Modelling and Identification

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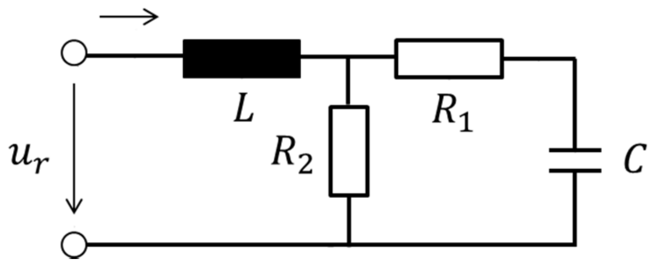
Electrical systems

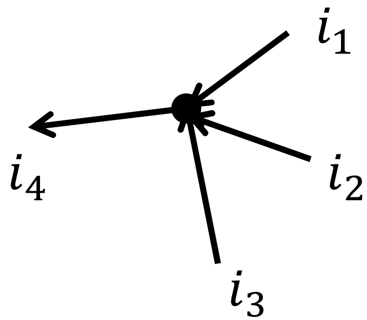
- Resistor: $u(t) = Ri(t),$
- Capacitor: $i(t) = C\frac{du}{dt}$
- Inductor: $u(t) = L\frac{di}{dt}$
- **Kirchhoff voltage law**: The sum of voltages along an **arbitrary closed path** in the circuit is 0.

$$\sum_j u_j(t) = 0$$



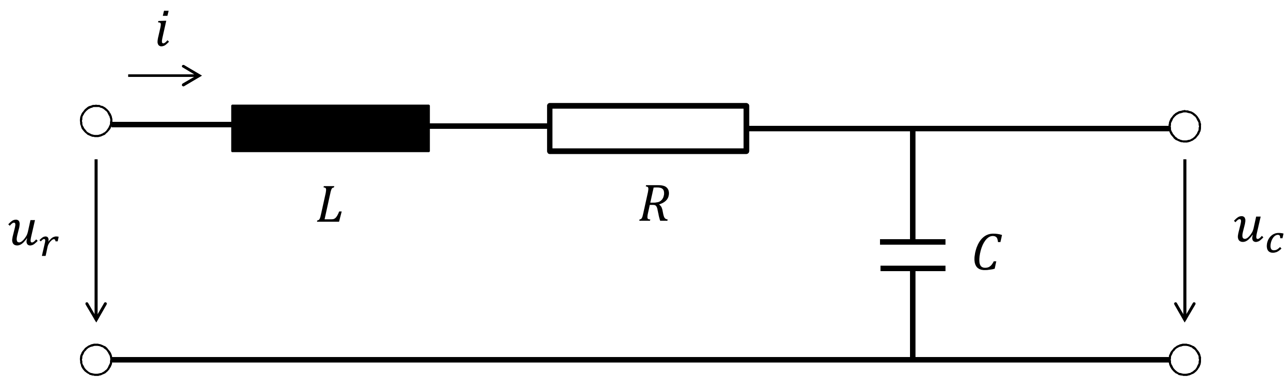
- **Kirchhoff current law**: The sum of the currents at **a node** is 0.

$$\sum_j i_j(t) = 0$$



Electrical systems

Example: Input: u_r, Output: u_c



Kirchhoff voltage law:

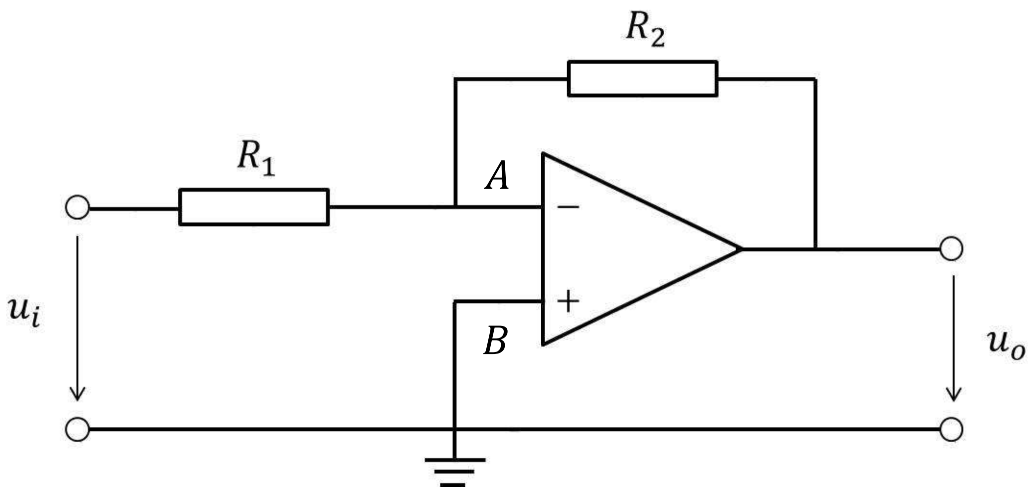
$$L\frac{di}{dt} + Ri + u_c = u_r$$

Capacitor:

$$i = C\frac{du_c}{dt}$$

$$LC\frac{d^2u_c}{dt^2} + RC\frac{du_c}{dt} + u_c = u_r$$

Example: Input: u_i, Output: u_o



Kirchhoff current law:

$$\frac{u_i}{R_1} + \frac{u_o}{R_2} = 0$$

$$u_0 = -\frac{R_2}{R_1} u_i$$

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Electromagnetic systems

- **Law of motors:** A wire in a magnetic field that carries a current will have a force exerted on it

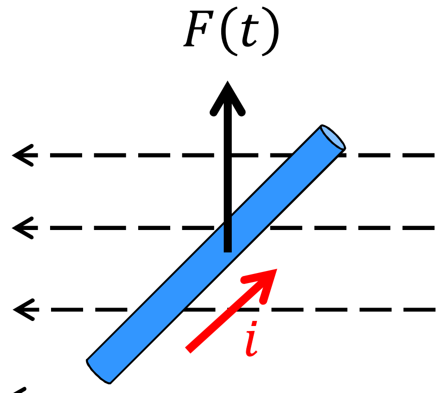
$$F(t) = Bli$$

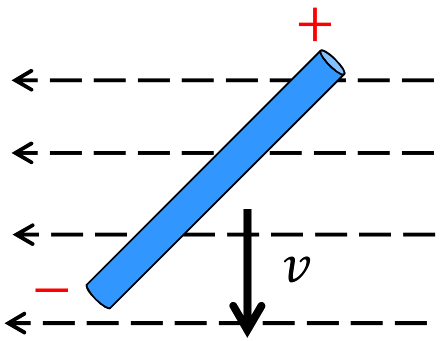
B: flux density of the magnetic field

l: length of the conductor

- **Law of generators:** A voltage will be induced in a wire that moves relative to the magnetic field

$$e(t) = Blv$$





Electromagnetic systems

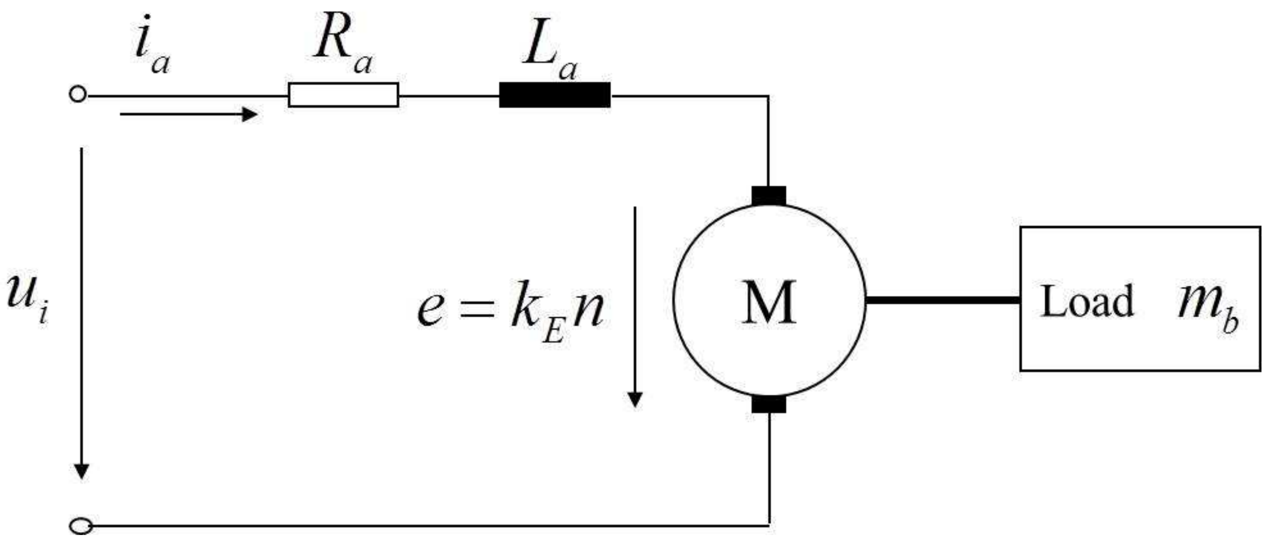
DC motor:

- Often used as actuator in control systems
- Torque exerted on the rotor:

$$M(t) = k_I i_a$$

- The voltage induced:

$$e(t) = k_E n$$



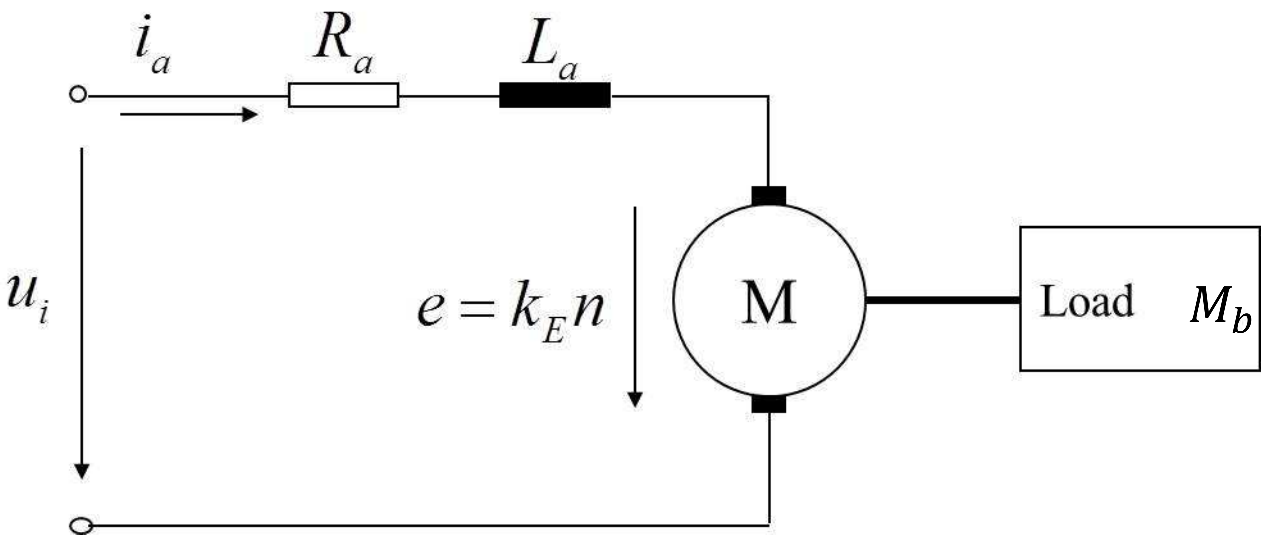
Model of DC motor:

Kirchhoff voltage law

$$u_i(t) = R_a i_a(t) + L_a \frac{di_a(t)}{dt} + k_E n(t)$$

Torque balance

$$J\frac{dn}{dt} = k_I i_a - M_b$$



Thermal systems

The behaviour of a thermal system is described by

- Temperature T
- Heat flow rate q

Element laws:

- **Thermal resistence**

The heat flow rate from a body of temperature T_1 to a body of temperature T_2 is

$$q(t) = \frac{1}{R}(T_1(t) - T_2(t))$$

R: Thermal resistance of the path between two bodies. Unit: $K \cdot s/J$

➢ Thermal capacitance

The rate of temperature change is related to the instantaneous net heat flow rate into the body by

$$\frac{dT}{dt} = \frac{1}{C}\left(q_{in}(t) - q_{out}(t)\right)$$

C: Thermal capacitance of the object. Unit: J/K.

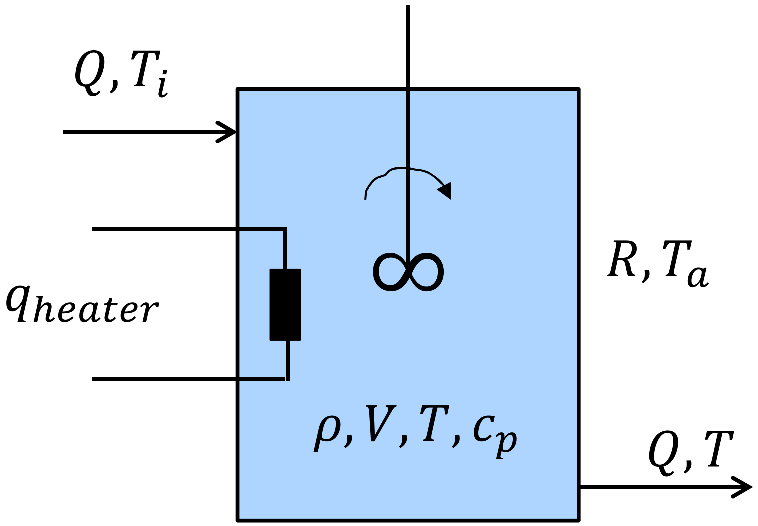
For an object with mass m and specific heat c_p, the heat capacitance is

$$C = mc_p$$

Assumptions: The thermal gradients within the object is not too big. Otherwise, a distributed parameter model is needed.

Thermal systems

Example 1: electrically heated stirred tank



Q: volume flow rate

q_{heater}: heat rate of the heater

Consider the temperature change of the liquid in the tank

$$\frac{dT}{dt} = \frac{1}{C}(q_{in}(t) - q_{out}(t))$$

$$C = mc_p = \rho V c_p$$

The heat flow rate entering the tank

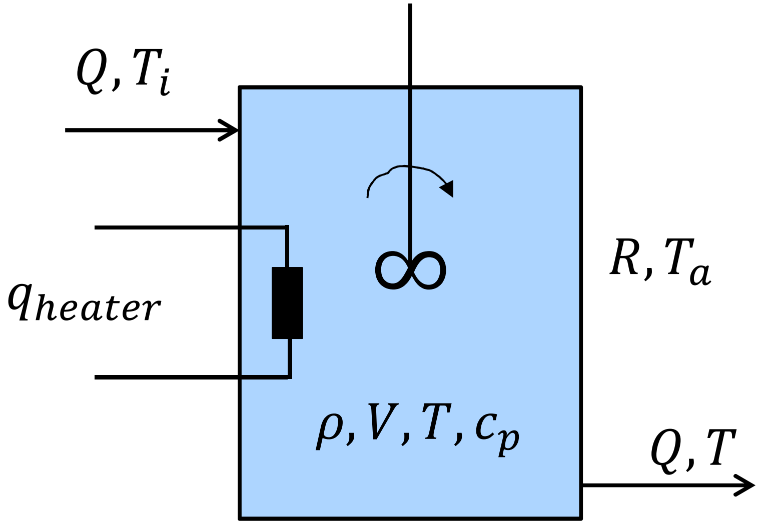
$$q_{in}(t) = q_{heater} + \rho Q c_p T_i(t)$$

The heat flow rate leaving the tank

$$q_{out}(t) = \rho Q c_p T(t) + \frac{1}{R}\big(T(t) - T_a(t)\big)$$

Thermal systems

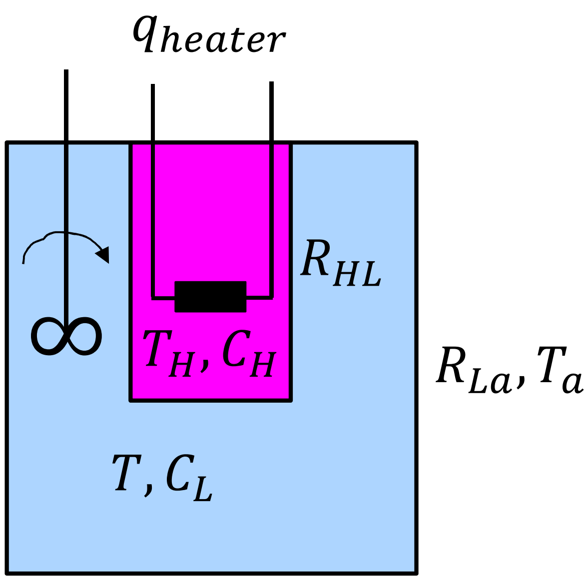
Example 1: electrically heated stirred tank



Total model:

$$\rho V c_p \frac{dT}{dt} = \rho Q c_p (T_i(t) - T(t)) - \frac{1}{R}\big(T(t) - T_a(t)\big) + q_{heater}$$

Example 2: tank with heater for batch processing (Close, 2002)



Consider the temperature change of the liquid in the tank

$$\frac{dT}{dt} = \frac{1}{C_L}(q_{HL}(t) - q_{La}(t))$$

Consider the temperature change of the heating element

$$\frac{dT_H}{dt} = \frac{1}{C_H}(q_{heater}(t) - q_{HL}(t))$$

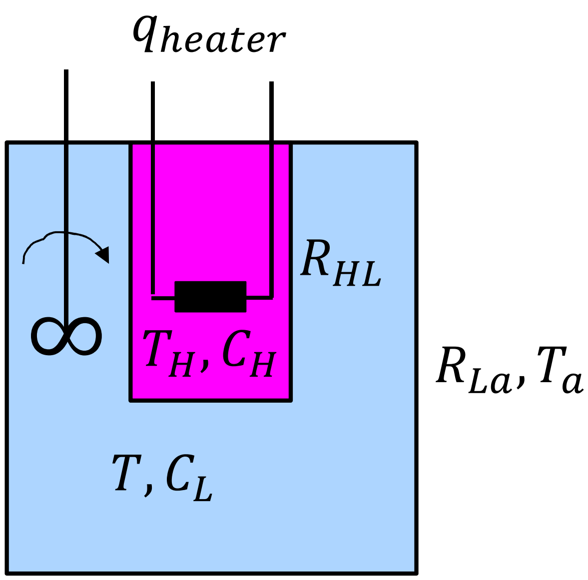
Heat flow from liquid to atmosphere

$$q_{La}(t) = \frac{1}{R_{La}}(T - T_a)$$

Heat flow from heating element to liquid

$$q_{HL}(t) = \frac{1}{R_{HL}}(T_H - T)$$

Example 2: tank with heater for batch processing (Close, 2002)

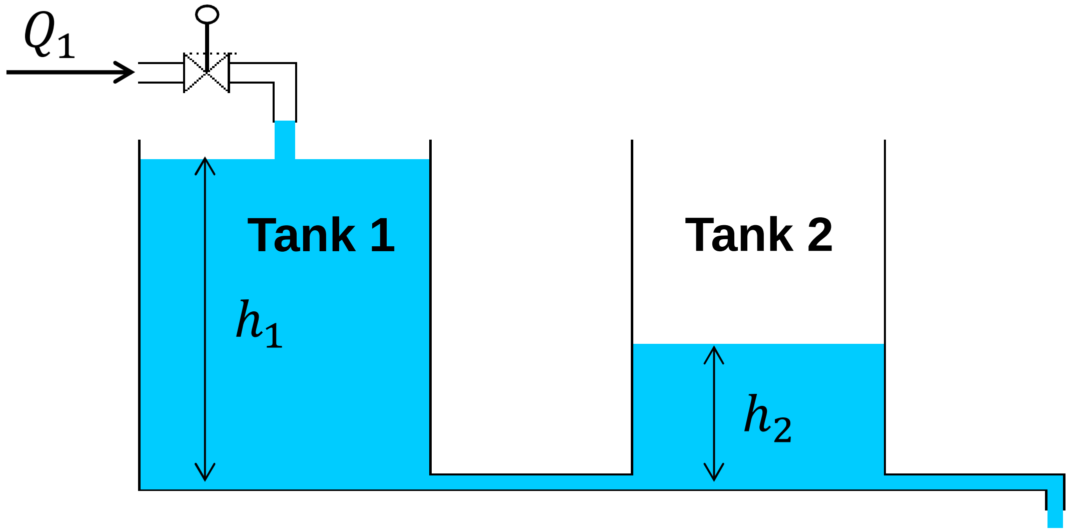


Total model:

$$\frac{dT_H}{dt} = \frac{1}{C_H}\left(q_{heater}(t) - \frac{1}{R_{HL}}(T_H - T)\right)$$

$$\frac{dT}{dt} = \frac{1}{C_L}\left(\frac{1}{R_{HL}}(T_H - T) - \frac{1}{R_{La}}(T - T_a)\right)$$

Example: fluid system

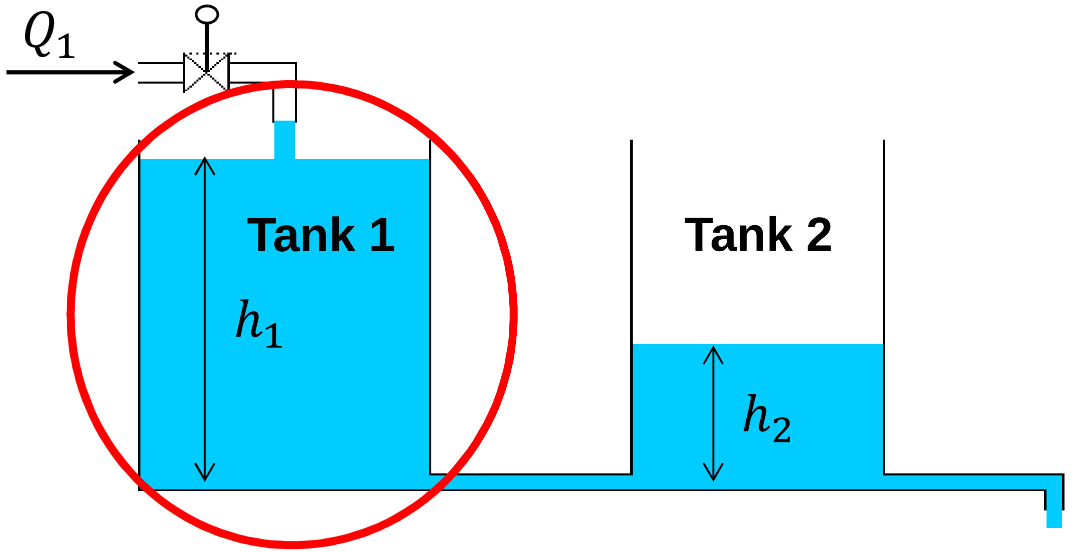


Q_1: volume flow rate

A_{12}, A_{20}: cross section of the pipelines

A_1, A_2: cross section of the tanks

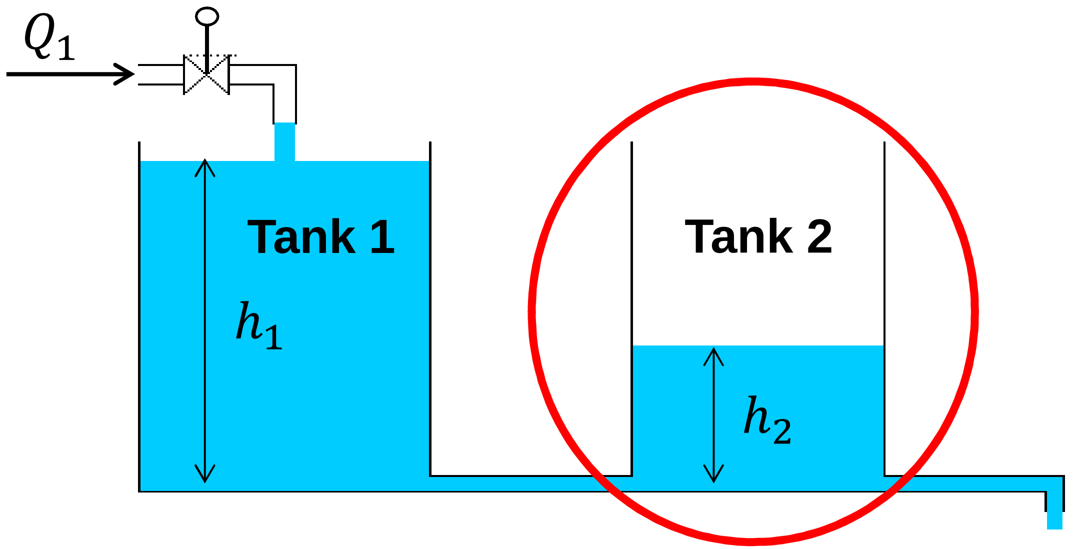
Example: fluid system



Mass balance for Tank 1

$$A_1\dot{h}_1 = Q_1 - Q_{12}$$

Example: fluid system

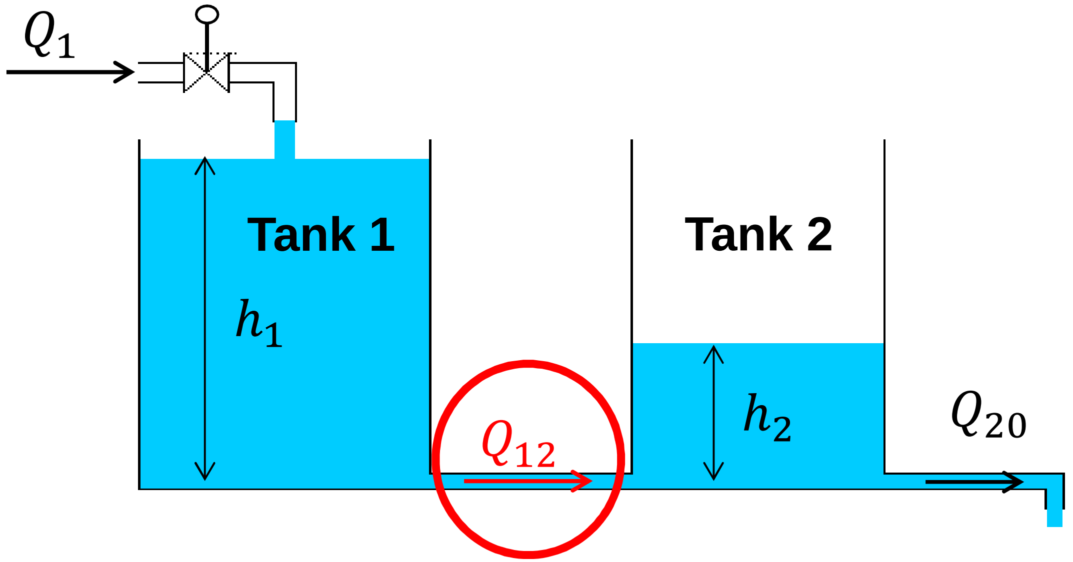


Mass balance for Tank 1 $A_1\dot{h}_1 = Q_1 - Q_{12}$

Mass balance for Tank 2 $A_2\dot{h}_2 = Q_{12} - Q_{2o}$

Coupling between two subsystems?

Example: fluid system



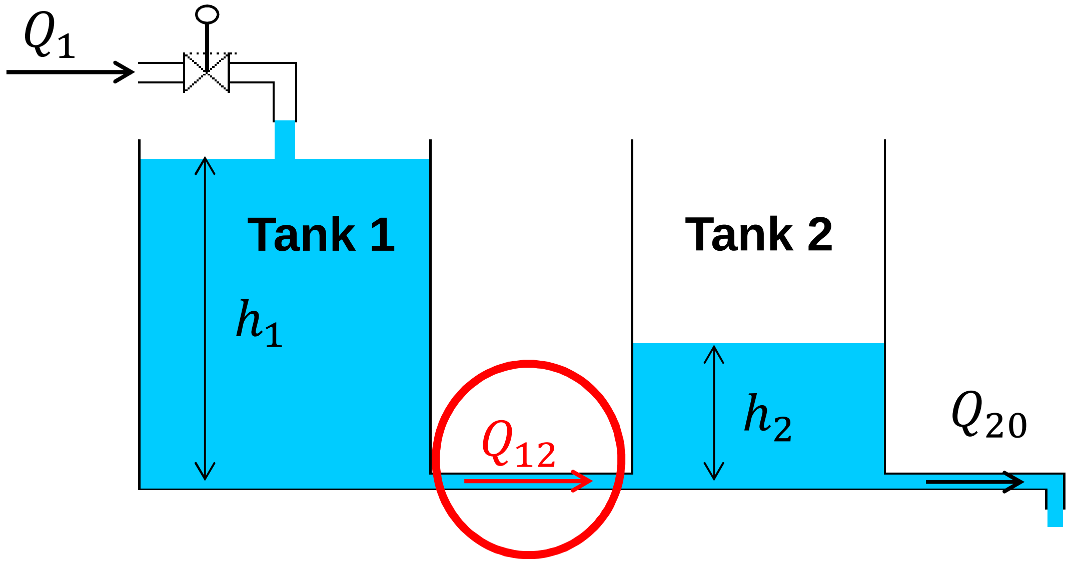
Mass balance for Tank 1 $\quad A_1\dot{h}_1 = Q_1 - Q_{12}$

Mass balance for Tank 2 $\quad A_2\dot{h}_2 = Q_{12} - Q_{2o}$

Torricelli‘s law $\quad Q_{12} = aA_{12}sgn(h_1 - h_2)\sqrt{2g|h_1 - h_2|}$

Torricelli‘s law $\quad Q_{2o} = aA_{2o}\sqrt{2gh_2}$

Example: fluid system



System model

$$\begin{cases} A_1\dot{h}_1 = Q_1 - aA_{12}sgn(h_1 - h_2)\sqrt{2g|h_1 - h_2|} \\ A_2\dot{h}_2 = aA_{12}sgn(h_1 - h_2)\sqrt{2g|h_1 - h_2|} - aA_{2o}\sqrt{2gh_2} \end{cases}$$