UNIT- I

1

Set Theory and Logic

Q. What is Discrete Mathematics?

→ Discrete Mathematics :→

Discrete mathematics is mathematics that deals with discrete objects.
Discrete objects are those which are separated from each other, which are not connected to each other.
Eg. Finite mathematics, Integers, people, house etc.

The main goal of this course is to provide students with an opportunity to gain an understanding of theoretical foundation of computer science.

Discrete mathematics is the study of mathematical structures that are fundamentally discrete rather than continuous.

Discrete mathematics excludes topics in "continuous mathematics" such as calculus & analysis.

Set of objects studied in discrete mathematics can be finite or infinite.

Book - Discrete Mathematical structures
Author - Kolman, busby, Page No - 2

Q. Define set? How it can be represented?

→ * Sets :→

- A set is any well defined collection of objects called the elements or members of the set.

- Well defined just means that it is possible to decide if a given objects belongs to the collection or not.

→ * Representation of set

We use uppercase letters such as A, B, C to denote sets & lowercase letters such as a, b, c, x, y, z to denote the members of set.

We represent set in two form

i] Tabular form of set.

Eg. Set V of all vowels in english

i.e $V = \{a, e, i, o, u\}$

ii] Rule Method

$B = \{x : x \text{ is an even integer}, x > 0\}$

: denotes such that

, denotes "and"

Let $A = \{1, 3, 5, 7\}$ then
$1 \in A$, $3 \in A$, $2 \notin A$

Q. Define universal set?

*→ Universal set :→

If there are some sets under consideration then there happens to be a fixed set which contains each one of the given sets. such a fixed set is known as the universal set & denoted by "U"

if $A = \{1, 2, 3, 4\}$, $B = \{2, 3, 5\}$, $C = \{3, 6, 7\}$

then $U = \{1, 2, 3, 4, 5, 6, 7\}$

Q. Define Empty set?

*→ Empty set :-

If set consisting no element at all is called an empty set or null set.

$A = \{\phi\}$

Ref. Book – Discrete Mathematics
Author – S. K. Chakrobourthy
page No :- 1.2-6

Q. Define subset?

→ * Subset :-

If A & B are two sets such that every ~~[illegible]~~ element in a set A is also an element of a set B then A is called a subset of B

i.e $A \subseteq B$

Q. Define Powerset

⇒ * Power set :→

consider the set $A = \{a, b\}$, the subsets of A are ϕ, $\{a\}$, $\{b\}$ & $\{a, b\}$. Then the family of all the subsets of A is called the power set of A, which is denoted by $P(A)$, thus, $P(A) = \{\phi, \{a\}, \{b\}, \{a, b\}\}$.

Symbolically, $P(A) = \{x : x \text{ is a subset of } A\}$

eg. consider $A = \phi$ Then $P(A) = \{\phi\}$ & contains no element.

Ref. Book – Discrete Mathematics
Author – Swapan Kumar Charobourthy
Page NO – 1.2.9

Q. Define Cardinality of a set?

* Cardinality of a set

The number of distict elements contained in a finite set is called the cardinality or the cardinal number of the set. The cardinality of a set is denoted by various notations like, $n(A)$ or card (A), $|A|$, & A.

eg. Cardinality of empty set, ϕ, is 0 & is denoted by $n(\phi) = 0$, Let $A = \{2, 3, 5, 7, 11\}$. then $n(A) = 5$.

9. Explain Naive set theory?

* Naive set theory :→

Naive set theory is one of several theories of sets used in the discussion of the ~~fundamed~~ foundation of mathematics.

Unlike axiomatic set theories, which are defined using a formal logic, naive set theory is defined informally in natural language. It describes the aspects of mathematical sets familier in discrete mathematics (for eg. venn diagram & symbolic reasoning about their boolean algebra), & suffices for the everyday usage of set theory concept in contemporary mathematics.

- A Naive theory is considered to be a non-formalised theory, that is a theory that uses a natural language to describe a sets & operations on sets.

The words and, or, if, then, not,

for some, for every are useful to study set naively.

- The first developement of set theory was a naive set theory. It was created at the end of the 19th century by Georg Cantor (Cantorian)

* Axiomatic theories:

Axiomatic set theory was developed in response to these early attempts to understand sets, with the goal of determining precisely what opera-tion were allocated & when is allocated.

Q. ~~Explain~~ What are the diff. operations of sets?

* Operations on sets.

Sets can be combined in many different ways so as to produce new sets.

1] Union of sets :-

Let A & B be two sets. Then the union of A & B is set of all those element which are neither in set A or in set B or in both sets

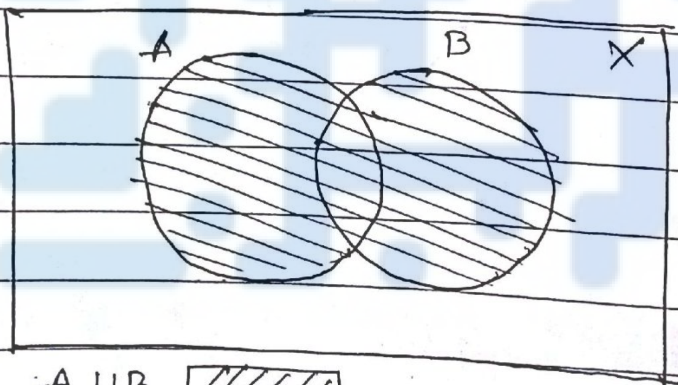
Union of sets A & B is denoted by $A \cup B$

which is read as 'A union B'

symbolically

$$A \cup B = \{x ; x \in A \text{ or } x \in B\}$$

In mathematics, when we use 'or', i.e $x \in A$ or $x \in B$, we do not exclude the possibility that x is an element of both A & B



A UB

Examples :-

1. $A = \{1, 2, 3\}$, $B = \{x, y, z\}$

$A \cup B = \{1, 2, 3, x, y, z\}$

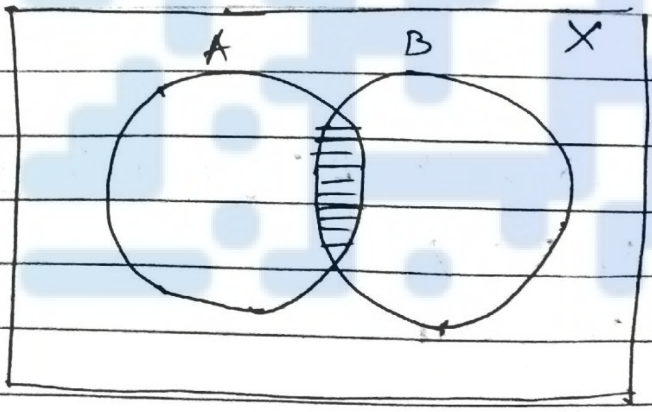
2. $A = \{1, 2, 3, 4\}$, $B = \{1, 3\}$, $B \subseteq A$

$A \cup B = \{1, 2, 3, 4\} = A$

* Intersection of sets :-

Let A & B be two sets. the intersection of A & B is the set of all elements which are in A & also in B. i.e set of common elements of A & B is Intersection of sets A & B. We denote the intersection of A & B by $A \cap B$

$A \cap B = \{x : x \in A \text{ and } x \in B\}$



$A \cap B$

Eq. $A = \{1,2,3,4\}$, $B = \{x, y, z\}$

$A \cap B = \phi$

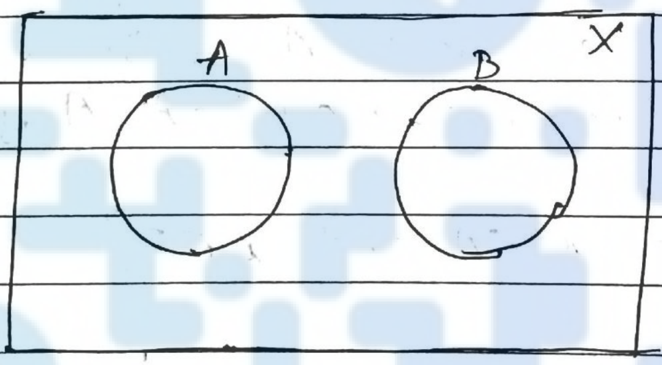
(ii) $A = \{1,2,3,4\}$, $B = \{1,3\}$, $B \subseteq A$

$A \cap B = \{1,3\} = B$

Q. Define Disjoint set?

* Disjoint sets

Set A & set B are called disjoint sets if no element is common to A and B i.e. A and B are disjoint then $A \cap B = \phi$



$A \cap B = \phi$

eq.

1. $A = \{1,3,5,7\}$, $B = \{2,4,6,8\}$

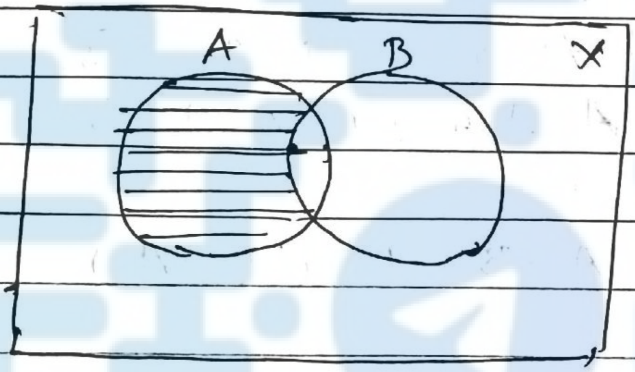
$A \cap B = \phi$

Q Define Difference of two sets?

* Difference of two sets :

Let A & B be two sets. the difference of A & B is denoted by '$A \sim B$' or '$A - B$' is set of all those elements of A which are not in B

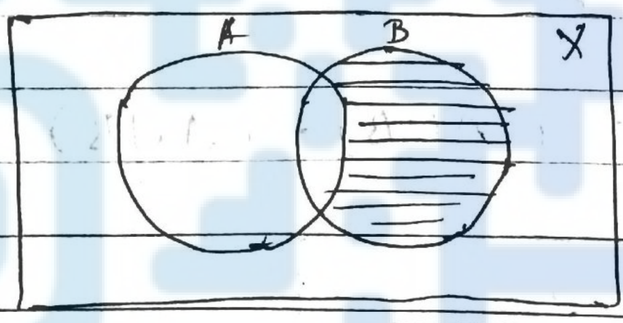
$$A - B = \{x ; x \in A \text{ and } x \in B\}$$



$A - B =$ 

similarly,

$$B - A = \{y, y \in B \text{ and } y \in A\}$$



$B - A =$ 

Ex. $A = \{1,2,3,4\}$, $B = \{4,5,6,7\}$

$A - B = \{1,2,3\}$

$B - A = \{5,6,7\}$

$A \cap B = \{4\}$

$A \cup B = \{1,2,3,4,5,6,7\}$

Q. Explain symmetric difference of two set?

* Symmetric difference of two sets.

Let A & B be two sets then symmetric difference of two sets A & B is denoted by $A \Delta B$ or $A \oplus B$

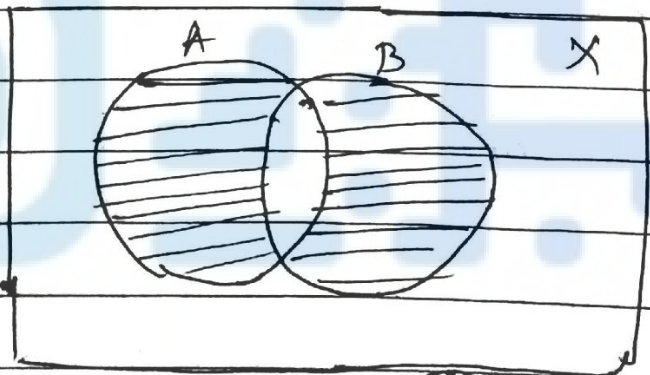
$$A \oplus B = (A - B) \cup (B - A)$$

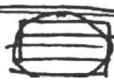
hence

if $x \in A \oplus B$

$\Rightarrow$ x is an element of exactly one of A & B or $x \in$ either A or $x \in B$ but x does not belongs to both.

i.e. $A \oplus B = (A \cup B) - (A \cap B)$



$A \oplus B =$ 

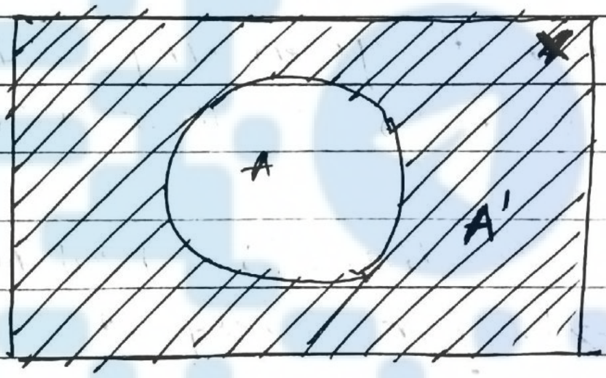
Q. Define Complement of set?

* Complement of set

Let A be any set, then the complement of set A is denoted by A' or $\bar{A}$ complement of A is set of those element which are in universal set X but are not in set A

$\therefore \; A' = X - A$

$A' = \{x : x \in X \text{ and } x \notin A\}$



$A' =$ 

Ref. Book :- Discrete Mathematics
Author – S. K. Chakroborthy
Page No – 1.5

Q. Explain diff. types of sets?

* Types of sets :→

1. Bounded and Unbounded sets :→

We say that a subset D of the real numbers

- Unbounded above if given any real number r we can find $d \in D$ so that $d > r$;

- bounded above if there is a real number U so that $d \leq U$ for every $d \in D$, & U is said to be an upper bound for D.

- Unbounded below if given any real number r we can find $d \in D$ so that $d < r$;

- bounded below if there is a real number L so that $d \geq L$ for every $d \in D$ and L is said to be a lower bound for D

- bounded if it is bounded above and below.

for eg. the +ve integers are bounded below & unbounded above. the -ve real numbers are bounded above and bounded below.

If D is bounded above, we say that the real number U is the least upper bound of D if both of the following are true:

- U is an upper bound of D;
- if U' is any other upper bound of D then $U \leq U'$

Similarly, if D is bounded below, so say that the real number L is the greatest lower bound of D if both the following are true:

- L is a Lower bound of D;
- if L' is any other Lower bound of D then $L \geq L'$

URL :- www.math.ucla.edu > Handout-359-433

2. Countable and Uncountable sets

If a rule is such that it associates with each element $a \in A$, one & only one element $b \in B$ then this rule is called one to one. Also, if for each element $b \in B$, there exists exactly one element $a \in A$, then this rule is called one to one & onto, which is also known as one to one correspondance

A set which is not finite is called an infinite set. A set is said to be countably infinite, if there exists one to one correspondance betn the elements in the set & elements in N; there is, if the set is

* Finite & Infinite set / countably finite & uncountable infinite set

Countably infinity, we can make a list of its members in such a way that each one corresponds uniquely to a natural no's.

A Countably infinite set is also reffered as denumerable. The set of non-negative even integers is countably infinite.

- A set which is either finite or denumerable is called countable. A set which is not countable is called as countable.

- A set which not countably infinite is called uncountably infinit set.

~~consider~~ consider the set given below.

A = { polygons with less than 8 slide

B = { The notes in western music }

C = { even numbers betn 1 & 10 }

D = { prime numbers }

E = { squate numbers.

By listing the elements of the above sets, we obtain the following.

Here the number of elements of the set

x is denoted by $n(x)$

A = {Triangle, Quadilateral, Pentagon, Hexagon, Heptagon}

$\Rightarrow n(A) = 5$

B = {P, Q, R, S, T, U, V} $n(B) = 7$

C = {2, 4, 6, 8} $n(C) = 4$

D = {2, 3, 5, 7, 11, 13, ..} $n(D) = ?$

E = {1, 4, 9, 16, 25, ..} $n(E) = ?$

If the number of elements in a set is finite ~~set~~ that set is said to be a finite set.

If the number of elements in a set is Infinite that set is said to be an infinite set.

Ref. URL → www. springer. com>content>document

math. uga. edu/~pete/settheorypart1

* Cantor's diagonal Argument

Cantor's diagonal method is elegant, powerful, & simple. It has been the source of fundamental & fruitful theorems as well as devastating, & ultimately, fruitful paradoxes.

* Cantor's proof :-

Suppose that $f: N \to [0, 1]$ is any function. Make a table of value of f, where the 1^{st} row contains the decimal expansion of $f(1)$. The 2^{nd} row contains the decimal exponsion of $f(2)$ the n^{th} row contains the decimal expansion of $f(n)$.

n	$f(n)$
1	0 . $\underline{3}$ 1 4 1 5 9 2 6 5 3 ...
2	0 . 3 $\underline{7}$ 3 7 3 7 3 7 3 7 ...
3	0 . 1 4 $\underline{2}$ 8 5 7 1 4 2 8 ...
4	0 . 7 0 7 $\underline{1}$ 0 6 7 8 1 1 ...
5	0 . 3 7 5 0 $\underline{0}$ 0 0 0 0 0 ...
⋮	⋮

The highlighted digits are 0.37210...
suppose that we add 1 to each of these digits, to get the number
0.48321...

[Ref. URL - www.math.ku.edu/~njpik/cantor
- www.msc.uky.edu/fall06/pdfs]

* Propositional Logic :-

It is a language that use for Reasoning
We start with logic of sentences called propositional logic & elements of logic, relationships betn propositions & reasoning.

* Logic :-
Logic is analysis of language which consists of signs.

Logic is a set of rules (axioms) which we can use to draw valid conclusions.

* PROPOSITIONS :→
A propositions or statements is a declarative sentence which is either true or false but not both.

example
1. Pune is capital of India
2. Mars is a planet
3. 9 > 13

4. $y + 8 = 12$
5. Bring that book.
6. $x \in A$
7. There are 12 months in a year.

1, 3 → false
2, 7 → True

4, 5, 6 → Not false not true not a statement.

* Open statement :-

Sentence that contains one or more variable such that when certain value are substituted for the variables we get statements

Example :-

1. $x + 7 = 9$
2. $x + 3 = 5$
3. $3x + 2 > 8$

Q. What type of sentence is

1] $4 + 3 = 15$

2] $3 - 4 = 20$

Q. What value of x following setences will become true statement

1] $3x + 9 = 15$

2] $x + 6 = 8$

3] $x + 1 > 5$

4] $x + 2 < 8$

5] $5x \geq 25$

6] $5x \leq 25$

Ans:. ① 2 , 2] 2, 3] $x \geq 5$

4] $x < 6$ or $x \leq 5$

5] $x \geq 5$ 6] $x \leq 5$

Q. Which of the following statements are true

1] $x + 4 = 6$ when $x = 2$ ✓

2] $x + 4 \neq 6$ when $x = 2$ ✗

3] $x + 5 \neq 8$ when $x = 3$ ✗

4) $2x + 4y = 14$ when $x = 1, y = 3$ ✓

5] $3x + 5y = 11$ when $x = 0, y = 2$ ✗

6] $5 \in \{4, 2, x\}$ when $x = 5$ ✓

[Ref. Book – S. K. Chakraborty
Discrete mathematics
Page No – 122]

Q. What are different Logical connectives?

* Logical connectives

Every statement must be either true or false but not both.

I] Compound statement :-
Two or more statements can be combined to produce a new statement. These statements are called compound statement.

i] Negation :-
Negation ≡ NOT
" it is not the case that"
If 'p' denotes a statement then negation of p is denoted by "$\sim p$" or ($\bar{p}$)

Truth table :-

p	$\sim p$
T	F
F	T

$p \rightarrow$ Gopal is intelligent
statement :- Gopal is not intelligent
propositional form = $\sim p$

Examples

1] If p is statement
"I am going for a walk" then
$\sim p$ is the statement
"I am not going for a walk"
or
"It is not the case that I am
going for a walk"

2] If p is statement
"I like to read"
$\sim p$ " I don't like to read"

2] Conjunction (And or $\wedge$)

If p and q are the statement the
compound statement "p and q" is
called as the 'p conjunction q"

examples
Let us consider the statements
p: The sun is shining
q: The birds the singing
then $p \wedge q$ is the statement
The sun is shining and the
birds are singing.

Truth Table:

$p \wedge q$

p	q	$p \wedge q$
T	T	T
F	F	F
T	F	F
F	T	F

3] Disjunction ('$\vee$' or 'OR')

If p & q are statements then the compound statement "p or q" is called disjunction of p and q.

Example

1] consider if

p : I will purchase a dress

q : I will purchase a book

then $p \vee q$

I will purchase a dress or I will purchase a book.

2] Consider if

p: a is equal to 5

q : b is equal to 7

the $p \vee q$

a is equl to 5 or b is equal to 7

$p \vee q$

p	q	$p \vee q$
F	F	F
F	T	T
T	F	T
F	T	T

4] Conditional ("If... then")

if p and q are statement then compound statement "If p then q" denoted by $p \rightarrow q$ is called conditional statement.

Ex.
Let p : Hari works hard.
q : Hari will pass the exam
then
$p \rightarrow q$: If hari works hard then he will pass the exam.

Truth table

$p \rightarrow q$

p	q	$p \rightarrow q$
T	T	T
T	F	F
F	T	T
F	F	T

5] Bicondional (Double Implication)
(If and only if)

if p and q are two propositions then $p \rightarrow q$ and $q \rightarrow p$ is called biconditional or double implication
$p \leftrightarrow q$.

p	q	$p \rightarrow q$	$q \rightarrow p$	$p \leftrightarrow p$
T	T	T	T	T
T	F	F	T	F
F	T	T	F	F
F	F	T	T	T

6] Contrapositive of Implication

The implication $\sim q \rightarrow \sim p$ is called the contrapositive of the implication $p \rightarrow q$

Truth table

$p \rightarrow q$

p	q	$p \rightarrow q$
T	T	T
T	F	F
F	T	T
F	F	T

p	q	$p \rightarrow q$	$\sim p$	$\sim q$	$\sim p \rightarrow \sim q$
T	T	T	F	F	T
T	F	F	F	T	T
F	T	T	T	F	F
F	F	T	T	T	T

p	q	$p \rightarrow q$	$\sim q$	$\sim p$	$\sim q \rightarrow \sim p$
T	T	T	F	F	T
T	F	F	T	F	F
F	T	T	F	T	T
F	F	T	F	T	T

Q. Using the following statements

p : Mohan is rich.
q : Mohan is happy

Write following statement in symbolic form.

1] Mohan is rich but unhappy

$p \wedge \sim q$

2] Mohan is poor but happy

$\sim p \wedge q$

3) Mohan is neither rich nor happy

$\sim (p \vee q)$ $\quad$ $\sim p \wedge \sim q$

4) Mohan is poor or he is both rich and unhappy

$\sim p \vee (p \wedge \sim q)$

[Ref. Book - Discrete mathematical structures with appl^ns to computer science
Author - J. P. Tremblay
R. Manohar
Page No - 8]

Q Using the following statements
p : Rajani is tall
q : Rajani is beautiful
Write the following statement in symbolic form

→

1] Rajani is tall and beautiful
$p \wedge q$

2] Rajani is tall but not beautiful
$p \wedge \sim q$

3] It is false that Rajani is short or beautiful
$\sim(\sim p \vee q)$

4) Rajani is tall or Rajani is short and beautiful
$p \vee (\sim p \wedge q)$

Q. Express following statements in propositional form:

1] There are many clouds in the sky but it did not rain

p: There are many clouds in the sky
q: It rain
$\therefore p \wedge \sim q$

2] I will get first class if and only if I study well and score above 80 in mathematics
p: I will get first class
q: I study well
r: score above 80 in math

$\therefore p \leftrightarrow (q \wedge r)$

3] Computers are cheap but softwares are costly
p: computers are cheap
q: softwares are costly

$\therefore p \wedge q$

4) It is very hot and humid or Ramesh is having heart problem
p: It is very hot
q: It is very humid
r: Ramesh is having heart problem
$\therefore (p \wedge q) \vee r$.

5] In small restaurants the food is good and service is poor

p: In small restaurant food is good

q: service is poor

$\Rightarrow \quad p \wedge q$

Q. Using the following propositions

p: I am bored

q: I am waiting for one hour

r: There is no bus.

Translate the following int English

1] $(q \vee r) \rightarrow p$

If I am waiting for one hour or there is no bus, then I get bored.

2) $\sim q \rightarrow \sim p$

if I am not waiting for one houe then I am not bored.

3) $(q \rightarrow p) \vee (r \rightarrow p)$

If I am waiting for one houe then I am bored, or if there is no bus then I am bored.

Propotional Calculus

consider the following statement

p: He is intelligent

q: He is lazy

r: He is rich

write in symbolic form

i] He is neither intelligent nor lazy

$\sim p \wedge \sim q$

ii] It is false that he is intelligent but not lazy

$\sim (p \wedge \sim p)$

iii] He is intelligent or lazy but not rich

$(p \vee q) \wedge \sim r$

iv) It is false that he is intelligent or lazy but not rich

$\sim ((p \vee q) \wedge \sim r)$

v) It is not true that he is not rich

$\sim (\sim r)$

vi] He is rich or else he is both intelligent & lazy

$r \vee (p \wedge q)$

Propositional Calculus :→

Construct truth table

1] $(\sim p \vee q) \rightarrow q$

p	q	$\sim p$	$\sim p \vee q$	$(\sim p \vee q) \rightarrow q$
T	T	F	T	T
T	F	F	F	T
F	T	T	T	T
F	F	T	T	F

2) $(\sim p \rightarrow r) \wedge (p \vee q)$

p	q	r	$\sim p$	$(\sim p \rightarrow r)$	$(p \vee r)$	$(\sim p \rightarrow r) \wedge (p \vee q)$
T	T	T	F	T	T	T
T	T	F	F	T	T	T
T	F	T	F	T	T	T
T	F	F	F	T	T	T
F	T	T	T	T	T	T
F	T	F	T	F	T	F
F	F	T	T	T	F	F
F	F	F	T	F	F	F

Q. If $p \to q$ is false, determine the truth table of $(\sim p\ (p \wedge q)) \to q$

p	q	$p \to q$
T	T	T
T	F	F
F	T	T
F	F	T

p	q	$p \wedge q$	$\sim p(p \wedge q)$	$\sim(p \wedge q) \to q$
T	F	F	T	F

Truth value of $\sim (p \wedge q) \to q$ is false.

Q. If p and q are false propositions, find the truth value of

$(p \vee q) \wedge (\sim p \vee \sim q)$

p	q	$(p \vee q)$	$\sim p$	$\sim q$	$\sim p \vee \sim q$	$(p \vee q) \wedge (\sim p \vee \sim q$
F	F	F	T	T	T	F

* Tautology
* contradiction
* Contingency

* Tautologies :→

A proposition p is a tautology if it is true under all circumstances.
It means it contains only T in the final column of its truth table

Q. Prove that the statement $(p \to q) \leftrightarrow (\sim q \to \sim p)$ is a tautology

p	q	$p \to q$	$\sim p$	$\sim q$	$\sim q \to \sim p$	$(p \to q) \leftrightarrow (\sim q \to \sim p)$
T	T	T	F	F	T	T
T	F	F	F	T	F	T
F	T	T	T	F	T	T
F	F	T	T	T	T	T

As the final column contains all T's So it is tautology

* Contradiction :-

A statement that is always false is called contradiction

It means it contains only F in the final column of its truth table

Q. Show that the statement $p \wedge \sim p$ is a contradiction

p	$\sim p$	$p \wedge \sim p$
T	F	F
F	T	F

last column contains all F's so it is contradiction

* Contingency :->

A statement that can be either true or false depending on the truth values of its variables is called contigency

Prove that $(p \to q) \to (p \wedge q)$ is a contingency

p	q	$p \to q$	$p \wedge q$	$(p \to q) \to (p \wedge q)$
T	T	T	T	T
T	F	F	F	T
F	T	T	F	F
F	T	T	F	F

Value of final column depends on the truth value of variables so it is contingency.

Q. Define Logical Equivalence?

Logical Equivalence

The propositions A and B are logically equivalent if and only if they have same truth value for every choice of truth values of simple propositions involved in them

we denote this fact by $A \equiv B$

eg. Prove that $(p \vee q) \wedge \sim p \equiv \sim p \wedge q$.

p	q	$\sim p$	$p \vee q$	$(p \vee q) \wedge \sim p$	$\sim p \wedge q$
T	T	F	F	F	F
T	F	F	F	F	F
F	T	T	T	T	T
F	F	T	T	F	F

From the table truth values of $(p \vee q) \wedge \sim p$ and $\sim p \wedge q$ are same for each choice of p and q.

Hence $(p \vee q) \wedge \sim p$ is equivalent to $\sim p \wedge q$

i.e $(p \vee q) \wedge \sim p \cong \sim p \wedge q$

Logical Identities :→

1] DeMorgans laws

i] $\sim(p \vee q) = \sim p \wedge \sim q$

ii] $\sim(p \wedge q) = \sim p \vee \sim q$

2] Associative laws.

i] $p \vee (q \vee r) = (p \vee q) \vee r$

ii] $p \wedge (q \wedge r) = (p \wedge q) \wedge r$

3] Commutative laws

i] $p \vee q = q \wedge p$

ii] $p \wedge q = q \wedge p$

4) Idempotent laws

i] $p \vee p = p$

ii] $p \wedge p = p$

5] Double Negation

$\sim(\sim p) = p$

6] Distributive laws

$p \vee (q \wedge r) = (p \vee q) \wedge (p \vee r)$

$p \wedge (q \vee r) = (p \wedge q) \vee (p \wedge r)$

7) Absorption Laws .

i] $p \vee (p \wedge q) = p$

ii] $p \wedge (p \vee q) = p$

* Normal Forms :→

Ref. Book. Discrete ~~mathematical structures~~ with appl^ns Author – J. P. Tremblay Page No – 50

1] Disjunction Normal form (dnf)
2] Conjunction Normal form (cnf)

* Disjunction Normal Form (dnf)

dnf is disjunction (v) of fundamental conjunctions (∧)
i.e P, $P \wedge q$, $\sim P \wedge q$, $\sim P \wedge \sim q$, $P \wedge \sim q$, $q \wedge \sim P$, $P \wedge \sim P$.

Examples :.
1] $(P \wedge q) \vee P \vee (P \wedge \sim P)$
11] $(P \wedge q) \vee (r \wedge q) \vee (\sim r \wedge P)$

* conjunctive Normal Form (cnf)

conjunctive normal form is a conjunction (∧) of fundamental disjunctions (v)

$P \vee q$, $P \vee \sim q$, $\sim P \vee q$, $\sim P \vee \sim q$, $\sim q \vee q$ are fundamental disjunctions

Examples :.
1] $(P \vee q) \wedge (q \vee r) \wedge (\sim P \vee \sim r)$
2] $P \wedge (\sim q \vee \sim r)$

Q. Find cnf and dnf for the following expression

$(p \rightarrow q) \wedge (q \rightarrow p)$

$(\sim p \vee q) \wedge (\sim q \vee p)$ cnf

* $(p \rightarrow q) \wedge (q \rightarrow p)$

$(\sim p \vee q) \wedge (\sim q \vee p)$

$(\sim p \wedge \sim q) \vee (\sim p \wedge p) \vee (q \wedge \sim q) \vee (q \wedge p)$

$(\sim p \wedge \sim q) \vee F \vee F \vee (q \wedge p)$

$(\sim p \wedge \sim q) \vee (q \wedge p)$ dnf.

* dnf obtain

$(p \rightarrow q) \wedge (\sim p \wedge q)$

$(\sim p \vee q) \wedge (\sim p \wedge q)$

$(\sim p \wedge \sim p \wedge q) \vee (q \wedge \sim p \wedge q)$.

$(\sim p \wedge q) \vee (q \wedge \sim p)$ dnf

* $(p \wedge p \rightarrow q)) \rightarrow p$

$(p \wedge (\sim p \vee q)) \rightarrow p$

$\sim(p \wedge (\sim p \vee q)) \vee p$

$\sim p \vee \sim(\sim p \vee q) \vee p$

$\sim p \vee (p \wedge \sim q) \vee p$ dnf

Q. obtain cnf form of the following:

$p \wedge (p \rightarrow q)$

$p \wedge (\sim p \vee q)$ cnf

Q. Explain Applications of propositional logic

→

* Translation of sentence

General Rule for translation

→ Look for the patterns corresponding to logical connectives in the sentence & use them to define elementary propositions.

Eg. You can have free coffee if you are senior citizen and it is tuesday

a : You can have free coffee
b : You are senior citizen
c : It is tuesday

$(B \land c) \rightarrow a$

ii) If you can study well. you will get good marks and eligible for campus placement

a = you can study well
b = You will get good marks
c = Eligible for campus placement

$a \rightarrow (B \land C)$

* Mathematical Induction :→

Mathematical induction is a powerful technique in a mathematics; specially in number theory. In mathematics, we are often required to generalise a particular solution. In order to do this we look for a pattern in the particular solution.

Mathematical induction generalises this pattern of solutions by proving that it is always possible to extend the solution for a group that is one larger than the previous. Mathematical induction is an important tool in algorithm verification, to check wheather a program statement is loop varient, that is wheather it is true before & after every pass thr a programming loop.

* Statement of principle of mathematical inducⁿ

→ Let $P(n)$ be a statement involving a natural number n

1. If $P(n)$ is true for $n = n_0$ &
2. Assuming $P(K)$ is true, $(k \geq n_0)$ we prove $P(k+1)$ is also true,

then $P(n)$ is true for all natural numbers $n \geq n_0$

Eq. 1 Prove by mathematical induction for $n \geq 1$

$$1.2 + 2.3 + 3.4 + \dots n(n+1) = \frac{n(n+1)(n+2)}{3}$$

→ Basis of Induction :-

for $n=1$

$$1.2 = \frac{1.2.3}{3}$$

Induction step

Assume that, $P(K)$ is true

i.e. $1.2 + 2.3 + 3.4 + \dots k(k+1) = \frac{k(k+1)(k+2)}{3}$

Then we have

$$[1.2 + 2.3 + 3.4 + \dots k[k+1]] + (k+1)(k+2)$$

$$= \frac{k(k+1)(k+2)}{3} + \cancel{(k+1)(k+2)}$$

$$= \frac{(k+1)(k+2)(k+3)}{3}$$

Hence assuming $P(k)$ is true, $P(k+1)$ is also true, Therefore $P(n)$ is true for all $n \geq 1$

2] Show that n^3+2n is divisible by 3 for all $n \geq 1$

→

1. Basis of induction

For $n=1$

we have $1^3+2\cdot1 = 3$ is divisible by 3

2. Induction step :

Assume that k^3+2k is divisible by 3.

Then we have $(k+1)^3 + 2(k+1)$

$= k^3+3k^2+3k+1+2k+2$

$= (k^3+2k) + 3(k^2+k+1)$

k^3+2k is divisible by 3 and also $3(k^2+k+1)$

Hence assuming $P(k)$ is true. $P(k+1)$ is also true. Therefore $P(n)$ is true for all $n \geq 1$

3] Prove that 8^n-3^n is a multiple of 5 by mathematical induction for $n \geq 1$

→ Basis of Induction :

For $n=1$

$8^1 - 3^1 = 5$

~~9b+~~ it is multiple of 5

2. Induction step :

Assume that $P(k)$ is true:

i.e. $8^k - 3^k$ is multiple of 5 say $5r$

i.e. $8^k - 3^k = 5r$

where r is an integer

Then we have

$$8^{k+1} - 3^{k+1} = 8^k \cdot 8 - 3^k \cdot 3$$
$$= 8^k \cdot (5+3) - 3^k \cdot 3$$
$$= 8^k \cdot 5 + (8^k \cdot 3 - 3^k \cdot 3)$$
$$= 8^k \cdot 5 + 3(8^k - 3^k)$$

$8^k \cdot 5$ is multiple of 5 & also $8^k - 3^k$ is multiple of 5

Therefore $8^{k+1} - 3^{k+1}$ is multiple of 5

Hence assuming $P(k)$ is true, $P(k+1)$ is also true, Therefore $P(n)$ is true for all $n \geq 1$.

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ON and Above.

Q. Define Multisets?

* Multisets :-

set is a collection of distict objects. Multiset is collection of objects that are not necessarily distict.

eg.

1. {1, 1, 2, 2, 3, 3, 3, 3}
2. {1, 2, 3, 4}
3. {1, 2, 3, 4, 4}

* Multiplicity of an element in a multiset

multiplicity of an element in a mutiset is the number of times element present in the multiset.

Hence the multiplicity of 2 in eq. 1 above is 2 & multiplicity of 3 in eq. 1 is 4, multiplicity of 1 in eq. 1 is 2

* cardinality of multiset

The total number of elements in a multiset, including the repeated membership is the cardinality of the multiset. The no. of times an element belongs to the multiset is the multiplicity of that element.

eq. in the multiset

$\{a, a, a, b, b, c, c, d\}$

the multiplicities of element a, b, c, d are 3, 2, 2, 1 respectively & the cardinality of the multiset is 8

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bikash kanti sarkar

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* Venn Diagrams.

- We often use pictures in mathematics. The relation betn sets can be conveniently illustrated by certain diagram called venn diagram.

- In venn diagram a universal set X is represented by a large reactangle and subsets of X by regions enclosed within a closed curve generally circles, lying in the region enclosed by the rectangle.

- If a sets B is a subset of A, the circle representing B is drawn inside the circle representing A. If A & B are disjoint sets, then the circles represnting A & B are drawn in such a way that they have no common area.

- If A & B are not disjoint, then the circles representing A & B are drawn in such a way that they have some common area

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