Examination

Modelling and Identification / Modellbildung und Identifikation

Date:	20th March, 2015
Duration of examination:	120 Minutes
Points:	100

Please write legibly!

You must document how you obtain the answer to qualify for full credits.

This examination consists of **6 problems**. First, **check that your copy contains all 6 problems**.

Admitted aids: indelible pens ruler and means of drawing, non-programmable calculator, provided answer sheets. Having **non-admitted aids present** after the distribution of the examination tasks also constitutes as an attempt to cheat and leads to the non-approval of your examination.

Write your name and matriculation number on each page of the answer sheets including the cover sheet

Use only the provided answer sheet for the answers. Only the answer sheets will be collected.

Write an explanation for all answers and give the approach for your calculations. The absence of an explanation or the approach has an influence on the assessment of the answer. Answers only consisting “Yes” or “No” will not gain points.

Problem 1: Theoretic Modelling (5+8+14 points)

Figure 1 shows a circuit of a solar cell. The current i_E (in Ampere), which is delivered from the solar cell, depends nonlinear on the voltage u_E (in Volt) of the solar cell. The relationship is shown in the characteristic curve in image 2.

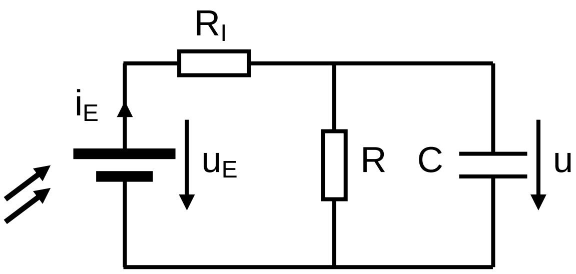

Figure 1: Equivalent network

a) Derive the function $i_E(u_E)$ based on the characteristic curve. Use the quadratic approach with $i_E = \alpha \cdot u_E^{\ 2} + \gamma$.

b) Derive the differential equation $\frac{\partial u}{\partial t} = f(u, u_E, C, R, \alpha, \gamma)$.

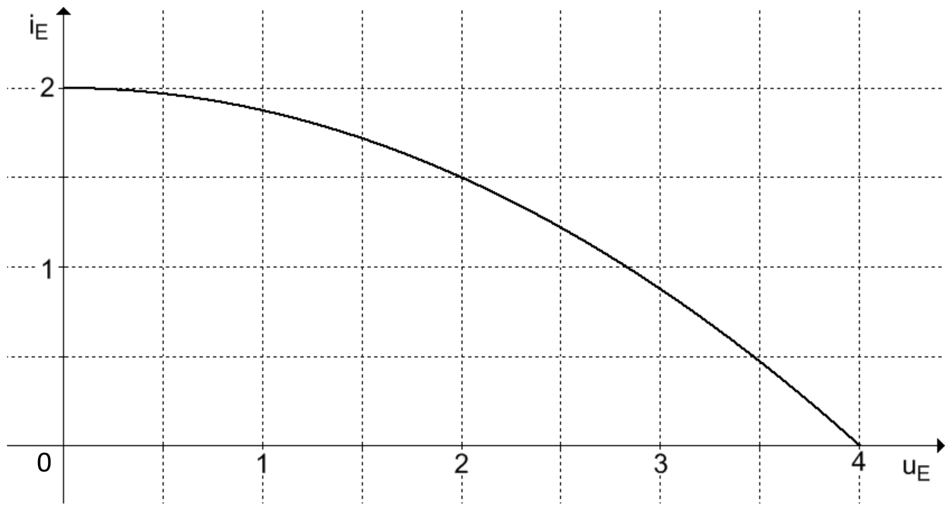

Figure 2: Characteristic curve

c) Given the mechanical model of a train shown in figure 3, derive a state-space model with traction force of the locomotive *f(t)* as an input and velocity of the coach *v(t)* as an output. Use state x_1 as the speed of the locomotive, state x_2 as the force on the spring and x_3 as the speed of the coach.

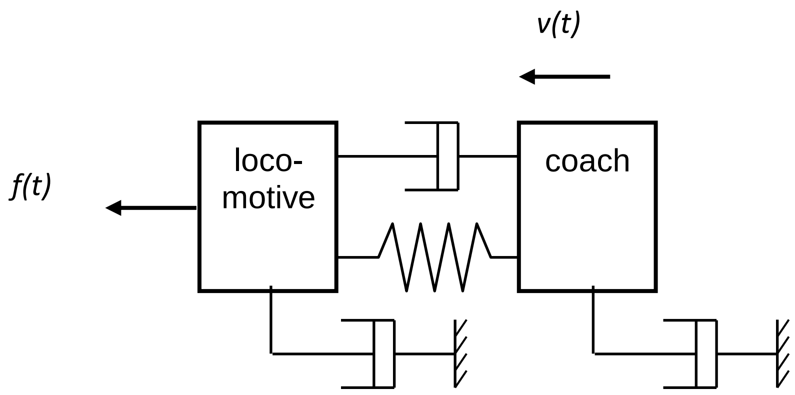

Figure 3: Mechanical model of a train

Problem 2: Experimental Modelling (8+5 points)

a) Given the result of an step response experiment in figure 4 use Küpfmüller approach to determine an mathematical model of the unknown system. The same figure is given also on the answer sheets.

b) Which assumptions are made in Küpfmüller approach and argue if they are valid in this case?

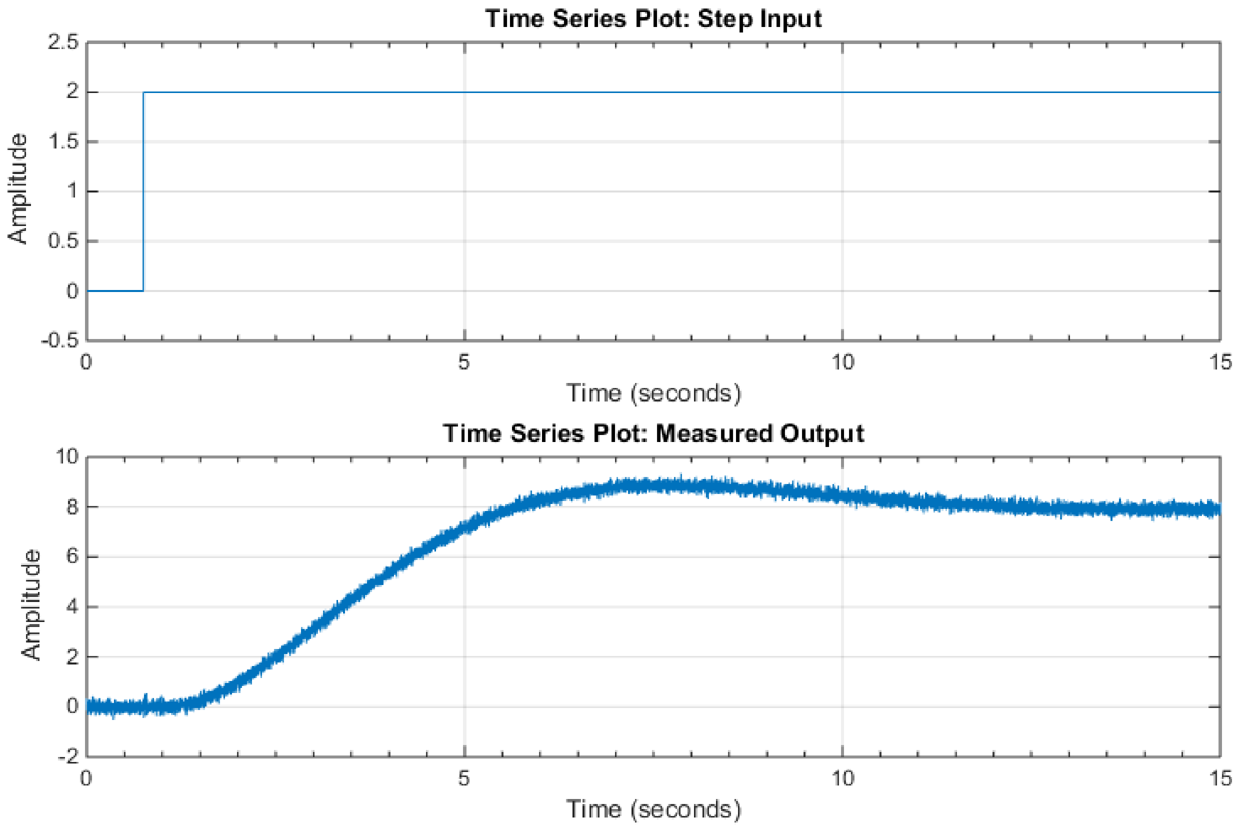

Figure 4: Step Response

Problem 3: Least-Square Method (9+5 points)

The unknown nonlinear but static behaviour of a system is supposed to be approximated by a polynomial of second order. For this purpose measurements data is available in table 1.

x	0.5	2.5	5	6	7.5	9.5	10.5	12
y	1.5	2	4	5	7	9.5	10.5	11

Table 1

a) Calculate the coefficients a, b, c of the optimal least square fit for a polynomial of second order.

$$y = ax^2 + bx + c$$

b) Sketch your resulting function into the prepared place in the answer sheet and argue whether the model assumption of second order was good.

Problem 4: Identification of dynamic systems (4+10+6 points)

The unknown linear dynamic system is supposed to be approximated by a model in form of

$$y(k) = b_1 u(k-1) + b_2 u(k-2) + b_3 u(k-3) + v(k)$$

where y is the system output, u is the control input, $v(k) = w(k) + c_1 w(k-1)$, $w(k)$ is a white noise.

To identify the model parameters the system was excited and measurement data dare given in Table 2.

k	1	2	3	4	5	6	7	8	9	10	11
u(k)	0	1	1	0	1	1	0	0	1	1	1
y(k)	0.11	0.16	0.73	2.81	-0.08	-2.51	2.27	-0.15	-2.76	-0.19	3.68

Table 2

a) Is the noise $v(k)$ white noise or coloured noise?

b) Assume that the least-square approach is used to identify the parameters b_1, b_2, b_3 in the model. Show how this data can be used for least-square identification. For this purpose derive the equation and solve it for the parameters b_1, b_2, b_3 up to the point where you would need to invert the matrix.

c) What are the statistical properties of this least-square estimate?

Problem 5: Prediction Error Method (5+6+4 points)

The unknown linear dynamic behaviour of a system is supposed to be approximated as follows:

$$y(k) + a_1 y(k-1) + a_2 y(k-2) = b_1 u(k-1) + b_2 u(k-2) + w(k) + c_1 w(k-1).$$

where y is the system output, u is the control input and $w(k)$ is white noise.

a) What is the optimal predictor for this model?

b) Formulate the optimisation problem.

c) How can the optimisation problem obtained in b) be solved?

Problem 6: Subspace Method (4+7 points)

The unknown linear dynamic behaviour of a **SISO** system is supposed to be approximated by a state-space model (A, B, C, D). For this purpose the system was excited and measurement data were generated. Based on singular value decomposition, the extended observability matrix Γ_S was obtained as follows:

$$\Gamma_s = \begin{bmatrix} -5.563 & 0.259 \\ -2.002 & -0.569 \\ -0.621 & -0.766 \\ -0.105 & -0.737 \\ 0.072 & -0.639 \\ 0.120 & -0.531 \\ 0.121 & -0.433 \\ 0.107 & -0.349 \\ 0.089 & -0.280 \\ 0.073 & -0.225 \\ 0.059 & -0.180 \end{bmatrix}$$

a) Determine the matrix C of the system based on Γ_S.

b) Show how to calculate the system matrix A based on Γ_S.