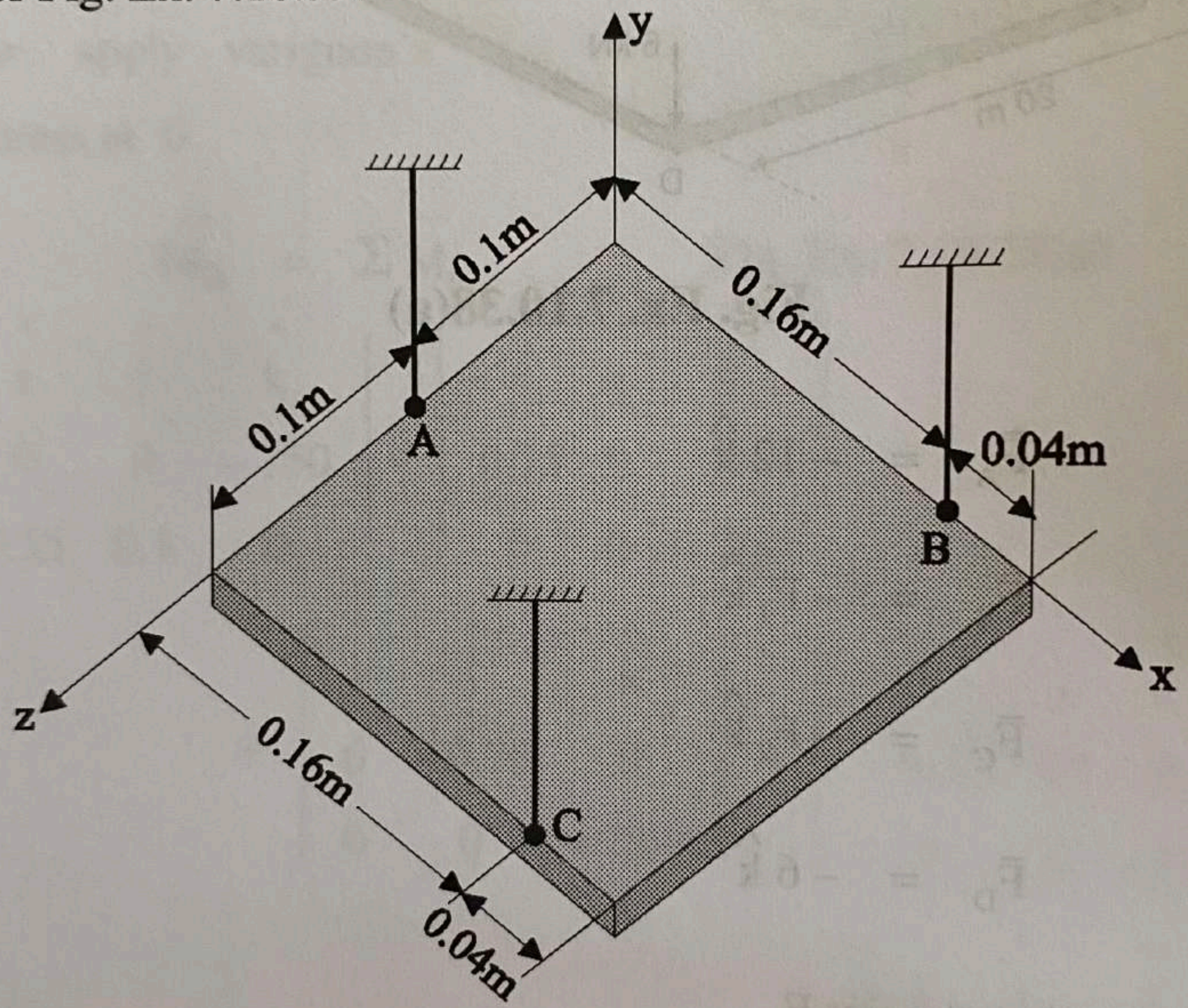
Solving above equations,

We get, $\quad \mathbf{T_{AB} = 1316.435\ N}$

$\mathbf{T_{AC} = 1892.95\ N}$

$\mathbf{T_{AD} = 1211.63\ N}$

A square plate of size 0.2 m × 0.2 m having weight 250 N is supported by three vertical wires. Determine tension in each wire. Refer Fig. Ex. 7.10.39.



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TYPE V : EXAMPLES BASED ON EQUILIBRIUM OF PARALLEL FORCES IN SPACE

Ex. 7.10.39

A square plate of size 0.2 m × 0.2 m having weight 250 N is supported by three vertical wires. Determine tension in each wire. Refer Fig. Ex. 7.10.39.

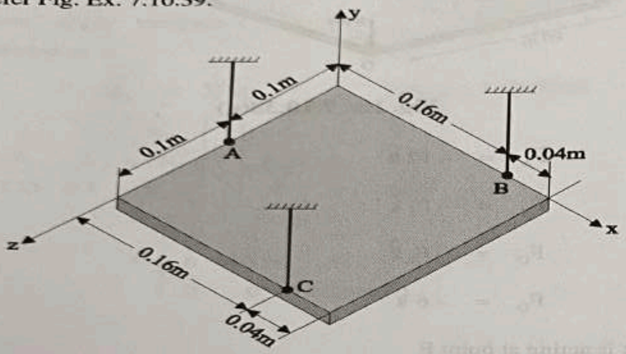


Fig. Ex. 7.10.39

☑ **Soln. :** F.B.D. of plate is shown in Fig. Ex. 7.10.39(a).

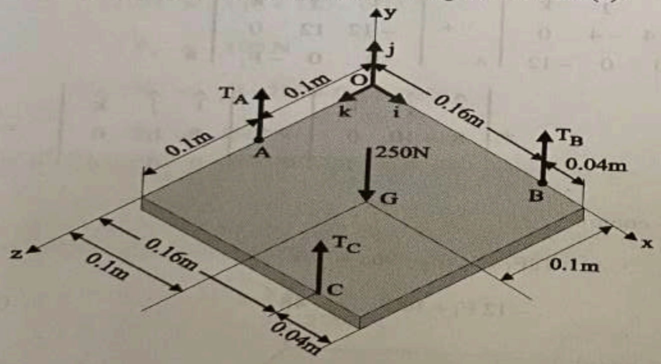


Fig. Ex. 7.10.39(a)

Force vectors	Co-ordinates of points of application
$\bar{T}_A = T_A \cdot j$	A (0, 0, 0.1)
$\bar{T}_B = T_B\ j$	B (0.16, 0, 0)
$\bar{T}_C = T_C \cdot j$	C (0.16, 0, 0.2)
$\bar{W} = -250\ j$	G (0.1, 0, 0.1)

Apply conditions of equilibrium,

$\Sigma F_x = 0,\ \Sigma F_z = 0$ (No forces in these directions)

$$\Sigma F_y = 0 \Rightarrow \quad T_A + T_B + T_C - 250 = 0$$

$$\therefore \quad T_A + T_B + T_C = 250 \qquad \text{...(1)}$$

Now, use $\Sigma \bar{M} = 0$ (Taking moment at origin)

$$\therefore \begin{vmatrix} i & j & k \\ 0 & 0 & 0.1 \\ 0 & T_A & 0 \end{vmatrix} + \begin{vmatrix} i & j & k \\ 0.16 & 0 & 0 \\ 0 & T_B & 0 \end{vmatrix} + \begin{vmatrix} i & j & k \\ 0.16 & 0 & 0.2 \\ 0 & T_C & 0 \end{vmatrix} + \begin{vmatrix} i & j & k \\ 0.1 & 0 & 0.1 \\ 0 & -250 & 0 \end{vmatrix} = 0$$

$$i(-0.1\ T_A) + k(0.16\ T_B) + i(-0.2\ T_C) + k(0.16\ T_C) + i(25) + k(-25) = 0$$

Adding all i and k co-efficients,

$$-0.1\ T_A - 0.2\ T_C + 25 = 0$$

$$\therefore T_A + 2\ T_C = 250 \qquad \text{...(2)}$$

and, $\quad 0.16\ T_B + 0.16\ T_C - 25 = 0$

$$\therefore \quad T_B + T_C = 156.25 \qquad \text{...(3)}$$

Solving Equations (1), (2) and (3)

We get $\quad T_A = 93.75$ N ...Ans.

$T_B = 78.125$ N ...Ans.

$T_C = 78.125$ N ...Ans.

Ex. 7.10.40

A homogeneous semi circular plate of weight 10 kN and radius 1m is supported in horizontal plane by three vertical wires as shown in Fig. Ex. 7.10.40. Determine the tensions in the wires.

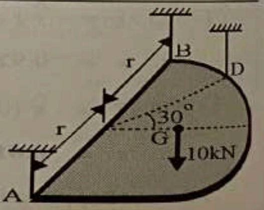
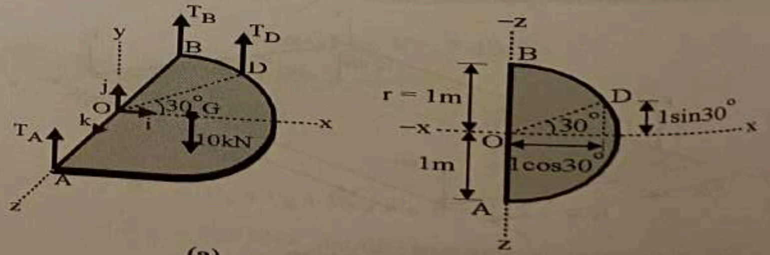


Fig. Ex. 7.10.40

☑ **Soln. :** Force vectors and co-ordinates of points of application.



(a) (b)

Fig. Ex. 7.10.40