AI Assignment 2 (i)

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Approach

Graph Topology

The chosen topology is the Erdős–Rényi model, a model for generating random graphs. This model is parametrised by two parameters: n, the number of nodes and p, the probability for any pair of nodes to be connected by an edge.

The Erdős–Rényi model is popular for its simplicity. In this model, edges are created independently of each other, making the presence of one edge not influence the likelihood of another. The simplicity of the model allows for more focus on the colouring algorithm itself, without the additional complexity that a more structured graph might introduce.

Key Parameters

Number of Nodes (n): 100 nodes were used for the graph. This size is substantial enough to provide interesting colouring challenges while remaining computationally manageable.

Probability of Edge Creation (p): A probability of 0.1 was used to ensure the graph was sparse. This means that each node, on average, connects to 10% of the other nodes, reducing the likelihood of creating a dense graph where nearly every node is connected to every other node. A function was also implemented to ensure that each node has a minimum of 2 connections if the number of nodes was to be reduced, each node would still have at least 2 connections if possible.

Number of Colours: 10 colours were provided for the nodes, which is based on the heuristic that square root n generally gives a reasonable number of colours to start with for a graph of size n. This is not the minimal number of colours needed for a graph of this size but is chosen to ensure a high probability of finding a solution.

Algorithm Description

1. **Initialisation:** Assign a random colour to each node from a fixed set of colours.
2. **Iteration:** Each node examines the colours of its immediate neighbours.
3. **Adjustment:** If a node shares a colour with any of its neighbours, it selects a new colour at random from the remaining colours.
4. **Conflict Check:** The algorithm continues iterating until either there are no conflicts.
5. **Termination:** The algorithm stops when no two adjacent nodes share the same colour.

Experimentation & Results

Experimentation Methodology:

- Used an Erdős–Rényi model graph with 100 nodes *(n=100)* and a connection probability of 0.1*(p=0.1)*.
- Initial colour assignment was random, choosing from a selection of 10 different colours.
- Implemented a decentralised algorithm where each node independently adjusts its colour based on the colours of its neighbours to resolve conflicts.

Results Analysis:

- The algorithm was able to reduce the number of colour conflicts consistently (although only 1 iteration was needed for 100 nodes), leading to a conflict-free colouring of the graph.
- The graph below shows a clear decrease in the number of colouring conflicts in a graph from over 40 to 0 as the iterations progress from 0 to 1. This indicates that the decentralised graph colouring algorithm is effectively resolving conflicts. The consistent downward trajectory without any increase in conflicts suggests there are no oscillations or regressions in the algorithm's performance.

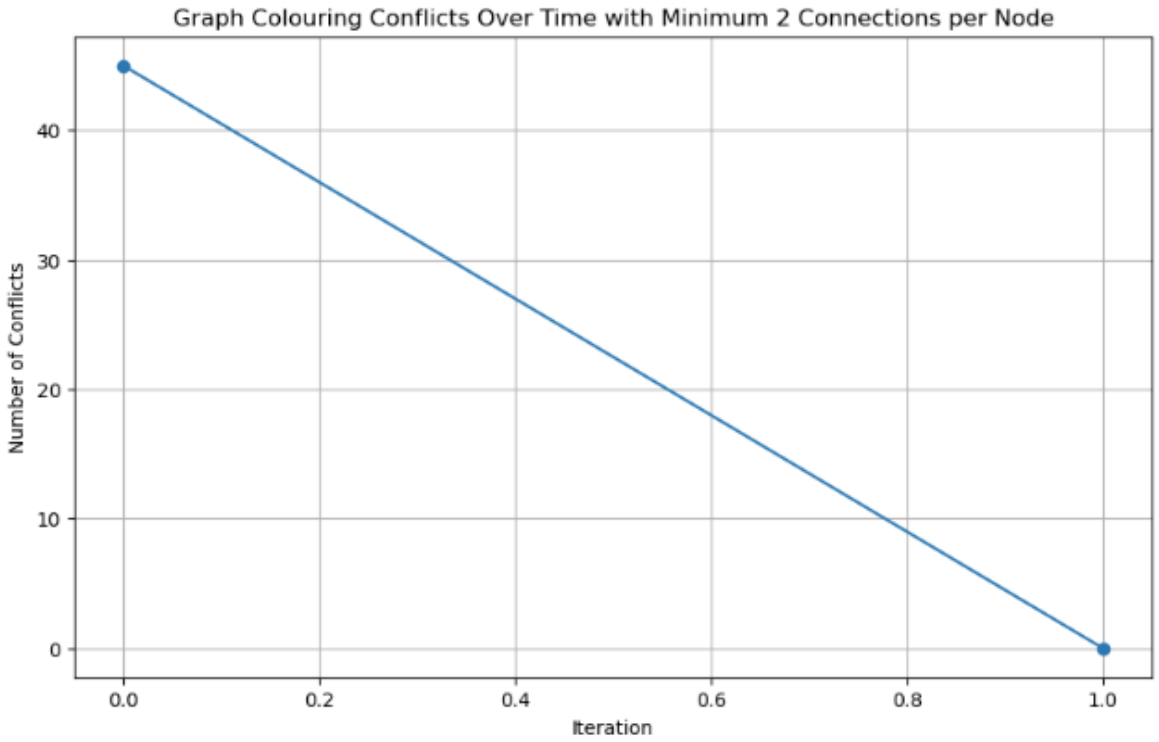

Figure 1: Graph showing conflicts being resolved over iterations

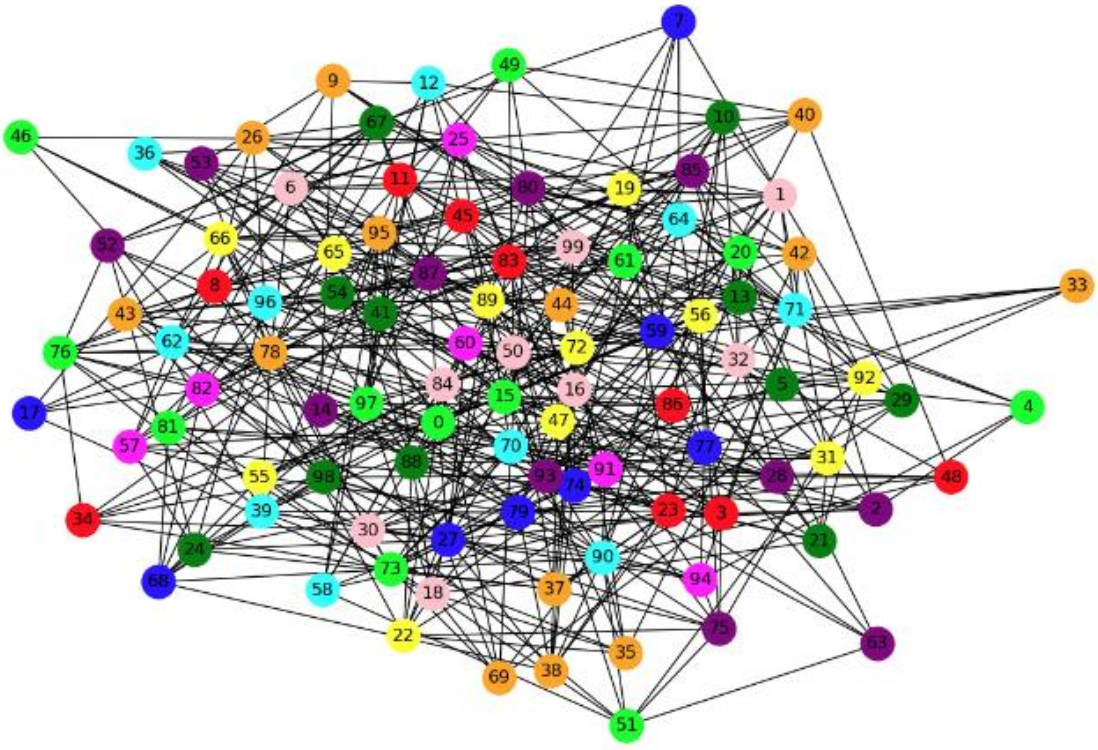

Figure 2: Visualisation of generated graph colouring