UNIT 2

Distributed Forces and Friction

Introduction

- The earth exerts a gravitational force on each of the particles forming a body. These forces can be replace by a single equivalent force equal to the weight of the body and applied at the *center of gravity* for the body.

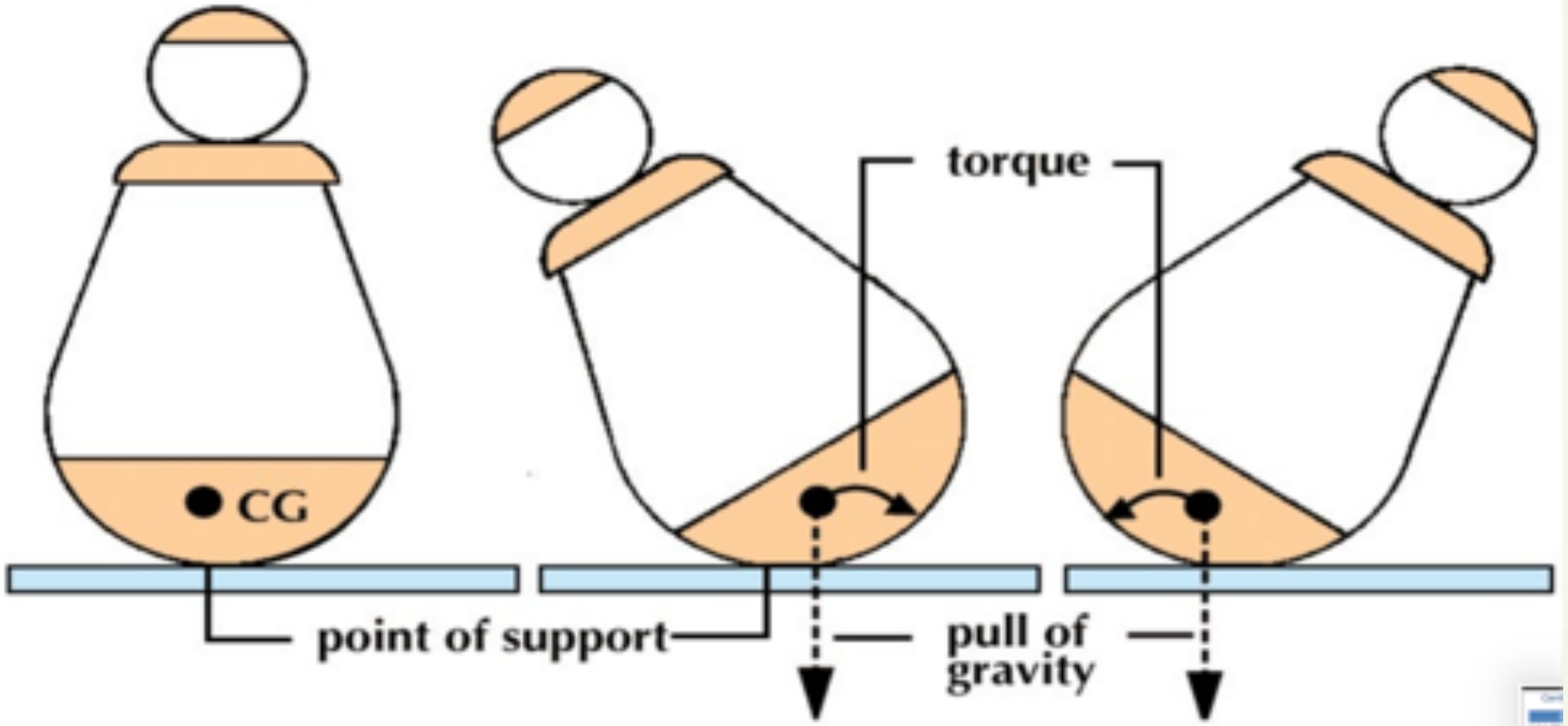
- The *centroid of an area* is analogous to the center of gravity of a body. The concept of the *first moment of an area* is used to locate the centroid.

- Determination of the area of a *surface of revolution* and the volume of a *body of revolution* are accomplished with the *Theorems of Pappus-Guldinus*.

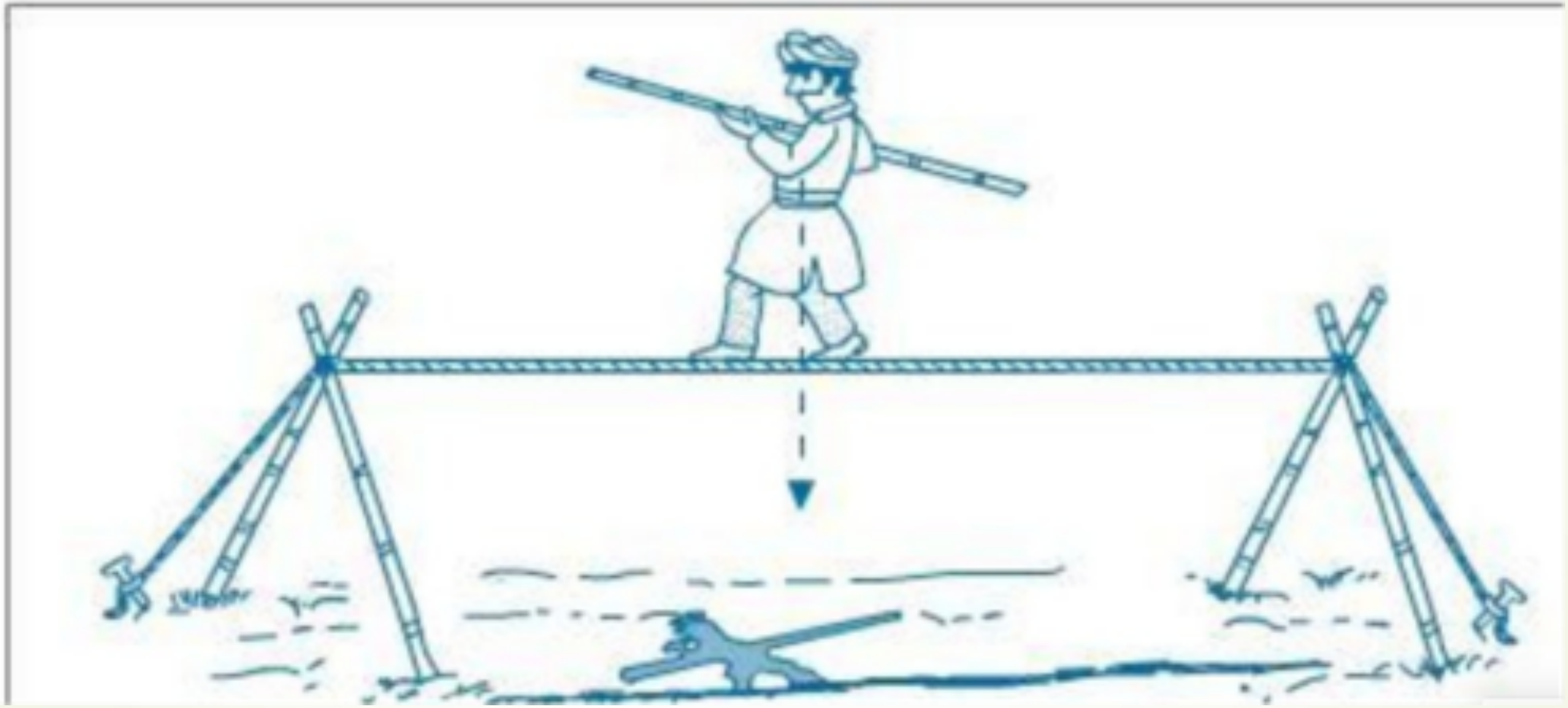
Centroid and Centre of Gravity

Centroid	Center of Gravity
• It is defined as a point about which the entire line, area or volume is assumed to be concentrated. • It is related to distribution of length, area and volume.	• It is defined as a point about which the entire weight of the body is assumed to be concentrated. • Center of mass. • It is related to distribution of mass.

Examples

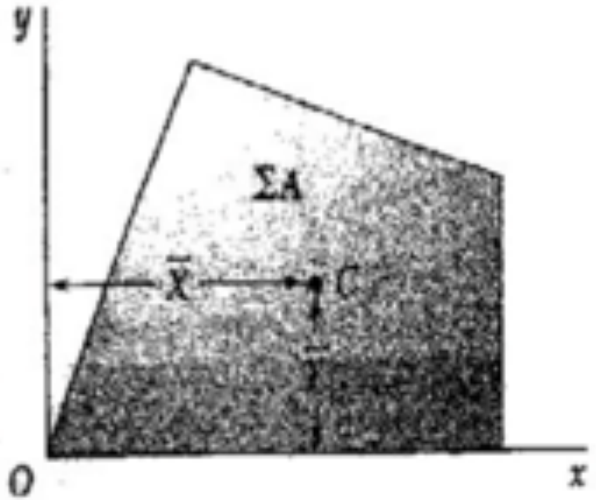


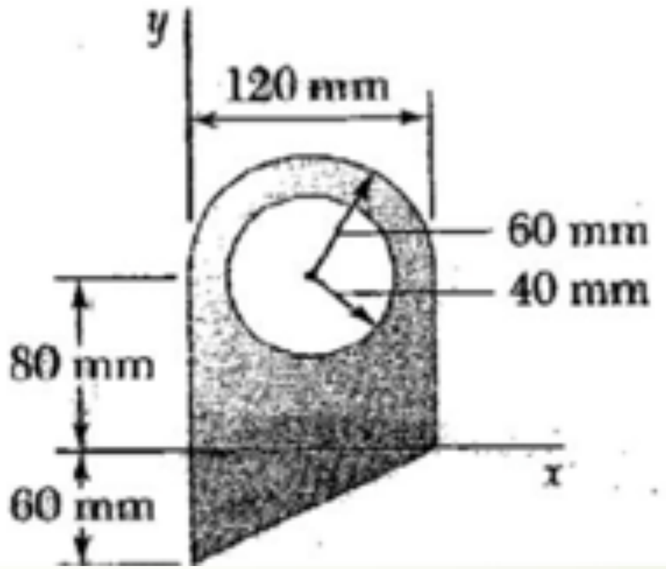
Examples



Axis of Reference

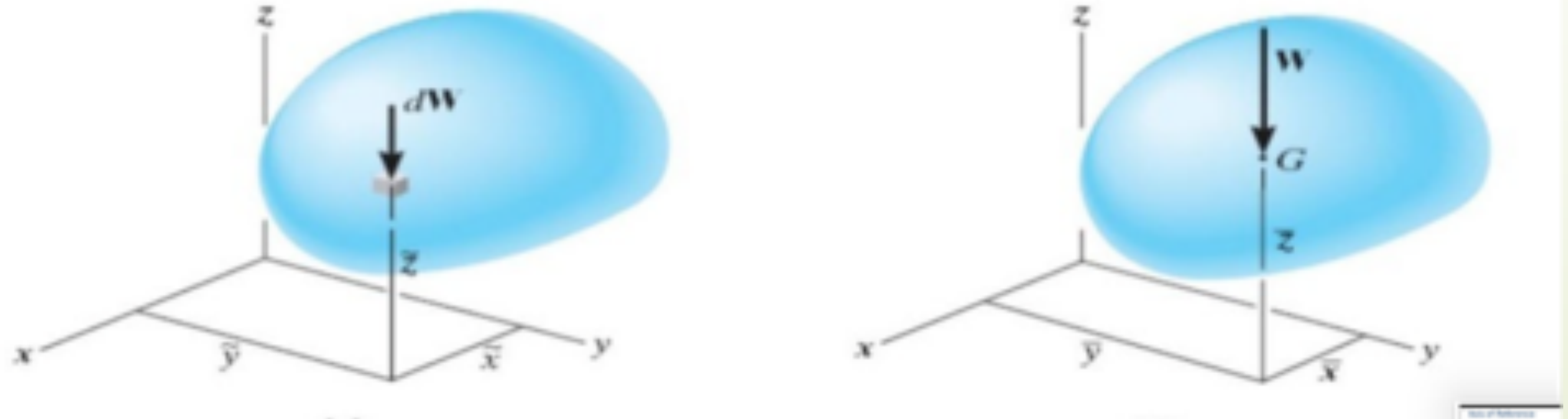
The centre of gravity of a body is always calculated with reference to some assumed axis known as axis of reference (or sometimes with reference to some point of reference). The axis of reference, of plane figures, is generally taken as the lowest line of the figure for calculating $\bar{y}$ and the left line of the figure for calculating $\bar{x}$.





Center of Gravity

- Consider system of n particles fixed within a region of space.
- The weights of the particles can be replaced by a single (equivalent) resultant weight having defined point G of application



(a)

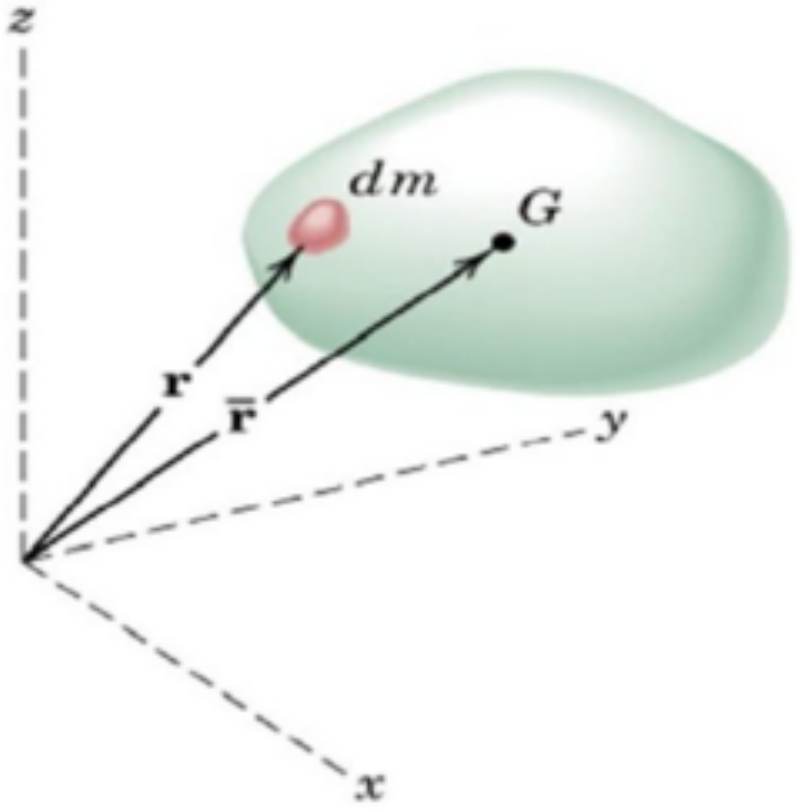
(b)

Center of Gravity

- Resultant weight = total weight of n particles , $W_R = \int dW$
- Sum of moments of weights of all the particles about x, y, z axes = moment of resultant weight about these axes
- moments about the x axis, $\bar{x}W_R = \tilde{x}_1W_1 + \tilde{x}_2W_2 + \ldots + \tilde{x}_nW_n$
- moments about y axis, $\bar{y}W_R = \tilde{y}_1W_1 + \tilde{y}_2W_2 + \ldots + \tilde{y}_nW_n$
- Generally,

$$\bar{x} = \frac{\int \tilde{x}\,dW}{\int dW}; \qquad \bar{y} = \frac{\int \tilde{y}\,dW}{\int dW}; \qquad \bar{z} = \frac{\int \tilde{z}\,dW}{\int dW}$$

Center of mass

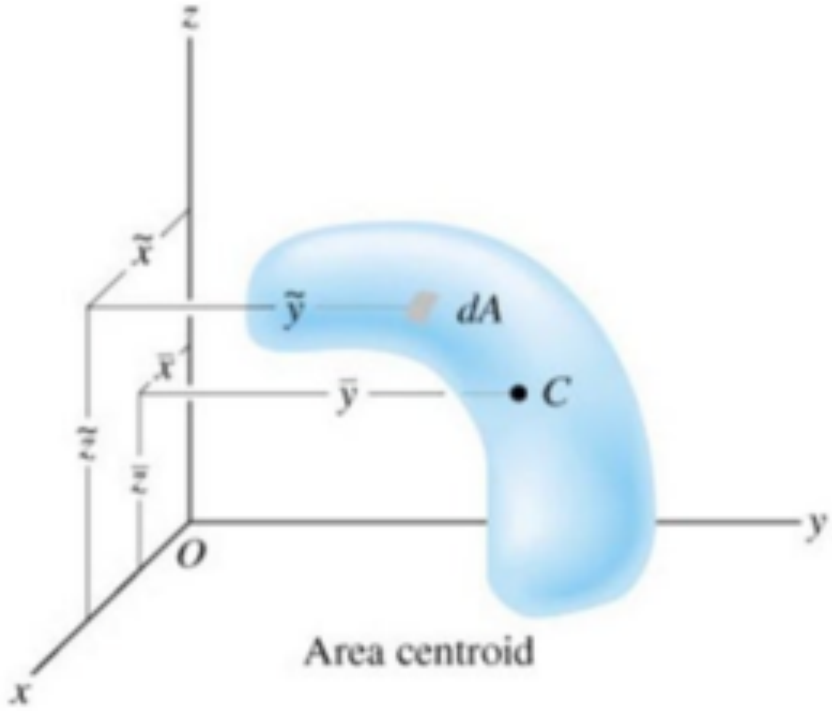


$$\bar{x} = \frac{\int \tilde{x}\, dm}{\int dm}$$

$$\bar{y} = \frac{\int \tilde{y}\, dm}{\int dm}$$

$$\bar{z} = \frac{\int \tilde{z}\, dm}{\int dm}$$

Center of Area / Area centroid

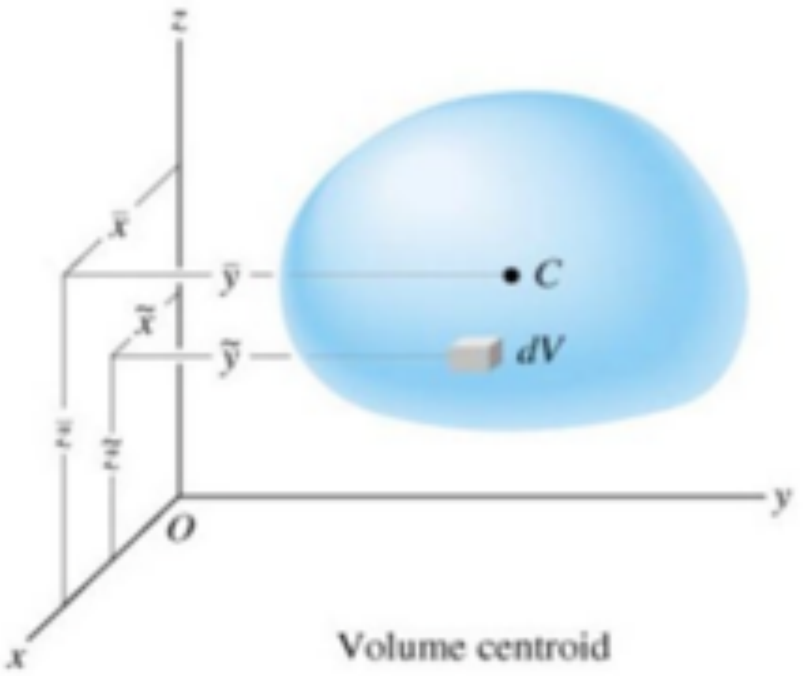


$$\bar{x} = \frac{\int_A \tilde{x}\, dA}{\int_A dA}$$

$$\bar{y} = \frac{\int_A \tilde{y}\, dA}{\int_A dA}$$

$$\bar{z} = \frac{\int_A \tilde{z}\, dA}{\int_A dA}$$

Center of Volume / Volume centroid



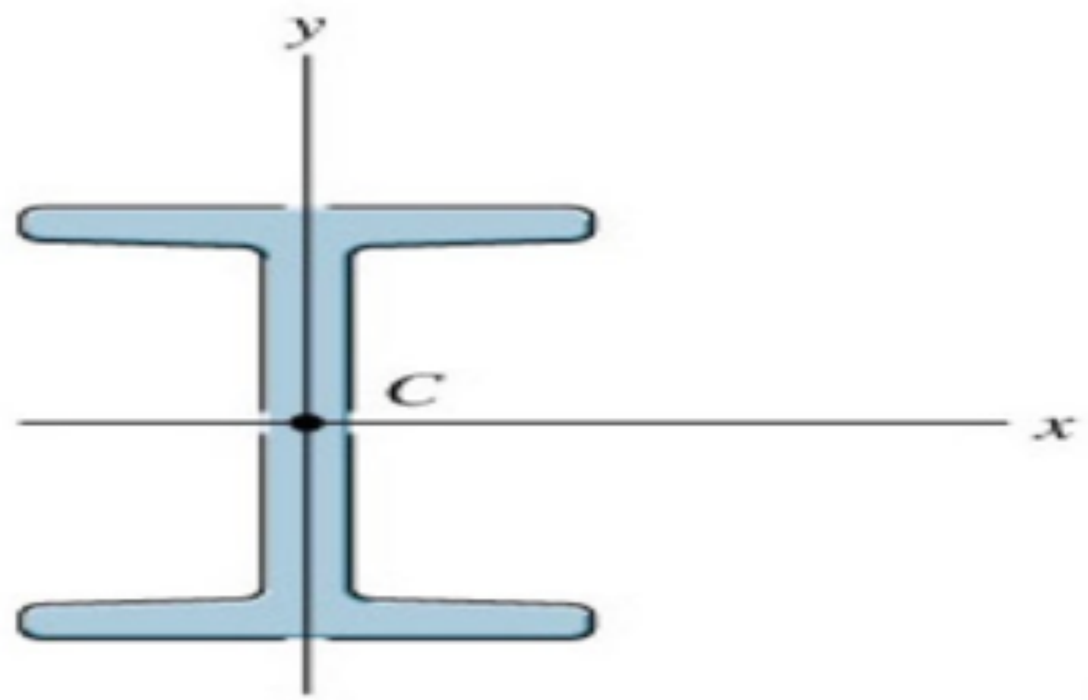
Volume centroid

$$\bar{x} = \frac{\int_V \tilde{x}\, dV}{\int_V dV}$$

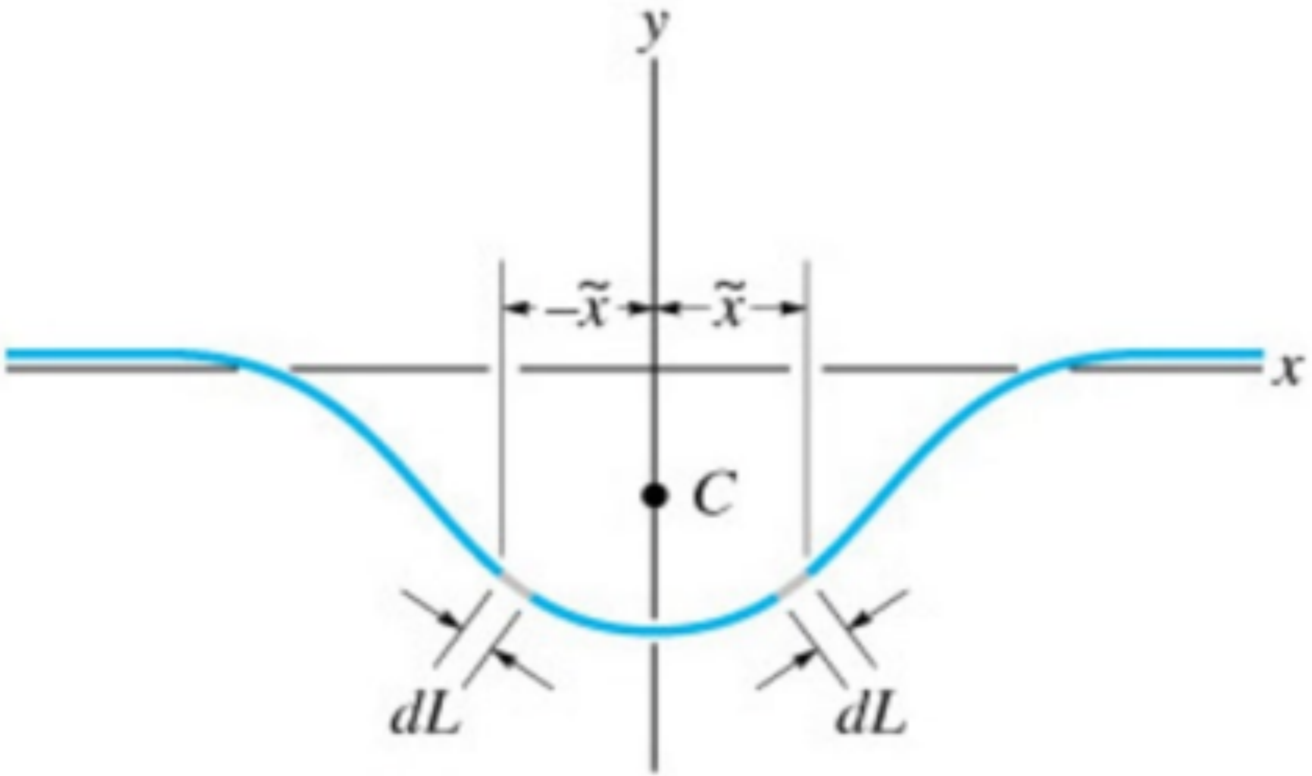
$$\bar{y} = \frac{\int_V \tilde{y}\, dV}{\int_V dV}$$

$$\bar{z} = \frac{\int_V \tilde{z}\, dV}{\int_V dV}$$

Axis of symmetry

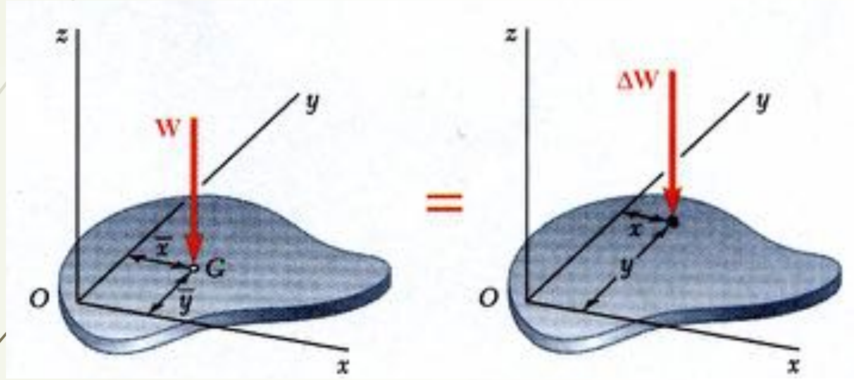


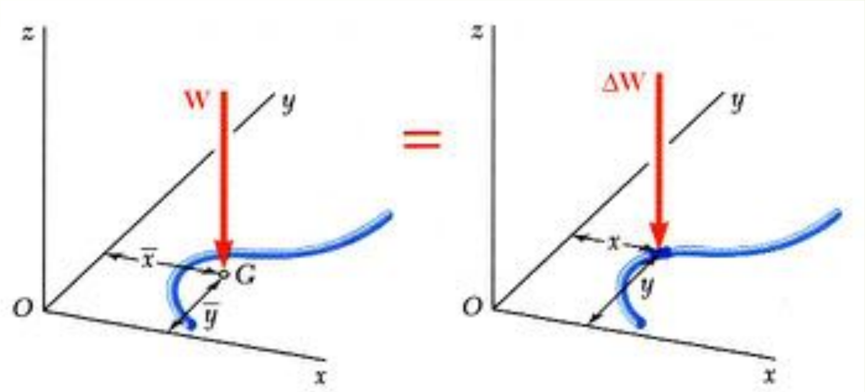
Axis of symmetry



Center of Gravity of a 2D Body

- Center of gravity of a plate
- Center of gravity of a wire





$$\sum M_y \quad \bar{x}W = \sum x\Delta W$$

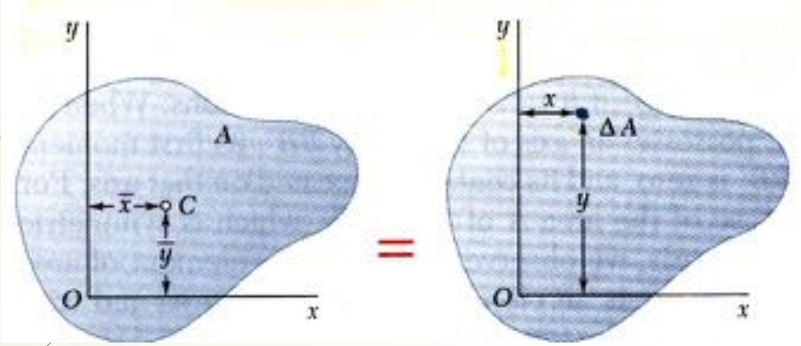
$$= \int x\,dW$$

$$\sum M_y \quad \bar{y}W = \sum y\Delta W$$

$$= \int y\,dW$$

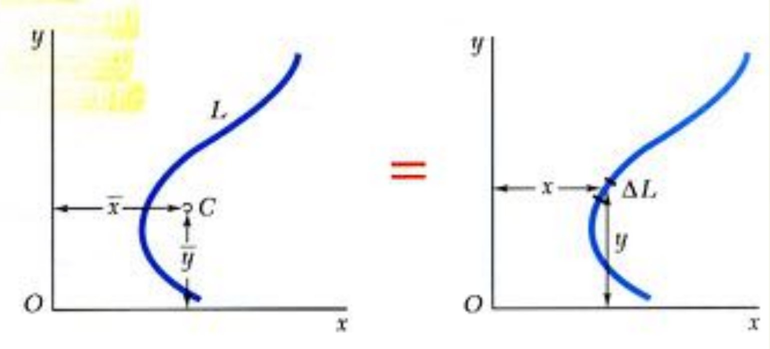
Centroids and First Moments of Areas and Lines

- Centroid of an area



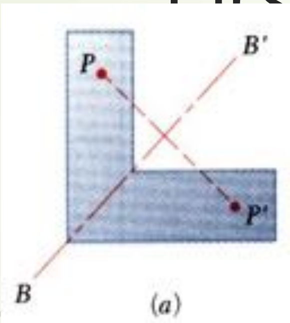
$$\bar{x}W = \int x\,dW$$
$$\bar{x}(\gamma A t) = \int x(\gamma t)\,dA$$
$$\bar{x}A = \int x\,dA = Q_y$$
$$= \text{first moment with respect to } y$$
$$\bar{y}A = \int y\,dA = Q_x$$
$$= \text{first moment with respect to } x$$

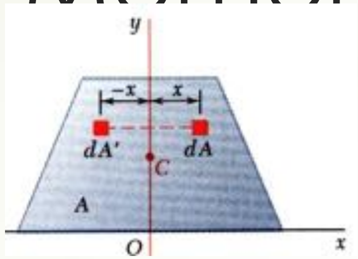
- Centroid of a line

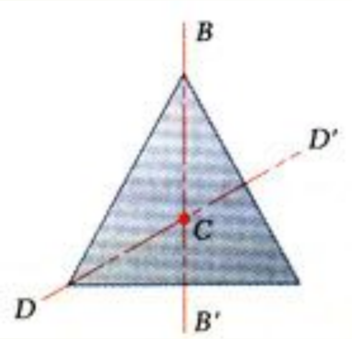


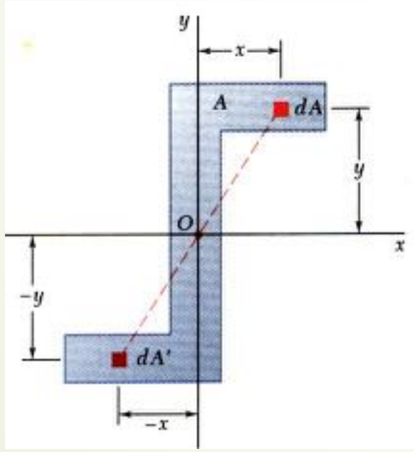
$$\bar{x}W = \int x\,dW$$
$$\bar{x}(\gamma L a) = \int x(\gamma a)\,dL$$
$$\bar{x}L = \int x\,dL$$
$$\bar{y}L = \int y\,dL$$

First Moments of Areas and Lines



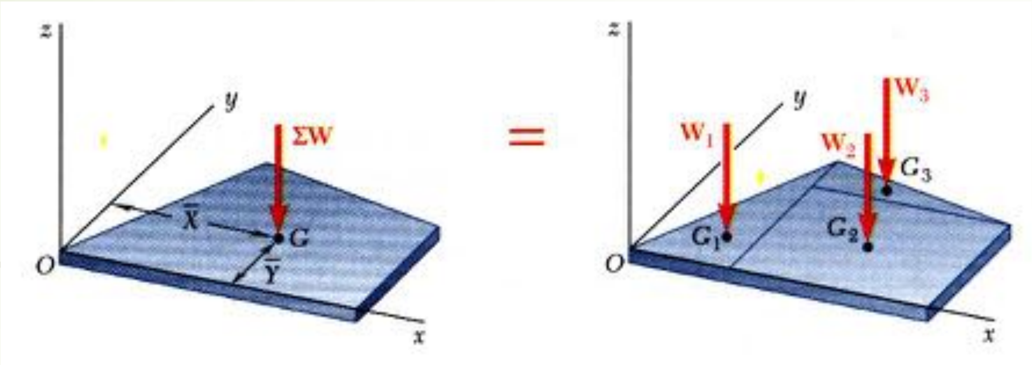






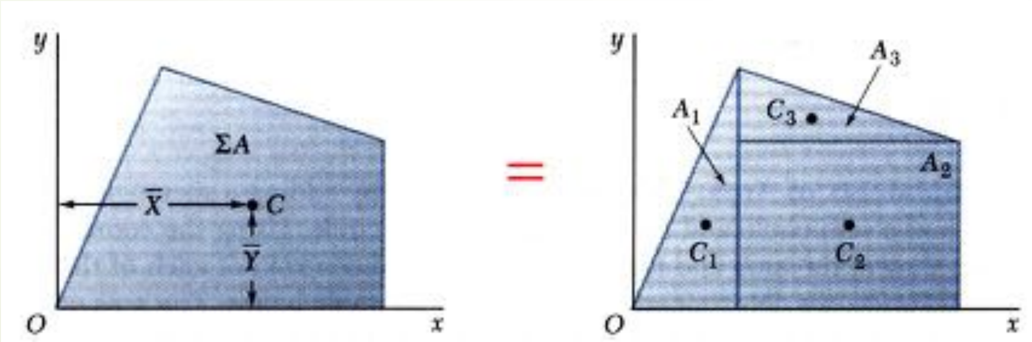
- An area is symmetric with respect to an axis *BB'* if for every point *P* there exists a point *P'* such that *PP'* is perpendicular to *BB'* and is divided into two equal parts by *BB'*.
- The first moment of an area with respect to a line of symmetry is zero.
- If an area possesses a line of symmetry, its centroid lies on that axis
- If an area possesses two lines of symmetry, its centroid lies at their intersection.
- An area is symmetric with respect to a center *O* if for every element *dA* at (*x*,*y*) there exists an area *dA'* of equal area at (-*x*,-*y*).
- The centroid of the area coincides with the center of symmetry.

Composite Plates and Areas



- Composite plates

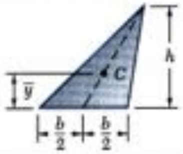
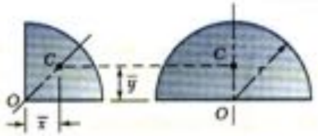
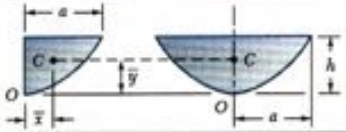
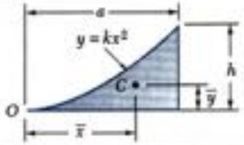
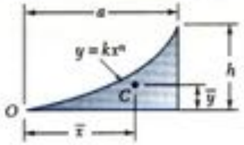
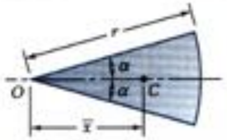
$$\bar{X}\sum W = \sum \bar{x} W$$
$$\bar{Y}\sum W = \sum \bar{y} W$$



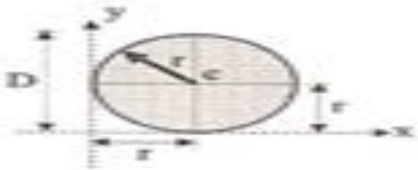
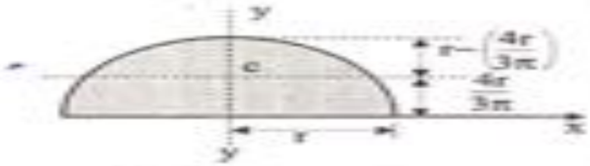
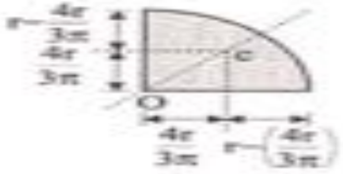
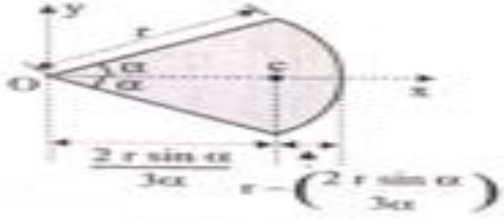
- Composite area

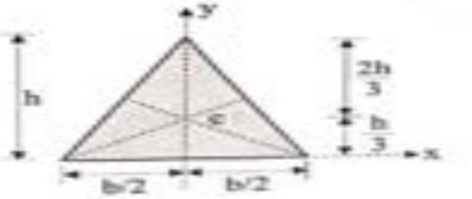
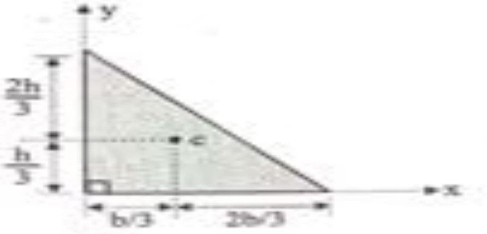
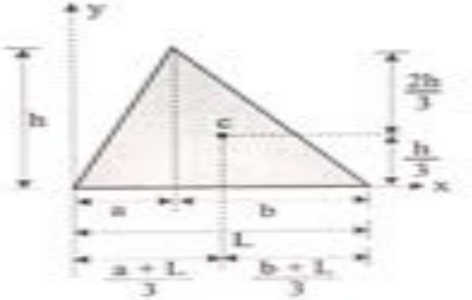
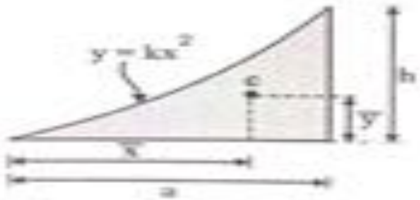
$$\bar{X}\sum A = \sum \bar{x} A$$
$$\bar{Y}\sum A = \sum \bar{y} A$$

Centroids of Common Shapes of Areas

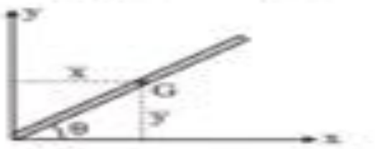
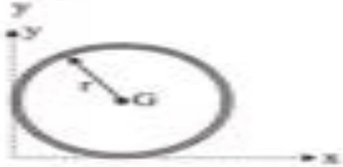
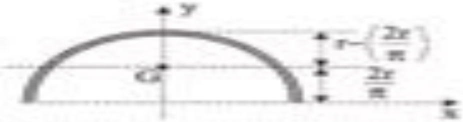
Shape		$\bar{x}$	$\bar{y}$	Area
Triangular area			$\frac{h}{3}$	$\frac{bh}{2}$
Quarter-circular area		$\frac{4r}{3\pi}$	$\frac{4r}{3\pi}$	$\frac{\pi r^2}{4}$
Semicircular area		0	$\frac{4r}{3\pi}$	$\frac{\pi r^2}{2}$
Quarter-elliptical area		$\frac{4a}{3\pi}$	$\frac{4b}{3\pi}$	$\frac{\pi ab}{4}$
Semielliptical area		0	$\frac{4b}{3\pi}$	$\frac{\pi ab}{2}$
Semiparabolic area		$\frac{3a}{8}$	$\frac{3h}{5}$	$\frac{2ah}{3}$
Parabolic area		0	$\frac{3h}{5}$	$\frac{4ah}{3}$
Parabolic spandrel		$\frac{3a}{4}$	$\frac{3h}{10}$	$\frac{ah}{3}$
General spandrel		$\frac{n+1}{n+2}a$	$\frac{n+1}{4n+2}h$	$\frac{ah}{n+1}$
Circular sector		$\frac{2r\sin\alpha}{3\alpha}$	0	αr^2

Centroid and Areas of Some Common Plane Areas

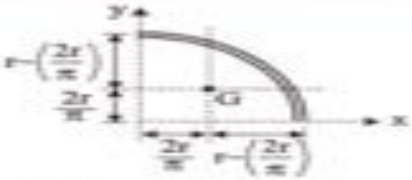
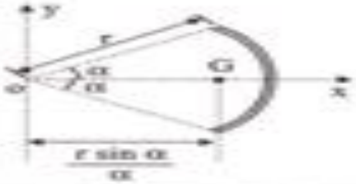
Sr. No.	Name	Shape	Area	x	y
1.	Rectangle		$b \times d$	$b/2$	$d/2$
2.	Square		a^2 OR $\frac{(\text{Diagonal})^2}{2}$	$\frac{a}{2}$	$\frac{a}{2}$
3.	Circle		πr^2 OR $\frac{\pi}{4} D^2$	r	r
4.	Semi circle		$\frac{\pi r^2}{2}$	0	$\frac{4r}{3\pi}$
5.	Quarter circle		$\frac{\pi r^2}{4}$	$\frac{4r}{3\pi}$	$\frac{4r}{3\pi}$
6.	Sector of a circle		αr^2 (α in radian) Note : α is semiangle	$\frac{2r\sin\alpha}{3\alpha}$ (α in degree) (α in radian)	0

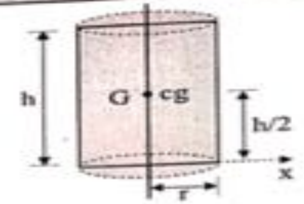
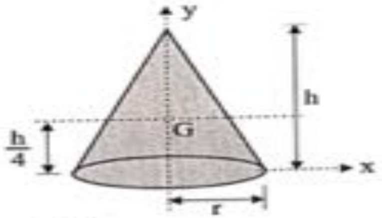
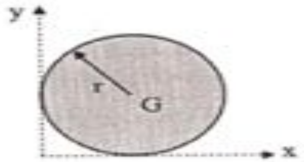
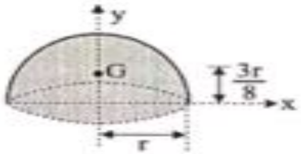
Sr. No.	Name	Shape	Area	x	y
7.	Triangle				
(a)	Symmetrical triangle		$\frac{1}{2} \times b \times h$	0	h/3
(b)	Right angled triangle		$\frac{1}{2} \times b \times h$	b/3	h/3
(c)	Unsymmetrical triangle		$\frac{1}{2} \times L \times h$	$\frac{a+L}{3}$	$\frac{h}{3}$
8.	Trapezoid		$\left(\frac{a+b}{2}\right)h$	—	—
9.	Parabolic spandrel		$\frac{ah}{3}$	$\frac{3a}{4}$	$\frac{3h}{10}$

Some common line segments

Sr. No.	Name	Shape	Area	x	y
10.	Parabolic area		$\frac{4ah}{3}$	0	$\frac{3h}{5}$
11.	A straight line		L	L/2	0
12.	An inclined line		L	$\frac{L}{2}\cos\theta$	$\frac{L}{2}\sin\theta$
13.	Circular Arc		$2\pi r$	r	r
14.	Semi. circular Arc		πr	0	$\frac{2r}{\pi}$

Some Common solid bodies : [only for reference]

Sr. No.	Name	Shape	Length	x	y
15.	Quarter circular Arc		$\frac{\pi r}{2}$	$\frac{2r}{\pi}$	$\frac{2r}{\pi}$
16.	An arc of a circle		$2r\alpha$	$\frac{r\sin\alpha}{\alpha}$	0

Sr. No.	Name	Shape	Length	x	y
17.	A solid cylinder		$\pi r^2 h$	0	h/2
18.	A solid right circular cone		$\frac{1}{3}\pi r^2 h$	0	$\frac{h}{4}$
19.	Sphere		$\frac{4}{3}\pi r^3$	r	r
20.	Hemi-sphere		$\frac{2\pi r^3}{3}$	0	$\frac{3r}{8}$

3) Centroid of line's:

$$\bar{x} = \frac{l_1 x_1 + l_2 x_2 + l_3 x_3 + \cdots l_n x_n}{l_1 + l_2 + l_3 + \cdots + l_n}$$

$$\bar{y} = \frac{l_1 y_1 + l_2 y_2 + l_3 y_3 + \cdots + l_n y_n}{l_1 + l_2 + l_3 + \cdots l_n}$$

4) Centre of gravity of solid (vol^m) body:

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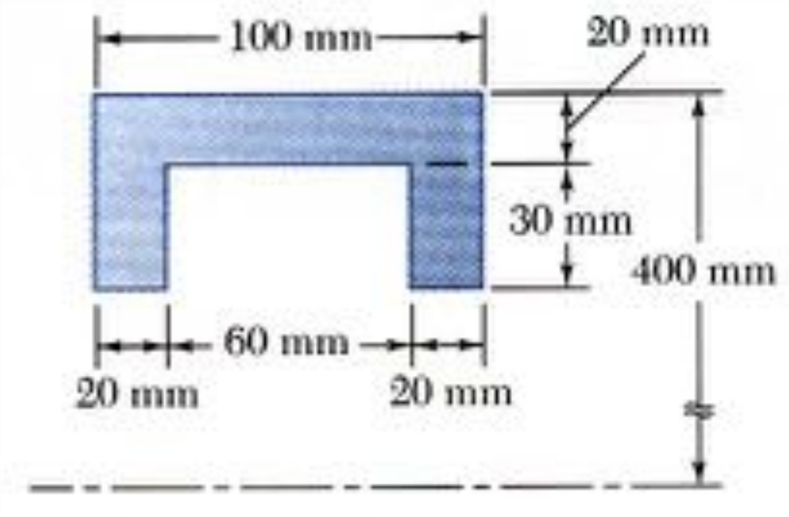
Date: / /

$$\bar{x} = \frac{V_1 x_1 + V_2 x_2 + \cdots V_n x_n}{V_1 + V_2 + \cdots V_n}$$

$$\bar{y} = \frac{V_1 y_1 + V_2 y_2 + \cdots V_n y_n}{V_1 + V_2 + \cdots V_n}$$

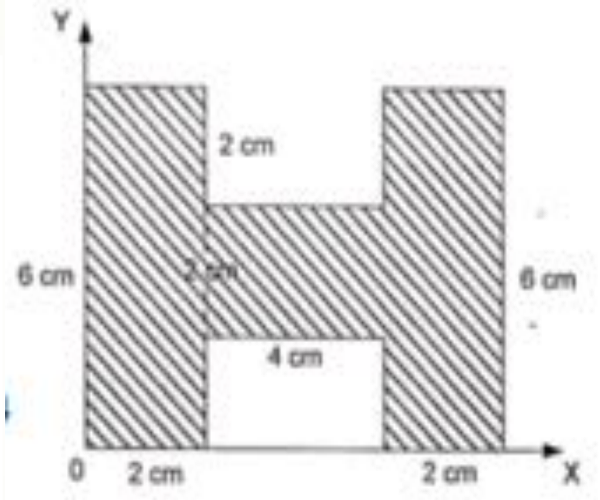
Sample Problem



Determine the coordinate of the center of gravity of the object as shown in the figure below.

Solution :

- Divide the object into three parts.
- Area of part 1 (A_1) = (2)(6) =
- The center point lies on the x axis (x_1) =
- The center point lies on the y axis (y_1) =
- Area of part 2 (A_2) = (4)(2) = 8 cm^2
- The center point lies on the x axis (x_2) =
- The center point lies on the y axis (y_2) =
- Area of part 3 (A_3) =



Solution :

- Divide the object into three parts.
- Area of part 1 (A_1) = (2)(6) = 12 cm^2
- The center point lies on the x axis (x_1) = 1/2 (2) = 1 cm
- The center point lies on the y axis (y_1) = 1/2 (6) = 3 cm
- Area of part 2 (A_2) = (4)(2) = 8 cm^2
- The center point lies on the x axis (x_2) = 2 + (1/2)(4) = 2 + 2 = 4 cm
- The center point lies on the y axis (y_2) = 2 + (1/2)(2) = 2 + 1 = 3 cm
- Area of part 3 (A_3) = (2)(6) = 12

The center point lies on the x axis (x_3) = 2 + 4 + (1/2)(2) = 2 + 4 + 1 = 7 cm
The center point lies on the y axis (y_3) = 1/2 (6) = 3 cm
Coordinate of the center of gravity at x axis :

$$x = \frac{x_1 A_1 + x_2 A_2 + x_3 A_3}{A_1 + A_2 + A_3}$$

$$x = \frac{(1)(12) + (4)(8) + (7)(12)}{12 + 8 + 12}$$

$$x = \frac{12 + 32 + 84}{32}$$

$$x = \frac{128}{32}$$

$$x = 4\ cm$$

Coordinate of the center of gravity at y axis :

$$y = \frac{y_1 A_1 + y_2 A_2 + y_3 A_3}{A_1 + A_2 + A_3}$$

$$y = \frac{(3)(12) + (3)(8) + (3)(12)}{12 + 8 + 12}$$

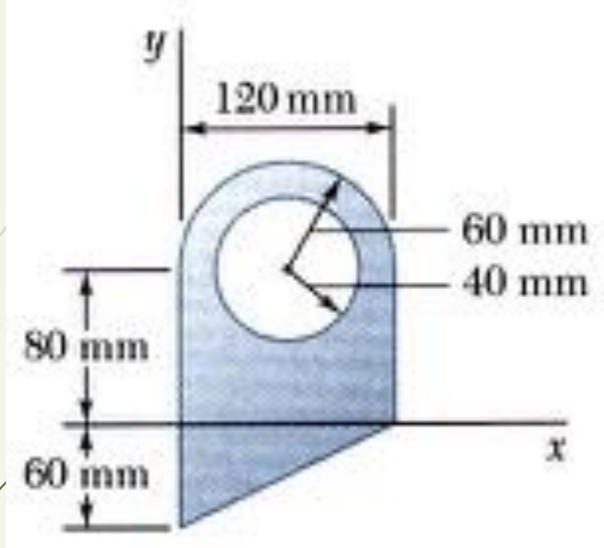
$$y = \frac{36 + 24 + 36}{32}$$

$$y = \frac{96}{32}$$

$$y = 3 cm$$

Coordinate of the center of gravity of the object is at x axis and y axis (x , y) = (4, 3)

Sample Problem

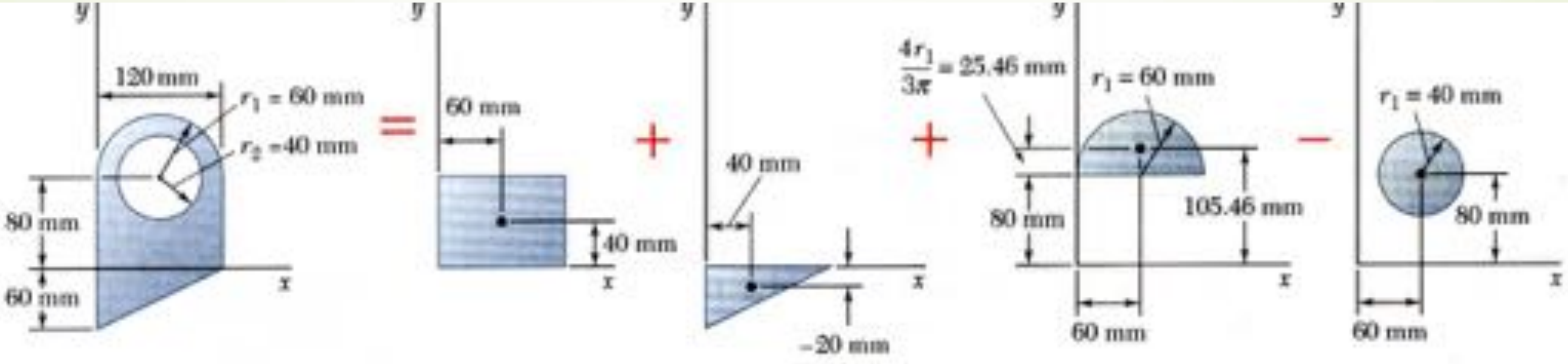


For the plane area shown, determine the first moments with respect to the *x* and *y* axes and the location of the centroid.

<u>SOLUTION</u>:

- Divide the area into a triangle, rectangle, and semicircle with a circular cutout.
- Calculate the first moments of each area with respect to the axes.
- Find the total area and first moments of the triangle, rectangle, and semicircle. Subtract the area and first moment of the circular cutout.
- Compute the coordinates of the area centroid by dividing the first moments by the total area.

Sample Problem 5.1



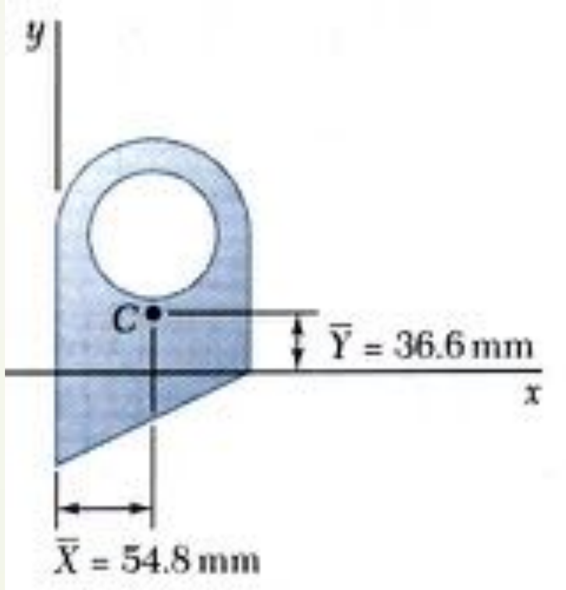
Component	A, mm²	$\bar{x}$, mm	$\bar{y}$, mm	$\bar{x}A$, mm³	$\bar{y}A$, mm³
Rectangle	$(120)(80) = 9.6 \times 10^3$	60	40	$+576 \times 10^3$	$+384 \times 10^3$
Triangle	$\frac{1}{2}(120)(60) = 3.6 \times 10^3$	40	−20	$+144 \times 10^3$	-72×10^3
Semicircle	$\frac{1}{2}\pi(60)^2 = 5.655 \times 10^3$	60	105.46	$+339.3 \times 10^3$	$+596.4 \times 10^3$
Circle	$-\pi(40)^2 = -5.027 \times 10^3$	60	80	-301.6×10^3	-402.2×10^3
	$\Sigma A = 13.828 \times 10^3$			$\Sigma\bar{x}A = +757.7 \times 10^3$	$\Sigma\bar{y}A = +506.2 \times 10^3$

- Find the total area and first moments of the triangle, rectangle, and semicircle. Subtract the area and first moment of the circular cutout.

$$\boxed{\begin{aligned} Q_x &= +506.2 \times 10^3 \text{ mm}^3 \\ Q_y &= +757.7 \times 10^3 \text{ mm}^3 \end{aligned}}$$

Sample Problem 5.1

- Compute the coordinates of the area centroid by dividing the first moments by the total area.



$$\bar{X} = \frac{\sum \bar{x} A}{\sum A} = \frac{+757.7 \times 10^3\ \text{mm}^3}{13.828 \times 10^3\ \text{mm}^2}$$

$$\boxed{\bar{X} = 54.8\ \text{mm}}$$

$$\bar{Y} = \frac{\sum \bar{y} A}{\sum A} = \frac{+506.2 \times 10^3\ \text{mm}^3}{13.828 \times 10^3\ \text{mm}^2}$$

$$\boxed{\bar{Y} = 36.6\ \text{mm}}$$

Second moment of area- MOMENT OF INERTIA OF AREA

Consider an area of a surface consists of large number of small elements of area dA each. The area integral of all such elements can be written mathematically as under :

$$\int_A (dA) = A$$

Referring Figure 4.37, the Area Moment of Inertia of elemental area dA about x axis, in its plane is defined as

$$I_{x\,(\text{Element})} = dA \times y^2$$

$$I_{y\,(\text{Element})} = dA \times x^2$$

Since, the axis x lies in the plane of element, these are also called as axial moment of inertia of the element dA.

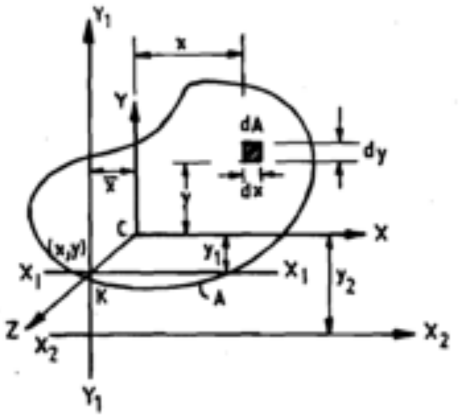


Figure 4.37

Polar moment of inertia of dA about z axis perpendicular to plane of A,

$$I_{z\,(\text{Element})} = dA\,(x^2 + y^2)$$

* Moment of Inertia :–

Inertia :-

MI – associated in plane bodies

MI – second moment of area.

– It is a tendency of body to continue in its state of rest or of uniform motion.

moment of Inertia = 2nd moment of area

$= y^2 \, dA$

or $= x^2 \, dA$ } elemental

I_{xx}	$= \int y^2 \, dA$
I_{yy}	$= \int x^2 \, dA$

Note:- 1) Moment of Inertia is purely mathematical term. Although it is important incase of various engg. design because M.I. shows 'Resistance' to rotation.

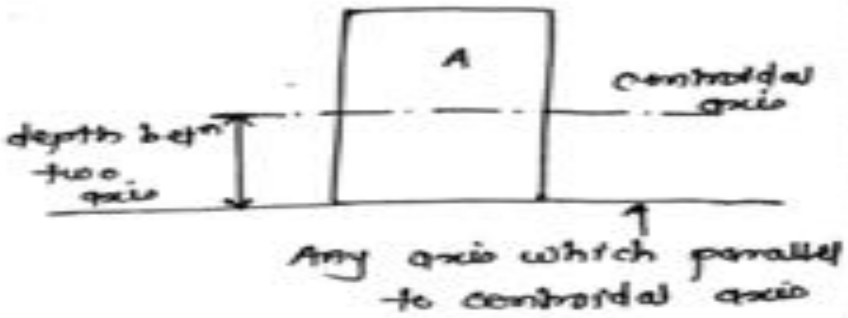
2) When M.I. of body or section is more its resistance to rotation is more.

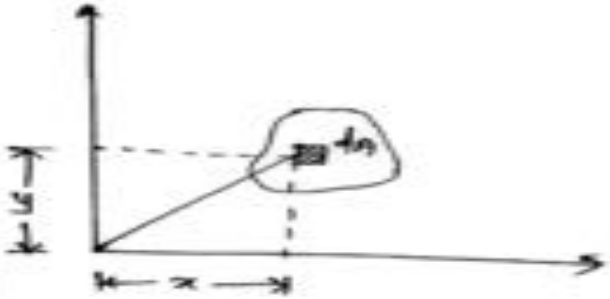
3) Generally the axis about which tendency of bending is more should be design appropriately against that bending effect.

* Theorem of M.I :-

parallel axis thrm
(Transfer thrm)

$$\boxed{I_{parallel} = \underset{\text{centroidal axis}}{I} + Ah^2}$$





$I_{xx} = \int y^2 \, dA.$

$I_{yy} = \int x^2 \, dA.$

$I_p = I_{zz} = \int r^2 \, dA.$

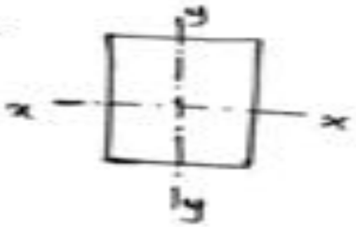
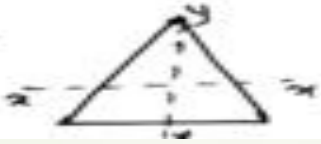
$x^2 + y^2 = r^2$

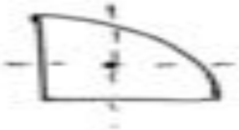
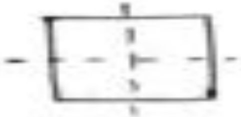
$\int x^2 dA + \int y^2 dA = \int r^2 dA.$

$I_{yy} + I_{xx} = I_{zz}$

$$\boxed{I_{polar} = I_{zz} = I_{xx} + I_{yy}.}$$

* M.I. of various plane fig.

Name	fig.	I_{xx}	I_{yy}
1) Rectangle		$\frac{bd^3}{12}$	$\frac{db^3}{12}$
2) Triangle		$\frac{bh^3}{36}$	

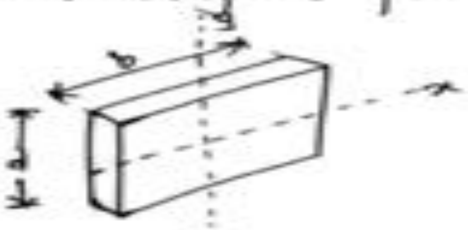
		I_{xx}	I_{yy}
3) Circle		$\frac{\pi d^4}{64}$	$\frac{\pi d^4}{64}$
4) Semi circle		$I_{xx} = 0.11\, r^4$	
5) Quarter circle		$0.55\, r^4$	
6) square		$\frac{b^4}{12}$	$\frac{b^4}{12}$

* Mass moment of Inertia for various shapes of bodies.

1) Uniform rod of length l - $ml^2/12$

m = entire mass of rod.

2) Rectangular plate of size $(a \times b)$ & thick 't'

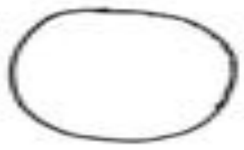


$I_{yy} = \frac{Mb^2}{12}$

$I_{xx} = \frac{Ma^2}{12}$

$I_P = I_{zz} = \frac{M}{12}(a^2+b^2)$

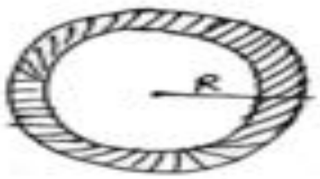
3) Circular plate — $\frac{mr^2}{4}$



$$I_{xx} = I_{yy} = \frac{Mr^2}{4}$$

$$I_{zz} = I_p = \frac{mr^2}{2}.$$

4) Circular ring of uniform cross-section.



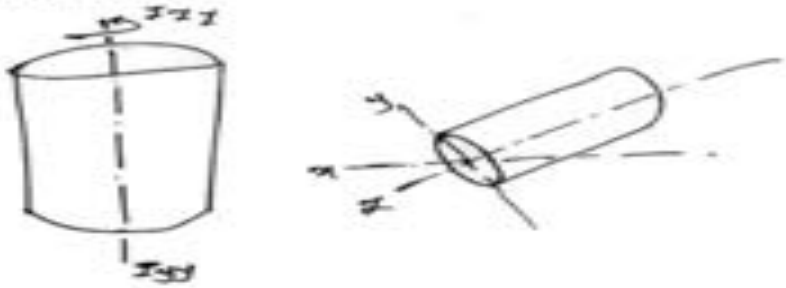
$$I_{xx} = I_{yy} = \frac{Mr^2}{2}$$

$$I_p = I_{zz} = Mr^2$$

5) Solid sphere of radius 'r'

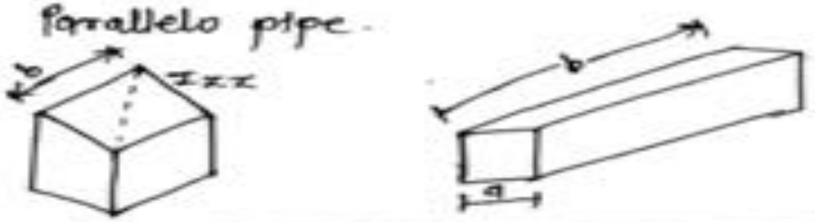
$$I_{xx} = I_{yy} = \frac{2}{5} mr^2$$

6) cylinder



$$I_{zz} = \frac{mr^2}{2}.$$

7) Parallelo pipe.



$$I_{zz} = \frac{M}{12}(a^2+b^2)$$

Perpendicular *Axis* Theorem

Polar Moment of Inertia of A about z axis passing through C. Referring Figure 4.37, where axis ZC is perpendicular to the plane of area A, we have

$$I_{z(A)} = \int_A dA\,(x^2 + y^2)$$

i.e.,

$$I_{z(A)} = I_{y(A)} + I_{x(A)}$$

Moment of Inertia of the area A about any axis X_1X_1 shown in Figure 4.37 is given by

$$I_{(X_1X_1)} = \int dA \times \text{(square of distance from } X_1X_1 \text{ axis)}$$

$$= \int dA \times (y + y_1)^2$$

where y_1 is the perpendicular distance between X_1X_1 and CX. Thus for a given axis X_1X_1, y_1 is constant.

Similarly,

$$I_{X_2X_2} = \int dA \times (y + y_2)^2$$

4.5.3 Parallel Axis Theorem

With reference to Figure 4.37 , it is noted that first moment of elemental area dA about centroidal axis is given by $(dA \times y)$. By the definition of centroid C of the area, it is further noted that

$$\int_A dA \times Y = 0$$

This means that horizontal plate of area A gets balanced about axis CX. When Moment of Inertia of areas are computed about any random axis X_1X_1, then

$$I_{(X_1X_1)} = \int dA(y + y_1)^2$$

$$= \int dA(y^2 + 2yy_1 + y_1^2)$$

$$= \int dA \times y^2 + 2y_1 \int dA \times y + y_1^2 \int dA$$

$$= I_{cx} + 0 + A(y_1)^2$$

Similarly, $$I_{X_2X_2} = I_{cx} + A(y_2)^2.$$

This equation is termed as parallel axis theorem whereby it is observed that out of all axes parallel to centroidal axis, CX, the Moment of Inertia about the centroidal axis is minimum, for a given direction of the axis.

Similarly, Referring Figure 4.37,

$$I_{Y_1Y_1} = I_{cy} + A(x_1)^2$$

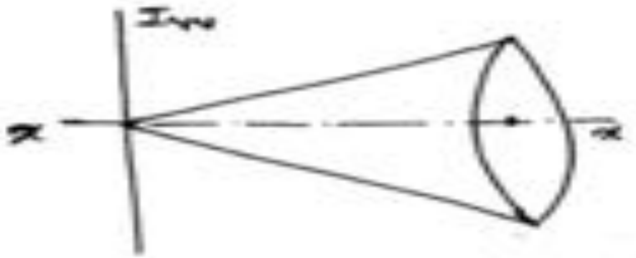
where x_1 is the perpendicular distance between the axes y_1 y_1 and CY.

$$I_{Z_1Z_1} = G_{cz} + A(x_1^2 + y_1^2)$$

where the perpendicular distance between axes z_1z_1 passing through point $K(x_1, y_1)$ and CZ is

$$r_1 = \sqrt{x_1^2 + y_1^2}$$

8) Solid cone

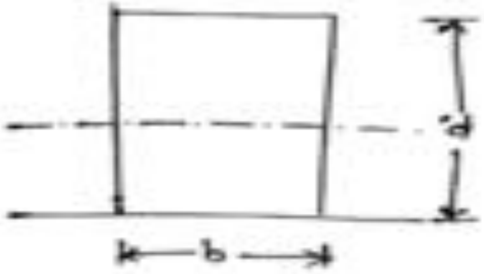


$$I_{xx} = \frac{3}{10} Mr^2$$

$$I_{yy} = \frac{3}{5} M \left(h^2 + \frac{R^2}{4} \right)$$

Que 1) findout m.I. for rectangular section about an axis parallel to the centroidal axis passing through the base.

→



$$I_{xx} = I_G + Ah^2$$

$$= \frac{bd^3}{12} + (b \times d) \cdot \left(\frac{d}{2}\right)^2$$

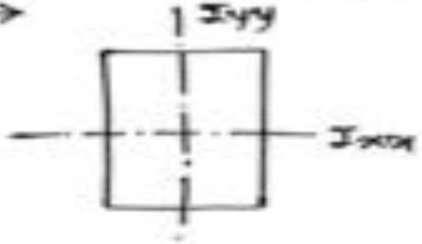
$$= \frac{bd^3}{12} + \frac{bd^3}{4}$$

$$= \frac{bd^3 + 3(bd^3)}{12}$$

$$= \frac{4\,bd^3}{12} = \frac{bd^3}{3}.$$

Que 2) Rectangular section about an axis passing through centroid & perpendicular to its plane (polar axis)

→



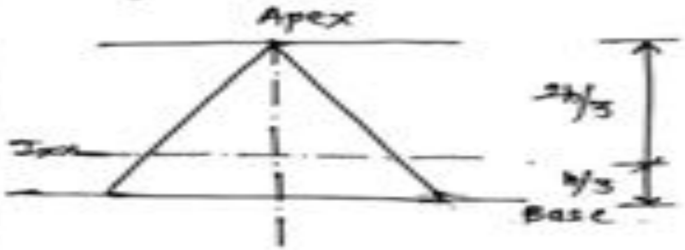
$$I_{zz} = I_{xx} + I_{yy}$$

$$= \frac{bd^3}{12} + \frac{db^3}{12}$$

$$= \frac{bd^3 + db^3}{12}$$

Que 3) Triangular section about an axis parallel to centroidal axis passing through.

i) base ii) Apex.



i) $I_G(\text{Base}) = I_G + Ah^2$

$$= \frac{bh^3}{36} + \left(\frac{1}{2}\times b\times h\right)\times\left(\frac{h}{3}\right)^2$$

$$= \frac{bh^3}{36} + \frac{bh^3}{18}$$

$$= \frac{3bh^3}{36} =$$

$$I_G(\text{Base}) = \frac{bh^3}{12}$$

ii) $I_G(\text{Apex}) = \frac{bh^3}{36} + \left(\frac{1}{2}\times b\times h\right)\times\left(\frac{2}{3}h\right)^2$

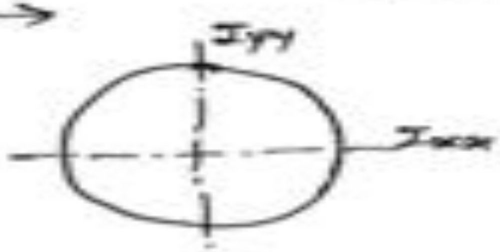
$$= \frac{bh^3}{36} + \frac{4bh^3}{18}$$

$$= \frac{9bh^3}{36} = \frac{bh^3}{4}$$

$$I_G(\text{Apex}) = \frac{bh^3}{4}.$$

Que 4> Circular section about an axis passing through centroid perpendicular to plane.

→



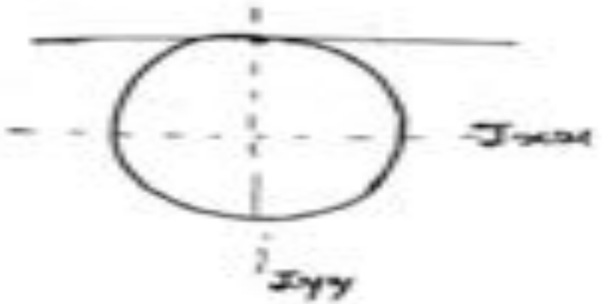
$$I_{zz} = I_{xx} + I_{yy}$$

$$= \frac{\pi}{64} d^4 + \frac{\pi}{64} d^4$$

$$I_{zz} = \frac{\pi d^4}{32}.$$

Que 5> circular section about an axis parallel to centroidal axis being tangent to the section within the plane of section.

→



$$I_{xx} = I_G + Ah^2$$

$$= \frac{\pi}{64} d^4 + \left(\frac{\pi}{4} d^2\right)\left(\frac{d}{2}\right)^2$$

$$= \frac{\pi}{64} d^4 + \frac{\pi d^4}{16}$$

$$I_{xx} = \frac{5\pi d^4}{64}$$

Que 6> Circular section about an axis parallel to polar axis touching the circumference & perpendicular to the plane plane of circle.

→

$$I_{xx} = I_G + Ah^2$$

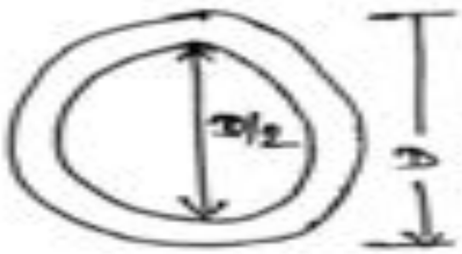
$$= \frac{\pi}{32}d^4 + \left(\frac{\pi}{4}d^2\right)\cdot\left(\frac{d}{2}\right)^2$$

$$= \frac{\pi}{32}d^4 + \frac{\pi d^4}{16}$$

$$= \frac{3\pi d^4}{32}$$

Que 7> Hollow circular section of inside diameter half of the outside diameter

→



$$\frac{\pi}{64}D^4 - \frac{\pi}{64}\left[D^4 - (D/2)^4\right]$$

$$= 15 \cdot \frac{\pi D^4}{64}$$

* Radius of Gyration :-

It is a math term def^n as $k = \sqrt{\frac{I}{A}}$

$I \propto k^2$

$k \propto \sqrt{I}$

$I = Ak^2$

$k = \sqrt{\frac{I}{A}}$

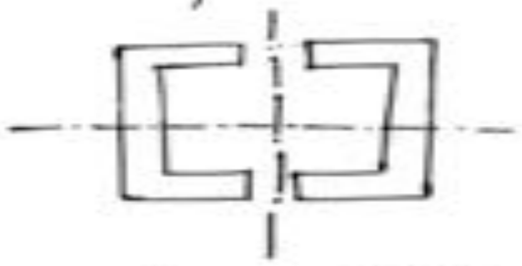
$k \propto \sqrt{\frac{bd^3}{12}}$

$k \propto \sqrt{b}$

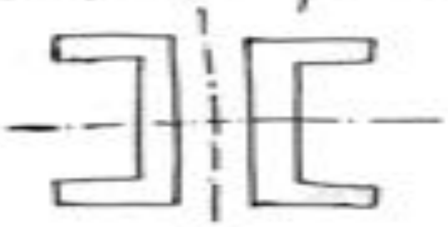
$k \propto d\sqrt{d}$

- It denotes resistance of material against rotation & bending
- Two sect^n having same area can be conclude tobe stronger or weaker on basis of radius of gyration.
- In simple words whenever mass concentration is away from centroid radius of gyration increase

Ex. Back to back channel section are weaker as compaired to channel section place end to end.



Stronger sect^n



Weaker sect^n

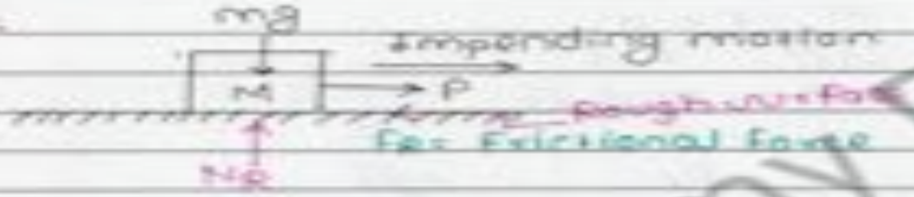
Friction

The force which opposes the movement or the tendency of movement is called **Frictional force or simply friction**. It is due to the resistance to motion offered by minutely projecting particles at the contact surfaces. However, there is a limit beyond which the magnitude of this force cannot increase.

* Friction *

Defn: When a body moves or tends to move over another body, a force opposing the motion develops at contact surface. This force which oppose the moment or tendency of moment is called frictional force or simply friction.

*** 1) Block Friction:**



Note:

- Frictional force acts opposite to the direction of motion.
- If surface is perfectly smooth then $F = 0$
- When two bodies are in contact and are in motion then the frictional force acts opposite to the dirn of relative motion.
- Frictional force is dependent on the material of contact surface (i.e. the nature of contact surface) and independent upon area of contact surface.

★ Types of Friction:

1) Dry Friction: (columb's Friction)

• Dry friction develops when the unlubricated surface of two solids are in contact under the condition of sliding or a tendency to slide.

2) Fluid Friction:

- Fluid Friction develop when adjacent layers in a fluid are moving at a different velocities.

note: Depends upon velocity gradient and viscosity of Fluid.

- Fluid friction depends upon relative velocity betn two layers.

* limiting equ^m cond^n OR Impending Motion OR At the verge of motion :-

Def^n: The condition just before start of motion is called limiting equ^m condition.

- At limiting equ^m condition,

$$\Sigma F_x = 0$$
$$\Sigma F_y = 0$$

* limiting Frictional force OR max^m Frictional force : (F_{max})

It is the max^m Frictional force developed at limiting equ^m condition called max^m F.F.

mathematically

$$\boxed{F_{max} = \mu_s N_R}$$

where μ_s = coefficient of static friction

N_R = normal reac^n

condition of sliding or a tendency to slide.

2) Fluid Friction:

- Fluid friction develop when adjacent layers in a fluid are moving at a different velocities.

note: Depends upon velocity gradient and viscosity of fluid.

- Fluid friction depends upon relative velocity betn two layers.

* Limiting equ^m cond^n or Impending Motion or At the verge of motion:

Def^n: The condition just before start of motion is called limiting equ^m condition.

- At limiting equ^m condition,

$$\Sigma F_x = 0$$
$$\Sigma F_y = 0$$

* Limiting frictional force or max^m frictional force: (Fmax)

It is the max^m frictional force developed at limiting equ^m condition called max^m F.F.

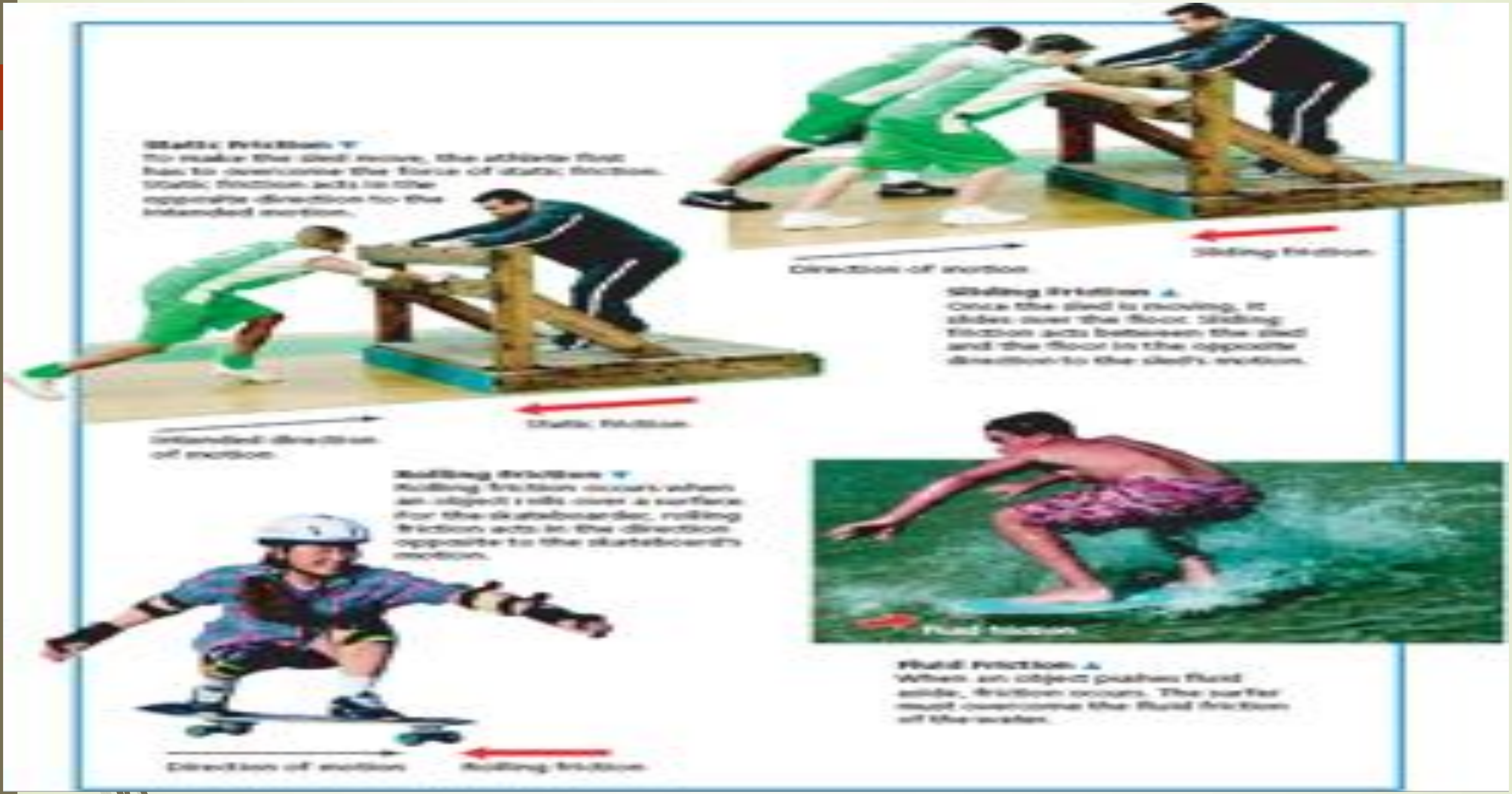
mathematically,

$$\boxed{F_{max} = \mu_s N_R}$$

where, μ_s = coefficient of static friction
N_R = normal reac^n

Friction :

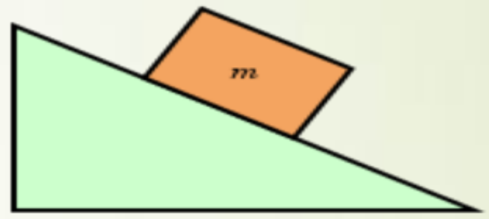
The force that two surfaces exert on each other when they rub against each other is called friction.



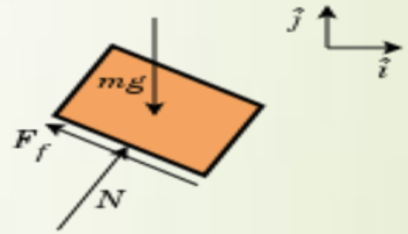
Technical terms :

1. Free body diagram(fbd) : A body isolated from its surrounding and shown by forces acting on it.
2. Static friction :The friction that acts on objects that are not moving is called as static friction.
3. Kinetic friction : Friction force acts on moving object is called as kinetic friction.
4. Impending motion: It is the state of body, where body is on the verge of motion but not in motion. In this state static friction reaches its upper limit. Friction at impending motion is called as limiting friction.

A block on a ramp

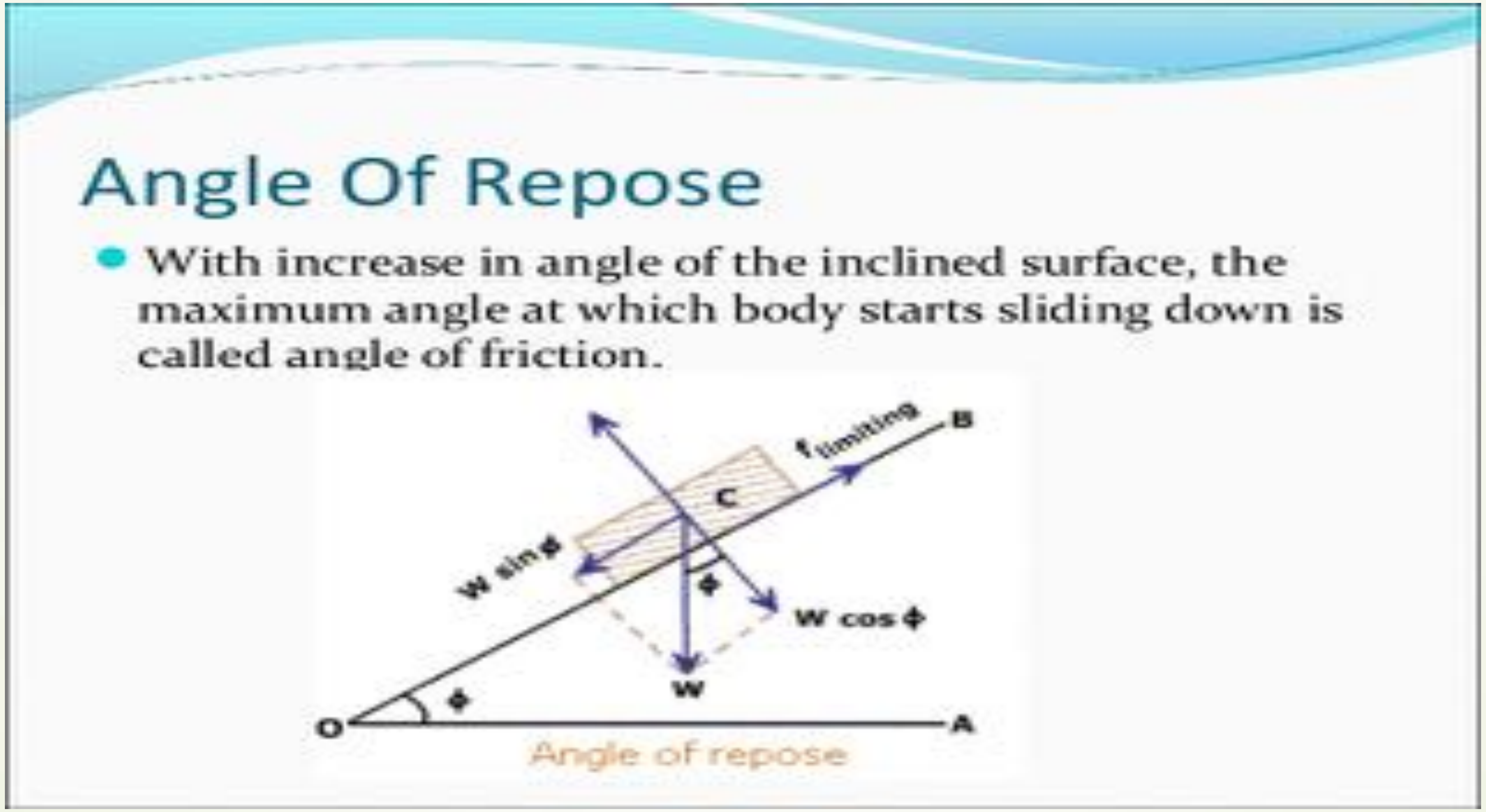


Free body diagram of just the block

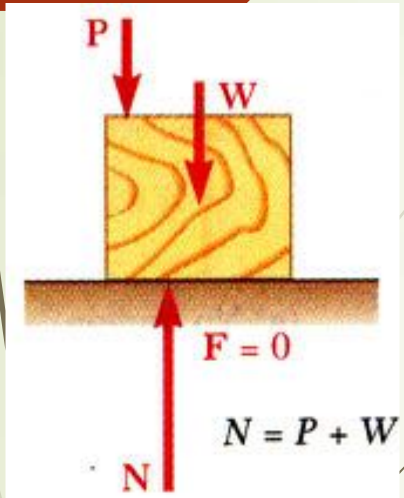


Technical terms:

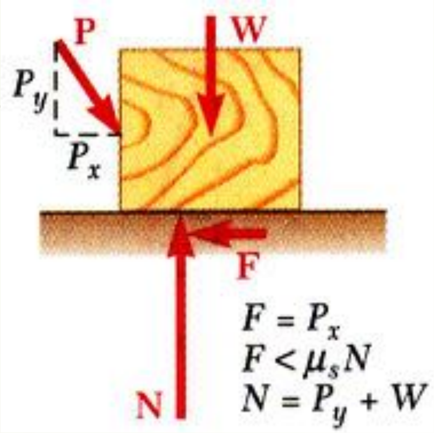
5. Angle of repose : **It is the maximum angle at which an object can rest on an inclined plane without sliding down. It is also defined as angle at which body is in the <u>impending motion due to its self weight</u>. It** can range from 0° to 90°



•Four situations can occur when a rigid body is in contact with a horizontal surface:

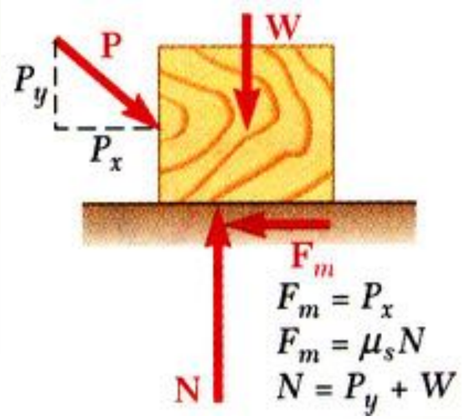


- No friction, ($P_x = 0$)



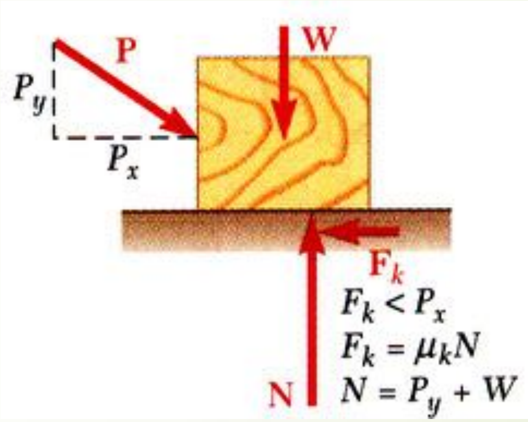
- No motion, ($P_x < F_m$)

Applied force is not enough to cause the motion



Impending motion ($P_x = F_m$)

Body is just about to slide



- Motion, ($P_x > F_m$)

Body is in the motion

Types of Friction

- If the applied force is more than this limit, there will be movement of one body over the other. This limiting value of frictional force when the motion is impending, it is known as **Limiting Friction**.
- When the applied force is less than the limiting friction, the body remains at rest and such frictional force is called **Static Friction**, which will be having any value between zero and the limiting friction.
- If the value of applied force exceeds the limiting friction, the body starts moving over the other body and the frictional resistance experienced by the body while moving is known as **Dynamic Friction**. Dynamic friction is less than limiting friction.

- Dynamic friction is classified into following two types:
  - a) Sliding friction
  - b) Rolling friction
- Sliding friction is the friction experienced by a body when it slides over the
other body.
- Rolling friction is the friction experienced by a body when it rolls over a surface.
- It is experimentally found that the magnitude of limiting friction bears a constant ratio to the normal reaction between two surfaces and this ratio is called **Coefficient of Friction**.

Laws of friction :

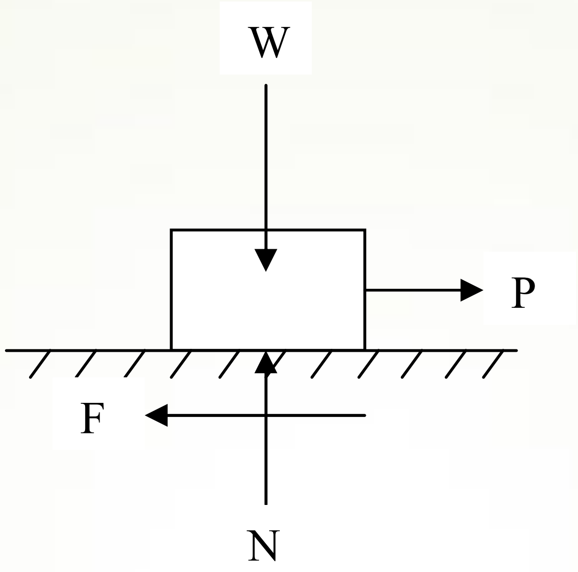
I. The friction of the moving object is proportional and perpendicular to the normal force.

II. The friction experienced by the object is dependent on the nature of the surface it is in contact with.

III. Friction is independent of the area of contact.

IV. Kinetic friction is independent of velocity.

V. The coefficient of static friction is greater than the coefficient of kinetic friction.



$$\text{Coefficient of friction} = \frac{F}{N}$$

where F is limiting friction and N is normal reaction between the contact surfaces.

Coefficient of friction is denoted by μ.

Thus, $\mu = \dfrac{F}{N}$

Angle of friction

Consider the block shown in figure resting on a horizontal surface and subjected to horizontal pull P. Let F be the frictional force developed and N the normal reaction. Thus, at contact surface the reactions are F and N. They can be graphically combined to get the reaction R which acts at angle θ to normal reaction. This angle θ called the angle of friction is given by

$$\tan\theta = \frac{F}{N}$$

As P increases, F increases and hence θ also increases. θ can reach the maximum value α when F reaches limiting value. At this stage,

$$\tan\alpha = \frac{F}{N} = \mu$$

This value of α is called Angle of Limiting Friction. Hence, the angle of limiting friction may be defined as the angle between the resultant reaction and the normal to the plane on which the motion of the body is impending.

★ limiting equ^m cond^n OR Impending Motion OR At the verge of motion :-

Def^n: The condition just before start of motion is called limiting equ^m condition.

- At limiting equ^m condition,

$$\Sigma F_x = 0$$
$$\Sigma F_y = 0$$

★ limiting Frictional force OR max^m Frictional force: (f_{max})

It is the max^m frictional force developed at limiting equ^m condition, called max^m f.f.

mathematically,

$$\boxed{f_{max} = \mu_s N_R}$$

where, μ_s : coefficient of static friction
N_R : normal rea^n

2) Static Friction (f) :- (F < Fmax)

• when the applied force is less than the limiting friction the body remain at rest and frictional force developed in such condition is called static friction.

note: Static friction is actual friction force developed for a given condn.

• range of static friction. $0 \le F$, $0 < F < F_{max}$

3) Dynamic or kinetic friction (F_k) :- ($F > F_{max}$) (motion → F_k)

• The frictional force betn the two surface when the body is in the motion is called dynamic or kinetic friction.

note: 1) If $F < F_{max}$, the body is in static equm.

$\Sigma F_x = 0$

$\Sigma F_y = 0$

2) If $F = F_{max}$, the body is in limiting equm

$\Sigma F_x = 0$

$\Sigma F_y = 0$

3) If $F > F_{max}$, the body is in motion.

(practically this condition does not exist)

which means,

$F = F_k = \mu_k \cdot N_R$

in motion

★ Coeffient of Friction :-

• Coeffient of friction is the ratio of frictional force to the normal reacn.

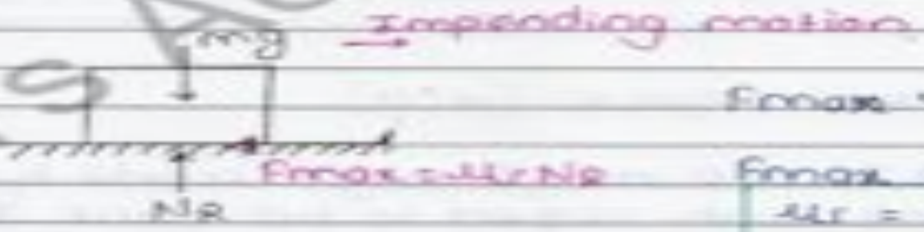
★ Types of coeffient of friction:

1) coe. of static friction. (μ_s)

• It is a ratio of limiting frictional force to the normal reacn.

• By experimental evidence it is prove that limiting frictional force is directly proportional to normal reaction at limiting eqm condition.

$\therefore F_{max} \propto N_R$



$F_{max} \propto N_R$

$F_{max} = \mu_s N_R$

$$\boxed{\mu_s = \frac{F_{max}}{N_R}}$$

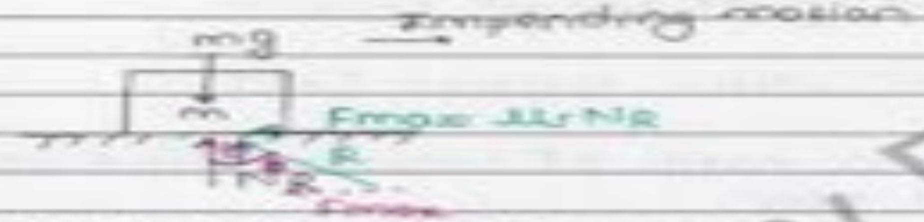
2) coe. of kinetic friction (μ_k):

• It is the ratio of kinetic frictional force to normal reacn.

$$\mu_k = \frac{F_k}{N_R}$$

* Angle of Friction (Φ) :-

Angle made by the resultant of limiting frictional force F_{max} and normal reacⁿ N_R with the normal reacⁿ.



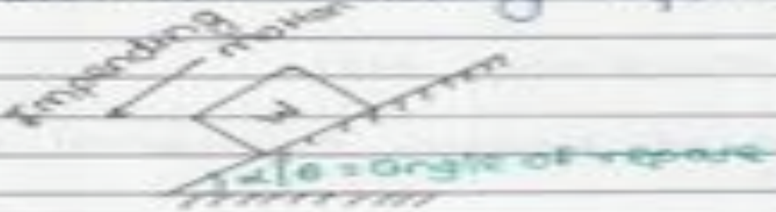
* Relation betⁿ coefficient of static friction and angle of friction at limiting equᵐ condition.

$$\tan\phi = \frac{F_{max}}{N_R}$$

$$\tan\phi = \frac{\mu_s N_R}{N_R} \therefore \boxed{\mu_s = \tan\phi} \rightarrow$$ for only limiting condⁿ

* Angle of repose: [α / θ]

"It is the maxᵐ angle made by the inclined plane with respect to hzl plane at which body remain in limiting equᵐ condition.



* Relation betⁿ coefficient of static friction and angle of repose at limiting equᵐ condⁿ :-

$\phi = \alpha$

$\boxed{\mu_s = \tan\alpha}$

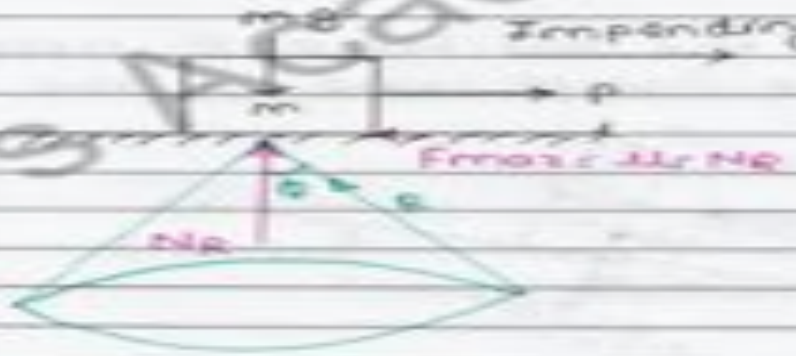
* Relation betn μ_s, ϕ and α at limiting equm condn:

$\mu_s = \tan\phi = \tan\alpha$

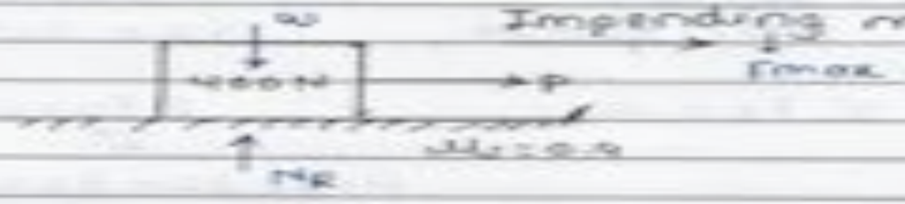
Note: At limiting equm condition angle of friction ϕ = angle of repose α.

* Cone of friction:

If the dirn of applied force is gradually changed through 360°, the resultant R generates a right circular cone with semi vertex angle equal to ϕ called cone of friction.



Ex:1) Find the min force reqd to cause impending motion



* Impending → F_{max}

By limiting equ^m cond^n.

$\Sigma F_y = 0$
$+\uparrow$

$N_R - 400 = 0$

$N_R = 400\,N$

$\Sigma F_x = 0$
$+\rightarrow$

$P - \mu_s N_R = 0$

$P = \mu_s N_R$

$= 0.4 \times 400$

$= 160\,N$.

Ex: 2) Determine the hz force 'P' needed to just start moving the 300 N crate up the plane shown in fig.

Sol^n.

$\Sigma F_y = 0$

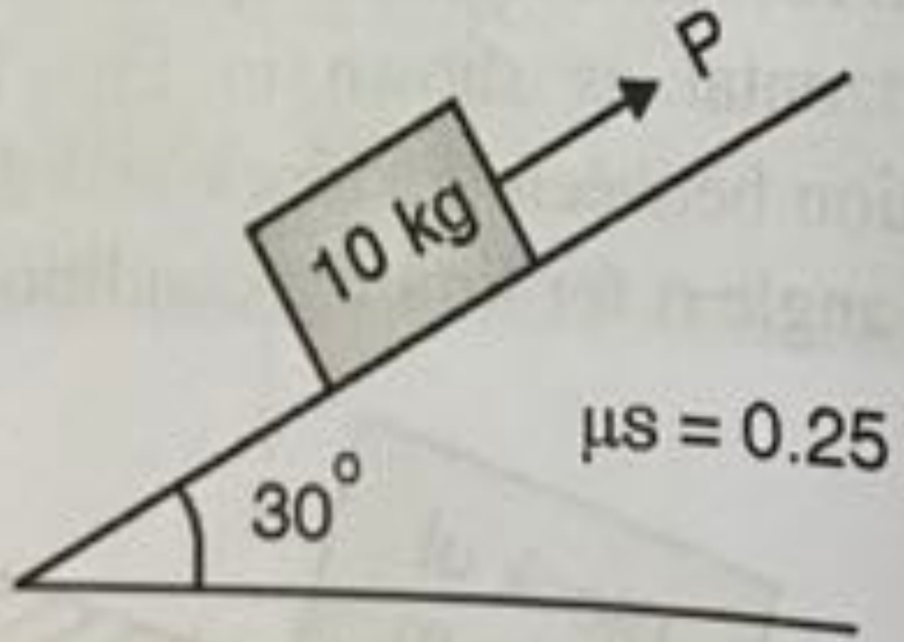
$N_R - 300\cos 30 = 0$ $N_R - P\sin 30 - 300\cos 30 = 0$

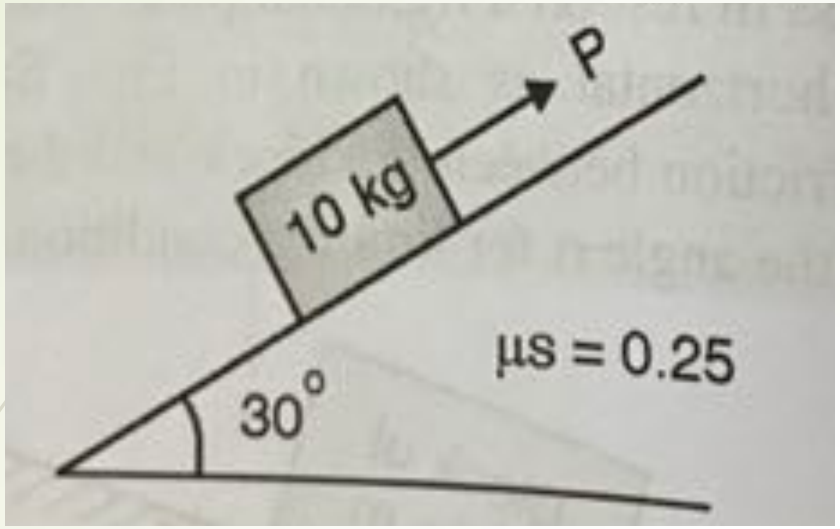
$N_R = 300\,N$ $N_R = 0.5P + 150\sqrt{3}$ --- (1)

$\Sigma F_x = 0$ $\Sigma F_x = 0$
$\rightarrow$

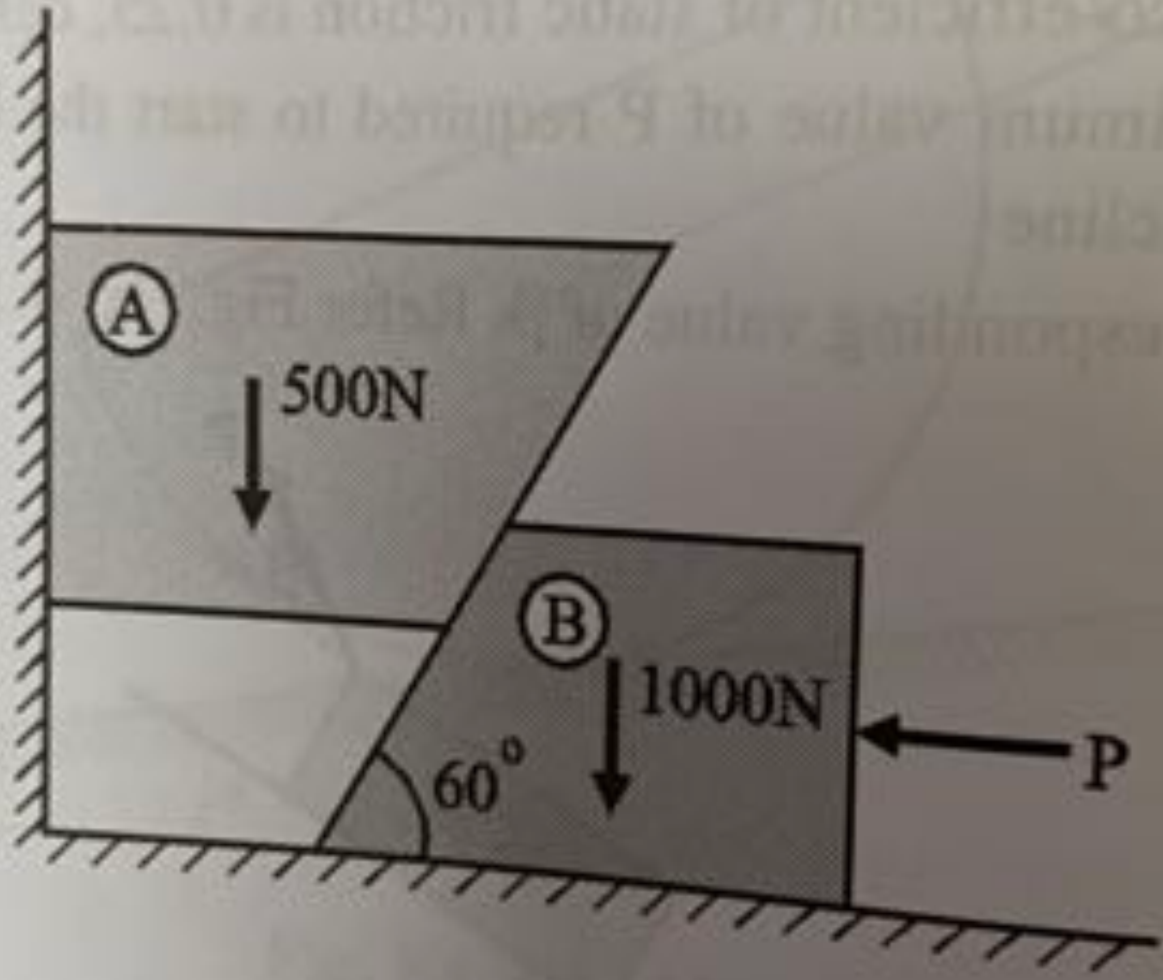
$P\cos 30 - 300\sin 30 - 0.3 N_R = 0$

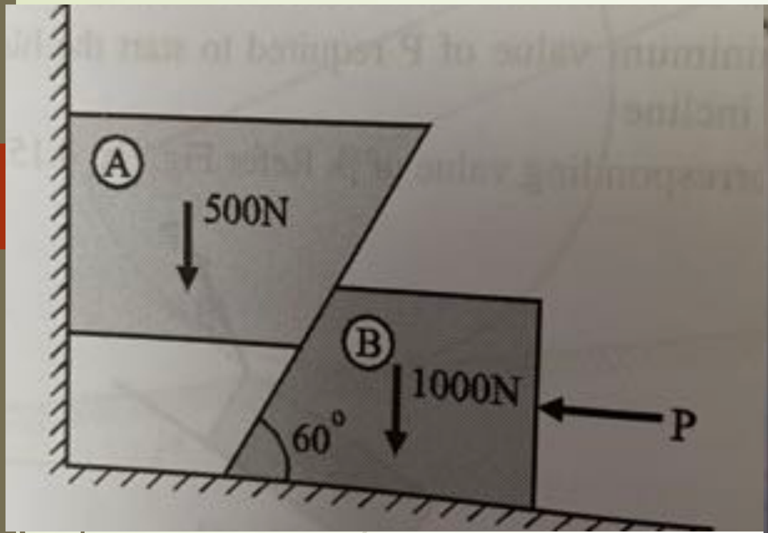
A block of mass 10 kg rests on an inclined plane as shown in Fig. Ex. 4.15.20. If the coefficient of static friction between the block and plane is $\mu_s = 0.25$, determine the maximum force P to maintain equilibrium.



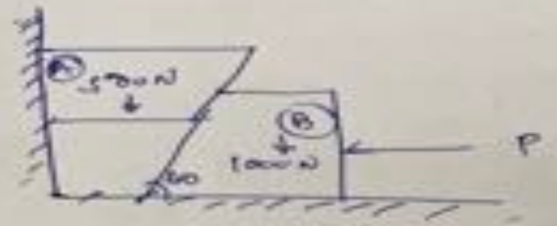

P
10 kg
μs = 0.25
30°

Two blocks A and B are resting against a wall and the floor as shown in Fig. Ex. 4.15.24. Find horizontal force P applied to the lower block that will **hold the system in equilibrium.** Take $\mu = 0.25$ at floor, $\mu = 0.3$ at wall and $\mu = 0.2$ between the blocks.

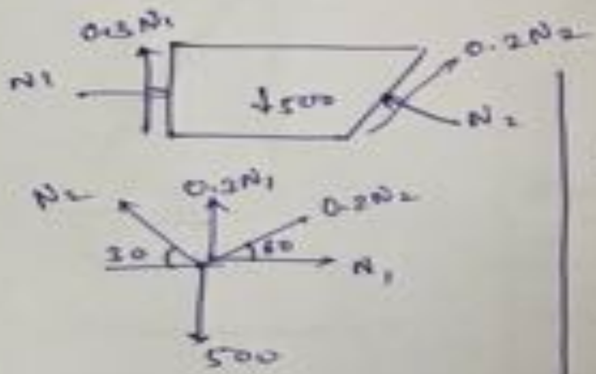




Soln



① FBD (A)



$\Sigma F_x = 0$

$N_1 + 0.2 N_2 \cos 60 - N_2 \cos 30 = 0$

$N_1 = 0.766 N_2$ — ①

$\Sigma F_y = 0$

$-500 + 0.2 N_2 \sin 60 + 0.3 N_1 + N_2 \sin 30 = 0$

Put $N_1 = 0.766 N_2$ — ②

$\boxed{N_2 = 553.7 \text{ N}}$

$\Rightarrow \boxed{N_1 = 424.1 \text{ N}}$

FBD (B)

$\Sigma F_x = 0$

$-0.25 N_3 - P - 0.2 N_2 \cos 60 + N_2 \cos 30 = 0$ — ③

$\Sigma F_y = 0$

$N_3 - 0.2 N_2 \sin 60 - 1000 - N_2 \sin 30 = 0$

$\Rightarrow N_3 = 1000 + 0.673 N_2$ — ④

$\boxed{N_3 = 1372 \text{ N}}$

Put all values in eq ③ i.e. $\Sigma F_x = 0$

$\Rightarrow \boxed{P = 81 \text{ N}}$

Ladder Friction

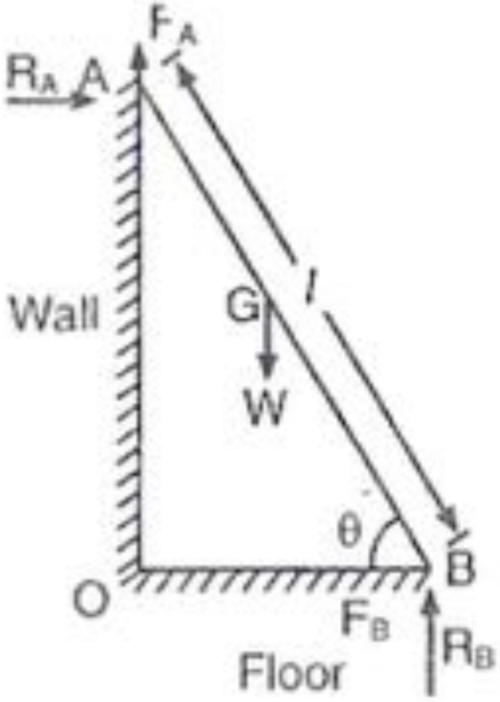
let us consider a ladder AB of length l and weight W resting on a wall and floor making an angle ∂ with the floor.

Let μ_a and μ_B are the coefficients of friction of wall and ladder, and floor and ladder respectively.

When **the lower end** of ladder tends to slip away from the wall, then the direction of frictional force F_B will be towards the wall.

When **the upper end** of the ladder moves Downward, the frictional force F_A will act upward.

R_A & R_B are the normal reactions at A and B respectiv



Ladder Friction

The ladder is in equilibrium under the action of the following forces:

a) Weight of the ladder

b) Normal reaction R_A at A

c) Frictional force F_A at A

d) Normal reaction R_B at B

e) Frictional force F_B at B

Resolving the forces horizontally, we get

$$\sum F_x = 0$$

$$R_A = F_B$$

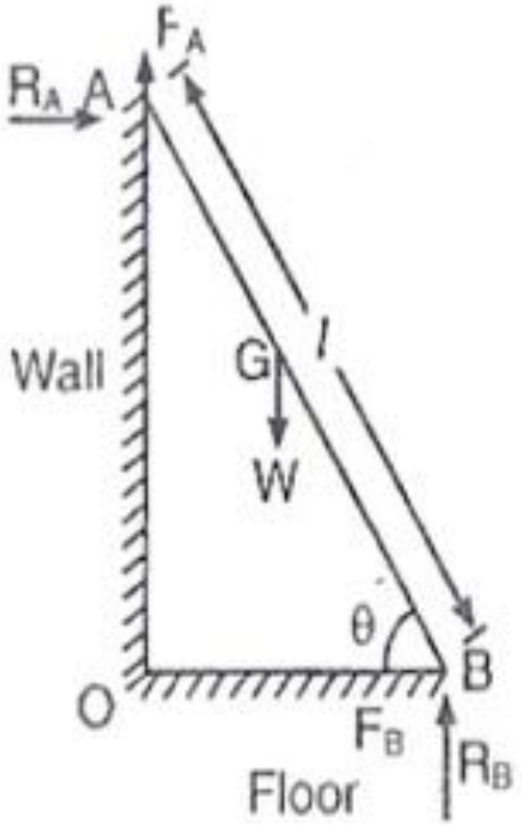
Resolving the forces vertically, we get

$$\sum Fy = 0$$

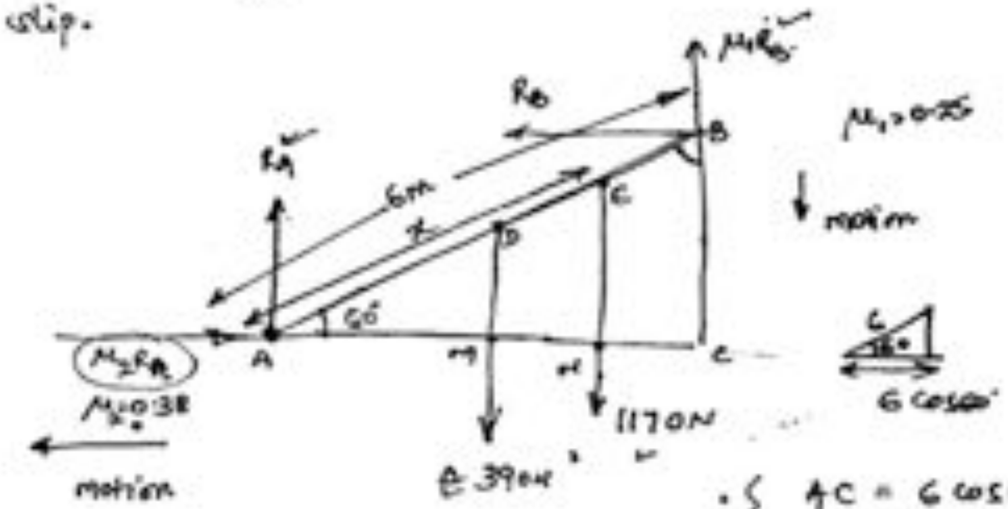
$$R_B + F_A = W$$

Moment about point B, we get

$$R_A \times l\sin\Theta + F_A \times l\cos\Theta = \frac{W.l\cos\Theta}{2}$$



P6: A ladder of weight 390 N and 6 m long is placed against a vertical wall
30° with wall. The co-efficient of friction between the ladder and wall is 0.25 and
that between ladder and floor is 0.38. Find how high a man of weight 1170 N
can ascend; before the ladder begin to slip.



$\Sigma F_x = 0$

$0.38 R_A - R_B = 0$

$0.38 R_A = R_B \quad \text{—— (1)}$

$0.38 R_A - R_B = 0$

$\Sigma F_y = 0$

$R_A + 0.25 R_B - 390 - 1170 = 0$

$R_A + 0.25 R_B = 1170 + 390$

$= 1560 \quad \text{—— (2)}$

$\Sigma M_A = 0$

$\boxed{R_A = 1424.65\,N}$ $\boxed{R_B = 541.36\,N}$

$390\,AM + 1170\,AN - R_B \times BC - \mu_1 R_B \times AC = 0$

$390 \times 3\cos 60° + 1170 \times x\cos 60° - 541.36 \times 6\sin 60° - 0.25 \times 541.36 \times 6\cos 60° = 0$

$1170\, x \cos 60° = 1445.1$

$x \cos 60° = 1.235$

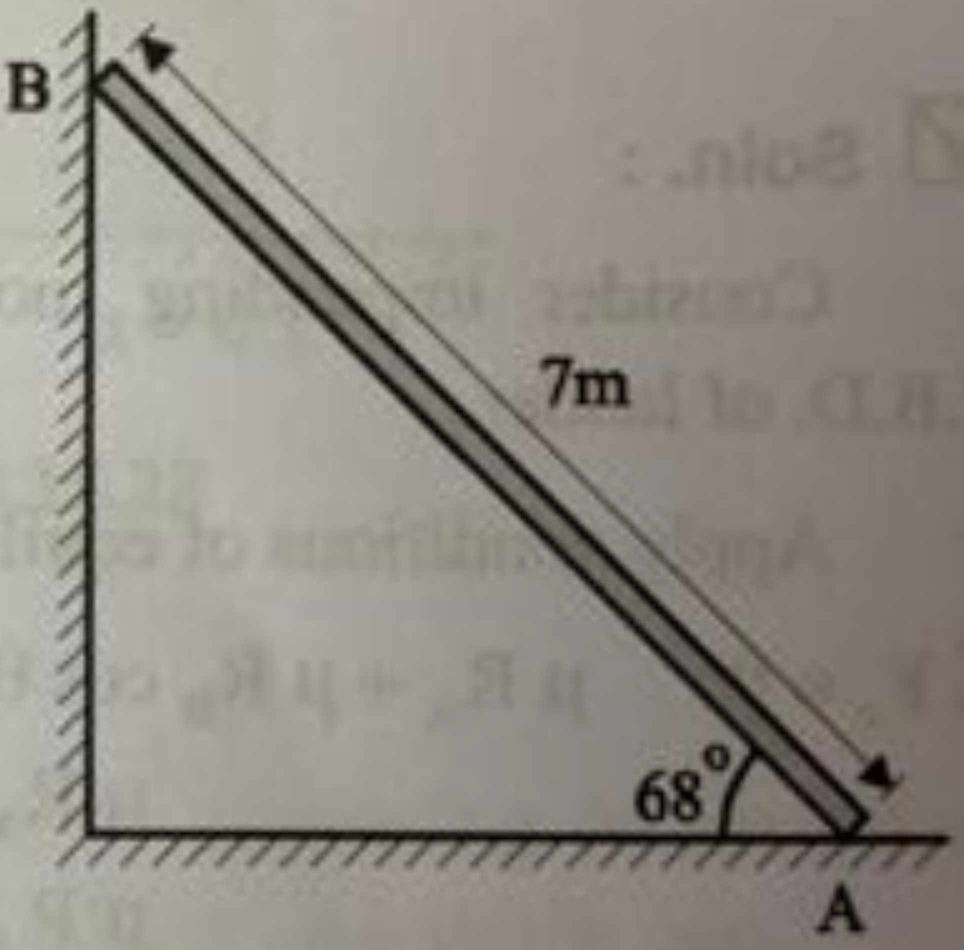
$\boxed{x = 2.47\,m}$ Ans.

$\therefore$ AC = 6 cos60°, BC = 6 sin60°, AM = 3 cos60°, AN = x cos60°

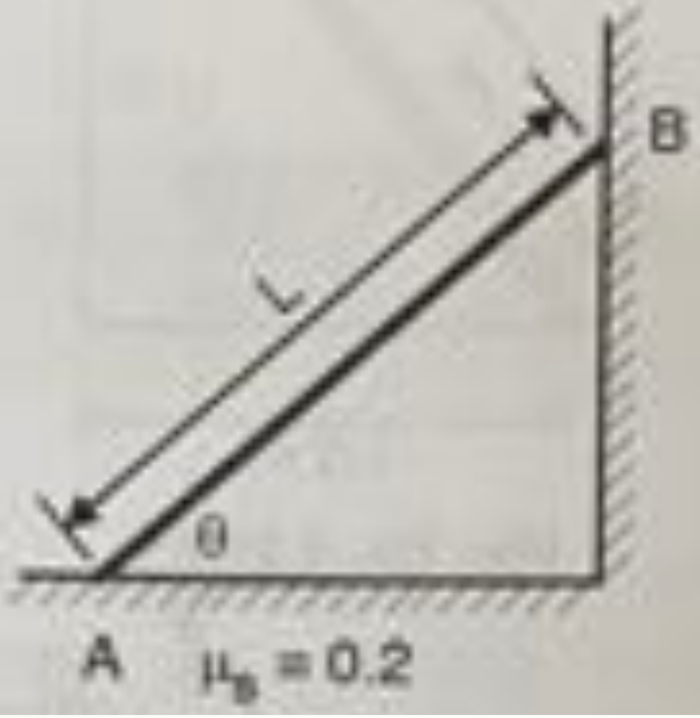
clockwise (+ve)
ACW (−ve)

390 (CW)
1170 (CW)
R_B (ACW)
$\mu_1 R_B$ (ACW)

A ladder of length 7 m leans against a wall as shown in Fig. Ex. 4.15.32. Assuming that the co-efficient of static friction μ_s is zero at B. Determine the smallest value of μ_s at A for which equilibrium is maintained.



The uniform rod having a weight W and length L is supported at its ends A and B as shown in Fig. Ex. 4.15.34, where the coefficient of static friction $\mu_s = 0.2$. Determine the greatest angle θ so that the rod does not slip. Refer Fig. Ex. 4.15.34.

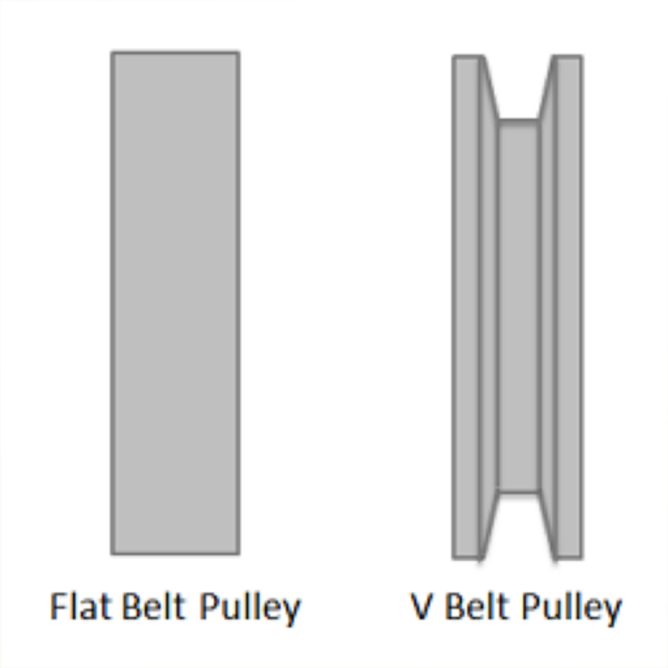


Belt Friction

- In any system where a belt or a cable is wrapped around a pulley or some other cylindrical surface, we have the potential for friction between the belt or cable and the surface it is in contact with.
- In some cases, such as a rope over a tree branch being used to lift an object, the friction forces represent a loss.
- In other cases such as a belt-driven system, these friction forces are put to use transferring power from one pulley to another pulley.

Friction in Belts

A flat belt is any system where the pulley or surface only interacts with the bottom surface of the belt or cable. If the belt or cable instead fits into a groove, then it is considered a V belt.



Derive Relation for Flat belt

Consider a flat belt passing over a fixed drum and belt is just about to slide towards right (impending motion)

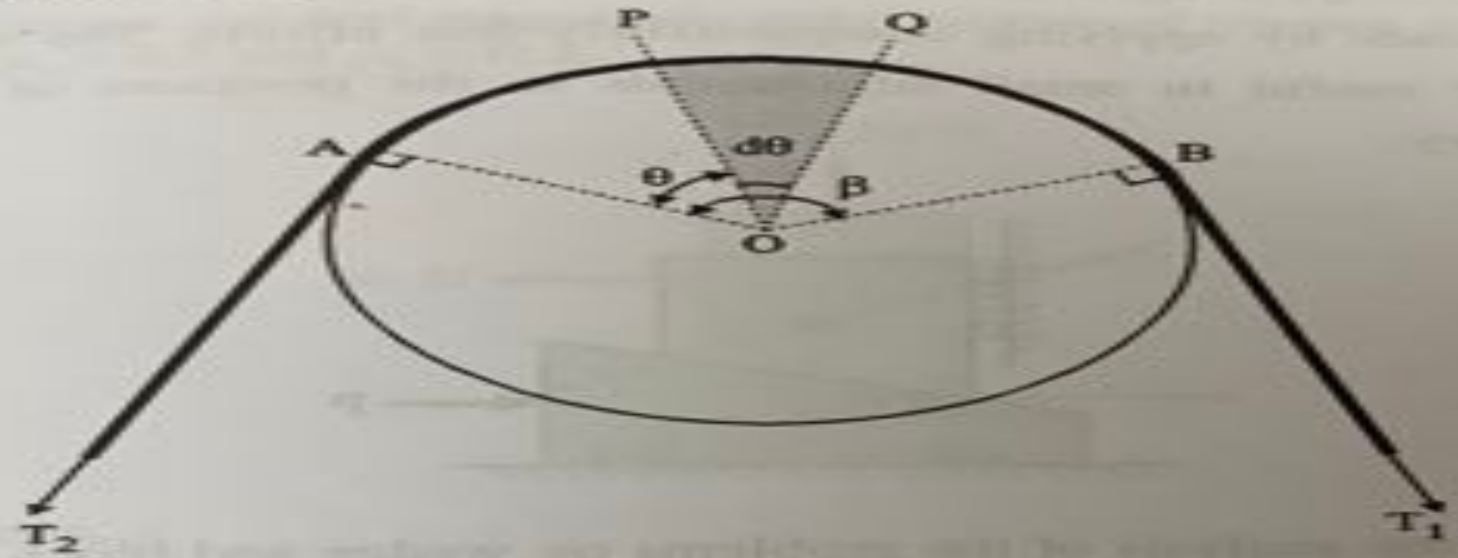
Let, T_1 = Tight side (greater tension)

T_2 = Slack side (smaller tension)

β = Lap Angle OR angle of contact **in Radians**.

μ_s = Co-efficient of static friction.

Now, consider a small elemental part PQ of the belt subtending an angle $d\theta$ at centre.



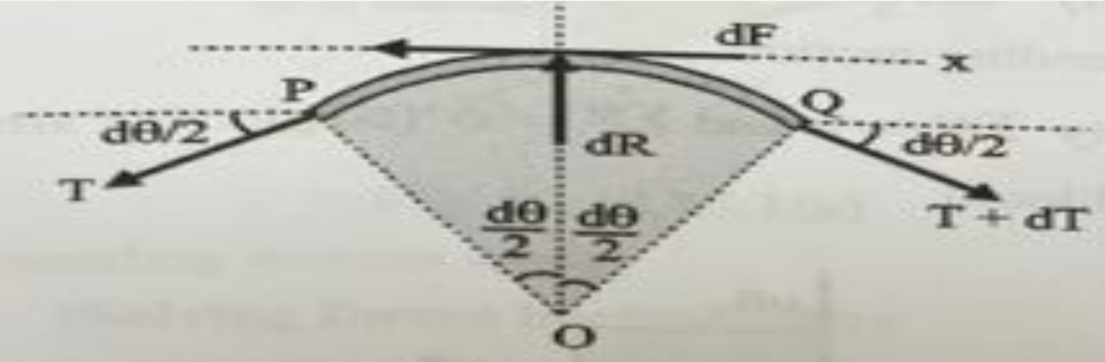


Fig. 4.13.2

The normal reaction offered by centre of drum is dR and frictional force between belt material and rim of pulley is dF.

Now for impending motion apply conditions of equilibrium.

$$\Sigma F_x = 0 \qquad -T\cos\frac{d\theta}{2} + (T + dT)\cos\frac{d\theta}{2} - dF = 0$$

$$\therefore \; dT \cdot \cos\frac{d\theta}{2} = dF$$

but $\frac{d\theta}{2}$ is a small angle so, $\cos\frac{d\theta}{2} = 1$

$$\therefore \; dT = dF$$

$$\mathbf{dT = \mu_s \cdot dR} \qquad \text{...(4.13.1)}$$

$$\Sigma F_y = 0 \qquad -T \cdot \sin\frac{d\theta}{2} + (T + dT) \cdot \sin\frac{d\theta}{2} + dR = 0$$

$$\therefore \; 2T \cdot \sin\frac{d\theta}{2} + dT \cdot \sin\frac{d\theta}{2} = dR$$

As dT sin dθ/2 is very small quantity neglecting it and for small angle $\frac{d\theta}{2}$ putting $\sin\frac{d\theta}{2} = \frac{d\theta}{2}$

$$\text{We get,} \quad 2T \cdot \frac{d\theta}{2} = dR$$

$$\therefore \quad T \cdot d\theta = dR$$

Substituting value of dR in Equation (4.13.1)

$$\therefore \quad dT = \mu_s \cdot T \cdot d\theta$$

$$\therefore \quad \frac{dT}{T} = \mu_s \cdot d\theta$$

Integrating above from A to B.

We have when $\theta = 0$, $T = T_2$ and when $\theta = \beta$, $T = T_1$

$$\therefore \quad \int_{T_2}^{T_1} \frac{dT}{T} = \mu_s \int_0^{\beta} d\theta$$

$$\therefore \quad [\ln T]_{T_2}^{T_1} = \mu_s \cdot [\theta]_0^{\beta}$$

$$\therefore \quad \ln T_1 - \ln T_2 = \mu_s \cdot \beta$$

$$\therefore \quad \ln \frac{T_1}{T_2} = \mu_s \cdot \beta$$

$$\therefore \quad \frac{T_1}{T_2} = e^{\mu_s \beta}$$

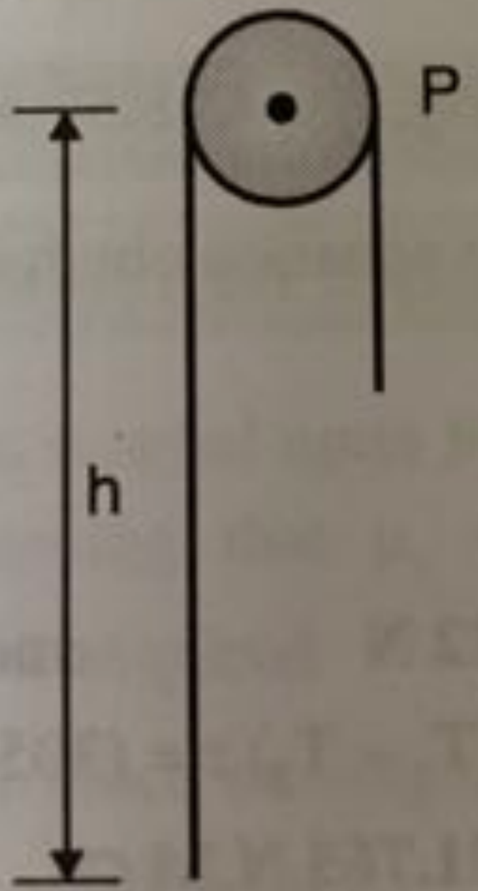
$$\therefore \quad \text{In general,} \quad \frac{\text{Tight side}}{\text{Slack side}} = e^{\mu_s \beta}$$

1. The above relation is used only when belt is about to slip.
2. If belt is actually slipping use $\dfrac{\text{Tight side}}{\text{Slack side}} = e^{\mu_k \cdot \beta}$
3. The lap angle or angle of contact β must be in radians.
4. Tight side represents the larger tension in part of belt which pulls and slack side represents the smaller tension in part which resists.

Fire point

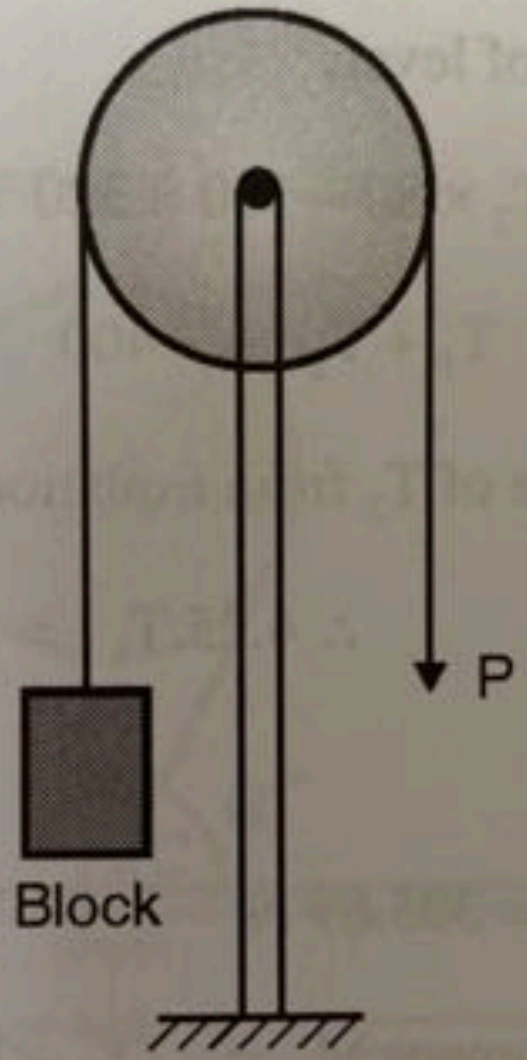
The relation $\dfrac{T_1}{T_2} = e^{\mu\beta}$ is independent of r, its use is not restricted to circular contact surfaces, it may also be used for a surface of arbitrary shape.

A chain having a weight of 1.5 N/m and a total length of 10 m is suspended over a peg P as shown in Fig. Ex. 4.15.55. If the coefficient of static friction between the peg and cord is $\mu_s = 0.25$, determine the largest length h which one side of the suspended cord can have without causing motion. Neglect the size of peg.



SPPU - Q. 6(b), Dec.16, Q. 8(b), Dec. 17, 6 Marks

A cable is passing over the disc of belt friction apparatus at a lap angle 180° as shown in Fig. Ex. 4.15.54. If the weight of block is 500 N, determine the range of force P to maintain equilibrium.



$$= e^{0.25 \times \pi}$$

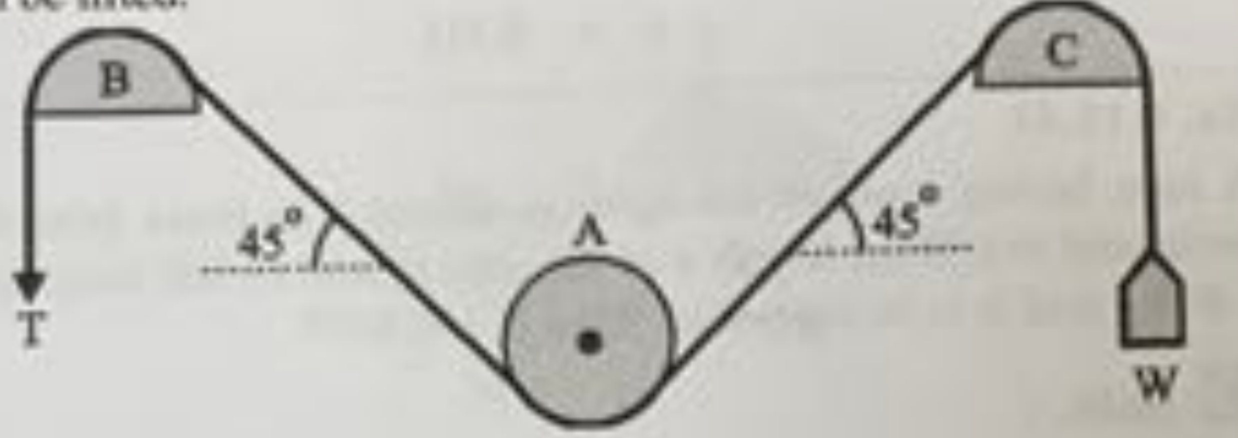
$$\pi = \frac{22}{7} = 3.14$$

$$\Rightarrow = e^{(0.25 \times 3.14)}$$

$$= e^{0.785}$$ (put this in calculator)

$$= 2.19$$

The maximum tension that can be developed in belt is 600 N. If the pulley A is free to rotate and coefficient of static friction at fixed drums B and C is 0.25. Determine the largest mass of block that can be lifted.



Example 7.10. *Find the moment of inertia of a T-section with flange as 150 mm × 50 mm and web as 150 mm × 50 mm about X-X and Y-Y axes through the centre of gravity of the section.*

Solution. The given *T*-section is shown in Fig. 7.14.

First of all, let us find out centre of gravity of the section. As the section is symmetrical about *Y-Y* axis, therefore its centre of gravity will lie on this axis. Split up the whole section into two rectangles *viz*., 1 and 2 as shown in figure. Let bottom of the web be the axis of reference.

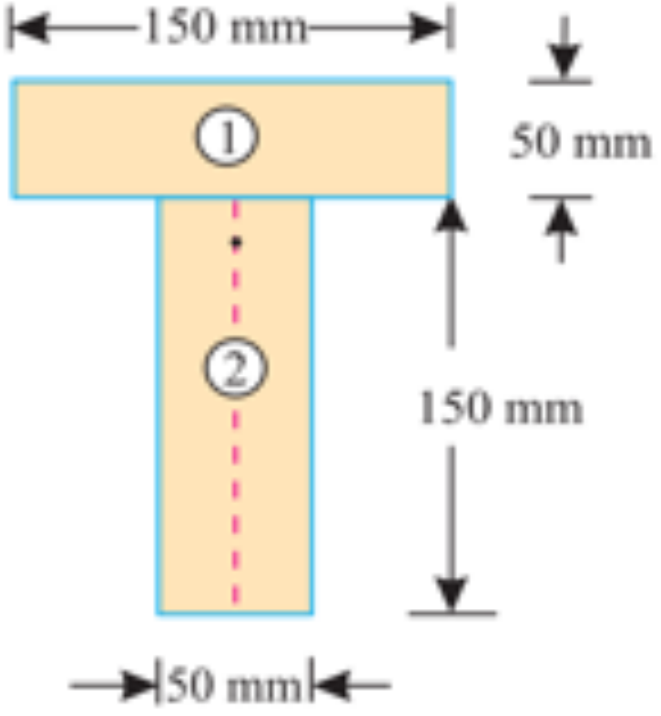


Fig. 7.14.

(*i*) *Rectangle* (1)

$$a_1 = 150 \times 50 = 7500 \text{ mm}^2$$

and $$y_1 = 150 + \frac{50}{2} = 175 \text{ mm}$$

(*ii*) *Rectangle* (2)

$$a_2 = 150 \times 50 = 7500 \text{ mm}^2$$

and $$y_2 = \frac{150}{2} = 75 \text{ mm}$$

Example 8.2. *A body, resting on a rough horizontal plane, required a pull of 180 N inclined at 30° to the plane just to move it. It was found that a push of 220 N inclined at 30° to the plane just moved the body. Determine the weight of the body and the coefficient of friction.*

Solution. Given: Pull = 180 N; Push = 220 N and angle at which force is inclined with horizontal plane (α) = 30°

Let W = Weight of the body

R = Normal reaction, and

μ = Coefficient of friction.

First of all, consider a pull of 180 N acting on the body. We know that in this case, the force of friction (F_1) will act towards left as shown in Fig. 8.3. (*a*).

Resolving the forces horizontally,

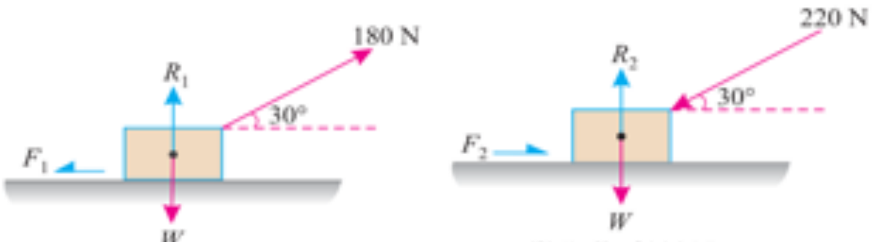
$$F_1 = 180 \cos 30° = 180 \times 0.866 = 155.9 \text{ N}$$

and now resolving the forces vertically,

$$R_1 = W - 180 \sin 30° = W - 180 \times 0.5 = W - 90 \text{ N}$$

We know that the force of friction (F_1),

$$155.9 = \mu R_1 = \mu (W - 90) \quad ...(i)$$



(*a*) Pull of 180 N

(*b*) Pull of 220 N

Fig. 8.3.

Now consider a push of 220 N acting on the body. We know that in this case, the force of friction (F_2) will act towards right as shown in Fig. 8.3 (*b*).

Resolving the forces horizontally,

$$F_2 = 220 \cos 30° = 220 \times 0.866 = 190.5 \text{ N}$$

and now resolving the forces horizontally,

$$R_2 = W + 220 \sin 30° = W + 220 \times 0.5 = W + 110 \text{ N}$$

We know that the force of friction (F_2)

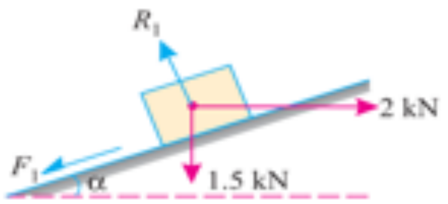
Example 8.9. *A load of 1.5 kN, resting on an inclined rough plane, can be moved up the plane by a force of 2 kN applied horizontally or by a force 1.25 kN applied parallel to the plane. Find the inclination of the plane and the coefficient of friction.*

Solution. Given: Load (W) = 1.5 kN; Horizontal effort (P_1) = 2 kN and effort parallel to the inclined plane (P_2) = 1.25 kN.

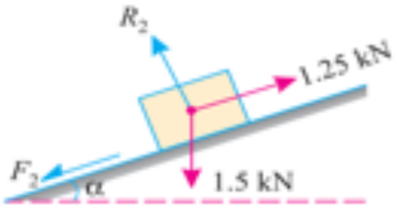
Inclination of the plane

Let α = Inclination of the plane, and

ϕ = Angle of friction.



(*a*) Horizontal force



(*b*) Force parallel to the plane

Fig. 8.14.

First of all, consider the load of 1.5 kN subjected to a horizontal force of 2 kN as shown in Fig. 8.14 (*a*). We know that when the force is applied horizontally, then the magnitude of the force, which can move the load up the plane.

$$P = W \tan(\alpha + \phi)$$

or

$$2 = 1.5 \tan(\alpha + \phi)$$

$$\therefore \quad \tan(\alpha + \phi) = \frac{2}{1.5} = 1.333 \quad \text{or} \quad (\alpha + \phi) = 53.1°$$

Now consider the load of 1.5 kN subjected to a force of 1.25 kN along the plane as shown in Fig. 8.14 (*b*). We Know that when the force is applied parallel to the plane, then the magnitude of the force, which can move the load up the plane,

$$P = W \times \frac{\sin(\alpha + \phi)}{\cos \phi}$$

sin 53.1° 0.8 1.2

Example 9.2. *A ladder 5 meters long rests on a horizontal ground and leans against a smooth vertical wall at an angle 70° with the horizontal. The weight of the ladder is 900 N and acts at its middle. The ladder is at the point of sliding, when a man weighing 750N stands on a rung 1.5 metre from the bottom of the ladder.*

Calculate the coefficient of friction between the ladder and the floor.

Solution. Given: Length of the ladder (l) = 5 m; Angle which the ladder makes with the horizontal (α) = 70°; Weight of the ladder (w_1) = 900 N; Weight of man (w_2) = 750 N and distance between the man and bottom of ladder = 1.5 m.

Forces acting on the ladder are shown in Fig. 9.3.

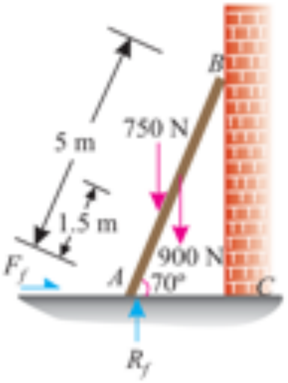


Fig. 9.3.

Let μ_f = Coefficient of friction between ladder and floor and

R_f = Normal reaction at the floor.

Resolving the forces vertically,

$$R_f = 900 + 750 = 1650 \text{ N} \quad ...(i)$$

$\therefore$ Force of friction at A

$$F_f = \mu_f \times R_f = \mu_f \times 1650 \quad ...(ii)$$

Now taking moments about B, and equating the same,

$$R_f \times 5 \sin 20° = (F_f \times 5 \cos 20°) + (900 \times 2.5 \sin 20°) + (750 \times 3.5 \sin 20°)$$

$$= (F_f \times 5 \cos 20°) + (4875 \sin 20°)$$

$$= (\mu_f \times 1650 \times 5 \cos 20°) + 4875 \sin 20°$$

and now substituting the values of R_f and F_f from equations (*i*) and (*ii*)

$$1650 \times 5 \sin 20° = (\mu_f \times 1650 \times 5 \cos 20°) + (4875 \sin 20°)$$

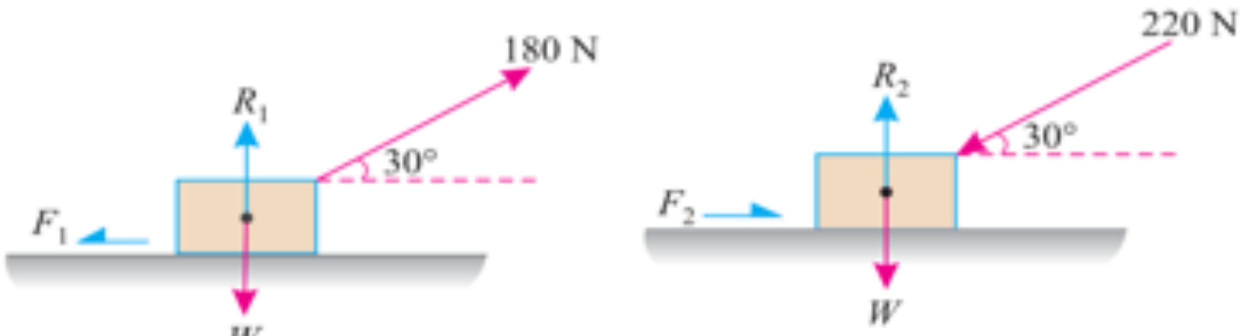
Dividing both sides by 5 sin 20°,

$$1650 = (\mu_f \times 1650 \cot 20°) + 975$$

$$= (\mu_f \times 1650 \times 2.7475) + 975 = 4533\ \mu_f + 975$$

$$\therefore \quad \mu_f = \frac{1650 - 975}{4533} = 0.15 \quad \textbf{Ans.}$$

Example 8.2. *A body, resting on a rough horizontal plane, required a pull of 180 N inclined at 30° to the plane just to move it. It was found that a push of 220 N inclined at 30° to the plane just moved the body. Determine the weight of the body and the coefficient of friction.*



(*a*) Pull of 180 N

(*b*) Pull of 220 N

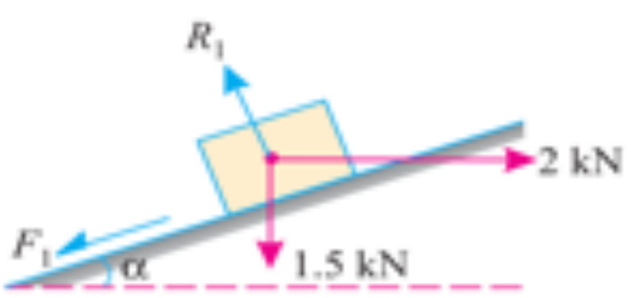
Example 8.9. *A load of 1.5 kN, resting on an inclined rough plane, can be moved up the plane by a force of 2 kN applied horizontally or by a force 1.25 kN applied parallel to the plane. Find the inclination of the plane and the coefficient of friction.*

Solution. Given: Load (W) = 1.5 kN; Horizontal effort (P_1) = 2 kN and effort parallel to the inclined plane (P_2) = 1.25 kN.

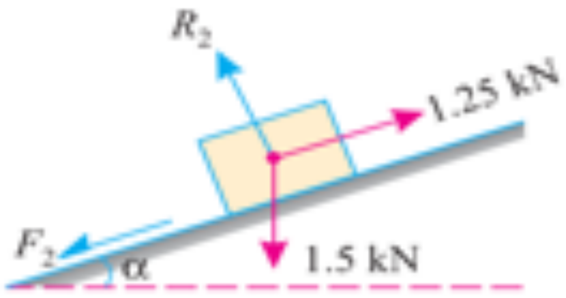
Inclination of the plane

Let α = Inclination of the plane, and

ϕ = Angle of friction.



(*a*) Horizontal force



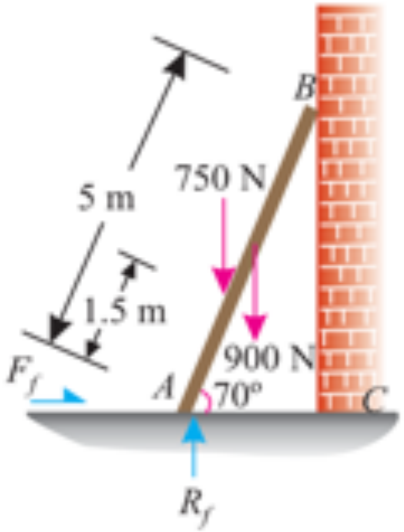
(*b*) Force parallel to the plane

Example 9.2. *A ladder 5 meters long rests on a horizontal ground and leans against a smooth vertical wall at an angle 70° with the horizontal. The weight of the ladder is 900 N and acts at its middle. The ladder is at the point of sliding, when a man weighing 750N stands on a rung 1.5 metre from the bottom of the ladder.*

Calculate the coefficient of friction between the ladder and the floor.

Solution. Given: Length of the ladder (l) = 5 m; Angle which the ladder makes with the horizontal (α) = 70°; Weight of the ladder (w_1) = 900 N; Weight of man (w_2) = 750 N and distance between the man and bottom of ladder = 1.5 m.

Forces acting on the ladder are shown in Fig. 9.3.



Let μ_f = Coefficient of friction between ladder and floor and

R_f = Normal reaction at the floor.

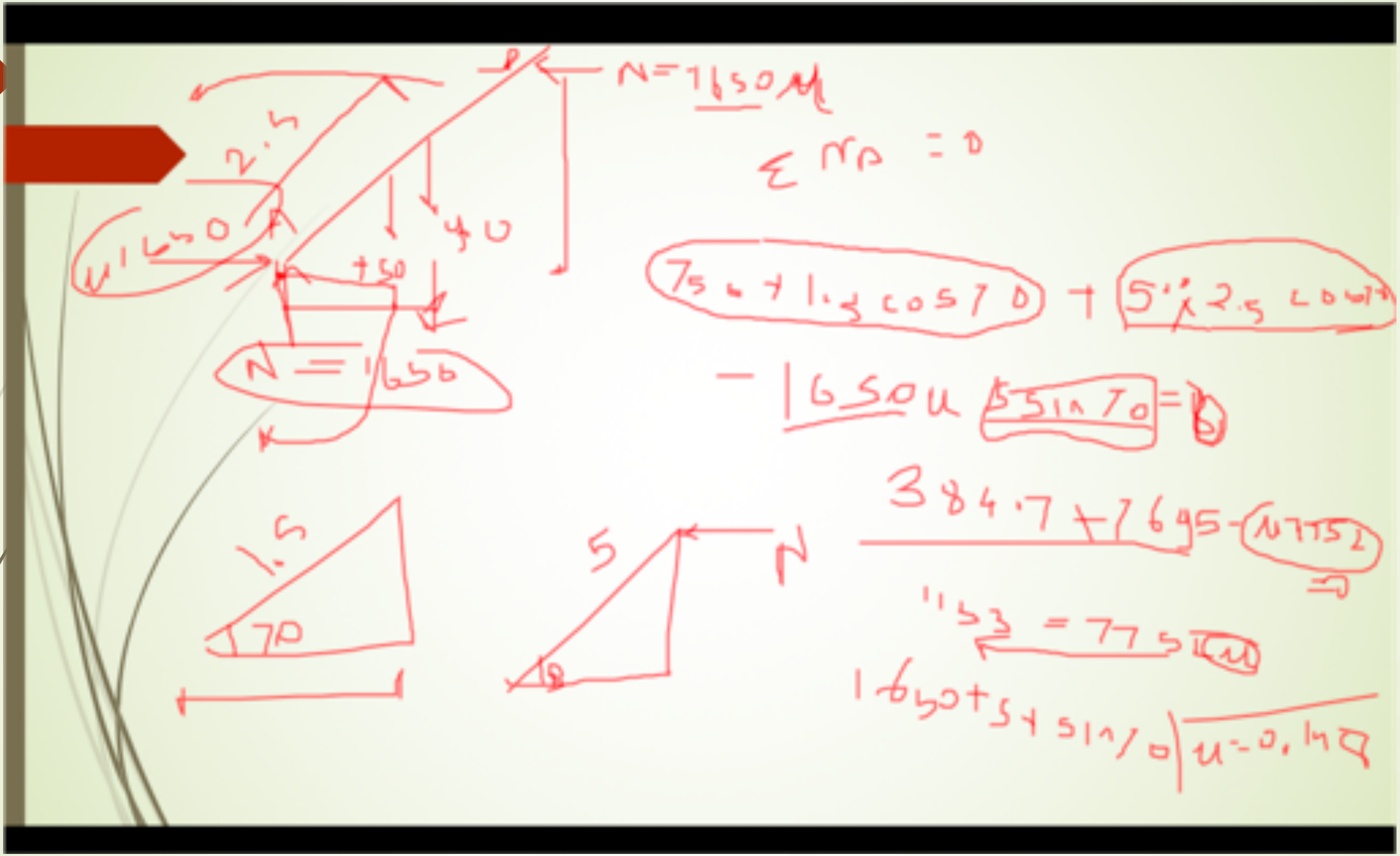
Resolving the forces vertically,

$$R_f = 900 + 750 = 1650 \text{ N} \quad \text{...}(i)$$

$\therefore$ Force of friction at A

$$F_f = \mu_f \times R_f = \mu_f \times 1650 \quad \text{...}(ii)$$

Now taking moments about B and equating the same



N=1650N
2.5
N = 1650
1.5
70
5
N

$$x(110+x) = 190 \quad \text{— } \boxed{2}$$

$$x(x-90) = 155 \quad \text{— } 1$$

$$\frac{110+x}{(x-90)} = 1.22$$

$$110+x = 1.22(x-90)$$

$$110+x = 1.22x - 109.8$$

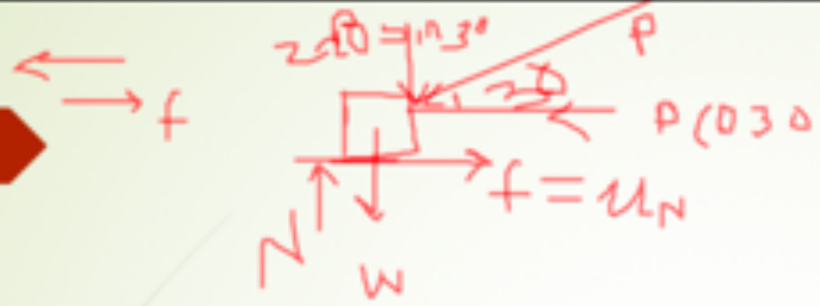
$$110+109.8 = 0.22x = 2??$$

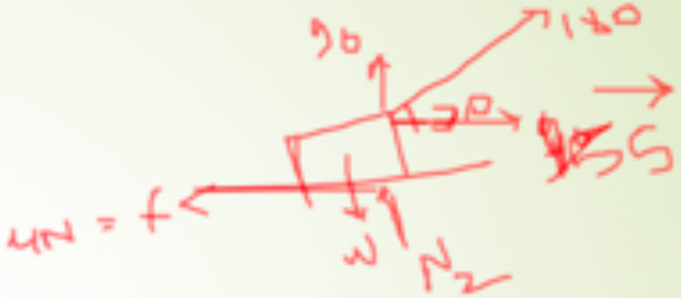
$$x = \frac{2??}{0.22}$$

$$x = 995$$

$$\frac{a}{b} = k$$

$$a = bk$$





$\Sigma F_y = 0$

$N - W - 220 \sin 30 = 0$

$N_1 = 110 + W$ (1)

$\Sigma F_x = 0$

$-220 \cos 30 + \mu N = 0$

$\mu N = 190.5$

$\mu (110 + W) = 190.5$

$$\boxed{110\mu + W\mu = 190.5}$$

$\Sigma F_y = 0$

$N_2 + 90 - W = 0$

$N_2 = (W - 90)$

$\Sigma F_x = 0$

$-\mu N_2 + 155 = 0$

$\mu (W - 90) = 155$

$\mu W - 90\mu = 155$