A Modified Null Space Strategy to Avoid Straight Edge Obstacles and Solve Consensus Problem

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May 1, 2014

Overview

Behaviors

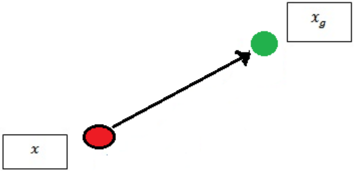

Figure: Go To Goal

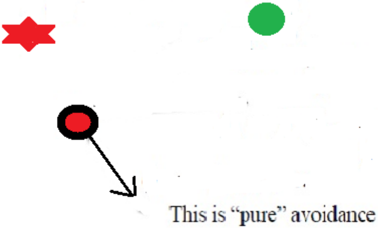

Figure: Avoid Obstacle

$$x' = K_{GTG}(x_g - x)$$

Where x_g is the position vector of the goal and x is the position vector of robot

$$x' = K_{OA}(x - x_{ob})$$

where x is the position vector of the mobile robot , x_{ob} is the position vector of the obstacle

Follow Wall

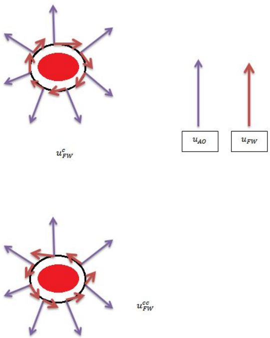

Figure: Follow wall

$$u_{FW} = \alpha\, \mathbf{R}(\pm\pi/2) u_{AO}$$

where α is scalar and $\mathbf{R}(\theta)$ is the rotation matrix:

$$R(\theta) = \begin{bmatrix} cos\theta & -sin\theta \\ sin\theta & \cos\theta \end{bmatrix}$$

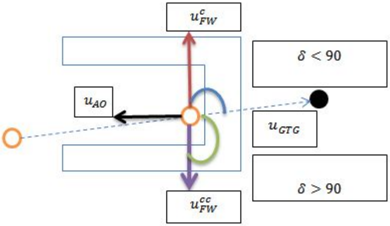

Figure: Whether to Follow Walls Clockwise or Anti-Clockwise?

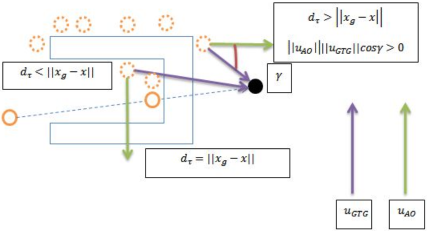

Figure: When to stop following walls

When to stop following walls?

1. Enough progress is made towards the goal
2. The inner product between go to goal and avoid obstacle control signal is positive.

General NSB and its need

When ever there are conflicting tasks involved with a control stratergy there is a genuine need for prioritising the tasks before we actually apply the stratergy.
This helps in

- 1) taking the robot to desired target.
- 2) filtering out the component of lower priority conflicting task.
- 3) increasing efficiency and reliability of the system.

$$\sigma_i = f(\mathbf{x}_i) \tag{1}$$

where $\sigma_i \in \mathbb{R}^m$ denotes the task to be controlled for the i^{th} robot.
Equation 1 is called direct kinematics equation

Considering the neighboring robots static, and σ_i to be differentiable,

$$\dot{\sigma_i} = \mathbf{J}_i(\mathbf{x}_i)\dot{\mathbf{x}}_i, \tag{2}$$

By inverting the locally linear mapping in 2, motion references $\mathbf{x}_{d_i}(t)$ for the i^{th} robot starting from desired value $\sigma_{d_i}(t)$ can be obatined

$$\dot{\mathbf{x}}_{d_i} = \mathbf{v}_{d_i} = \mathbf{J}_i^{\dagger}\sigma_{d_i}, \tag{3}$$

where $\mathbf{J}_i^{\dagger} = \mathbf{J}_i^T \left(\mathbf{J}_i\mathbf{J}_i^T\right)^{-1}$ denotes the penrose pseudo inverse of $\mathbf{J}_i$. Discrete time integration of robot's reference velocity results in numerical drift of the reconstructed position of robot and hence Closed Loop Inverse Kinmeatics version of the algorithm is used to counteract the undesired numerical drift to yield

$$\dot{\mathbf{x}}_{d_i} = \mathbf{v}_{d_i} = \mathbf{J}_i^{\dagger}\left(\dot{\sigma}_{d_i} + \mathbf{\Lambda}\tilde{\sigma}\right), \tag{4}$$

where $\mathbf{\Lambda}$ is a positive definite matrix and

$$\tilde{\sigma} = \sigma_d - \sigma. \tag{5}$$

Formulation of Task functions

- Goal Oriented Task Function

$$\mathbf{v}_i = \mathbf{K}_{GTG} \cdot (\mathbf{x}_g - \mathbf{x}_i) = \mathbf{u}_{GTG_i}, \tag{6}$$

where $\mathbf{K}_{GTG}$ is a positive definite matrix.

- Obstacle Avoidance Task Function

$$\mathbf{u}_{OA_i} = \mathbf{J}_i^{\dagger} k_{OA} \left(d - ||\mathbf{x}_i - \mathbf{x}_{ob}|| \right), \tag{7}$$

where $k_{OA} \in \mathbb{R}^+$.

NSB with two tasks GTG and OA

Let V_1 be the velocity of priorty 1 task, i.e., Obstacle Avoidance task. Also V_2 be the velocity of priorty 2 task , i.e., Go to Goal.

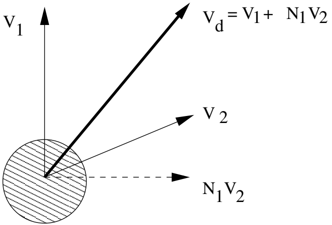

Figure: Top View of Sample Robot, with OA and GTG velocities.

Therefore the final equation with two tasks OA and GTG is given by

$$\mathbf{V}_d = \mathbf{V}_{OA} + \mathbf{N}_1 \cdot \mathbf{V}_{GTG} \tag{8}$$

Consensus problem

- A multi robot system (MRS) consists of N mobile robots and let $\mathbf{x} = [\mathbf{x}_1^T \ \mathbf{x}_2^T \cdots \mathbf{x}_N^T]^T$, where $\mathbf{x}_i \in \mathbb{R}^2$ denotes the position coordinates of i^{th} robot.
- A task can be modelled as driving the MRS from an initial state $\mathbf{x}_0$ to a final state $\mathbf{x}_f$
- Let initial state be $\mathbf{x}_0$ and final state be $\mathbf{x}_f = [\mathbf{c}^T \ \mathbf{c}^T \cdots \mathbf{c}^T]^T$, where $\mathbf{c} \in \mathbb{R}^2$

$$\mathbf{u}_i = \dot{\mathbf{x}}_i = k_p(\mathbf{c} - \mathbf{x}_i),$$

$$\dot{\mathbf{x}} = k_p\left(\mathbf{A}\mathbf{x} + \mathbf{b}\right)$$

$$\mathbf{A} = \begin{bmatrix} -1 & 0 & 0 & \cdots & 0 \\ 0 & -1 & 0 & \cdots & 0 \\ \vdots & & & & \\ 0 & \cdots & 0 & -1 & 0 \\ 0 & 0 & 0 & \cdots & -1 \end{bmatrix} \text{ and } \mathbf{b} = \begin{bmatrix} \mathbf{c} \\ \mathbf{c} \\ \vdots \\ \mathbf{c} \end{bmatrix}, \qquad (9)$$

where $\mathbf{A} \in \mathbb{R}^{2N \times 2N}$ and $\mathbf{b} \in \mathbb{R}^{2N}$

Consensus problem

The eigen values associated with system matrix of the aforementioned LTI system are strictly negative (-1 here) and $||\mathbf{b}|| < H < \infty$, therefore the above system satisfies BIBO stability criterion and the state converges to some point $\mathbf{c}^*$ given by:

$$\mathbf{c}^* = \lim_{t \to \infty} \mathbf{x}(t), \tag{10}$$

where t denotes time.

Flocking

The control strategy should, therefore drive the robots of MRS such that:

1. All the robots flock around the consensus point.
2. None of the robot collides with an obstacle.
3. None of the robot collide with any of it's neighbors. Concretely, it should abide to the constraint depicted in equation

$$\mathbf{x}_i(t^*) \neq \mathbf{x}_j(t^*) \;\; \forall \; i \neq j \tag{11}$$

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Modes of Operation

1. **No obstacle and no other robot sensed**

$$\mathbf{u}_i = \mathbf{u}_{GTG_i}.$$

2. **Obstacle sensed and no other robot sensed**

$$\mathbf{u}_i = \mathbf{u}_{AO_i} + \mathbf{N}_{1_i} \cdot \mathbf{u}_{GTG_i}$$

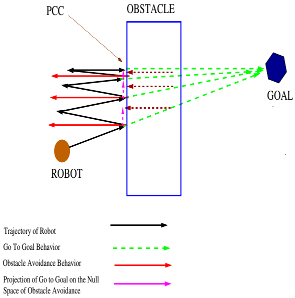
3. **Obstacle and other robot sensed**

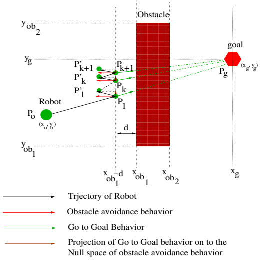
$$\mathbf{u}_i = \mathbf{u}_{AO_i} + \mathbf{N}_{1_i} \cdot \mathbf{u}_{RA_{ij}}$$

4. **No obstacle sensed but other Robot sensed**

$$\mathbf{u}_i = \mathbf{u}_{RA_{ij}} + \mathbf{N}_{ij} \cdot \mathbf{u}_{GTG_i}$$

Failure of Conventional NSB strategy for single mobile robot





$$\lim_{k\to\infty} \mathbf{p}_k = \begin{bmatrix} x_{ob_1} - d \\ y_g \end{bmatrix}$$

$$\lim_{k\to\infty} \mathbf{p}'_k = \begin{bmatrix} x_{ob_1} - d - \beta\Delta t \\ y_g \end{bmatrix}$$

Refer Section 4.2, Chapter 4,

Modified NSB based control strategy for MRS

1. **No obstacle and no other robot sensed**

$$\mathbf{u}_i = \mathbf{u}_{GTG_i}.$$

2. **Obstacle sensed and no other robot sensed**

$$\mathbf{v}_{d_i} = \begin{cases} \mathbf{u}^{C}_{FW_i} \text{ if } \mathbf{u}^{C}_{FW_i} \cdot \mathbf{u}_{GTG_i} > 0 \\ \mathbf{u}^{CC}_{FW_i} \text{ if } \mathbf{u}^{CC}_{FW_i} \cdot \mathbf{u}_{GTG_i} > 0 \end{cases}.$$

3. **Obstacle and other robot sensed**

$$\mathbf{u}_i = \mathbf{u}_{FW_i} + \mathbf{N}_{1_i} \cdot \mathbf{u}_{RA_{ij}}$$

4. **No obstacle sensed but other Robot sensed**

$$\mathbf{u}_i = \mathbf{u}_{RA_{ij}} + \mathbf{N}_{ij} \cdot \mathbf{u}_{GTG_i}$$

Conclusion

- Identified the failure of conventional NSB control strategy
- Modified NSB proposed

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