Q2. Given a hypothetical situation where a random person (say X) has two particular preferences when it comes to holiday-destination-choices or restaurant-choices. The probability that X sticks to their first preference is 0.75 and the probability that they continue with their second preference is 0.55, and on an average they opt for 2 holidays per year or eat out twice a year. If we assume that their first preference of holiday-destination or restaurant has a 55 market share of their respective business domains, what would your prediction of their market shares be after 2 years? Your answer must include transition matrices and diagrams to model choice changes.

Answer :-

1 Introduction and Assumptions

This problem models a random person's (X) holiday-destination or restaurant preferences as a **Markov Chain (MC)** to predict the market share of their first preference after two years. A Markov Chain is suitable because the decision on any given trip (or meal) depends only on the choice made in the immediately preceding one, reflecting a state transition based on fixed probabilities[cite: 1380, 1381].

We define two states for Person X's choice:

- **State 1 ($\mathbf{S}_1$):** Opting for the **First Preference** (P1).
- **State 2 ($\mathbf{S}_2$):** Opting for the **Second Preference** (P2).

1.1 Problem Parameters and Key Assumptions

1. The problem involves a total of $N = 2$ choices per year (2 holidays or 2 meals out)[cite: 22]. The prediction spans 2 years, resulting in $k = 4$ transitions (or time steps).
2. **Given Transition Probabilities:**
   - Probability of sticking to the first preference ($S_1 \to S_1$): $P(S_1|S_1) = 0.75$[cite: 22].
   - Probability of continuing with the second preference ($S_2 \to S_2$): $P(S_2|S_2) = 0.55$[cite: 22].
3. **Initial State Vector (π_0):** The initial market share (or probability distribution) for the first trip/meal is based on the 55% market share of the first preference[cite: 24].

$$\pi_0 = \begin{pmatrix} P(S_1) & P(S_2) \end{pmatrix} = \begin{pmatrix} 0.55 & 0.45 \end{pmatrix}$$

2 Methodology: Transition Matrix Construction

2.1 Calculation of Missing Probabilities

Since the columns in a transition matrix must sum to 1 (all possibilities are accounted for, characteristic of a probability distribution), we calculate the probability of switching:

- Switching from P1 to P2 ($S_1 \to S_2$):

$$P(S_2|S_1) = 1 - P(S_1|S_1) = 1 - 0.75 = 0.25$$

- Switching from P2 to P1 ($S_2 \to S_1$):

$$P(S_1|S_2) = 1 - P(S_2|S_2) = 1 - 0.55 = 0.45$$

2.2 Transition Matrix (T)

The matrix **T** defines the likelihood of moving from current state (rows) to the next state (columns):

$$\mathbf{T} = \begin{pmatrix} P(S_1|S_1) & P(S_2|S_1) \\ P(S_1|S_2) & P(S_2|S_2) \end{pmatrix} = \begin{pmatrix} 0.75 & 0.25 \\ 0.45 & 0.55 \end{pmatrix}$$

2.3 Markov Chain Diagram

3 Calculation and Final Solution

The distribution of choices after k steps (π_k) is calculated by applying the transition matrix $\mathbf{T}$ to the initial state vector π_0 for k iterations:

$$\pi_k = \pi_0 \mathbf{T}^k$$

We are predicting the market share after $k = 4$ choices (2 years, 2 choices/year)[cite: 22].

3.1 Step 1: Calculate $\mathbf{T}^4$

Instead of matrix exponentiation, we calculate iteratively:

After 1 Year (2 choices, $k = 2$): $\pi_2 = \pi_0 \mathbf{T}^2$

$$\mathbf{T}^2 = \begin{pmatrix} 0.75 & 0.25 \\ 0.45 & 0.55 \end{pmatrix} \begin{pmatrix} 0.75 & 0.25 \\ 0.45 & 0.55 \end{pmatrix} = \begin{pmatrix} (0.75 \times 0.75) + (0.25 \times 0.45) & (0.75 \times 0.25) + (0.25 \times 0.55) \\ (0.45 \times 0.75) + (0.55 \times 0.45) & (0.45 \times 0.25) + (0.55 \times 0.55) \end{pmatrix}$$

$$\mathbf{T}^2 = \begin{pmatrix} 0.5625 + 0.1125 & 0.1875 + 0.1375 \\ 0.3375 + 0.2475 & 0.1125 + 0.3025 \end{pmatrix} = \begin{pmatrix} 0.675 & 0.325 \\ 0.585 & 0.415 \end{pmatrix}$$

After 2 Years (4 choices, $k = 4$): $\mathbf{T}^4 = \mathbf{T}^2 \mathbf{T}^2$

$$\mathbf{T}^4 = \begin{pmatrix} 0.675 & 0.325 \\ 0.585 & 0.415 \end{pmatrix} \begin{pmatrix} 0.675 & 0.325 \\ 0.585 & 0.415 \end{pmatrix}$$

$$P(S_1|S_1)_{\text{after 4}} = (0.675 \times 0.675) + (0.325 \times 0.585) = 0.455625 + 0.190125 = 0.64575$$

$$P(S_2|S_1)_{\text{after 4}} = (0.675 \times 0.325) + (0.325 \times 0.415) = 0.219375 + 0.134875 = 0.35425$$

$$\mathbf{T}^4 = \begin{pmatrix} 0.64575 & 0.35425 \\ 0.6186 & 0.3814 \end{pmatrix}$$

3.2 Step 2: Calculate Final Distribution (π_4)

$$\pi_4 = \pi_0 \mathbf{T}^4 = \begin{pmatrix} 0.55 & 0.45 \end{pmatrix} \begin{pmatrix} 0.64575 & 0.35425 \\ 0.6186 & 0.3814 \end{pmatrix}$$

$$P(S_1)_{\text{after 4}} = (0.55 \times 0.64575) + (0.45 \times 0.6186)$$

$$P(S_1)_{\text{after 4}} = 0.3551625 + 0.27837$$

$$P(S_1)_{\text{after 4}} = 0.6335325$$

3.3 Final Solution

The predicted market share for Person X's first preference after two years (4 steps) is **63.35**%.

$$\pi_4 = \begin{pmatrix} 0.6335 & 0.3665 \end{pmatrix}$$