Complex Numbers for Audio Signal Processing

Valerio Velardo

Join the community!

thesoundofai.slack.com

Why bother with complex numbers?

Why bother with complex numbers?

- Fourier transform -> magnitude and phase

Why bother with complex numbers?

- Fourier transform -> magnitude and phase
- Magnitude is a real number

Why bother with complex numbers?

- Fourier transform -> magnitude and phase
- Magnitude is a real number
- … something with magnitude + phase?

COMPLICATED NUMBERS?

NO SIRaj! IT’S *COMPLEX NUMBERS*

The genesis of CNs

The genesis of CNs

sqrt(-1)

The genesis of CNs

sqrt(-1)

i^2=-1

Our first complex number

$$c = a + ib$$

$$a, b \in \mathbb{R}$$

Our first complex number

Real part

$$c = \boxed{a} + ib$$

$$a, b \in \mathbb{R}$$

Our first complex number

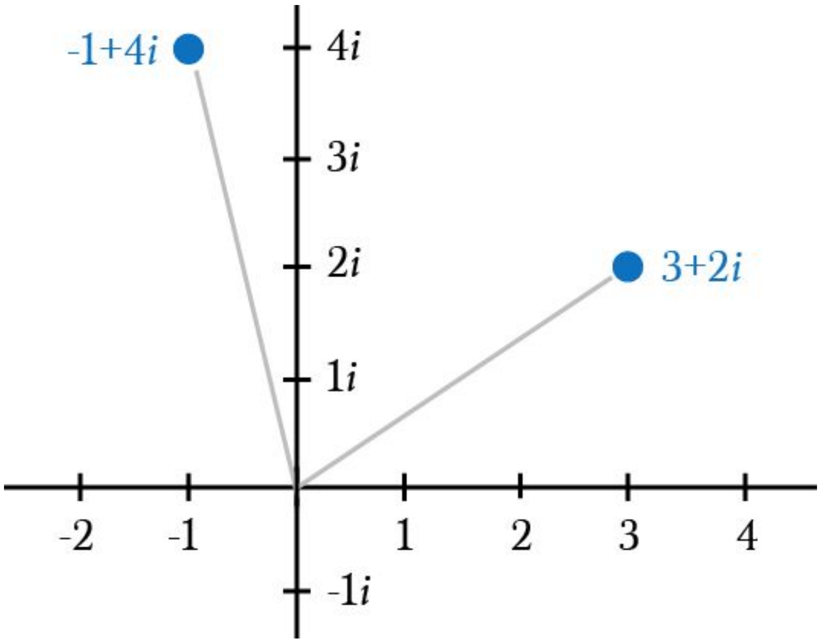
Real part

Imaginary part

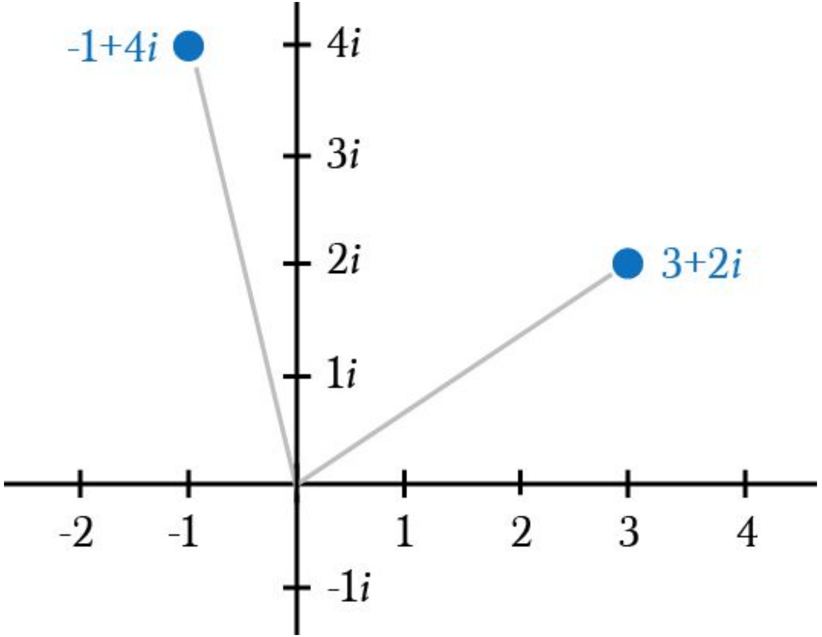
$$c = \boxed{a} + \boxed{ib}$$

$$a, b \in \mathbb{R}$$

Plotting complex numbers

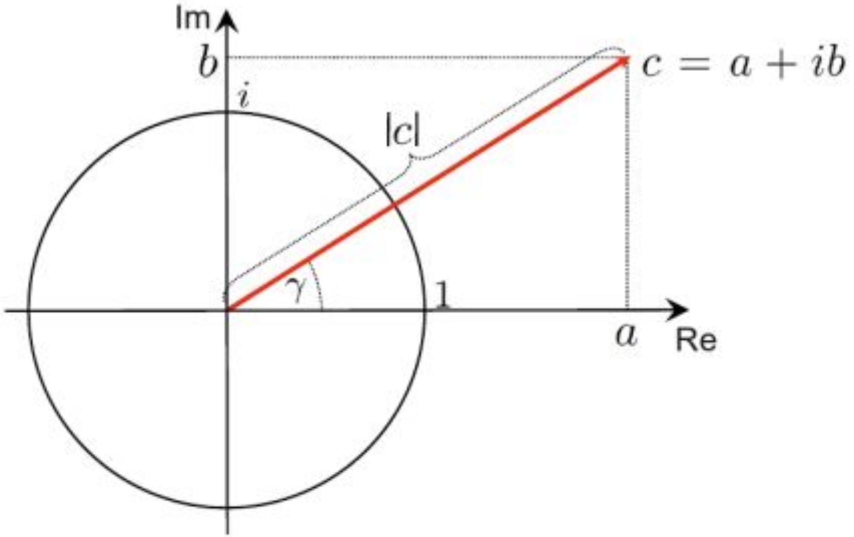


Plotting complex numbers

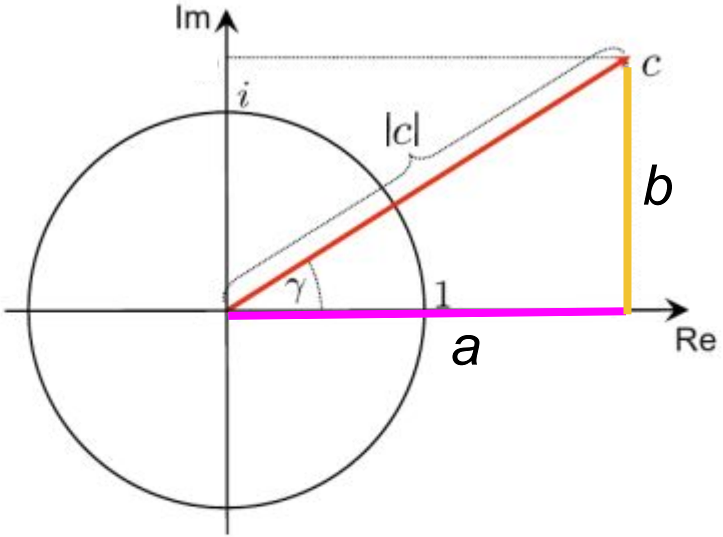


Cartesian coordinates

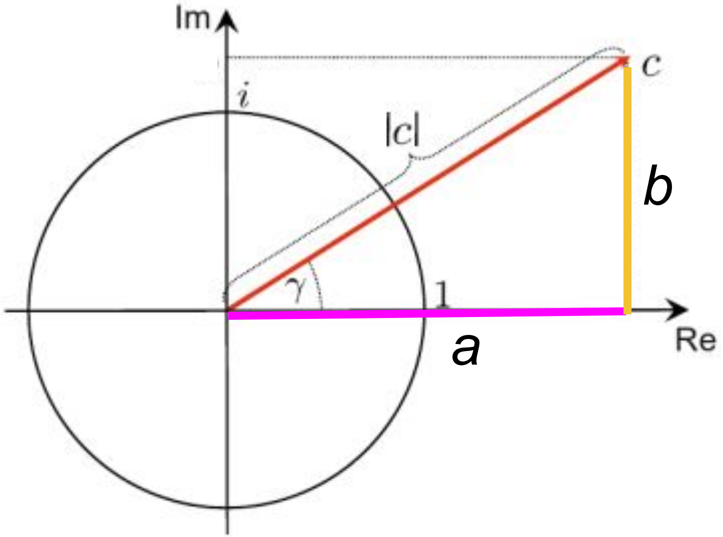
Polar coordinate representation



Polar coordinate representation

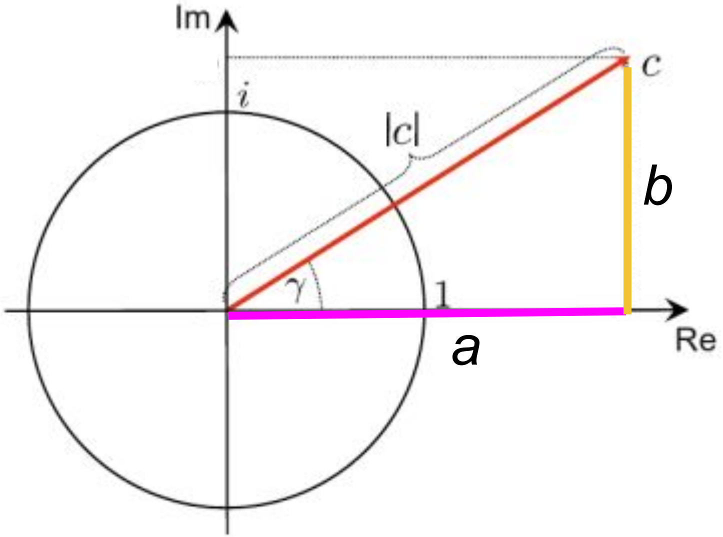


Polar coordinate representation



$$|c|^2 = a^2 + b^2$$

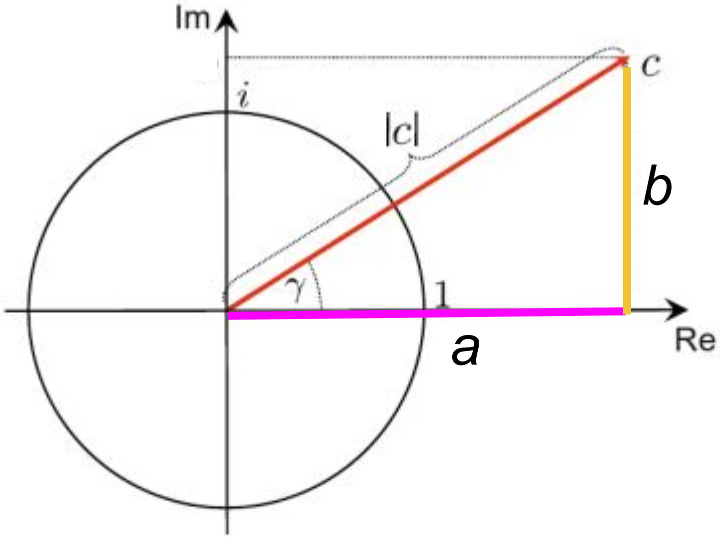
Polar coordinate representation



$$|c|^2 = a^2 + b^2$$

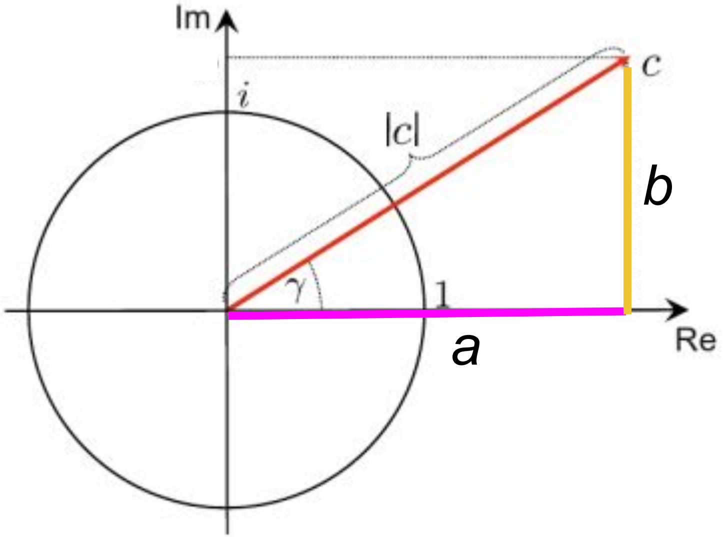
$$|c| = \sqrt{a^2 + b^2}$$

Polar coordinate representation



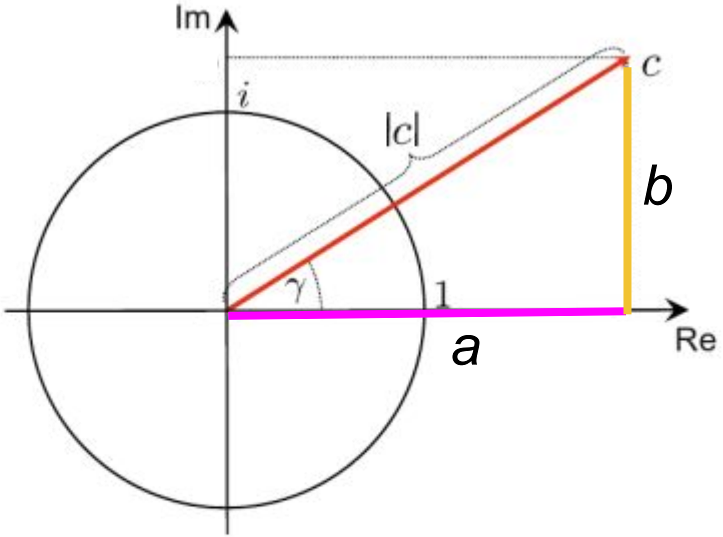
$$\cos(\gamma) = \frac{a}{|c|}$$

Polar coordinate representation



$$\cos(\gamma) = \frac{a}{|c|} \qquad \sin(\gamma) = \frac{b}{|c|}$$

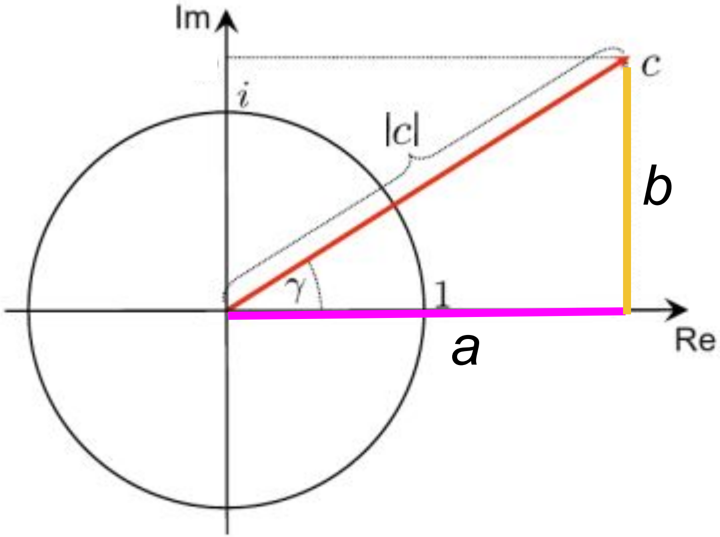
Polar coordinate representation



$$\cos(\gamma) = \frac{a}{|c|} \qquad \sin(\gamma) = \frac{b}{|c|}$$

$$\frac{\sin(\gamma)}{\cos(\gamma)} = \frac{b}{a}$$

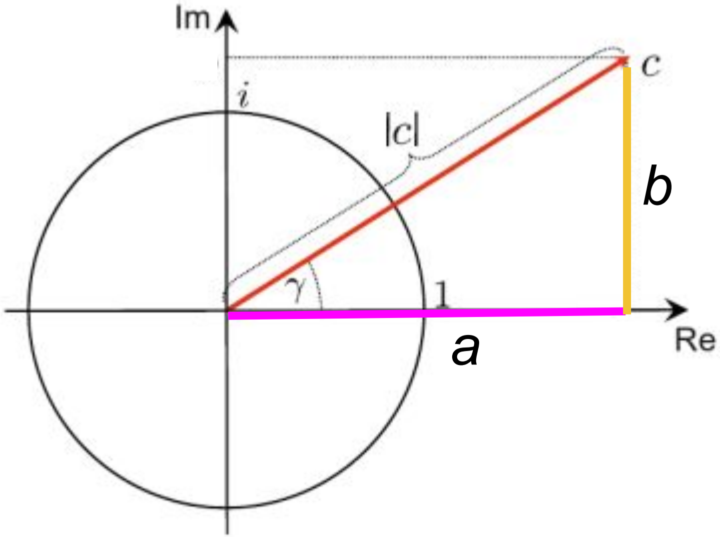
Polar coordinate representation



$$\cos(\gamma) = \frac{a}{|c|} \qquad \sin(\gamma) = \frac{b}{|c|}$$

$$\tan(\gamma) \quad \boxed{\frac{\sin(\gamma)}{\cos(\gamma)}} = \frac{b}{a}$$

Polar coordinate representation

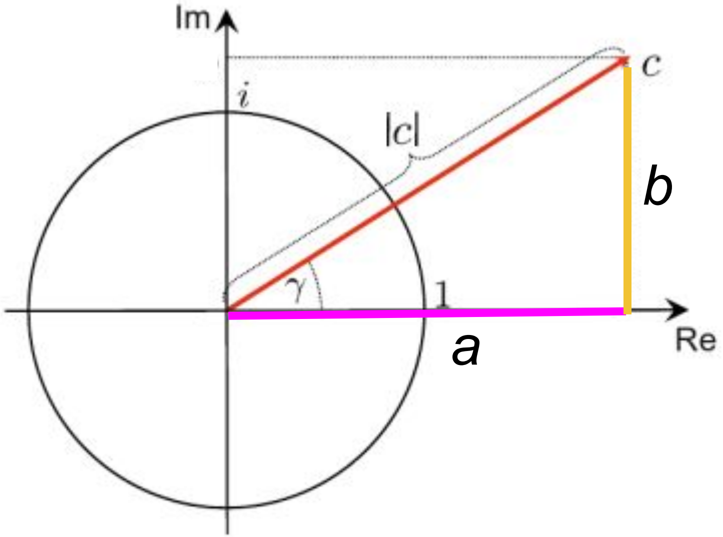


$$\cos(\gamma) = \frac{a}{|c|} \qquad \sin(\gamma) = \frac{b}{|c|}$$

$$\frac{\sin(\gamma)}{\cos(\gamma)} = \frac{b}{a}$$

$$\boxed{\gamma = \arctan\left(\frac{b}{a}\right)}$$

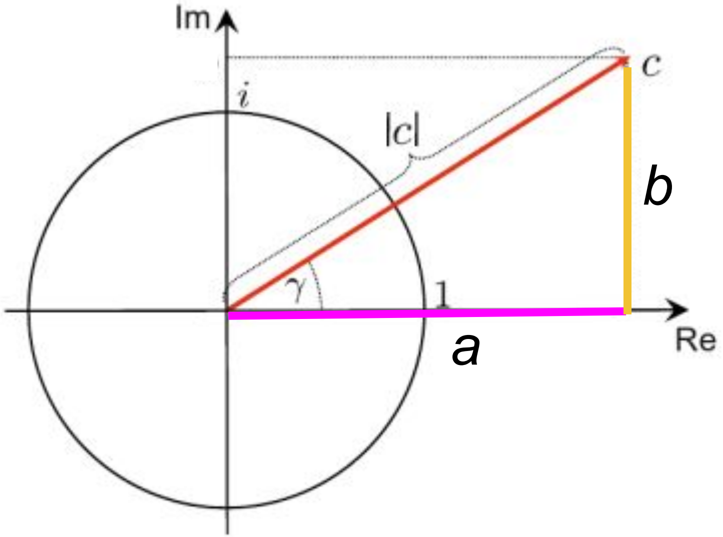
Polar coordinate representation



$$\gamma = \arctan\left(\frac{b}{a}\right)$$

$$|c| = \sqrt{a^2 + b^2}$$

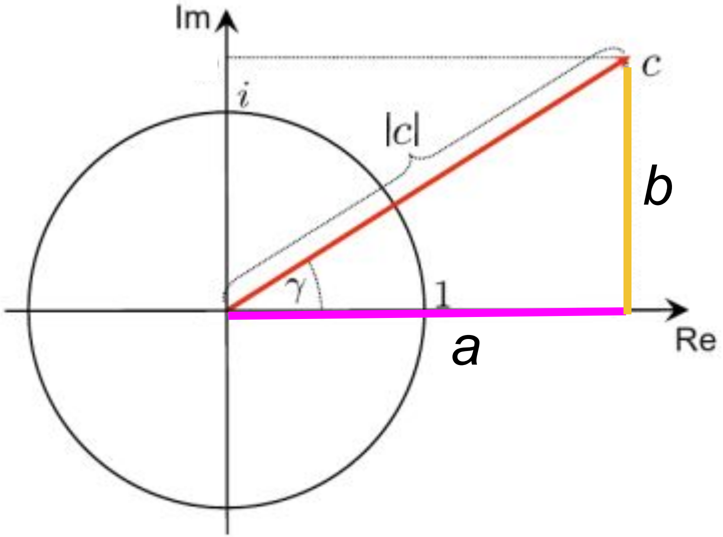
Polar coordinate representation



$$\gamma = \arctan\left(\frac{b}{a}\right)$$
$$|c| = \sqrt{a^2 + b^2}$$

$$a = |c| \cdot \cos(\gamma) \quad b = |c| \cdot \sin(\gamma)$$

Polar coordinate representation

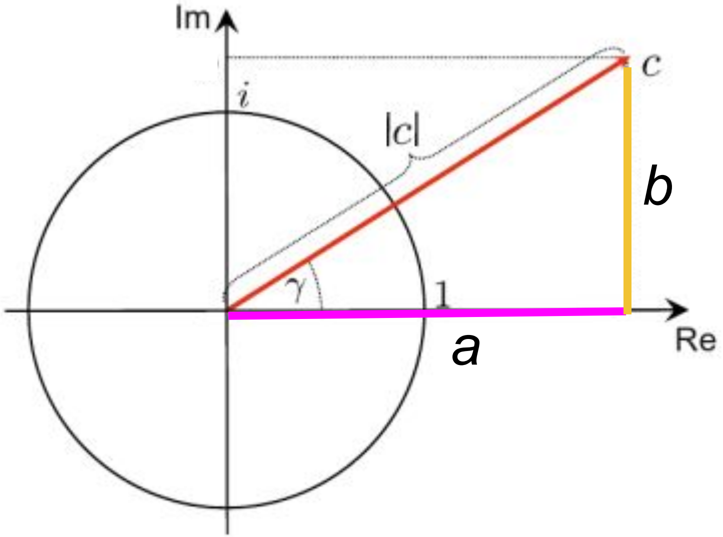


$$\boxed{\begin{aligned}\gamma &= \arctan\left(\frac{b}{a}\right)\\ |c| &= \sqrt{a^2+b^2}\end{aligned}}$$

$$a = |c| \cdot \cos(\gamma) \quad b = |c| \cdot \sin(\gamma)$$

$$c = a + ib$$

Polar coordinate representation



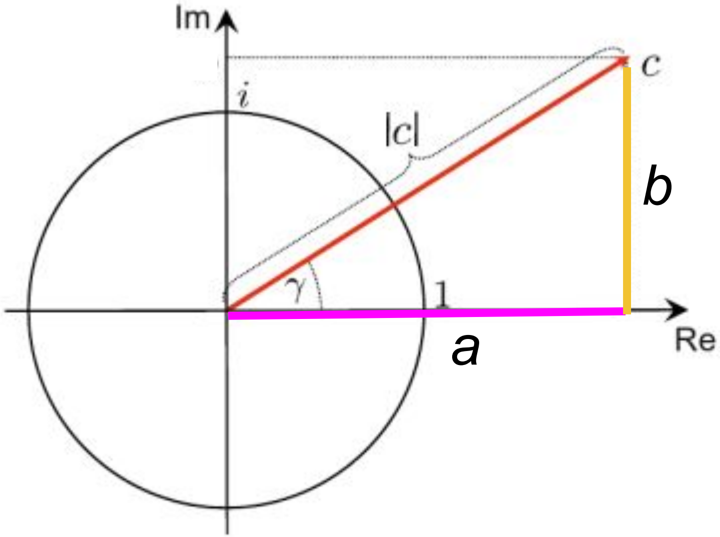
$$\gamma = \arctan\left(\frac{b}{a}\right)$$

$$|c| = \sqrt{a^2 + b^2}$$

$$a = \boxed{|c| \cdot \cos(\gamma)} \quad b = |c| \cdot \sin(\gamma)$$

$$c = \boxed{a} + ib$$

Polar coordinate representation

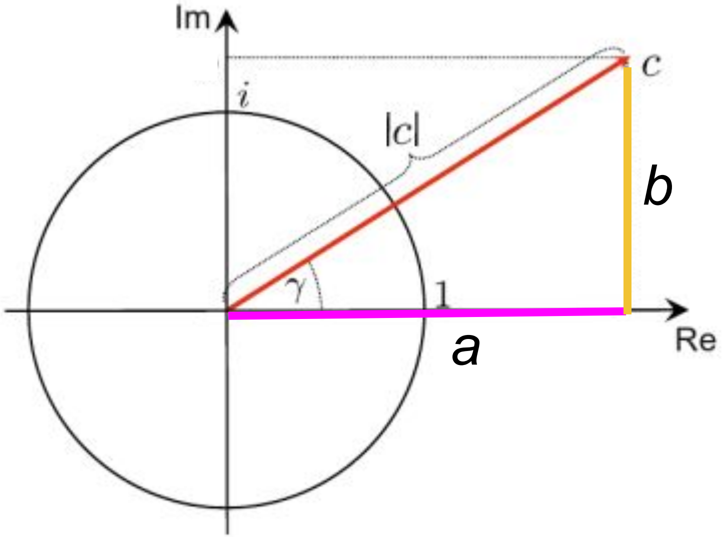


$$\boxed{\begin{aligned}\gamma &= \arctan\left(\frac{b}{a}\right)\\ |c| &= \sqrt{a^2+b^2}\end{aligned}}$$

$$a = \boxed{|c| \cdot \cos(\gamma)} \quad b = \boxed{|c| \cdot \sin(\gamma)}$$

$$c = \boxed{a} + i\boxed{b}$$

Polar coordinate representation



$$\gamma = \arctan\left(\frac{b}{a}\right)$$
$$|c| = \sqrt{a^2 + b^2}$$

$$a = |c| \cdot \cos(\gamma) \quad b = |c| \cdot \sin(\gamma)$$

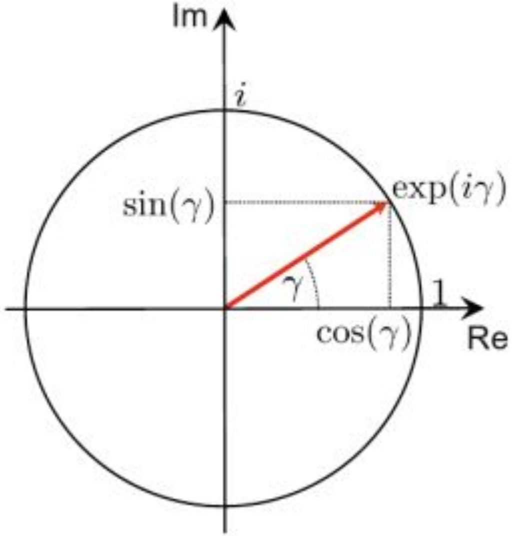
$$c = a + ib$$

$$\boxed{c = |c| \cdot (\cos(\gamma) + i\sin(\gamma))}$$

Euler formula

$$e^{i\gamma} = \cos(\gamma) + i\sin(\gamma)$$

Euler formula



$$e^{i\gamma} = \cos(\gamma) + i\sin(\gamma)$$

Euler identity

$$e^{i\pi} + 1 = 0$$

Euler identity

$$\boxed{e^{i\pi}} + 1 = 0$$

-1

Euler identity

$$\boxed{e^{i\pi}} + 1 = 0$$

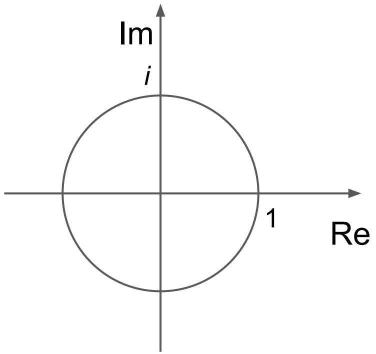
-1

$$e^{i\gamma} = \cos(\gamma) + i\sin(\gamma)$$

Euler identity

$$\boxed{e^{i\pi}} + 1 = 0$$

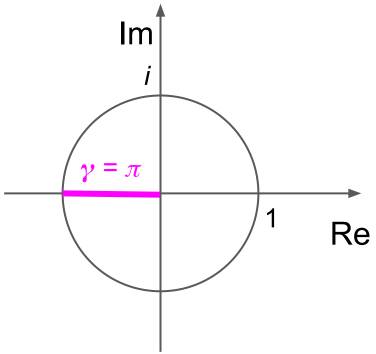
-1



Euler identity

$$\boxed{e^{i\pi}} + 1 = 0$$

-1



Polar coordinates 2.0

$$c = |c| \cdot (\cos(\gamma) + i \sin(\gamma))$$

$$e^{i\gamma} = \cos(\gamma) + i \sin(\gamma)$$

Polar coordinates 2.0

$$c = |c| \cdot \left(\boxed{\cos(\gamma) + i\sin(\gamma)}\right)$$

$$\boxed{e^{i\gamma}} = \cos(\gamma) + i\sin(\gamma)$$

Polar coordinates 2.0

$$c = |c| \cdot (\cos(\gamma) + i\sin(\gamma))$$

$$e^{i\gamma} = \cos(\gamma) + i\sin(\gamma)$$

$$\boxed{c = |c| \cdot e^{i\gamma}}$$

Polar coordinates interpretation

$$c = |c| \cdot e^{i\gamma}$$

Polar coordinates interpretation

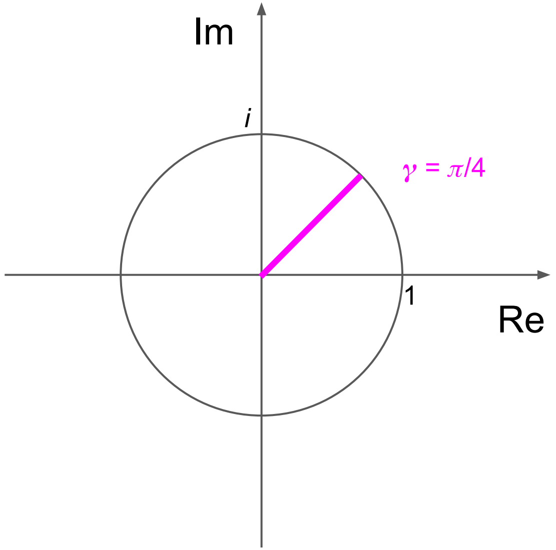
$$c = |c| \cdot \boxed{e^{i\gamma}}$$

Direction of a number
in the complex plane

Polar coordinates interpretation

$$c = |c| \cdot \boxed{e^{i\gamma}}$$

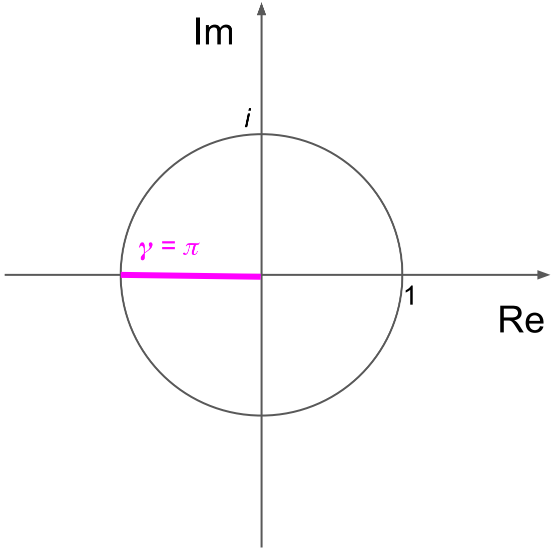
Direction of a number
in the complex plane



Polar coordinates interpretation

$$c = |c| \cdot \boxed{e^{i\gamma}}$$

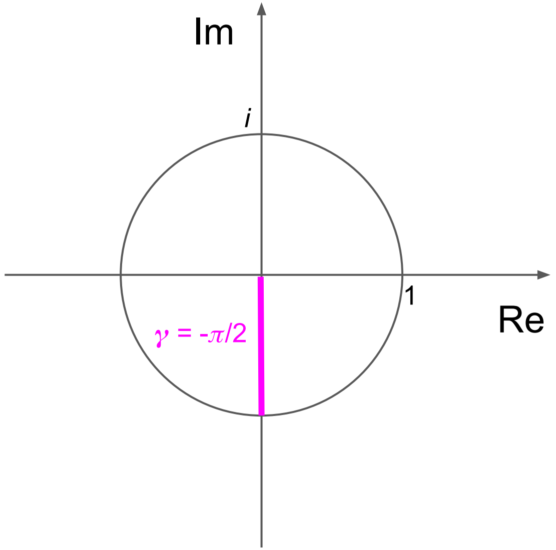
Direction of a number
in the complex plane



Polar coordinates interpretation

$$c = |c| \cdot \boxed{e^{i\gamma}}$$

Direction of a number
in the complex plane



Polar coordinates interpretation

$$c = \boxed{|c|} \cdot \boxed{e^{i\gamma}}$$

Scales distance from origin

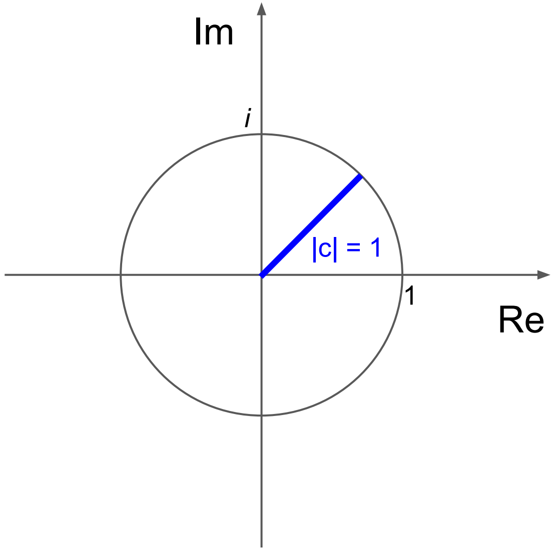
Direction of a number in the complex plane

Polar coordinates interpretation

$$c = \boxed{|c|} \cdot \boxed{e^{i\gamma}}$$

Scales distance from origin

Direction of a number in the complex plane

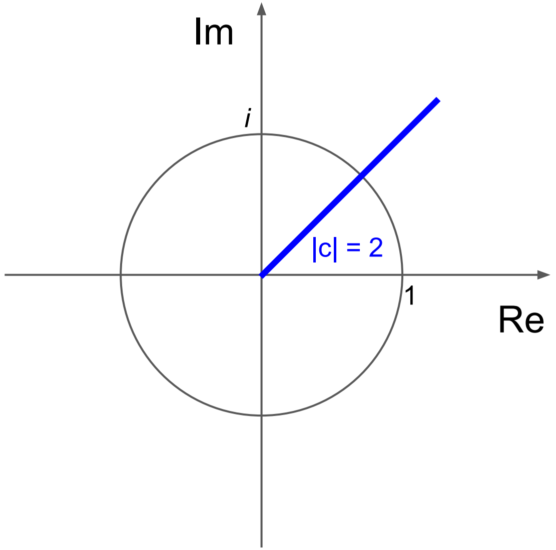


Polar coordinates interpretation

$$c = \boxed{|c|} \cdot \boxed{e^{i\gamma}}$$

Scales distance from origin

Direction of a number in the complex plane

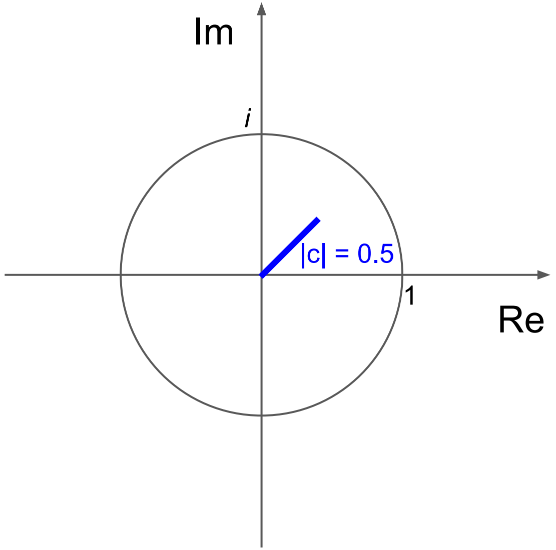


Polar coordinates interpretation

$$c = \boxed{|c|} \cdot \boxed{e^{i\gamma}}$$

Scales distance from origin

Direction of a number in the complex plane



What’s up next?

- Complex representation of Fourier transform