Topics: Normal distribution, Functions of Random Variables

1. The time required for servicing transmissions is normally distributed with μ = 45 minutes and σ = 8 minutes. The service manager plans to have work begin on the transmission of a customer's car 10 minutes after the car is dropped off and the customer is told that the car will be ready within 1 hour from drop-off. What is the probability that the service manager cannot meet his commitment?

    A. 0.3875
    B. 0.2676
    C. 0.5
    D. 0.6987

Ans: B

Given data:

Mean =55(45+10)

SD= 8

x=60

probability that the service manager meets his commitment:

z = (60-55)/8 =5/8=0.625

probability that the service manager doesn't meets his commitment:

P = (1- z) = 1-(0.625) =0.265

2.The current age (in years) of 400 clerical employees at an insurance claims processing center is normally distributed with mean μ = 38 and Standard deviation σ =6. For each statement below, please specify True/False. If false, briefly explain why.

    A. More employees at the processing center are older than 44 than between 38 and 44.

    B. A training program for employees under the age of 30 at the center would be expected to attract about 36 employees.

    Ans:

A. **False**

        explanation in ipnyb set2_Q2

B. **True**

        explanation in ipnyb set2_Q2

3.If $X_1 \sim N(\mu, \sigma^2)$ and $X_2 \sim N(\mu, \sigma^2)$ are *iid* normal random variables, then what is the difference between $2 X_1$ and $X_1 + X_2$? Discuss both their distributions and parameters.

Ans:

      X1 = N(mean , sqrt(SD))
      X2 = N (mean, sqrt (SD))

   X1 and X2 has no difference between them

  Similarly, 2(X1) == (X1 + X2)
  Therefore, have no difference both are same, the parameters are MEAN and VARIANCE

4.Let $X \sim N(100, 20^2)$. Find two values, *a* and *b*, symmetric about the mean, such that the probability of the random variable taking a value between them is 0.99.

    A. 90.5, 105.9
    B. 80.2, 119.8
    C. 22, 78
    D. 48.5, 151.5
    E. 90.1, 109.9

Ans:

    A. **D** 48.5, 151.5

    Explanation in set2_Q4 ipnyb

5.Consider a company that has two different divisions. The annual profits from the two divisions are independent and have distributions $Profit_1 \sim N(5, 3^2)$ and $Profit_2 \sim N(7, 4^2)$ respectively. Both the profits are in $ Million. Answer the following questions about the total profit of the company in Rupees. Assume that $1 = Rs. 45

A. Specify a Rupee range (centered on the mean) such that it contains 95% probability for the annual profit of the company.
B. Specify the 5th percentile of profit (in Rupees) for the company
C. Which of the two divisions has a larger probability of making a loss in a given year?

Ans:

Company total profit =profit1+ profit2

$=P \sim N(5, 3\text{^}2) + P\sim N(7, 4\text{^}2)$

$=P\sim N(12, 25) = P\sim N(12,5\text{^}2)$

So here mean = 12 , standard deviation = 5 ,

A) 95% probability for the annual profit of the company is
We know Z = (X – mu) / sigma
X = Z * sigma + mu
We know 95% of Z score is 1.96
Now the range = 12 – 1.96 * 5 = 2.2 \$ = 2.2 * 45 =99 M
Another one is = 12 + 1.96* 5 = 22.8 \$ = 22.8 * 45 = 1026 M

B) 5th percentile of profit (in Rupees) for the company is
Here 5% of Z score is - 1.645
Now the 5% probability is = (12 – 1.645 * 5) = 3.78 \$ = 3.78 * 45 = 170.1M

C) first division