

INFORMATION ASSURANCE & SECURITY

Defn: Information Security - Protection of info and its critical elements, incl. the systems & hardware that use, store & transmit that info.

— Committee on National Security Systems (CNSC)

Security Goals :-

- Confidentiality -
- Integrity
- Availability

- Confidentiality - Data should not be available to 3rd party users - only to authorized users/entities.
- Integrity - While reading, data must not be altered.
Any changes taking place must be made by authorized users.
Whatever data is transmitted, the data should not be changed midway. If changed, there should be a mech to identify it.
- Availability - Data should be accessible/available whenever needed.

Attacks :-

- Threats to Confidentiality
 - Snooping
 - Traffic analysis

- Integrity
 - Modification
 - Masquerading
 - Replaying
 - Repudiation

- Availability
 - Dos
 - Distributed Dos

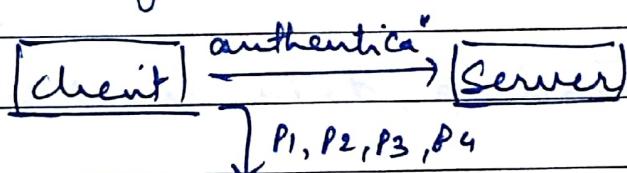
→ Traffic Analysis - Predicting data by monitoring.
Ex: Suppose you have voltage 1001..

There will be a sequence - can be predicted.

• Monitor traffic through a router, identify packets, sequence of packets based on id, source, dest, can get complete data

→ Sniffing - Will be able to read data in unencrypted form.

→ Replaying -



Authentication message
contains, say 4 parameters

Attacker parallelly redirect parameters to himself
Server replies to client and authenticates it
Client does work, session closed.

Now without modifying any content, attacker sends message to server.

Server doesn't know whether it's a replay attack or genuine message.

How to prevent? Server must be able to identify if it is replayed message or original.

① Include one extra parameter - timestamp

↳ to verify if it is fresh or old.

↳ timestamp should incl. date & time

↳ Disadv: Time synchronization bet "systems"

^{Extra}
④ A no. at once - Parameter - a no. can be sent only once.

- Masquerading - Behaving / imposing as genuine user - gives username & password.
- Modification - login to genuine user acc & change data
- Repudiation - ex: send a mail to someone, he reads mail and deletes it & says he didn't recv it. But now, logs are available, everything is traceable.
If sender / receiver denies sending / receiving
- DOS : Service denied to genuine user b/c system is engaged in checking a crafted message
ex: Launch a DOS attack by a 3-way handshake

Sync

Sync + Ack

Ack

Attacker never completes handshake :: resources exhausted.

Services & Mechanisms

Security Attack: Any action that compromises the security of info owned by org.

Security Service ① Mech / procedure to enhance the security of data processing systems & info transfers of an org.

② Intended to counter security attacks

detected
prevent
recover

Security Mech → so used to find, if attack has occurred
prevent security attack, recover data from attack.

⇒ Security Services

- ① ↳ Data confidentiality - to prevent attack
possible for confidentiality
 - ② ↳ Data Integrity - Att. To prevent attack
possible for integrity
 - anti-change
 - anti-replay demonstrating
 - ③ ↳ Authentication - identity of user
 - (HTTPS) - peer to peer (peer entity) - connection oriented comm
 - (HTTP) - data origin = no need to prove identity
- Server verifies from which source data is originated
- L-#
- ④ ↳ Non repudiation
 - proof of origin
 - proof of delivery
 - ⑤ ↳ Access Control - Prevent unauthorized access of data

⇒ Security Mechanisms :-

- ① ↳ Encipherment - Encryption + decryption
→ for confidentiality of data
 Defn → To convert readable info / data into non-readable format
 This uses an encryption algorithm

Plain Encryption, Cipher (Encrypted data)
 (readable) + key algo Text (Non Readable)

Encryption algo uses a key

At the dest machine, cipher text must be converted back to plain text : Decryption.

Decryption + key \rightarrow Plain text

\rightarrow Encipherment is 5 tuples: { E, D, P, C, K }

② ↳ Data Integrity

- when data is received, must maintain original form
- while sending data, data should be received the way it was sent. If there is any modification - receiver should identify it.
- any change to data must be made by authorized user only.

use checksum mech. to check if data is intact.

M + checksum : can verify if data integrity (Message) is maintained or not.

Ex Hash Algorithm.

hash (plain text) \rightarrow output (hashdigest)

- Attach hashdigest with message (original data)

- one way hash algorithm: not able to get back original plain text from hashdigest i.e impractical to get back original digest

\rightarrow M+checksum is stored. } for
 $h(m) \rightarrow \text{digest}$ } stored data

\rightarrow M+checksum sent to recv. } for transmitted
recv calculates checksum } data
and verifies if it is the same as the sent message. } make sure both use same
hash algo.

- ③ ↳ Digital Signature - Technique for to identify authenticity. If same user is sending different messages with diff content - diff users signatures generated for same user.
- Message + signature - sent from one entity to the other.
 - This is applicable for public key cryptosystem (asymmetric)
 - Every message has unique signature even if all messages belong to same user/entity
 - M + Signature sent to receiver
 - Receiver does not verify integrity
 - Signature used to identify whether the sign belongs to message or not.
 - Receiver identifies if signature belongs to particular message.
 - Checks if message has come from a certain entity
 - one key used for identifying signature user.

- ④ ↳ Authentication Exchange
- Mutual Authentication - both parties verify authentication of both users.
 - One side authentication - only client has to verify authentication to server.
- Some parameters are exchanged during authentication - called Authentication Exchange Parameters.

- ⑤ ↳ Traffic Padding
- Add padding bits to message i.e useless/bogus bits to confuse traffic analyzer - mitigate attacks due to traffic analysis.

⑥ ↳ Routing Control

If a big message has to be transferred, file can't be accommodated in single IP packet \therefore split in diff packets \rightarrow same original packet - same source & dest.

use source routing concept i.e sender specifies path to be taken by each packet-

If all n packets have same path - attack poss.

If diff packets are sent using different paths, not poss for attacker to attack all routers.

⑦ ↳ Notarization

Third party is involved to send packet msg from one party to another.

To avoid repudiation, third party introduced so that no party can deny.

⑧ ↳ Access Control

Service	Mech
Confidentiality	①, ⑥.
Integrity	①, ③, ②.
Authentication	①, ③, ④.
Non-repudiation	②, ③, ⑦.
Access Control	⑧.

Defns:

- Plaintext - original message / readable form
- Ciphertext - coded message (with E, K).
- cipher - algo for transforming P.T to C.T
- key - info used in cipher known only to sender, recv.
- encipher : $P.T \rightarrow C.T$
- decipher $C.T \rightarrow P.T$

- cryptography - study of encryption principle methods
 - cryptanalysis (codebreaking) - procedure to derive PT from CT; study of principles of deciphering ciphertext without knowing key ; finding what key is ~~without~~ C.T
 - cryptology - field of cryptography + cryptanalysis

\Rightarrow Cryptosystem

If \mathcal{O} is a five tuple (P, C, K, E, D) such that

4 P :

4c:

↳ :

46

4 D

\Rightarrow Requirements for secure use of Encryption Algo:

① Strong encryption algo - opponent should be unable to decrypt C.T or discover key even if opponent has a no. of C.T + P.T that produce each C.T

② Secret key known only to sender & receiver
maintain Secrecy'

\Rightarrow Type 1:

① Symmetric Encryption

- Conventional / private key / single key

• Cender & new - common key.

- all classical encryption algos are sym.

• was the only type before 1970.

② Asymmetric Encryption

```

graph LR
    A[Plaintext] -- "E. algo + key1" --> B["[ ]"]
    B -- "Transmitted" --> C["[ ]"]
    C -- "D algo + key2" --> D[Plain Text off]
    
```

Key 1 → key shared by sender & receiver

• Same key used for encryption & decryption.

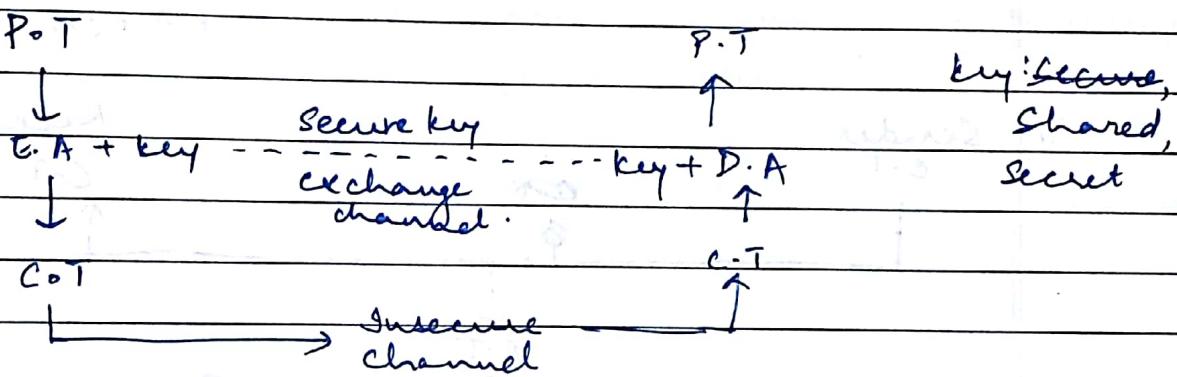
Note

All E. algos based on 2 principles:

- Substitution in which each element in P.T (bit/letter/group of bits or letters) is mapped into another element, and
- transposition: elements of P.T are rearranged.
(no info should be lost)

(Most systems involve multiple stages of subs & transp.)

→ General Idea of Symmetric Key C.S.:-



Encryption: $C = E_K(P)$ Decryption: $P = D_K(C)$

$$D_K(E_K(x)) = F_K(D_K(x)) = x.$$

(Proof).

8/11/18

Block cipher: Encryp" & decryp" takes place in blocks.

Stream cipher: bit by bit.

⇒ Objective of attacking an encryption system is to recover key and not just derive C.T from P.T.

2 methods:

i) Cryptanalysis

ii) Brute force - try all keys available in domain.

Cryptanalysis

- ① Cryptography: Science & art of creating secret codes
- ② Cryptanalysis: Science & art of breaking those codes

Cryptanalysis attacks:- (to derive keys)

- i) cipher text only - attacker only has set of C.T + algo details
- ii) plain text only - Attacker has P.T + corresponding C.T + algo details - can derive key.
- iii) chosen plaintext - similar to (ii)
- iv) chosen ciphertext - similar to (i).

attacker has to choose
P.T + C.T

ii) Sender

C.T



C.T



Receiver

C.T

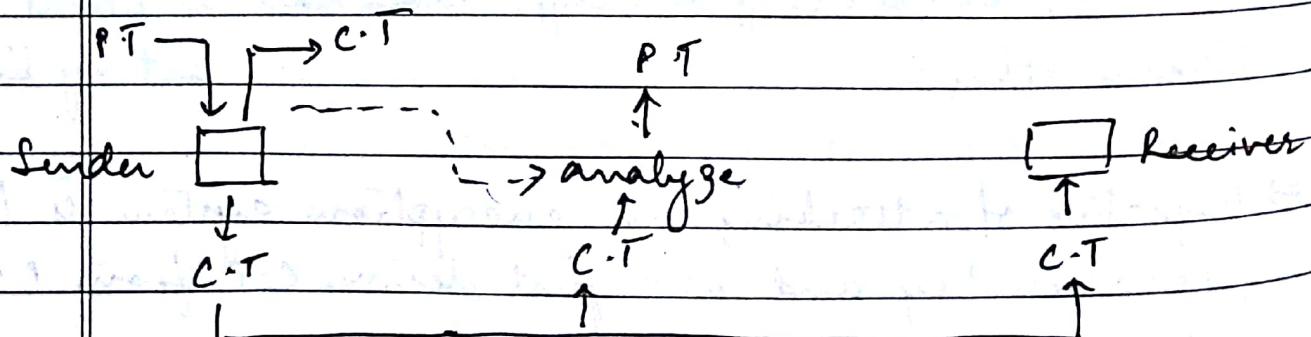


Previous pair

analyze <---| P.T ↔ C.T |---

P.T

iii) attacker chooses some P.T, finds C.T for this I/P
in some way (Lia's system)



iv) chosen cipher text analysis

↳ Pair created from chosen cipher text

* Substitution cipher:

→ Each char of P.T substituted by another char.

→ Types:

i) Monoalphabetic } cymns

ii) Polyalphabetic }

i) Relation betⁿ chars of P.T & C.T is one to one.

Ex: Hello → blood (Since both 'l' subs by 'o')

ii) Relation betⁿ P.T char & C.T char is one to many.

⇒ autokey cipher → polyalphabetic

↳ P.T char used as key

↳ to start, only one key value is used. then from P.T

$$\hookrightarrow P = P_1 P_2 P_3 \dots$$

$$C = C_1 C_2 C_3 \dots$$

$$k = k_1 \cdot P_1 \cdot P_2 \dots \quad (\text{key value + chars of P.T})$$

⇒ Additive cipher: monoalphabetic, shift, caesar

Ex of autokey:

$$C_i = (P_i + k_i) \% 26 \quad P_i = (C_i - k_i) \bmod 26.$$

P.T : a t t a c k

P value: 0 19 19 0 2 10

key stream: 12 0 19 19 0 2

C value: 12 19 12 19 2 12

C.T : M T M T C M

\Rightarrow Vigenere cipher

\rightarrow POLY

stream

block cipher

\rightarrow divide P.T into a block of fixed size.

\rightarrow Block size depends upon key size

\rightarrow use key & block for encryption

$$\text{Encryption: } c_i = p_i + k_i$$

$$\text{Decryption: } p_i = c_i - k_i$$

Ex: She is ~~listening~~ \rightarrow P.T

6 char keyword \rightarrow PASCAL

\therefore length of block = PASCAL = 6.

She is. listening

P values: 18 7 4 8 18 11

key stream 15 0 18 2 0 11

Add \rightarrow
C.T values

7 7 22 10 18 22

C.T \rightarrow H H W K S W

\Rightarrow Hill Cipher

\rightarrow Block cipher

\rightarrow E.A takes m successive P.T letters and substitutes them for m cipher text letters.

Prerequisites

\rightarrow Add to While choosing K, K inverse must exist (used for decryption) else decryption is not an inverse of encryption

\rightarrow uses matrix multiplication operation

key:

$$K = \begin{bmatrix} k_{11} & \dots & k_{1n} \\ k_{21} & & k_{2n} \\ \vdots & & \\ k_{m1} & & \end{bmatrix}$$

Say,
P.T domain size
restricted to 26
Characters.

$C \cdot T = K \times P \cdot T$ (multiplication of key and plain text)

To check if 'K' exists:

K has an inverse iff $\det K$ is invertible in \mathbb{Z}_{26} iff $\gcd(\det K, 26) = 1$.

$$K^{-1} = (\det K) \times \begin{pmatrix} k_{22} & -k_{12} \\ -k_{21} & k_{11} \end{pmatrix} \text{ where } K = \begin{bmatrix} k_{11} & k_{12} \\ k_{21} & k_{22} \end{bmatrix}$$

$$C_1 = (K_{11} P_1 + K_{12} P_2 + K_{13} P_3) \bmod 26.$$

$$C_2 = (K_{21} P_1 + K_{22} P_2 + K_{23} P_3) \bmod 26.$$

:

$$C = E(K, P) = KP \bmod 26 \quad (\text{all matrices})$$

$$P = D(K, C) = K^{-1}C \bmod 26 = K^{-1}KP = P.$$

* Transposition cipher

→ characters moved around.

→ reorders symbols

→ does not substitute one char for the other.

→ types:

i) keyless - no key used

ii) keyed - key used.

iii) combining (i), (ii)

⇒ Rail Fence Technique.

→ Arrange P.T in zigzag way.

→ keyless

Ex:

"meet me at the park"

m e m a t e a h
e t e t h p n

→ C.T: memataketethpe.

④ decide a block size → say 4.

meet m n t a e e h r e a e k + t p
meat → ct ↓
+ help
ark

⑤

use key value:

Say:

E ↓ 3 | 1 | 4 | 5 | 2 | ↑ D
E ↓ 1 | 2 | 3 | 4 | 5 |

- Divide plain text into blocks.
- Block size = key size.

3rd char in PT move to 1st etc

~~a tt~~
a | t | t | a | c | k | i | s | t | o | n | u | l | g | h | t |
t | a | a | c | t |

If size of last block \lt block size → add padding character

Combining key & keyless approaches:-

- write row by row

- read col by col

- Shift / shuffle columns using key i.e 3rd col goes to 1st etc

- read col by col.

e n e m y
a t t a c c shuffle
k s t o n
i g h t z
extra

e e m y n
t a a c t
t k o n s
h i t z g

keymed: 31452
12345

C.T : et-th eak i maotycznznts g

To decrypt: Reverse process.

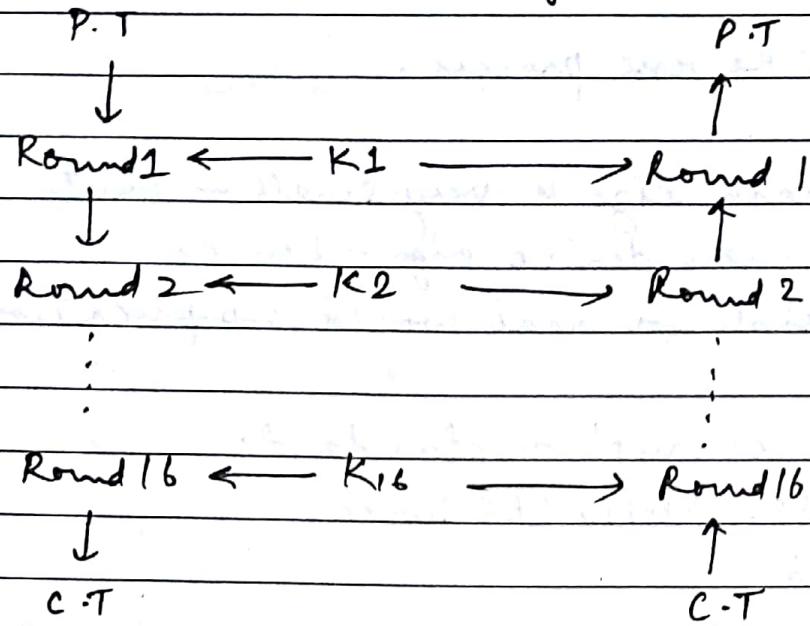
If Since domain size is very small - brute force attack very easy ∵ can derive algo + key easily.
∴ Not possible for real world applications.

- DES: Data encryption standard.
- Double DES: Apply DES twice
- Triple DES

* DES

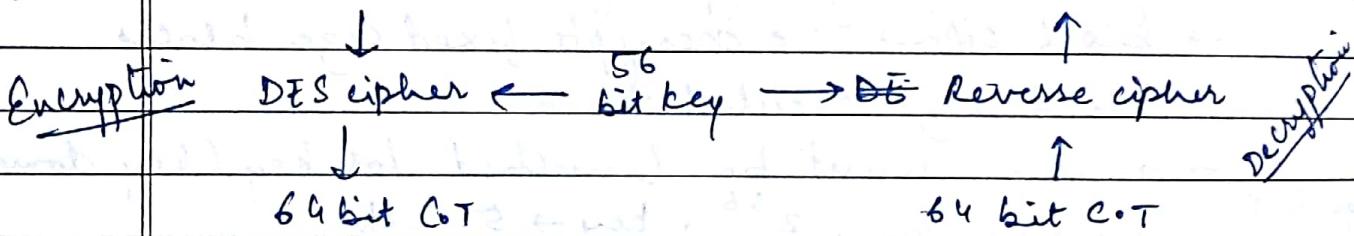
- Symmetric key cryptosystem
- Block cipher i.e encrypts fixed size blocks
block size decided.
- reduce
brute
force
- domain must be launched for key (key domain)
key domain: 2^{56} , key \rightarrow 56 bits
 $P.T \rightarrow 64$ bits
- P.T \rightarrow many steps \rightarrow C.T
all steps are same: performed numerous times
i.e same internal structure for all rounds
- ~~ideal no = 16~~
→ More no. of rounds \Rightarrow Time to convert P.T to C.T is more. If no. of rounds greater than 16 -
possible to break in.
- ∴ No. of rounds = 16 enough to secure data
- Different O/P of 1st round \rightarrow I/P of 2nd round.
O/P of 2nd round \rightarrow I/P of 3rd round
- In each round, key used is different
∴ Same procedure but diff I/P & key.
- From 56 bit key, 48 bit round key derived.
different for each round.

→ Cond": All round keys are different.



Internal structure of all rounds is same.

10/11/18 64 bit plain text 64 bit P.T.



⇒ General structure of DES

- ↳ 2 permutations (P-boxes) called initial & final permutations - just bit / position change.
- ↳ final & initial permutation are inverse of each other.

↳ ^{they} do not add to strength of DES

64 bit P.T.

Initial permutation → 48 bit key

Round 1 ← key 1 ←

Round key generator

← 56 bit key

Round 16 ← key 16 ←

final permutation ← 64 bit C.T.

→ Keys for each round are different - generated from 56 bit key - by round key generator, for each round 48 bit key is generated from 56 bit key.

* Initial & final permutation steps

Initial: 58th bit → 1st bit (goes to)

1st bit → 8th bit

Final: 1st bit → 58th bit

Reverse

Initial: Rows 1, 2, 3, 4 → even nos. Rows 5, 6, 7, 8 → odd nos.

Initial:

8	10	18	16	2
20	12	4		
22	14	6		
29	16	8		
17	9	1		
19	11	3		
21	13	5		
23	15	7		

Final: Alternative cols filled from bottom to top
starting from col 2.

40	8	18	16	24
4	1	11	3	
34	3	11	3	
3	2	10	1	
33	1	4	9	17

Ex Find O/P of initial permutation box when 1/4 is given in hexadecimal as:

0x0002 0000 0000 0001

Convert to binary → 64 bit in a bit stream only

2 bits are 1 → 15th & 64th bit

⇒ 64th bit in P.T becomes 25th bit after permutation

Since in the table it is in the 25th place.

& 15th bit becomes 13rd bit since it is at position 63 in matrix.

Starts from US

18 Habs

~~8 bit~~ 16 bit Date: / /

Date: / /

In binary

000 - - -

001

1574

64-th bit

Now check table : 15th bit becomes 63rd
64th bit 25th

* ~~10~~ In each round - also same, key & 1/f changes for each round!

In each round →

64 bits divided into 2 parts : 32 bit size of each part
↳ left part & right part

Left block involved in exclusive operation

Right " possesses processed with ~~say~~ round by
O/P of right block now is processed with left block
Then swap left block with right block.

Now in next round, left block is processed with new round key.

expansion Key is 48 bits and right block is 32 bits \rightarrow can't exor.
 \therefore Expansion is done, then exor.

Again there is exclusive operation with left block
This time 48 bit key is mapped to 32 bit so that it
can exclusively be or-ed

To compress 48 bit \rightarrow 32 bit : use S-box. i.e a set of S boxes. Each S-box takes 6 bit I/P and produces 4 bit outputs.

∴ 8 5-boxes required.

48 bit key divided into chunks of 6 bits.
Each chunk goes to S-box.

{ Right + key
 { Right + left
 swap

How does S-box work?

Each S-box maintains its own table & converts
6 bit \rightarrow 4 bit.

Then it goes to permutation box - no compression or expansion.

\rightarrow Concentration is on right block.

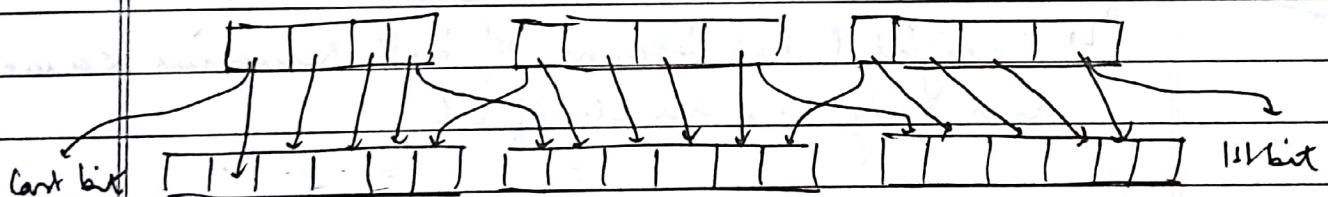
Expansion: How 32 bits \rightarrow 48 bits?

Arrange 32 bits in blocks of 4 bits.

32	1	2	3	4	5
4	5	6	7	8	9
8	9	10	11	12	13
12	13	14	15	16	17
16	17	18	19	20	21

Expansion
Permutation
Box

Each block of 4 bits expanded into 6 bits.



S box : Compression 6 bit \rightarrow 4 bit

To identify any row in table \rightarrow 2 bits reqd

col 4 bits

- Each S-box maintains its own table.
- Table 6-bit I/P, 1st bit & last bit represents row no.. Remaining 4 bits represent col no..
- In every row, values range from 0 to 15

Ex: S box I/P is 100011

row no.	1	1	1	1	1	1	1	1	1	1	1	1	1	1	1	1
col no.	1	1	1	1	1	1	1	1	1	1	1	1	1	1	1	1
1	0	0	0	1	1											
2	1	1	1	1	1											
3	1	1	1	1	1											
4	1	1	1	1	1											
5	1	1	1	1	1											
6	1	1	1	1	1											
7	1	1	1	1	1											
8	1	1	1	1	1											
9	1	1	1	1	1											
10	1	1	1	1	1											
11	1	1	1	1	1											
12	1	1	1	1	1											
13	1	1	1	1	1											
14	1	1	1	1	1											
15	1	1	1	1	1											
16	1	1	1	1	1											

Intersection of row & col \rightarrow o/p of s-box

s-box table starts from 0.

00	01	10	11	12	13	14	15
00	01	10	11	00	01	10	11
01	02	11	10	01	02	11	10
10	11	00	01	10	11	00	01
11	12	01	00	11	12	01	00

12 \rightarrow 0/1 i.e. 1100

For 011011, row \rightarrow 01 = 1
 col \rightarrow 1101 = 13 } 5 (from table).

And 5: 0100

111118

After S box: P-box, Permutation box,

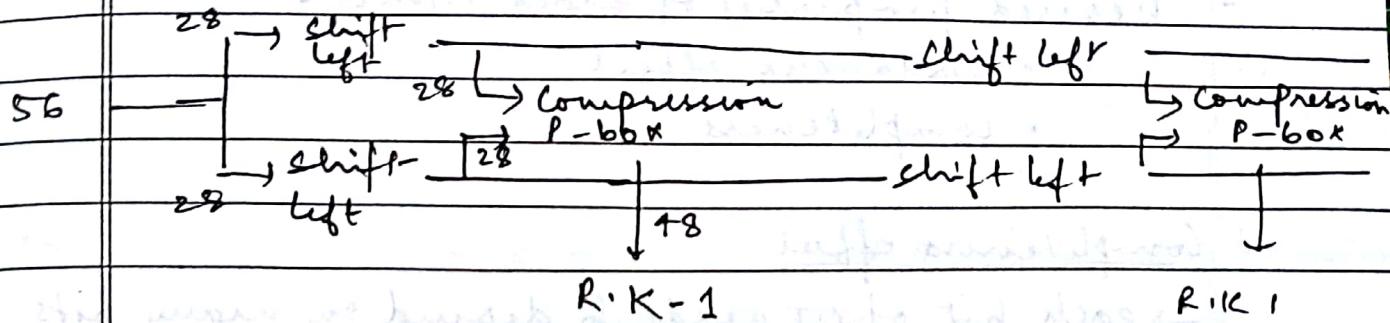
\hookrightarrow straight P-box b.c. no. of bits remains same.
 and only bits are shuffled

O/p of final permutation is cipher text.

Decryption operation:

- starts from round 16 - key 16 used first i.e. only encryption removed first
- initial & final permutation cancel each other
- expansion P box and straight P box used same for all rounds.
- * The 56 bit key (original) \rightarrow 48 bit key (new)
 flow?

Divide 56 bit key into 2 parts of size 28 bits each.
 In each round perform a left shift, right shift
 on both parts
 ↳ compression P-box which generates 48 bit



Shifting :- In Round 1, 2, 9, 16 → bits are shifted by one position
 other rounds → bits shifted by 2 positions

After R1, Till round 8 → Total no. of bits shifted is even
 After → odd
 Round 16 → 28 bits totally shifted.

- Before ~~the~~ round starts, key goes through permutation choice table

- Permutation choice 1 :

- Refer initial permutation table to calculate PC-1

I/P: 58 50 42 34 26 18

58 49

Subtract i from i^{th} row of I/P

Table is 8×7

Only 1st 4 elements of i^{th} row of I/P are used

Continue with 5th row. Add 6 to all elements.

Date: / /

* Avalanche effect

- If a single bit changes in IP or key, there is a significant change in C.T.
- Approx half bits change in C.T.
- Desired properties of block cipher:
 - avalanche effect
 - completeness

* Completeness effect

- each bit of C.T needs to depend on many bits of P.T.
- The diffusion & confusion produced by P-boxes & S-boxes in DES shows completeness

⇒ Design criteria of S-box :-

- entries of each row are permutations of values from 0 to 15.
- single bit change in IP, two bits will change in OP of S-box.

⇒ Design criteria of P-boxes :-

- B. 4 o/p's from each S-box go to diff S-boxes

56 bit key
derived by
removing parity bit from
64 bit key.

papergrid

Date: / /

⇒ Weakness in keys

Domain of keys = 2^{56} , some keys should not be used.

i) Weak keys - set of keys that produce same round key in all 16 rounds.

Ex: 00000000 00000000
00000000 FFFFFFFF (keys divided into 2 halves).

Note O/P of 1st round of decryption = O/P of 15th round of encryption

If weak keys are used, O/P after every round of decryption = P.T.

(Weak keys)

ii) Semi weak keys. - creates 2 different round keys are generated from 56 bit key and each of them is repeated 8 times (in a random order)
- first & second key in pair - RKS generated by both are the same, but order is different.
may not be same.

Total no. of semi weak keys = ⑫

iii) Possible weak keys - ⑯ such keys.

- produces 4 different round keys.
- 16 RKS are divided into 4 groups & each group is made of 4 same RKS.

Ex Assume RK1 & RK2. What is the prob. of selecting a weak, semi-weak or poss weak keys if 56 bit key is semi weak.

~~X~~

~~RK1~~

$$\frac{4+12+48}{2^{56}}$$

~~VAGY~~

Ex 000...00 00...0 - one of the possible
 28 bits 28 bits weak keys.

17/1/18

56 bit key [Brute force attack] : Try one key at a time from key domain.
 Attacker only has cipher text.
 He has to find out P.K.

~~56~~~~2~~2⁵⁶ possibilities for K :

k_1	key →
k_2	
:	
:	
:	

56 bits

Attacker tries to decrypt with K_1 . If P.T does not make sense : drop K_1 and take K_2 .

$$D_{K_1}[C_1] = P_1 \rightarrow \text{meaningless}$$

$$\text{then } D_{K_2}[C_1] = P_2$$

$$2^{56} = \text{no. of attempts} \cdot (\text{max})$$

→ Depends on the no. of systems used, processing speed etc.

→ If a network system is used, DES would be broken.
 ∵ Single stage / key DES is not secure.

Later it was proved, one need not try 2^{56} times.

They can try 2^{55} attempts (half)

This is done by using:

key complement concept.

* Key Complement Property of DES

Q.S Given a bitstream x . Prove that

$$DES_{\bar{k}}(\bar{x}) = \overline{DES_k(x)}$$

\bar{x} → denotes complement of x

\bar{k} → key

\Rightarrow Key Complement property :-

In the key domain, definitely half the keys are complement of the other half.

$$C = E(K, P) \rightarrow \bar{C} = E(\bar{K}, \bar{P})$$

Proof :-

Properties of exor :

$$\begin{aligned} \textcircled{1}. \quad \bar{x} \oplus \bar{y} &= \bar{x}\bar{y} + \bar{x}y \\ &= x \oplus y \end{aligned}$$

$$\textcircled{2}. \quad \bar{x} \oplus y = \bar{x}\bar{y} + xy = \bar{x} \oplus y$$

Assume P.T is complemented & key is complemented.
This complemented P.T is divided into two halves $\rightarrow L \& R$

Complemented R (exor) Complemented Key
 $\Rightarrow R \text{ exor key } (\text{Same} \xrightarrow{\text{from prop 1}})$

Then complement of L is exor with O/P of R exor key.

from prop $\textcircled{2} \rightarrow$ O/P of this 2nd exor is complement of left \oplus (right \oplus key).

C_L - C.T of left block

C_R - C.T of right block

For 2nd exor $\{ C_L, C_R = P_R, P_L \oplus F[P_R, K] \}$ takes place.
 Since swap
 \uparrow
 I/P to function

Same steps apply for complement.

P_R, P_L complemented

K complemented

$$C_L, C_R = \overline{P_L}, \overline{P_L} \oplus F[P_L, K] - \text{Same v/r as previous}$$

$$= \overline{P_R}, \overline{P_L} \oplus F[P_R, K]$$

Same for all 16 rounds

Concl: If key is complemented & P.T is complemented, O/P is the same as if it is not complemented.

\therefore One need not try 2^{56} combinations.

Just try 2^{55} for deriving key.



Double DES

- Enhancement for DES.

- 2 DES

64 bit PT



DES cipher

↓

64 bit IMP

middle

ter

↓

DES cipher

↓

64 bit

C.T

- Same rounds. (internal comp. of DES are same)

- encrypt given P.T 2 times

- O/P of 1st encryption, encrypt further - once more

- final C.T

- If key K_1 is used in both encryptions, there are chances that it may get cancelled.

- After 2nd DES - C.T

- If same key used in 1st stage of DES and 2nd stage of DES - chances of getting cancelled.

\therefore use different keys in 1st & 2nd stage.

(K_1) (K_2)

C.T

- At time of decryption, K_2 used first, then K_1 is used.

- Meet in the Middle Attack :- (Known PT attack)

$K_1, K_2 \rightarrow 56$ bits each \therefore thought complexity is 2^{112}
(brute force)

However, the complexity \uparrow only till 2^{57}
This was proved later

- For known P.T attack, attacker can break double DES in 2^{57} attacks.
- i.e. attacker has P.T & corresponding C.T
- Attacker takes P.T and a key from key domain
- Stores all poss C.Ts. middle texts after encryption.
- C.T is taken and decrypted it with key from key domain and stores decryption O/P in another table - (middle text)
- Middle text of encryption & decryption should be same
- \therefore Max encryptions for Middle text = 2^{56}
decryptions = 2^{56}
- \therefore Total = $2 \times 2^{56} = 2^{57}$
- Can derive k_1 & k_2 - both

$M_1 = E_{k_1}(P)$ $M_2 = D_{k_2}(C)$ known
PT & CT

- If $M_1 = M_2 \rightarrow k_1, k_2$ same - found
- sort tables in which M is stored. If they match, corresponding k_1, k_2 found.

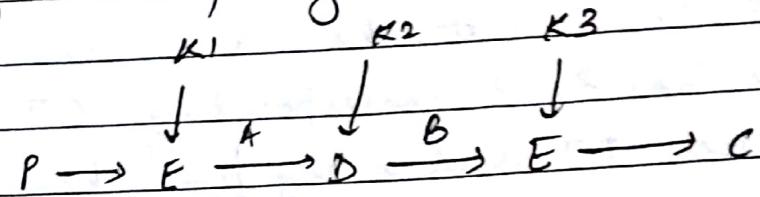
* Triple DES

- no modification in internal structure of DES
- no. of stages only varies
- 3 stages are: encryption - decryption - encryption
(for encryption: k_1, k_2, k_3)
for decryption: decryption - encryption - decryption
(k_3, k_2, k_1)
- $C = E_{k_3}[D_{k_2}[E_{k_1}[P]]]$

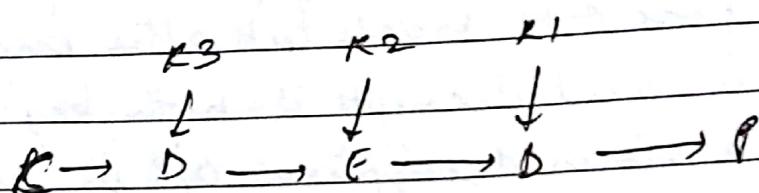
i) Triple DES with 2 keys

ii) $\overbrace{\quad\quad\quad}^3$

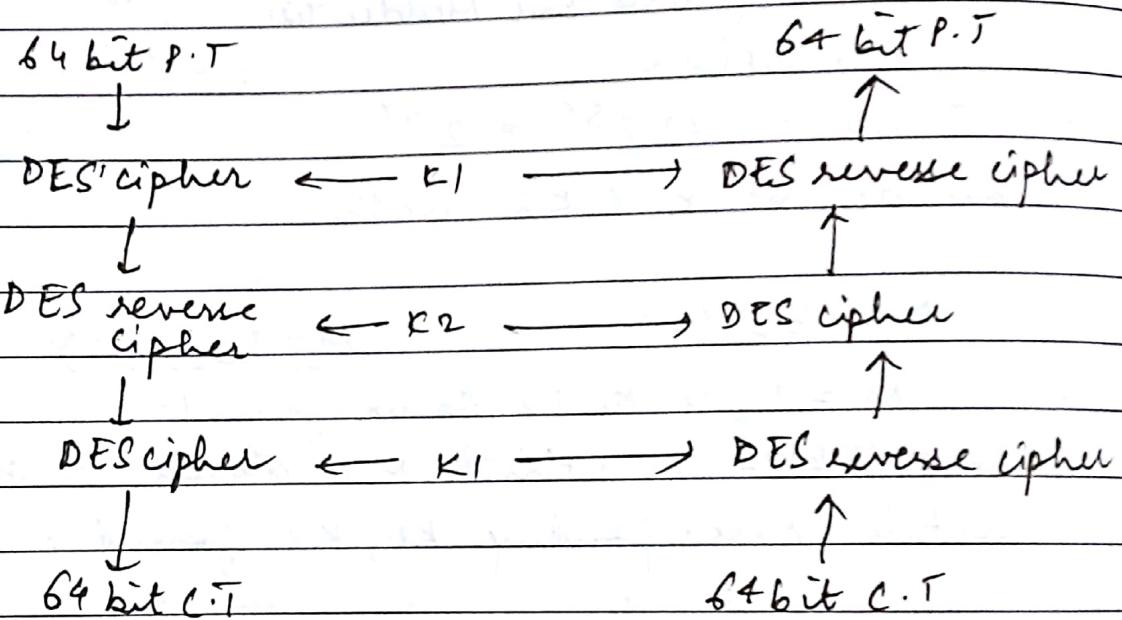
- effective key length = 168 bits



Triple DES
encryption
with 3 keys.



\Rightarrow Triple DES with 2 keys:-



If $K_1 = K_2 \Rightarrow$ single DES bcc. first 2 stages i.e. encryption and decryption cancel out.
 $\therefore K_1 \neq K_2$

Q.S. How can an adversary take advantage of ~~tation~~ ~~complement~~ ~~combinational~~ property of DES for 3 DES with 2 keys?

Solu-

$$\text{DES}_{\overline{k}} [\overline{x}] = \overline{c} = \text{DES}_k [x]$$

For triple DES with 2 keys:

$$\text{DES}_{\overline{k_1, k_2}} [\overline{x}] = \overline{c} \rightarrow \text{Prove. How?}$$

$\overline{DES}_{K_1}[x] = \overline{DES}_{K_1}[x] \rightarrow$ O/P of 1st stage of encryption
 ↳ used in end stage iff.

$$\overline{DES}_{K_2}^{-1}[\overline{DES}_{K_1}[x]] = \overline{DES}_{K_2, K_1}[x]$$

for every round

↳ O/P of 2nd round.

$$\overline{DES}_{K_1}[\overline{DES}_{K_2, K_1}[x]] = \overline{DES}_{K_1, K_2, K_1}[x]$$

With this, how many attempts are needed to break the system?

$$(2^{110})^3$$

~~56~~
~~2 × 3~~

$$K_1 \cdot K_2 \cdot K_3$$

$\uparrow \quad \uparrow \quad \uparrow$

$2^{55} \quad \underbrace{2^{55}}_{2^{55}} \quad (\text{no need to try complements})$

22/01/18 (Problems)

Design criteria applicable for all S-boxes.

4 design criteria for S-box :- Take 2 I/Ps. In these 2 I/Ps only 3rd & 4th bit should differ (1st, 2nd, 5th, 6th bit same). For these 2 I/Ps, if O/P differs ^{at least} by 2 bits → Design criteria 4.

Check 4th design criterion for S-box 2 using following pair of I/Ps :-

a) 001100 & 110000

\downarrow \downarrow (middle 2 bits diff)
 $\underline{\underline{000000}}$ $\underline{\underline{111100}}$

row → 00

col → 0110

6th design criterion for S-box 5:-

32-bit, 6-bit I/P pairs

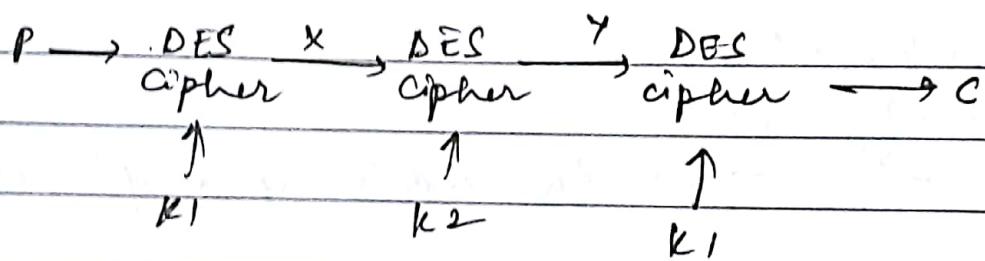
32, 6-bit I/P word pairs such that $x_i \oplus x_j \neq 000000$

These 32 I/P pairs create 32 4-bit O/P word pairs

For each pair compute O/P of S-box

$$d = y_1 \oplus y_2 \rightarrow \text{for all } 32 \text{ pairs}$$

Q5 Devise a meet in middle attack for triple DES.
with 2 keys.



Is it possible to derive K_1, K_2, K_3 in 3DES?

Solution

In 2DES we need to know PT & CT for MIM attack

Even in 3DES assume attacker has some pairs of PT & CT

For MIM attack with 3DES

X → guess

- find K1 (first)

Then MIM for 2DES (remaining
2DES ciphers)

Take random
64 bit-guess it is X

key domain $\rightarrow 2^{64}$

Decrypt X with
different keys.

* Cipher Block Modes

- diff ways to transmit data
- assume all blocks are 64 bits i.e message is multiple of 64.
- All blocks are encrypted with same key k .
- gives n C.T blocks.
- If attacker can decrypt one block, he can decrypt all blocks.
- C.T depends on something else (beside key) which is diff each time.

Suppose message divided into 5 blocks — P_1 to P_5 .

$$E_k[P_1] \rightarrow C_1, E_k[P_2] \rightarrow C_2, E_k[P_3] \rightarrow C_3.$$

$$E_k[P_4] \rightarrow C_4, E_k[P_5] \rightarrow C_5$$

Since same key is used for encryption, if 2 P.T blocks are same corresponding C.T blocks will be the same.

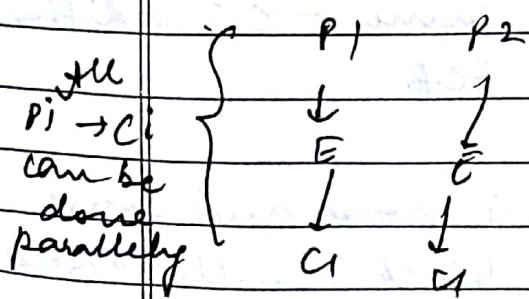
∴ The attacker only needs key, then he can decrypt rest of the blocks.

Modes of Operations categorized into :

① ECB - Electronic Code Book :- similar to telephone directory

Divide msg into blocks of 64 bits. — encrypt each block — transfer

Adv :- Parallel encryption DES.



(i) one round key generation is enough

(ii) If noise is there, i.e. bit inversion takes place in

~~if~~ - one block, the effect will be there
for only block - rest of them are unaffected

(ii) Disadv - If attacker derives key ; the attacker
can decrypt rest of the blocks.

(iii) 2 blocks of P.T same \therefore 2 blocks of C.T same
To overcome this \rightarrow CBC

(2)

CBC - Cipher Block Chaining :-

- One block attached to another.
- Chaining is applied to C.T.
- To process block P_2 , o/p of P_1 is needed and so on.
- Dependant on C.T of previous block
- To encrypt first block, initialization vector (IV)
~~key~~ is used. (IV) \rightarrow must be a secret
like key.
- Take XOR with IV in round 1, then apply DES.
- o/p CT of round $(i-1)$ is exor-ed in round i
with P.T i .

$$\text{Encryption} : C_0 = I.V$$

$$C_i = E(K, P_i \oplus C_{i-1})$$

$$\text{Decryption} : P_i = D(K, C_i) \oplus I.V$$

$$P_i = D(K, C_i) \oplus C_{i-1}$$

- Same key used for encryption
- even if blocks of P.T are same - C.T is different
- Overcomes drawbacks of ECB.

Disadvantages :- (i) If there is noise and some
bits are interchanged in block 2 - this affect
rest of the blocks after that \rightarrow only ^{two} ~~one~~ blocks
are affected

i.e. offsets that block & block after that.

94/01/18

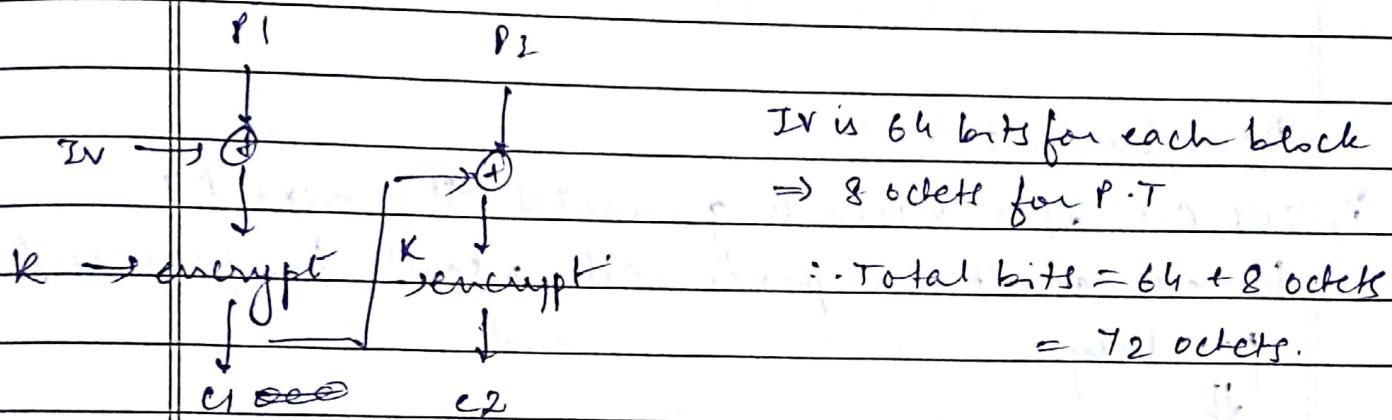
Prob 1 :- 64 octets of message - use DES in CBC mode - how many octets are sent now?

$$\text{Soln: } 64 \text{ octets} = 64 \times 8 \text{ bits}$$

Total data is divided into blocks of 64 bits.

$$\therefore \text{No. of blocks} = 8 \rightarrow 8 \text{ blocks of C.T after}$$

IV also needs to be transferred. encryption



Prob 2 : N 64 bit blocks

$$\text{a) No. of DES encryption operations} = N * 16$$

as DES must be performed on N blocks 16

times (16 rounds).

16 round keys for processing one block. Same round keys for processing \sim one block.

\Rightarrow 16 round keys to process one block and there are N blocks.

$$1 \text{ block} \rightarrow 16 \text{ rounds} \quad P_1, P_2, P_3, \dots, P_N$$

$(16 * N)$? Refer pp

$$\text{Prob 3: } 32 \text{ octets} = 32 \times 8 \text{ bits}$$

MAC - Message Authentication Code.

For integrity we add ICV:

32 octets ~~+ EC~~. D + checksum. (ICV).

data (ICV: integrity checksum value)

Date: / /

32 octets + MAC / generated using CBC DES .
 data ICV Data = 32×8 bits = 256 bits
 NO. of blocks = $\frac{256}{64}$ = 4 blocks

Each block encrypted using CBC DES

blocks of C1 C2 C3 C4 + MAC MAC = 8 octets
 { 32 octets (last c.T block) IV = 64 bits
 const 64 bits = 8 octets.

Drawbacks of CBC :

- i) Till C.T of a block is generated, the next P.T block can't be processed. \therefore can't be processed parallelly.

use Modification : CFB

④

Cipher Feedback Mode

- IV used as shift register - no circular shift, just shift
- stream cipher i.e. used to encrypt bytes of data, not blocks
- O/P of 1st encryption of IV using key value
- 64 bits in IV shifted by 's' no. of bits left, empty space occupied by O/P of previous block
- P.T is bit or byte
- A byte of P.T is XOR-ed with byte O/P of encryption
- O/P of a round goes to next round,
- select most significant n-bits
- In CBC and CFB : even if P.T blocks are the same - C.T will be different (as O/P of previous is used & shift reg contents are different)

Drawback: If some bits are inverted during transmission in C.T. how many blocks are affected?

Suppose in shift reg. 8 bits are shifted and C.T is put in place of shift in IV, how long does C.T stay in IV? - All those blocks are corrupted?

↳ 8 blocks

$$\therefore 1 + 8 = 9 \text{ blocks affected}$$

Prob 1: C.T bit corrupted - case 1

P.T bit corrupted - case 2

EBC - only that block is affected } Case 1 - C.T
CBC - affects 2 blocks } affected

If P.T gets corrupted in CBC - All blocks are affected at receiver's side.

For CFB, if shift by 8 bits - 9 blocks affected

2 bits - 33 blocks affected

$$\left(\frac{64}{2} \rightarrow IV = 32 + 1 \right)$$

↑ original

Drawback: Parallel not possible



Solution : OFB .

④ Output Feedback Mode

- output of encryption is taken (not C.T) and uses it in next shift register.

- OFB → O/P after encryption is taken in next block.

CBC, CFB → C.T used.

- stream cipher.

- O/P of 1st encryption used as feedback for next encryption
- no dependency.

(5)

Counter Mode

- IV is incremented by 1 or n from block to block automatically without depending on previous O/P
- counter incremented for each block.
- keep incrementing IV - no shift.

25/01/18

ASYMMETRIC KEY CRYPTOSYSTEM (PUBLIC)
(2 KEY CRYPTOSYSTEM)

- ① → In symmetric, both sender & receiver must have key - a major problem is distribution of key between sender & receiver.
- ② → In symm., not poss to generate signature or verify that message has reached intended user.
→ 2 keys used :- public key - encrypt
private " - decrypt
→ For digital signature :-
private key - to generate signature
public " " verify

Ex RSA - used in many real world applications

- send receiver generates pair of keys - public & private
- recvr. announces the public key - anyone can obtain it. (private key not shared)
- sender knows public key and encrypts data using this key.

- If someone intercepts the message, they won't be able to decrypt.
 - Asymmetric - because diff keys are used for encryption & decryption
 - ① Public key - encrypt & verify
private - decrypt & verify generally
 - Sender can encrypt data, can not decrypt it.
 - Only receiver can decrypt it.
 - Anyone can verify signatures (since key is public)
- Note: These Algos depend on 2 keys :-
- Receiver's public key & private key are related to each other.
 - It should be computationally feasible to perform encryption by / decryption by knowing public / private key

(iii)

Prob. Limitations of symm → ① distribution of key
② signature

3 categories of asymm:

- 1) Encryption / decryption
- 2) Key distrn
- 3) Signatures

More Properties :-

- Trapdoor secret
- one way function - function such that if $y = f(x)$, and if y is known, it is not possible to get back x i.e. f^{-1} is not feasible.
i.e. easy to compute $f(x)$ but not $f^{-1}(y)$
you can get x back from y by knowing some secrets i.e. trapdoor secrets.
- Secrets used to get back original message → trapdoor secret

factorization Ex: $n = p \times q$ prime nos. Easy to compute n
prob. Not for p, q (esp. if they are big)

discrete logarithm problem Ex: $y \equiv x^k \pmod{n}$ → easy to compute
congruent to y .
If we know trapdoor L'
such that $k \times k' \equiv 1 \pmod{\phi(n)}$
then we can use $x = y^{k'} \pmod{n}$ to find x

RSA CRYPTOSYSTEM

- best for one way trapdoor functions
- DES takes characters and produces characters
- RSA takes integers integers
- public & private key required

Step 1: Generating pair of keys

i) generate 2 prime nos. - p, q . (p, q must be large)

To check if generated integer is prime no. or not

a) check divisibility by 2

b) if not divisible by 2, use primality alg.
to check (P)

ii) $N = p \times q$ (4 bit \times 4 bit) $\Rightarrow N$ is greater than 4 bits

most be large: $\underbrace{q}_{\text{large}} \quad \underbrace{512}_{\text{large}} \quad \Rightarrow \quad \underbrace{\text{"}}_{\text{large}} \quad \underbrace{\text{"}}_{\text{large}} \quad \underbrace{512}_{\text{large}}$

When value of N is more, it leads to factoring problem. (-time taken more)

iii) choose e and d . Compute $\phi(N)$ → values ^{quotient} too

Euler's quotient: Take prime nos, p . count from -1 to p .

Find if each of those integers & p are co-prime or not

Ex) $p = 7$. Find $\text{GCD}(1, 7) \rightarrow 1 \therefore 1, 7$ are coprime

$\text{GCD}(2, 7) \rightarrow 1 \quad 2, 7$ are coprime

$\text{GCD}(3, 7) \rightarrow 1$

$\text{GCD}(4, 7) \rightarrow 1$

$\phi(N) \Rightarrow$ the integer lesser than N and co-prime to N .
Date: 1/1/2018

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\therefore for 1 to p we get $p-1$ co-primes.
Euler's genoertiv: No. of co-primes is $p-1$ if p is prime and if N is non-prime
If p is prime no., $\phi(p) = p-1$

$\phi(N) = (p-1) \times (q-1) = \cancel{p} \times \cancel{q}$ ($\because p, q$ are prime \therefore odd)
even even [How many co-primes exist from 1 to N ?]

iv) Generate pair of keys - public & private.

- generate an integer no. e verify if inverse (multiplicative) exists or not wrt $\phi(N)$

$$e \cdot d \equiv 1 \pmod{\phi(N)} \Rightarrow \text{inverse exists.}$$

\therefore Now: (i.e. $e \cdot d \pmod{n} = 1$)

$(e, N) \rightarrow$ public key

$(d, N) \rightarrow$ private key.

29/1/18 Derive e & $d \rightarrow$ choose any 1 value ~~e, d~~ $< \phi(n)$

$$e \cdot d \equiv 1 \pmod{\phi(N)}$$

B If this cond' satisfies, e is multiplicative inverse of d and vice versa.

(e, N) - public (d, N) - private

v) Announce public key. It is advised to disclose $p, q, \phi(n)$.

Public key announced as (e, n) . Private key retained by entity.

Receives generate signature and sends it.

vi) Encryption: Convert P.T to integers and
 $c = P^e \pmod{n}$

vii) Decryption: $P = c^d \pmod{n}$

Ex $p=7, q=11$
 $\therefore N = 77 \quad \phi(77) = (p-1)(q-1) = 60$.
 choose e, d such that
 $e \cdot d \text{ mod } \phi(N) = 1$.

$e \rightarrow 13$ Then $d = 37$.

If $P \cdot T = 5$, $13 \cdot 37 \text{ mod } 60 = 1$.

↳ encrypted : $5^{13} \text{ mod } 77 = 26$.
 $C \cdot T = 26$

↳ decrypted : $26^{37} \text{ mod } 77 = \boxed{5} \leftarrow P \cdot T$

$\phi(N) = 60, \phi(N)^* \rightarrow \mathbb{Z}_{60}^*$.

~~problem~~

Why should e & d be less than $\phi(N)$.

Suppose $e =$ more

If message to be sent is less than ~~N~~, $M < N$,

Ex : $\overset{M}{5} = 128 \quad C = 128^{13} \text{ mod } 77$

But $P \neq 128$.

∴ if you want to transmit something greater than N ~~or~~, divide M into blocks.

$$\begin{array}{|c|c|c|} \hline 1 & 2 & | 8 \\ \hline \end{array} \quad \begin{matrix} \uparrow & \uparrow \\ 12 < 77 & 8 < 77 \checkmark \end{matrix}$$

→ ~~Encrypt~~ Encrypt 12 and 8 separately.

Encrypt - $12^{13} \text{ mod } 77 = 12$

Decrypt - $12^{37} \text{ mod } 77 = 12$

$8^{13} \text{ mod } 77 = 50$.

$50^{37} \text{ mod } 77 = 12$.

~~Note~~ : For alphabets - integer values are assigned between 00 and 25. (two digit)

Ex : NO $\rightarrow N=13$
 $O=14 \quad \therefore P \cdot T = \underline{1314} \leftarrow \text{concatenated}$

$$1 < e < \phi(n)$$

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Date: / /

Q3 If $e = 31$, $n = 3599$, $d = ?$

$$31 + d \bmod 3599 = 1$$

extended
extracted

Euclidean algo provides inverse of no. given e, n ?
↳ If $\gcd(\phi(n), e) = 1$, then inverse of e ($i.e.$) exists.

else inverse does not exist.

If $\gcd(a, b) = 1$

and if we find 2 nos. such that $sxa + txb = 1$

Then t is inverse of b .

$$r_1 \leftarrow \phi(n) (a)$$

$$s_1 = 1 \quad t_2 = 0$$

$$r_2 \leftarrow e \quad (b).$$

$$t_1 = 0 \quad t_2 = 1.$$

while($r_2 \neq 0$)

{

$$q \leftarrow r_1 / r_2;$$

After this, compute :-

$$r \leftarrow r_1 - q \times r_2$$

$$s_1 \leftarrow s_2 \quad Sxa + txb = \gcd(a, b).$$

$$s_2 \leftarrow r_2$$

If this gives $\gcd(a, b) = 1$

$$r_2 \leftarrow r$$

then inverse of b exists

$$s \leftarrow s_1 - q \times s_2;$$

wrt $a - t$ is inverse of b

$$s_1 \leftarrow s_2$$

$$s_2 \leftarrow s$$

If it is not 1, inverse of

$$t \leftarrow t_1 - q \times t_2$$

b wrt a does not exist.

$$t_1 \leftarrow t_2$$

$$t_2 \leftarrow t$$

}

$$\gcd(a, b) \leftarrow r_1$$

$$s \leftarrow s_1$$

$$t \leftarrow t_1.$$

In RSA, public key of given user is $e=31$, $n=3599$

Find private key of user. $d=?$

Ans: Step 1: Compute $\phi(n)$ by $n=p \times q$.

Fact Step 2: Factorize n into p, q .

$$\begin{aligned} p &= 59, \\ q &= 61 \end{aligned} \quad \left\{ \begin{array}{l} 59 \times 61 \\ = 3599 \end{array} \right.$$

$$\therefore \phi(n) = 60 \times 58 = 3480$$

$$= 3480.$$

$$ed \equiv 1 \pmod{\phi(n)}$$

$$31 \cdot d \equiv 1 \pmod{3480}$$

$$\boxed{d = 3031}$$

$$\gcd(3031, 3480) = 1$$

* Attacks on RSA:-

① ~~factoring attack~~

- n small, attack possible. If n is large, diff to factor
- message domain is small.

② Chosen C-T attack

- objective is not to get 'd', but to get M from C-T.
- It is assumed that sender sends message to receiver and receiver sends an ack message back.
- Suppose attacker intercepts and gets C-T for which publickey is (e, N)
- goal is to derive M from C
- Attacker takes random no. $r : cr^e \pmod{N}$ is found. b inverse should exist
- send this to receiver, decryts using (d, N) with some
- Bob decrypts using:

$$[C \cdot r^e]^d \pmod{N} = c^d \cdot r^{ed} \pmod{N}$$

Actual message: $M \bmod N$

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Date: / /

$$= M \cdot 1 \bmod N.$$

$$C = M \cdot d$$

$d \bmod N \rightarrow \text{inverse}$.

Receiver sends back this $M \cdot r \bmod N$

Attacker intercepts again and uses inverse:

$$M \cdot r \cdot r^{-1} \bmod N = M \bmod N.$$

Attacker gets message M . ✓

③ Probabilistic Attack / → Plaintext Attack
Cyclic

- Given e, N, C it is easy to derive M . $(C, e, N) \rightarrow M$
- Keep encrypting ' C ' again & again to derive C again
- $C = M^e \bmod N$.

$$C_1 = C^e \bmod N \quad \text{If } C_1 = C, \text{ stop} \cdot \text{else:}$$

$$C_2 = C_1^e \bmod N \quad \text{If } C_2 = C, \text{ stop} \cdot \text{else:}$$

:

e. If $C_n = C_{n+1}$ it means $C_n = M$.

- Proved that this will happen.

Q5 In public key cryptography, to intercept P.T. $C = 10$
sent to a user whose public key is $e = 5, N = 35$
what is the P.T?

Soln

$$C_1 = 10^5 \bmod 35 = 5$$

$$C_2 = 5^5 \bmod 35 \quad : \# 1 = 5$$

$$= 10 \quad \approx C.$$

31/1/18

Q5 Launched/demonstrate a chosen C-T attack
by making use of given data value of $n = 33$
and $e = 7$, $C = 14$.

Soln

$$33 = 3 \times 11 \quad p = 3, q = 11$$

$$\phi(n) = 2 \times 10 \\ = 20$$

$$7 \cdot d \bmod 20 = 1 \Rightarrow (d = 3)$$

choose s such that x^s exists w.r.t. $\text{mod } N$.

If $r = 2$, then $r^{-1} = 17$.

$$(2 \cdot d \text{ und } 38 = 1)$$

Attacker sends $C \cdot g^e \bmod n$

$$= 14 \cdot 2^7 \cdot \text{mod } 33 = 1792 \text{ mod } 33 \\ = (10).$$

Decryption takes place :- $(4 \cdot 2^7)^3 \text{ mod } 33$.

$$\begin{aligned} & \cancel{\left(+4^3 \right) \cdot \left(2^{6+21} \right) \bmod 33} \quad 10^3 \bmod 33 \\ & \uparrow \quad \downarrow \\ & M. \end{aligned}$$

Multiply with x^{-1}

~~This is Mr.~~

$$\begin{aligned} &= 10 \cdot x^{-1} \bmod 33 \\ &= 10 \cdot 17 \bmod 33 \\ &= \textcircled{5} M \end{aligned}$$

To verify $7^4 \equiv 5 \pmod{33} = 14 \neq 0 \therefore$ verified.

Attacks continued

④ Blind Signature Attack

M + Signature

to be computed - uses private key.

Signature = $M^d \bmod N$. where (d, N) is private key
(Encrypt message using private key of sender).

5

R

Receiver has to verify signature
i.e. verify if signature corresponds
to M.

M + Signature

1

M + Signs

This is done using public key of sender.

$\therefore (\text{Signature})^e \bmod N.$

$$\begin{aligned} &= (M^d)^e \bmod N = (M^{d \cdot e}) \bmod N \\ &= M \bmod N \\ &= M. \end{aligned}$$

Compare this M (from signature) with M sent by sender.

Verifies authenticity/signature, verifies that it is signed by actual genuine entity.

Suppose, attacker want to send a message.

Attacker sends message in a way such that the receiver should think that message came from genuine sender and not attacker.

\therefore He needs signature of sender.

↳ Almost similar to chosen CT attack.

Objective is to find signature from sender

\therefore Attacker generates a random no. such that inverse exists wrt N .

Attacker computes $M' = r^e M \bmod N$. → public key
↑
for this he needs ~~BS~~
signature

Attacker requests sender to sign M' .

If sender agrees to sign his private key

$$(M')^d \bmod N.$$

$$\begin{aligned} &= (r^e M)^d \bmod N = r^{ed} \cdot M^d \bmod N \\ &= r \cdot M^d \bmod N. \end{aligned}$$

M^d is the signature for M from sender.

\therefore To obtain value M^d , multiply r^{-1}

$$\begin{aligned} s \cdot M^d \bmod N &\xrightarrow{\times s^{-1}} s^{-1} \cdot s \cdot M^d \bmod N \\ &= 1 \cdot M^d \bmod N \\ &= \underline{M^d \bmod N}. \end{aligned}$$

Signature for M obtained ✓

∴ Now attacker can send message + signature to receiver

② Plaintext Attack

01/21/18

Congruence of Squares - To compute factors of N .

To factor $N \rightarrow N \equiv x^2 - y^2 = (x-y)(x+y)$

Suppose $x^2 \equiv y^2 \pmod{N} \rightarrow x^2 - y^2 \equiv y^2 \pmod{N}$?

Then $x^2 - y^2 \equiv 0 \pmod{N}$

compute $(x+y) \& (x-y) \pmod{N}$

2 simple
methods
by
 N is
small

$$\text{Ex: } 34^2 = 8^2 \pmod{91}$$

$$x^2 - y^2 \equiv y^2 \pmod{N}.$$

$$\text{Ex: If } N = 1649.$$

Step 1: ① Take some integer say 41.

$$\text{Find: } 41^2 \pmod{1649} = 32. \quad \text{--- ①}$$

Take another integer, say 43.

$$\therefore 43^2 \pmod{1649} = 200. \quad \text{--- ②.}$$

Step 2: Multiply 41, 43.

$$32 \times 200 = 6400 = (80)^2.$$

$$(41 \cdot 43)^2 \equiv (80)^2 \pmod{N}. \quad (\text{In the form of } x^2 \equiv y^2 \pmod{N})$$

$$\left. \begin{array}{l} 32 = 2^5 \times 5^0 \\ 200 = 2^3 \times 5^2 \end{array} \right\} (41 \cdot 43)^2 = 2^8 \times 5^2$$

↓

$$\begin{aligned} 6400 &= 2^8 \times 5^2 \\ &= (2^4 \times 5^1)^2 \end{aligned}$$

④

$$\therefore (41 \cdot 43)^2 \equiv (2^4 \times 5^1)^2 \pmod{1649}$$

$32 = 2^5 \times 5^0$ take exponents 5 and 0
and apply mod operations with bases 2.
 $\rightarrow 5 \bmod 2$ and $0 \bmod 5$.

$$32 \rightarrow \begin{bmatrix} 5 \\ 0 \end{bmatrix} \equiv \begin{bmatrix} 1 \\ 0 \end{bmatrix} \pmod{2} . 200 \rightarrow \begin{bmatrix} 3 \\ 2 \end{bmatrix} \equiv \begin{bmatrix} 1 \\ 0 \end{bmatrix} \pmod{2}$$

⑥ Common Modulus Attack

For encrypting $M \rightarrow$ use 2 public keys (e_1, N) and $(e_2, N) \rightarrow$ same N . (say it is for 2 diff users or same user with diff keys).

Is it poss to derive M from intercepted C.T?

~~$$M - \begin{cases} (e_1, N) & \text{public key}_1 \\ (e_2, N) & \text{public key}_2 \end{cases}$$

$\left[\begin{array}{l} e_1, e_2 \text{ must} \\ \text{be coprime} \end{array} \right]$~~

~~$$\begin{aligned} c_1 &\equiv M^{e_1} \pmod{N} \\ c_2 &\equiv M^{e_2} \pmod{N} \end{aligned} \quad \left\{ \begin{array}{l} \text{same message, same modulus,} \\ \text{diff. public keys.} \end{array} \right.$$

∴ there is possibility of deriving M .~~

Attacker must have 2 values in addition to c_1, c_2 ,
 $e_1, e_2, N \rightarrow x \& y$ such that :

$$x \cdot e_1 + y \cdot e_2 = 1$$

$$\begin{aligned} \text{Compute: } c_1^x \cdot c_2^y &\pmod{N} \\ &= (M^{e_1})^x \cdot (M^{e_2})^y \pmod{N} \\ &= M^{e_1x + e_2y} \pmod{N} \\ &= M \pmod{N} \end{aligned}$$

~~$c_1 \cdot x + e_2 \cdot y = 1$~~

∴ M derived

⑦ Plain Text Attack - Short Message Attack

Assumption : Message domain size is very small.

Assume message = 4 bits
 - Possible messages are: 0000, 0001, ..., 1111
 (e, N) publically known.

$$C = M^e \text{ mod } N$$

Q3 If C is intercepts C , one can easily identify M by exhaustive search method. As you know C, e, N - since the poss. messages are less. find $(M_i)^e \text{ mod } N$ and see if it matches C . Keep doing this till message is found.

$$(M_1)^e \text{ mod } N = C_1$$

$$\text{if } C \neq C_1,$$

$$(M_2)^e \text{ mod } N = C_2$$

$$\text{if } C \neq C_2$$

$$(M_n)^e \text{ mod } N = C_n$$

Message size should be small.

To mitigate this attack:
 add bogus/padding bits.
 so msg not readable to attacker even if he decy

Q3 Suppose Bob uses RSA cryptosystem with a large mod N for which factorization can not be found in a reasonable amount of time. Suppose Alice sends a message to Bob by representing each alphabetical character as an integer betⁿ 0 to 25 and then encrypting each no. separately using RSA with larger e and larger N .

Is this method secure?

Ans

→ Short message attack

→ encrypt one char at a time (stream cipher)

→ each no. is encrypted separately : M is a single character.

IM → Short message attack possible $\rightarrow 26$ possibilities only