# Model Predictive Control

## 1. Introduction to Model Predictive Control

Jun.-Prof. Dr.-Ing. Daniel Görges
Juniorprofessur für Elektromobilität
Technische Universität Kaiserslautern

## What is Model Predictive Control?

### Concept of Model Predictive Control

- **Prediction**
  - Predict the system behavior using a system model over a prediction horizon $N$

System model $\boldsymbol{x}(k+1) = \boldsymbol{f}(\boldsymbol{x}(k), \boldsymbol{u}(k), k)$

Input sequence $\boldsymbol{U}(k) = \begin{pmatrix} \boldsymbol{u}(k) \\ \boldsymbol{u}(k+1) \\ \vdots \\ \boldsymbol{u}(k+N-1) \end{pmatrix}$

Current state $\boldsymbol{x}(k)$

State sequence $\boldsymbol{X}(k) = \begin{pmatrix} \boldsymbol{x}(k+1) \\ \boldsymbol{x}(k+2) \\ \vdots \\ \boldsymbol{x}(k+N) \end{pmatrix}$

Predict
= Solve system model
= "Simulate" forward in time

TECHNISCHE UNIVERSITÄT KAISERSLAUTERN

# What is Model Predictive Control?

## Concept of Model Predictive Control

- **Optimization**
  - Optimize the system behavior for some cost function under some constraints using the prediction

Cost function $V_N\big(\boldsymbol{x}(k), \boldsymbol{U}(k)\big) = \sum_{i=0}^{N-1} \ell(\boldsymbol{x}(k+i), \boldsymbol{u}(k+i), k+i)$

Formulate

Optimization problem $\min_{\boldsymbol{U}(k)} V_N\big(\boldsymbol{x}(k), \boldsymbol{U}(k)\big)$

$$\text{subject to } \begin{cases} \boldsymbol{x}(k+i+1) = \boldsymbol{f}(\boldsymbol{x}(k+i), \boldsymbol{u}(k+i), k+i) \\ \boldsymbol{x}(k+i) \in \mathbb{X}(k+i) & \text{state constraints} \\ \boldsymbol{u}(k+i) \in \mathbb{U}(k+i) & \text{input constraints} \end{cases}$$

Solve

Optimal input sequence $\boldsymbol{U}^*(k) = \begin{pmatrix} \boldsymbol{u}^*(k) \\ \boldsymbol{u}^*(k+1) \\ \vdots \\ \boldsymbol{u}^*(k+N-1) \end{pmatrix}$



TECHNISCHE UNIVERSITÄT KAISERSLAUTERN

# What is Model Predictive Control?

## Concept of Model Predictive Control

- **Receding Horizon Implementation**
  - Implement the first element of the optimal input sequence $\boldsymbol{u}^*(k)$ to the system

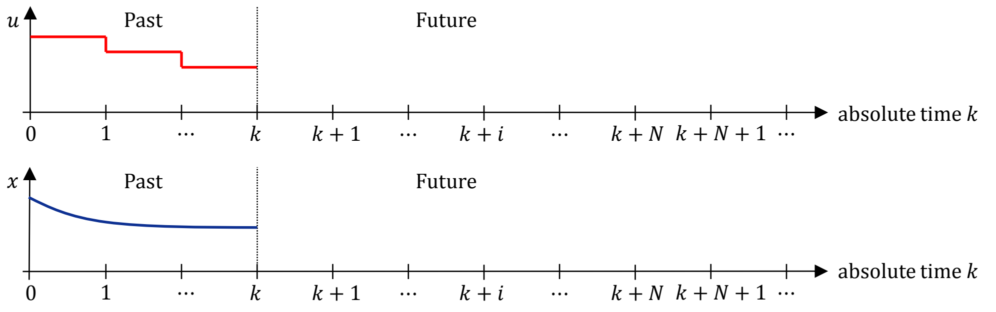
    Optimal input $\boldsymbol{u}(k) = (\boldsymbol{I} \quad \boldsymbol{0} \quad \cdots \quad \boldsymbol{0})\boldsymbol{U}^*(k) = \boldsymbol{u}^*(k)$

  - Throw away the remaining elements of the optimal input sequence $\boldsymbol{u}^*(k+1), \dots, \boldsymbol{u}^*(k+N-1)$
  - Repeat the prediction, optimization, and receding horizon implementation at time $k+1$

- **Remarks**
  - The receding horizon implementation transforms open-loop optimal control into feedback control
  - Feedback introduces robustness w.r.t. disturbances and uncertainties
  - Feedback compensates the finite control degrees of freedom due the finite prediction horizon and therefore improves the control performance
  - The receding horizon implementation “emulates” an infinite prediction horizon

# What is Model Predictive Control?

## Concept of Model Predictive Control



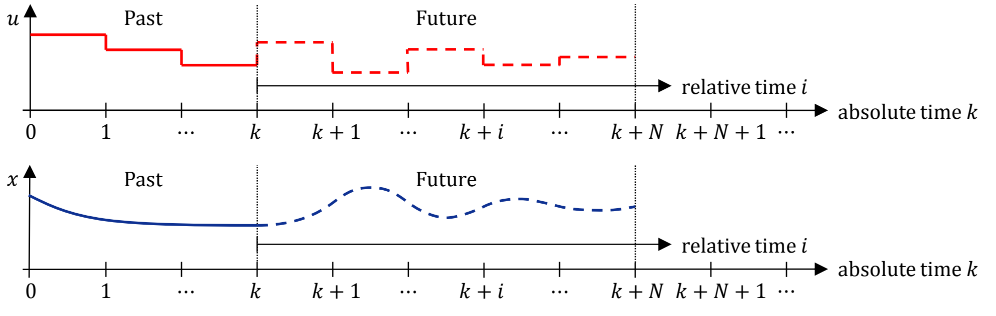

1. Measure the current state $\boldsymbol{x}(k)$


Jun.-Prof. Dr.-Ing. Daniel Görges
Juniorprofessur für Elektromobilität
Technische Universität Kaiserslautern




# What is Model Predictive Control?

## Concept of Model Predictive Control



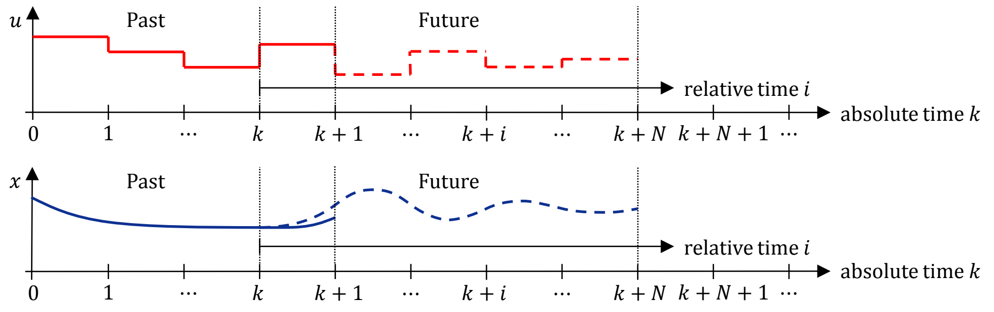

1. Measure the current state $\boldsymbol{x}(k)$
2. Solve the optimization problem to determine the optimal input sequence $\boldsymbol{U}^*(k)$ using the prediction


Jun.-Prof. Dr.-Ing. Daniel Görges
Juniorprofessur für Elektromobilität
Technische Universität Kaiserslautern

TECHNISCHE UNIVERSITÄT KAISERSLAUTERN

# What is Model Predictive Control?

## Concept of Model Predictive Control



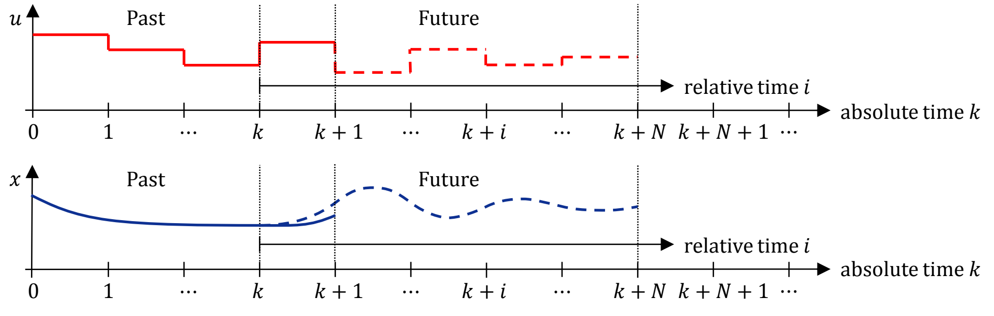

1. Measure the current state $\boldsymbol{x}(k)$
2. Solve the optimization problem to determine the optimal input sequence $\boldsymbol{U}^*(k)$ using the prediction
3. Implement the first element of the optimal input sequence $\boldsymbol{u}^*(k) = (\boldsymbol{I} \quad \boldsymbol{0} \quad \cdots \quad \boldsymbol{0})\boldsymbol{U}^*(k)$



TECHNISCHE UNIVERSITÄT KAISERSLAUTERN

# What is Model Predictive Control?

## Concept of Model Predictive Control

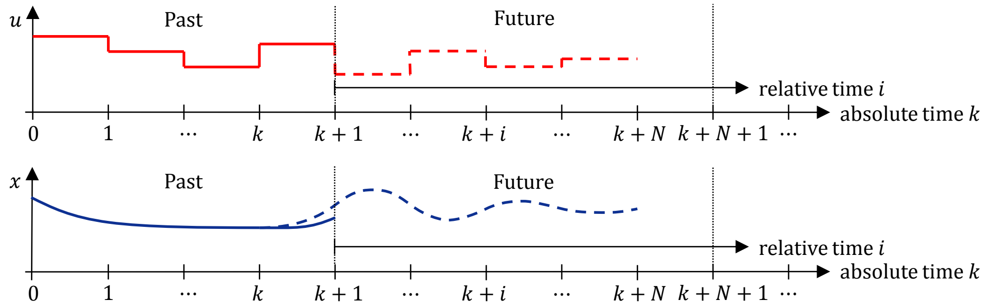



1. Measure the current state $\boldsymbol{x}(k)$
2. Solve the optimization problem to determine the optimal input sequence $\boldsymbol{U}^*(k)$ using the prediction
3. Implement the first element of the optimal input sequence $\boldsymbol{u}^*(k) = (\boldsymbol{I} \quad \boldsymbol{0} \quad \cdots \quad \boldsymbol{0})\boldsymbol{U}^*(k)$
4. Increment the time instant $k := k+1$ and go to 1.

TECHNISCHE UNIVERSITÄT KAISERSLAUTERN

# What is Model Predictive Control?

## Concept of Model Predictive Control



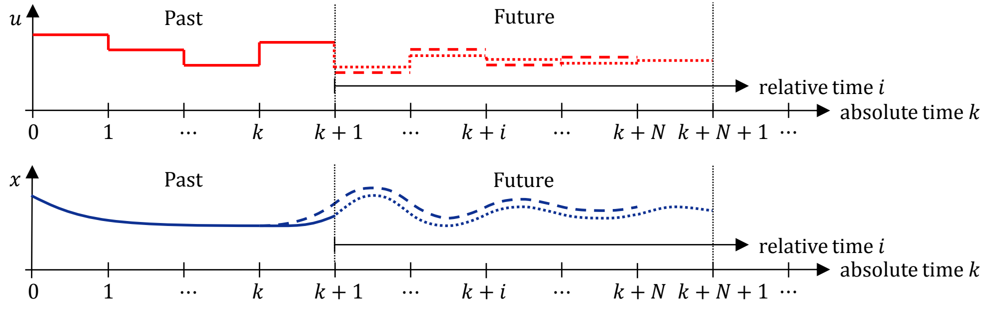

1. Measure the current state $\boldsymbol{x}(k+1)$




TECHNISCHE UNIVERSITÄT KAISERSLAUTERN


# What is Model Predictive Control?

## Concept of Model Predictive Control

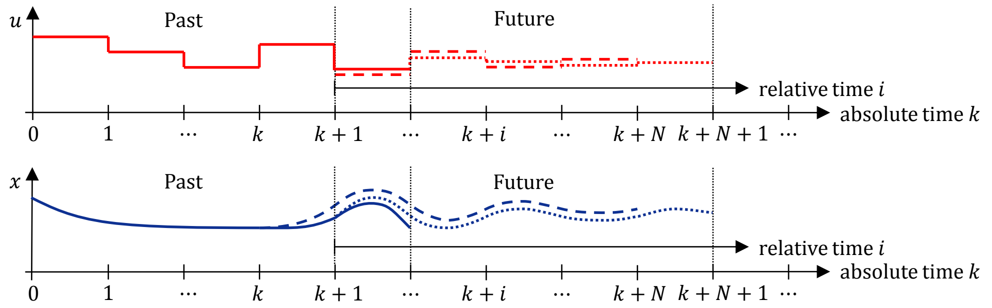



1. Measure the current state $\boldsymbol{x}(k+1)$
2. Solve the optimization problem to determine the optimal input sequence $\boldsymbol{U}^*(k+1)$ using the prediction

Technische Universität Kaiserslautern

# What is Model Predictive Control?

## Concept of Model Predictive Control



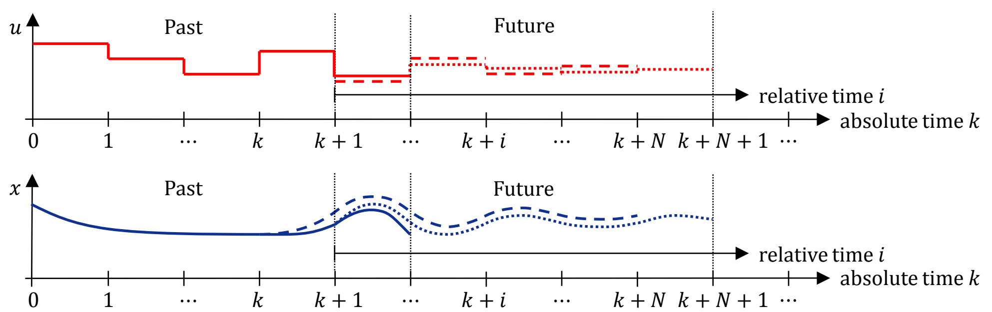

1. Measure the current state $\boldsymbol{x}(k+1)$
2. Solve the optimization problem to determine the optimal input sequence $\boldsymbol{U}^*(k+1)$ using the prediction
3. Implement the first element of the optimal input sequence $\boldsymbol{u}^*(k+1) = (\boldsymbol{I} \quad \boldsymbol{0} \quad \cdots \quad \boldsymbol{0})\boldsymbol{U}^*(k+1)$




Technische Universität Kaiserslautern

# What is Model Predictive Control?

## Concept of Model Predictive Control


1. Measure the current state $\boldsymbol{x}(k+1)$
2. Solve the optimization problem to determine the optimal input sequence $\boldsymbol{U}^*(k+1)$ using the prediction
3. Implement the first element of the optimal input sequence $\boldsymbol{u}^*(k+1) = (\boldsymbol{I} \quad \boldsymbol{0} \quad \cdots \quad \boldsymbol{0})\boldsymbol{U}^*(k+1)$
4. Increment the time instant $k+1 \coloneqq k+2$ and go to 1.

# MPC is like …

## … driving a Car

**System Model**
Longitudinal dynamics
Lateral dynamics
Vertical dynamics
…

**States**
Position
Speed
Orientation
…

**Inputs**
Accelerator pedal position
Brake pedal position
Steering angle
Gear
…


Source: Volvo

**Cost Function**
Fuel consumption
…

**Constraints**
Speed limits (hard)
Lane boundaries (soft[1])
Engine torque (hard) …

**Prediction Horizon**
Given by visual range
Receding due to driving

**Disturbances**
Road grade (predictable)
Traffic (not predictable[2]) …

**Uncertainties**
Vehicle mass …

Jun.-Prof. Dr.-Ing. Daniel Görges
Juniorprofessur für Elektromobilität
Technische Universität Kaiserslautern





# Why MPC?

## Model Predictive Control versus Traditional Optimal Control

**Model Predictive Control**

- Method generally applicable to many problems (MIMO; constraints; nonlinear, delay, uncertain, stochastic, time-varying and hybrid systems, …)
- Allows regarding input and state constraints (in a very systematic way)
- Provides feedback control law (robustness w.r.t. disturbances, uncertainties)
- Usually based on numerical optimization
- Usually based on online optimization (large computation time)

**Traditional Optimal Control ***

- Method must be tailored to the problem
- Allows regarding input constraints only (regarding state constraints very involved)
- Provides often no feedback control law (no robustness w.r.t. disturbances, uncertainties)
- Usually based on analytical optimization
- Usually based on offline optimization (small computation time)

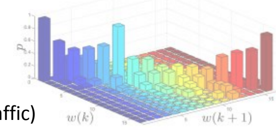
* Calculus of Variations, Pontryagin's Minimum Principle, (Dynamic Programming)

Jun.-Prof. Dr.-Ing. Daniel Görges
Juniorprofessur für Elektromobilität
Technische Universität Kaiserslautern

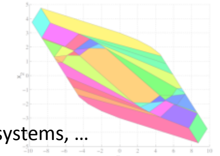
# Which Variants of MPC exist?

## Variants of Model Predictive Control

- **Linear Model Predictive Control (Linear Prediction Model, Linear Constraints)**
- **Nonlinear Model Predictive Control (Nonlinear Prediction Model, Linear or Nonlinear Constraints)**
- **Robust Model Predictive Control (Uncertain Prediction Model)**
- **Stochastic Model Predictive Control (Stochastic Prediction Model) ***
  - Energy management in hybrid vehicles (stochastic modeling of the traffic)
  - Energy management in power systems (stochastic modeling of renewable generation and load)
  - Portfolio optimization in financial systems (stochastic modeling of share prices, …)



- **Explicit Model Predictive Control**
  - Offline optimization
  - Systems with very fast dynamics (down to nanoseconds)
  - Mechatronic systems, vehicular systems, power systems, power electronic systems, …



Source: Multi-Parametric Toolbox (MPT)

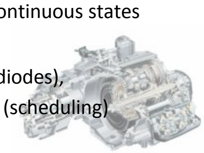


TECHNISCHE UNIVERSITÄT KAISERSLAUTERN

# Which Variants of MPC exist?

## Variants of Model Predictive Control

- **Fast Model Predictive Control ***
  - Optimization based on „fast" numerical optimization methods (fast gradient method, …)
  - Systems with fast dynamics (down to microseconds)
  - Mechatronic systems, vehicular systems, power systems, power electronic systems, …
- **Hybrid Model Predictive Control (Hybrid Prediction Model)**
  - Systems including continuous dynamics (differential or difference equations, continuous states and inputs) and discrete events (finite automata, discrete states and inputs)
  - Vehicles (manual gearbox, automatic gearbox), power electronics (transistors, diodes), process control systems (on/off valves), networked embedded control systems (scheduling)



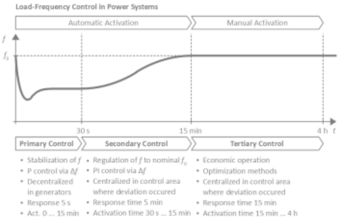
Source: www.kfz-tech.de

- **Distributed Model Predictive Control ***
  - Optimization distributed over several computers
  - Large-scale systems (power systems, transportation systems, irrigation systems, process control, …)

# Which Variants of MPC exist?

## Variants of Model Predictive Control

- **Hierarchical Model Predictive Control ***
  - Systems with different time constants
- **Economic Model Predictive Control ***
  - Optimization of economic objectives
- **Other Names in Academia and Industry**
  - Generalised Predictive Control (GPC), Adaptive Predictive Control (APC), Dynamic Matrix Control (DMC), Predictive Functional Control (PFC), Model Algorithmic Control (MAC), Extended Prediction Self Adaptive Control (EPSAC), Sequential Open Loop Optimization (SOLO), …
- **Generic Names**
  - Model Predictive Control (MPC), Receding Horizon Control (RHC)
  - These names will be used interchangeably in this lecture




* Very active research areas



# How about MPC and Industry?

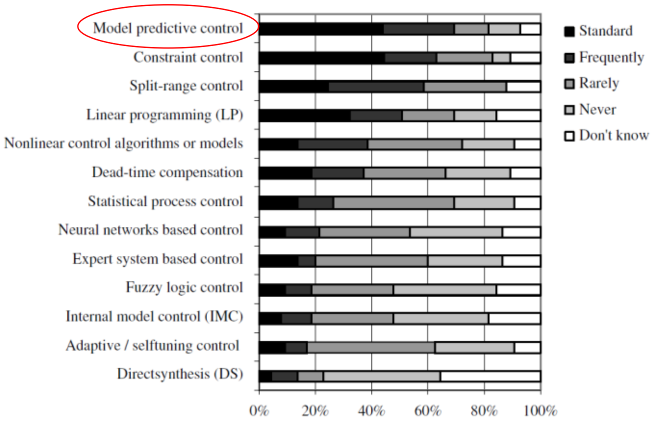
## Industrial Model Predictive Control

| Area | Aspen Technology | Honeywell Hi-Spec | Adersa[b] | Invensys | SGS[c] | Total |
|---|---|---|---|---|---|---|
| Refining | 1200 | 480 | 280 | 25 | | 1985 |
| Petrochemicals | 450 | 80 | — | 20 | | 550 |
| Chemicals | 100 | 20 | 3 | 21 | | 144 |
| Pulp and paper | 18 | 50 | — | — | | 68 |
| Air & Gas | — | 10 | — | — | | 10 |
| Utility | — | 10 | — | 4 | | 14 |
| Mining/Metallurgy | 8 | 6 | 7 | 16 | | 37 |
| Food Processing | — | — | 41 | 10 | | 51 |
| Polymer | 17 | — | — | — | | 17 |
| Furnaces | — | — | 42 | 3 | | 45 |
| Aerospace/Defense | — | — | 13 | — | | 13 |
| Automotive | — | — | 7 | — | | 7 |
| Unclassified | 40 | 40 | 1045 | 26 | 450 | 1601 |
| Total | 1833 | 696 | 1438 | 125 | 450 | 4542 |
| First App. | DMC:1985<br>IDCOM-M:1987<br>OPC:1987 | PCT:1984<br>RMPCT:1991 | IDCOM:1973<br>HIECON:1986 | 1984 | 1985 | |
| Largest App. | 603 × 283 Out × In | 225 × 85 | — | 31 × 12 | — | |

S. Joe Qin and Thomas A. Badgwell. A survey of industrial model predictive control technology. *Control Engineering Practice*, 11(7):733-764, 2003. (Snapshot from mid-1999)

# How about MPC and Industry?

## Advanced Process Control




Margret Bauer and Ian K. Craig. Economic assessment of advanced process control – A survey and framework. Journal of Process Control, 18(1):2-18, 2008.





# How about MPC and Industry?

## Other Applications? Of course!

- **Remarks**
  - Model predictive control has been limited for many years to process control with slow dynamics (seconds to hours) due to the online optimization and the associated large computation time
  - Model predictive control has been increasingly applied in recent years also to mechatronic systems, vehicular systems, power systems, power electronic systems, … with fast dynamics (milliseconds to nanoseconds) due to substantial advances in computer hardware (FPGAs, GPUs, …) and important developments in model predictive control algorithms (explicit, fast and distributed MPC)
- **Examples**
  - Mechatronic systems (nonlinear MPC of a crane, nonlinear MPC of a magnetic levitation system)
  - Vehicular systems (adaptive cruise control, eco-driving assistance for electric vehicles, …)
  - Power systems (load-frequency control, economic dispatch, unit commitment, …)
  - Power electronic systems

# How about MPC and Academia?

## Research on Model Predictive Control

- **Journals**
  - The term „Model Predictive Control“ is contained in
    57 of 413 papers (13.8 %) in Automatica (2012)
    39 of 435 papers (8.1 %) in IEEE Transactions on Automatic Control (2012)
    36 of 146 papers (24.7 %) in Control Engineering Practice (2012)
    39 of 189 papers (20.6 %) in IEEE Transactions on Control Systems Technology (2012)
- **Conferences**
  - There are special conferences like the IFAC Conference on Nonlinear Model Predictive Control
  - The term „Model Predictive Control“ is contained in
    205 of 1274 papers (16.1 %) at the 53rd IEEE Conference on Decision and Control (2013)
    63 of 1130 papers (5.6 %) at the 2013 American Control Conference
    16 of 247 papers (6.4 %) at the 2012 IEEE International Conference on Control Applications





# How about the History of MPC?

## History of Model Predictive Control

| | | | |
|---|---|---|---|
| 2009 | Rawlings et al.[9] | Economic MPC | Academia-driven<br>State-space models |
| 2009 | Richter et al.[8] | Fast MPC | |
| 2007 | Scattolini and Colaneri[7] | Hierarchical MPC | |
| 2005 | Muñoz de la Peña et al.[6] | Stochastic MPC | |
| 2002 | Camponogara et al. | Distributed MPC | |
| 2002 | Bemporad et al.[5] | Explicit MPC | |
| 2000 | Mayne et al.[4] | Unified stability theory | |
| 1999 | Bemporad and Morari[3] | Hybrid MPC | |
| 1996 | Kothare et al.[2] | Robust MPC | |
| 1990 | Bitmead et al. | First stability concepts (terminal cost) | |
| 1988 | Keerthi and Gilbert | First stability concepts (terminal constraint) | |
| 1980 | Cutler and Ramaker | Dynamic Matrix Control (DMC) | Industry-driven<br>Step- or impulse-response and transfer function models |
| 1978 | Richalet et al. | Identification and Command (IDCOM) | |
| 1967 | Lee and Markus | First concepts of MPC | |
| 1963 | Propoi | First concepts of MPC | |
| 1960 | Kalman[1] | Linear quadratic regulator (LQR) | |


J. B. Rawlings[4,9]


R. Scattolini[7]


M. Morari[2,3,5,8]


A. Bemporad[3,5,6]


R. E. Kalman[1]


D. Q. Mayne[4]

[4] Most downloaded paper from Automatica in the last 90 days

TECHNISCHE UNIVERSITÄT KAISERSLAUTERN

## What is in this Lecture?

### Lecture Outline

1. **Introduction to Model Predictive Control**
2. **Fundamentals of Discrete-Time Systems**
3. **Fundamentals of Optimization**
4. **Model Predictive Control without Constraints**
5. **Model Predictive Control with Constraints**
6. **Stability and Feasibility**
7. **Reference Tracking and Disturbance Rejection**
8. **Robust Model Predictive Control**



TECHNISCHE UNIVERSITÄT KAISERSLAUTERN

## Model Predictive Control

### Model Predictive Control

[BBM15] Francesco Borrelli, Alberto Bemporad, and Manfred Morari. *Predictive Control for Linear and Hybrid Systems*. Cambridge University Press, Cambridge, In Press. – www.mpc.berkeley.edu/mpc-course-material

[CB04] Eduardo F. Camacho and Carlos Bordons. *Model Predictive Control*. Springer, London, 2nd edition, 2004. – EIT 938/059 (Semesterapparat)

[DP04] Rainer Dittmar and Bernd-Markus Pfeiffer. *Modellbasierte prädiktive Regelung: Eine Einführung für Ingenieure*. Oldenbourg, München, 2004.

[GP11] Lars Grüne and Jürgen Pannek. *Nonlinear Model Predictive Control*. Springer, London, 2011.

[KC16] Basil Kouvaritakis and Mark Cannon. *Model Predictive Control: Classical, Robust and Stochastic*. Springer, Cham, 2016

[KH05] Wook Hyun Kwon and Soohee Han. *Receding Horizon Control: Model Predictive Control for State Models*. Springer, London, 2005.

## Model Predictive Control

[Mac02] Jan M. Maciejowski. *Predictive Control with Constraints*. Pearson Education, Harlow, 2002. – EIT 915/040 (Semesterapparat) *

[RM09] James B. Rawlings and David Q. Mayne. *Model Predictive Control: Theory and Design*. Nob Hill Publishing, Madison, WI, 2009. – EIT 938/062 (Sem.), http://jbrwww.che.wisc.edu/home/jbraw/mpc/

[Ros04] J. A. Rossiter. *Model-Based Predictive Control: A Practical Approach*. CRC Press, Boca Raton, FL, 2004. – EIT 938/061

## Optimization

[BV04] Stephen Boyd and Lieven Vandenberghe. *Convex Optimization*. Cambridge University Press, Cambridge, 2004. – www.stanford.edu/~boyd/cvxbook/ *

[PLB12] Markos Papageorgiou, Marion Leibold, and Martin Buss. *Optimierung: Statische, dynamische, stochastische Verfahren für die Anwendung*. Springer, Berlin, 3rd edition, 2012. – 3rd edition available as eBook, 1st and 2nd edition available under EIT 177/059, MAT Papa





## Optimization

[ŻC03] Stanislaw Żak and Edwin Chong. *An introduction to optimization*. Wiley, New York, NY, 2nd edition, 2003. – EIT 177/117, L EIT 102 *

## Discrete-Time Systems

[ÅW90] Karl J. Åström and Björn Wittenmark. *Computer-Controlled Systems: Theory and Design*. Prentice-Hall, Englewood Cliffs, NJ, 2nd edition, 1990. – EIT 915/075

[FPW97] Gene F. Franklin, John David Powell, and Michael L. Workman. *Digital Control of Dynamic Systems*. Addison-Wesley, Menlo Park, CA, 3rd edition, 1997. – EIT 938/021, L EIT 119 *

[Lun13] Jan Lunze. *Regelungstechnik 2: Mehrgrößensysteme, Digitale Regelung*. Springer, Berlin, 7th edition, 2013. – 7th edition available as eBook, other editions available under EIT 910/090-2, L EIT 229-2



* Recommended for the lecture