Written as per the latest syllabus prescribed by the Maharashtra State Bureau of Textbook Production and Curriculum Research, Pune.

Mathematics Part – II

STD. IX

Salient Features

- Written as per the new textbook.
- Exhaustive coverage of entire syllabus.
- Topic-wise distribution of textual questions and practice problems at the beginning of every chapter.
- Covers solutions to all Practice Sets and Problem Sets.
- Includes additional activities for practice.
- Includes additional problems and MCQs for practice.
- Indicative marks for all problems.
- Chapter-wise assessment for every chapter.
- Includes 'Challenging Questions (4 Marks)'.
- Includes 'One mark and Two marks questions (from Std. VIII)'.
- Constructions drawn with accurate measurements.

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PREFACE

Preparing this **'Mathematics Part - II'** book was a rollercoaster ride. We had a plethora of ideas, suggestions and decisions to ponder over. However, our basic premise was to keep this book in line with the new, improved syllabus and to provide students with an absolutely fresh material.

Mathematics Part - II covers several topics including basic concepts in geometry, logical proofs, trigonometry, co-ordinate geometry and surface area and volume. The study of these topics requires a deep and intrinsic understanding of concepts, terms and formulae. Hence, to ease this task, we present **'Std. IX: Mathematics Part - II'** – a complete and thorough guide, extensively drafted to boost the confidence of students.

For better understanding of different types of questions, Topic-Wise Distribution of Textual Questions and Practice Problems have been provided at the beginning of every chapter. Before each Practice Set, short and easy explanation of different concepts with illustrations for better understanding is given. Solutions and Answers to Textual Questions and Examples are provided in a lucid manner.

'Smart Check' given to enable the students to cross-check their answers.

'Apply your knowledge' covers all the textual activities and projects along with their answers.

'Multiple Choice Questions' and 'Additional Problems for Practice' include multiple unsolved problems for revision and in the process help the students to sharpen their problem solving skills. 'Solved examples' from textbook are also included in the book.

Every chapter ends with a 'Chapter Assessment'. This test stands as a testimony to the fact that the child has understood the chapter thoroughly.

'Activities for practice' includes additional activities along with their answers for the students to practice.

'Challenging Questions' include questions that fall out of the textbook, yet are core to the concerned subject. These questions would prepare the students for the Four Mark Questions that are supposed to be asked in the examination from outside the textbook.

Marks are provided for each and every problem. However, marks mentioned are indicative and are subject to change as per Maharashtra State Board's discretion.

All the diagrams are neat and have proper labelling. The book has a unique feature that all the constructions are as per the scale.

Since according to the paper pattern, weightage is accorded to the Std. VIII syllabus, we have aptly included them in a separate chapter i.e. 'One mark and Two marks questions (from Std. VIII)'.

The journey to create a complete book is strewn with triumphs, failures and near misses. If you think we've nearly missed something or want to applaud us for our triumphs, we'd love to hear from you.

Please write to us on : mail@targetpublications.org

A book affects eternity; one can never tell where its influence stops.

Best of luck to all the aspirants!

From,
Publisher

Edition: Second

Content

Note: *Solved examples from textbook are indicated by "+".*
Steps of construction are provided in Chapters for the students' understanding.

1 Basic Concepts in Geometry

Type of Problems	Practice Set	Q. Nos.
Co-ordinates of points and distance	1.1	Q.1, 2
	Practice Problems (Based on Practice Set 1.1)	Q.1, 5, 9
	1.2	Q.6
	Problem Set-1	Q.2, 3, 4, 6
Betweenness	1.1	Q.3, 4, 5, 6, 7
	Practice Problems (Based on Practice Set 1.1)	Q.2, 3, 4, 6, 7, 8
	Problem Set-1	Q.5
Congruent segments, Comparison of segments	1.2	Q.1, 4
	Practice Problems (Based on Practice Set 1.2)	Q.1
Midpoint of a segment	1.2	Q.2, 3
	Practice Problems (Based on Practice Set 1.2)	Q.2, 3
Ray, line	1.2	Q.5
Conditional statements and converse	1.3	Q.1, 2
	Practice Problems (Based on Practice Set 1.3)	Q.1, 2
	Problem Set-1	Q.7, 8
Antecedent (given part), Consequent (part to be proved)	Practice Problems (Based on Practice Set 1.3)	Q.3
	Problem Set-1	Q.9, 10

Let's Study

Basic concepts in geometry (Point, Line and Plane)

In Geometry, we have several undefined terms. Point, line and plane are few of these terms which are however basic concepts in Geometry.

From these three undefined terms, all other terms in Geometry can be defined.

i. A **point** can be any dot made by a sharp tip. It determines a location and it has no size.

ii. A **line** is defined as something that extends infinitely in both directions but has no width and is one dimensional.

iii. A **plane** extends infinitely in two dimensions.

iv. Lines and planes are set of points.

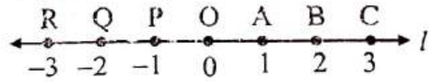
v. Each line and each plane contains infinite number of points.

Co-ordinates of Points and Distance

1. Co-ordinates of a Point:

The number associated with a point on the number line is called the co-ordinate of that point.

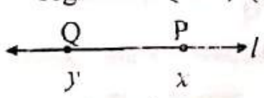
In the figure below, co-ordinate of point P is -1 and that of point B is 2.



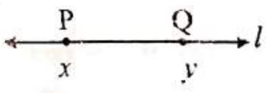
2. Distance between Two Points:

i. To find the distance between two points, consider their co-ordinates and subtract the smaller co-ordinate from the larger.

ii. The distance between points P and Q is denoted as d(P, Q). It is same as length of the segment PQ i.e., l(P, Q)



$(x > y)$



$(x < y)$

$\therefore$ $d(P, Q) = l(P, Q) = x - y$, if $x > y$

$d(P, Q) = l(P, Q) = y - x$, if $x < y$

Example :

For the number line shown below find

i. d(P, C) ii. d(R, P)

R Q P O A B C

−3 −2 −1 0 1 2 3 l

Solution:

i. Co-ordinate of the point P is −1.

Co-ordinate of the point C is 3.

Since, $3 > -1$

i.e., co-ordinate of point C > co-ordinate of point P

$\therefore$ d(P, C) = Co-ordinate of point C − Co-ordinate of point P

$\therefore$ $d(P, C) = 3 - (-1)$

$= 3 + 1$

$\therefore$ **d (P, C) = 4**

ii. Co-ordinate of the point R is −3.

Co-ordinate of the point P is −1.

Since, $-1 > -3$

i.e., co-ordinate of point P > co-ordinate of point R

$\therefore$ d(R, P) = Co-ordinate of point P − Co-ordinate of point R

$\therefore$ $d(R, P) = -1 - (-3)$

$= -1 + 3$

$\therefore$ **d(R, P) = 2**

Remember This

i. The distance between two points is obtained by subtracting the smaller co-ordinate from the larger co-ordinate.

ii. The distance between any two distinct points is a positive real number.

iii. If the two points are not distinct then the distance between them is zero.

Betweenness

1. If the points P, Q and R are three distinct collinear points, then there are three possibilities.

Case I: Point Q is between P and R.

P Q R

Case II: Point R is between P and Q.

P R Q

Case III: Point P is between R and Q.

R P Q

2. If d(P, Q) + d(Q, R) = d(P, R), then the point Q is said to be in between points P and R.

P Q R

3. The betweenness is written as P − Q − R or R − Q − P.

Example:

Three points A, B and C are such that d (A, B) = 6, d(A, C) = 8 and d(B, C) = 14. Find which of the point is between the other two.

Solution:

Given, d (A, B) = 6, d(A, C) = 8 and d(B, C) = 14.

B ← 6 → A ← 8 → C

d(B, C) = 14 ...(i)

d(A, B) + d(A, C) = 6 + 8

$\therefore$ d(A, B) + d(A, C) = 14 ...(ii)

$\therefore$ d(B, C) = d(A, B) + d(A, C) ...[From (i) and (ii)]

$\therefore$ **Point A is between the points B and C i.e., B − A − C or C − A − B.**

Practice Set 1.1

1. **Find the distances with the help of the number line given below. [1 Mark each]**

Q P K J H O A B C D E

−5 −4 −3 −2 −1 0 1 2 3 4 5 6

i. d(B, E)	ii. d(J, A)
iii. d(P, C)	iv. d(J, H)
v. d(K, O)	vi. d(O, E)
vii. d(P, J)	viii. d(Q, B)

Solution:

i. Co-ordinate of the point B is 2.

Co-ordinate of the point E is 5.

Since, $5 > 2$

$\therefore$ $d(B, E) = 5 - 2$

$\therefore$ **d(B, E) = 3**

ii. Co-ordinate of the point J is −2.

Co-ordinate of the point A is 1.

Since, $1 > -2$

$\therefore$ $d(J, A) = 1 - (-2)$

$= 1 + 2$

$\therefore$ **d(J, A) = 3**

iii. Co-ordinate of the point P is -4.
Co-ordinate of the point C is 3.
Since, $3 > -4$

$\therefore$ $d(P, C) = 3 - (-4)$
$= 3 + 4$

$\therefore$ **d(P, C) = 7**

iv. Co-ordinate of the point J is -2.
Co-ordinate of the point H is -1.
Since, $-1 > -2$

$\therefore$ $d(J, H) = -1 - (-2)$
$= -1 + 2$

$\therefore$ **d(J, H) = 1**

v. Co-ordinate of the point K is -3.
Co-ordinate of the point O is 0.
Since, $0 > -3$

$\therefore$ $d(K, O) = 0 - (-3)$
$= 0 + 3$

$\therefore$ **d(K, O) = 3**

vi. Co-ordinate of the point O is 0.
Co-ordinate of the point E is 5.
Since, $5 > 0$

$\therefore$ $d(O, E) = 5 - 0$

$\therefore$ **d(O, E) = 5**

vii. Co-ordinate of the point P is -4.
Co-ordinate of the point J is -2.
Since, $-2 > -4$

$\therefore$ $d(P, J) = -2 - (-4)$
$= -2 + 4$

$\therefore$ **d(P, J) = 2**

viii. Co-ordinate of the point Q is -5.
Co-ordinate of the point B is 2.
Since, $2 > -5$

$\therefore$ $d(Q, B) = 2 - (-5)$
$= 2 + 5$

$\therefore$ **d(Q, B) = 7**

2. If the co-ordinate of A is x and that of B is y, find d(A, B). **[1 Mark each]**

i. $x = 1, y = 7$ ii. $x = 6, y = -2$
iii. $x = -3, y = 7$ iv. $x = -4, y = -5$
v. $x = -3, y = -6$ vi. $x = 4, y = -8$

Solution:

i. Co-ordinate of point A is $x = 1$.
Co-ordinate of point B is $y = 7$.
Since, $7 > 1$

$\therefore$ $d(A, B) = 7 - 1$

$\therefore$ **d(A, B) = 6**

ii. Co-ordinate of point A is $x = 6$.
Co-ordinate of point B is $y = -2$.
Since, $6 > -2$

$\therefore$ $d(A, B) = 6 - (-2) = 6 + 2$

$\therefore$ **d(A, B) = 8**

iii. Co-ordinate of point A is $x = -3$.
Co-ordinate of point B is $y = 7$.
Since, $7 > -3$

$\therefore$ $d(A, B) = 7 - (-3) = 7 + 3$

$\therefore$ **d(A, B) = 10**

iv. Co-ordinate of point A is $x = -4$.
Co-ordinate of point B is $y = -5$.
Since, $-4 > -5$

$\therefore$ $d(A, B) = -4 - (-5)$
$= -4 + 5$

$\therefore$ **d(A, B) = 1**

v. Co-ordinate of point A is $x = -3$.
Co-ordinate of point B is $y = -6$.
Since, $-3 > -6$

$\therefore$ $d(A, B) = -3 - (-6)$
$= -3 + 6$

$\therefore$ **d(A, B) = 3**

vi. Co-ordinate of point A is $x = 4$.
Co-ordinate of point B is $y = -8$.
Since, $4 > -8$

$\therefore$ $d(A, B) = 4 - (-8)$
$= 4 + 8$

$\therefore$ **d(A, B) = 12**

3. From the information given below, find which of the point is between the other two. If the points are not collinear, state so. **[2 Marks each]**

i. $d(P, R) = 7$, $d(P, Q) = 10$, $d(Q, R) = 3$
ii. $d(R, S) = 8$, $d(S, T) = 6$, $d(R, T) = 4$
iii. $d(A, B) = 16$, $d(C, A) = 9$, $d(B, C) = 7$
iv. $d(L, M) = 11$, $d(M, N) = 12$, $d(N, L) = 8$
v. $d(X, Y) = 15$, $d(Y, Z) = 7$, $d(X, Z) = 8$
vi. $d(D, E) = 5$, $d(E, F) = 8$, $d(D, F) = 6$

Solution:

i. Given, $d(P, R) = 7$, $d(P, Q) = 10$, $d(Q, R) = 3$
$d(P, Q) = 10$ …(i)
$d(P, R) + d(Q, R) = 7 + 3 = 10$ …(ii)

$\therefore$ $d(P, Q) = d(P, R) + d(Q, R)$
…[From (i) and (ii)]

$\therefore$ Point R is between the points P and Q i.e., P – R – Q or Q – R – P.

$\therefore$ **Points P, R, Q are collinear**

ii. Given, $d(R, S) = 8$, $d(S, T) = 6$, $d(R, T) = 4$
$d(R, S) = 8$ ...(i)
$d(S, T) + d(R, T) = 6 + 4 = 10$ ...(ii)
$\therefore$ $d(R, S) \neq d(S, T) + d(R, T)$
...[From (i) and (ii)]
$\therefore$ **The given points are not collinear.**

iii. Given, $d(A, B) = 16$, $d(C, A) = 9$, $d(B, C) = 7$
$d(A, B) = 16$ ...(i)
$d(C, A) + d(B, C) = 9 + 7 = 16$ ...(ii)
$\therefore$ $d(A, B) = d(C, A) + d(B, C)$
...[From (i) and (ii)]
$\therefore$ Point C is between the points A and B.
i.e., A – C – B or B – C – A.
$\therefore$ **Points A, C, B are collinear**

iv. Given, $d(L, M) = 11$, $d(M, N) = 12$, $d(N, L) = 8$
$d(M, N) = 12$ ...(i)
$d(L, M) + d(N, L) = 11 + 8 = 19$ ...(ii)
$\therefore$ $d(M, N) \neq d(L, M) + d(N, L)$
...[From (i) and (ii)]
$\therefore$ **The given points are not collinear.**

v. Given, $d(X, Y) = 15$, $d(Y, Z) = 7$, $d(X, Z) = 8$
$d(X, Y) = 15$ ...(i)
$d(X, Z) + d(Y, Z) = 8 + 7 = 15$ ...(ii)
$\therefore$ $d(X, Y) = d(X, Z) + d(Y, Z)$
...[From (i) and (ii)]
$\therefore$ Point Z is between the points X and Y i.e., X – Z – Y or Y – Z – X.
$\therefore$ **Points X, Z, Y are collinear**

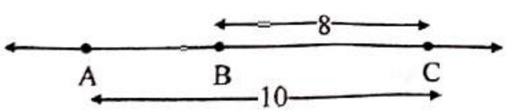
vi. Given, $d(D, E) = 5$, $d(E, F) = 8$, $d(D, F) = 6$
$d(E, F) = 8$ ...(i)
$d(D, E) + d(D, F) = 5 + 6 = 11$ ...(ii)
$\therefore$ $d(E, F) \neq d(D, E) + d(D, F)$
...[From (i) and (ii)]
$\therefore$ **The given points are not collinear.**

4. On a number line, points A, B and C are such that d(A, C) = 10, d(C, B) = 8. Find d(A, B) considering all possibilities.
[3 Marks]

Solution:

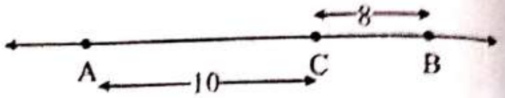
Given, $d(A, C) = 10$, $d(C, B) = 8$.

Case I: Points A, B, C are such that, A – B – C.



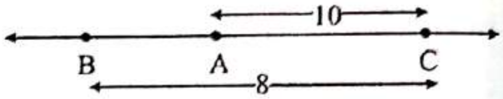
$\therefore$ $d(A, C) = d(A, B) + d(B, C)$
$\therefore$ $10 = d(A, B) + 8$
$\therefore$ $d(A, B) = 10 - 8$
$\therefore$ $d(A, B) = 2$

Case II: Points A, B, C are such that, A – C – B.



$\therefore$ $d(A, B) = d(A, C) + d(C, B)$
$= 10 + 8$
$\therefore$ $d(A, B) = 18$

Case III: Points A, B, C are such that, B – A – C.



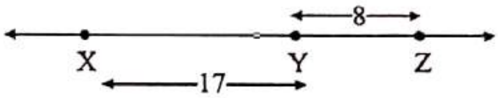
From the diagram,
d (A, C) > d(B, C)
Which is not possible
$\therefore$ Point A is not between B and C.
$\therefore$ **d(A, B) = 2 or d(A, B) = 18.**

5. Points X, Y, Z are collinear such that d(X, Y) = 17, d(Y, Z) = 8, find d(X, Z).
[3 Marks]

Solution:

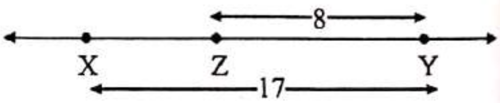
Given, $d(X, Y) = 17$, $d(Y, Z) = 8$

Case I: Points X, Y, Z are such that, X – Y – Z.



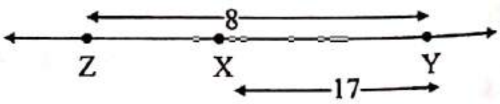
$\therefore$ $d(X, Z) = d(X, Y) + d(Y, Z)$
$= 17 + 8$
$\therefore$ $d(X, Z) = 25$

Case II: Points X, Y, Z are such that, X – Z – Y.



$\therefore$ $d(X, Y) = d(X, Z) + d(Z, Y)$
$\therefore$ $17 = d(X, Z) + 8$
$\therefore$ $d(X, Z) = 17 - 8$
$\therefore$ $d(X, Z) = 9$

Case III: Points X, Y, Z are such that, Z – X – Y.



From the diagram,
d(X, Y) > d (Y, Z)
Which is not possible
$\therefore$ Point X is not between Z and Y.
$\therefore$ **d(X, Z) = 25 or d(X, Z) = 9.**

6. **Sketch proper figure and write the answers of the following questions. [2 Marks each]**

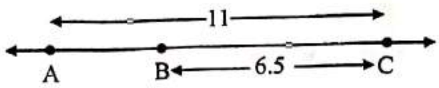
i. If A – B – C and l(AC) = 11, l(BC) = 6.5, then l(AB) = ?

ii. If R – S – T and l(ST) = 3.7, l(RS) = 2.5, then l(RT) = ?

iii. If X – Y – Z and $l(XZ) = 3\sqrt{7}$, $l(XY) = \sqrt{7}$, then l(YZ) = ?

Solution:

i. Given, l(AC) = 11, l(BC) = 6.5



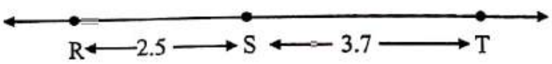
$l(AC) = l(AB) + l(BC)$ …[A – B – C]

∴ $11 = l(AB) + 6.5$

∴ $l(AB) = 11 - 6.5$

∴ $\boldsymbol{l(AB) = 4.5}$

ii. Given, l(ST) = 3.7, l(RS) = 2.5

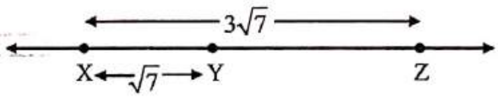


$l(RT) = l(RS) + l(ST)$ …[R – S – T]

$= 2.5 + 3.7$

∴ $\boldsymbol{l(RT) = 6.2}$

iii. $l(XZ) = 3\sqrt{7}$, $l(XY) = \sqrt{7}$



$l(XZ) = l(XY) + l(YZ)$ …[X – Y – Z]

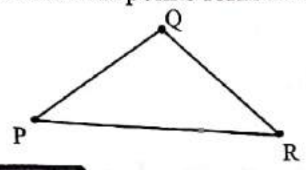
∴ $3\sqrt{7} = \sqrt{7} + l(YZ)$

∴ $l(YZ) = 3\sqrt{7} - \sqrt{7}$

∴ $\boldsymbol{l(YZ) = 2\sqrt{7}}$

7. **Which figure is formed by three non-collinear points? [1 Mark]**

Ans: Three non-collinear points form a triangle.



Let's Study

1. **Line Segment:**

i. The union set of point A, point B and points between A and B is called segment AB, written as seg AB.

ii. seg AB and seg BA denote the same line segment.

iii. The points A and B are called the end points of seg AB.

iv. A line segment is a subset of a line.

2. **Length of a Line Segment:**

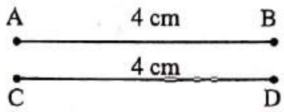
The distance between the end points of a line segment is called as the length of the segment. It is denoted by l(AB).

Note: i. $l(AB) = d(A, B)$

ii. $l(AB) = 4$ is also written as AB = 4

3. **Congruent Segments:**

Two line segments are said to be congruent, if they are of the same length.



If $l(AB) = l(CD) = 4$ cm, then seg AB ≅ seg CD.

Note: If we have to consider the length of segment AB, we write only AB or l(AB).

4. **Properties of Congruent Segments:**

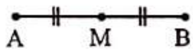
i. Reflexivity: seg AB ≅ seg AB

ii. Symmetry: If seg AB ≅ seg CD, then seg CD ≅ seg AB.

iii. Transitivity: If seg AB ≅ seg CD and seg CD ≅ seg PQ, then seg AB ≅ seg PQ.

5. **Midpoint of a Segment:**

The point M is said to be the midpoint of seg AB, if A – M – B and seg AM ≅ seg MB.



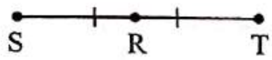
∴ $l(AM) = l(MB) = \frac{1}{2}\, l(AB)$

Note: Every line segment has one and only one midpoint.

Example : Point R is the midpoint of seg ST. If ST = 16, then find length of RS.

Solution:

Point R is the midpoint of seg ST and l(ST) = 16. …[Given]



$l(RS) = \frac{1}{2}\, l(ST)$ …[∵ R is midpoint of seg ST]

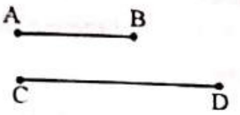
∴ $l(RS) = \frac{1}{2} \times 16 = 8$

∴ $\boldsymbol{l(RS) = 8}$

6. **Comparison of Segments:**

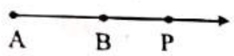
If l(AB) < l(CD), then we say that seg AB is smaller than seg CD.

This is written as seg AB < seg CD or seg CD > seg AB.



7. **Ray:**

i. Suppose A and B are two distinct points, then the union set of all the points on seg AB and the point P on the line AB such that A – B – P, is called ray AB.



ii. Point A is called as the end point of ray AB.

iii. The ray is a subset of a line.

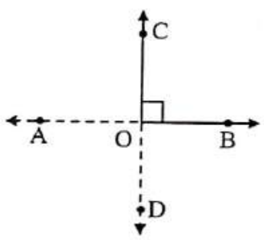
8. **Line:**

i. The union set of points on ray AP and opposite ray of ray AP is called line AP.

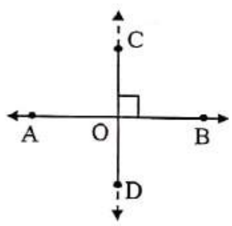
ii. The set of points of seg AP is a subset of points of line AP

9. **Perpendicularity of Segments and Rays:**

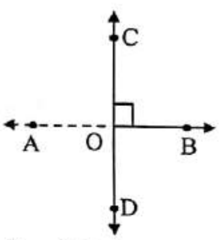
Two rays or two segments or a ray and a segment are said to be perpendicular to each other, if the lines containing them are perpendicular to each other.



ray OC ⊥ ray OB



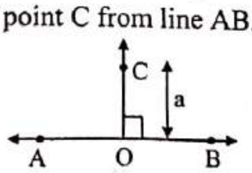
seg CD ⊥ line AB



line CD ⊥ ray OB

10. **Distance of a point from a line :**

i. If seg CO ⊥ line AB and point O lies on line AB, then the length of seg CO is called the distance of point C from line AB.



ii. Point O is called the foot of the perpendicular.

iii. If l(CO) = a, the point C is at a distance of a unit from line AB

Practice Set 1.2

1. **The following table shows points on a number line and their co-ordinates. Decide whether the pair of segments given below the table are congruent or not.**

[3 Marks each]

Point	A	B	C	D	E
Co-ordinate	–3	5	2	–7	9

i. seg DE and seg AB

ii. seg BC and seg AD

iii. seg BE and seg AD

Solution:

i. Co-ordinate of the point E is 9.
Co-ordinate of the point D is –7.
Since, 9 > –7

∴ d(D, E) = 9 – (–7) = 9 + 7 = 16

∴ l(DE) = 16 … (i)

Co-ordinate of the point A is –3.
Co-ordinate of the point B is 5.
Since, 5 > –3

∴ d(A, B) = 5 – (–3) = 5 + 3 = 8

∴ l(AB) = 8 …(ii)

∴ l(DE) ≠ l(AB) …[From (i) and (ii)]

∴ **seg DE and seg AB are not congruent.**

ii. Co-ordinate of the point B is 5.
Co-ordinate of the point C is 2.
Since, 5 > 2

∴ d(B, C) = 5 – 2 = 3

∴ l(BC) = 3 … (i)

Co-ordinate of the point A is –3.
Co-ordinate of the point D is –7.
Since, –3 > –7

∴ d(A, D) = –3 – (–7) = –3 + 7 = 4

∴ l(AD) = 4 …(ii)

∴ l(BC) ≠ l(AD) …[From (i) and (ii)]

∴ **seg BC and seg AD are not congruent.**

iii. Co-ordinate of the point E is 9.
Co-ordinate of the point B is 5.
Since, 9 > 5

∴ d(B, E) = 9 – 5 = 4

∴ l(BE) = 4 … (i)

Co-ordinate of the point A is –3.
Co-ordinate of the point D is –7.
Since, –3 > –7

∴ $d(A, D) = -3 - (-7) = 4$

∴ $l(AD) = 4$ ...(ii)

∴ $l(BE) = l(AD)$ ...[From (i) and (ii)]

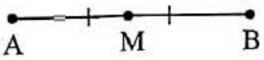
∴ **seg BE and seg AD are congruent.**
i.e, seg BE ≅ seg AD

---

2. Point M is the midpoint of seg AB. If AB = 8, then find the length of AM. **[2 Marks]**

Solution:

Point M is the midpoint of seg AB and $l(AB) = 8$. ...[Given]



$l(AM) = \frac{1}{2} l(AB)$

...[∵ M is midpoint of seg AB]

∴ $l(AM) = \frac{1}{2} \times 8 = 4$

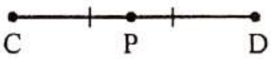
∴ $\mathbf{l(AM) = 4}$

---

3. Point P is the midpoint of seg CD. If CP = 2.5, find l(CD). **[2 Marks]**

Solution:

Point P is the midpoint of seg CD and $l(CP) = 2.5$ ...[Given]



$l(CP) = \frac{1}{2} l(CD)$

...[∵ P is midpoint of seg CD]

∴ $2.5 = \frac{1}{2} \times l(CD)$

∴ $l(CD) = 2.5 \times 2$

∴ $\mathbf{l(CD) = 5}$

---

4. If AB = 5 cm, BP = 2 cm and AP = 3.4 cm, compare the segments. **[2 Marks]**

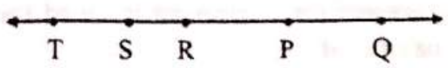
Solution:

Given, $l(AB) = 5$ cm, $l(BP) = 2$ cm,
$l(AP) = 3.4$ cm ...[Given]
Since, $2 < 3.4 < 5$

∴ $l(BP) < l(AP) < l(AB)$
i.e., seg BP < seg AP < seg AB

---

5. Write the answers to the following questions with reference to the figure given below: **[1 Mark each]**



i. Write the name of the opposite ray of ray RP
ii. Write the intersection set of ray PQ and ray RP.
iii. Write the union set of ray PQ and ray QR.
iv. State the rays of which seg QR is a subset.
v. Write the pair of opposite rays with common end point R.
vi. Write any two rays with common end point S.
vii. Write the intersection set of ray SP and ray ST.

Ans:

i. Ray RS or ray RT
ii. Ray PQ
iii. Line QR
iv. Ray QR, ray QS, ray QT, ray RQ, ray SQ, ray TQ
v. Ray RP and ray RS,
ray RQ and ray RT
vi. Ray ST, ray SR
vii. Point S

[Note: *Questions iv, v, vi have more than one answers. Students may write answers other than the ones given.*]

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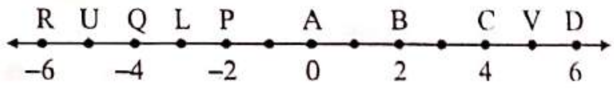
6. Answer the questions with the help of figure given below.



i. State the points which are equidistant from point B. **[2 Marks]**

ii. Write a pair of points equidistant from point Q. **[2 Marks]**

iii. Find d(U,V), d(P,C), d(V,B), d(U, L). **[2 Marks for each distance]**

Ans:

i. Points equidistant from point B are
  a. A and C,
  because $d(B, A) = d(B, C) = 2$
  b. D and P,
  because $d(B, D) = d(B, P) = 4$

---

ii. Points equidistant from point Q are
  a. L and U,
  because $d(Q, L) = d(Q, U) = 1$
  b. P and R,
  because $d(P, Q) = d(Q, R) = 2$

---

iii. a. Co-ordinate of the point U is –5.
Co-ordinate of the point V is 5.
Since, $5 > -5$

∴ $d(U, V) = 5 - (-5)$
$= 5 + 5$

∴ $\mathbf{d(U, V) = 10}$

b. Co-ordinate of the point P is –2.
Co-ordinate of the point C is 4.
Since, $4 > -2$
$\therefore$ $d(P, C) = 4 - (-2)$
$= 4 + 2$
$\therefore$ **d(P, C) = 6**

c. Co-ordinate of the point V is 5.
Co-ordinate of the point B is 2.
Since, $5 > 2$
$\therefore$ $d(V, B) = 5 - 2$
$\therefore$ **d(V, B) = 3**

d. Co-ordinate of the point U is –5.
Co-ordinate of the point L is –3.
Since, $-3 > -5$
$\therefore$ $d(U, L) = -3 - (-5)$
$= -3 + 5$
$\therefore$ **d(U, L) = 2**

Let's Study

Conditional statements and converse

1. **Conditional Statements:**
i. Any statement stated in the 'if-then' form is said to be a conditional statement.
ii. The part of statement, which follows 'if' is called **antecedent** and that which follows 'then' is called **consequent**.

Example:
General Statement: Two intersecting lines are contained in one plane.
Conditional Statement: If two lines intersect each other, then they are contained in one plane.

2. **Converse of a Statement:**
i. A statement obtained by interchanging the antecedent and consequent is called the converse of the original statement.

Example:
Conditional statement:
If two lines intersect, then they are in a plane.
Converse: If two lines are in a plane, then they intersect each other.

ii. If a property is true, then its converse may or may not be true.

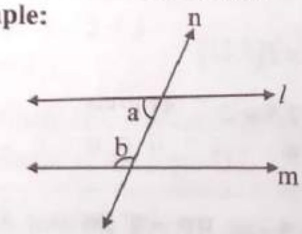
Example:
Conditional statement:
If a number is a prime number, then it is even or odd.
Converse: If a number is even or odd then it is a prime number.
Here, the given statement is true but its converse is not true.

Proofs

Till 300 B.C., Pythagoras and his group discovered many geometric properties and developed the theory of geometry to a great extent. At that time, Euclid, a teacher of mathematics at Alexandria in Egypt, brought about a revolutionary change in the outlook of the study of geometry. He organized the entire knowledge of geometry in such a way that if we assume some simple and obvious facts as true, then the other facts can be derived by logical reasoning.

1. **Postulates:** The self evident geometrical statements which are accepted by all are called postulates.
2. **Theorems:** Properties which can be proved logically are called theorems.
3. **The five postulates of Euclid:**
i. There are infinite lines passing through a point.
ii. There is one and only one line passing through two points.
iii. A circle of given radius can be drawn by taking any point as its centre.
iv. All right angles are congruent to each other.
v. If two interior angles formed on one side of a transversal of two lines add up to less than two right angles, then the lines produced in that direction intersect each other.

Example:



In the given figure $\angle a$ and $\angle b$ are interior angles formed on one side of transversal n.
If $\angle a + \angle b < 90° + 90°$
i.e., $\angle a + \angle b < 180°$
then lines l and m will be produced in the direction of $\angle a$ and $\angle b$, intersecting each other.

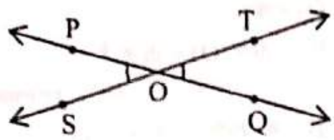
4. **Proof:**
i. The logical argument made to prove a theorem is called its proof.
ii. When we are going to prove that a conditional statement is true, its antecedent is called **given part** and the consequent is called **the part to be proved**.

5. **Types of proofs:**

i. **Direct proof:** If from an antecedent, we reach upto the consequent using axioms or previously proved theorems, then it is called a direct proof.

Example:

Theorem : The opposite angles formed by two intersecting lines are of equal measures.



Given : Line PQ and line ST intersect at point O such that P – O – Q, S – Q – T.

To prove : i. $\angle POT = \angle SOQ$
ii. $\angle POS = \angle QOT$

Proof : $\angle POS + \angle POT = 180°$ ...(i)[Angles in linear pair]

$\angle POS + \angle SOQ = 180°$ ...(ii)[Angles in linear pair]

$\angle POS + \angle POT = \angle POS + \angle SOQ$ ...[From (i) and (ii)]

$\therefore$ $\angle POT = \angle SOQ$ ... [Eliminating $\angle POS$]

Similarly, we can prove that $\angle POS = \angle QOT$

ii. **Indirect proof:** In this method, we suppose that the consequent is false and proceed logically and arrive at a step which contradicts what is given (antecedent) or some well known fact and then we accept that the consequent is true.

Example:

Statement : A prime number greater than 2 is odd.

Conditional statement : If p is a prime number greater than 2 then it is odd.

Given : p is a prime number greater than 2.

$\therefore$ 1 and p are the only divisors of p.

To prove : p is an odd number.

Proof : Let us assume that p is not an odd number.

$\therefore$ p is an even number

$\therefore$ divisor of p is 2 ...(i)

But it is given that p is a prime number greater than 2

$\therefore$ 1 and p are the only divisors of p ...(ii)

Statements (i) and (ii) are contradictory.

$\therefore$ our assumption , that p is not an odd number is false.

This proves that a prime number greater than 2 is odd.

Practice Set 1.3

1. **Write the following statements in 'if-then' form.** **[1 Mark each]**

i. The opposite angles of a parallelogram are congruent.

ii. The diagonals of a rectangle are congruent.

iii. In an isosceles triangle, the segment joining the vertex and the midpoint of the base is perpendicular to the base.

Ans:

i. If a quadrilateral is a parallelogram, then its opposite angles are congruent.

ii. If a quadrilateral is a rectangle, then its diagonals are congruent.

iii. If a triangle is isosceles triangle, then segment joining the vertex of a triangle and midpoint of the base is perpendicular to the base.

2. **Write converses of the following statements.** **[1 Mark each]**

i. The alternate angles formed by two parallel lines and their transversal are congruent.

ii. If a pair of the interior angles made by a transversal of two lines are supplementary, then the lines are parallel.

iii. The diagonals of a rectangle are congruent.

Ans:

i. If the alternate angles made by two lines and their transversal are congruent, then the two lines are parallel.

ii. If two parallel lines are intersected by a transversal, then the interior angles formed by the transversal are supplementary.

iii. If the diagonals of a quadrilateral are congruent, then that quadrilateral is a rectangle.

Problem Set – 1

1. **Select the correct alternative answer for the questions given below.** **[1 Mark each]**

i. How many midpoints does a segment have ?
(A) only one (B) two
(C) three (D) many

ii. How many points are there in the intersection of two distinct lines ?
(A) infinite (B) two
(C) one (D) not a single

iii. How many lines are determined by three distinct points?
(A) two (B) three
(C) one or three (D) six

iv. Find d(A, B), if co-ordinates of A and B are – 2 and 5 respectively.
(A) –2 (B) 5
(C) 7 (D) 3

v. If P – Q – R and d(P, Q) = 2, d(P, R) = 10, then find d(Q, R).
(A) 12 (B) 8
(C) $\sqrt{96}$ (D) 20

Answers:

i. (A) ii. (C) iii. (C)
iv. (C) v. (B)

Hints:

iv. Since, $5 > -2$

$\therefore$ $d(A, B) = 5 - (-2) = 5 + 2 = 7$

v. $d(P, R) = d(P, Q) + d(Q, R)$

$\therefore$ $10 = 2 + d(Q, R)$

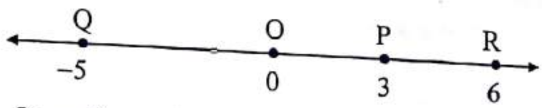
$\therefore$ $d(Q, R) = 8$

---

2. On a number line, co-ordinates of P, Q, R are 3, – 5 and 6 respectively. State with reason whether the following statements are true or false. **[2 Marks each]**

i. $d(P, Q) + d(Q, R) = d(P, R)$
ii. $d(P, R) + d(R, Q) = d(P, Q)$
iii. $d(R, P) + d(P, Q) = d(R, Q)$
iv. $d(P, Q) - d(P, R) = d(Q, R)$

Solution:



Co-ordinate of the point P is 3.
Co-ordinate of the point Q is –5.
Since, $3 > -5$
$d(P, Q) = 3 - (-5) = 3 + 5$

$\therefore$ $d(P, Q) = 8$

Co-ordinate of the point Q is –5.
Co-ordinate of the point R is 6.
Since, $6 > -5$
$d(Q, R) = 6 - (-5) = 6 + 5$

$\therefore$ $d(Q, R) = 11$

Co-ordinate of the point P is 3.
Co-ordinate of the point R is 6.
Since, $6 > 3$
$d(P, R) = 6 - 3$

$\therefore$ $d(P, R) = 3$

i. $d(P, Q) + d(Q, R) = 8 + 11$
$= 19$ ...(i)
$d(P, R) = 3$ ...(ii)

$\therefore$ $d(P, Q) + d(Q, R) \neq d(P, R)$ ...[From (i) and (ii)]

$\therefore$ **The given statement is false.**

ii. $d(P, R) + d(R, Q) = 3 + 11$
$= 14$ ...(i)
$d(P, Q) = 8$ ...(ii)

$\therefore$ $d(P, R) + d(R, Q) \neq d(P, Q)$ ...[From (i) and (ii)]

$\therefore$ **The given statement is false.**

iii. $d(R, P) + d(P, Q) = 3 + 8$
$= 11$ ...(i)
$d(R, Q) = 11$ ...(ii)

$\therefore$ $d(R, P) + d(P, Q) = d(R, Q)$ ...[From (i) and (ii)]

$\therefore$ **The given statement is true.**

iv. $d(P, Q) - d(P, R) = 8 - 3$
$= 5$ ...(i)
$d(Q, R) = 11$ ...(ii)

$\therefore$ $d(P, Q) - d(P, R) \neq d(Q, R)$ ...[From (i) and (ii)]

$\therefore$ **The given statement is false.**

---

3. Co-ordinates of some pairs of points are given below. Hence find the distance between each pair. **[1 Mark each]**

i. 3, 6
ii. –9, –1
iii. –4, 5
iv. 0, – 2
v. $x + 3, x - 3$
vi. –25, –47
vii. 80, –85

Solution:

i. Co-ordinate of first point is 3.
Co-ordinate of second point is 6.
Since, $6 > 3$

$\therefore$ Distance between the points $= 6 - 3 = \mathbf{3}$

ii. Co-ordinate of first point is –9.
Co-ordinate of second point is –1.
Since, $-1 > -9$

$\therefore$ Distance between the points $= -1 - (-9)$
$= -1 + 9 = \mathbf{8}$

iii. Co-ordinate of first point is –4.
Co-ordinate of second point is 5.
Since, $5 > -4$

$\therefore$ Distance between the points $= 5 - (-4)$
$= 5 + 4 = \mathbf{9}$

iv. Co-ordinate of first point is 0.
Co-ordinate of second point is –2.
Since, $0 > -2$

$\therefore$ Distance between the points $= 0 - (-2)$
$= 0 + 2$
$= \mathbf{2}$

v. Co-ordinate of first point is $x + 3$.
Co-ordinate of second point is $x - 3$.
Since, $x + 3 > x - 3$

$\therefore$ Distance between the points $= x + 3 - (x - 3)$
$= x + 3 - x + 3$
$= 3 + 3$
$= 6$

---

vi. Co-ordinate of first point is -25.
Co-ordinate of second point is -47.
Since, $-25 > -47$

$\therefore$ Distance between the points $= -25 - (-47)$
$= -25 + 47$
$= 22$

---

vii. Co-ordinate of first point is 80.
Co-ordinate of second point is -85.
Since, $80 > -85$

$\therefore$ Distance between the points $= 80 - (-85)$
$= 80 + 85$
$= 165$

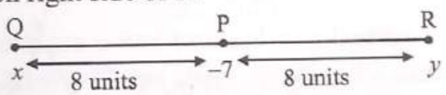
---

4. Co-ordinate of point P on a number line is -7. Find the co-ordinates of points on the number line which are at a distance of 8 units from point P. [3 Marks]

Solution:

Let point Q be at a distance of 8 units from P and on left side of P

Let point R be at a distance of 8 units from P and on right side of P.



i. Let the co-ordinate of point Q be x.
Co-ordinate of point P is -7.
Since, point Q is to the left of point P.

$\therefore$ $-7 > x$

$\therefore$ $d(P, Q) = -7 - x$

$\therefore$ $8 = -7 - x$

$\therefore$ $x = -7 - 8$

$\therefore$ $x = -15$

ii. Let the co-ordinate of point R be y.
Co-ordinate of point P is -7.
Since, point R is to the right of point P.

$\therefore$ $y > -7$

$\therefore$ $d(P, R) = y - (-7)$

$\therefore$ $8 = y + 7$

$\therefore$ $8 - 7 = y$

$\therefore$ $y = 1$

$\therefore$ **The co-ordinates of the points at a distance of 8 units from P are -15 and 1.**

5. Answer the following questions. **[1 Mark each]**

i. If A – B – C and d(A, C) = 17, d(B, C) = 6.5, then d (A, B) = ?

ii. If P – Q – R and d(P, Q) = 3.4, d(Q, R) = 5.7, then d(P, R) = ?

Solution:

i. Given, (A, C) = 17, d(B, C) = 6.5
d(A, C) = d(A, B) + d(B, C) ...[A–B–C]

$\therefore$ 17 = d(A, B) + 6.5

$\therefore$ d(A, B) = 17 – 6.5

$\therefore$ **d(A, B) = 10.5**

---

ii. Given, d(P, Q) = 3.4, d(Q, R) = 5.7
d(P, R) = d(P, Q) + d(Q, R) ...[P–Q–R]
= 3.4 + 5.7

$\therefore$ **d(P, R) = 9.1**

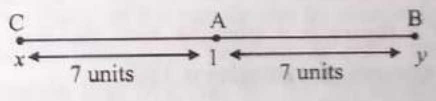
---

6. Co-ordinate of point A on a number line is 1. What are the co-ordinates of points on the number line which are at a distance of 7 units from A ? [3 Marks]

Solution:

Let point C be at a distance of 7 units from A and on left side of A

Let point B be at a distance of 7 units from A and on right side of A.



i. Let the co-ordinate of point C be x.
Co-ordinate of point A is 1.
Since, point C is to the left of point A.

$\therefore$ $1 > x$

$\therefore$ $d(A, C) = 1 - x$

$\therefore$ $7 = 1 - x$

$\therefore$ $x = 1 - 7$

$\therefore$ $x = -6$

ii. Let the co-ordinate of point B be y.
Co-ordinate of point A is 1.
Since, point B is to the right of point A.

$\therefore$ $y > 1$

$\therefore$ $d(A, B) = y - 1$

$\therefore$ $7 = y - 1$

$\therefore$ $7 + 1 = y$

$\therefore$ $y = 8$

$\therefore$ **The co-ordinates of the points at a distance of 7 units from A are -6 and 8.**

2 Parallel Lines

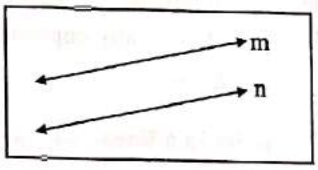
Type of Problems	Practice Set	Q. Nos.
Parallel lines, interior angle theorem, corresponding angle theorem, alternate angle theorem	2.1	Q.1, 2, 3, 4, 5
	Practice Problems (Based on Practice Set 2.1)	Q.1, 2, 3, 4, 5, 6
	Problem Set- 2	Q.3, 5, 6, 7
Test for parallel lines (Interior angles test, alternate angles test, corresponding angles test)	2.2	Q.1, 2, 3, 4, 5, 6
	Practice Problems (Based on Practice Set 2.2)	Q.1, 2, 3, 4, 5
	Problem Set- 2	Q.4, 8
Complementary angles, Supplementary angles	Problem Set- 2	Q.2

Let's Recall

1. **Parallel lines:**

   Lines in the same plane which do not intersect each other are called parallel lines.

   In the given figure, line m || line n.

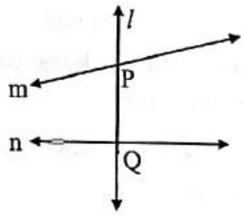


2. **Transversal:**

   If a line intersects two lines in two distinct points, then that line is called a transversal of those two lines.

   In the given figure, line l intersects lines m and n at points P and Q respectively.

   Hence, line l is a transversal of line m and line n.

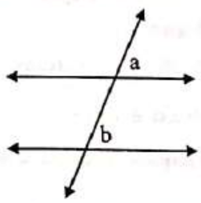


Angles formed by two lines and their transversal:

1. **Corresponding angles:**

   If the arms on the transversal of a pair of angles are in the same direction and the other arms are on the same side of the transversal, then it is called a pair of corresponding angles.

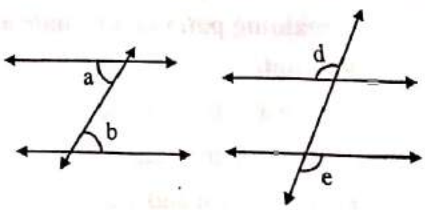
   In the adjacent figure, $\angle a$ and $\angle b$ is a pair of corresponding angles.



2. **Alternate angles:**

   A pair of angles which are on the opposite sides of the transversal such that their arms on the transversal are in opposite directions is called a pair of alternate angles.

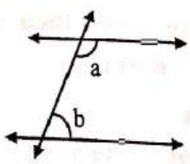
   In the adjacent figure, $\angle a$ and $\angle b$ is a pair of alternate interior angles, and $\angle d$ and $\angle e$ is pair of alternate exterior angles.



3. **Interior angles:**

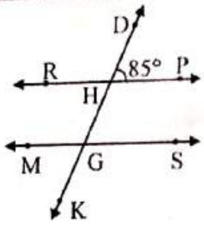
   A pair of angles which are on the same side of the transversal and lie between the two given lines is called a pair of interior angles.

   In the adjacent figure, $\angle a$ and $\angle b$ is a pair of interior angles on the same sides of the transversal.



Practice Set 2.1

1. **In the given figure, line RP || line MS and line DK is their transversal. ∠DHP = 85°. Find the measures of following angles. [3 Marks]**



i. ∠RHD ii. ∠PHG iii. ∠HGS iv. ∠MGK

Solution:

i.	∠DHP = 85°	(i)	[Given]
	∠DHP + ∠RHD = 180°		[Angles in a linear pair]
∴	85° + ∠RHD = 180°		
∴	∠RHD = 180° – 85°		
∴	**∠RHD = 95°**	(ii)	
ii.	∠PHG = ∠RHD		[Vertically opposite angles]
∴	**∠PHG = 95°**		[From (ii)]
iii.	line RP || line MS and line DK is their transversal.		
∴	∠HGS = ∠DHP		[Corresponding angles]
∴	**∠HGS = 85°**	(iii)	[From (i)]
iv.	∠MGK = ∠HGS		[Vertically opposite angles]
∴	**∠MGK = 85°**		[From (iii)]

Smart Check

There is more than one method of finding measure of these angles.

Example:

If you have ∠DHP = 85°

∴ ∠PHG + 85° = 180° ...[Angles in linear pair]

∴ ∠PHG = 180° – 85°

∴ ∠PHG = 95°

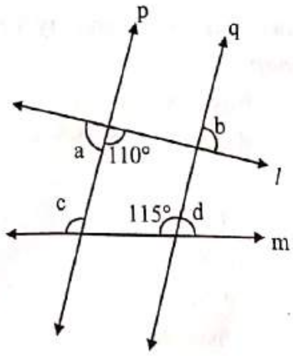
Now, ∠HGS = ∠DHP ...[Corresponding angles]

∴ ∠HGS = 85°

Now, ∠MGK = ∠DHP ...[Alternate exterior angles]

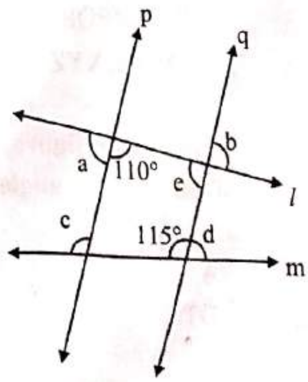
∴ ∠MGK = 85°

2. **In the given figure, line p || line q and line *l* and line m are transversals.**
Measures of some angles are shown. Hence find the measures of ∠a, ∠b, ∠c, ∠d. [3 Marks]



Solution:

i.	110° + ∠a = 180°	[Angles in a linear pair]
∴	∠a = 180° – 110°	
∴	**∠a = 70°**	
ii.	consider ∠e as shown in the figure line p || line q, and line *l* is their transversal.	
	∠e + 110° = 180°	[Interior angles]
∴	∠e = 180° – 110°	
∴	∠e = 70°	
	But, ∠b = ∠e	[Vertically opposite angles]
∴	**∠b = 70°**	
iii.	line p || line q, and line m is their transversal.	
∴	**∠c = 115°**	[Corresponding angles]



iv. $115° + \angle d = 180°$ [Angles in a linear pair]

$\therefore \angle d = 180° - 115°$

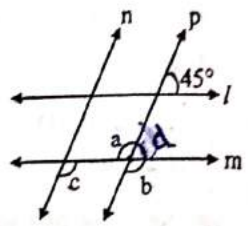
$\therefore \angle d = 65°$

Smart Check

Finding angles using other methods.

$\angle b = \angle a = 70°$ ...[Alternate exterior angles]

3. In the given figure, line l || line m and line n || line p. Find $\angle a$, $\angle b$, $\angle c$ from the given measure of an angle. **[3 Marks]**



Solution:

i. consider $\angle d$ as shown in the figure

line l || line m, and line p is their transversal.

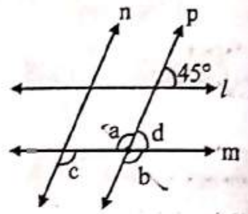
$\therefore \angle d = 45°$ [Corresponding angles]

Now, $\angle d + \angle b = 180°$ [Angles in a linear pair]

$\therefore 45° + \angle b = 180°$

$\therefore \angle b = 180° - 45°$

$\therefore$ **$\angle b = 135°$** (i)



ii. $\angle a = \angle b$ [Vertically opposite angles]

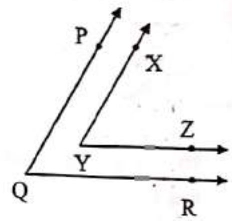
$\therefore$ **$\angle a = 135°$** [From (i)]

iii. line n || line p, and line m is their transversal.

$\therefore \angle c = \angle b$ [Corresponding angles]

$\therefore$ **$\angle c = 135°$** [From (i)]

4. In the given figure, sides of $\angle PQR$ and $\angle XYZ$ are parallel to each other. Prove that, $\angle PQR \cong \angle XYZ$. **[3 Marks]**



Given: Ray YZ || ray QR and ray YX || ray QP

To prove: $\angle PQR \cong \angle XYZ$

Construction: Extend ray YZ in the opposite direction. It intersects ray QP at point S.

Proof:

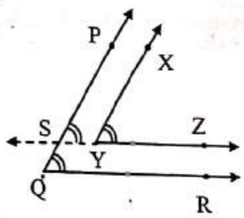
Ray YX || ray QP [Given]

$\therefore$ Ray YX || ray SP and seg SY is their transversal [P–S–Q]

$\therefore \angle XYZ \cong \angle PSY$ (i) [Corresponding angles]

ray YZ || ray QR [Given]

ray SZ || ray QR and seg PQ is their transversal. [S–Y–Z]

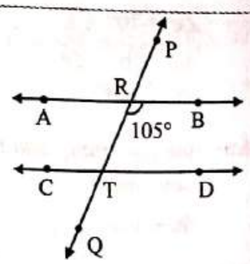


$\therefore \angle PSY \cong \angle SQR$ [Corresponding angles]

$\therefore \angle PSY \cong \angle PQR$ (ii) [P–S–Q]

$\therefore$ **$\angle PQR \cong \angle XYZ$** [From (i) and (ii)]

5. In the given figure, line AB || line CD and line PQ is transversal. Measure of one of the angles is given. Hence find the measures of the following angles. **[3 Marks]**



i. $\angle ART$ ii. $\angle CTQ$

iii. $\angle DTQ$ iv. $\angle PRB$

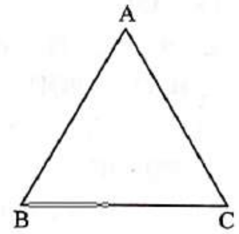
Solution:

i.	$\angle BRT = 105°$	(i)	[Given]
	$\angle ART + \angle BRT = 180°$		[Angles in a linear pair]
$\therefore$	$\angle ART + 105° = 180°$		
$\therefore$	$\angle ART = 180° - 105°$		
$\therefore$	$\mathbf{\angle ART = 75°}$	(ii)	
ii.	line AB ∥ line CD and line PQ is their transversal.		
$\therefore$	$\angle CTQ = \angle ART$		[Corresponding angles]
$\therefore$	$\mathbf{\angle CTQ = 75°}$		[From (ii)]
iii.	line AB ∥ line CD and line PQ is their transversal.		
$\therefore$	$\angle DTQ = \angle BRT$		[Corresponding angles]
$\therefore$	$\mathbf{\angle DTQ = 105°}$		[From (i)]
iv	$\angle PRB = \angle ART$		[Vertically opposite angles]
$\therefore$	$\mathbf{\angle PRB = 75°}$		[From (ii)]

Let's Study

Use of properties of parallel lines

Theorem: The sum of the measures of all the angles of a triangle is 180°.

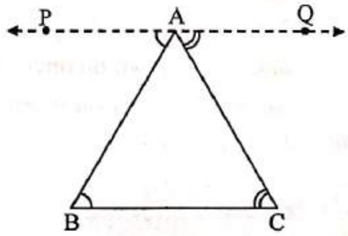


Given: ΔABC is any triangle.

To prove: $\angle ABC + \angle ACB + \angle BAC = 180°$.

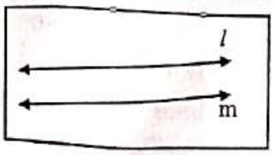
Construction: Draw a line parallel to seg BC and passing through A. On the line take points P and Q such that, P – A – Q.

Proof:



	line PQ ∥ side BC and seg AB is their transversal.		[Construction]
$\therefore$	$\angle ABC = \angle PAB$	(i)	[Alternate angles]
	line PQ ∥ side BC and seg AC is their transversal.		[Construction]
$\therefore$	$\angle ACB = \angle QAC$	(ii)	[Alternate angles]
	Adding (i) and (ii), we get		
	$\angle ABC + \angle ACB = \angle PAB + \angle QAC$		
$\therefore$	$\angle ABC + \angle ACB + \angle BAC$		
	$= \angle PAB + \angle QAC + \angle BAC$		[Adding $\angle BAC$ to both sides]
	$= \angle PAB + \angle BAC + \angle QAC$		
	$= \angle PAC + \angle QAC$		[$\because \angle PAB + \angle BAC = \angle PAC$]
	$= 180°$		[Angles in a linear pair]
$\therefore$	$\angle ABC + \angle ACB + \angle BAC = 180°$		
$\therefore$	The sum of measures of all angles of a triangle is 180°.		

Try This



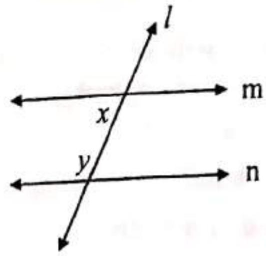
1. In the given figure, how will you decide whether line *l* and line m are parallel or not? (Textbook pg. no. 19)

Ans: In the figure, we observe that line *l* and line m are coplanar and do not intersect each other.

$\therefore$ **Line *l* and line m are parallel lines.**

Practice Set 2.2

1. **In the given figure, $y = 108°$ and $x = 71°$.**
Are the lines m and n parallel? Justify? **[1 Mark]**



Solution:

$y = 108°, x = 71°$ ...[Given]

$x + y = 71° + 108°$

$= 179°$

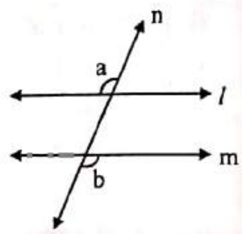
$\therefore$ $x + y \neq 180°$

$\therefore$ The angles x and y are not supplementary.

$\therefore$ The angles do not satisfy the interior angles test for parallel lines

$\therefore$ **line m and line n are not parallel lines.**

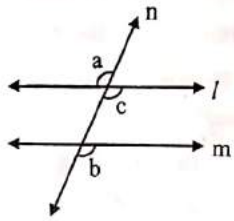
2. **In the given figure, if $\angle a \cong \angle b$ then prove that line $l \parallel$ line m. [2 Marks]**



Given: $\angle a \cong \angle b$

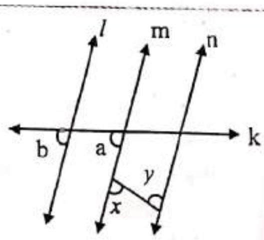
To prove: line $l \parallel$ line m

Proof:



consider $\angle c$ as shown in the figure		
$\angle a \cong \angle c$	(i)	[Vertically opposite angles]
But, $\angle a \cong \angle b$	(ii)	[Given]
$\therefore$ $\angle b \cong \angle c$		[From (i) and (ii)]
But, $\angle b$ and $\angle c$ are corresponding angles on lines l and m when line n is the transversal.		
$\therefore$ **line $l \parallel$ line m**		[Corresponding angles test]

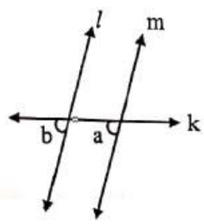
3. **In the given figure, if $\angle a \cong \angle b$ and $\angle x \cong \angle y$, then prove that line $l \parallel$ line n.** **[3 Marks]**

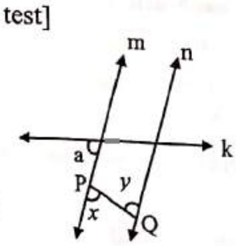


Given: $\angle a \cong \angle b$ and $\angle x \cong \angle y$

To prove: line $l \parallel$ line n

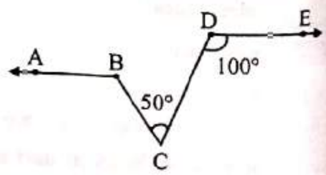
Proof:





$\angle a \cong \angle b$		[Given]
But, $\angle a$ and $\angle b$ are corresponding angles on lines l and m when line k is the transversal.		
$\therefore$ line $l \parallel$ line m	(i)	[Corresponding angles test]
$\angle x \cong \angle y$		[Given]
But, $\angle x$ and $\angle y$ are alternate angles on lines m and n when seg PQ is the transversal.		
$\therefore$ line m $\parallel$ line n	(ii)	[Alternate angles test]
$\therefore$ From (i) and (ii),		
line $l \parallel$ line m $\parallel$ line n		
i.e., line $l \parallel$ line n		

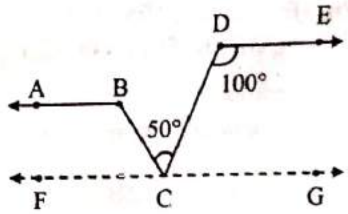
4. **In the given figure, if ray BA $\parallel$ ray DE, $\angle C = 50°$ and $\angle D = 100°$. Find the measure of $\angle ABC$.** **[3 Marks]**
(Hint: Draw a line passing through point C and parallel to line AB.)

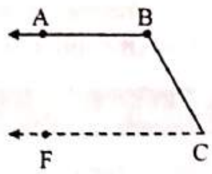


Solution:

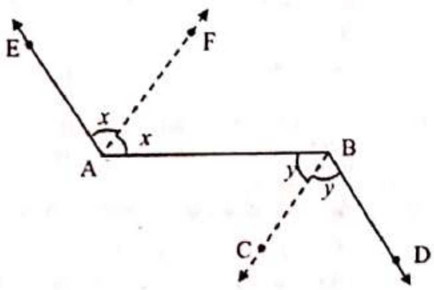
Draw a line FG passing through point C and parallel to line AB		
line FG $\parallel$ ray BA	(i)	[Construction]

	Ray BA ∥ ray DE	(ii)	[Given]
∴	line FG ∥ ray BA ∥ ray DE	(iii)	[From (i) and (ii)]
	line FG ∥ ray DE		[From (iii)]
	and seg DC is their transvensal		
∴	$\angle DCF = \angle EDC$		[Alternate angles]
∴	$\angle DCF = 100^\circ$		[$\because \angle D = 100^\circ$]
	Now, $\angle DCF = \angle BCF + \angle BCD$		[Angle addition property]
∴	$100^\circ = \angle BCF + 50^\circ$		
∴	$100^\circ - 50^\circ = \angle BCF$		
∴	$\angle BCF = 50^\circ$	(iv)	
	Now, line FG ∥ ray BA and seg BC is their transversal.		
∴	$\angle ABC + \angle BCF = 180^\circ$		[Interior angles]
∴	$\angle ABC + 50^\circ = 180^\circ$		[From (iv)]
∴	$\angle ABC = 180^\circ - 50^\circ$		
∴	$\mathbf{\angle ABC = 130^\circ}$		





5. In the given figure, ray AE ∥ ray BD, ray AF is the bisector of ∠EAB and ray BC is the bisector of ∠ABD. Prove that line AF ∥ line BC. **[3 Marks]**

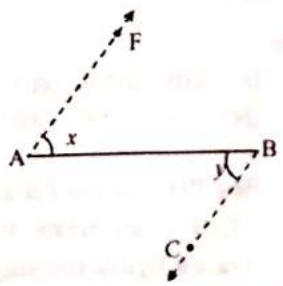


Given: Ray AE ∥ ray BD, and ray AF and ray BC are the bisectors of ∠EAB and ∠ABD respectively.

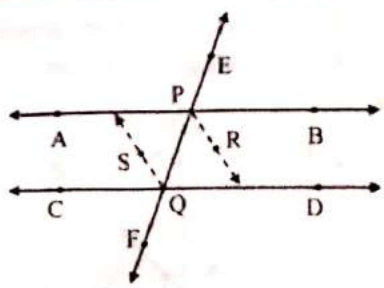
To prove: line AF ∥ line BC

Proof:

	Ray AE ∥ ray BD and seg AB is their transversal.		
∴	$\angle EAB = \angle ABD$	(i)	[Alternate angles]
	$\angle FAB = \frac{1}{2}\angle EAB$		[Ray AF bisects ∠EAB]
∴	$2\angle FAB = \angle EAB$	(ii)	
	$\angle CBA = \frac{1}{2}\angle ABD$		[Ray BC bisects ∠ABD]
∴	$2\angle CBA = \angle ABD$	(iii)	
∴	$2\angle FAB = 2\angle CBA$		[From (i), (ii) and (iii)]
∴	$\angle FAB = \angle CBA$		
	But, ∠FAB and ∠ABC are alternate angles on lines AF and BC when seg AB is the transversal.		
∴	**line AF ∥ line BC**		[Alternate angles test]



6. A transversal EF of line AB and line CD intersects the lines at points P and Q respectively. Ray PR and ray QS are parallel and bisectors of ∠BPQ and ∠PQC respectively. Prove that line AB ∥ line CD. **[3 Marks]**



Given: Ray PR ∥ ray QS
Ray PR and ray QS are the bisectors of ∠BPQ and ∠PQC respectively.

To prove: line AB ∥ line CD

Proof:

Ray PR ∥ ray QS and seg PQ is their transversal.		
$\angle RPQ = \angle SQP$	(i)	[Alternate angles]
$\angle RPQ = \frac{1}{2}\angle BPQ$	(ii)	[Ray PR bisects $\angle BPQ$]
$\angle SQP = \frac{1}{2}\angle PQC$	(iii)	[Ray QS bisects $\angle PQC$]
∴ $\frac{1}{2}\angle BPQ = \frac{1}{2}\angle PQC$		[From (i), (ii) and (iii)]
∴ $\angle BPQ = \angle PQC$		
But, $\angle BPQ$ and $\angle PQC$ are alternate angles on lines AB and CD when line EF is the transversal.		
∴ **line AB ∥ line CD**		[Alternate angles test]

Problem Set – 2

1. Select the correct alternative and fill in the blanks in the following statements. **[1 Mark each]**

i. If a transversal intersects two parallel lines then the sum of interior angles on the same side of the transversal is ________.
(A) 0° (B) 90° (C) 180° (D) 360°

ii. The number of angles formed by a transversal of two lines is ________.
(A) 2 (B) 4 (C) 8 (D) 16

iii. A transversal intersects two parallel lines. If the measure of one of the angles is 40°, then the measure of corresponding angle is ________
(A) 40° (B) 140° (C) 50° (D) 180°

iv. In ΔABC, $\angle A = 76°$, $\angle B = 48°$, then $\angle C$ = ________.
(A) 66° (B) 56° (C) 124° (D) 28°

v. Two parallel lines are intersected by a transversal. If measure of one of the alternate interior angles is 7… then the measure of the other angle is ________.
(A) 105° (B) 15° (C) 75° (D) 45°

Answers:

i. (C) ii. (C) iii. (A) iv. (B)
v. (C)

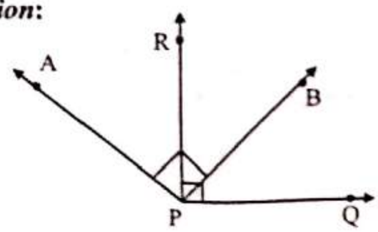
Hints:

iv. In ΔABC, $\angle A + \angle B + \angle C = 180°$
∴ $\angle C = 180° - 76° - 48° = 56°$

2. Ray PQ and ray PR are perpendicular to each other. Points B and A are in the interior and exterior of $\angle QPR$ respectively. Ray PB and ray PA are perpendicular to each other.
Draw a figure showing all these rays and write -

i. A pair of complementary angles **[1 Mark]**
ii. A pair of supplementary angles **[1 Mark]**
iii. A pair of congruent angles. **[2 Marks]**

Solution:



3 Triangles

Type of Problems	Practice Set	Q. Nos.
Sum of the measures of the angles of a triangle	3.1	Q.2, 3, 4, 5, 6, 7, 8, 9
	Practice Problems (Based on Practice Set 3.1)	Q.1, 2, 4, 5(i), 6, 7, 8
Theorem of remote interior angles of a triangle, property of an exterior angle of triangle	3.1	Q.1
	Practice Problems (Based on Practice Set 3.1)	Q. 3, 5(ii, iii, iv), 9
	Problem set-3	Q.8
Congruence of triangles, Properties of congruence of a triangle	3.2	Q.1, 2, 3, 4, 5, 6
	Practice Problems (Based on Practice Set 3.2)	Q.1, 2, 3, 4, 5
	3.4	Q.8
Isosceles triangle theorem and its converse	3.1	Q.10
	3.3	Q.1
	Practice Problems (Based on Practice Set 3.3)	Q.1, 2, 3, 4, 5, 6, 7
	3.4	Q.5, 6
	Problem set-3	Q.2, 4, 7
Median of a triangle, property of median drawn on hypotenuse of right triangle	3.3	Q.2, 3, 4
	Practice Problems (Based on Practice Set 3.3)	Q.8
Perpendicular bisector theorem, angle bisector theorem	3.4	Q.1, 2
	Practice Problems (Based on Practice Set 3.4)	Q.1
Properties of inequalities of sides and angles of a triangle	3.4	Q.3, 4, 7
	Practice Problems (Based on Practice Set 3.4)	Q. 2, 3, 4, 5, 6, 7, 8
	Problem set-3	Q.3, 5, 6
Similar triangles	3.5	Q.1, 2, 3
	Practice Problems (Based on Practice Set 3.5)	Q.1, 2, 3, 4, 5

Let's Study

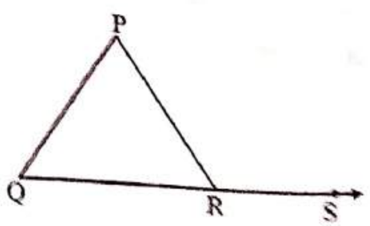
Exterior angle:

Exterior angle of a triangle is formed when one side of a triangle is extended.

If the side QR of ΔPQR is extended at point S such that Q – R – S, then $\angle PRS$ is called an exterior angle of ΔPQR at R.

$\angle PQR$ and $\angle QPR$ are called remote interior angles.

Every triangle has 6 exterior angles, two at each vertex.



In ΔRTS,

$\angle TRS + \angle RTS + \angle TSR = \boxed{180°}$ ...[Sum of the measures of the angles of a triangle is 180°]

$\therefore$ $\boxed{70°} + \angle RTS + \boxed{20°} = 180°$

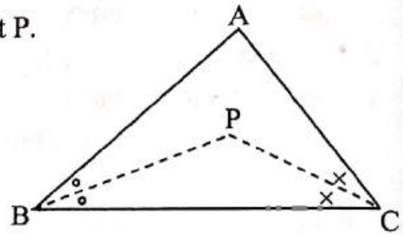
$\therefore$ $\angle RTS + 90° = 180°$

$\therefore$ $\angle RTS = 180° - 90°$

$\therefore$ $\angle RTS = \boxed{90°}$

2. In the given figure, bisectors of $\angle B$ and $\angle C$ of ΔABC intersect at point P. Prove that $\angle BPC = 90° + \frac{1}{2}\angle BAC$.

Complete the proof by filling in the blanks. (Textbook pg. no.27)



Proof:

In ΔABC,

$\angle BAC + \angle ABC + \angle ACB = \boxed{180°}$ ...[Sum of the measures of the angles of a triangle is 180°]

$\therefore$ $\frac{1}{2}\angle BAC + \frac{1}{2}\angle ABC + \frac{1}{2}\angle ACB = \frac{1}{2} \times \boxed{180°}$ ... $\left[\text{Multiplying each term by } \frac{1}{2}\right]$

$\therefore$ $\frac{1}{2}\angle BAC + \angle PBC + \angle PCB = 90°$

$\therefore$ $\angle PBC + \angle PCB = 90 - \frac{1}{2}\angle BAC$ ...(i)

In ΔBPC,

$\angle BPC + \angle PBC + \angle PCB = 180°$ ...[Sum of measures of angles of a triangle]

$\therefore$ $\angle BPC + \boxed{90° - \frac{1}{2}\angle BAC} = 180°$ ...[From (i)]

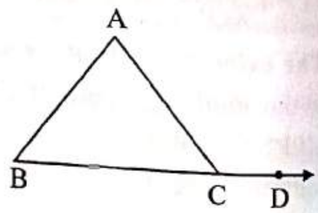
$\therefore$ $\angle BPC = 180° - \left(90° - \frac{1}{2}\angle BAC\right)$

$= 180° - 90° + \frac{1}{2}\angle BAC$

$= 90° + \frac{1}{2}\angle BAC$

Practice Set 3.1

1. In the adjoining figure, $\angle ACD$ is an exterior angle of ΔABC. $\angle B = 40°$, $\angle A = 70°$. Find the measure of $\angle ACD$. [1 Mark]



Solution:

$\angle A = 70°$, $\angle B = 40°$ [Given]

$\angle ACD$ is an exterior angle of ΔABC. [Given]

$\therefore$ $\angle ACD = \angle A + \angle B$ [Theorem of remote interior angles]

$= 70° + 40°$

$\therefore$ $\mathbf{\angle ACD = 110°}$

Smart Check

$\angle ACB + \angle ACD = 180°$ ...[Angles in a linear pair]

$\therefore$ $\angle ACB + 110° = 180°$

$\therefore$ $\angle ACB = 180° - 110° = 70°$

If the sum of the angles of ΔABC is 180°, then our answer is correct.

$\angle ABC + \angle ACB + \angle BAC = 40° + 70° + 70° = 180°$

2. **In ΔPQR, $\angle P = 70°$, $\angle Q = 65°$, then find $\angle R$.** **[1 Mark]**

Solution:

$\angle P = 70°$, $\angle Q = 65°$	[Given]
In ΔPQR,	
$\angle P + \angle Q + \angle R = 180°$	[Sum of the measures of the angles of a triangle is 180°]
$\therefore$ $70° + 65° + \angle R = 180°$	
$\therefore$ $\angle R = 180° - 70° - 65°$	
$\therefore$ $\angle R = 45°$	

Smart Check

If the sum of the angles of ΔPQR is 180°, then our answer is correct.
$\angle P + \angle Q + \angle R = 70° + 65° + 45° = 180°$

3. **The measures of angles of a triangle are $x°$, $(x - 20)°$, $(x - 40)°$. Find the measure of each angle.** **[2 Marks]**

Solution:

The measures of the angles of a triangle are $x°$, $(x - 20)°$, $(x - 40)°$.	[Given]
$\therefore$ $x° + (x - 20)° + (x - 40)° = 180°$	[Sum of the measures of the angles of a triangle is 180°]
$\therefore$ $3x - 60 = 180$	
$\therefore$ $3x = 180 + 60$	
$\therefore$ $3x = 240$	
$\therefore$ $x = \frac{240}{3}$	
$\therefore$ $x = 80°$	
$\therefore$ The measures of the remaining angles are $x - 20° = 80° - 20° = 60°$, $x - 40° = 80° - 40° = 40°$	
$\therefore$ **The measures of the angles of the triangle are 80°, 60° and 40°.**	

4. **The measure of one of the angles of a triangle is twice the measure of its smallest angle and the measure of the other is thrice the measure of the smallest angle. Find the measures of the three angles.** **[3 Marks]**

Solution:

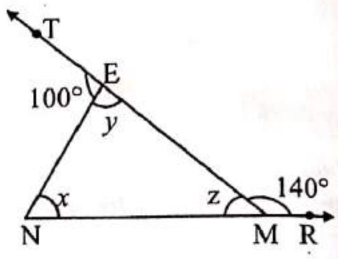
Let the measure of the smallest angle be $x°$.	
One of the angles is twice the measure of the smallest angle.	
$\therefore$ Measure of that angle = $2x°$	
Another angle is thrice the measure of the smallest angle.	
$\therefore$ Measure of that angle = $3x°$	
$\therefore$ The measures of the remaining two angles are $2x°$ and $3x°$.	
Now, $x° + 2x° + 3x° = 180°$	[Sum of the measures of the angles of a triangle is 180°]
$\therefore$ $6x = 180$	
$\therefore$ $x = \frac{180}{6}$	
$\therefore$ $x° = 30°$	

∴ The measures of the remaining angles are $2x^\circ = 2 \times 30^\circ = 60^\circ$,

$3x^\circ = 3 \times 30^\circ = 90^\circ$

∴ **The measures of the three angles of the triangle are 30°, 60° and 90°.**

5. In the adjoining figure, measures of some angles are given. Using the measures, find the values of x, y, z. [3 Marks]



Solution:

i. $\angle NET = 100^\circ$ and $\angle EMR = 140^\circ$ [Given]

$\angle EMN + \angle EMR = 180^\circ$ [Angles in a linear pair]

∴ $z + 140^\circ = 180^\circ$

∴ $z = 180^\circ - 140^\circ$

∴ $z = 40^\circ$

ii. Also, $\angle NET + \angle NEM = 180^\circ$ [Angles in a linear pair]

∴ $100^\circ + y = 180^\circ$

∴ $y = 180^\circ - 100^\circ$

∴ $y = 80^\circ$

iii. In ΔENM,

$\angle ENM + \angle NEM + \angle EMN = 180^\circ$ [Sum of the measures of the angles of a triangle is 180°]

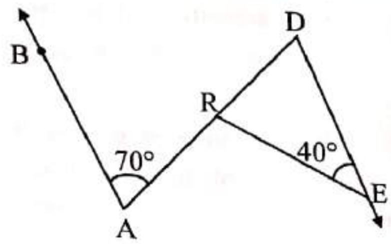
∴ $x + 80^\circ + 40^\circ = 180^\circ$

∴ $x = 180^\circ - 80^\circ - 40^\circ$

∴ $x = 60^\circ$

∴ $\boldsymbol{x = 60^\circ, y = 80^\circ, z = 40^\circ}$

6. In the adjoining figure, line AB || line DE. Find the measures of ∠DRE and ∠ARE using given measures of some angles. [3 Marks]



Solution:

i. $\angle BAD = 70^\circ$, $\angle DER = 40^\circ$ [Given]

line AB || line DE and seg AD is their transversal.

∴ $\angle EDA = \angle BAD$ [Alternate angles]

∴ $\angle EDA = 70^\circ$ (i)

In ΔDRE,

$\angle EDR + \angle DER + \angle DRE = 180^\circ$ [Sum of the measures of the angles of a triangle is 180°]

∴ $70^\circ + 40^\circ + \angle DRE = 180^\circ$ [From (i) and D–R–A]

∴ $\angle DRE = 180^\circ - 70^\circ - 40^\circ$

∴ $\angle DRE = 70^\circ$

ii. $\angle DRE + \angle ARE = 180^\circ$ [Angles in a linear pair]

∴ $70^\circ + \angle ARE = 180^\circ$

∴ $\angle ARE = 180^\circ - 70^\circ$

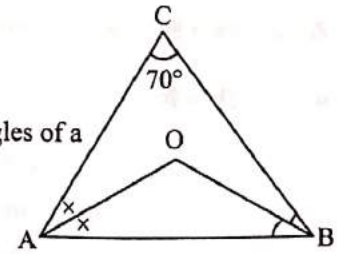
∴ $\angle ARE = 110^\circ$

∴ $\boldsymbol{\angle DRE = 70^\circ, \angle ARE = 110^\circ}$

7. **In ΔABC, bisectors of $\angle A$ and $\angle B$ intersect at point O. If $\angle C = 70^\circ$, find the measure of $\angle AOB$.**

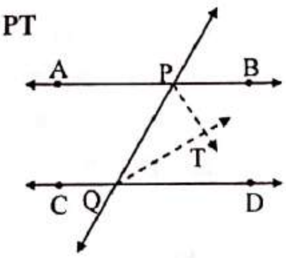
[3 Marks]

***Solution*:**



	$\angle OAB = \angle OAC = \frac{1}{2}\angle BAC$	(i)	[Seg AO bisects $\angle BAC$]
	$\angle OBA = \angle OBC = \frac{1}{2}\angle ABC$	(ii)	[Seg BO bisects $\angle ABC$]
	In ΔABC,		
	$\angle BAC + \angle ABC + \angle ACB = 180^\circ$		[Sum of the measures of the angles of a triangle is 180°]
∴	$\angle BAC + \angle ABC + 70^\circ = 180^\circ$		
∴	$\angle BAC + \angle ABC = 180^\circ - 70^\circ$		
∴	$\angle BAC + \angle ABC = 110^\circ$		
∴	$\frac{1}{2}(\angle BAC) + \frac{1}{2}(\angle ABC) = \frac{1}{2} \times 110^\circ$		$\left[\text{Multiplying both sides by } \frac{1}{2}\right]$
∴	$\angle OAB + \angle OBA = 55^\circ$	(iii)	[From (i) and (ii)]
	In ΔOAB,		
	$\angle OAB + \angle OBA + \angle AOB = 180^\circ$		[Sum of the measures of the angles of a triangle is 180°]
∴	$55^\circ + \angle AOB = 180^\circ$		[From (iii)]
∴	$\angle AOB = 180^\circ - 55^\circ$		
∴	$\mathbf{\angle AOB = 125^\circ}$		

8. **In the adjoining figure, line AB ‖ line CD and line PQ is the transversal. Ray PT and ray QT are bisectors of $\angle BPQ$ and $\angle PQD$ respectively. Prove that m $\angle PTQ = 90^\circ$.** **[3 Marks]**



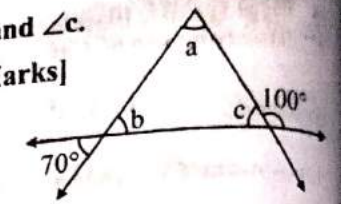
Given: line AB ‖ line CD and line PQ is the transversal. ray PT and ray QT are the bisectors of $\angle BPQ$ and $\angle PQD$ respectively.

To prove: $m\angle PTQ = 90^\circ$

***Proof*:**

	$\angle TPB = \angle TPQ = \frac{1}{2}\angle BPQ$	(i)	[Ray PT bisects $\angle BPQ$]
	$\angle TQD = \angle TQP = \frac{1}{2}\angle PQD$	(ii)	[Ray QT bisects $\angle PQD$]
	line AB ‖ line CD and line PQ is their transversal.		[Given]
∴	$\angle BPQ + \angle PQD = 180^\circ$		[Interior angles]
∴	$\frac{1}{2}(\angle BPQ) + \frac{1}{2}(\angle PQD) = \frac{1}{2} \times 180^\circ$		$\left[\text{Multiplying both sides by } \frac{1}{2}\right]$
∴	$\angle TPQ + \angle TQP = 90^\circ$	(iii)	[From (i) and (ii)]
	In ΔPTQ,		
	$\angle TPQ + \angle TQP + \angle PTQ = 180^\circ$		[Sum of the measures of the angles of a triangle is 180°]
∴	$90^\circ + \angle PTQ = 180^\circ$		[From (iii)]
∴	$\angle PTQ = 180^\circ - 90^\circ$		
	$= 90^\circ$		
∴	$\mathbf{m\angle PTQ = 90^\circ}$		

9. Using the information in the adjoining figure, find the measures of $\angle a$, $\angle b$ and $\angle c$. **[3 Marks]**



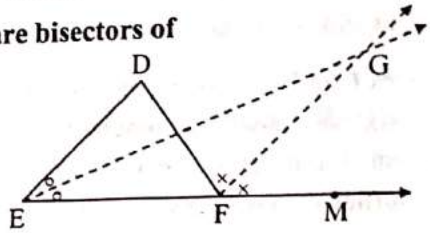
Solution:

i.	$\angle c + 100° = 180°$	[Angles in a linear pair]
$\therefore$	$\angle c = 180° - 100°$	
$\therefore$	$\angle c = 80°$	
ii.	$\angle b = 70°$	[Vertically opposite angles]
iii.	$\angle a + \angle b + \angle c = 180°$	[Sum of the measures of the angles of a triangle is 180°]
$\therefore$	$\angle a + 70° + 80° = 180°$	
$\therefore$	$\angle a = 180° - 70° - 80°$	
$\therefore$	$\angle a = 30°$	
$\therefore$	$\mathbf{\angle a = 30°, \angle b = 70°, \angle c = 80°}$	

10. In the adjoining figure, line DE || line GF, ray EG and ray FG are bisectors of $\angle DEF$ and $\angle DFM$ respectively. Prove that,

i. $\angle DEG = \frac{1}{2}\angle EDF$

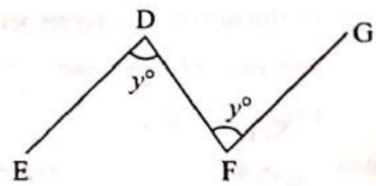
ii. EF = FG

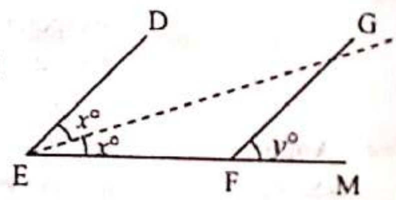


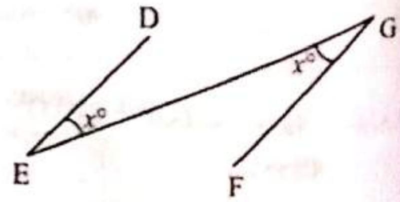
[4 Marks]

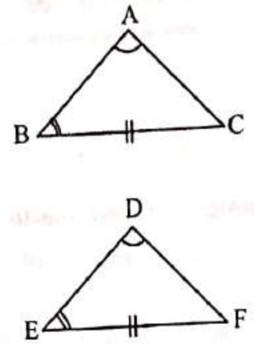
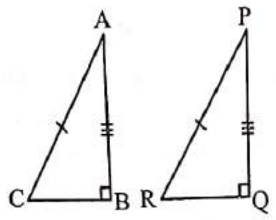
Proof:

i	$\angle DEG = \angle FEG = x°$	(i)	[Ray EG bisects $\angle DEF$]
	$\angle GFD = \angle GFM = y°$	(ii)	[Ray FG bisects $\angle DFM$]
	line DE || line GF		[Given]
	and DF is their transversal.		
$\therefore$	$\angle EDF = \angle GFD$		[Alternate angles]
$\therefore$	$\angle EDF = y°$	(iii)	[From (ii)]
	line DE || line GF		[Given]
	and EM is their transversal.		
$\therefore$	$\angle DEF = \angle GFM$		[Corresponding angles]
$\therefore$	$\angle DEG + \angle FEG = \angle GFM$		[Angle addition property]
$\therefore$	$x° + x° = y°$		[From (i) and (ii)]
$\therefore$	$2x° = y°$		
$\therefore$	$x° = \frac{1}{2}y°$		
$\therefore$	$\mathbf{\angle DEG = \frac{1}{2}\angle EDF}$		[From (i) and (iii)]
ii.	line DE || line GF		[Given]
	and GE is their transversal.		
$\therefore$	$\angle DEG = \angle FGE$	(iv)	[Alternate angles]
$\therefore$	$\angle FEG = \angle FGE$	(v)	[From (i) and (iv)]
$\therefore$	In ΔFEG,		
	$\angle FEG = \angle FGE$		[From (v)]
$\therefore$	**EF = FG**		[Converse of isosceles triangle theorem]







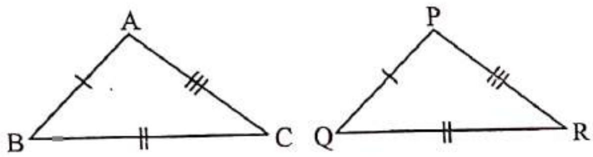
Side – Angle – Angle (SAA) Or Angle – Angle – Side (AAS)	For a given correspondence, when two angles and a non-included side of one triangle are congruent to the corresponding angles and the non-included side of other triangle, then the two triangles are congruent.		In ΔABC and ΔDEF, if seg $BC \cong$ seg EF, $\angle BAC \cong \angle EDF$, and $\angle ABC \cong \angle DEF$, then $\Delta ABC \cong \Delta DEF$
Hypotenuse-side	Two right angled triangles are congruent, if the hypotenuse and a side of one triangle are congruent to the hypotenuse and the corresponding side of the other triangle.		In right angled ΔABC and ΔPQR, if $\angle ABC = \angle PQR = 90°$ hypotenuse $AC \cong$ hypotenuse PR, and seg $AB \cong$ seg PQ, then $\Delta ABC \cong \Delta PQR$

[**Note:** Corresponding sides of congruent triangles in short is written as c.s.c.t. and corresponding angles of congruent triangles in short is written as c.a.c.t.]

Practice Set 3.2

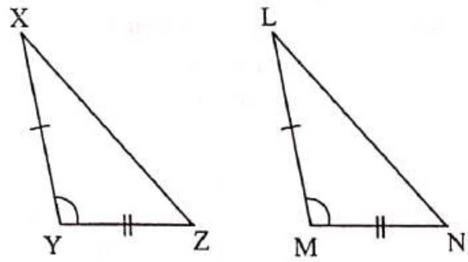
1. **In each of the examples given below, a pair of triangles is shown. Equal parts of triangles in each pair are marked with the same signs. Observe the figures and state the test by which the triangles in each pair are congruent.** **[1 Mark each]**

i.



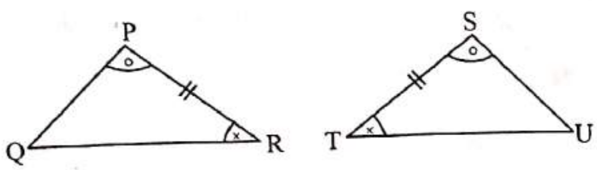
By **SSS** test
$\Delta ABC \cong \Delta PQR$

ii.



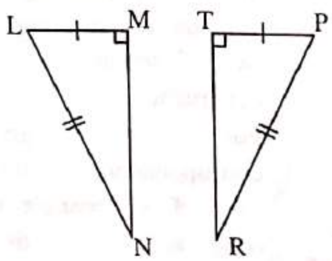
By **SAS** test
$\Delta XYZ \cong \Delta LMN$

iii.



By **ASA** test
$\Delta PRQ \cong \Delta STU$

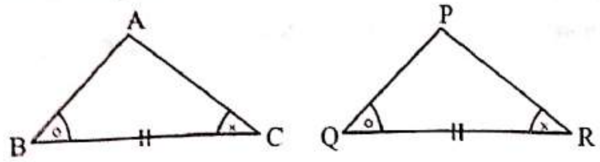
iv.



By **hypotenuse side** test
$\Delta LMN \cong \Delta PTR$

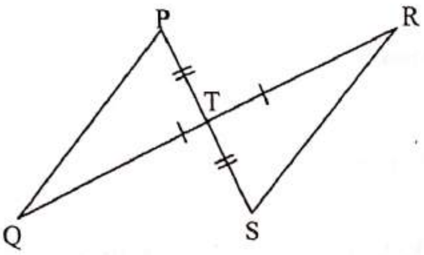
2. **Observe the information shown in pairs of triangles given below. State the test by which the two triangles are congruent. Write the remaining congruent parts of the triangles.** **[2 Marks each]**

i.



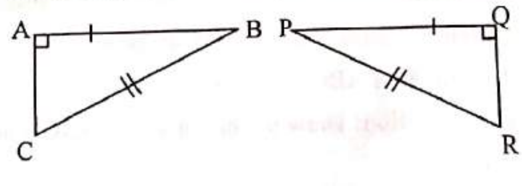
	From the information shown in the figure,	
	In ΔABC and ΔPQR,	
	$\angle ABC \cong \angle PQR$	
	seg BC $\cong$ seg QR	
	$\angle ACB \cong \angle PRQ$	
$\therefore$	$\Delta ABC \cong \Delta PQR$	[$\boxed{\text{ASA}}$ test]
$\therefore$	$\angle BAC \cong \boxed{\angle QPR}$	[Corresponding angles of congruent triangles]
	seg AB $\cong$ $\boxed{\text{seg PQ}}$ and $\boxed{\text{seg AC}}$ $\cong$ seg PR	[Corresponding sides of congruent triangles]

ii.



	From the information shown in the figure,	
	In ΔPTQ and ΔSTR,	
	seg PT $\cong$ seg ST	
	$\angle PTQ \cong \angle STR$	[Vertically opposite angles]
	seg TQ $\cong$ seg TR	
$\therefore$	$\Delta PTQ \cong \Delta STR$	[$\boxed{\text{SAS}}$ test]
$\therefore$	$\angle TPQ \cong \boxed{\angle TSR}$ and $\boxed{\angle TQP} \cong \angle TRS$	[Corresponding angles of congruent triangles]
	seg PQ $\cong$ $\boxed{\text{seg SR}}$	[Corresponding sides of congruent triangles]

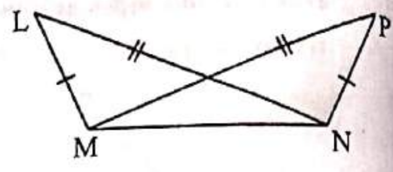
3. **From the information shown in the figure, state the test assuring the congruence of ΔABC and ΔPQR. Write the remaining congruent parts of the triangles.** **[2 Marks]**



***Solution*:**

	In ΔBAC and ΔPQR,	
	seg BA $\cong$ seg PQ seg BC $\cong$ seg PR $\angle BAC = \angle PQR = 90°$	[Given]
$\therefore$	$\Delta BAC \cong \Delta PQR$	**[Hypotenuse side test]**
$\therefore$	**seg AC $\cong$ seg QR**	[c.s.c.t.]
	$\angle ABC \cong \angle QPR$ and **$\angle ACB \cong \angle QRP$**	[c.a.c.t.]

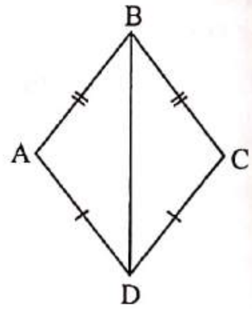
4. **As shown in the adjoining figure, in ΔLMN and ΔPNM, LM = PN, LN = PM. Write the test which assures the congruence of the two triangles. Write their remaining congruent parts.** **[2 Marks]**



Solution:

	In ΔLMN and ΔPNM,	
	seg LM $\cong$ seg PN	[Given]
	seg LN $\cong$ seg PM	
	seg MN $\cong$ seg NM	[Common side]
$\therefore$	ΔLMN $\cong$ ΔPNM	**[SSS test]**
$\therefore$	**$\angle$LMN $\cong$ $\angle$PNM,**	**[c.a.c.t.]**
	$\angle$MLN $\cong$ $\angle$NPM, and	
	$\angle$LNM $\cong$ $\angle$PMN	

5. **In the adjoining figure, seg AB $\cong$ seg CB and seg AD $\cong$ seg CD. Prove that ΔABD $\cong$ ΔCBD.** **[2 Marks]**

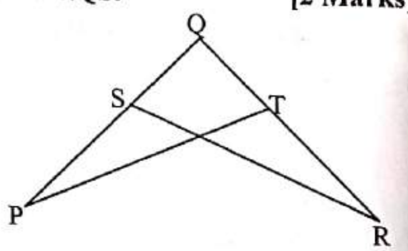


Proof:

	In ΔABD and ΔCBD,	
	seg AB $\cong$ seg CB	[Given]
	seg AD $\cong$ seg CD	
	seg BD $\cong$ seg BD	[Common side]
$\therefore$	**ΔABD $\cong$ ΔCBD**	[SSS test]

6. **In the adjoining figure, $\angle$P $\cong$ $\angle$R, seg PQ $\cong$ seg RQ. Prove that ΔPQT $\cong$ ΔRQS.** **[2 Marks]**

Proof:



	In ΔPQT and ΔRQS,	
	$\angle$P $\cong$ $\angle$R	[Given]
	seg PQ $\cong$ seg RQ	
	$\angle$Q $\cong$ $\angle$Q	[Common angle]
$\therefore$	**ΔPQT $\cong$ ΔRQS**	[ASA test]

Let's Study

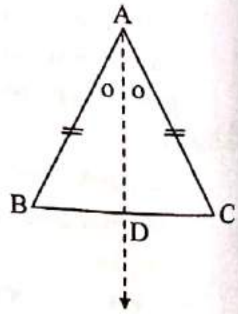
Isosceles triangle theorem

Theorem: If two sides of a triangle are congruent, then the angles opposite to them are congruent.

Given: In ΔABC, seg AB $\cong$ seg AC

To prove: $\angle$ABC $\cong$ $\angle$ACB

Construction: Draw the bisector of $\angle$BAC which intersects side BC at point D.



Proof:

	In ΔABD and ΔACD,	
	seg AB $\cong$ seg AC	[Given]
	$\angle$BAD $\cong$ $\angle$CAD	[Construction]
	seg AD $\cong$ seg AD	[Common side]
$\therefore$	ΔABD $\cong$ ΔACD	$\boxed{\text{[SAS test]}}$
$\therefore$	$\angle$ABD $\cong$ $\boxed{\angle ACD}$	[c.a.c.t.]
$\therefore$	$\angle$ABC $\cong$ $\angle$ACB	[B – D – C]

In ΔABC and ΔECB,

	seg $AB \cong$ seg EC	[From (i)]
	$\angle ABC \cong \angle ECB$	[Each angle is of measure 90°]
	seg $BC \cong$ seg CB	[Common side]
$\therefore$	$\Delta ABC \cong \Delta ECB$	[SAS test]
$\therefore$	seg $AC \cong$ seg EB	[c. s. c. t.]
$\therefore$	$AC = EB$ ... (ii)	
	But, $BD = \frac{1}{2}(EB)$ ... (iii)	[Construction]
$\therefore$	$BD = \frac{1}{2}AC$	[From (ii) and (iii)]

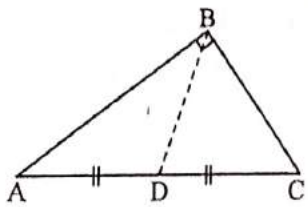
Try This

1. In the given figure, ΔABC is a right angled triangle. seg BD is the median on hypotenuse. Measure the lengths of the following segments.

i. AD   ii. DC   iii. BD

From the measurements verify that $BD = \frac{1}{2}AC$.

(Textbook pg. no. 37)

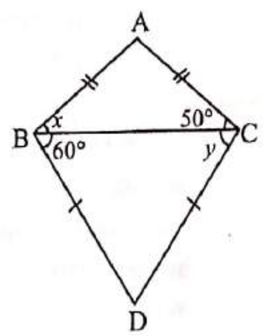


Solution:

	$AD = DC = BD = 1.9$ cm	
	$AC = AD + DC$	[A – D – C]
	$= 1.9 + 1.9$	
	$= 2 \times 1.9$ cm	
$\therefore$	$AC = 2 \times BD$	
$\therefore$	$BD = \frac{1}{2}AC$	

Practice Set 3.3

1. Find the values of x and y using the information shown in the given figure. Find the measures of $\angle ABD$ and $\angle ACD$. **[3 Marks]**



Solution:

i.	$\angle ACB = 50^\circ$	[Given]
	In ΔABC, seg $AC \cong$ seg AB	[Given]
$\therefore$	$\angle ABC \cong \angle ACB$	[Isosceles triangle theorem]
$\therefore$	$x = 50^\circ$	
ii.	$\angle DBC = 60^\circ$	[Given]
	In ΔBDC, seg $BD \cong$ seg DC	[Given]
$\therefore$	$\angle DCB \cong \angle DBC$	[Isosceles triangle theorem]
$\therefore$	$y = 60^\circ$	
iii.	$\angle ABD = \angle ABC + \angle DBC$	[Angle addition property]
	$= 50^\circ + 60^\circ$	
$\therefore$	$\angle ABD = 110^\circ$	
iv.	$\angle ACD = \angle ACB + \angle DCB$	[Angle addition property]
	$= 50^\circ + 60^\circ$	
$\therefore$	$\angle ACD = 110^\circ$	
$\therefore$	$\mathbf{x = 50^\circ, y = 60^\circ,}$	
	$\mathbf{\angle ABD = 110^\circ, \angle ACD = 110^\circ}$	

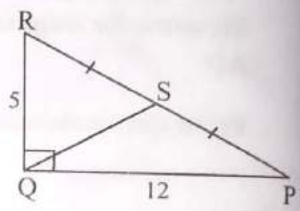
2. **The length of hypotenuse of a right angled triangle is 15. Find the length of median on its hypotenuse.** **[1 Mark]**

Solution:

Length of hypotenuse = 15	[Given]
Length of median on the hypotenuse	
$= \frac{1}{2} \times$ length of hypotenuse	[In a right angled triangle, the length of the median on the hypotenuse is half the length of the hypotenuse]
$= \frac{1}{2} \times 15 = 7.5$	
$\therefore$ **The length of the median on the hypotenuse is 7.5 units.**	

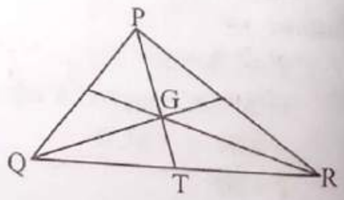
3. **In ΔPQR, $\angle$Q = 90°, PQ = 12, QR = 5 and QS is a median. Find l(QS).** **[2 Marks]**

Solution:



i. PQ = 12, QR = 5	[Given]
In ΔPQR, $\angle$Q = 90°	[Given]
$\therefore$ $PR^2 = QR^2 + PQ^2$	[Pythagoras theorem]
$= 5^2 + 12^2$	
$= 25 + 144$	
$\therefore$ $PR^2 = 169$	
$\therefore$ PR = 13 units	[Taking square root of both sides]
ii. In right angled ΔPQR, seg QS is the median on hypotenuse PR.	
$\therefore$ $QS = \frac{1}{2} PR$	[In a right angled triangle, the length of the median on the hypotenuse is half the length of the hypotenuse]
$= \frac{1}{2} \times 13$	
$\therefore$ **l(QS) = 6.5 units**	

4. **In the given figure, point G is the point of concurrence of the medians of ΔPQR. If GT = 2.5, find the lengths of PG and PT.** **[2 Marks]**



Solution:

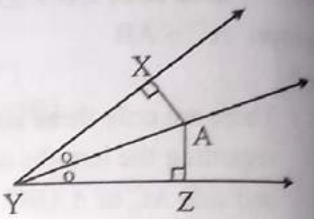
i. In ΔPQR, G is the point of concurrence of the medians.	[Given]
The centroid divides each median in the ratio 2 : 1.	
PG : GT = 2 : 1	
$\therefore$ $\frac{PG}{GT} = \frac{2}{1}$	
$\therefore$ $\frac{PG}{2.5} = \frac{2}{1}$	
$\therefore$ PG = 2 × 2.5	
$\therefore$ PG = 5 units	
ii. Now, PT = PG + GT	[P – G – T]
= 5 + 2.5	
$\therefore$ PT = 7.5 units	
$\therefore$ **l(PG) = 5 units, l(PT) = 7.5 units**	

Proof:

	In ΔACD,		
	$AD = AC$	(i)	[Construction]
$\therefore$	$\angle ACD = \angle ADC$		[Isosceles triangle theorem]
$\therefore$	$\angle ACD + \angle ACB > \angle ADC$		[Adding $\angle ACB$ to one side]
$\therefore$	$\angle BCD > \angle ADC$		[Angle addition property]
$\therefore$	$\angle BCD > \angle BDC$		[B – A – D]
$\therefore$	Side BD > Side BC		[Side opposite to greater angle is greater]
$\therefore$	$BA + AD > BC$		[$\because BD = BA + AD$]
$\therefore$	$BA + AC > BC$		[From (i)]
	Similarly, we can prove that $AB + BC > AC$, $AC + BC > AB$.		

Practice Set 3.4

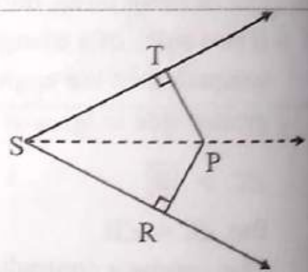
1. In the adjoining figure, point A is on the bisector of $\angle XYZ$. If $AX = 2$ cm, then find AZ. **[2 Marks]**



Solution:

	$AX = 2$ cm	[Given]
	Point A lies on the bisector of $\angle XYZ$.	[Given]
$\therefore$	Point A is equidistant from the sides of $\angle XYZ$.	[Every point on the bisector of an angle is equidistant from the sides of the angle]
$\therefore$	$AZ = AX$	
$\therefore$	**$AZ = 2$ cm**	

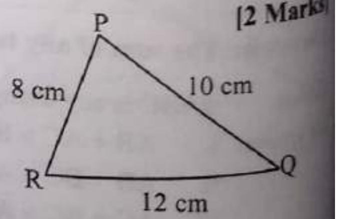
2. In the adjoining figure, $\angle RST = 56°$, seg $PT \perp$ ray ST, seg $PR \perp$ ray SR and seg $PR \cong$ seg PT. Find the measure of $\angle RSP$. State the reason for your answer. **[2 Marks]**



Solution:

	seg $PT \perp$ ray ST, seg $PR \perp$ ray SR seg $PR \cong$ seg PT	[Given]
$\therefore$	Point P lies on the bisector of $\angle TSR$	[Any point equidistant from the sides of an angle is on the bisector of the angle]
$\therefore$	Ray SP is the bisector of $\angle RST$.	
	$\angle RSP = 56°$	[Given]
$\therefore$	$\angle RSP = \frac{1}{2} \angle RST$	
	$= \frac{1}{2} \times 56°$	
$\therefore$	**$\angle RSP = 28°$**	

3. In ΔPQR, $PQ = 10$ cm, $QR = 12$ cm, $PR = 8$ cm. Find out the greatest and the smallest angle of the triangle. **[2 Marks]**



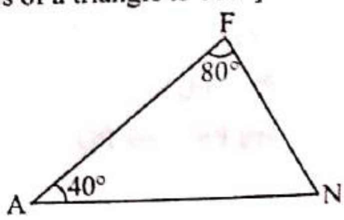
Solution:

	In ΔPQR, $PQ = 10$ cm, $QR = 12$ cm, $PR = 8$ cm	[Given]
	Since, $12 > 10 > 8$	
$\therefore$	$QR > PQ > PR$	
$\therefore$	$\angle QPR > \angle PRQ > PQR$	[Angle opposite to greater side is greater]
$\therefore$	**In ΔPQR, $\angle QPR$ is the greatest angle and $\angle PQR$ is the smallest angle.**	

4. In ΔFAN, $\angle F = 80°$, $\angle A = 40°$. Find out the greatest and the smallest side of the triangle. State the reason. **[2 Marks]**

Solution:

	In ΔFAN,	
	$\angle F + \angle A + \angle N = 180°$	[Sum of the measures of the angles of a triangle is $180°$]
$\therefore$	$80° + 40° + \angle N = 180°$	
$\therefore$	$\angle N = 180° - 80° - 40°$	
$\therefore$	$\angle N = 60°$	
	Since, $80° > 60° > 40°$	
$\therefore$	$\angle F > \angle N > \angle A$	
$\therefore$	$AN > FA > FN$	[Side opposite to greater angle is greater]
$\therefore$	**In ΔFAN, AN is the greatest side and FN is the smallest side.**	

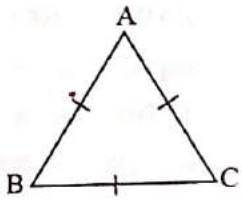


5. Prove that an equilateral triangle is equiangular. **[3 Marks]**

Given: ΔABC is an equilateral triangle.

To prove: ΔABC is equiangular

i.e. $\angle A \cong \angle B \cong \angle C$



Proof:

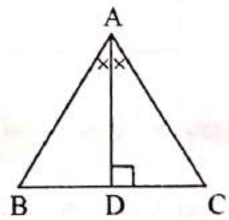
	seg AB $\cong$ seg BC $\cong$ seg AC	(i)	[Sides of an equilateral triangle]
	In ΔABC,		
	seg AB $\cong$ seg BC		[From (i)]
$\therefore$	$\angle C \cong \angle A$	(ii)	[Isosceles triangle theorem]
	In ΔABC,		
	seg BC $\cong$ seg AC		[From (i)]
$\therefore$	$\angle A \cong \angle B$	(iii)	[Isosceles triangle theorem]
$\therefore$	$\angle A \cong \angle B \cong \angle C$		[From (ii) and (iii)]
$\therefore$	**ΔABC is equiangular.**		

6. Prove that, if the bisector of $\angle BAC$ of ΔABC is perpendicular to side BC, then ΔABC is an isosceles triangle. **[3 Marks]**

Given: Seg AD is the bisector of $\angle BAC$.

seg AD $\perp$ seg BC

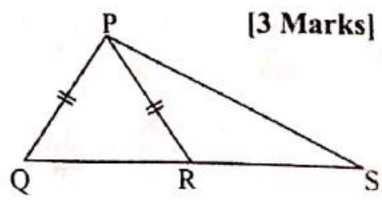
To prove: ΔABC is an isosceles triangle.



Proof:

	In ΔABD and ΔACD,	
	$\angle BAD \cong \angle CAD$	[seg AD is the bisector of $\angle BAC$]
	seg AD $\cong$ seg AD	[Common side]
	$\angle ADB \cong \angle ADC$	[Each angle is of measure $90°$]
$\therefore$	$\Delta ABD \cong \Delta ACD$	[ASA test]
$\therefore$	seg AB $\cong$ seg AC	[c. s. c. t.]
$\therefore$	**ΔABC is an isosceles triangle.**	

7. In the adjoining figure, if seg PR $\cong$ seg PQ, show that seg PS > seg PQ. **[3 Marks]**

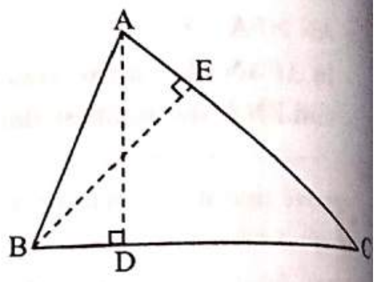


Proof:

	In ΔPQR,		
	seg PR $\cong$ seg PQ		[Given]
$\therefore$	$\angle PQR \cong \angle PRQ$	(i)	[Isosceles triangle theorem]
	$\angle PRQ$ is the exterior angle of ΔPRS.		

∴	$\angle PRQ > \angle PSR$	(ii)	[Property of exterior angle]
∴	$\angle PQR > \angle PSR$		[From (i) and (ii)]
	i.e. $\angle Q > \angle S$	(iii)	
	In ΔPQS,		
	$\angle Q > \angle S$		[From (iii)]
∴	$PS > PQ$		[Side opposite to greater angle is greater]
∴	**seg PS > seg PQ**		

8. In the adjoining figure, in ΔABC, seg AD and seg BE are altitudes and AE = BD. Prove that seg AD ≅ seg BE. [2 Marks]



***Proof*:**

	In ΔADB and ΔBEA,	
	seg BD ≅ seg AE	[Given]
	$\angle ADB = \angle BEA = 90°$	[Given]
	seg AB ≅ seg BA	[Common side]
∴	$\Delta ADB \cong \Delta BEA$	[Hypotenuse-side test]
∴	**seg AD ≅ seg BE**	[c. s. c. t.]

Let's Study

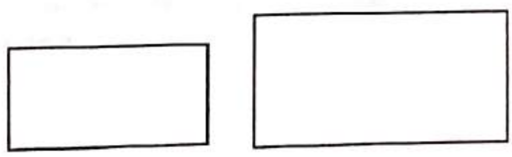
Similar triangles

Similar figures:

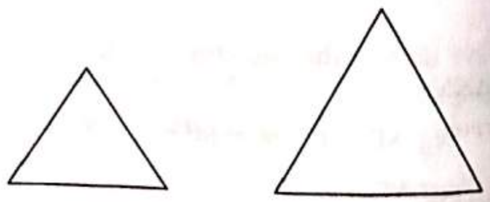
Figures which have same shape but not necessarily the same size are called similar figures.

Examples:

i.



ii.



Similarity of triangles

For a given one-to-one correspondence between two triangles, if

i. their corresponding angles are congruent and

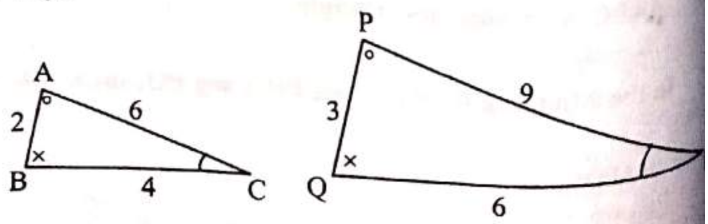
ii. their corresponding sides are in the same proportion, then the correspondence is known as similarity and the two triangles are said to be similar.

In the adjoining figures, for correspondence ABC ↔ PQR

i. $\angle A \cong \angle P$, $\angle B \cong \angle Q$, $\angle C \cong \angle R$

ii. $\frac{AB}{PQ} = \frac{2}{3}$, $\frac{BC}{QR} = \frac{4}{6} = \frac{2}{3}$, $\frac{AC}{PR} = \frac{6}{9} = \frac{2}{3}$

i.e., $\frac{AB}{PQ} = \frac{BC}{QR} = \frac{AC}{PR}$



Hence, ΔABC and ΔPQR are similar triangles. It is written as $\Delta ABC \sim \Delta PQR$.

Try This

1. We have learnt that if two triangles are equiangular then their sides are in proportion. What do you think if two quadrilaterals are equiangular? Are their sides in proportion? Draw different figures and verify. Verify the same for other polygons.

(Textbook pg. no. 50)

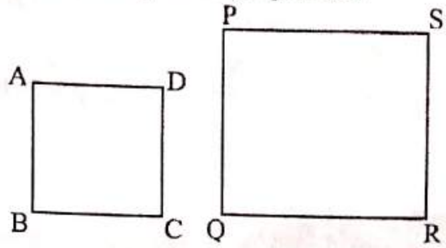
Ans: If two quadrilaterals are equiangular then their sides will not necessarily be in proportion.

Case 1: The two quadrilaterals are of the same type.

Consider squares ABCD and PQRS.

$\angle A = \angle P$, $\angle B = \angle Q$, $\angle C = \angle R$, $\angle D = \angle S$

$\frac{AB}{PQ} = \frac{BC}{QR} = \frac{CD}{RS} = \frac{AD}{PS}$



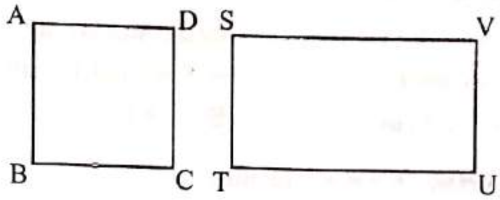
Case 2: The two quadrilaterals are of different types.

Consider square ABCD and rectangle STUV.

$\angle A = \angle S$, $\angle B = \angle T$, $\angle C = \angle U$, $\angle D = \angle V$

Now, $\frac{AB}{ST} = \frac{CD}{UV}$ and $\frac{BC}{TU} = \frac{AD}{SV}$

But $\frac{AB}{ST} \neq \frac{BC}{TU}$



[Students should draw different quadrilaterals and verify.]

Practice Set 3.5

1. If $\Delta XYZ \sim \Delta LMN$, write the corresponding angles of the two triangles and also write the ratios of corresponding sides.

Solution: **[2 Marks]**

$\Delta XYZ \sim \Delta LMN$	[Given]
$\therefore$ $\left.\begin{array}{l}\angle X \cong \angle L \\ \angle Y \cong \angle M \\ \angle Z \cong \angle N\end{array}\right\}$	[Corresponding angles of similar triangles]
$\frac{XY}{LM} = \frac{YZ}{MN} = \frac{XZ}{LN}$	[Corresponding sides of similar triangles]

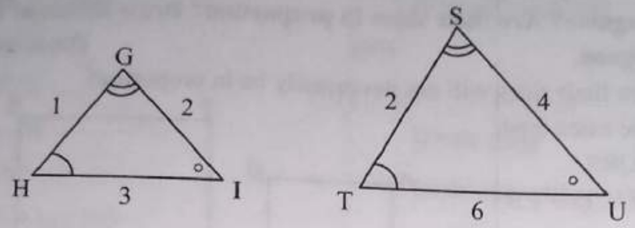
2. In ΔXYZ, XY = 4 cm, YZ = 6 cm, XZ = 5 cm. If $\Delta XYZ \sim \Delta PQR$ and PQ = 8 cm, then find the lengths of remaining sides of ΔPQR.

Solution: **[2 Marks]**

i. $\Delta XYZ \sim \Delta PQR$		[Given]
$\therefore \frac{XY}{PQ} = \frac{YZ}{QR} = \frac{XZ}{PR}$		[Corresponding sides of similar triangles]
$\therefore \frac{4}{8} = \frac{6}{QR} = \frac{5}{PR}$	(i)	
Now, $\frac{4}{8} = \frac{6}{QR}$		
$\therefore QR = \frac{6 \times 8}{4}$		
$\therefore$ QR = 12 cm		
ii. Also, $\frac{4}{8} = \frac{5}{PR}$		[From (i)]
$\therefore PR = \frac{5 \times 8}{4}$		
$\therefore$ PR = 10 cm		
$\therefore$ **QR = 12 cm , PR = 10cm**		

3. **Draw a sketch of a pair of similar triangles. Label them. Show their corresponding angles by the same signs. Show the lengths of corresponding sides by numbers in proportion.** **[2 Marks]**

Ans:



$\Delta GHI \sim \Delta STU$

Problem Set – 3

1. **Choose the correct alternative answer for the following questions.** **[1 Mark each]**

i. If two sides of a triangle are 5 cm and 1.5 cm, the length of its third side cannot be ______
(A) 3.7 cm (B) 4.1 cm (C) 3.8 cm (D) 3.4 cm

ii. In ΔPQR, if $\angle R > \angle Q$, then ______.
(A) $QR > PR$ (B) $PQ > PR$ (C) $PQ < PR$ (D) $QR < PR$

iii. In ΔTPQ, $\angle T = 65°$, $\angle P = 95°$. Which of the following is a true statement?
(A) $PQ < TP$ (B) $PQ < TQ$ (C) $TQ < TP < PQ$ (D) $PQ < TP < TQ$

Answers:

i. (D) ii. (B) iii. (B)

Hints:

i. Sum of the lengths of two sides of a triangle > length of the third side
Here, $1.5 \text{ cm} + 3.4 \text{ cm} = 4.9 \text{ cm} < 5 \text{ cm}$
∴ Third side $\neq 3.4$ cm

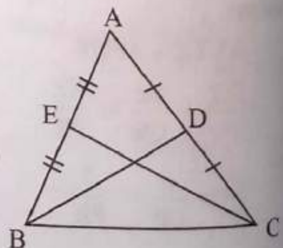
iii. $\angle Q = 180° - (95° + 65°) = 20°$
∴ $\angle Q < \angle T < \angle P$
∴ $PT < PQ < TQ$

2. **ΔABC is isosceles in which $AB = AC$. Seg BD and seg CE are medians. Show that $BD = CE$.** **[3 Marks]**

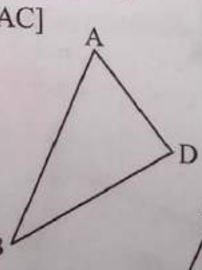
Given: In isosceles ΔABC, $AB = AC$.
seg BD and seg CE are the medians of ΔABC.

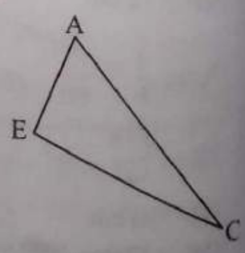
To prove: $BD = CE$

Proof:



$AE = \frac{1}{2} AB$	(i)	[E is the midpoint of side AB]
$AD = \frac{1}{2} AC$	(ii)	[D is the midpoint of side AC]
Also, $AB = AC$	(iii)	[Given]
∴ $AE = AD$	(iv)	[From (i), (ii) and (iii)]
In ΔADB and ΔAEC,		
seg $AB \cong$ seg AC		[Given]
$\angle BAD \cong \angle CAE$		[Common angle]
seg $AD \cong$ seg AE		[From (iv)]
∴ $\Delta ADB \cong \Delta AEC$		[SAS test]
∴ seg $BD \cong$ seg CE		[c.s.c.t.]
∴ **$BD = CE$**		





4 Constructions of Triangles

Type of Problems	Practice Set	Q. Nos.
Constructions of triangles	Problem Set- 4	Q.3
To construct a triangle when its base, an angle adjacent to the base and the sum of the lengths of remaining sides is given.	4.1	Q.1, 2, 3, 4
	Practice Problems (Based on Practice Set 4.1)	Q.1, 2, 3
	Problem Set- 4	Q.1
To construct a triangle when its base, angle adjacent to the base and difference between the remaining sides is given.	4.2	Q.1, 2, 3
	Practice Problems (Based on Practice Set 4.2)	Q.1, 2, 3, 4, 5
	Problem Set- 4	Q.4
To construct a triangle, if its perimeter, base and the angles which include the base are given.	4.3	Q.1, 2, 3
	Practice Problems (Based on Practice Set 4.3)	Q.1, 2, 3, 4
	Problem Set- 4	Q.2

Let's Recall

In the previous standard we have studied the following triangle constructions.

i. To construct a triangle when its three sides are given.

ii. To construct a triangle when its base and two adjacent angles are given.

iii. To construct a triangle when two sides and the included angle are given.

iv. To construct a right angled triangle when its hypotenuse and one side is given.

Perpendicular bisector Theorem:

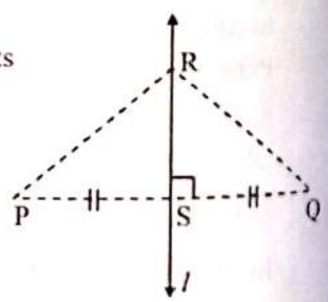
1. Every point on the perpendicular bisector of a segment is equidistant from its end points.

In the given figure,

line l is perpendicular bisector of seg PQ.

Point R lies on line l

$\therefore$ RP = RQ



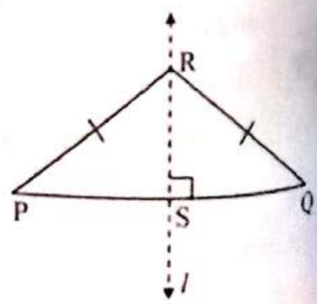
2. Every point equidistant from the end points of a segment is on the perpendicular bisector of the segment.

In the given figure ,

Point R is equidistant from the points P and Q.

$\therefore$ RP = RQ

$\therefore$ Point R lies on perpendicular bisector of seg PQ.



Let's Study

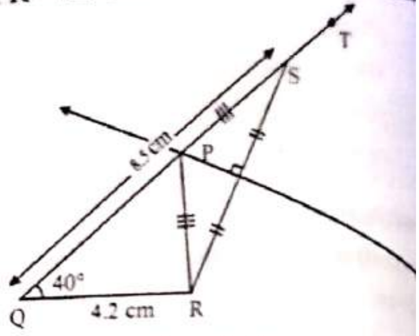
Constructions of triangles

To construct a triangle, three conditions are required. Out of three sides and three angles of a triangle if two parts and some additional information about them is given, then we can construct a triangle using them.

1. **Construct ΔPQR, in which $QR = 4.2$ cm, $m\angle Q = 40°$ and $PQ + PR = 8.5$ cm.** [4 Ma

Ans:

	As shown in the rough figure draw seg QR = 4.2 cm		
	Draw a ray QT making an angle of 40° with QR		
	Take a point S on ray QT, such that QS = 8.5 cm		
	Now, QP + PS = QS		[Q–P–S]
∴	QP + PS = 8.5 cm	(i)	
	Also, PQ + PR = 8.5 cm	(ii)	[Given]
∴	QP + PS = PQ + PR		[From (i) and (ii)]
∴	PS = PR		
∴	Point P is on the perpendicular bisector of seg SR		
∴	The point of intersection of ray QT and perpendicular bisector of seg SR is point P.		

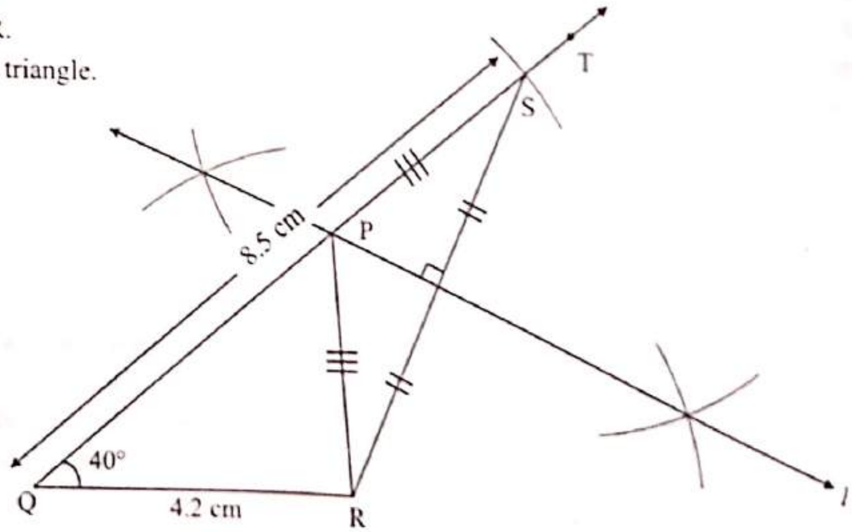


Rough figure

Steps of construction:

i. Draw seg QR of length 4.2 cm.
ii. Draw ray QT, such that $\angle RQT = 40°$.
iii. Mark point S on ray QT such that $l(QS) = 8.5$ cm.
iv. Join points R and S.
v. Draw perpendicular bisector of seg RS intersecting ray QT. Name the point as P.
vi. Join the points P and R.

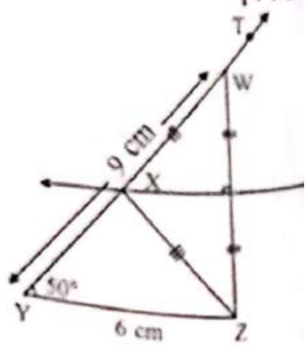
Hence, ΔPQR is the required triangle.



2. **Construct ΔXYZ, in which $YZ = 6$ cm, $XY + XZ = 9$ cm, $\angle XYZ = 50°$.** [4 Ma

Ans:

	As shown in the rough figure draw seg YZ = 6 cm		
	Draw a ray YT making an angle of 50° with YZ		
	Take a point W on ray YT, such that YW = 9 cm		
	Now, YX + XW = YW		[Y–X–W]
∴	YX + XW = 9 cm	(i)	



Rough figure

	Also, $XY + XZ = 9$ cm	(ii) [Given]
∴	$YX + XW = XY + XZ$	[From (i) and (ii)]
∴	$XW = XZ$	
∵	Point X is on the perpendicular bisector of seg WZ	
∴	The point of intersection of ray YT and perpendicular bisector of seg WZ is point X.	

Steps of construction:

i. Draw seg YZ of length 6 cm.

ii. Draw ray YT, such that $\angle ZYT = 50°$.

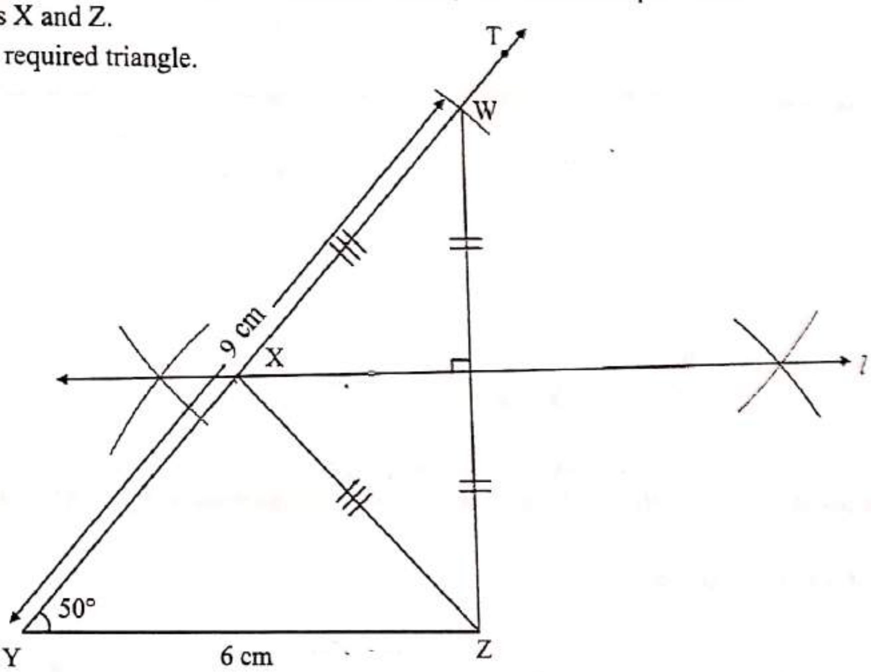
iii. Mark point W on ray YT such that $l(YW) = 9$ cm.

iv. Join points W and Z.

v. Draw perpendicular bisector of seg WZ intersecting ray YT. Name the point as X.

vi. Join the points X and Z.

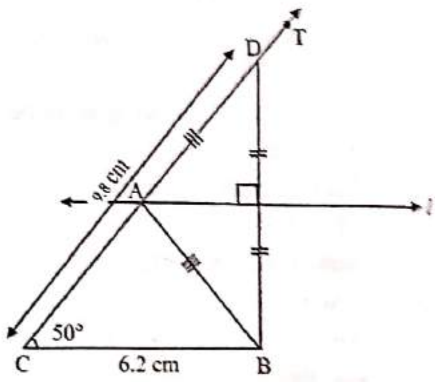
Hence, ΔXYZ is the required triangle.



3. Construct ΔABC, in which $BC = 6.2$ cm, $\angle ACB = 50°$, $AB + AC = 9.8$ cm. **[4 Marks]**

Ans:

	As shown in the rough figure draw seg $CB = 6.2$ cm		
	Draw a ray CT making an angle of 50° with CB		
	Take a point D on ray CT, such that $CD = 9.8$ cm		
	Now, $CA + AD = CD$		[C–A–D]
∴	$CA + AD = 9.8$ cm	(i)	
	Also, $AB + AC = 9.8$ cm	(ii)	[Given]
∴	$CA + AD = AB + AC$		[From (i) and (ii)]
∴	$AD = AB$		
∴	Point A is on the perpendicular bisector of seg DB		
∴	The point of intersection of ray CT and perpendicular bisector of seg DB is point A.		



Rough figure

Steps of construction:

i. Draw seg BC of length 6.2 cm.

ii. Draw ray CT, such that $\angle BCT = 50°$.

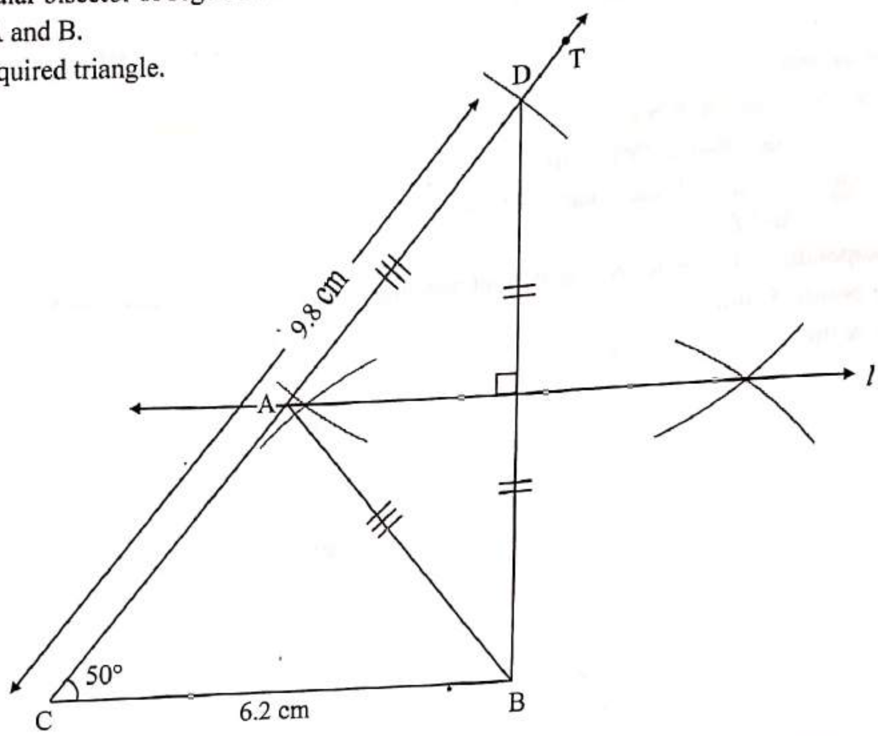
iii. Mark point D on ray CT such that $l(CD) = 9.8$ cm.

iv. Join points D and B.

v. Draw perpendicular bisector of seg DB intersecting ray CT. Name the point as A.

vi. Join the points A and B.

Hence, ΔABC is the required triangle.



4. Construct ΔABC, in which $BC = 3.2$ cm, $\angle ACB = 45°$ and perimeter of ΔABC is 10 cm. **[4 Marks]**

Solution:

Perimeter of $\Delta ABC = AB + BC + AC$

$\therefore$ $10 = AB + 3.2 + AC$

$\therefore$ $AB + AC = 10 - 3.2$

$\therefore$ $AB + AC = 6.8$ cm ... (i)

Now, In ΔABC

$BC = 3.2$ cm, $\angle ACB = 45°$ and

$AB + AC = 6.8$ cm

As shown in the rough figure draw seg $BC = 3.2$ cm

Draw a ray CT making an angle of 45° with CB

Take a point D on ray CT, such that $CD = 6.8$ cm

Now, $CA + AD = CD$ ... (ii) [C–A–D]

$\therefore$ $CA + AD = 6.8$ cm ... (iii) [From (i)]

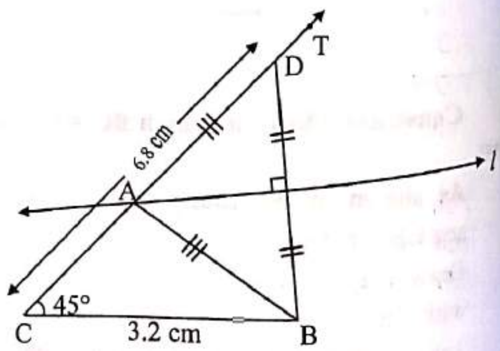
Also, $AB + AC = 6.8$ cm

$\therefore$ $CA + AD = AB + AC$ [From (ii) and (iii)]

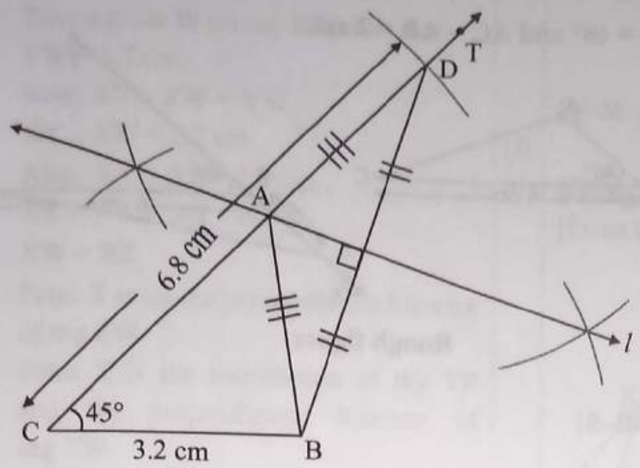
$\therefore$ $AD = AB$

$\therefore$ Point A is on the perpendicular bisector of seg DB

$\therefore$ The point of intersection of ray CT and perpendicular bisector of seg DB is point A.



Rough figure



Steps of construction:

i. Draw seg BC of length 3.2 cm.

ii. Draw ray CT, such that $\angle BCT = 45°$.

iii. Mark point D on ray CT such that $l(CD) = 6.8$ cm.

iv. Join points D and B.

v. Draw perpendicular bisector of seg DB intersecting ray CT. Name the point as A.

vi. Join the points A and B.

Hence, ΔABC is the required triangle.

Let's Study

Construction II

To construct a triangle when its base, angle adjacent to the base and difference between the remaining sides is given.

Example 1: Construct ΔABC in which base BC = 8 cm, $\angle ABC = 45°$ and AB – AC = 3 cm.

Explanation:

Here, AB – AC = 3 cm

$\therefore$ AB > AC

As shown in the rough figure draw seg BC = 8 cm

Draw a ray BP making an angle of 45° with BC

Take a point D on ray BP, such that BD = 3 cm.

Now, AB – AD = BD [B–D–A]

$\therefore$ AB – AD = 3cm ... (i)

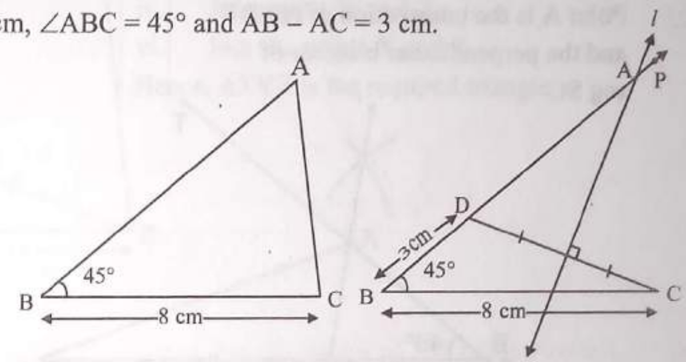
Also, AB – AC = 3cm ... (ii) [Given]

$\therefore$ AB – AD = AB – AC [From (i) and (ii)]

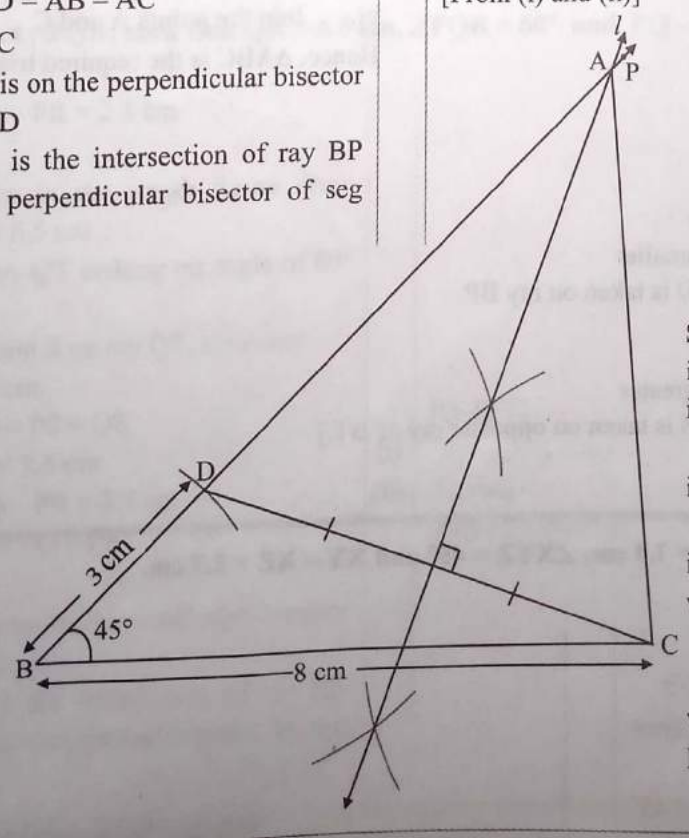
$\therefore$ AD = AC

$\therefore$ Point A is on the perpendicular bisector of seg CD

$\therefore$ Point A is the intersection of ray BP and the perpendicular bisector of seg CD



Rough figure



Steps of construction:

i. Draw seg BC of length 8 cm.

ii. Draw ray BP, such that $\angle CBP = 45°$.

iii. Take point D on ray BP such that $l(BD) = 3$ cm.

iv. Join the points C and D.

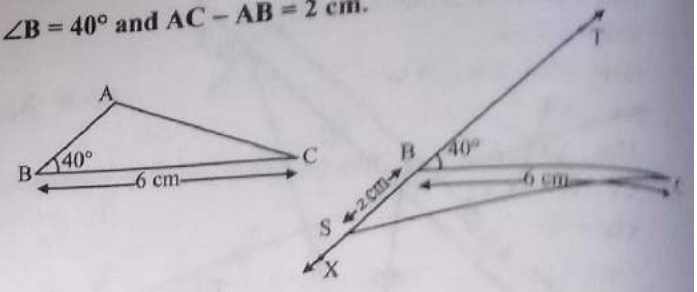
v. Draw perpendicular bisector of seg CD intersecting ray BP. Name that point as A.

vi. Join the points A and C.

Hence, ΔABC is the required triangle.

2. **Construct ΔABC in which base $BC = 6$ cm, $\angle B = 40°$ and $AC - AB = 2$ cm.**

Explanation:



Rough figure

Here, $AC - AB = 2$ cm

$\therefore$ $AC > AB$

As shown in the rough figure draw seg $BC = 6$ cm

Draw a ray BT making an angle of $40°$ with BC.

Take a point S on opposite ray of BT, such that $BS = 2$ cm.

Now, $AS - AB = BS$ [A–B–S]

$\therefore$ $AS - AB = 2$ cm ...(i)

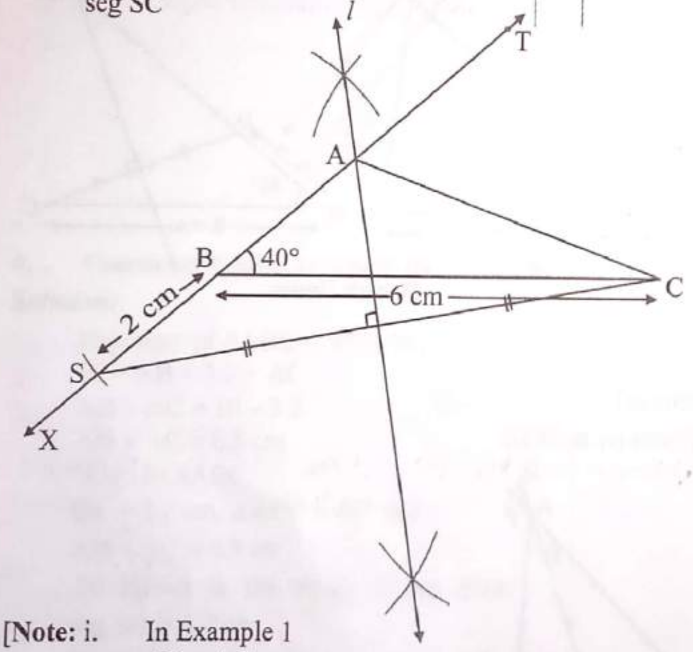
Also, $AC - AB = 2$ cm ...(ii) [Given]

$\therefore$ $AS - AB = AC - AB$ [From (i) and (ii)]

$\therefore$ $AS = AC$

$\therefore$ Point A is on the perpendicular bisector of seg SC

$\therefore$ Point A is the intersection of ray BT and the perpendicular bisector of seg SC



Steps of construction:

i. Draw seg BC of length 6 cm.

ii. Draw ray BT, such that $\angle CBT = 40°$. Draw opposite ray BX of ray BT.

iii. Take point S on ray BX such that $l(BS) = 2$ cm.

iv. Join the points S and C.

v. Draw the perpendicular bisector of seg SC intersecting ray BT. Name the point as A.

vi. Join the points A and C.

Hence, ΔABC is the required triangle.

[Note: i. In Example 1
$AB > AC$
i.e., side opposite to $\angle B$ is smaller
$\therefore$ distance of 3 cm i.e., point D is taken on ray BP.
ii. In Example 2
$AC > AB$
i.e., side opposite to $\angle B$ is greater
$\therefore$ distance of 2 cm i.e., point S is taken on opposite ray of BT.]

Practice Set 4.2

1. **Construct ΔXYZ, such that $YZ = 7.4$ cm, $\angle XYZ = 45°$ and $XY - XZ = 2.7$ cm.** [4 Marks]

Ans:

Here, $XY - XZ = 2.7$ cm

$\therefore$ $XY > XZ$

As shown in the rough figure draw seg $YZ = 7.4$ cm

Draw a ray YP making an angle of $45°$

with YZ

Take a point W on ray YP, such that YW = 2.7 cm.

Now, XY − XW = YW [Y–W–X]

∴ XY − XW = 2.7 cm ... (i)

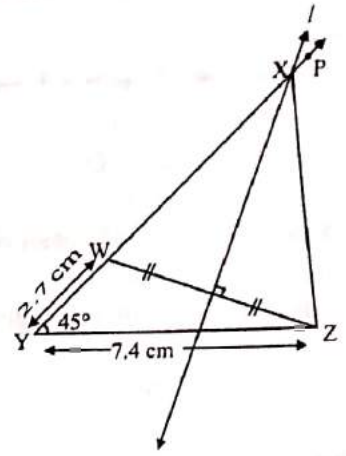
Also, XY − XZ = 2.7 cm ... (ii) [Given]

∴ XY − XW = XY − XZ [From (i) and (ii)]

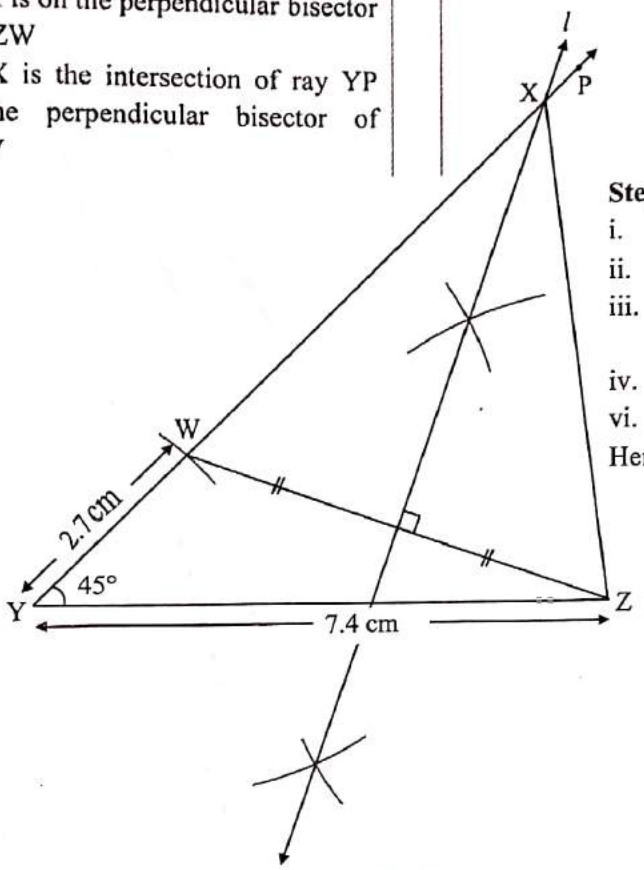
∴ XW = XZ

∴ Point X is on the perpendicular bisector of seg ZW

∴ Point X is the intersection of ray YP and the perpendicular bisector of seg ZW



Rough figure



Steps of construction:

i. Draw seg YZ of length 7.4 cm.

ii. Draw ray YP, such that ∠ZYP = 45°.

iii. Mark point W on ray YP such that *l*(YW) = 2.7 cm.

iv. Join points W and Z.

vi. Join the points X and Z.

Hence, ΔXYZ is the required triangle.

2. Construct ΔPQR, such that QR = 6.5 cm, ∠PQR = 60° and PQ − PR = 2.5 cm. **[4 Marks]**

Ans:

Here, PQ − PR = 2.5 cm

∴ PQ > PR

As shown in the rough figure draw seg QR = 6.5 cm

Draw a ray QT making on angle of 60° with QR

Take a point S on ray QT, such that QS = 2.5 cm.

Now, PQ − PS = QS [Q–S–T]

∴ PQ − PS = 2.5 cm ... (i)

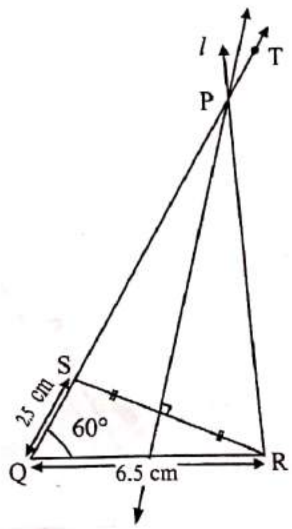
Also, PQ − PR = 2.5 cm ... (ii) [Given]

∴ PQ − PS = PQ − PR [From (i) and (ii)]

∴ PS = PR

∴ Point P is on the perpendicular bisector of seg RS

∴ Point P is the intersection of ray QT and the perpendicular bisector of seg RS



Rough figure

Steps of construction:

i. Draw seg QR of length 6.5 cm.

ii. Draw ray QT, such that $\angle RQT = 60°$.

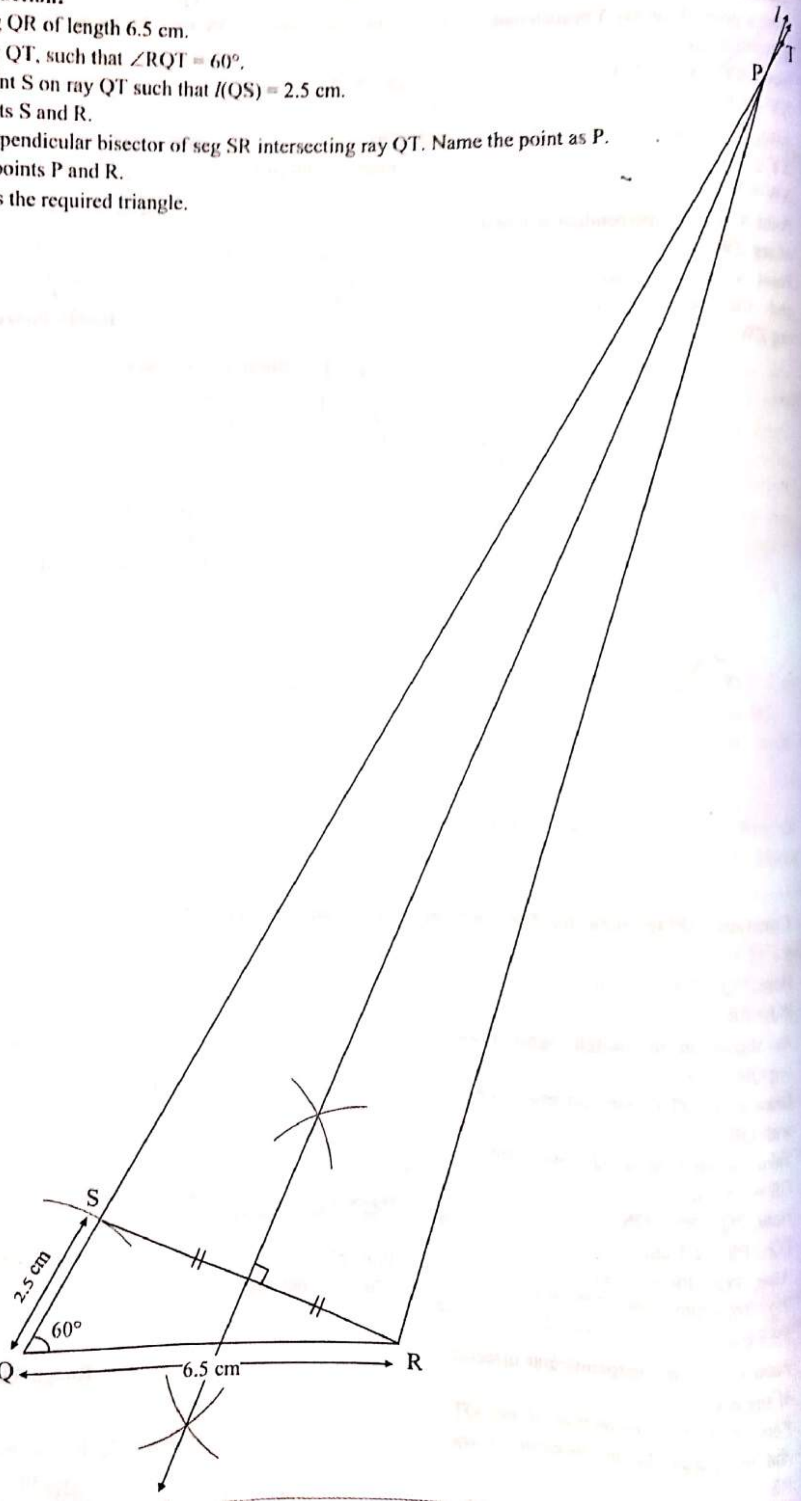
iii. Mark point S on ray QT such that $l(QS) = 2.5$ cm.

iv. Join points S and R.

v. Draw perpendicular bisector of seg SR intersecting ray QT. Name the point as P.

vi. Join the points P and R.

Hence, ΔPQR is the required triangle.



3. Construct ΔABC, such that $BC = 6$ cm, $\angle ABC = 100°$ and $AC - AB = 2.5$ cm. **[4 Marks]**

Ans:

Here, $AC - AB = 2.5$ cm

$\therefore$ $AC > AB$

As shown in the rough figure draw seg $BC = 6$ cm

Draw a ray BT making an angle of $100°$ with BC.

Take a point D on opposite ray of BT, such that $BD = 2.5$ cm.

Now, $AD - AB = BD$ ... [A–B–D]

$\therefore$ $AD - AB = 2.5$ cm ... (i)

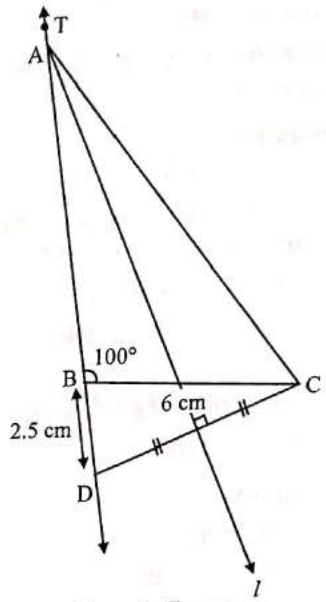
Also, $AC - AB = 2.5$ cm ... (ii) [Given]

$\therefore$ $AD - AB = AC - AB$ ... [From (i) and (ii)]

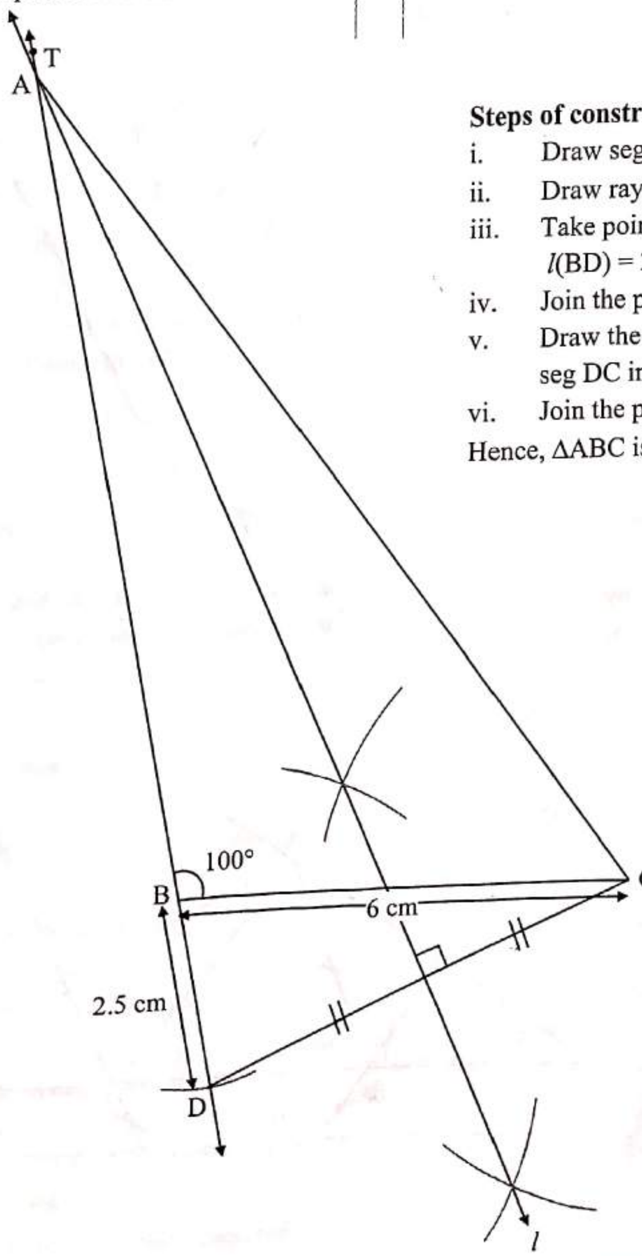
$\therefore$ $AD = AC$

$\therefore$ Point A is on the perpendicular bisector of seg DC

$\therefore$ Point A is the intersection of ray BT and the perpendicular bisector of seg DC



Rough figure



Steps of construction:

i. Draw seg BC of length 6 cm.

ii. Draw ray BT, such that $\angle CBT = 100°$.

iii. Take point D on opposite ray of BT such that $l(BD) = 2.5$ cm.

iv. Join the points D and C.

v. Draw the perpendicular bisector of seg DC intersecting ray BT. Name the point as A.

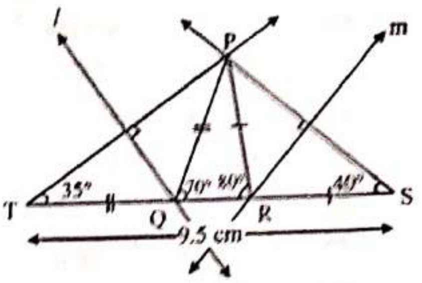
vi. Join the points A and C.

Hence, ΔABC is the required triangle.

Practice Set 4.3

1. Construct ΔPQR, in which $\angle Q = 70°$, $\angle R = 80°$ and $PQ + QR + PR = 9.5$ cm. **[4 Marks]**

Solution:



Rough figure

i. As shown in the figure, take point T and S on line QR, such that		
$QT = PQ$ and $RS = PR$	(i)	
$QT + QR + RS = TS$		[T–Q–R, Q–R–S]
$\therefore$ $PQ + QR + PR = TS$	(ii)	[From (i)]
Also,		
$PQ + QR + PR = 9.5$ cm	(iii)	[Given]
$\therefore$ $TS = 9.5$ cm		[From (ii) and (iii)]
ii. In ΔPQT		
$PQ = QT$		[From (i)]
$\therefore$ $\angle QPT = \angle QTP = x°$	(iv)	[Isosceles triangle theorem]
In ΔPQT, $\angle PQR$ is the exterior angle.		
$\therefore$ $\angle QPT + \angle QTP = \angle PQR$		[Remote interior angles theorem]
$\therefore$ $x + x = 70°$		[From (iv)]
$\therefore$ $2x = 70°$		
$\therefore$ $x = 35°$		
$\therefore$ $\angle PTQ = 35°$		
$\therefore$ $\angle T = 35°$		
Similarly, $\angle S = 40°$		
iii. Now, in ΔPTS		
$\angle T = 35°$, $\angle S = 40°$ and $TS = 9.5$ cm		
Hence, ΔPTS can be drawn.		
iv. Since, $PQ = TQ$,		
$\therefore$ Point Q lies on perpendicular bisector of seg PT.		
Also, $RP = RS$		
$\therefore$ Point R lies on perpendicular bisector of seg PS.		
$\therefore$ Points Q and R can be located by drawing the perpendicular bisector of PT and PS respectively.		
$\therefore$ ΔPQR can be drawn.		

Steps of construction:

i. Draw seg TS of length 9.5 cm.

ii. From point T draw ray making angle of 35°.

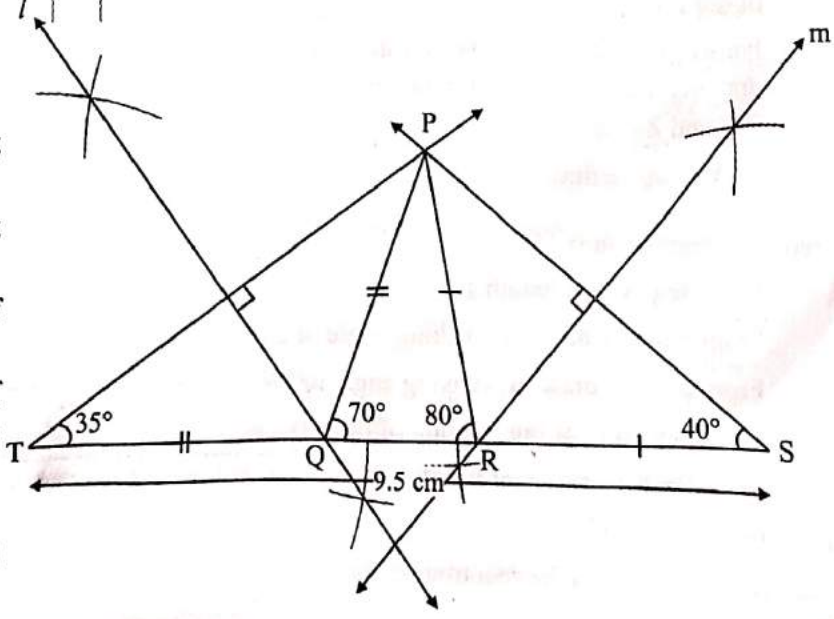
iii. From point S draw ray making angle of 40°.

iv. Name the point of intersection of two rays as P.

v. Draw the perpendicular bisector of seg PT and seg PS intersecting seg TS in Q and R respectively.

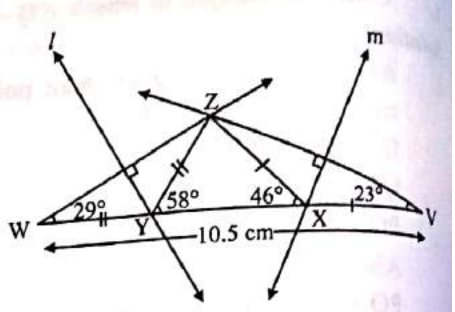
vi. Join PQ and PR.

Hence, ΔPQR is the required triangle.



2. **Construct ΔXYZ, in which $\angle Y = 58°$, $\angle X = 46°$ and perimeter of triangle is 10.5 cm.** **[4 Marks]**

Solution:



Rough figure

i. As shown in the figure, take point W and V on line YX, such that

$YW = ZY$ and $XV = ZX$ ... (i)

$YW + YX + XV = WV$ [W–Y–X, Y–X–V]

$\therefore$ $ZY + YX + ZX = WV$ ... (ii) [From (i)]

Also,

$ZY + YX + ZX = 10.5$ cm ... (iii) [Given]

$\therefore$ $WV = 10.5$ cm [From (ii) and (iii)]

ii. In ΔZWY

$ZY = YW$ [From (i)]

$\therefore$ $\angle YZW = \angle YWZ = x°$ ... (iv) [Isosceles triangle theorem]

In ΔZYW, $\angle ZYX$ is the exterior angle.

$\therefore$ $\angle YZW + \angle YWZ = \angle ZYX$ [Remote interior angles theorem]

$\therefore$ $x + x = 58°$ [From (iv)]

$\therefore$ $2x = 58°$

$\therefore$ $x = 29°$

$\therefore$ $\angle ZWY = 29°$

$\therefore$ $\angle W = 29°$

Similarly, $\angle V = 23°$

iii. Now, in ΔZWV

$\angle W = 29°$, $\angle V = 23°$ and

$WV = 10.5$ cm

Hence, ΔZWV can be drawn.

iv. Since, $ZY = YW$

$\therefore$ Point Y lies on perpendicular bisector of seg ZW.

Also, $ZX = XV$

$\therefore$ Point X lies on perpendicular bisector of seg ZV.

$\therefore$ Points Y and X can be located by drawing the perpendicular bisector of ZW and ZV respectively.

$\therefore$ ΔXYZ can be drawn.

Steps of construction:

i. Draw seg WV of length 10.5 cm.

ii. From point W draw ray making angle of 29°.

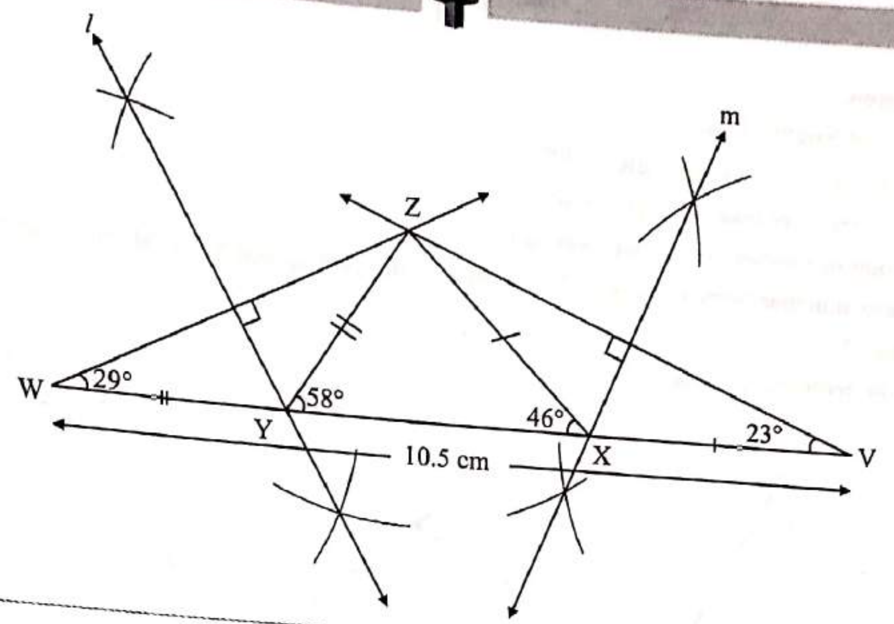
iii. From point V draw ray making angle of 23°.

iv. Name the point of intersection of two rays as Z.

v. Draw the perpendicular bisector of seg WZ and seg VZ intersecting seg WV in Y and X respectively.

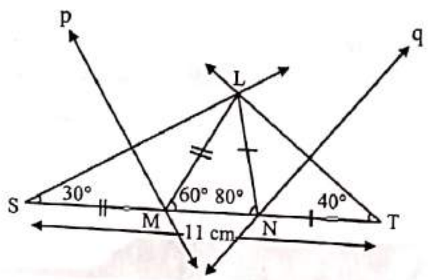
vi. Join ZY and ZX.

Hence, ΔXYZ is the required triangle.



3. **Construct ΔLMN, in which $\angle M = 60°$, $\angle N = 80°$ and $LM + MN + NL = 11$ cm.** **[4 Marks]**

***Solution*:**



Rough figure

i. As shown in the figure, take point S and T on line MN, such that

$MS = LM$ and $NT = LN$ ...(i)

$MS + MN + NT = ST$ [S–M–N, M–N–T]

$\therefore$ $LM + MN + LN = ST$ ...(ii) [From (i)]

Also,

$LM + MN + LN = 11$ cm ...(iii) [Given]

$\therefore$ $ST = 11$ cm [From (ii) and (iii)]

ii. In ΔLSM

$LM = MS$ [From (i)]

$\therefore$ $\angle MLS = \angle MSL = x°$ ...(iv) [Isosceles triangle theorem]

In ΔLMS, $\angle LMN$ is the exterior angle.

$\therefore$ $\angle MLS + \angle MSL = \angle LMN$ [Remote interior angles theorem]

$\therefore$ $x + x = 60°$ [From (iv)]

$\therefore$ $2x = 60°$

$\therefore$ $x = 30°$

$\therefore$ $\angle LSM = 30°$

$\therefore$ $\angle S = 30°$

Similarly, $\angle T = 40°$

iii. Now, in ΔLST

$\angle S = 30°$, $\angle T = 40°$ and $ST = 11$ cm

Hence, ΔLST can be drawn.

iv. Since, $LM = MS$

$\therefore$ Point M lies on perpendicular bisector of seg LS.

Also $LN = NT$

$\therefore$ Point N lies on perpendicular bisector of seg LT.

$\therefore$ Points M and N can be located by drawing the perpendicular bisector of LS and LT respectively.

$\therefore$ ΔLMN can be drawn.

Steps of construction:

i. Draw seg ST of length 11 cm.

ii. From point S draw ray making angle of 30°.

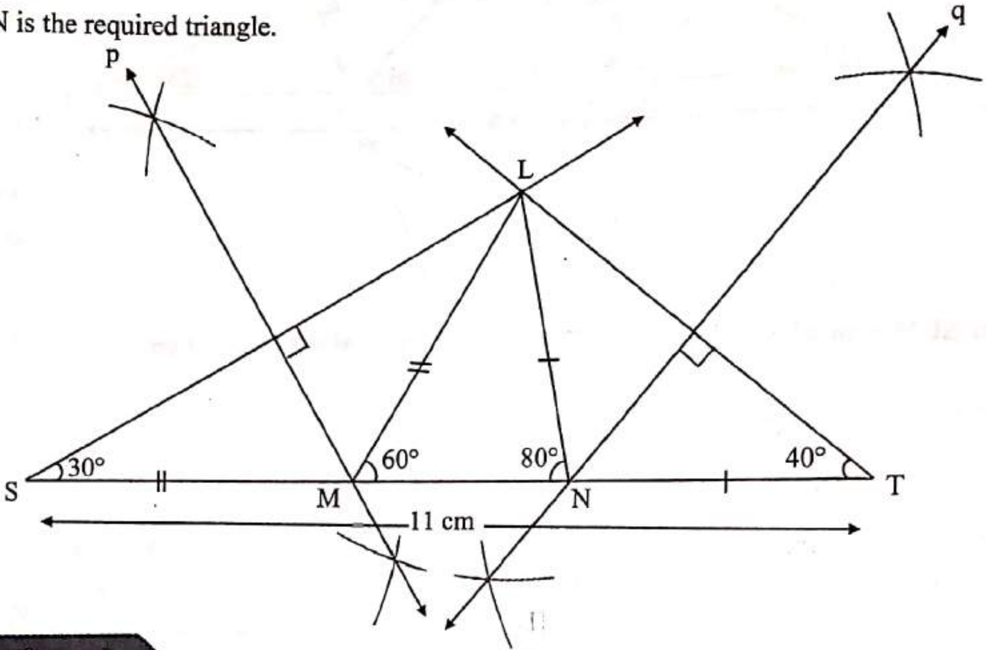
iii. From point T draw ray making angle of 40°.

iv. Name the point of intersection of two rays as L.

v. Draw the perpendicular bisector of seg LS and seg LT intersecting seg ST in M and N respectively.

vi. Join LM and LN.

Hence, ΔLMN is the required triangle.



Problem Set – 4

1. Construct ΔXYZ, such that $XY + XZ = 10.3$ cm, $YZ = 4.9$ cm, $\angle XYZ = 45°$. [4 Marks]

Ans:

As shown in the rough figure draw seg $YZ = 4.9$ cm

Draw a ray YT making an angle of 45° with YZ

Take a point W on ray YT, such that $YW = 10.3$ cm

Now, $YX + XW = YW$ [Y–X–W]

$\therefore$ $YX + XW = 10.3$ cm ....(i)

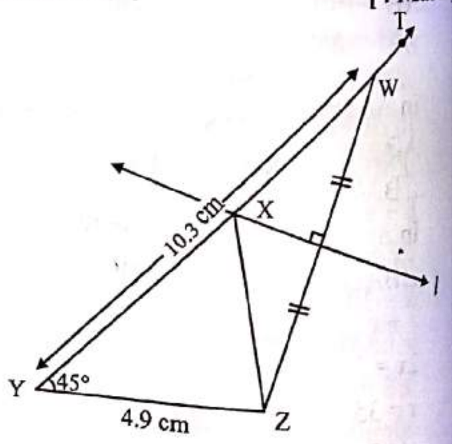
Also, $XY + XZ = 10.3$ cm ....(ii) [Given]

$\therefore$ $YX + XW = XY + XZ$ [From (i) and (ii)]

$\therefore$ $XW = XZ$

$\therefore$ Point X is on the perpendicular bisector of seg WZ

$\therefore$ The point of intersection of ray YT and perpendicular bisector of seg WZ is point X.



Rough figure

Steps of construction:

i. Draw seg YZ of length 4.9 cm.

ii. Draw ray YT, such that $\angle ZYT = 75°$.

iii. Mark point W on ray YT such that $l(YW) = 10.3$ cm.

iv. Join points W and Z.

v. Draw perpendicular bisector of seg WZ intersecting ray YT. Name the point as X.

vi. Join the points X and Z.

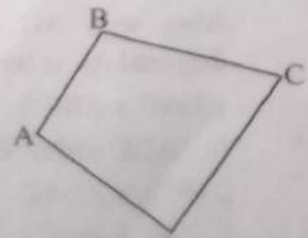
Hence, ΔXYZ is the required triangle.

5 Quadrilaterals

Type of Problems	Practice Set	Q. Nos.
Parallelogram and its properties	5.1	Q.1, 2, 3, 4, 5, 6, 7
	Practice Problems (Based on Practice Set 5.1)	Q.1, 2, 3, 4, 5, 6, 7, 8, 9
	Problem Set- 5	Q.4
Tests for parallelogram	5.2	Q.1, 2, 3, 4, 5
	Practice Problems (Based on Practice Set 5.2)	Q.1, 2, 3
	Problem Set- 5	Q.7
Rectangle and its properties	5.3	Q.1
	Practice Problems (Based on Practice Set 5.3)	Q.1, 2
	Problem Set- 5	Q.2, 6
Rhombus and its properties	5.3	Q.2, 4
	Practice Problems (Based on Practice Set 5.3)	Q. 3, 4, 5, 6
	Problem Set- 5	Q.5
Square and its properties	5.3	Q.3
	Practice Problems (Based on Practice Set 5.3)	Q. 7
	Problem Set- 5	Q.3
Properties of rectangle, rhombus, square and parallelogram	5.3	Q.5
Trapezium and its properties, isosceles trapezium	5.4	Q.1, 2, 3
	Practice Problems (Based on Practice Set 5.4)	Q.1, 2, 3, 4, 5, 6
Theorem of midpoints of two sides of a triangle and its converse	5.5	Q.1, 2, 3, 4
	Practice Problems (Based on Practice Set 5.5)	Q.1, 2, 3, 4, 5, 6, 7
	Problem Set- 5	Q.8, 9

Let's Recall

1. **Write the following pairs considering □ABCD.** (Textbook pg. no. 57)



Pairs of adjacent sides: i. **AB, AD** ii. **AD, DC**
iii. **DC, BC** iv. **BC, AB**

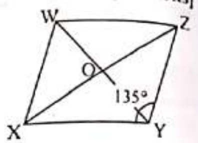
Pairs of adjacent angles: i. **$\angle A$, $\angle B$** ii. **$\angle C$, $\angle D$**
iii. **$\angle B$, $\angle C$** iv. **$\angle D$, $\angle A$**

Pairs of opposite sides: i. **AB, DC** ii. **AD, BC**

Pairs of opposite angles: i. **$\angle A$, $\angle C$** ii. **$\angle B$, $\angle D$**

Practice Set 5.1

1. Diagonals of a parallelogram WXYZ intersect each other at point O. If $\angle XYZ = 135°$, then what is the measure of $\angle XWZ$ and $\angle YZW$? If $l(OY) = 5$ cm, then $l(WY) = ?$ **[3 Marks]**



Solution:

i. $\angle XYZ = 135°$ [Given]

$\square$WXYZ is a parallelogram. [Given]

$\angle XWZ = \angle XYZ$ [Opposite angles of a parallelogram]

$\therefore$ $\angle XWZ = 135°$ ...(i)

ii. $\angle YZW + \angle XYZ = 180°$ [Adjacent angles of a parallelogram are supplementary]

$\therefore$ $\angle YZW + 135° = 180°$ [From (i)]

$\therefore$ $\angle YZW = 180° - 135°$

$\therefore$ $\angle YZW = 45°$

iii. $l(OY) = 5$ cm [Given]

$l(OY) = \frac{1}{2}\, l(WY)$ [Diagonals of a parallelogram bisect each other]

$\therefore$ $l(WY) = 2 \times l(OY)$

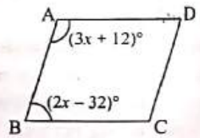
$= 2 \times 5$

$\therefore$ $l(WY) = 10$ cm

$\therefore$ **$\angle XWZ = 135°$, $\angle YZW = 45°$, $l(WY) = 10$ cm**

2. In a parallelogram ABCD, if $\angle A = (3x + 12)°$, $\angle B = (2x - 32)°$, then find the value of x and the measures of $\angle C$ and $\angle D$. **[3 Marks]**

Solution:



i. $\square$ABCD is a parallelogram. [Given]

$\therefore$ $\angle A + \angle B = 180°$ [Adjacent angles of a parallelogram are supplementary]

$\therefore$ $(3x + 12)° + (2x - 32)° = 180°$

$\therefore$ $3x + 12 + 2x - 32 = 180$

$\therefore$ $5x - 20 = 180$

$\therefore$ $5x = 180 + 20$

$\therefore$ $5x = 200$

$\therefore$ $x = \frac{200}{5}$

$\therefore$ $x = 40$

ii. $\angle A = (3x + 12)°$

$= [3(40) + 12]°$

$= (120 + 12)° = 132°$

$\angle B = (2x - 32)°$

$= [2(40) - 32]°$

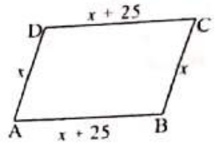
$= (80 - 32)° = 48°$

$\therefore$ $\left.\begin{array}{l}\angle C = \angle A = 132° \\ \angle D = \angle B = 48°\end{array}\right\}$ [Opposite angles of a parallelogram]

$\therefore$ **The value of x is 40, and the measures of $\angle C$ and $\angle D$ are 132° and 48° respectively.**

3. Perimeter of a parallelogram is 150 cm. One of its sides is greater than the other side by 25 cm. Find the lengths of all sides. **[3 Marks]**

Solution:



i. Let $\square$ABCD be the parallelogram and the length of AD be x cm.

One side is greater than the other by 25 cm.

$\therefore$ AB = $x + 25$ cm

$\left.\begin{array}{l}AD = BC = x \text{ cm} \\ AB = DC = (x + 25) \text{ cm}\end{array}\right\}$ [Opposite sides of a parallelogram]

ii. Perimeter of $\square$ABCD = 150 cm [Given]

$\therefore$ AB + BC + DC + AD = 150

$\therefore$ $(x + 25) + x + (x + 25) + x = 150$

$\therefore$ $4x + 50 = 150$

$\therefore$ $4x = 150 - 50$

$\therefore$ $4x = 100$

$\therefore$ $x = \frac{100}{4}$

$\therefore$ $x = 25$

iii. AD = BC = x = 25 cm

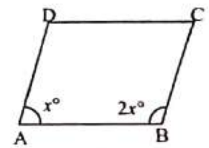
AB = DC = $x + 25 = 25 + 25$

= 50 cm

$\therefore$ **The lengths of the sides of the parallelogram are 25 cm, 50 cm, 25 cm and 50 cm.**

4. If the ratio of measures of two adjacent angles of a parallelogram is 1 : 2, find the measures of all angles of the parallelogram. **[3 Marks]**

Solution:



i. Let $\square$ABCD be the parallelogram.

The ratio of measures of two adjacent angles of a parallelogram is 1:2.

Let the common multiple be x.

$\therefore$ $\angle A = x°$ and $\angle B = 2x°$

$\angle A + \angle B = 180°$ [Adjacent angles of a parallelogram are supplementary]

$\therefore$ $x + 2x = 180$

$\therefore$ $3x = 180$

$\therefore$ $x = \frac{180}{3}$

$\therefore$ $x = 60$

ii. $\angle A = x° = 60°$

$\angle B = 2x° = 2 \times 60° = 120°$

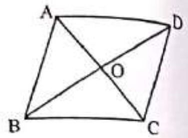
$\left.\begin{array}{l}\angle A = \angle C = 60° \\ \angle B = \angle D = 120°\end{array}\right\}$ [Opposite angles of a parallelogram]

$\therefore$ **The measures of the angles of the parallelogram are 60°, 120°, 60° and 120°.**

5. Diagonals of a parallelogram intersect each other at point O. If AO = 5, BO = 12 and AB = 13, then show that □ABCD is a rhombus. **[3 Marks]**

Given: AO = 5, BO = 12 and AB = 13.

To prove: □ABCD is a rhombus.



Proof:

$AO = 5, BO = 12, AB = 13$ [Given]

$AO^2 + BO^2 = 5^2 + 12^2$

$= 25 + 144$

∴ $AO^2 + BO^2 = 169$ (i)

$AB^2 = 13^2 = 169$ (ii)

∴ $AB^2 = AO^2 + BO^2$ [From (i) and (ii)]

∴ ΔAOB is a right-angled triangle. [Converse of Pythagoras theorem]

∴ ∠AOB = 90°

∴ seg AC ⊥ seg BD (iii) [A–O–C]

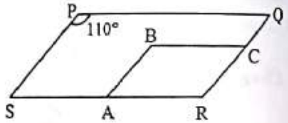
In parallelogram ABCD,

seg AC ⊥ seg BD [From (iii)]

∴ **□ABCD is a rhombus.** [A parallelogram is a rhombus if its diagonals are perpendicular to each other]

---

6. In the adjoining figure, □PQRS and □ABCR are two parallelograms. If ∠P = 110°, then find the measures of all the angles of □ABCR. **[3 Marks]**



Solution:

□PQRS is a parallelogram. [Given]

∴ ∠R = ∠P [Opposite angles of a parallelogram]

∴ ∠R = 110° (i)

□ABCR is a parallelogram. [Given]

∴ ∠A + ∠R = 180° [Adjacent angles of a parallelogram are supplementary]

∴ ∠A + 110° = 180° [From (i)]

∴ ∠A = 180° − 110°

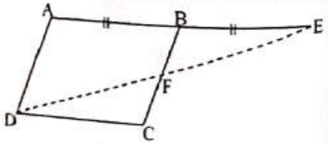
∴ ∠A = 70°

∴ ∠C = ∠A = 70°, ∠B = ∠R = 110° [Opposite angles of a parallelogram]

∴ **∠A = 70°, ∠B = 110°, ∠C = 70°, ∠R = 110°**

---

7. In the adjoining figure, □ABCD is a parallelogram. Point E is on the ray AB such that BE = AB, then prove that line ED bisects seg BC at point F. **[3 Marks]**



Given: □ABCD is a parallelogram.

BE = AB

To prove: Line ED bisects seg BC at point F i.e. FC = FB

Proof:

□ABCD is a parallelogram. [Given]

seg AB ≅ seg DC (i) [Opposite sides of a parallelogram]

seg AB ≅ seg BE (ii) [Given]

∴ seg DC ≅ seg BE (iii) [From (i) and (ii)]

side DC ∥ side AB [Opposite sides of a parallelogram]

i.e. side DC ∥ seg AE [A–B–E]

and seg DE is their transversal.

∴ ∠CDE ≅ ∠AED [Alternate angles]

∴ ∠CDF ≅ ∠BEF (iv) [D–F–E, A–B–E]

In ΔDFC and ΔEFB,

seg DC ≅ seg EB [From (iii)]

∠CDF ≅ ∠BEF [From (iv)]

∠DFC ≅ ∠EFB [Vertically opposite angles]

∴ ΔDFC ≅ ΔEFB [SAA test]

∴ FC = FB [c.s.c.t]

∴ **Line ED bisects seg BC at point F.**

Let's Recall

Tests for parallel lines:

When a transversal intersects two lines, then the two lines will be parallel if

i. a pair of corresponding angles is congruent, or

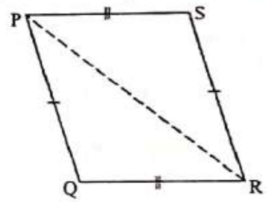
ii. a pair of alternate angles is congruent, or

iii. a pair of interior angles is supplementary.

Let's Study

Tests for parallelogram

Theorem: If pairs of opposite sides of a quadrilateral are congruent, then that quadrilateral is a parallelogram.



Given: In □PQRS,

side PS ≅ side QR, and

side PQ ≅ side SR

To prove: □PQRS is a parallelogram.

Construction: Draw diagonal PR.

Proof:

In ΔSPR and ΔQRP,

side SP ≅ side QR, side SR ≅ side QP [Given]

side PR ≅ side RP [Common side]

∴ ΔSPR ≅ ΔQRP [SSS test]

∴ ∠SPR ≅ ∠QRP [c.a.c.t.]

∠PRS ≅ ∠RPQ [c.a.c.t.]

∠SPR and ∠QRP are alternate angles formed by the transversal PR of seg PS and seg QR.

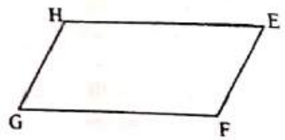
∴ side PS ∥ side QR (i) [Alternate angles test]

Similarly, ∠PRS and ∠RPQ are the alternate angles formed by transversal PR of seg PQ and seg SR.

∴ side PQ ∥ side SR (ii) [Alternate angles test]

∴ □PQRS is a parallelogram. [From (i) and (ii)]

Theorem: If both the pairs of opposite angles of a quadrilateral are congruent, then it is a parallelogram.



Given: In □EFGH,

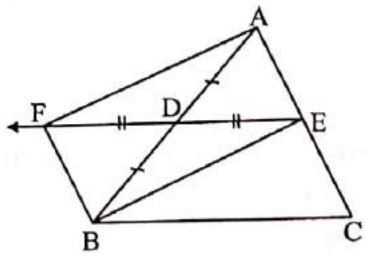
∠E ≅ ∠G and ∠F ≅ ∠H

To prove: □EFGH is a parallelogram.

∴	$\Delta CBD \cong \Delta ADB$		[SAS test]
∴	$\angle CDB \cong \angle ABD$		[c.a.c.t.]
	$\angle CDB$ and $\angle ABD$ are a pair of alternate angles formed by transversal BD of seg CD and seg BA.		
∴	seg CD ∥ seg BA	(iii)	[Alternate angles test]
∴	□ABCD is a parallelogram.		[From (i) and (iii)]

Textual Activity

1. **Points D and E are the midpoints of side AB and side AC of ΔABC respectively. Point F is on ray ED such that ED = DF. Prove that □AFBE is a parallelogram. For this example write 'given' and 'to prove' and complete the proof.**
(Textbook pg. no. 66)



Given: D and E are the midpoints of side AB and side AC respectively.
ED = DF

To prove: □AFBE is a parallelogram.

Proof:

	seg AB and seg EF are the $\boxed{\text{diagonals}}$ of □AFBE.	
	seg AD ≅ seg DB	$\boxed{\text{[Given]}}$
	seg $\boxed{\text{DE}}$ ≅ seg $\boxed{\text{DF}}$	[Given]
∴	Diagonals of □AFBE $\boxed{\text{bisect}}$ each other.	
∴	□AFBE is a parallelogram.	$\boxed{\text{[By test of parallelogram]}}$

Remember This

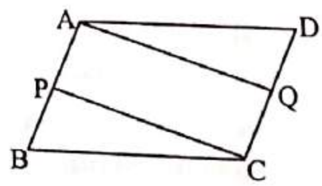
Tests for parallelogram: A quadrilateral is a parallelogram, if
i. pairs of its opposite sides are congruent.
ii. pairs of its opposite angles are congruent.
iii. its diagonals bisect each other.
iv. a pair of its opposite sides is parallel and congruent.

Try This

1. **Lines in a note book are parallel. Using these lines how can we draw a parallelogram ?**
(Textbook pg. no. 65)

Ans: If we connect any two horizontal lines of the note book using vertical or oblique/slanting parallel lines, we will get a parallelogram.

Practice Set 5.2



1. **In the adjoining figure, □ABCD is a parallelogram, P and Q are midpoints of sides AB and DC respectively, then prove □APCQ is a parallelogram.** **[3 Marks]**

Given: □ABCD is a parallelogram. P and Q are the midpoints of sides AB and DC respectively.

To prove: □APCQ is a parallelogram.

Proof:

$AP = \frac{1}{2}AB$ ... (i) [P is the midpoint of side AB]

$QC = \frac{1}{2}DC$ ... (ii) [Q is the midpoint of side CD]

□ABCD is a parallelogram. [Given]

$\therefore$ AB = DC [Opposite sides of a parallelogram]

$\therefore$ $\frac{1}{2}AB = \frac{1}{2}DC$ $\left[\text{Multiplying both sides by } \frac{1}{2}\right]$

$\therefore$ AP = QC ... (iii) [From (i) and (ii)]

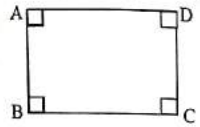
Also, AB ∥ DC [Opposite angles of a parallelogram]

i.e. AP ∥ QC ... (iv) [A – P – B, D – Q – C]

From (iii) and (iv),

□APCQ is a parallelogram. [A quadrilateral is a parallelogram if its opposite sides is parallel and congruent]

2. Using opposite angles test for parallelogram, prove that every rectangle is a parallelogram. [2 Marks]



Given: □ABCD is a rectangle.

To prove: Rectangle ABCD is a parallelogram.

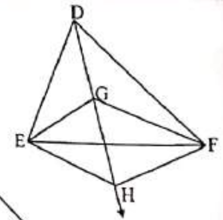
Proof:

□ABCD is a rectangle. [Given]

$\therefore$ $\left.\begin{array}{l}\angle A \cong \angle C = 90^\circ \\ \angle B \cong \angle D = 90^\circ\end{array}\right\}$ [Angles of a rectangle]

$\therefore$ **Rectangle ABCD is a parallelogram.** [A quadrilateral is a parallelogram, if pairs of its opposite angles are congruent]

3. In the adjoining figure, G is the point of concurrence of medians of ΔDEF. Take point H on ray DG such that D–G–H and DG = GH, then prove that □GEHF is a parallelogram. [3 Marks]

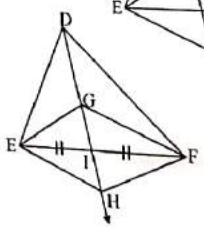


Given: Point G (centroid) is the point of concurrence of the medians of ΔDEF.

DG = GH

To prove: □GEHF is a parallelogram.

Proof:



Let ray DH intersect seg EF at point I such that E–I–F.

$\therefore$ seg DI is the median of ΔDEF.

$\therefore$ EI = FI ... (i)

Point G is the centroid of ΔDEF.

$\therefore$ $\frac{DG}{GI} = \frac{2}{1}$ [Centroid divides each median in the ratio 2 : 1]

$\therefore$ DG = 2(GI)

$\therefore$ GH = 2(GI) [DG = GH]

$\therefore$ GI + HI = 2(GI) [G–I–H]

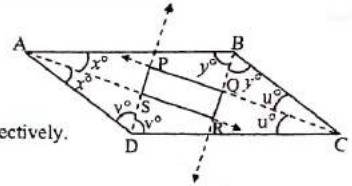
$\therefore$ HI = 2(GI) – GI

$\therefore$ HI = GI ... (ii)

From (i) and (ii),

□GEHF is a parallelogram. [A quadrilateral is a parallelogram, if its diagonals bisect each other]

4. Prove that quadrilateral formed by the intersection of angle bisectors of all angles of a parallelogram is a rectangle. [4 Marks]



Given: □ABCD is a parallelogram.

Rays AS, BQ, CQ and DS bisect ∠A, ∠B, ∠C and ∠D respectively.

To prove: □PQRS is a rectangle.

Proof:

$\angle BAS = \angle DAS = x^\circ$ ... (i) [ray AS bisects ∠A]

$\angle ABQ = \angle CBQ = y^\circ$ ... (ii) [ray BQ bisects ∠B]

$\angle BCQ = \angle DCQ = u^\circ$ ... (iii) [ray CQ bisects ∠C]

$\angle ADS = \angle CDS = v^\circ$ ... (iv) [ray DS bisects ∠D]

□ABCD is a parallelogram. [Given]

$\therefore$ $\angle A + \angle B = 180^\circ$ [Adjacent angles of a parallelogram are supplementary]

$\therefore$ $\angle BAS + \angle DAS + \angle ABQ + \angle CBQ = 180^\circ$ [Angle addition property]

$\therefore$ $x^\circ + x^\circ + y^\circ + y^\circ = 180$ [From (i) and (ii)]

$\therefore$ $2x^\circ + 2y^\circ = 180$

$\therefore$ $x + y = 90^\circ$ ... (v) [Dividing both sides by 2]

Also, $\angle A + \angle D = 180^\circ$ [Adjacent angles of a parallelogram are supplementary]

$\therefore$ $\angle BAS + \angle DAS + ADS + \angle CDS = 180^\circ$ [Angle addition property]

$\therefore$ $x^\circ + x^\circ + v^\circ + v^\circ = 180^\circ$

$\therefore$ $2x^\circ + 2v^\circ = 180^\circ$

$\therefore$ $x^\circ + v^\circ = 90^\circ$ ... (vi) [Dividing both sides by 2]

In ΔARB,

$\angle RAB + \angle RBA + \angle ARB = 180^\circ$ [Sum of the measures of the angles of a triangle is 180°]

$\therefore$ $x^\circ + y^\circ + \angle SRQ = 180^\circ$ [A – S – R, B – Q – R]

$\therefore$ $90^\circ + \angle SRQ = 180^\circ$ [From (v)]

$\therefore$ $\angle SRQ = 180^\circ - 90^\circ = 90^\circ$ ... (vii)

Similarly, we can prove

$\angle SPQ = 90^\circ$ ... (viii)

In ΔASD,

$\angle ASD + \angle SAD + \angle SDA = 180^\circ$ [Sum of the measures of angles a triangle is 180°]

$\therefore$ $\angle ASD + x^\circ + v^\circ = 180^\circ$

$\therefore$ $\angle ASD + 90^\circ = 180^\circ$ [From (vi)]

$\therefore$ $\angle ASD = 180^\circ - 90^\circ = 90^\circ$

$\angle PSR = \angle ASD$ [Vertically opposite angles]

$\therefore$ $\angle PSR = 90^\circ$ ... (ix)

Similarly we can prove

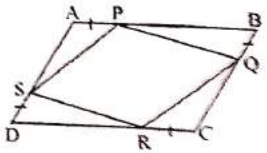
$\angle PQR = 90^\circ$ ... (x)

$\therefore$ In □PQRS,

$\angle SRQ = \angle SPQ = \angle PSR = \angle PQR = 90^\circ$ [From (vii), (viii), (ix), (x)]

$\therefore$ **□PQRS is a rectangle.** [Each angle is of measure 90°]

5. In the adjoining figure, if points P, Q, R, S are on the sides of parallelogram such that AP = BQ = CR = DS, then prove that □PQRS is a parallelogram. [3 Marks]



Given: □ABCD is a parallelogram.

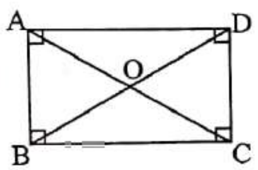
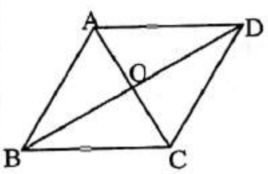
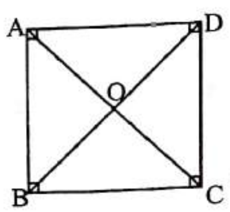
AP = BQ = CR = DS

To prove: □PQRS is a parallelogram.

Proof:

	□ABCD is a parallelogram.		[Given]
∴	$\angle B = \angle D$	(i)	[Opposite angles of a parallelogram]
	Also, $AB = CD$		[Opposite sides of a parallelogram]
∴	$AP + BP = DR + CR$		[A–P–B, D–R–C]
∴	$AP + BP = DR + AP$		[$AP = CR$]
∴	$BP = DR$	(ii)	
	In ΔPBQ and ΔRDS,		
	seg $BP \cong$ seg DR		[From (ii)]
	$\angle PBQ \cong \angle RDS$		[From (i)]
	seg $BQ \cong$ seg DS		[Given]
∴	$\Delta PBQ \cong \Delta RDS$		[SAS test]
∴	seg $PQ \cong$ seg RS	(iii)	[c.s.c.t]
	Similarly, we can prove that		
	$\Delta PAS \cong \Delta RCQ$		
∴	seg $PS \cong$ seg RQ	(iv)	[c.s.c.t]
	From (iii) and (iv),		
	□PQRS is a parallelogram.		[A quadrilateral is a parallelogram, if pairs of its opposite angles are congruent]

Let's Study

Quadrilateral	Type	Definition	Properties	Example
 	Rectangle	If all angles of a quadrilateral are right angles, it is called a rectangle.	i. Opposite sides are congruent. ii. Diagonals are congruent. iii. Diagonals bisect each other.	In rectangle ABCD, i. side $AB \cong$ side DC, side $BC \cong$ side AD ii. diagonal $AC \cong$ diagonal BD iii. $l(BO) = l(DO)$ and $l(AO) = l(CO)$
 	Rhombus	If all the sides of a quadrilateral are congruent, it is called a rhombus.	i. Opposite angles are congruent. ii. Diagonals bisect each other and they are perpendicular to each other. iii. Diagonals bisect the opposite angles.	In rhombus ABCD, i. $\angle ABC \cong \angle ADC$, $\angle BAD \cong \angle BCD$ ii. $l(BO) = l(DO)$ and $l(AO) = l(CO)$ diagonal $BD \perp$ diagonal AC iii. Angles made by diagonal AC $m\angle BAC = m\angle DAC$ $m\angle BCA = m\angle DCA$ Angles made by diagonal BD $m\angle ABD = m\angle CBD$ $m\angle ADB = m\angle CDB$
 	Square	If all the sides and all the angles of a quadrilateral are congruent, it is called a square.	i. Diagonals are congruent. ii. Diagonals bisect each other and they are perpendicular to each other. iii. Diagonals bisect the opposite angles.	In square ABCD, i. diagonal $AC \cong$ diagonal BD ii. $l(BO) = l(DO)$ and $l(AO) = l(CO)$ diagonal $BD \perp$ diagonal AC iii. Angles made by diagonal AC: $m\angle BAC = m\angle DAC$ $m\angle BCA = m\angle DCA$ Angles made by diagonal BD: $m\angle ABD = m\angle CBD$ $m\angle ADB = m\angle CDB$

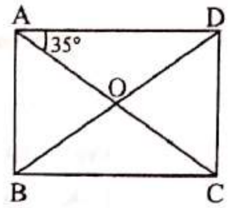
Remember This

i. Diagonals of a rectangle and a square are congruent.
ii. Diagonals of a rhombus and a square are perpendicular bisectors of each other.
iii. Diagonals of a rectangle bisect each other.
iv. Diagonals of a rhombus and a square bisect the pairs of opposite angles.

Practice Set 5.3

1. Diagonals of a rectangle ABCD intersect at point O. If AC = 8 cm, then find BO and if $\angle CAD = 35°$, then find $\angle ACB$.

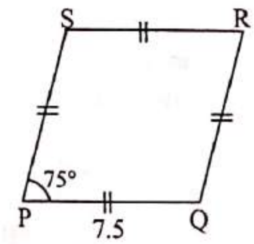
Solution: **[2 Marks]**



i. $AC = 8$ cm	(i)	[Given]
□ABCD is a rectangle.		[Given]
∴ $BD = AC$		[Diagonals of a rectangle are congruent]
∴ $BD = 8$ cm		[From (i)]
$BO = \frac{1}{2} BD$		[Diagonals of a rectangle bisect each other]
∴ $BO = \frac{1}{2} \times 8$		
∴ $BO = 4$ cm		
ii. side AD ∥ side BC and seg AC is their transversal.		[Opposite sides of a rectangle are parallel]
∴ $\angle ACB = \angle CAD$		[Alternate angles]
∴ $\angle ACB = 35°$		[$\because \angle CAD = 35°$]
∴ **BO = 4 cm, $\angle ACB = 35°$**		

2. In a rhombus PQRS, if PQ = 7.5 cm, then find QR. If $\angle QPS = 75°$, then find the measures of $\angle PQR$ and $\angle SRQ$. **[2 Marks]**

Solution:

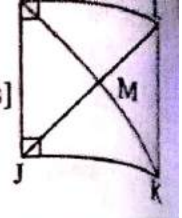


i. $PQ = 7.5$ cm	[Given]
□PQRS is a rhombus.	[Given]
∴ $QR = PQ$	[Sides of a rhombus are congruent]
∴ $QR = 7.5$ cm	
ii. $\angle QPS = 75°$	[Given]
$\angle QPS + \angle PQR = 180°$	[Adjacent angles of a rhombus are supplementary]
∴ $75° + \angle PQR = 180°$	
∴ $\angle PQR = 180° - 75°$	
∴ $\angle PQR = 105°$	
iii. $\angle SRQ = \angle QPS$	[Opposite angles of a rhombus]
∴ $\angle SRQ = 75°$	
∴ **QR = 7.5 cm, $\angle PQR = 105°$, $\angle SRQ = 75°$**	

3. Diagonals of a square IJKL intersects at point M. Find the measures of $\angle IMJ$, $\angle JIK$ and $\angle LJK$. **[3 Marks]**

Solution:

i. □IJKL is a square.	[Given]
∴ seg IK ⊥ seg JL	[Diagonals of a square are perpendicular to each other]



$\therefore \quad \angle IMJ = 90^\circ$

$\angle JIL = 90^\circ$ ... (i) [Angle of a square]

ii. $\angle JIK = \frac{1}{2}\angle JIL$ [Diagonals of a square bisect the opposite angles]

$\therefore \quad \angle JIK = \frac{1}{2}(90^\circ)$ [From (i)]

$\therefore \quad \angle JIK = 45^\circ$

$\angle IJK = 90^\circ$ ... (ii) [Angle of a square]

iii. $\angle LJK = \frac{1}{2}\angle IJK$ [Diagonals of a square bisect the opposite angles]

$\therefore \quad \angle LJK = \frac{1}{2}(90^\circ)$ [From (ii)]

$\therefore \quad \angle LJK = 45^\circ$

$\therefore$ **$\angle IMJ = 90^\circ$, $\angle JIK = 45^\circ$, $\angle LJK = 45^\circ$**

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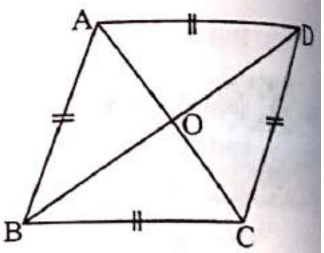
4. **Diagonals of a rhombus are 20 cm and 21 cm respectively, then find the side of rhombus and its perimeter.** **[3 Marks]**

***Solution*:**



i. Let □ABCD be the rhombus.

$AC = 20$ cm, $BD = 21$ cm

$AO = \frac{1}{2}\ AC$ [Diagonals of a rhombus bisect each other]

$= \frac{1}{2} \times 20 = 10$ cm ... (i)

Also, $BO = \frac{1}{2}\ BD$ [Diagonals of a rhombus bisect each other]

$= \frac{1}{2} \times 21 = \frac{21}{2}$ cm ... (ii)

ii. In ΔAOB, $\angle AOB = 90^\circ$ [Diagonals of a rhombus are prependicular to each other]

$\therefore \quad AB^2 = AO^2 + BO^2$ [Pythagoras theorem]

$= (10)^2 + \left(\frac{21}{2}\right)^2$ [From (i) and (ii)]

$= 100 + \frac{441}{4}$

$= \frac{400+441}{4}$

$\therefore \quad AB^2 = \frac{841}{4}$

$\therefore \quad AB = \sqrt{\frac{841}{4}}$ [Taking square root of both sides]

$= \frac{29}{2} = 14.5$ cm

iii. Perimeter of □ABCD

$= 4 \times AB = 4 \times 14.5 = 58$ cm

$\therefore$ **The side and perimeter of the rhombus are 14.5 cm and 58 cm respectively.**

5. **State with reasons whether the following statements are 'true' or 'false'.** **[1 Mark each]**

i. Every parallelogram is a rhombus.

ii. Every rhombus is a rectangle.

iii. Every rectangle is a parallelogram.

iv. Every square is a rectangle.

v. Every square is a rhombus.

vi. Every parallelogram is a rectangle.

Ans:

i. False.

All the sides of a rhombus are congruent, while the opposite sides of a parallelogram are congruent.

ii. False.

All the angles of a rectangle are congruent, while the opposite angles of a rhombus are congruent.

iii. True.

The opposite sides of a parallelogram are parallel and congruent. Also, its opposite angles are congruent.
The opposite sides of a rectangle are parallel and congruent. Also, all its angles are congruent.

iv. True.

The opposite sides of a rectangle are parallel and congruent. Also, all its angles are congruent.
All the sides of a square are parallel and congruent. Also, all its angles are congruent.

v. True.

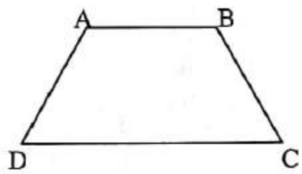
All the sides of a rhombus are congruent. Also, its diagonals are perpendicular bisectors of each other.
All the sides of a square are congruent. Also, its diagonals are perpendicular bisectors of each other.

vi. False.

All the angles of a rectangle are congruent, while the opposite angles of a parallelogram are congruent.

Let's Study

Trapezium



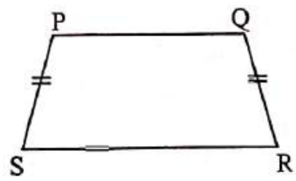
i. When only one pair of opposite sides of a quadrilateral is parallel, then it is called a trapezium.

In the adjoining figure, □ABCD is a trapezium as side AB || side DC.

ii. In a trapezium, only two pairs of angles are supplementary.

For the adjoining figure, $\angle A + \angle D = 180°$ and $\angle B + \angle C = 180°$.

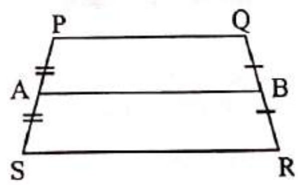
But, $\angle A + \angle B \neq 180°$ and $\angle C + \angle D \neq 180°$.



Isosceles Trapezium:

A trapezium in which non-parallel sides are congruent is called an isosceles trapezium.

In the adjoining figure, □PQRS is an isosceles trapezium as seg PQ || seg SR and seg PS ≅ seg QR.



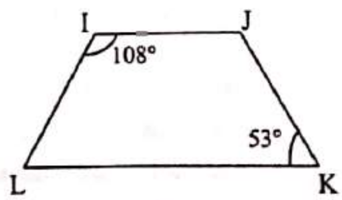
Median of the trapezium:

The segment joining the midpoints of non parallel sides of a trapezium is called the **median** of the trapezium

In the adjoining figure, seg AB is the median of the trapezium as A and B are the midpoints of non-parallel sides PS and QR respectively.

Practice Set 5.4

1. **In □IJKL, side IJ || side KL, $\angle I = 108°$ and $\angle K = 53°$, then find the measures of $\angle J$ and $\angle L$.** **[2 Marks]**



Solution:

i. $\angle I = 108°$ [Given]

side IJ || side KL [Given]

and side IL is their transveral.

∴ $\angle I + \angle L = 180°$ [Interior angles]

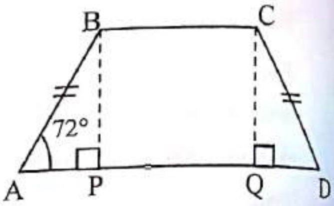
∴ $108° + \angle L = 180°$

∴ $\angle L = 180° - 108° = 72°$

ii.	$\angle K = 53^\circ$		[Given]
	side IJ $\parallel$ side KL		[Given]
	and side JK is their transveral.		
$\therefore$	$\angle J + \angle K = 180^\circ$		[Interior angles]
$\therefore$	$\angle J + 53^\circ = 180^\circ$		
$\therefore$	$\angle J = 180^\circ - 53^\circ = 127^\circ$		
$\therefore$	**$\angle L = 72^\circ$, $\angle J = 127^\circ$**		

2. In □ABCD, side BC $\parallel$ side AD, side AB $\cong$ side DC. If $\angle A = 72^\circ$, then find the measures of $\angle B$ and $\angle D$. [3 Marks]

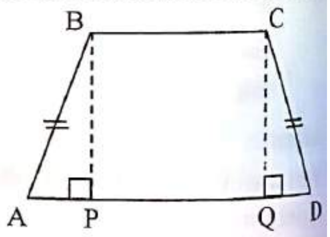
Construction: Draw seg BP $\perp$ side AD, A – P – D, seg CQ $\perp$ side AD, A – Q – D.



***Solution*:**

i.	$\angle A = 72^\circ$		[Given]
	In □ABCD, side BC $\parallel$ side AD		[Given]
	and side AB is their transversal.		
$\therefore$	$\angle A + \angle B = 180^\circ$		[Interior angles]
$\therefore$	$72^\circ + \angle B = 180^\circ$		
$\therefore$	$\angle B = 180^\circ - 72^\circ = 108^\circ$		
ii.	In ΔBPA and ΔCQD,		
	$\angle BPA \cong \angle CQD$		[Each angle is of measure 90°]
	Hypotenuse AB $\cong$ Hypotenuse DC		[Given]
	seg BP $\cong$ seg CQ		[Perpendicular distance between two parallel lines]
$\therefore$	$\Delta BPA \cong \Delta CQD$		[Hypotenuse side test]
$\therefore$	$\angle BAP \cong \angle CDQ$		[c. a. c. t.]
$\therefore$	$\angle A = \angle D$		
$\therefore$	$\angle D = 72^\circ$		
$\therefore$	**$\angle B = 108^\circ$, $\angle D = 72^\circ$**		

3. In □ABCD, side BC < side AD, side BC $\parallel$ side AD and if side BA $\cong$ side CD, then prove that $\angle ABC \cong \angle DCB$. [3 Marks]



Given: side BC < side AD, side BC $\parallel$ side AD, side BA $\cong$ side CD

To prove: $\angle ABC \cong \angle DCB$

Construction: Draw seg BP $\perp$ side AD, A – P – D
seg CQ $\perp$ side AD, A – Q – D

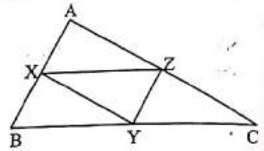
***Proof*:**

	In ΔBPA and ΔCQD,		
	$\angle BPA \cong \angle CQB$		[Each angle is of measure 90°]
	Hypotenuse BA $\cong$ Hypotenuse CD		[Given]
	seg BP $\cong$ seg CQ		[Perpendicular distance between two parallel lines]
$\therefore$	$\Delta BPA \cong \Delta CQD$		[Hypotenuse side test]
$\therefore$	$\angle BAP \cong \angle CDQ$		[c. a. c. t.]
$\therefore$	$\angle A = \angle D$	(i)	
	Now, side BC $\parallel$ side AD		[Given]
	and side AB is their transversal.		
$\therefore$	$\angle A + \angle B = 180^\circ$	(ii)	[Interior angles]
	Also, side BC $\parallel$ side AD		[Given]
	and side CD is their transversal.		
$\therefore$	$\angle C + \angle D = 180^\circ$	(iii)	[Interior angles]
$\therefore$	$\angle A + \angle B = \angle C + \angle D$		[From (ii) and (iii)]
$\therefore$	$\angle A + \angle B = \angle C + \angle A$		[From (i)]
$\therefore$	$\angle B = \angle C$		
$\therefore$	**$\angle ABC \cong \angle DCB$**		

	seg CF ∥ seg BA and seg AC is their transversal.	[Construction]
∴	∠BAC ≅ ∠FCA	[Alternate angles]
∴	∠DAE ≅ ∠FCE ...(vi)	[B-D-A, A-E-C]
	In ΔADE and ΔCFE,	
	seg AD ≅ seg CF	[From (v)]
	∠AED ≅ ∠CEF	[Vertically opposite angles]
	∠DAE ≅ ∠FCE	[From (vi)]
∴	ΔADE ≅ ΔCFE	[SAA test]
∴	seg AE ≅ seg CE	[c.s.c.t.]
∴	line *l* bisects side AC.	

Practice Set 5.5

1. In the adjoining figure, points X, Y, Z are the midpoints of side AB, side BC and side AC of ΔABC respectively. AB = 5 cm, AC = 9 cm and BC = 11 cm. Find the lengths of XY, YZ, XZ. **[3 Marks]**

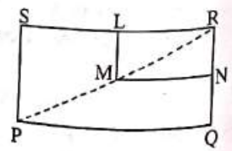


Solution:

i.	AC = 9 cm	[Given]
	Points X and Y are the midpoints of sides AB and BC respectively.	[Given]
∴	$XY = \frac{1}{2}AC$	[Midpoint theorem]
	$= \frac{1}{2} \times 9 = 4.5$ cm	
ii.	AB = 5 cm	[Given]
	Points Y and Z are the midpoints of sides BC and AC respectively.	[Given]
∴	$YZ = \frac{1}{2}AB$	[Midpoint theorem]
	$= \frac{1}{2} \times 5 = 2.5$ cm	
iii.	BC = 11 cm	[Given]
	Points X and Z are the midpoints of sides AB and AC respectively.	[Given]
∴	$XZ = \frac{1}{2}BC$	[Midpoint theorem]
	$= \frac{1}{2} \times 11 = 5.5$ cm	
∴	**l(XY) = 4.5 cm, l(YZ) = 2.5 cm, l(XZ) = 5.5 cm**	

2. In the adjoining figure, □PQRS and □MNRL are rectangles. If point M is the midpoint of side PR, then prove that,

i. SL = LR ii. $LN = \frac{1}{2}SQ$. **[4 Marks]**

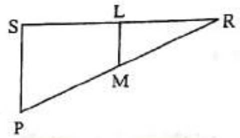


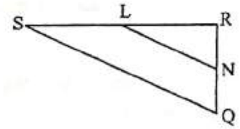
Given: □PQRS and □MNRL are rectangles. M is the midpoint of side PR.

To prove: i. SL = LR ii. $LN = \frac{1}{2}$ (SQ)

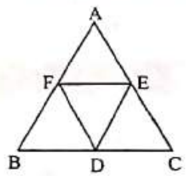
Proof:

i.	□PQRS and □MNRL are rectangles.	[Given]
∴	∠S = ∠L = 90°	[Angles of rectangles]
	∠S and ∠L form a pair of corresponding angles on sides SP and LM when SR is their transversal.	
∴	seg ML ∥ seg PS ...(i)	[Corresponding angles test]
	In ΔPRS,	
	Point M is the midpoint of PR and	[Given]
	seg ML ∥ seg PS.	[From (i)]
∴	Point L is the midpoint of seg SR. ...(ii)	[Converse of midpoint theorem]
∴	**SL = LR**	
ii.	Similarly for ΔPRQ, we can prove that,	
	Point N is the midpoint of seg QR. ...(iii)	
	In ΔRSQ,	
	Points L and N are the midpoints of seg SR and seg QR respectively.	[From (ii) and (iii)]
∴	**$LN = \frac{1}{2}SQ$**	[Midpoint theorem]





3. In the adjoining figure, ΔABC is an equilateral triangle. Points F, D and E are midpoints of side AB, side BC, side AC respectively. Show that ΔFED is an equilateral triangle. **[3 Marks]**



Given: ΔABC is an equilateral triangle.
Points F, D and E are midpoints of side AB, side BC, side AC respectively.

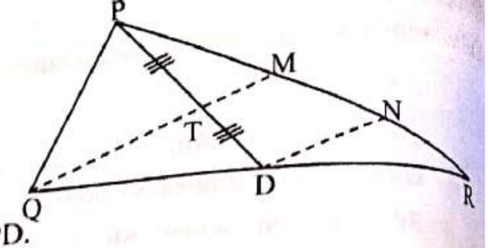
To prove: ΔFED is an equilateral triangle.

Proof:

	ΔABC is an equilateral triangle.	[Given]
∴	AB = BC = AC ...(i)	[Sides of an equilateral triangle]
	Points F and D are the midpoints of sides AB and BC respectively.	
∴	$FD = \frac{1}{2}AC$ ...(ii)	[Midpoint theorem]
	Points D and E are the midpoints of sides BC and AC respectively.	
∴	$DE = \frac{1}{2}AB$ ...(iii)	[Midpoint theorem]
	Points F and E are the midpoints of sides AB and AC respectively.	
∴	$FE = \frac{1}{2}BC$ ...(iv)	[Midpoint theorem]
∴	FD = DE = FE	[From (i), (ii), (iii) and (iv)]
∴	**ΔFED is an equilateral triangle.**	

4. **In the adjoining figure, seg PD is a median of ΔPQR. Point T is the midpoint of seg PD. Produced QT intersects PR at M. Show that $\frac{PM}{PR} = \frac{1}{3}$.** **[4 Marks]**

[Hint: Draw DN || QM]



Given: seg PD is a median of ΔPQR. Point T is the midpoint of seg PD.

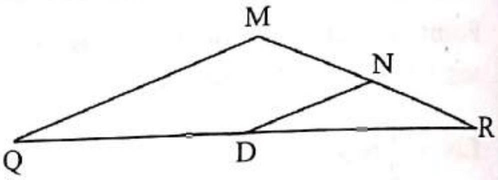
To prove: $\frac{PM}{PR} = \frac{1}{3}$

Construction: Draw seg DN || seg QM such that P–M–N and M–N–R.

Proof:

In ΔPDN,
Point T is the midpoint of seg PD ... [Given]
and seg TM || seg DN ... [Construction and Q–T–M]
$\therefore$ Point M is the midpoint of seg PN. ... [Converse of midpoint theorem]
$\therefore$ PM = MN ... (i)

In ΔQMR,
Point D is the midpoint of seg QR ... [Given]
and seg DN || seg QM ... [Construction]
$\therefore$ Point N is the midpoint of seg MR. ... [Converse of midpoint theorem]
$\therefore$ RN = MN ... (ii)
$\therefore$ PM = MN = RN ... (iii) [From (i) and (ii)]
Now, PR = PM + MN + RN ... [P–M–N, M–N–R]
$\therefore$ PR = PM + PM + PM ... [From (iii)]
$\therefore$ PR = 3PM
$\therefore$ $\frac{PM}{PR} = \frac{1}{3}$



Problem Set – 5

1. **Choose the correct alternative answer and fill in the blanks.** **[1 Mark each]**

i. If all pairs of adjacent sides of a quadrilateral are congruent, then it is called ________.
(A) rectangle (B) parallelogram (C) trapezium (D) rhombus

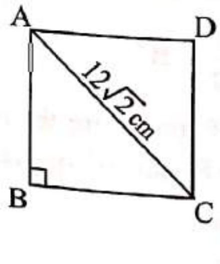
ii. If the diagonal of a square is $12\sqrt{2}$ cm, then the perimeter of square is ________.
(A) 24 cm (B) $24\sqrt{2}$ cm (C) 48 cm (D) $48\sqrt{2}$ cm

iii. If opposite angles of a rhombus are $(2x)^\circ$ and $(3x - 40)^\circ$, then the value of x is ________.
(A) 100° (B) 80° (C) 160° (D) 40°

Answers:
i. (D) ii. (C) iii. (D)

Hints:

ii. In ΔABC,
$AC^2 = AB^2 + BC^2$ ...[Pythagoras theorem]
$\therefore$ $(12\sqrt{2})^2 = AB^2 + AB^2$ ...[$\because$ AB = BC]
$\therefore$ $AB^2 = \frac{12^2 \times 2}{2} = 12^2$
$\therefore$ AB = 12 cm
$\therefore$ Perimeter of $\square$ABCD = 4 × 12 = 48 cm



iii. $2x = 3x - 40$ ...[Opposite angles of a rhombus]
$\therefore$ $x = 40^\circ$

6 Circle

Type of Problems	Exercise	Q. Nos.
Some terms related with a circle	Practice Problems (Based on Practice Set 6.1)	Q.1
The perpendicular drawn from the centre of a circle on its chord bisects the chord, and its converse	6.1	Q.1, 2, 3, 4, 5, 6
	Practice Problems (Based on Practice Set 6.1)	Q.2, 3, 4, 5, 6, 7, 8, 9, 10, 11
	Problem Set-6	Q.4, 6
Relation between congruent chords of a circle and their distances from the centre, properties of congruent chords	6.2	Q.1, 2, 3
	Practice Problems (Based on Practice Set 6.2)	Q.1, 2, 3, 4
	Problem Set-6	Q.5
To construct the incircle of a triangle	6.3	Q.1, 3
	Practice Problems (Based on Practice Set 6.3)	Q.1, 2, 6
To construct the circumcircle of a triangle	6.3	Q.2, 4, 5
	Practice Problems (Based on Practice Set 6.3)	Q.3, 4, 7
To construct the incircle and circumcircle of a triangle	Practice Problems (Based on Practice Set 6.3)	Q.5
	Problem Set-6	Q.2, 3

Let's Study

Circle

The set of points in a plane which are equidistant from a fixed point in the plane is called a circle.

Some terms related with a circle:

i. **Centre:** The fixed point is called the centre of the circle.

ii. **Radius:** The segment joining the centre of the circle and a point on the circle is called the radius of the circle.

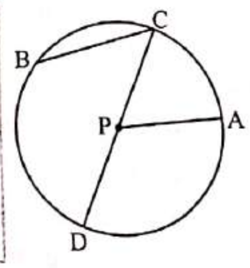
iii. **Chord:** A line segment joining any two points on the circle is called a chord of the circle.

iv. **Diameter:** A chord passing through the centre of the circle is called the diameter of the circle. A diameter is the longest chord of the circle.

In given figure, observe the circle with center P. With reference to this figure, complete the following table.

(Textbook pg. no. 76)

seg BC, seg DC	seg PA	seg DC	seg PC, seg PA, seg PD	Point P	∠DPA, ∠CPA	∠CPA
chord	**radius**	diameter	radius	centre	central angle	**central angle**



$\therefore$ $\Delta AOP \cong \Delta BOP$ [SSS test]

$\therefore$ $\angle OPA \cong \angle OPB$ ... (i) [c.a.c.t.]

But, $\angle OPA + \angle OPB = 180°$ ... (ii) [Angles in a linear pair]

$\therefore$ $\angle OPB + \angle OPB = 180°$ [From (i) and (ii)]

$\therefore$ $2(\angle OPB) = 180°$

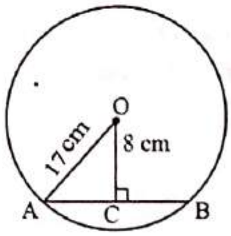
$\therefore$ $\angle OPB = \frac{180°}{2} = 90°$

$\therefore$ seg OP $\perp$ chord AB

Example: The distance of a chord from the centre of a circle is 8 cm. If the radius of the circle is 17 cm, then find the length of the chord.

Given: In a circle with centre O,
OA is radius and AB is its chord,
seg OC $\perp$ chord AB, A–C–B
AO = 17 cm, OC = 8 cm

To Find: Length of the chord AB



Solution:

i. In ΔOAC, $\angle C = 90°$

$\therefore$ $OA^2 = OC^2 + AC^2$ [Pythagoras theorem]

$\therefore$ $17^2 = 8^2 + AC^2$

$\therefore$ $AC^2 = 17^2 - 8^2$

$= (17 + 8)(17 - 8)$ [$a^2 - b^2 = (a + b)(a - b)$]

$= 25 \times 9$

$\therefore$ $AC = \sqrt{25 \times 9}$ [Taking square root on both sides]

$\therefore$ $AC = 5 \times 3 = 15$ cm ... (i)

ii. Now, $AC = \frac{1}{2}AB$ [Perpendicular drawn from the centre of a circle to the chord bisects the chord]

$\therefore$ $AB = 2(AC)$

$= 2 \times 15$ [From (i)]

$\therefore$ **AB = 30 cm**

Remember This

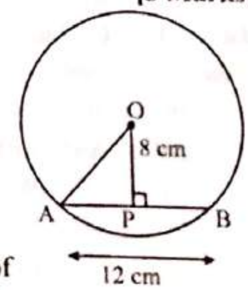
i. The perpendicular drawn from the centre of a circle on its chord bisects the chord.
ii. The segment joining the centre of a circle and the midpoint of its chord is perpendicular to the chord.

Practice Set 6.1

1. **Distance of chord AB from the centre of a circle is 8 cm. Length of the chord AB is 12 cm. Find the diameter of the circle.** **[3 Marks]**

Given: In a circle with centre O,
OA is radius and AB is its chord,
seg OP $\perp$ chord AB, A–P–B
AB = 12 cm, OP = 8 cm

To Find: Diameter of the circle



Solution:

i. $AP = \frac{1}{2}AB$ [Perpendicular drawn from the centre of a circle to the chord bisects the chord.]

$\therefore$ $AP = \frac{1}{2} \times 12 = 6$ cm ... (i)

ii. In ΔOPA, $\angle$OPA = 90°

$\therefore$ $OA^2 = OP^2 + AP^2$ [Pythagoras theorem]

$= 8^2 + 6^2$ [From (i)]

$= 64 + 36$

$\therefore$ $OA^2 = 100$

$\therefore$ $OA = \sqrt{100}$ [Taking square root on both sides]

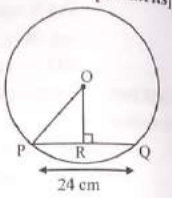
$= 10$ cm

iii. Radius (r) = 10 cm

$\therefore$ Diameter = 2r = 2 × 10 = 20 cm

$\therefore$ **The diameter of the circle is 20 cm.**

2. Diameter of a circle is 26 cm and length of a chord of the circle is 24 cm. Find the distance of the chord from the centre. **[3 Marks]**



Given: In a circle with centre O,
PO is radius and PQ is its chord,
seg OR ⊥ chord PQ, P–R–Q
PQ = 24 cm, diameter (d) = 26 cm

To Find: Distance of the chord from the centre (OR)

Solution:

i. Radius (OP) = $\frac{d}{2} = \frac{26}{2} = 13$ cm ...(i)

$\therefore$ $PR = \frac{1}{2}PQ$ [Perpendicular drawn from the centre of a circle to the chord bisects the chord.]

$= \frac{1}{2} \times 24 = 12$ cm ...(ii)

ii. In ΔORP, $\angle$ORP = 90°

$\therefore$ $OP^2 = OR^2 + PR^2$ [Pythagoras theorem]

$\therefore$ $13^2 = OR^2 + 12^2$ [From (i) and (ii)]

$\therefore$ $169 = OR^2 + 144$

$\therefore$ $OR^2 = 169 - 144$

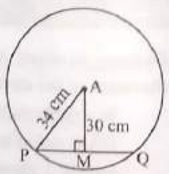
$\therefore$ $OR^2 = 25$

$\therefore$ $OR = \sqrt{25}$ [Taking square root on both sides]

$= 5$ cm

$\therefore$ **The distance of the chord from the centre of the circle is 5 cm.**

3. Radius of a circle is 34 cm and the distance of the chord from the centre is 30 cm, find the length of the chord. **[3 Marks]**



Given: In a circle with centre A,
PA is radius and PQ is chord,
seg AM ⊥ chord PQ, P–M–Q
AP = 34 cm, AM = 30 cm

To Find: Length of the chord (PQ)

Solution:

i. In ΔAMP, $\angle$AMP = 90°

$\therefore$ $AP^2 = AM^2 + PM^2$ [Pythagoras theorem]

$\therefore$ $34^2 = 30^2 + PM^2$

$\therefore$ $PM^2 = 34^2 - 30^2$

$\therefore$ $PM^2 = (34 - 30)(34 + 30)$ [$a^2 - b^2 = (a - b)(a + b)$]

$= 4 \times 64$

$\therefore$ $PM = \sqrt{4 \times 64}$ [Taking square root on both sides]

$= 2 \times 8 = 16$ cm ...(i)

ii. Now, $PM = \frac{1}{2}(PQ)$ [Perpendicular drawn from the centre of a circle to the chord bisects the chord.]

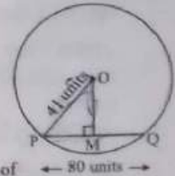
$\therefore$ $16 = \frac{1}{2}(PQ)$ [From (i)]

$\therefore$ $PQ = 16 \times 2$

$= 32$ cm

$\therefore$ **The length of the chord of the circle is 32 cm.**

4. Radius of a circle with centre O is 41 units. Length of a chord PQ is 80 units, find the distance of the chord from the centre of the circle. **[3 Marks]**



Given: In a circle with centre O,
OP is radius and PQ is its chord.
seg OM ⊥ chord PQ, P–M–Q
OP = 41 units, PQ = 80 units,

To Find: Distance of the chord from the centre of the circle(OM)

Solution:

i. $PM = \frac{1}{2}$ (PQ) [Perpendicular drawn from the centre of a circle to the chord bisects the chord.]

$= \frac{1}{2}(80) = 40$ units ...(i)

ii. In ΔOMP, $\angle$OMP = 90°

$\therefore$ $OP^2 = OM^2 + PM^2$ [Pythagoras theorem]

$\therefore$ $41^2 = OM^2 + 40^2$ [From (i)]

$\therefore$ $OM^2 = 41^2 - 40^2$

$= (41 - 40)(41 + 40)$ [$a^2 - b^2 = (a - b)(a + b)$]

$= (1)(81)$

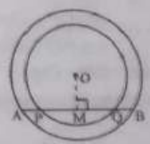
$\therefore$ $OM^2 = 81$

$\therefore$ $OM = \sqrt{81}$ [Taking square root on both sides]

$= 9$ units

$\therefore$ **The distance of the chord from the centre of the circle is 9 units.**

5. In the adjoining figure, centre of two circles is O. Chord AB of bigger circle intersects the smaller circle in points P and Q. Show that AP = BQ. **[3 Marks]**



Given: Two concentric circles having centre O.

To prove: AP = BQ

Construction: Draw seg OM ⊥ chord AB, A–M–B

Proof:

For smaller circle,

seg OM ⊥ chord PQ [Construction, A–P–M, M–Q–B]

$\therefore$ PM = MQ ...(i) [Perpendicular drawn from the centre of the circle to the chord bisects the chord.]

For bigger circle,

seg OM ⊥ chord AB [Construction]

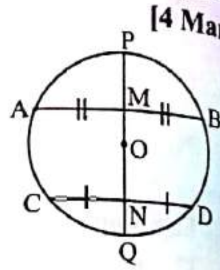
$\therefore$ AM = MB [Perpendicular drawn from the centre of the circle to the chord bisects the chord.]

$\therefore$ AP + PM = MQ + QB [A–P–M, M–Q–B]

$\therefore$ AP + MQ = MQ + QB [From (i)]

$\therefore$ **AP = BQ**

6. **Prove that, if a diameter of a circle bisects two chords of the circle then those two chords are parallel to each other.** **[4 Marks]**



Given: O is the centre of the circle.
seg PQ is the diameter.
Diameter PQ bisects the chords AB and CD in points M and N respectively.

To prove: chord AB || chord CD.

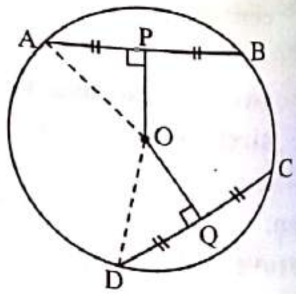
Proof:

Diameter PQ bisects the chord AB in point M		[Given]
∴ seg AM ≅ seg BM		
∴ seg OM ⊥ chord AB		[Segment joining the centre of a circle and the midpoint of its chord is perpendicular to the chord, P–M–O, O–N–Q]
∴ $\angle OMA = 90°$	(i)	
Also, diameter PQ bisects the chord CD in point N		[Given]
∴ seg CN ≅ seg DN		
seg ON ⊥ chord CD		[Segment joining the centre of a circle and the midpoint of its chord is perpendicular to the chord, P–M–O, O–N–Q]
∴ $\angle ONC = 90°$	(ii)	
Now, $\angle OMA + \angle ONC = 90° + 90°$ $= 180°$		[From (i) and (ii)]
But, ∠OMA and ∠ONC form a pair of interior angles on lines AB and CD when seg MN is their transversal.		
∴ **chord AB || chord CD**		[Interior angles test]

Let's Study

Properties of congruent chords

Theorem: Congruent chords of a circle are equidistant from the centre of the circle.



Given: Point O is the centre of the circle.
chord AB ≅ chord CD
seg OP ⊥ chord AB, A–P–B
seg OQ ⊥ chord CD, D–Q–C

To prove: OP = OQ

Construction: Draw seg OA and seg OD.

Proof:

seg OP ⊥ chord AB and seg OQ ⊥ chord CD		[Given]
∴ $AP = \frac{1}{2} AB$	(i)	[Perpendicular drawn from the centre of the circle to the chord bisects the chord]
$DQ = \frac{1}{2} CD$	(ii)	
AB = CD	(iii)	[Given]
∴ AP = DQ		[From (i), (ii) and (iii)]
i.e. seg AP ≅ seg DQ	(iv)	[Segments of equal lengths]
In ΔAPO and ΔDQO,		
∠APO ≅ ∠DQO		[Each is of 90°]
seg AP ≅ seg DQ		[From (iv)]

$\therefore \quad XU^2 = 37^2 - 12^2$

$= (37 + 12)(37 - 12)$

$= 49 \times 25$ [$a^2 - b^2 = (a + b)(a - b)$]

$\therefore \quad XU = \sqrt{49 \times 25}$ [Taking square root on both sides]

$\therefore \quad XU = 7 \times 5 = 35$ cm ... (ii)

iii. Now, $XU = \frac{1}{2} UV$ [Perpendicular drawn from the centre of the circle to the chord bisects the chord]

$\therefore \quad 35 = \frac{1}{2} UV$ [From (ii)]

$\therefore \quad UV = 35 \times 2$

$= 70$ cm

$\therefore \quad ST = UV = 70$ cm [chord ST $\cong$ chord UV]

$\therefore$ **The lengths of the two chords are 70 cm each.**

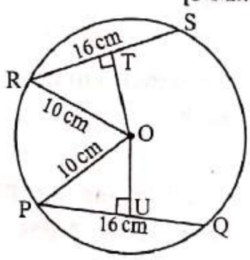
Remember This

i. Congruent chords of a circle are equidistant from the centre of the circle.
ii. The chords of a circle equidistant from the centre of a circle are congruent.
iii. Congruent chords in congruent circles are equidistant from their respective centres.
iv. Chords of congruent circles which are equidistant from their respective centres are congruent.

Practice Set 6.2

1. **Radius of circle is 10 cm. There are two chords of length 16 cm each. What will be the distance of these chords from the centre of the circle ?** **[3 Marks]**

Given: In a circle with centre O,
OR and OP are radii and RS and PQ are its congruent chords.
PQ = RS = 16 cm,
OR = OP = 10 cm
seg OU $\perp$ chord PQ, P–U–Q
seg OT $\perp$ chord RS, R–T–S



To find: Distance of chords from centre of the circle.

Solution:

i. $PU = \frac{1}{2}(PQ)$ [Perpendicular drawn from the centre of the circle to the chord bisects the chord.]

$\therefore \quad PU = \frac{1}{2} \times 16 = 8$ cm ... (i)

ii. In ΔOUP, $\angle OUP = 90°$

$\therefore \quad OP^2 = OU^2 + PU^2$ [Pythagoras theorem]

$\therefore \quad 10^2 = OU^2 + 8^2$ [From (i)]

$\therefore \quad 100 = OU^2 + 64$

$\therefore \quad OU^2 = 100 - 64$

$= 36$

$\therefore \quad OU = \sqrt{36}$ [Taking square root on both sides]

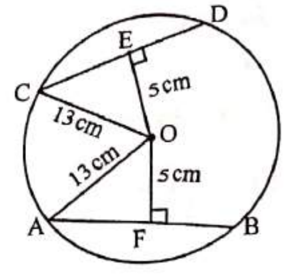
$\therefore \quad OU = 6$ cm

iii. Now, $OT = OU$ [Congruent chords of a circle are equidistant from the centre.]

$\therefore \quad OT = OU = 6$ cm

$\therefore$ **The distance of the chords from the centre of the circle is 6 cm.**

2. **In a circle with radius 13 cm, two equal chords are at a distance of 5 cm from the centre. Find the lengths of the chords.** **[3 Marks]**



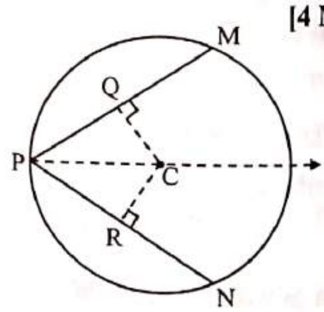
Given: In a circle with cente O,
OA and OC are the radii and
AB and CD are its congruent chords.
OA = OC = 13 cm
OE = OF = 5 cm
seg OE ⊥ chord CD, C–E–D
seg OF ⊥ chord AB, A–F–B

To find: length of the chords

***Solution*:**

i.	In ΔAFO, $\angle AFO = 90°$		
∴	$AO^2 = AF^2 + FO^2$		[Pythagoras theorem]
∴	$13^2 = AF^2 + 5^2$		
∴	$169 = AF^2 + 25$		
∴	$AF^2 = 169 - 25$		
∴	$AF^2 = 144$		
∴	$AF = \sqrt{144}$		[Taking square root on both sides]
∴	$AF = 12$ cm	(i)	
ii.	Now, $AF = \frac{1}{2}$ AB		[Perpendicular drawn from the centre of the circle to the chord bisects the chord.]
∴	$12 = \frac{1}{2}$ (AB)		[From (i)]
∴	$AB = 12 \times 2 = 24$ cm		
∴	CD = AB = 24 cm		[chord AB ≅ chord CD]
∴	**The lengths of the two chords are 24 cm each.**		

3. **Seg PM and seg PN are congruent chords of a circle with centre C. Show that the ray PC is the bisector of ∠NPM.** **[4 Marks]**



Given: Point C is the centre of the circle.
chord PM ≅ chord PN

To prove: Ray PC is the bisector of ∠NPM.

Construction: Draw seg CR ⊥ chord PN, P–R–N
seg CQ ⊥ chord PM, P–Q–M

***Proof*:**

	chord PM ≅ chord PN		[Given]
	seg CR ⊥ chord PN seg CQ ⊥ chord PM		[Construction]
∴	seg CR ≅ seg CQ	(i)	[Congruent chords are equidistant from the centre]
	In ΔPRC and ΔPQC,		
	∠PRC ≅ ∠PQC		[Each is of 90°]
	seg CR ≅ seg CQ		[From (i)]
	seg PC ≅ seg PC		[Common side]
∴	ΔPRC ≅ ΔPQC		[Hypotenuse side test]
∴	∠RPC ≅ ∠QPC		[c. a. c. t.]
∴	∠NPC ≅ ∠MPC		[N – R – P, M – Q – P]
∴	**Ray PC is the bisector of ∠NPM.**		

Remember This

i. To construct the incircle of a triangle, draw the bisectors of any two angles of the triangle.

ii. To construct the circumcircle of a triangle, draw the perpendicular bisectors of any two sides of the triangle.

iii. The incentre of any triangle lies in the interior of the triangle.

iv. Position of circumcentre of different triangles:

Type of triangle	Acute-angled triangle	Right-angled triangle	Obtuse-angled triangle
Position of circumcentre	**Interior** of the triangle	**On the hypotenuse**	**Exterior** of the triangle

v. The perpendicular bisectors and angle bisectors of an equilateral triangle are coincedent.

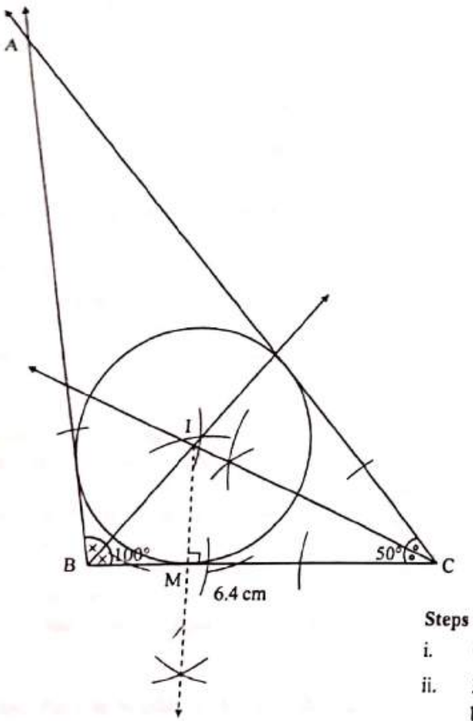
vi. The incentre and circumcentre of an equilateral triangle coincide.

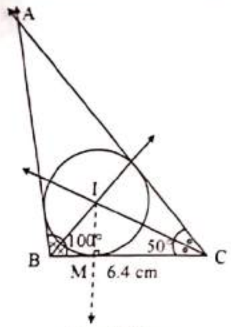
vii. The ratio of the radii of circumcircle and incircle of an equilateral triangle is 2 : 1.

Practice Set 6.3

1. Construct ΔABC such that $\angle B = 100°$, BC = 6.4 cm, $\angle C = 50°$ and construct its incircle. **[3 Marks]**

Ans:





Rough Figure

Steps of construction:

i. Construct ΔABC of the given measurement

ii. Draw the bisectors of $\angle B$ and $\angle C$. Let these bisectors intersect at point I.

iii. Draw a perpendicular IM on side BC. Point M is the foot of the perpendicular.

iv. With I as centre and IM as radius, draw a circle which touches all the three sides of the triangle.

2. Construct ΔPQR such that $\angle P = 70°$, $\angle R = 50°$, QR = 7.3 cm and construct its circumcircle. **[3 Marks]**

Solution:

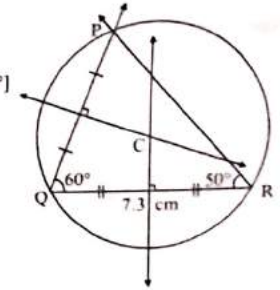
In ΔPQR,

$m\angle P + m\angle Q + m\angle R = 180°$ ...[Sum of the measures of the angles of a triangle is 180°]

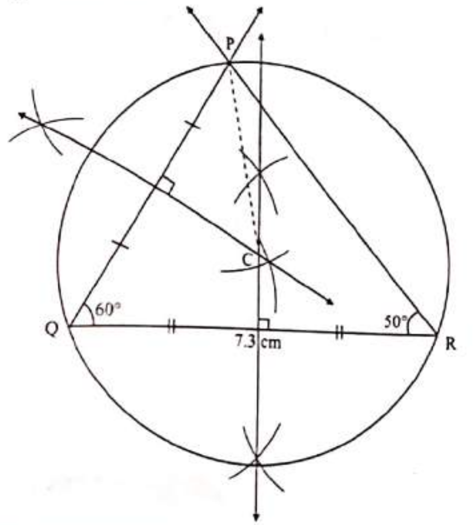
$\therefore 70° + m\angle Q + 50° = 180°$

$\therefore m\angle Q = 180° - 70° - 50°$

$\therefore m\angle Q = 60°$



Rough Figure



Steps of construction:

i. Construct Δ PQR of the given measurement.

ii. Draw the perpendicular bisectors of side PQ and side QR of the triangle.

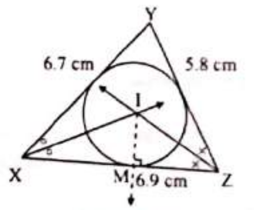
iii. Name the point of intersection of the perpendicular bisectors as point C.

iv. Join seg CP

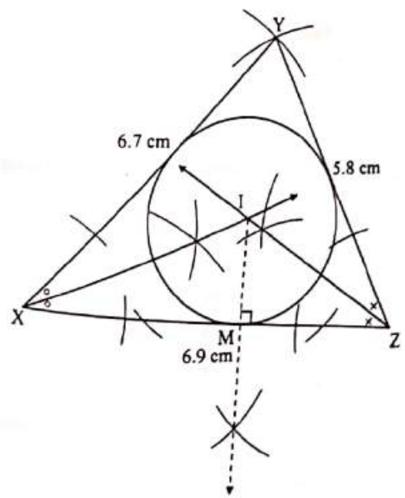
v. With C as centre and CP as radius, draw a circle which passes through the three vertices of the triangle.

3. Construct ΔXYZ such that XY = 6.7 cm, YZ = 5.8 cm, XZ = 6.9 cm. Construct its incircle. **[3 Marks]**

Ans:



Rough Figure



Steps of construction:

i. Construct ΔXYZ of the given measurement

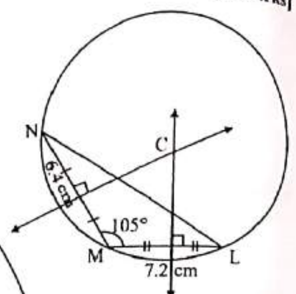
ii. Draw the bisectors of $\angle X$ and $\angle Z$. Let these bisectors intersect at point I.

iii. Draw a perpendicular IM on side XZ. Point M is the foot of the perpendicular.

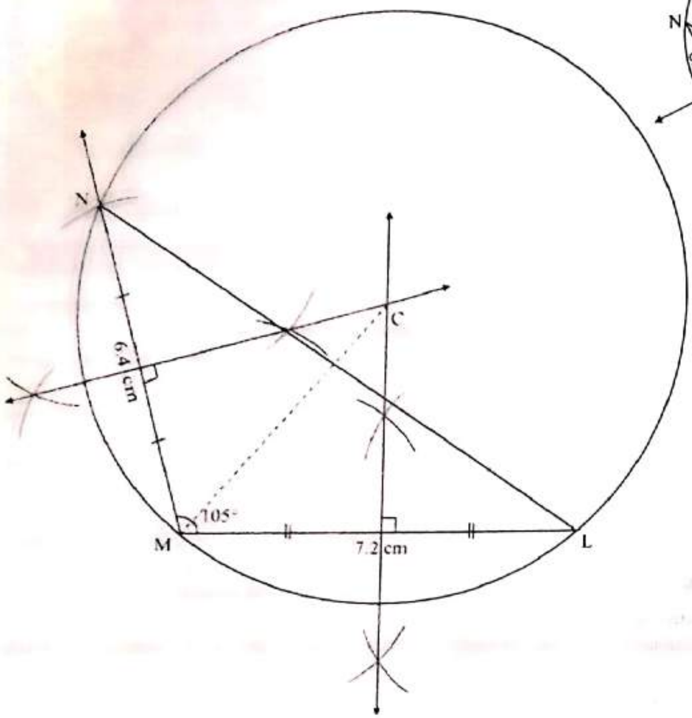
iv. With I as centre and IM as radius, draw a circle which touches all the three sides of the triangle.

4. **In ΔLMN, LM = 7.2 cm, ∠M = 105°, MN = 6.4 cm, then draw ΔLMN and construct its circumcircle.** **[3 Marks]**

Ans:



Rough Figure

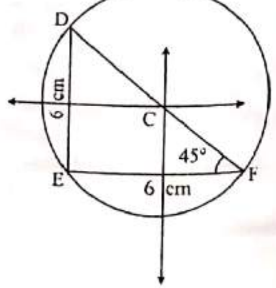


Steps of construction:

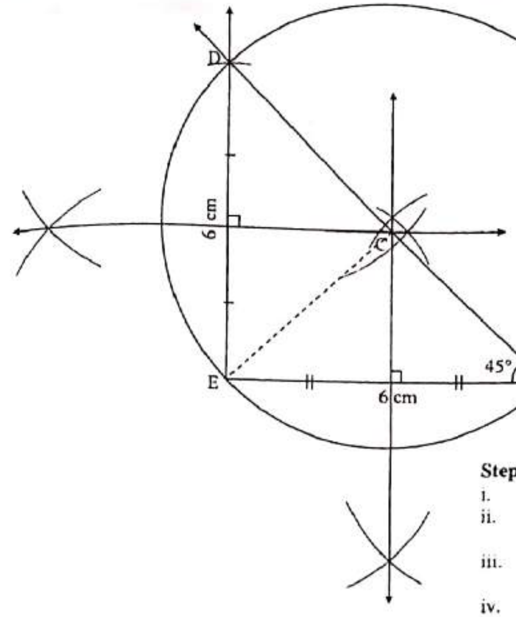
i. Construct Δ LMN of the given measurement.
ii. Draw the perpendicular bisectors of side MN and side ML of the triangle.
iii. Name the point of intersection of the perpendicular bisectors as point C.
iv. Join seg CM
v. With C as centre and CM as radius, draw a circle which passes through the three vertices of the triangle.

5. **Construct ΔDEF such that DE = EF = 6 cm, ∠F = 45° and construct its circumcircle.** **[3 Marks]**

Ans:



Rough Figure



Steps of construction:

i. Construct Δ DEF of the given measurement.
ii. Draw the perpendicular bisectors of side DE and side EF of the triangle.
iii. Name the point of intersection of the perpendicular bisectors as point C.
iv. Join seg CE
v. With C as centre and CE as radius, draw a circle which passes through the three vertices of the triangle.

Problem Set – 6

I. Choose correct alternative answer and fill in the blanks. **[1 Mark each]**

i. Radius of a circle is 10 cm and distance of a chord from the centre is 6 cm. Hence, the length of the chord is ______.
(A) 16 cm (B) 8 cm (C) 12 cm (D) 32 cm

ii. The point of concurrence of all angle bisectors of a triangle is called the ______.
(A) centroid (B) circumcentre (C) incentre (D) orthocentre

iii. The circle which passes through all the vertices of a triangle is called ______.
(A) circumcircle (B) incircle (C) congruent circle (D) concentric circle

iv. Length of a chord of a circle is 24 cm. If distance of the chord from the centre is 5 cm, then the radius of that circle is______.
(A) 12 cm (B) 13 cm (C) 14 cm (D) 15 cm

v. The length of the longest chord of the circle with radius 2.9 cm is ______.
(A) 3.5 cm (B) 7 cm (C) 10 cm (D) 5.8 cm

vi. Radius of a circle with centre O is 4 cm. If l(OP) = 4.2 cm, say where point P will lie.
(A) on the centre (B) inside the circle (C) outside the circle (D) on the circle

vii. The lengths of parallel chords which are on opposite sides of the centre of a circle are 6 cm and 8 cm. If radius of the circle is 5 cm, then the distance between these chords is ______.
(A) 2 cm (B) 1 cm (C) 8 cm (D) 7 cm

Answers:

i. (A)	ii. (C)	iii. (A)	iv. (B)
v. (D)	vi. (C)	vii. (D)	

7 Co-ordinate Geometry

Type of Problems	Practice Set	Q. Nos.
Axes, origin, quadrants, the co-ordinates of a point in a plane	7.1	Q.1, 2
	Practice Problems (Based on Practice Set 7.1)	Q.1, 2, 4
	Practice Problems (Based on Practice Set 7.2)	Q.7
	Problem Set- 7	Q.2, 3, 5
To plot the points on the graph paper	7.1	Q.3
	Practice Problems (Based on Practice Set 7.1)	Q.3
	7.2	Q.1, 7
	Practice Problems (Based on Practice Set 7.2)	Q.1, 9
	Problem Set- 7	Q.4
Lines parallel to the X-axis and the Y-axis	7.2	Q.2, 3, 4, 5, 6
	Practice Problems (Based on Practice Set 7.2)	Q.2, 3, 4, 5, 6, 8
	Problem Set- 7	Q.6, 7
Graph of linear equations	7.2	Q.8, 9
	Practice Problems (Based on Practice Set 7.2)	Q.10, 11, 12

Let's Study

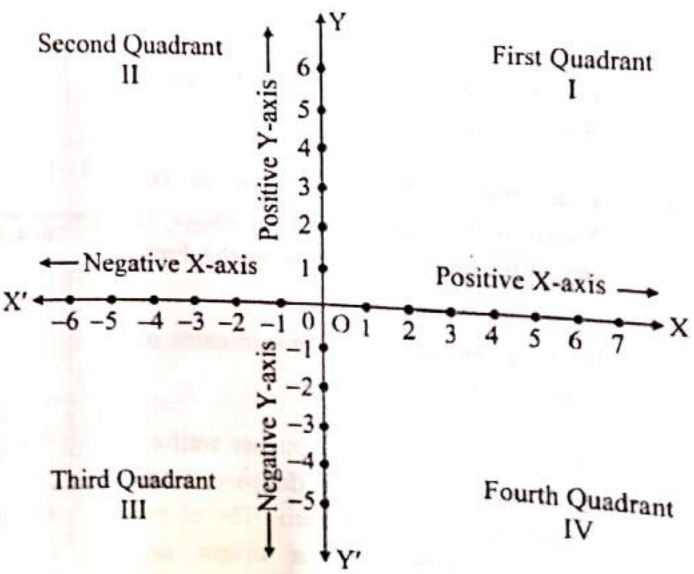
Axes, Origin, Quadrants

1. **Co-ordinate Geometry:**
It is a branch of geometry which sets up a definite correspondence between the position of a point in a plane and a pair of real numbers called co-ordinates.

2. **Axes:**
   i. **X-axis:** To locate a point in a plane, a horizontal number line is drawn in the plane. This number line is called the X-axis.
   ii. **Y-axis:** The number line intersecting the X-axis at point marked O and perpendicular to the X-axis, is called the Y-axis.

3. **Origin:** The point of intersection of X-axis and Y-axis is called the origin and is denoted by O.
On the X-axis, positive numbers are shown on the right of O and negative numbers on the left of O.
On the Y-axis, positive numbers are shown above O and negative numbers below O.

As shown in the figure below, the quadrants are numbered in the anticlockwise direction.



4. **Quadrants:** The X and Y axes divide the plane into four parts, each of which is called a quadrant.

Practice Set 7.1

1. State in which quadrant or on which axis do the following points lie.

i. A(–3, 2)   ii. B(–5, –2)
iii. K(3.5, 1.5)   iv. D(2, 10)
v. E(37, 35)   vi. F(15, –18)
vii. G(3, –7)   viii. H(0, –5)
ix. M(12, 0)   x. N(0, 9)
xi. P(0, 2.5)   xii. Q(–7, –3)

[½ Mark each]

Ans:

Sr. No.	Point	x co-ordinate	y co-ordinate	Quadrant/Axis
i.	A(–3, 2)	Negative	Positive	Quadrant II
ii.	B(–5, –2)	Negative	Negative	Quadrant III
iii.	K(3.5, 1.5)	Positive	Positive	Quadrant I
iv.	D(2, 10)	Positive	Positive	Quadrant I
v.	E(37, 35)	Positive	Positive	Quadrant I
vi.	F(15, –18)	Positive	Negative	Quadrant IV
vii.	G(3, –7)	Positive	Negative	Quadrant IV
viii.	H(0, –5)	0	Negative	Y – axis
ix.	M(12, 0)	Positive	0	X – axis
x.	N(0, 9)	0	Positive	Y – axis
xi.	P(0, 2.5)	0	Positive	Y – axis
xii.	Q(–7, –3)	Negative	Negative	Quadrant III

2. In which quadrant are the following points?

i. whose both co-ordinates are positive.
ii. whose both co-ordinates are negative.
iii. whose x co-ordinate is positive and the y co-ordinate is negative.
iv. whose x co-ordinate is negative and y co-ordinate is positive.

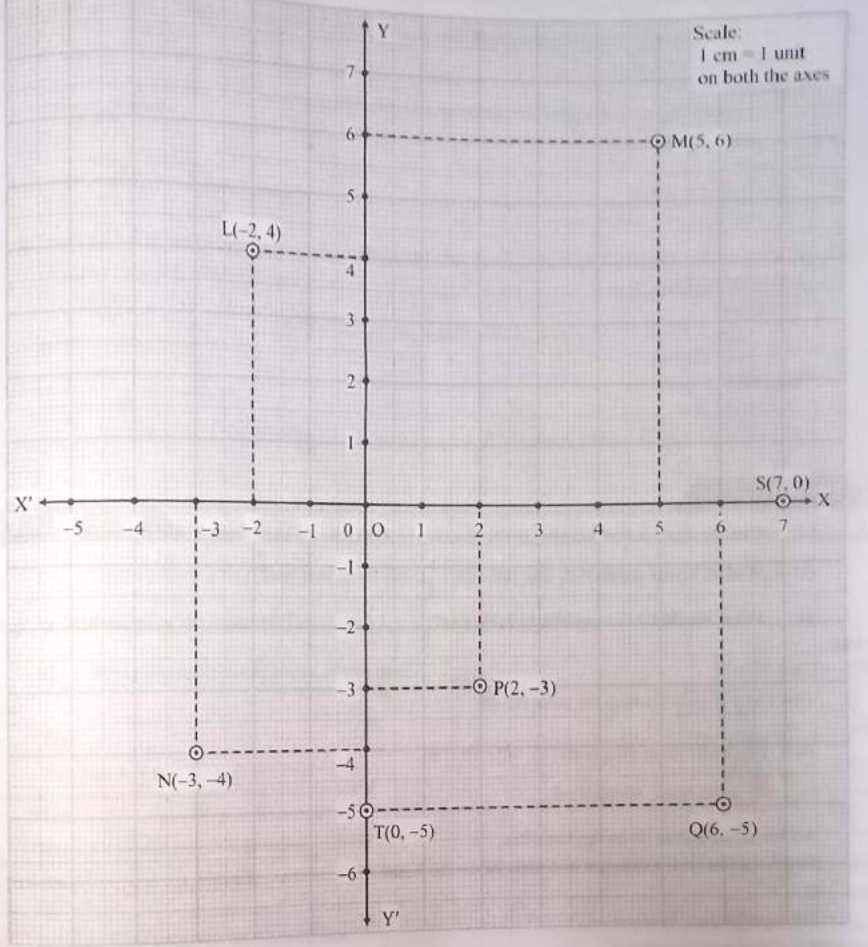
[2 Marks]

Ans: i. Quadrant I
ii. Quadrant III
iii. Quadrant IV
iv. Quadrant II

3. Draw the co-ordinate system on a plane and plot the following points.
L(–2, 4), M(5, 6), N(–3, –4), P(2, –3), Q(6, –5), S(7, 0), T(0, –5)

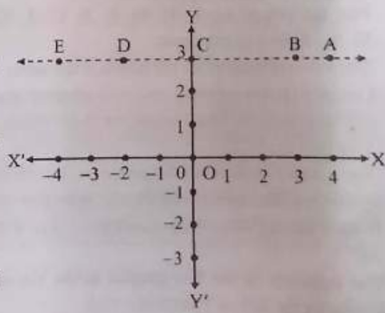
[3 Marks]

Ans:



Let's Study

Lines parallel to the X-axis



Plot the points A(4, 3), B(3, 3), C(0, 3), D(–2, 3) and E(–4, 3) on a graph paper.
The y co-ordinate of all the points is the same.
On joining the points, you will observe that the points are collinear and the line formed by these points is parallel to the X-axis.
The y co-ordinate of all the points on line AE is 3.
Therefore, the equation of line AE is $y = 3$.
If the y co-ordinate of any point is 3, then it will be on the line AE.
The equation of the line parallel to the X-axis at a distance of 3 units above the X-axis is $y = 3$.

Try This

1. **On a graph paper, plot the points (0, 1), (1, 3), (2, 5). Are they collinear? If so, draw the line that passes through them.**

i. Through which quadrants does this line pass ?

ii. Write the co-ordinates of the point at which it intersects the Y-axis.

iii. Show any point in the third quadrant which lies on this line. Write the co-ordinates of the point.

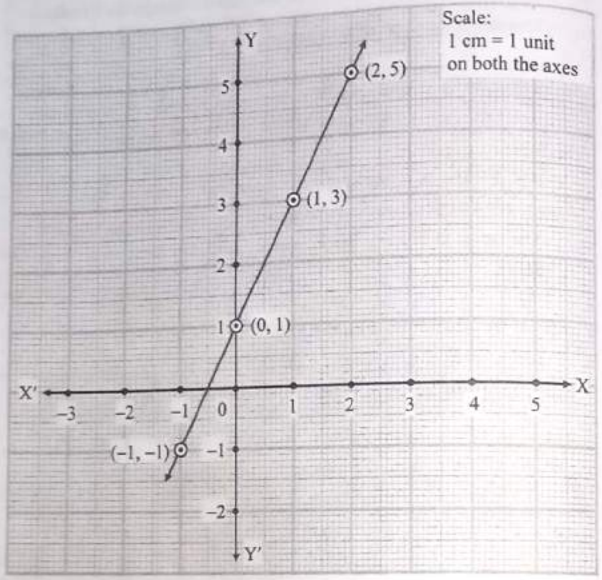
(Textbook pg. no. 96)

Ans:

i. The line passes through the quadrants I, II and III.

ii. The line intersects the Y-axis at (0, 1).

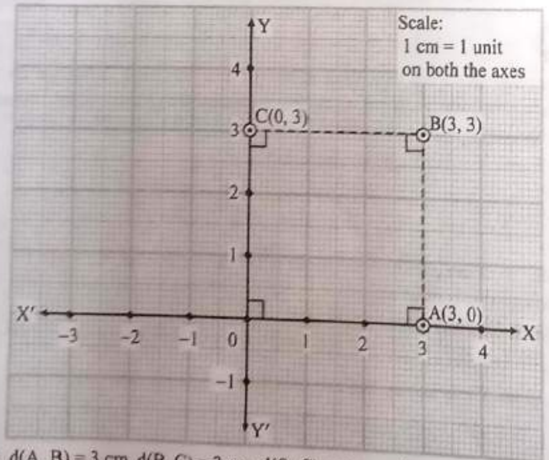
iii. (–1, –1)



Practice Set 7.2

1. **On a graph paper plot the points A(3, 0), B(3, 3), C(0, 3). Join A, B and B, C. What is the figure formed?** **[4 Marks]**

Solution:



d(O, A) = 3 cm, d(A, B) = 3 cm, d(B, C) = 3 cm, d(O, C) = 3 cm and each angle of □OABC is 90°.

∴ **□OABC is a square.**

2. **Write the equation of the line parallel to the Y-axis at a distance of 7 units from it to its left.** **[1 Mark]**

Solution:

The equation of a line parallel to the Y-axis is x = a.

Since, the line is at a distance of 7 units to the left of Y-axis.

∴ a = –7

∴ **x = –7 is the equation of the required line.**

3. **Write the equation of the line parallel to the X-axis at a distance of 5 units from it and below the X-axis.** **[1 Mark]**

Solution:

The equation of a line parallel to the X-axis is y = b.

Since, the line is at a distance of 5 units below the X-axis.

∴ b = –5

∴ **y = –5 is the equation of the required line.**

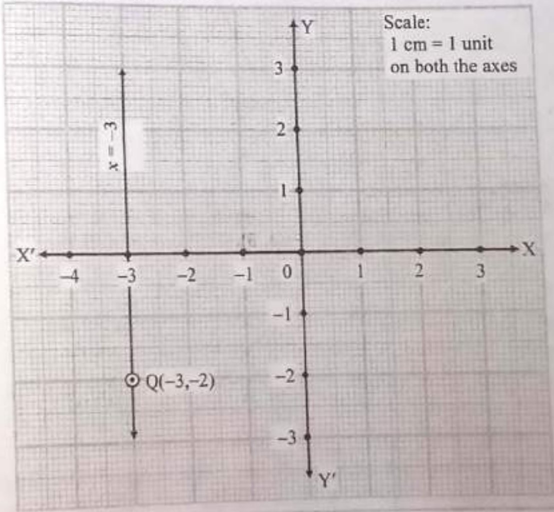
4. **The point Q(–3, –2) lies on a line parallel to the Y-axis. Write the equation of the line and draw its graph.** **[2 Marks]**

Solution:

The equation of a line parallel to the Y-axis is x = a.

Here, a = –3

∴ **x = –3 is the equation of the required line.**



5. **Y-axis and line x = – 4 are parallel lines. What is the distance between them?** **[2 Marks]**

Solution:

Equation of Y-axis is x = 0.

Equation of the line parallel to the Y-axis is x = – 4. ...[Given]

∴ Distance between the Y-axis and the line x = – 4 is 0 – (– 4) ...[0 > –4]

= 0 + 4

= 4 units

∴ **The distance between the Y-axis and the line x = – 4 is 4 units.**

[**Note:** *The question is modified as X-axis cannot be parallel to the line x = – 4.*]

6. **Which of the equations given below have graphs parallel to the X-axis, and which ones have graphs parallel to the Y-axis?** **[1 Mark each]**

i. $x = 3$ ii. $y - 2 = 0$ iii. $x + 6 = 0$ iv. $y = -5$

Ans:

i. The equation of a line parallel to the Y-axis is $x = a$.

$\therefore$ **The line $x = 3$ is parallel to the Y-axis.**

ii. $y - 2 = 0$

$\therefore$ $y = 2$

The equation of a line parallel to the X-axis is $y = b$.

$\therefore$ **The line $y - 2 = 0$ is parallel to the X-axis.**

iii. $x + 6 = 0$

$\therefore$ $x = -6$

The equation of a line parallel to the Y-axis is $x = a$.

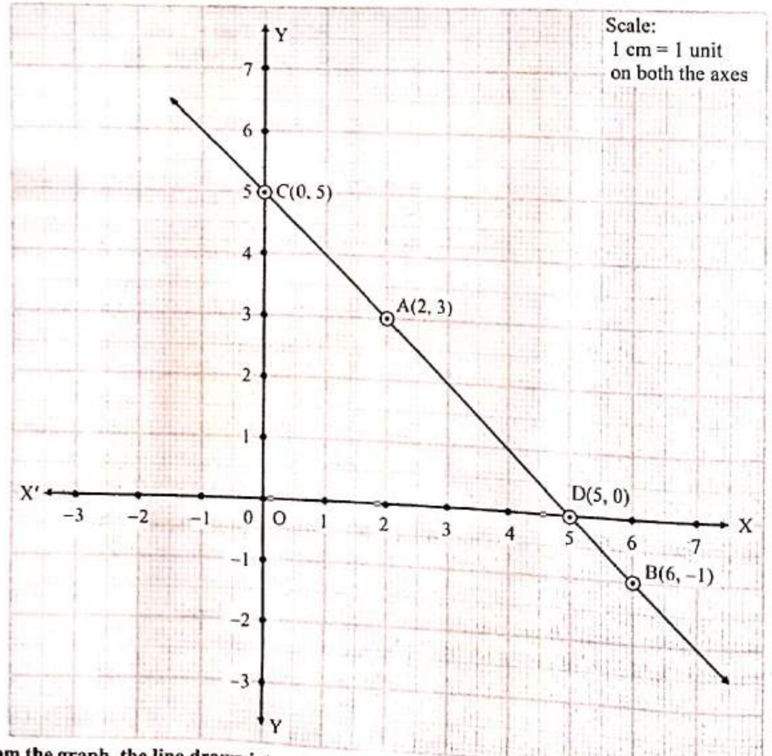
$\therefore$ **The line $x + 6 = 0$ is parallel to the Y-axis.**

iv. The equation of a line parallel to the X-axis is $y = b$.

$\therefore$ **The line $y = -5$ is parallel to the X-axis.**

7. **On a graph paper, plot the points A(2, 3), B(6, –1) and C(0, 5). If these points are collinear, then draw the line which includes them. Write the co-ordinates of the points at which the line intersects the X-axis and the Y-axis.** **[3 Marks]**

Solution:



From the graph, the line drawn intersects the X-axis at D(5, 0) and the Y-axis at C(0, 5).

8. **Draw the graphs of the following equations on the same system of co-ordinates. Write the co-ordinates of their points of intersection.**

$x + 4 = 0, y - 1 = 0, 2x + 3 = 0, 3y - 15 = 0$ **[4 Marks]**

Solution:

i. $x + 4 = 0$

$\therefore$ $x = -4$

ii. $y - 1 = 0$

$\therefore$ $y = 1$

iii. $2x + 3 = 0$

$\therefore$ $2x = -3$

$\therefore$ $x = \frac{-3}{2}$

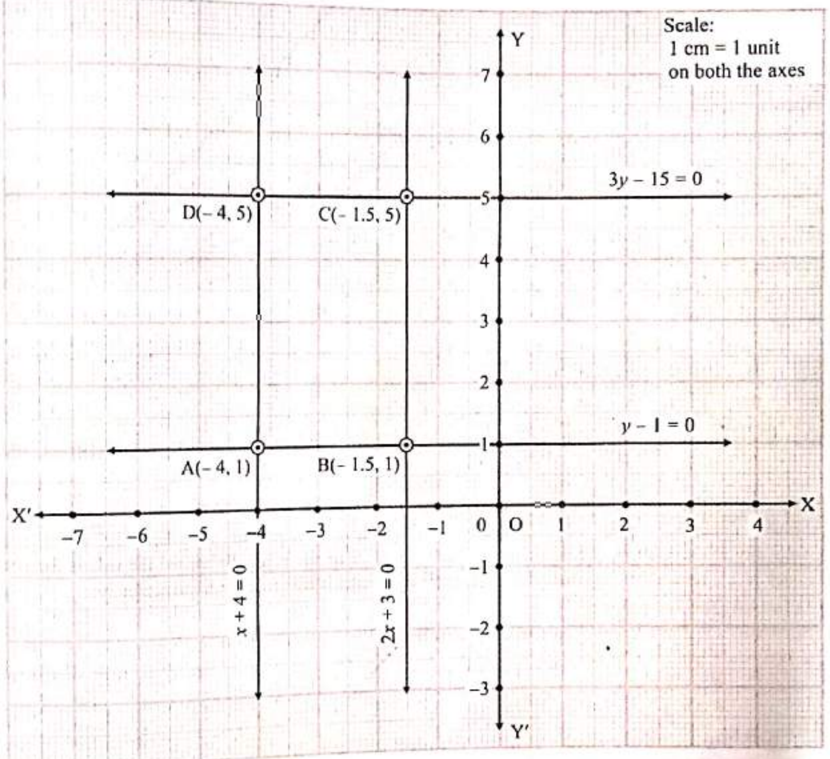
$\therefore$ $x = -1.5$

iv. $3y - 15 = 0$

$\therefore$ $3y = 15$

$\therefore$ $y = \frac{15}{3}$

$\therefore$ $y = 5$



The co-ordinates of the point of intersection of $x + 4 = 0$ and $y - 1 = 0$ are **A(–4, 1).**

The co-ordinates of the point of intersection of $y - 1 = 0$ and $2x + 3 = 0$ are **B(–1.5, 1).**

The co-ordinates of the point of intersection of $3y - 15 = 0$ and $2x + 3 = 0$ are **C(–1.5, 5).**

The co-ordinates of the point of intersection of $x + 4 = 0$ and $3y - 15 = 0$ are **D(–4, 5).**

9. Draw the graphs of the equations given below.

i. $x + y = 2$

ii. $3x - y = 0$

iii. $2x + y = 1$

[3 Marks each]

Solution:

i. $x + y = 2$

$\therefore$ $y = 2 - x$

When $x = 0$,

$y = 2 - x$

$= 2 - 0$

$= 2$

When $x = 1$,

$y = 2 - x$

$= 2 - 1$

$= 1$

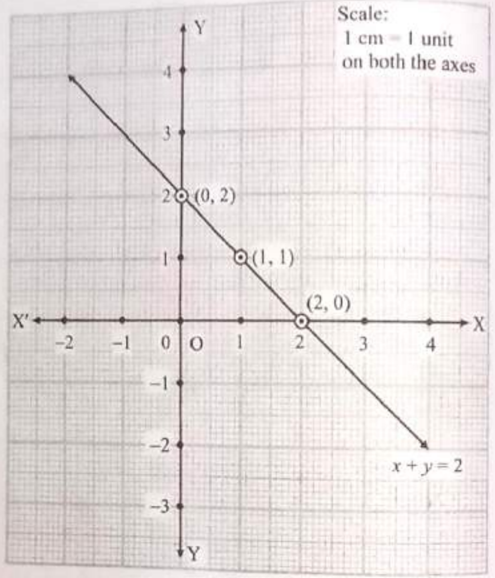
When $x = 2$,

$y = 2 - x$

$= 2 - 2$

$= 0$

x	0	1	2
y	2	1	0
(x, y)	(0, 2)	(1, 1)	(2, 0)



Smart Check

Take any other point on the line with equation $x + y = 2$ on the graph. Substitute the values for x and y to check if it satisfies the equation. If it does, the line represents the given equation correctly.

Let us take the point (3, – 1).

Substitute $x = 3$ and $y = -1$ in the given equation.

L.H.S. $= x + y$

$= 3 - 1$

$= 2$

$=$ R.H.S.

Thus, the given equation is satisfied.

$\therefore$ The line represents the given equation correctly.

ii. $3x - y = 0$

$\therefore$ $y = 3x$

When $x = 0$,

$y = 3x$

$= 3(0)$

$= 0$

When $x = 1$,

$y = 3x$

$= 3(1)$

$= 3$

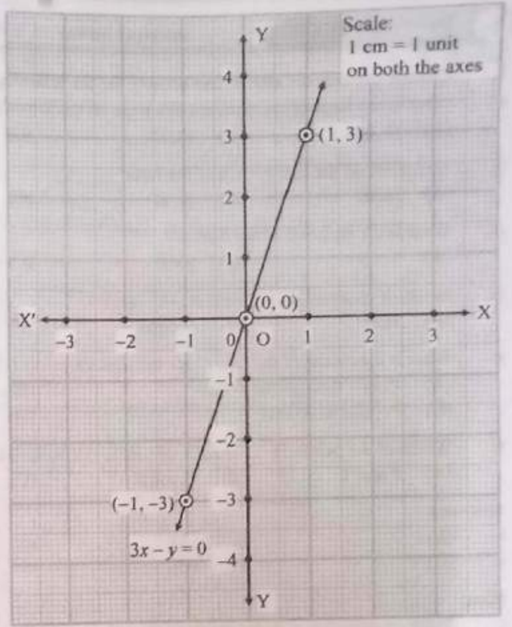
When $x = -1$,

$y = 3x$

$= 3(-1)$

$= -3$

x	0	1	–1
y	0	3	–3
(x, y)	(0, 0)	(1, 3)	(–1, –3)



iii. $2x + y = 1$

$\therefore$ $y = 1 - 2x$

When $x = 0$,

$y = 1 - 2x$

$= 1 - 2(0)$

$= 1 - 0$

$= 1$

When $x = 1$,

$y = 1 - 2x$

$= 1 - 2(1)$

$= 1 - 2$

$= -1$

When $x = -1$,

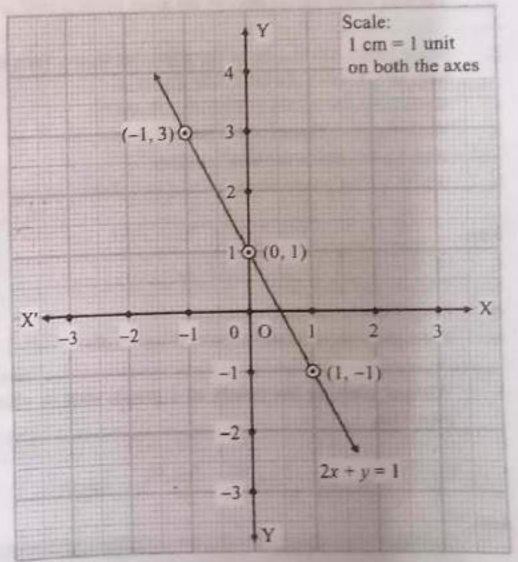
$y = 1 - 2x$

$= 1 - 2(-1)$

$= 1 + 2$

$= 3$

x	0	1	–1
y	1	–1	3
(x, y)	(0, 1)	(1, –1)	(–1, 3)



8 Trigonometry

Type of Problems	Practice Set	Q. Nos.
Trigonometric Ratios	8.1	Q.1, 2, 3, 4
	Practice Problems (Based on Practice Set 8.1)	Q.1, 2, 3
	8.2	Q.1, 3, 4
	Practice Problems (Based on Practice Set 8.2)	Q.1, 2, 5
Relations among trigonometric ratios	Problem Set- 8	Q.2, 3
	Practice Problems (Based on Practice Set 8.2)	Q.3, 4, 7, 8
Trigonometric Ratios of 0°, 30°, 45°, 60° and 90° angles	Problem Set- 8	Q.4, 5
	8.2	Q.2
	Practice Problems (Based on Practice Set 8.2)	Q.6

Let's Study

Introduction to Trigonometry

The word 'Trigonometry' is derived from the Greek words **Tri** meaning **three, gona** meaning **sides** and **metron** meaning **measure.**

Thus, Trigonometry means measurement of a triangle i.e., it is the study of relation between sides and angles of a triangle.

Trigonometry has great importance in the field of Navigation, Aerospace, engineering and other fields of science.

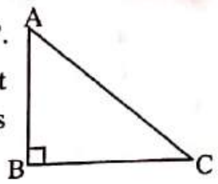
Let's Recall

The subject trigonometry starts with right-angled triangle, theorem of Pythagoras and similar triangles.

1. **Right-angled triangle:**

   A triangle with one right angle is called a right-angled triangle and the side opposite to the right angle is called the hypotenuse.

   In the given figure, $\angle B = 90°$. Hence, ΔABC is a right angled triangle, and AC is its hypotenuse.



2. **Pythagoras theorem:**

   In a right-angled triangle, the square of the hypotenuse is equal to the sum of the squares of the other two sides.

   In the figure given above, ΔABC is right angled at B, side AC is the hypotenuse and sides AB and BC are the other two sides.

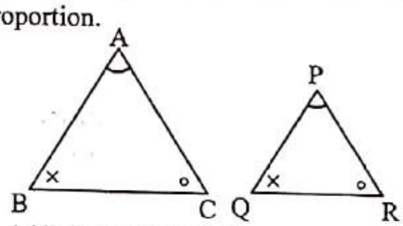
   $\therefore \quad AC^2 = AB^2 + BC^2$

3. **Similarity of Triangles:**

   Two triangles are said to be similar, if their

   i. corresponding angles are congruent and

   ii. corresponding sides are in the same proportion.



In ΔABC and ΔPQR, if

i. $\angle A \cong \angle P$, $\angle B \cong \angle Q$, $\angle C \cong \angle R$ and

ii. $\frac{AB}{PQ} = \frac{BC}{QR} = \frac{AC}{PR}$,

then $\Delta ABC \sim \Delta PQR$.

Let's Study

Terms related to right-angled triangle

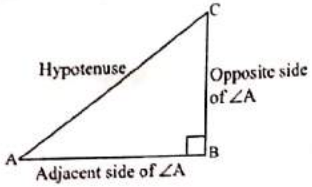
In a right angled triangle,

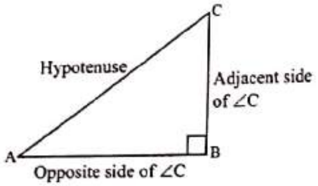
i. the side opposite to the right angle is called the **hypotenuse.**

ii. for any acute angle, the side opposite to it is called the **opposite side.**

iii. for any acute angle, the side adjacent to it other than the hypotenuse is called the **adjacent side.**

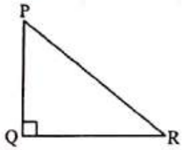
In right angled ΔABC, $\angle B = 90°$, $\angle A$ and $\angle C$ are acute angles.





Example:

In the figure given below, ΔPQR is a right angled triangle. Write the names of sides opposite and adjacent to $\angle P$ and $\angle R$.



(Textbook pg. no. 102)

Ans:

In right angled ΔPQR,

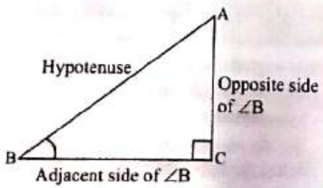
i. side opposite to $\angle P$ = **QR**

ii. side opposite to $\angle R$ = **PQ**

iii. side adjacent to $\angle P$ = **PQ**

iv. side adjacent to $\angle R$ = **QR**

Trigonometric ratios



In right angled ΔABC, $\angle C = 90°$. The trigonometric ratios of $\angle B$ can be defined as follows:

i. Sine ratio of $\angle B = \dfrac{\text{Opposite side of } \angle B}{\text{Hypotenuse}} = \dfrac{AC}{AB}$

ii. Cosine ratio of $\angle B = \dfrac{\text{Adjacent side of } \angle B}{\text{Hypotenuse}} = \dfrac{BC}{AB}$

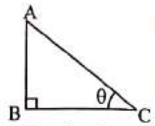
iii. Tangent ratio of $\angle B = \dfrac{\text{Opposite side of } \angle B}{\text{Adjacent side of } \angle B} = \dfrac{AC}{BC}$

The ratios defined above can be written as sin B, cos B and tan B respectively.

Note:

i. The measures of acute angles of a triangle can be written using Greek letters θ (theta), α (alpha), β (beta), etc.

ii. In the given figure, the measure of acute angle $\angle C$ of ΔABC is shown by the symbol θ.



Hence, we can write the ratios sin C, cos C and tan C as sin θ, cos θ and tan θ respectively.

Practice Set 8.1

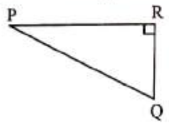
1. In the given figure, $\angle R$ is the right angle of ΔPQR. Write the following ratios.

i. sin P

ii. cos Q

iii. tan P

iv. tan Q



[2 Marks]

Ans:

i. $\sin P = \dfrac{\text{Opposite side of } \angle P}{\text{Hypotenuse}} = \dfrac{QR}{PQ}$

ii. $\cos Q = \dfrac{\text{Adjacent side of } \angle Q}{\text{Hypotenuse}} = \dfrac{QR}{PQ}$

iii. $\tan P = \dfrac{\text{Opposite side of } \angle P}{\text{Adjacent side of } \angle P} = \dfrac{QR}{PR}$

iv. $\tan Q = \dfrac{\text{Opposite side of } \angle Q}{\text{Adjacent side of } \angle Q} = \dfrac{PR}{QR}$

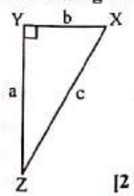
2. In the right angled ΔXYZ, $\angle XYZ = 90°$ and a, b, c are the lengths of the sides as shown in the figure. Write the following ratios.

i. sin X

ii. tan Z

iii. cos X

iv. tan X



[2 Marks]

Ans:

i. $\sin X = \dfrac{\text{Opposite side of } \angle X}{\text{Hypotenuse}} = \dfrac{YZ}{XZ} = \dfrac{a}{c}$

ii. $\tan Z = \dfrac{\text{Opposite side of } \angle Z}{\text{Adjacent side of } \angle Z} = \dfrac{XY}{YZ} = \dfrac{b}{a}$

iii. $\cos X = \dfrac{\text{Adjacent side of } \angle X}{\text{Hypotenuse}} = \dfrac{XY}{XZ} = \dfrac{b}{c}$

iv. $\tan X = \dfrac{\text{Opposite side of } \angle X}{\text{Adjacent side of } \angle X} = \dfrac{YZ}{XY} = \dfrac{a}{b}$

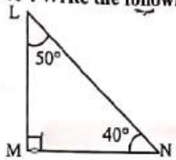
3. In right angled ΔLMN, $\angle LMN = 90°$, $\angle L = 50°$ and $\angle N = 40°$. Write the following ratios.

i. sin 50°

ii. cos 50°

iii. tan 40°

iv. cos 40°



[2 Marks]

Ans:

i. $\sin 50° = \dfrac{\text{Opposite side of } 50°}{\text{Hypotenuse}} = \dfrac{MN}{LN}$

ii. $\cos 50° = \dfrac{\text{Adjacent side of } 50°}{\text{Hypotenuse}} = \dfrac{LM}{LN}$

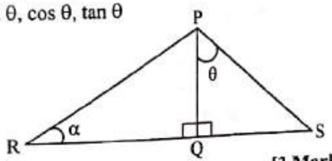
iii. $\tan 40° = \dfrac{\text{Opposite side of } 40°}{\text{Adjacent side of } 40°} = \dfrac{LM}{MN}$

iv. $\cos 40° = \dfrac{\text{Adjacent side of } 40°}{\text{Hypotenuse}} = \dfrac{MN}{LN}$

4. In the given figure, $\angle PQR = 90°$, $\angle PQS = 90°$, $\angle PRQ = \alpha$ and $\angle QPS = \theta$. Write the following trigonometric ratios.

i. sin α, cos α, tan α

ii. sin θ, cos θ, tan θ



[3 Marks]

Ans:

i. In ΔPQR,

$\sin \alpha = \dfrac{\text{Opposite side of } \alpha}{\text{Hypotenuse}} = \dfrac{PQ}{PR}$

$\cos \alpha = \dfrac{\text{Adjacent side of } \alpha}{\text{Hypotenuse}} = \dfrac{RQ}{PR}$

$\tan \alpha = \dfrac{\text{Opposite side of } \alpha}{\text{Adjacent side of } \alpha} = \dfrac{PQ}{RQ}$

ii. In ΔPQS,

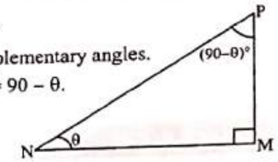
$\sin \theta = \dfrac{\text{Opposite side of } \theta}{\text{Hypotenuse}} = \dfrac{QS}{PS}$

$\cos \theta = \dfrac{\text{Adjacent side of } \theta}{\text{Hypotenuse}} = \dfrac{PQ}{PS}$

$\tan \theta = \dfrac{\text{Opposite side of } \theta}{\text{Adjacent side of } \theta} = \dfrac{QS}{PQ}$

Let's Study

Relation among Trigonometric Ratios



In ΔPMN, $\angle M = 90°$

$\angle P$ and $\angle N$ are complementary angles.

If $\angle N = \theta$, then $\angle P = 90 - \theta$.

$\sin \theta = \dfrac{\text{Opposite side of } \theta}{\text{Hypotenuse}} = \dfrac{PM}{PN}$ …(i)

$\cos \theta = \dfrac{\text{Adjacent side of } \theta}{\text{Hypotenuse}} = \dfrac{NM}{PN}$ …(ii)

$\tan \theta = \dfrac{\text{Opposite side of } \theta}{\text{Adjacent side of } \theta} = \dfrac{PM}{NM}$ …(iii)

$\sin (90 - \theta) = \dfrac{\text{Opposite side of } (90 - \theta)}{\text{Hypotenuse}}$

$= \dfrac{NM}{PN}$ …(iv)

$\cos (90 - \theta) = \dfrac{\text{Adjacent side of } (90 - \theta)}{\text{Hypotenuse}}$

$= \dfrac{PM}{PN}$ …(v)

$\tan (90 - \theta) = \dfrac{\text{Opposite side of } (90 - \theta)}{\text{Adjacent side of } (90 - \theta)}$

$= \dfrac{NM}{PM}$ …(vi)

$\therefore$ $\mathbf{\sin \theta = \cos (90 - \theta)}$ …[From (i) and (v)]

$\mathbf{\cos \theta = \sin (90 - \theta)}$ …[From (ii) and (iv)]

$\tan \theta \times \tan (90 - \theta) = \dfrac{PM}{NM} \times \dfrac{NM}{PM}$ …[From (iii) and (vi)]

$\therefore$ $\mathbf{\tan \theta \times \tan (90 - \theta) = 1}$

$\dfrac{\sin \theta}{\cos \theta} = \dfrac{\frac{PM}{PN}}{\frac{NM}{PN}}$ …[From (i) and (ii)]

$= \dfrac{PM}{PN} \times \dfrac{PN}{NM}$

$= \dfrac{PM}{NM} = \tan \theta$ …[From (iii)]

$\therefore$ $\mathbf{\dfrac{\sin \theta}{\cos \theta} = \tan \theta}$

$\therefore \quad \left(\frac{PQ}{PR}\right)^2 + \left(\frac{QR}{PR}\right)^2 = 1$

$\therefore \quad (\sin\theta)^2 + (\cos\theta)^2 = 1$ ...[From (i) and (ii)]

Remember This

i. $(\sin\theta)^2$ means 'square of $\sin\theta$',
i.e., $(\sin\theta)^2 = \sin^2\theta$

ii. $\sin^2\theta + \cos^2\theta = 1$ is true for any value of θ. It is called a basic trigonometric identity.

a. $0 \le \sin\theta \le 1,\ 0 \le \sin^2\theta \le 1$

b. $0 \le \cos\theta \le 1,\ 0 \le \cos^2\theta \le 1$

Try This

1. **Verify that the equation '$\sin^2\theta + \cos^2\theta = 1$' is true when $\theta = 0°$ or $\theta = 90°$.**

(Textbook pg. no. 112)

Solution:

$\sin^2\theta + \cos^2\theta = 1$

i. If $\theta = 0°$,

L.H.S. $= \sin^2\theta + \cos^2\theta$
$= \sin^2 0° + \cos^2 0°$
$= 0 + 1$ ...[$\because \sin 0° = 0, \cos 0° = 1$]
$= 1$
$=$ R.H.S.

$\therefore$ $\mathbf{\sin^2\theta + \cos^2\theta = 1}$

ii. If $\theta = 90°$,

L.H.S. $= \sin^2\theta + \cos^2\theta$
$= \sin^2 90° + \cos^2 90°$
$= 1 + 0$ ...[$\because \sin 90° = 1, \cos 90° = 0$]
$= 1$
$=$ R.H.S.

$\therefore$ $\mathbf{\sin^2\theta + \cos^2\theta = 1}$

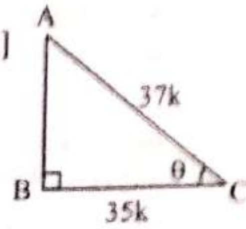
Practice Set 8.2

1. **In the following table, a ratio is given in each column. Find the remaining two ratios in the column and complete the table.**
[3 Marks each]

Sr. No.	i.	ii.	iii.	iv.	v.	vi.	vii.	viii.	ix.
$\sin\theta$		$\frac{11}{61}$		$\frac{1}{2}$				$\frac{3}{5}$	
$\cos\theta$	$\frac{35}{37}$				$\frac{1}{\sqrt{3}}$				
$\tan\theta$			1			$\frac{21}{20}$	$\frac{8}{15}$		$\frac{1}{2\sqrt{2}}$

Solution:

i. $\cos\theta = \frac{35}{37}$ ...(i)[Given]



In right angled ΔABC,
$\angle C = \theta$.

$\cos\theta = \frac{\text{Adjacent side of }\theta}{\text{Hypotenuse}}$

$\therefore \quad \cos\theta = \frac{BC}{AC}$ ...(ii)

$\therefore \quad \frac{BC}{AC} = \frac{35}{37}$ ...[From (i) and (ii)]

Let the common multiple be k.

$\therefore$ BC = 35k and AC = 37k

Now, $AC^2 = AB^2 + BC^2$ ...[Pythagoras theorem]

$\therefore \quad (37k)^2 = AB^2 + (35k)^2$

$\therefore \quad 1369k^2 = AB^2 + 1225k^2$

$\therefore \quad AB^2 = 1369k^2 - 1225k^2$
$= 144k^2$

$\therefore \quad AB = \sqrt{144k^2}$ ...[Taking square root of both sides]
$= 12k$

$\therefore \quad \sin\theta = \frac{\text{Opposite side of }\theta}{\text{Hypotenuse}} = \frac{AB}{AC} = \frac{12k}{37k} = \mathbf{\frac{12}{37}}$

$\tan\theta = \frac{\text{Opposite side of }\theta}{\text{Adjacent side of }\theta} = \frac{AB}{BC} = \frac{12k}{35k} = \mathbf{\frac{12}{35}}$

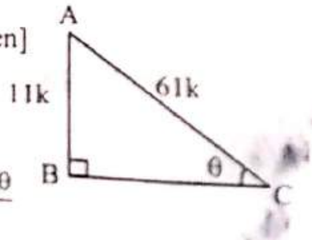
Smart Check

L.H.S. $= \sin^2\theta + \cos^2\theta$
$= \left(\frac{12}{37}\right)^2 + \left(\frac{35}{37}\right)^2$
$= \frac{144}{1369} + \frac{1225}{1369}$
$= \frac{1369}{1369} = 1 =$ R.H.S.

$\tan\theta = \frac{\sin\theta}{\cos\theta} = \frac{\frac{12}{37}}{\frac{35}{37}} = \frac{12}{37} \times \frac{37}{35} = \frac{12}{35}$

Thus, our answer is correct.

ii. $\sin\theta = \frac{11}{61}$ ...(i)[Given]



In right angled ΔABC,
$\angle C = \theta$.

$\sin\theta = \frac{\text{Opposite side of }\theta}{\text{Hypotenuse}}$

$\therefore \quad \sin\theta = \frac{AB}{AC}$ ...(ii)

$\therefore \quad \frac{AB}{AC} = \frac{11}{61}$ ...[From (i) and (ii)]

Let the common multiple be k.

$\therefore$ AB = 11k and AC = 61k

Now, $AC^2 = AB^2 + BC^2$ ...[Pythagoras theorem]

$\therefore$ $(61k)^2 = (11k)^2 + BC^2$

$\therefore$ $3721k^2 = 121k^2 + BC^2$

$\therefore$ $BC^2 = 3721k^2 - 121k^2$

$= 3600k^2$

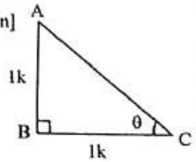
$\therefore$ $BC = \sqrt{3600k^2}$ ...[Taking square root of both sides]

$= 60k$

$\therefore$ $\cos\theta = \dfrac{\text{Adjacent side of }\theta}{\text{Hypotenuse}} = \dfrac{BC}{AC} = \dfrac{60k}{61k} = \dfrac{60}{61}$

$\tan\theta = \dfrac{\text{Opposite side of }\theta}{\text{Adjacent side of }\theta} = \dfrac{AB}{BC} = \dfrac{11k}{60k} = \dfrac{11}{60}$

iii. $\tan\theta = 1 = \dfrac{1}{1}$ ...(i)[Given]



In right angled ΔABC,

$\angle C = \theta$.

$\tan\theta = \dfrac{\text{Opposite side of }\theta}{\text{Adjacent side of }\theta}$

$\therefore$ $\tan\theta = \dfrac{AB}{BC}$ ...(ii)

$\therefore$ $\dfrac{AB}{BC} = \dfrac{1}{1}$ ...[From (i) and (ii)]

Let the common multiple be k.

$\therefore$ AB = 1k and BC = 1k.

Now, $AC^2 = AB^2 + BC^2$ ...[Pythagoras theorem]

$= k^2 + k^2$

$= 2k^2$

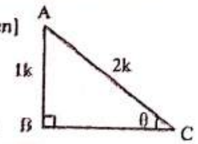
$\therefore$ $AC = \sqrt{2k^2}$ ...[Taking square root of both sides]

$= \sqrt{2}k$

$\therefore$ $\sin\theta = \dfrac{\text{Opposite side of }\theta}{\text{Hypotenuse}} = \dfrac{AB}{AC} = \dfrac{1k}{\sqrt{2}k} = \dfrac{1}{\sqrt{2}}$

$\cos\theta = \dfrac{\text{Adjacent side of }\theta}{\text{Hypotenuse}} = \dfrac{BC}{AC} = \dfrac{1k}{\sqrt{2}k} = \dfrac{1}{\sqrt{2}}$

iv. $\sin\theta = \dfrac{1}{2}$ ...(i)[Given]



In right angled ΔABC,

$\angle C = \theta$

$\sin\theta = \dfrac{\text{Opposite side of }\theta}{\text{Hypotenuse}}$

$\therefore$ $\sin\theta = \dfrac{AB}{AC}$ ...(ii)

$\therefore$ $\dfrac{AB}{AC} = \dfrac{1}{2}$ ...[From (i) and (ii)]

Let the common multiple be k.

$\therefore$ AB = 1k and AC = 2k

Now, $AC^2 = AB^2 + BC^2$ ...[Pythagoras theorem]

$\therefore$ $(2k)^2 = k^2 + BC^2$

$\therefore$ $4k^2 = k^2 + BC^2$

$\therefore$ $BC^2 = 4k^2 - k^2 = 3k^2$

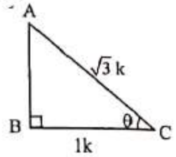
$\therefore$ $BC = \sqrt{3k^2}$ ...[Taking square root of both sides]

$= \sqrt{3}k$

$\therefore$ $\cos\theta = \dfrac{\text{Adjacent side of }\theta}{\text{Hypotenuse}} = \dfrac{BC}{AC} = \dfrac{\sqrt{3}k}{2k} = \dfrac{\sqrt{3}}{2}$

$\tan\theta = \dfrac{\text{Opposite side of }\theta}{\text{Adjacent side of }\theta} = \dfrac{AB}{BC} = \dfrac{1k}{\sqrt{3}k} = \dfrac{1}{\sqrt{3}}$

v. $\cos\theta = \dfrac{1}{\sqrt{3}}$ ...(i)[Given]



In right angled ΔABC,

$\angle C = \theta$.

$\cos\theta = \dfrac{\text{Adjacent side of }\theta}{\text{Hypotenuse}}$

$\therefore$ $\cos\theta = \dfrac{BC}{AC}$ ...(ii)

$\therefore$ $\dfrac{BC}{AC} = \dfrac{1}{\sqrt{3}}$ ...[From (i) and (ii)]

Let the common multiple be k.

$\therefore$ BC = 1k and AC = $\sqrt{3}$ k

Now, $AC^2 = AB^2 + BC^2$ ...[Pythagoras theorem]

$\therefore$ $(\sqrt{3}k)^2 = AB^2 + k^2$

$\therefore$ $3k^2 = AB^2 + k^2$

$\therefore$ $AB^2 = 3k^2 - k^2 = 2k^2$

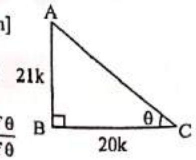
$\therefore$ $AB = \sqrt{2k^2}$ ...[Taking square root of both sides]

$= \sqrt{2}$ k

$\therefore$ $\sin\theta = \dfrac{\text{Opposite side of }\theta}{\text{Hypotenuse}} = \dfrac{AB}{AC} = \dfrac{\sqrt{2}k}{\sqrt{3}k} = \dfrac{\sqrt{2}}{\sqrt{3}}$

$\tan\theta = \dfrac{\text{Opposite side of }\theta}{\text{Adjacent side of }\theta} = \dfrac{AB}{BC} = \dfrac{\sqrt{2}k}{1k} = \sqrt{2}$

vi. $\tan\theta = \dfrac{21}{20}$ ...(i)[Given]



In right angled ΔABC,

$\angle C = \theta$.

$\tan\theta = \dfrac{\text{Opposite side of }\theta}{\text{Adjacent side of }\theta}$

$\therefore$ $\tan\theta = \dfrac{AB}{BC}$ ...(ii)

$\therefore$ $\dfrac{AB}{BC} = \dfrac{21}{20}$ ...[From (i) and (ii)]

Let the common multiple be k.

$\therefore$ AB = 21k and BC = 20k

Now, $AC^2 = AB^2 + BC^2$ ...[Pythagoras theorem]

$= (21k)^2 + (20k)^2$

$= 441k^2 + 400k^2$

$= 841k^2$

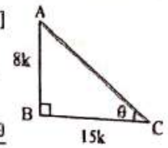
$\therefore$ $AC = \sqrt{841k^2}$ ...[Taking square root of both sides]

$= 29k$

$\therefore$ $\sin\theta = \dfrac{\text{Opposite side of }\theta}{\text{Hypotenuse}} = \dfrac{AB}{AC} = \dfrac{21k}{29k} = \dfrac{21}{29}$

$\cos\theta = \dfrac{\text{Adjacent side of }\theta}{\text{Hypotenuse}} = \dfrac{BC}{AC} = \dfrac{20k}{29k} = \dfrac{20}{29}$

vii. $\tan\theta = \dfrac{8}{15}$ ...(i)[Given]



In right angled ΔABC,

$\angle C = \theta$.

$\tan\theta = \dfrac{\text{Opposite side of }\theta}{\text{Adjacent side of }\theta}$

$\therefore$ $\tan\theta = \dfrac{AB}{BC}$ ...(ii)

$\therefore$ $\dfrac{AB}{BC} = \dfrac{8}{15}$ ...[From (i) and (ii)]

Let the common multiple be k.

$\therefore$ AB = 8k and BC = 15k

Now, $AC^2 = AB^2 + BC^2$ ...[Pythagoras theorem]

$= (8k)^2 + (15k)^2$

$= 64k^2 + 225k^2$

$= 289k^2$

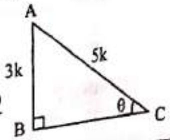
$\therefore$ $AC = \sqrt{289k^2}$ ...[Taking square root of both sides]

$= 17k$

$\therefore$ $\sin\theta = \dfrac{\text{Opposite side of }\theta}{\text{Hypotenuse}} = \dfrac{AB}{AC} = \dfrac{8k}{17k} = \dfrac{8}{17}$

$\cos\theta = \dfrac{\text{Adjacent side of }\theta}{\text{Hypotenuse}} = \dfrac{BC}{AC} = \dfrac{15k}{17k} = \dfrac{15}{17}$

viii. $\sin\theta = \dfrac{3}{5}$ ...(i)[Given]



In right angled ΔABC,

$\angle C = \theta$.

$\sin\theta = \dfrac{\text{Opposite side of }\theta}{\text{Hypotenuse}}$

$\therefore$ $\sin\theta = \dfrac{AB}{AC}$ ...(ii)

$\therefore$ $\dfrac{AB}{AC} = \dfrac{3}{5}$ ...[From (i) and (ii)]

Let the common multiple be k.

$\therefore$ AB = 3k and AC = 5k

Now, $AC^2 = AB^2 + BC^2$ ...[Pythagoras theorem]

$\therefore$ $(5k)^2 = (3k)^2 + BC^2$

$\therefore$ $25k^2 = 9k^2 + BC^2$

$\therefore$ $BC^2 = 25k^2 - 9k^2$

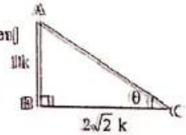
$\therefore$ $BC = \sqrt{16k^2}$ ...[Taking square root of both sides]

$= 4k$

$\cos\theta = \dfrac{\text{Adjacent side of }\theta}{\text{Hypotenuse}} = \dfrac{BC}{AC} = \dfrac{4k}{5k} = \dfrac{4}{5}$

$\tan\theta = \dfrac{\text{Opposite side of }\theta}{\text{Adjacent side of }\theta} = \dfrac{AB}{BC} = \dfrac{3k}{4k} = \dfrac{3}{4}$

ix. $\tan\theta = \dfrac{1}{2\sqrt{2}}$ ...(i)[Given]



In right angled ΔABC,

$\angle C = \theta$.

$\tan\theta = \dfrac{\text{Opposite side of }\theta}{\text{Adjacent side of }\theta}$

$\therefore$ $\tan\theta = \dfrac{AB}{BC}$ ...(ii)

$\therefore$ $\dfrac{AB}{BC} = \dfrac{1}{2\sqrt{2}}$ ...[From (i) and (ii)]

Let the common multiple be k.

$\therefore$ AB = 1k and BC = $2\sqrt{2}$ k

Now, $AC^2 = AB^2 + BC^2$ ...[Pythagoras theorem]

$= k^2 + (2\sqrt{2}\,k)^2$

$= k^2 + 8k^2$

$= 9k^2$

$\therefore$ $AC = \sqrt{9k^2}$ ...[Taking square root of both sides]

$= 3k$

$\therefore$ $\sin\theta = \dfrac{\text{Opposite side of }\theta}{\text{Hypotenuse}} = \dfrac{AB}{AC} = \dfrac{1k}{3k} = \dfrac{1}{3}$

$\cos\theta = \dfrac{\text{Adjacent side of }\theta}{\text{Hypotenuse}} = \dfrac{BC}{AC} = \dfrac{2\sqrt{2}k}{3k} = \dfrac{2\sqrt{2}}{3}$

Sr. No.	i.	ii.	iii.	iv.	v.	vi.	vii.	viii.	ix.
$\sin\theta$	$\frac{12}{37}$	$\frac{11}{61}$	$\frac{1}{\sqrt{2}}$	$\frac{1}{2}$	$\frac{\sqrt{2}}{\sqrt{3}}$	$\frac{21}{29}$	$\frac{8}{17}$	$\frac{3}{5}$	$\frac{1}{3}$
$\cos\theta$	$\frac{35}{37}$	$\frac{60}{61}$	$\frac{1}{\sqrt{2}}$	$\frac{\sqrt{3}}{2}$	$\frac{1}{\sqrt{3}}$	$\frac{20}{29}$	$\frac{15}{17}$	$\frac{4}{5}$	$\frac{2\sqrt{2}}{3}$
$\tan\theta$	$\frac{12}{35}$	$\frac{11}{60}$	1	$\frac{1}{\sqrt{3}}$	$\sqrt{2}$	$\frac{21}{20}$	$\frac{8}{15}$	$\frac{3}{4}$	$\frac{1}{2\sqrt{2}}$

2. Find the values of: **[2 Marks each]**

i. $5\sin 30° + 3\tan 45°$

ii. $\frac{4}{5}\tan^2 60° + 3\sin^2 60°$

iii. $2\sin 30° + \cos 0° + 3\sin 90°$

iv. $\frac{\tan 60°}{\sin 60° + \cos 60°}$

v. $\cos^2 45° + \sin^2 30°$

vi. $\cos 60° \times \cos 30° + \sin 60° \times \sin 30°$

Solution:

i. $\sin 30° = \frac{1}{2}$ and $\tan 45° = 1$

$5\sin 30° + 3\tan 45° = 5\left(\frac{1}{2}\right) + 3(1)$

$= \frac{5}{2} + 3$

$= \frac{5+6}{2}$

$\therefore$ **$5\sin 30° + 3\tan 45° = \frac{11}{2}$**

ii. $\tan 60° = \sqrt{3}$ and $\sin 60° = \frac{\sqrt{3}}{2}$

$\frac{4}{5}\tan^2 60° + 3\sin^2 60°$

$= \frac{4}{5}(\tan 60°)^2 + 3(\sin 60°)^2$

$= \frac{4}{5}(\sqrt{3})^2 + 3\left(\frac{\sqrt{3}}{2}\right)^2$

$= \frac{4}{5} \times 3 + 3 \times \frac{3}{4}$

$= \frac{12}{5} + \frac{9}{4}$

$= \frac{48+45}{20}$

$= \frac{93}{20}$

$\therefore$ **$\frac{4}{5}\tan^2 60° + 3\sin^2 60° = \frac{93}{20}$**

iii. $\sin 30° = \frac{1}{2}$, $\cos 0° = 1$ and $\sin 90° = 1$

$2\sin 30° + \cos 0° + 3\sin 90° = 2\left(\frac{1}{2}\right) + 1 + 3(1)$

$= 1 + 1 + 3$

$\therefore$ **$2\sin 30° + \cos 0° + 3\sin 90° = 5$**

iv. $\tan 60° = \sqrt{3}$, $\sin 60° = \frac{\sqrt{3}}{2}$ and $\cos 60° = \frac{1}{2}$

$\frac{\tan 60°}{\sin 60° + \cos 60°} = \frac{\sqrt{3}}{\frac{\sqrt{3}}{2} + \frac{1}{2}}$

$= \frac{\sqrt{3}}{\frac{\sqrt{3}+1}{2}}$

$= \sqrt{3} \times \frac{2}{\sqrt{3}+1}$

$\therefore$ **$\frac{\tan 60°}{\sin 60° + \cos 60°} = \frac{2\sqrt{3}}{\sqrt{3}+1}$**

v. $\cos 45° = \frac{1}{\sqrt{2}}$ and $\sin 30° = \frac{1}{2}$

$\cos^2 45° + \sin^2 30° = (\cos 45°)^2 + (\sin 30°)^2$

$= \left(\frac{1}{\sqrt{2}}\right)^2 + \left(\frac{1}{2}\right)^2$

$= \frac{1}{2} + \frac{1}{4}$

$= \frac{2+1}{4}$

$\therefore$ **$\cos^2 45° + \sin^2 30° = \frac{3}{4}$**

vi. $\cos 60° = \frac{1}{2}$, $\cos 30° = \frac{\sqrt{3}}{2}$, $\sin 60° = \frac{\sqrt{3}}{2}$ and $\sin 30° = \frac{1}{2}$

$\cos 60° \times \cos 30° + \sin 60° \times \sin 30°$

$= \frac{1}{2} \times \frac{\sqrt{3}}{2} + \frac{\sqrt{3}}{2} \times \frac{1}{2}$

$= \frac{\sqrt{3}}{4} + \frac{\sqrt{3}}{4}$

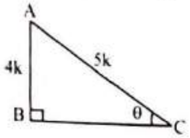
$= \frac{\sqrt{3}+\sqrt{3}}{4}$

$= \frac{2\sqrt{3}}{4} = \frac{\sqrt{3}}{2}$

$\therefore$ **$\cos 60° \times \cos 30° + \sin 60° \times \sin 30° = \frac{\sqrt{3}}{2}$**

3. If $\sin\theta = \frac{4}{5}$, then find $\cos\theta$. **[2 Marks]**

Solution:



$\sin\theta = \frac{4}{5}$ ...(i)[Given]

In right angled ΔABC,

$\angle C = \theta$.

$\sin\theta = \frac{\text{Opposite side of }\theta}{\text{Hypotenuse}}$

$\therefore \sin\theta = \frac{AB}{AC}$ ...(ii)

$\therefore \frac{AB}{AC} = \frac{4}{5}$ ...[From (i) and (ii)]

Let the common multiple be k.

$\therefore$ AB = 4k and AC = 5k

Now, $AC^2 = AB^2 + BC^2$ ...[Pythagoras theorem]

$\therefore (5k)^2 = (4k)^2 + BC^2$

$\therefore 25k^2 = 16k^2 + BC^2$

$\therefore BC^2 = 25k^2 - 16k^2 = 9k^2$

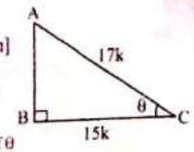
$\therefore BC = \sqrt{9k^2}$ ...[Taking square root of both sides]

$= 3k$

$\therefore \cos\theta = \frac{\text{Adjacent side of }\theta}{\text{Hypotenuse}} = \frac{BC}{AC} = \frac{3k}{5k} = \frac{3}{5}$

4. If $\cos\theta = \frac{15}{17}$, then find $\sin\theta$. **[2 Marks]**

Solution:



$\cos\theta = \frac{15}{17}$ ...(i)[Given]

In right angled ΔABC,

$\angle C = \theta$.

$\cos\theta = \frac{\text{Adjacent side of }\theta}{\text{Hypotenuse}}$

$\therefore \cos\theta = \frac{BC}{AC}$ ...(ii)

$\therefore \frac{BC}{AC} = \frac{15}{17}$ ...[From (i) and (ii)]

Let the common multiple be k.

$\therefore$ BC = 15k and AC = 17k

Now, $AC^2 = AB^2 + BC^2$ ...[Pythagoras theorem]

$\therefore (17k)^2 = AB^2 + (15k)^2$

$\therefore 289k^2 = AB^2 + 225k^2$

$\therefore AB^2 = 289k^2 - 225k^2$

$= 64k^2$

$\therefore AB = \sqrt{64k^2}$ ...[Taking square root of both sides]

$= 8k$

$\therefore \sin\theta = \frac{\text{Opposite side of }\theta}{\text{Hypotenuse}} = \frac{AB}{AC} = \frac{8k}{17k} = \frac{8}{17}$

Problem Set - 8

1. Choose the correct alternative answer for the following multiple choice questions. **[1 Mark each]**

i. Which of the following statements is true?

(A) $\sin\theta = \cos(90 - \theta)$

(B) $\cos\theta = \tan(90 - \theta)$

(C) $\sin\theta = \tan(90 - \theta)$

(D) $\tan\theta = \tan(90 - \theta)$

ii. Which of the following is the value of $\sin 90°$?

(A) $\frac{\sqrt{3}}{2}$ (B) 0

(C) $\frac{1}{2}$ (D) 1

iii. $2\tan 45° + \cos 45° - \sin 45° = ?$

(A) 0 (B) 1

(C) 2 (D) 3

iv. $\frac{\cos 28°}{\sin 62°} = ?$

(A) 2 (B) -1

(C) 0 (D) 1

Answers:

i. (A) ii. (D) iii. (C)

iv. (D)

Hints:

iii. $2\tan 45° + \cos 45° - \sin 45°$

$= 2(1) + \frac{1}{\sqrt{2}} - \frac{1}{\sqrt{2}} = 2$

iv. $\frac{\cos 28°}{\sin 62°} = \frac{\sin(90° - 28°)}{\sin 62°}$ ...[$\because \cos\theta = \sin(90 - \theta)$]

$= \frac{\sin 62°}{\sin 62°}$

$= 1$

9 Surface Area and Volume

Type of Problems	Practice Set	Q. Nos.
Cuboid	9.1	Q.1, 2, 5
	Practice Problems (Based on Practice Set 9.1)	Q.1, 2, 5
	Problem Set-9	Q.2
Cube	9.1	Q.3, 4, 6
	Practice Problems (Based on Practice Set 9.1)	Q. 3, 4, 6
	Problem Set-9	Q.5
Cylinder	9.1	Q.7, 8
	Practice Problems (Based on Practice Set 9.1)	Q.7, 8, 9
	Problem Set-9	Q.1, 8, 9
Cone	9.2	Q.1, 2, 3, 4, 5, 6, 7, 8, 9, 10
	Practice Problems (Based on Practice Set 9.2)	Q.1, 2, 3, 4, 5, 6, 7
	Problem Set-9	Q.3, 7
Sphere, hemisphere	9.3	Q.1, 2, 3, 4, 5
	Practice Problems (Based on Practice Set 9.3)	Q.1, 2, 3, 4, 5, 6, 7, 8, 9
	Problem Set-9	Q.4, 6

Let's Recall

1. **Cuboid:**

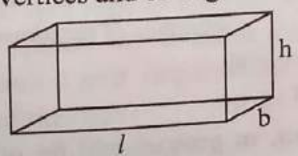
i. A box-shaped solid object is called a cuboid.

ii. A cuboid has six flat sides.

iii. All of its faces are rectangles and all angles are right angles.

iv. The opposite rectangular faces of a cuboid are identical.

v. It has 8 vertices and 12 edges.



If length, breadth and height of a cuboid are l, b and h respectively, then

i. Total surface area of a cuboid
$= \mathbf{2(lb + bh + hl)}$

ii. Area of vertical surfaces of a cuboid
$= \mathbf{2(l + b) \times h}$

iii. Volume of a cuboid $= \mathbf{l \times b \times h}$

Example:

The length, breadth and height of a room are 20 m, 15 m and 10 m respectively. Its vertical faces are to be painted. Find the cost of painting at the rate of ₹ 12 per sq. m. Also, find the volume of the room.

Given: For cuboidal room,
length (l) = 20 m, breadth (b) = 15 m,
height (h) = 10 m

To find: cost of painting and volume of room

Solution:

i. Area of vertical faces of the room
$= 2(l + b) \times h$
$= 2(20 + 15) \times 10$
$= 2 \times 35 \times 10$
$= 700$ sq. m.

ii. Rate of painting the wall = ₹ 12 per sq. m.

∴ Cost of painting
= Area of vertical faces × Rate of painting
$= 700 \times 12$
= ₹ 8400

iii. Volume of the room = $l \times b \times h$

$= 20 \times 15 \times 10$

$= 3000$ cubic m.

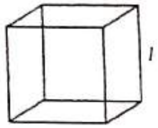
$\therefore$ **The cost of painting the room is ₹ 8400 and its volume is 3000 cubic m.**

2. Cube:

i. A box-shaped solid object which has six identical square faces is called a cube.

ii. All of the faces of a cube are squares and all the angles are right angles.

iii. A cube has 8 vertices and 12 edges.



If the length of the edge of a cube is l, then

i. Total surface area of a cube = $6l^2$

ii. Area of vertical surfaces of a cube = $4l^2$

iii. Volume of a cube = l^3

Example:

Two cubes each of volume 343 cubic cm are joined end to end. Find the total surface area of the resulting cuboid.

Given: Volume of cube = 343 cubic cm

To find: Total surface area of cuboid

Solution:

i. Volume of cube = l^3

$\therefore 343 = l^3$

$\therefore l^3 = 343$

$\therefore l = \sqrt[3]{343}$ ...[Taking cube root on both sides]

$\therefore l = 7$ cm

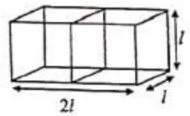
ii. For the new cuboid,

length (L) = $2l = 2 \times 7$

$= 14$ cm

breadth (B) = $l = 7$ cm

height (H) = $l = 7$ cm



$\therefore$ Total surface area of the cuboid

$= 2\,(LB + BH + LH)$

$= 2\,(14 \times 7 + 7 \times 7 + 14 \times 7)$

$= 2\,(98 + 49 + 98)$

$= 2 \times 245$

$= 490$ sq. cm.

Alternate method:

When we join two cubes end to end, we get total 10 surfaces of a cube.

ii. Total surface area of the cuboid = $10l^2$

$= 10 \times 7^2$

$= 490$ sq. cm.

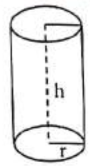
$\therefore$ **The total surface area of the resulting cuboid is 490 sq. cm.**

3. Cylinder:

i. A solid object with two identical flat ends that are circular and one curved surface is called a cylinder.

ii. It has two edges, at which the two plane surfaces meet with the curved surface.

iii. The edges of a cylinder are curved edges.



If the radius and height of a cylinder are r and h respectively, then

i. Curved surface area of a cylinder = $2\pi rh$

ii. Total surface area of a cylinder = $2\pi r(r + h)$

iii. Volume of a cylinder = $\pi r^2 h$

Example:

The diameter of a road roller is 3 m and its length is 4.9 m. Find the area of ground pressed by the road roller in 200 rotations.

Given: For road roller,
diameter (d) = 3 m, length (h) = 4.9 m

To find: Area of ground pressed by the road roller in 200 rotations

Solution:

i. Since, area of ground pressed in 1 rotation of road roller = curved surface area of road roller

Curved surface area of the road roller

$= 2\pi rh$

$= \pi dh$ ...[$\because 2r = d$]

$= \frac{22}{7} \times 3 \times 4.9$

$= 22 \times 3 \times 0.7 = 46.2$ sq. m.

ii. Area of ground pressed in 1 rotation

$= 46.2$ sq. m

$\therefore$ Area of ground pressed in 200 rotations

= Area of ground pressed in 1 rotation × Number of rotations of road roller

$= 46.2 \times 200$

$= 9240$ sq. m

$\therefore$ **9240 sq. m ground will be pressed in 200 rotations of the road roller.**

Remember This

1. Cuboid:

i. Total surface area of a cuboid $= 2(lb + bh + hl)$

ii. Area of vertical surfaces of a cuboid $= 2(l + b) \times h$

iii. Volume of a cuboid $= l \times b \times h$

2. Cube:

i. Total surface area of a cube $= 6l^2$

ii. Area of vertical surfaces of a cube $= 4l^2$

iii. Volume of a cube $= l^3$

3. Cylinder:

i. Curved surface area of a cylinder $= 2\pi rh$

ii. Total surface area of a cylinder $= 2\pi r(r + h)$

iii. Volume of a cylinder $= \pi r^2 h$

Practice Set 9.1

1. Length, breadth and height of a cuboid shape box of medicine is 20 cm, 12 cm and 10 cm respectively. Find the surface area of vertical faces and total surface area of this box. **[3 Marks]**

Given: For cuboid shape box of medicine,
length (l) = 20 cm, breadth (b) = 12 cm and height (h) = 10 cm.

To find: Surface area of vertical faces and total surface area of the box

Solution:

i. Surface area of vertical faces of the box

$= 2\,(l + b) \times h$

$= 2\,(20 + 12) \times 10$

$= 2 \times 32 \times 10$

$= 640$ sq.cm.

ii. Total surface area of the box

$= 2\,(lb + bh + lh)$

$= 2\,(20 \times 12 + 12 \times 10 + 20 \times 10)$

$= 2\,(240 + 120 + 200)$

$= 2 \times 560$

$= 1120$ sq.cm.

$\therefore$ **The surface area of vertical faces and total surface area of the box are 640 sq.cm. and 1120 sq.cm. respectively.**

2. Total surface area of a box of cuboid shape is 500 sq.unit. Its breadth and height is 6 unit and 5 unit respectively. What is the length of that box? **[3 Marks]**

Given: For cuboid shape box,
breadth (b) = 6 unit, height (h) = 5 unit
Total surface area = 500 sq. unit.

To find: Length of the box (l)

Solution:

Total surface area of the box $= 2\,(lb + bh + lh)$

$\therefore 500 = 2\,(6l + 6 \times 5 + 5l)$

$\therefore \frac{500}{2} = (11l + 30)$

$\therefore 250 = 11l + 30$

$\therefore 250 - 30 = 11l$

$\therefore 220 = 11l$

$\therefore \frac{220}{11} = l$

$\therefore l = 20$ units

$\therefore$ **The length of the box is 20 units.**

3. Side of a cube is 4.5 cm. Find the surface area of all vertical faces and total surface area of the cube. **[3 Marks]**

Given: Side of cube (l) = 4.5 cm

To find: Surface area of all vertical faces and the total surface area of the cube

Solution:

i. Area of vertical faces of cube $= 4l^2$

$= 4\,(4.5)^2$

$= 4 \times 20.25$

$= 81$ sq.cm.

ii. Total surface area of the cube $= 6l^2$

$= 6\,(4.5)^2$

$= 6 \times 20.25$

$= 121.5$ sq.cm.

$\therefore$ **The surface area of all vertical faces and the total surface area of the cube are 81 sq.cm. and 121.5 sq.cm. respectively.**

4. Total surface area of a cube is 5400 sq. cm. Find the surface area of all vertical faces of the cube. **[3 Marks]**

Given: Total surface area of cube
= 5400 sq.cm.

To find: Surface area of all vertical faces of the cube

Solution:

i. Total surface area of cube $= 6l^2$

$\therefore 5400 = 6l^2$

$\therefore \frac{5400}{6} = l^2$

$\therefore l^2 = 900$

ii. Area of vertical faces of cube $= 4l^2$

$= 4 \times 900$

$= 3600$ sq.cm.

$\therefore$ **The surface area of all vertical faces of the cube is 3600 sq.cm.**

5. Volume of a cuboid is 34.50 cubic metre. Breadth and height of the cuboid is 1.5 m and 1.15 m respectively. Find its length. **[3 Marks]**

Given: Breadth (b) = 1.5 m, height (h) = 1.15 m
Volume of cuboid = 34.50 cubic metre

To find: Length of the cuboid (l)

Solution:

Volume of cuboid $= l \times b \times h$

$\therefore \quad 34.50 = l \times b \times h$

$\therefore \quad 34.50 = l \times 1.5 \times 1.15$

$\therefore \quad l = \dfrac{34.50}{1.5 \times 1.15}$

$= \dfrac{34500}{15 \times 115}$

$= \dfrac{300}{15}$

$= 20$

$\therefore$ **The length of the cuboid is 20 m.**

6. **What will be the volume of a cube having length of edge 7.5 cm ?** **[2 Marks]**

Given: Length of edge of cube $(l) = 7.5$ cm

To find: Volume of a cube

Solution:

Volume of a cube $= l^3$

$= (7.5)^3$

$= 421.875$

≈ 421.88 cubic cm

$\therefore$ **The volume of the cube is 421.88 cubic cm.**

7. **Radius of base of a cylinder is 20 cm and its height is 13 cm, find its curved surface area and total surface area. ($\pi = 3.14$)** **[3 Marks]**

Given: Radius (r) = 20 cm, height (h) = 13 cm

To find: Curved surface area and the total surface area of the cylinder

Solution:

i. Curved surface area of cylinder

$= 2\pi rh$

$= 2 \times 3.14 \times 20 \times 13$

$= 1632.8$ sq.cm

ii. Total surface area of cylinder

$= 2\pi r(r + h)$

$= 2 \times 3.14 \times 20(20 + 13)$

$= 2 \times 3.14 \times 20 \times 33$

$= 4144.8$ sq.cm

$\therefore$ **The curved surface area and the total surface area of the cylinder are 1632.8 sq.cm and 4144.8 sq.cm respectively.**

8. **Curved surface area of a cylinder is 1980 cm^2 and radius of its base is 15 cm. Find the height of the cylinder. $\left(\pi = \dfrac{22}{7}\right)$** **[3 Marks]**

Given: Curved surface area of cylinder = 1980 sq.cm, radius (r) = 15 cm

To find: Height of the cylinder (h)

Solution:

Curved surface area of cylinder $= 2\pi rh$

$\therefore \quad 1980 = 2 \times \dfrac{22}{7} \times 15 \times h$

$\therefore \quad h = \dfrac{1980 \times 7}{2 \times 22 \times 15}$

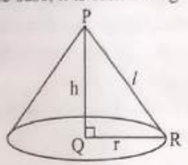
$\therefore \quad h = 21$ cm

$\therefore$ **The height of the cylinder is 21 cm.**

Let's Study

Terms related to a cone and their relation

A cone is a solid object which has one plane circular surface, i.e. its base and only one curved surface. It has only one vertex (apex). If the vertex is over the center of the base, it is called a right angled cone.



In the above figure, 'h' is the perpendicular height, 'l' is the slant height and 'r' is the radius of the base of the cone.

In ΔPQR, $\angle Q = 90°$

$\therefore \quad PR^2 = PQ^2 + QR^2$ ...[Pythagoras theorem]

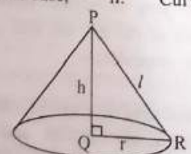
$\therefore \quad l^2 = h^2 + r^2$

$\therefore \quad (\text{Slant height})^2 = (\text{Perpendicular height})^2 + (\text{Base radius})^2$

Surface area of a cone

A cone has two surfaces:

i. Circular base, ii. Curved surface



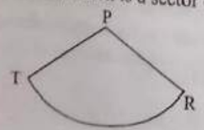
Area of circular base:

We can find the area of base of a cone as it is a circular base(circle).

$\therefore$ Area of circular base $= \pi r^2$

Area of curved surface:

By cutting the cone along the edge PR we get the net of the curved surface, which is a sector of a circle.



Curved surface area of a cone

= Area of sector of a circle

Total surface area

= curved surface area + area of the base

$= \pi rl + \pi r^2$

$\therefore$ Total surface area of cone $= \boldsymbol{\pi r(l + r)}$

[**Note:** *See the first activity of Apply your knowledge to know how to find curved surface area of cone.*]

Example:

The radius and height of a cone are 35 cm and 12 cm respectively. Find its curved surface area and total surface area.

Given: Radius (r) = 35 cm, height (h) = 12 cm

To find: Curved surface area and total surface area

Solution:

i. $l^2 = r^2 + h^2$

$= 35^2 + 12^2$

$= 1225 + 144$

$\therefore \quad l^2 = 1369$

$\therefore \quad l = \sqrt{1369}$ ...[Taking square root on both sides]

$= 37$ cm

ii. Curved surface area of cone

$= \pi rl$

$= \dfrac{22}{7} \times 35 \times 37$

$= 22 \times 5 \times 37$

$= 4070$ sq. cm

iii. Total surface area of cone

$= \pi r\,(l + r)$

$= \dfrac{22}{7} \times 35\,(37 + 35)$

$= \dfrac{22}{7} \times 35 \times 72$

$= 22 \times 5 \times 72$

$= 7920$ sq. cm.

$\therefore$ **The curved surface area and the total surface area of the cone are 4070 sq.cm and 7920 sq.cm. respectively.**

Volume of cone

If the base-radii and heights of cone and cylinder are equal, then

Volume of cylinder $= 3 \times$ Volume of cone

$\therefore \quad \pi r^2 h = 3 \times$ Volume of cone

$\therefore \quad$ Volume of cone $= \dfrac{1}{3}\pi r^2 h$

Example:

Perpendicular height of a cone is 21 cm and radius is 20 cm. Find the volume of a cone. ($\pi = 3.14$)

Given: Height (h) = 21 cm, radius (r) = 20 cm

To find: Volume of a cone

Solution:

Volume of cone $= \dfrac{1}{3}\pi r^2 h$

$= \dfrac{1}{3} \times \dfrac{22}{7} \times (20)^2 \times 21$

$= \dfrac{1}{3} \times 22 \times 20 \times 20 \times 3$

$= 22 \times 20 \times 20$

$= 8800$ cubic cm

$\therefore$ **The volume of a cone is 8800 cubic cm**

Remember This

i. Area of base of a cone $= \pi r^2$

ii. Curved surface area of a cone $= \pi rl$

iii. Total surface area of a cone $= \pi r\,(l + r)$

iv. Volume of a cone $= \dfrac{1}{3} \times \pi r^2 h$

Practice Set 9.2

1. **Perpendicular height of a cone is 12 cm and its slant height is 13 cm. Find the radius of the base of the cone.** **[2 Marks]**

Given: Height (h) = 12 cm, length (l) = 13 cm

To find: Radius of the base of the cone (r)

Solution:

$l^2 = r^2 + h^2$

$\therefore \quad 13^2 = r^2 + 12^2$

$\therefore \quad 169 = r^2 + 144$

$\therefore \quad 169 - 144 = r^2$

$\therefore \quad r^2 = 25$

$\therefore \quad r = \sqrt{25}$ ...[Taking square root on both sides]

$= 5$ cm

$\therefore$ **The radius of base of the cone is 5 cm.**

2. **Find the volume of a cone, if its total surface area is 7128 sq.cm and radius of base is 28 cm. $\left(\pi = \dfrac{22}{7}\right)$** **[4 Marks]**

Given: Radius (r) = 28 cm, Total surface area of cone = 7128 sq.cm

To find: Volume of the cone

Solution:

i. Total surface area of cone = $\pi r(l + r)$

$\therefore$ $7128 = \frac{22}{7} \times 28 \times (l + 28)$

$\therefore$ $7128 = 22 \times 4 \times (l + 28)$

$\therefore$ $l + 28 = \frac{7128}{22 \times 4}$

$\therefore$ $l + 28 = 81$

$\therefore$ $l = 81 - 28$

$\therefore$ $l = 53$ cm

ii. Now, $l^2 = r^2 + h^2$

$\therefore$ $53^2 = 28^2 + h^2$

$\therefore$ $2809 = 784 + h^2$

$\therefore$ $2809 - 784 = h^2$

$\therefore$ $h^2 = 2025$

$\therefore$ $h = \sqrt{2025}$ ...[Taking square root on both sides]

$= 45$ cm

$\therefore$ Volume of cone $= \frac{1}{3}\pi r^2 h$

$= \frac{1}{3} \times \frac{22}{7} \times 28^2 \times 45$

$= \frac{1}{3} \times \frac{22}{7} \times 28 \times 28 \times 45$

$= 22 \times 4 \times 28 \times 15$

$= 36960$ cubic.cm

$\therefore$ **The volume of the cone is 36960 cubic.cm.**

3. Curved surface area of a cone is 251.2 cm^2 and radius of its base is 8 cm. Find its slant height and perpendicular height. ($\pi = 3.14$) **[4 Marks]**

Given: Radius (r) = 8 cm, curved surface area of cone = 251.2 cm^2

To find: Slant height (l) and the perpendicular height (h) of the cone

Solution:

i. Curved surface area of cone = πrl

$\therefore$ $251.2 = 3.14 \times 8 \times l$

$\therefore$ $l = \frac{251.2}{3.14 \times 8}$

$= \frac{25120}{314 \times 8}$

$= \frac{3140}{314}$

$\therefore$ $l = 10$ cm

ii. Now, $l^2 = r^2 + h^2$

$\therefore$ $10^2 = 8^2 + h^2$

$\therefore$ $100 = 64 + h^2$

$\therefore$ $100 - 64 = h^2$

$\therefore$ $h^2 = 36$

$\therefore$ $h = \sqrt{36}$ ...[Taking square root on both sides]

$= 6$ cm

$\therefore$ **The slant height and the perpendicular height of the cone are 10 cm and 6 cm respectively.**

4. What will be the cost of making a closed cone of tin sheet having radius of base 6 m and slant height 8 m if the rate of making is ₹ 10 per sq.m? **[3 Marks]**

Given: Radius (r) = 6 m, length (l) = 8 m

To find: Total cost of making the cone

Solution:

i. To find the total cost of making the cone of tin sheet, first we need to find the total surface area of the cone.

Total surface area of the cone

$= \pi r(l + r)$

$= \frac{22}{7} \times 6 \times (8 + 6)$

$= \frac{22}{7} \times 6 \times 14$

$= 22 \times 6 \times 2 = 264$ sq.m

ii. Rate of making the cone = ₹ 10 per sq.m

$\therefore$ Total cost

= Total surface area × Rate of making the cone

$= 264 \times 10$

= ₹ 2640

$\therefore$ **The total cost of making the cone of tin sheet is ₹ 2640.**

5. Volume of a cone is 6280 cubic cm and base radius of the cone is 20 cm. Find its perpendicular height. ($\pi = 3.14$) **[3 Marks]**

Given: Radius (r) = 20 cm, Volume of cone = 6280 cubic cm

To find: Perpendicular height (h) of the cone

Solution:

Volume of cone $= \frac{1}{3}\pi r^2 h$

$\therefore$ $6280 = \frac{1}{3} \times 3.14 \times 20^2 \times h$

$\therefore$ $h = \frac{6280 \times 3}{3.14 \times 400}$

$= \frac{6280 \times 3}{314 \times 4}$

$= \frac{20 \times 3}{4} = 15$ cm

$\therefore$ **The perpendicular height of the cone is 15 cm.**

[Note: *The question has been modified according to the answer given in textbook.*]

6. Surface area of a cone is 188.4 sq.cm and its slant height is 10 cm. Find its perpendicular height ($\pi = 3.14$). **[3 Marks]**

Given: Length (l) = 10 cm, curved surface area of the cone = 188.4 sq.cm

To find: Perpendicular height (h) of the cone

Solution:

i. Curved surface area of the cone = πrl

$\therefore$ $188.4 = 3.14 \times r \times 10$

$\therefore$ $r = \frac{188.4}{3.14 \times 10}$

$= \frac{188.4}{31.4}$

$= \frac{1884}{314}$

$= 6$ cm

ii. Now, $l^2 = r^2 + h^2$

$\therefore$ $10^2 = 6^2 + h^2$

$\therefore$ $100 = 36 + h^2$

$\therefore$ $100 - 36 = h^2$

$\therefore$ $h^2 = 64$

$\therefore$ $h = \sqrt{64}$ ...[Taking square root on both sides]

$= 8$ cm

$\therefore$ **The perpendicular height of the cone is 8 cm.**

7. Volume of a cone is 1232 cm^3 and its height is 24 cm. Find the surface area of the cone. $\left(\pi = \frac{22}{7}\right)$ **[4 Marks]**

Given: Height (h) = 24 cm, Volume of cone = 1232 cm^3

To find: Surface area of the cone

Solution:

i. Volume of cone $= \frac{1}{3}\pi r^2 h$

$\therefore$ $1232 = \frac{1}{3} \times \frac{22}{7} \times r^2 \times 24$

$\therefore$ $r^2 = \frac{1232 \times 3 \times 7}{22 \times 24}$

$= \frac{56 \times 1 \times 7}{1 \times 8}$

$\therefore$ $r^2 = 49$

$\therefore$ $r = \sqrt{49}$ ...[Taking square root on both sides]

$= 7$ cm

ii. Now, $l^2 = r^2 + h^2$

$\therefore$ $l^2 = 7^2 + 24^2$

$= 49 + 576$

$= 625$

$\therefore$ $l = \sqrt{625}$ ...[Taking square root on both sides]

$= 25$ cm

iii. Curved surface area of cone = πrl

$= \frac{22}{7} \times 7 \times 25$

$= 22 \times 25$

$= 550$ sq.cm

$\therefore$ **The surface area of the cone is 550 sq.cm.**

[Note: *Here, we have taken $V = 1232$ cm^3, not 1212 cm^3 as $V = 1212$ cm^3 will not give the exact value of radius.*]

8. The curved surface area of a cone is 2200 sq.cm and its slant height is 50 cm. Find the total surface area of cone. $\left(\pi = \frac{22}{7}\right)$ **[3 Marks]**

Given: Length (l) = 50 cm, curved surface area of cone = 2200 sq.cm

To find: Total surface area of the cone

Solution:

i. Curved surface area of cone = πrl

$\therefore$ $2200 = \frac{22}{7} \times r \times 50$

$\therefore$ $r = \frac{2200 \times 7}{22 \times 50}$

$= \frac{100 \times 7}{50} = 14$ cm

ii. Total surface area of cone

$= \pi r(l + r)$

$= \frac{22}{7} \times 14 \times (50 + 14)$

$= \frac{22}{7} \times 14 \times 64$

$= 22 \times 2 \times 64$

$= 2816$ sq.cm

$\therefore$ **The total surface area of the cone is 2816 sq.cm.**

9. There are 25 persons in a tent which is conical in shape. Every person needs an area of 4 sq.m. of the ground inside the tent. If height of the tent is 18 m, find the volume of the tent. **[4 Marks]**

Given: For the tent, height (h) = 18m, number of people in the tent = 25, area required for each person = 4 sq.m

To find: Volume of the tent

Solution:

i. Every person needs an area of 4 sq.m. of the ground inside the tent.

Surface area of the base of the tent
= number of people in the tent × area required for each person
= 25 × 4
= 100 sq.m

ii. Surface area of the base of the tent = πr^2

$\therefore$ $100 = \pi r^2$

$\therefore$ $\pi r^2 = 100$

iii. Volume of the tent = $\frac{1}{3}\pi r^2 h$

$= \frac{1}{3} \times 100 \times 18$ ...[$\because \pi r^2 = 100$]

$= 100 \times 6$

$= 600$ cubic metre

$\therefore$ **The volume of the tent is 600 cubic metre.**

10. In a field, dry fodder for the cattle is heaped in a conical shape. The height of the cone is 2.1 m and diameter of base is 7.2 m. Find the volume of the heap of the fodder. If it is to be covered by polythene in rainy season then how much minimum polythene sheet is needed? $\left(\pi = \frac{22}{7} \text{ and } \sqrt{17.37} = 4.17\right)$

[4 Marks]

Given: Height of the heap (h) = 2.1 m.
diameter of the base (d) = 7.2 m

$\therefore$ Radius of the base (r) = $\frac{d}{2} = \frac{7.2}{2} = 3.6$ m

To find: Volume of the heap of the fodder and polythene sheet required

Solution:

i. Volume of the heap of fodder = $\frac{1}{3}\pi r^2 h$

$= \frac{1}{3} \times \frac{22}{7} \times (3.6)^2 \times 2.1$

$= \frac{1}{3} \times \frac{22}{7} \times 3.6 \times 3.6 \times 2.1$

$= 1 \times 22 \times 1.2 \times 3.6 \times 0.3$

$= 28.51$ cubic metre

ii. Now, $l^2 = r^2 + h^2$

$= (3.6)^2 + (2.1)^2$

$= 12.96 + 4.41$

$\therefore$ $l^2 = 17.37$

$\therefore$ $l = \sqrt{17.37}$ ...[Taking square root on both sides]

$= 4.17$ m

iii. Area of the polythene sheet needed to cover the heap of the fodder
= Curved surface area of the conical heap

$= \pi r l$

$= \frac{22}{7} \times 3.6 \times 4.17$

$= 47.18$ sq.m

$\therefore$ **The volume of the heap of the fodder is 28.51 cubic metre and a polythene sheet of 47.18 sq.m will be required to cover it.**

Let's Study

Surface area of a sphere

i. A sphere is defined as a ball-like shape body bound by a surface.

ii. Every point of a sphere is equidistant from a fixed point, called the centre.

iii. A sphere is a geometric shape in three-dimensional space.

iv. One half of a sphere is known as hemisphere.

Surface area of sphere and hemisphere:

If the radius of a sphere / hemisphere is r, then

i. Surface area of a sphere = $4\pi r^2$

ii. Surface area of a hemisphere = $2\pi r^2$

iii. Total surface area of a solid hemisphere
= Surface area of hemisphere + Area of circle
$= 2\pi r^2 + \pi r^2$
$= 3\pi r^2$

Volume of sphere and hemisphere:

If the radius of base of cone and that of hemisphere are same, then

i. Volume of hemisphere = 2 × Volume of cone

$= 2 \times \frac{1}{3}\pi r^2 h$

$= 2 \times \frac{1}{3}\pi r^2 \times r$ ... [$\because$ r = h]

$\therefore$ Volume of hemisphere = $\frac{2}{3}\pi r^3$

ii. Volume of sphere = 2 × volume of hemisphere

$= 2 \times \frac{2}{3}\pi r^3$

Volume of sphere = $\frac{4}{3}\pi r^3$

Examples:

1. Find the surface area and volume of a sphere of radius 2.8 cm.

Given: Radius (r) = 2.8 cm

To find: Surface area and volume of the sphere

Solution:

i. Surface area of sphere = $4\pi r^2$

$= 4 \times \frac{22}{7} \times 2.8^2$

$= 4 \times \frac{22}{7} \times 2.8 \times 2.8$

$= 4 \times 22 \times 0.4 \times 2.8$

$= 98.56$ sq cm.

ii. Volume of sphere = $\frac{4}{3}\pi r^3$

$= \frac{4}{3} \times \frac{22}{7} \times 2.8^3$

$= \frac{4}{3} \times \frac{22}{7} \times 2.8 \times 2.8 \times 2.8$

$= \frac{4}{3} \times 22 \times 0.4 \times 2.8 \times 2.8$

$= 91.99$ cubic cm.

$\therefore$ **The surface area and volume of the sphere are 98.56 sq.cm and 91.99 cubic cm respectively.**

2. The volume of a hemisphere is 486π cubic cm find its total surface area. ($\pi = 3.14$)

Given: Volume of hemisphere = 486π

To find: Total surface area of the hemisphere

Solution:

Here, to find the total surface area, first we need to find radius by using volume.

i. Volume of hemisphere = $\frac{2}{3}\pi r^3$

$\therefore$ $486\pi = \frac{2}{3}\pi r^3$

$\therefore$ $r^3 = \frac{486 \times 3}{2}$

$= 243 \times 3$

$\therefore$ $r^3 = 729$

$\therefore$ $r = \sqrt[3]{729}$ ...[Taking cube root on both sides]

$= 9$ cm

ii. Total surface area of hemisphere = $3\pi r^2$

$= 3 \times 3.14 \times 9^2$

$= 763.02$ sq. cm.

$\therefore$ **The total surface area of the hemisphere is 763.02 sq. cm.**

Remember This

1. Sphere:

i. Surface area of a sphere = $4\pi r^2$

ii. Volume of sphere = $\frac{4}{3}\pi r^3$

2. Hemisphere:

i. Surface area of a hemisphere = $2\pi r^2$

ii. Total surface area of a solid hemisphere = $3\pi r^2$

iii. Volume of hemisphere = $\frac{2}{3}\pi r^3$

Practice Set 9.3

1. Find the surface areas and volumes of spheres of the following radii

[3 Marks each]

i. 4 cm **ii. 9 cm**

iii. 3.5 cm (π = 3.14)

i. **Given:** Radius (r) = 4 cm

To find: Surface area and volume of sphere

Solution:

Surface area of sphere = $4\pi r^2$

$= 4 \times 3.14 \times 4^2$

$\therefore$ **Surface area of sphere = 200.96 sq.cm**

Volume of sphere = $\frac{4}{3}\pi r^3$

$= \frac{4}{3} \times 3.14 \times 4^3$

$\therefore$ **Volume of sphere = 267.95 cubic cm**

ii. **Given:** Radius (r) = 9 cm

To find: Surface area and volume of sphere

Solution:

Surface area of sphere = $4\pi r^2$

$= 4 \times 3.14 \times 9^2$

$\therefore$ **Surface area of sphere = 1017.36 sq.cm**

Volume of sphere = $\frac{4}{3}\pi r^3$

$= \frac{4}{3} \times 3.14 \times 9^3$

$= \frac{4}{3} \times 3.14 \times 9 \times 9 \times 9$

$= 4 \times 3.14 \times 3 \times 9 \times 9$

$\therefore$ **Volume of sphere = 3052.08 cubic cm**

iii. **Given:** Radius (r) = 3.5 cm

To find: Surface area and volume of sphere

Solution:

Surface area of sphere = $4\pi r^2$

$= 4 \times 3.14 \times (3.5)^2$

$\therefore$ **Surface area of sphere = 153.86 sq.cm**

$$\text{Volume of sphere} = \frac{4}{3}\pi r^3$$
$$= \frac{4}{3} \times 3.14 \times (3.5)^3$$

$\therefore$ **Volume of sphere = 179.50 cubic cm**

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2. If the radius of a solid hemisphere is 5 cm, then find its curved surface area and total surface area. ($\pi = 3.14$) [3 Marks]

Given: Radius (r) = 5 cm

To find: Curved surface area and total surface area of hemisphere

Solution:

i. Curved surface area of hemisphere $= 2\pi r^2$

$= 2 \times 3.14 \times 5^2$

$= 2 \times 3.14 \times 25$

$= 50 \times 3.14$

$= 157$ sq.cm.

ii. Total surface area of hemisphere $= 3\pi r^2$

$= 3 \times 3.14 \times 5^2$

$= 235.5$ sq.cm.

$\therefore$ **The curved surface area and total surface area of hemisphere are 157 sq.cm. and 235.5 sq.cm. respectively.**

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3. If the surface area of a sphere is 2826 cm^2 then find its volume. ($\pi = 3.14$) [3 Marks]

Given: Surface area of sphere = 2826 sq.cm.

To find: Volume of sphere

Solution:

i. Surface area of sphere $= 4\pi r^2$

$\therefore$ $2826 = 4 \times 3.14 \times r^2$

$\therefore$ $r^2 = \frac{2826}{4 \times 3.14} = \frac{282600}{4 \times 314} = \frac{900}{4}$

$\therefore$ $r^2 = 225$

$\therefore$ $r = \sqrt{225}$ …[Taking square root on both sides]

$= 15$ cm

ii. Volume of sphere $= \frac{4}{3}\pi r^3$

$= \frac{4}{3} \times 3.14 \times 15^3$

$= \frac{4}{3} \times 3.14 \times 15 \times 15 \times 15$

$= 4 \times 3.14 \times 5 \times 15 \times 15$

$= 14130$ cubic cm.

$\therefore$ **The volume of the sphere is 14130 cubic cm.**

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4. Find the surface area of a sphere, if its volume is 38808 cubic cm. $\left(\pi = \frac{22}{7}\right)$ [3 Marks]

Given: Volume of sphere = 38808 cubic cm.

To find: Surface area of sphere

Solution:

i. Volume of sphere $= \frac{4}{3}\pi r^3$

$\therefore$ $38808 = \frac{4}{3} \times \frac{22}{7} \times r^3$

$\therefore$ $r^3 = \frac{38808 \times 3 \times 7}{4 \times 22}$

$= \frac{9702 \times 3 \times 7}{22}$

$\therefore$ $r^3 = 441 \times 21 = 21 \times 21 \times 21$

$\therefore$ $r = 21$ cm …[Taking cube root on both sides]

ii. Surface area of sphere $= 4\pi r^2$

$= 4 \times \frac{22}{7} \times 21^2$

$= 4 \times \frac{22}{7} \times 21 \times 21$

$= 4 \times 22 \times 3 \times 21$

$= 5544$ sq.cm.

$\therefore$ **The surface area of sphere is 5544 sq.cm.**

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5. Volume of a hemisphere is 18000π cubic cm. Find its diameter. [3 Marks]

Given: Volume of hemisphere $= 18000\pi$ cubic cm.

To find: Diameter of the hemisphere

Solution:

i. Volume of hemisphere $= \frac{2}{3}\pi r^3$

$\therefore$ $18000\pi = \frac{2}{3}\pi r^3$

$\therefore$ $18000 = \frac{2}{3} r^3$

$\therefore$ $r^3 = \frac{18000 \times 3}{2}$

$= 9000 \times 3$

$\therefore$ $r^3 = 27000$

$\therefore$ $r = \sqrt[3]{27000}$

…[Taking cube root on both sides]

$= 30$ cm

ii. Diameter $= 2r$

$= 2 \times 30 = 60$ cm

$\therefore$ **The diameter of the hemisphere is 60 cm.**

Problem Set – 9

1. If diameter of a road roller is 0.9 m and its length is 1.4 m, how much area of a field will be pressed in its 500 rotations? $\left(\pi = \frac{22}{7}\right)$ [3 Marks]