Methods of Soft Control
(Methoden der Soft-Control)

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Organisation of this course

Chapter 1: Introduction

Chapter 2: Fuzzy control

Chapter 3: Neural networks

Chapter 4: Evolutionary algorithms

Linguistic variables

- Recall the example of fuzzy controller used to control the amount of cooling medium in a drilling machine:

Rule 1: IF $Speed = very\ low$,
THEN $Amount\ of\ Cooling\ medium = very\ little$.

Rule 2: IF $Speed = low$,
THEN $Amount\ of\ Cooling\ medium = little$.

Rule 3: IF $Speed = middle$,
THEN $Amount\ of\ Cooling\ medium = normal$.

Rule 4: IF $Speed = high$,
THEN $Amount\ of\ Cooling\ medium = much$.

Rule 5: IF $Speed = very\ high$,
THEN $Amount\ of\ Cooling\ medium = very\ much$.

How to realize this fuzzy controller?

Linguistic variables

In the fuzzy description, instead of precise numerical values, the variables are described by linguistic terms.

Compare:

Description 1: His **temperature** is **39,5°C**. (precise value)

Description 2: His **temperature** is **high**. (fuzzy value)

Question:

In which description there is **more** information?

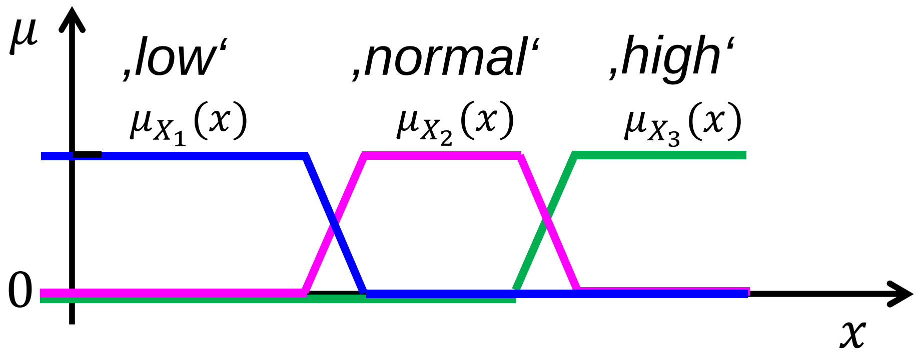
In description 1, we know exactly that the temperature is 39,5°C. In description 2, it is not clear whether the temperature is 39°C or 40°C. That means, some quatitative information is lost in description 2.

However, on the other side, in description 2, it implies that the temperature is „high“ in reference to the normal temperature. In comparison, in description 1, no such information can be read. That means, some qualitative information is contained in description 2.

Linguistic variables

A **linguistic variable** X (e.g. temperature) can take values among **linguistic terms** denoted by $X_1, X_2, \cdots, X_n$ (e.g. ‚low',‘normal',‘high'). Each lingustic term X_i correpsonds to a fuzzy set represented by a membership function $\mu_{X_i}(x)$.

Example:



$\mu_{X_1}(x), \mu_{X_2}(x), \mu_{X_3}(x)$ represent, respectively, the membership functions corresponding to the linguistic terms $X_1('low'), X_2('normal'), X_3('high')$.

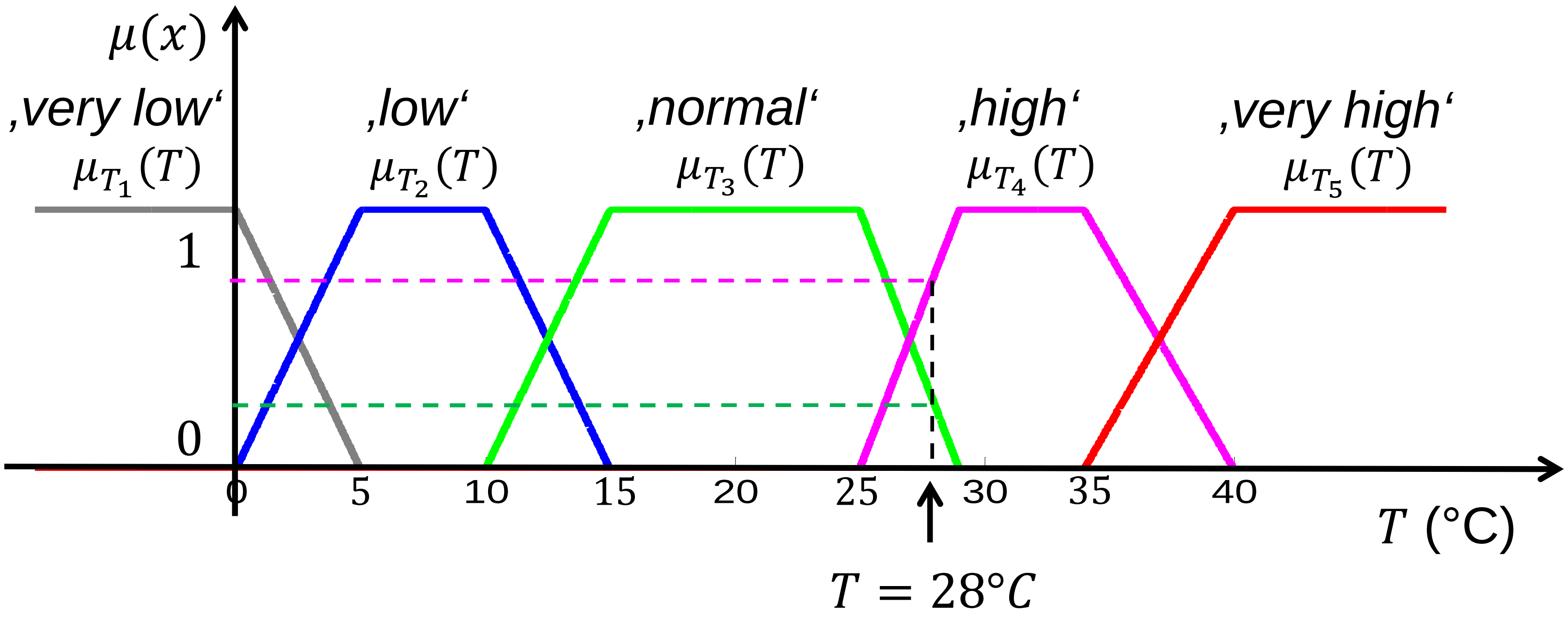
Fuzzification

Fuzzification is the mapping of the numerical variable $x \in \mathbf{R}$ to the linguistic terms $X_1, X_2, \cdots, X_n$ represented, respectively, by the membership functions $\mu_{X_1}(x), \mu_{X_2}(x), \cdots, \mu_{X_n}(x)$, i.e.

$$x \to \mu(x) = \begin{bmatrix} \mu_{X_1}(x) \\ \mu_{X_2}(x) \\ \vdots \\ \mu_{X_n}(x) \end{bmatrix}$$

Fuzzification

Example: The linguistic variable is Temperature



How to describe the membership functions $\mu_{T_1}(T), \cdots, \mu_{T_5}(T)$?

What is the fuzzified value of $T = 28°C$?

Fuzzification

The membership functions are piecewise linear and can be described by

$$\mu_{T_1}(T) = \begin{cases} 1, & if \;\; x \leq 0 \\ -0.2(x-5), & if \;\; 0 < x \leq 5 \\ 0, & if \;\; x > 5 \end{cases}$$

$$\mu_{T_2}(T) = \begin{cases} 0, & if \;\; x \leq 0 \\ 0.2x, & if \;\; 0 < x \leq 5 \\ 1, & if \;\; 5 < x \leq 10 \\ -0.2(x-15), & if \;\; 10 < x \leq 15 \\ 0, & if \;\; x > 15 \end{cases}$$

If $T = 28°C$, then $\mu(T) = \begin{bmatrix} \mu_{T_1}(T) \\ \mu_{T_2}(T) \\ \mu_{T_3}(T) \\ \mu_{T_4}(T) \\ \mu_{T_5}(T) \end{bmatrix} = \begin{bmatrix} \mu_{very\ low}(28) \\ \mu_{low}(28) \\ \mu_{normal}(28) \\ \mu_{high}(28) \\ \mu_{very\ high}(28) \end{bmatrix} = \begin{bmatrix} 0 \\ 0 \\ 0.25 \\ 0.75 \\ 0 \end{bmatrix}$

Fuzzy implication

Fuzzy rule:

$$\text{IF } \underbrace{X = A}_{\text{premise}}, \text{ THEN } \underbrace{Y = B}_{\text{conclusion}}.$$

Basic principle for the implication operator:

The membership function value of the conclusion should not be greater than the membership function value of the premise.

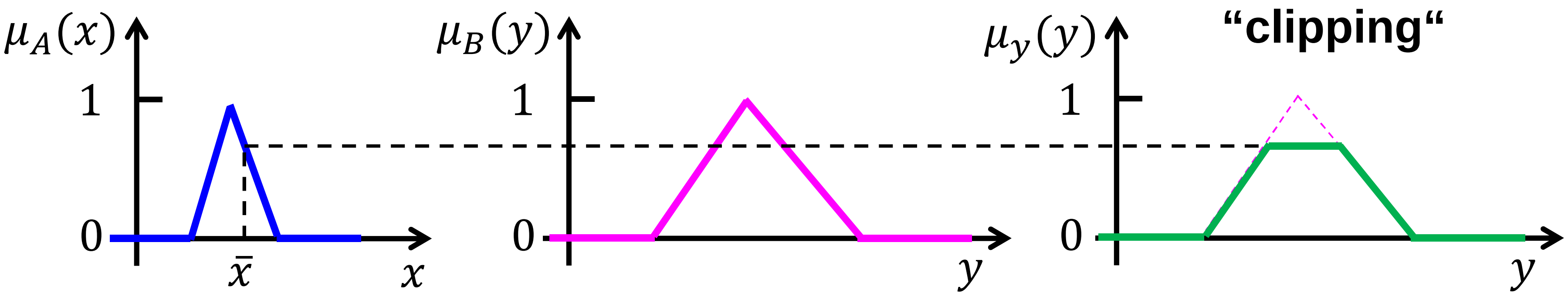
Implication operators:

- **Min-operator**: $\mu_y(y) = \min\ \{\mu_A(x), \mu_B(y)\}$
- **Algebraic product operator**: $\mu_y(y) = \mu_A(x) \cdot \mu_B(y)$

Fuzzy implication

Min-operator:

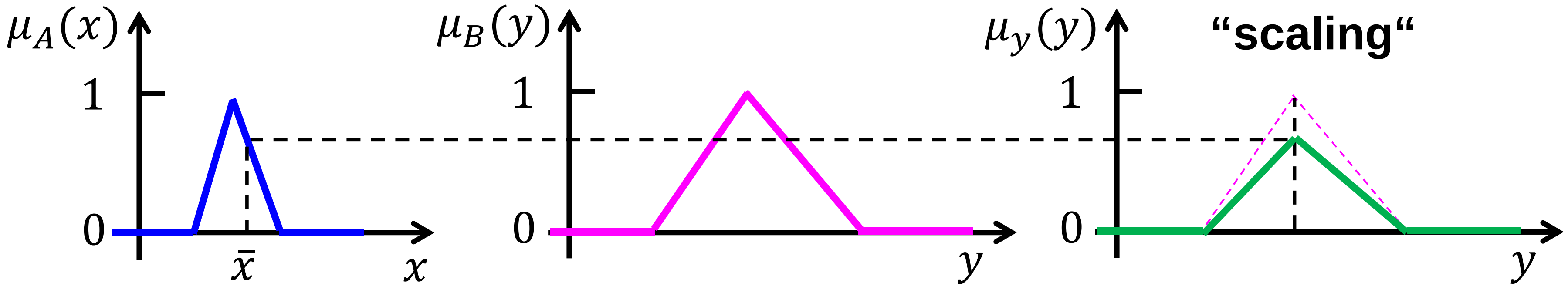
$$\mu_y(y) = \min\{\mu_A(\bar{x}), \mu_B(y)\}$$



Fuzzy implication

Algebraic product operator:

$$\mu_y(y) = \mu_A(\bar{x}) \cdot \mu_B(y)$$



Aggregation of sub-premises

Fuzzy rule:

$$\text{IF } \underbrace{X = A}_{\text{sub-premise 1}} \text{ AND } \underbrace{Y = B}_{\text{sub-premise 2}}, \text{ THEN } \underbrace{Z = C}_{\text{conclusion}}.$$

The premise of the fuzzy rule consists of several sub-premises. Thus, the grade of membership of the premise is calculated by

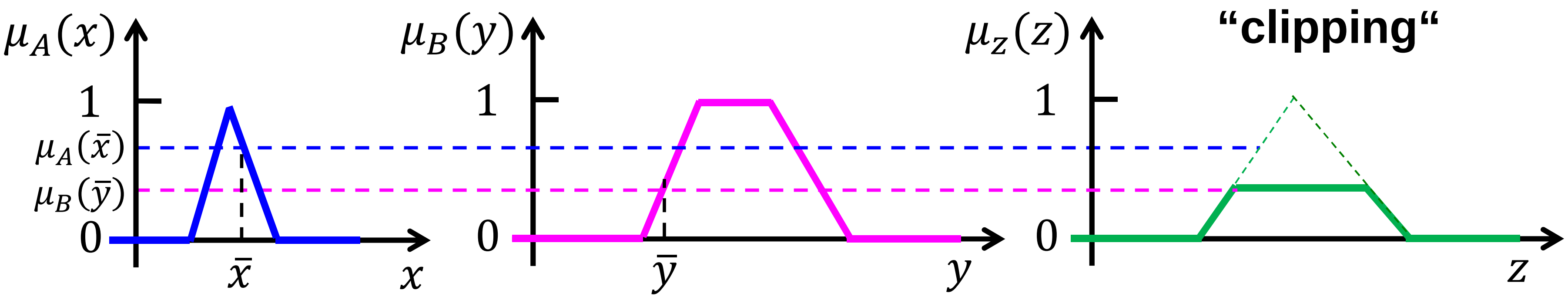
$$\mu_{agg}(x, y) = \mu_A(x) \cap \mu_B(y)$$

After that, the grade of membership of the conclusion is got by applying the implication operator.

Example: IF the **reactor temperature** is **high** and the **reactor pressure** is **low**, THEN the valve position is in the middle.

Aggregation of premises

Example:



In this example, for the given values of $\bar{x}, \bar{y}$, there is

$$\mu_B(\bar{y}) < \mu_A(\bar{x})$$

The grade of membership of the premise in the fuzzy rule is

$$\mu_{agg}(\bar{x}, \bar{y}) = \mu_A(\bar{x}) \cap \mu_B(\bar{y}) = \min\ \{\mu_A(\bar{x}), \mu_B(\bar{y})\} = \mu_B(\bar{y})$$

If the min-operator is chosen as implication operator, then the grade of membership of the conclusion is

$$\mu_z(z) = \min\{\mu_{agg}(\bar{x}, \bar{y}), \mu_C(z)\}$$

Accumulation of conclusions

Fuzzy rule base:

Rule 1: IF $X = A_1$, THEN $Z = C_1$.

Rule 2: IF $X = A_2$, THEN $Z = C_2$.

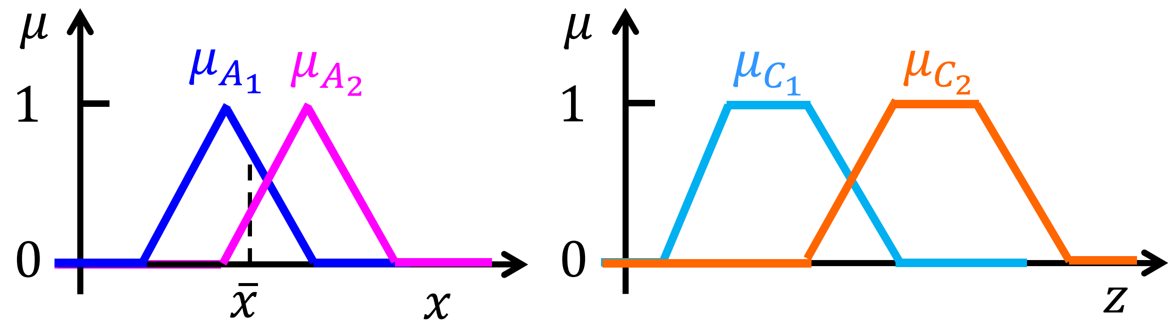
The fuzzy rules are combined by the fuzzy OR operator.

Hence, the grade of membership of the accumulated conclusion is

$$\mu_{acc}(z) = \mu_{R1}(z) \cup \mu_{R2}(z)$$

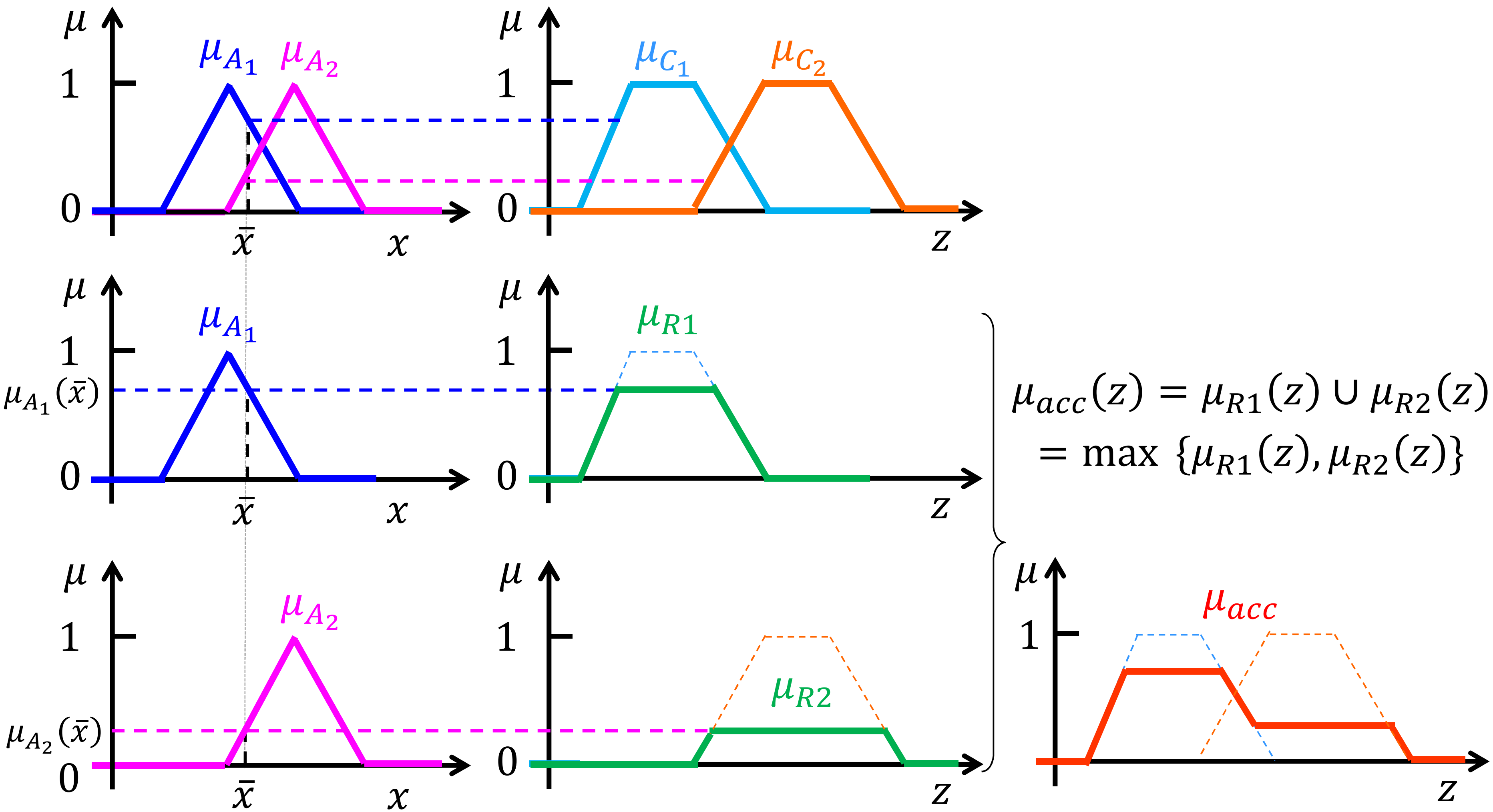
Accumulation of conclusions

Example: implication operator: min, OR-operator: max



Accumulation of conclusions

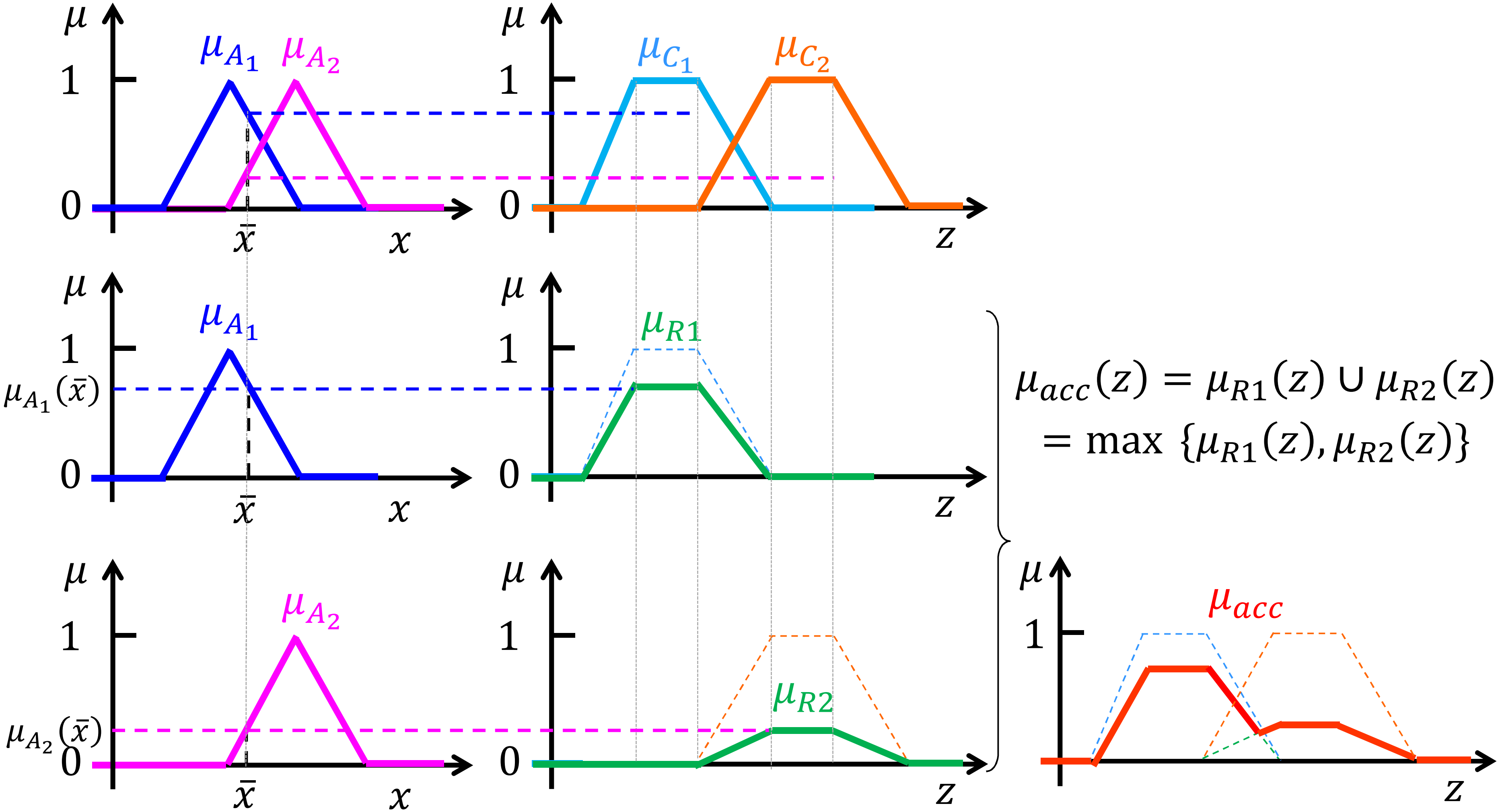
Example: implication operator: min, OR-operator: max



$$\mu_{acc}(z) = \mu_{R1}(z) \cup \mu_{R2}(z)$$
$$= \max\{\mu_{R1}(z), \mu_{R2}(z)\}$$

Accumulation of conclusions

Example: implication operator: algebraic product, OR-operator: max



$$\mu_{acc}(z) = \mu_{R1}(z) \cup \mu_{R2}(z) = \max\{\mu_{R1}(z), \mu_{R2}(z)\}$$

Accumulation of conclusions

Example: Fuzzy control of the amount of cooling medium in a drilling machine

Rule 1: IF $Speed = very\ low$,
THEN $Amount\ of\ Cooling\ medium = very\ little$.

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Rule 5: IF $Speed = very\ high$,
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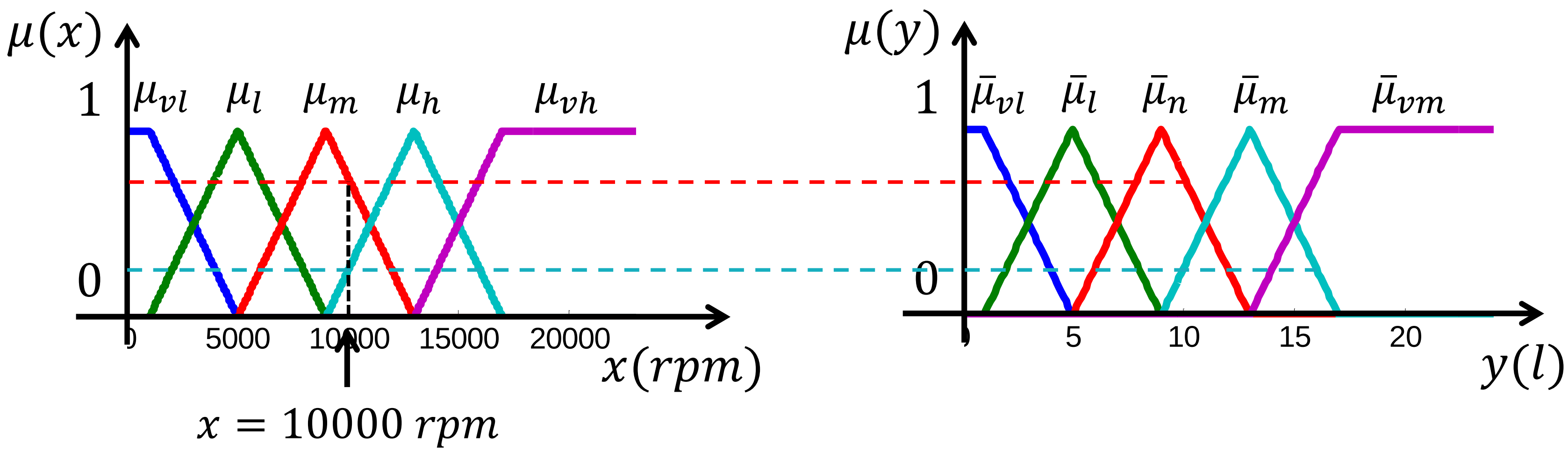
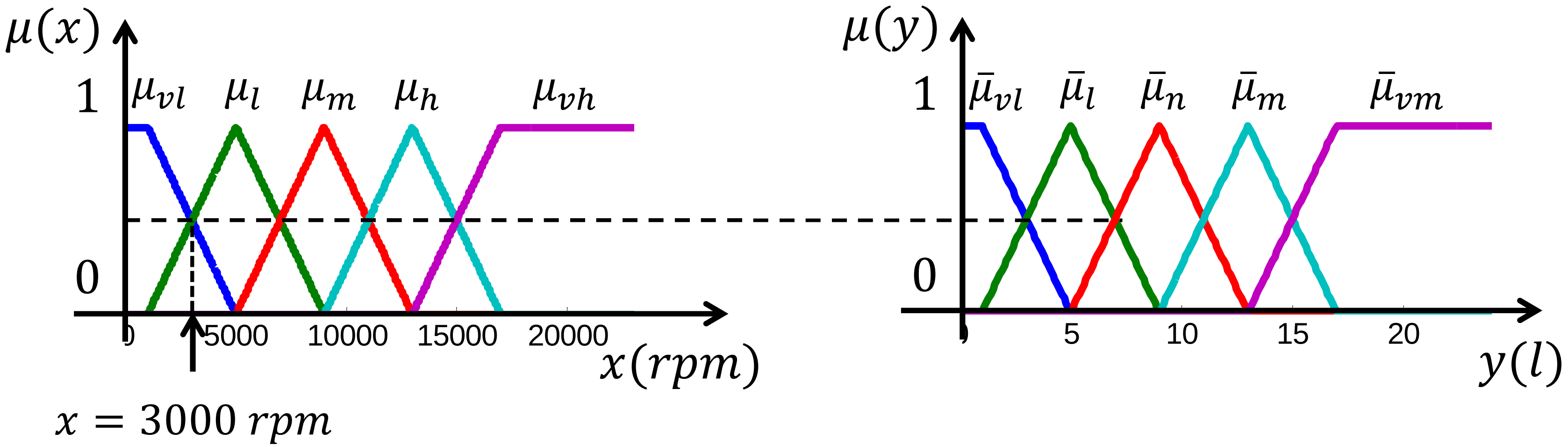
Determine the grade of membership of the accumulated conclusion at the given rotational speed.

Accumulation of conclusions

Case 1: Speed $x = 10000\ rpm$

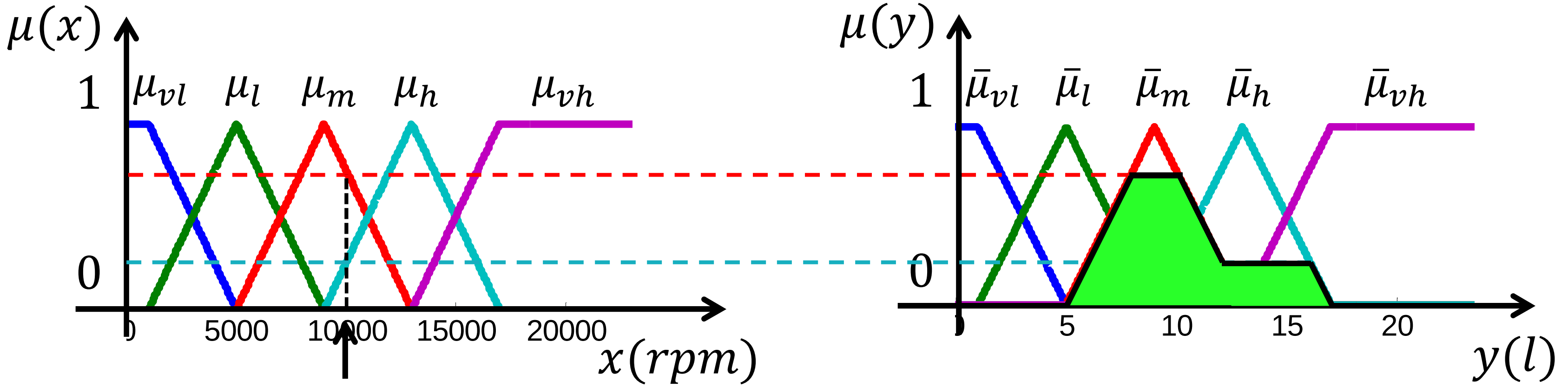


Case 2: Speed $v = 3000\ rpm$



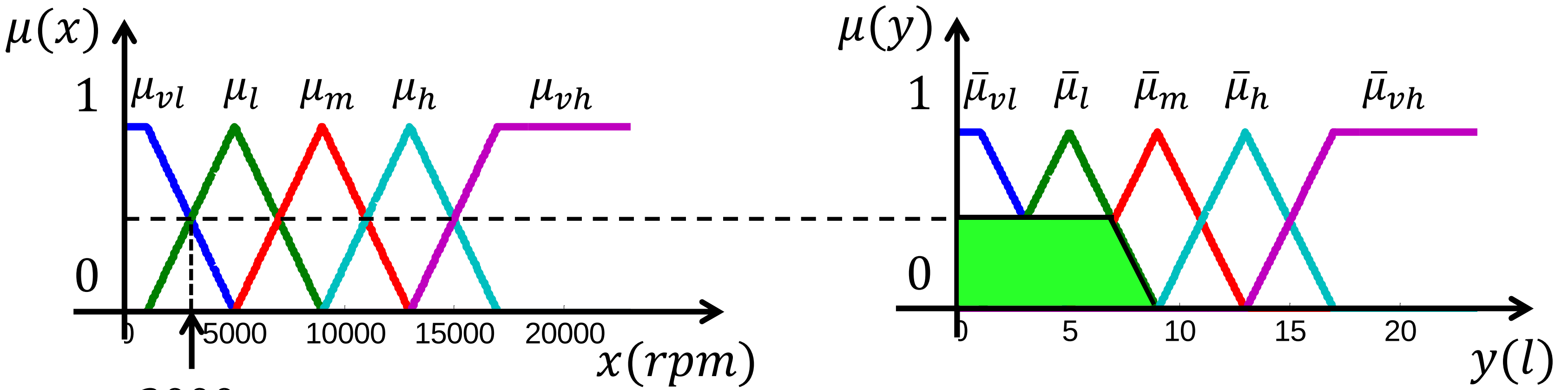
Accumulation of conclusions

Case 1: Speed $x = 10000\ rpm$



$x = 10000\ rpm$

Case 2: Speed $x = 3000\ rpm$



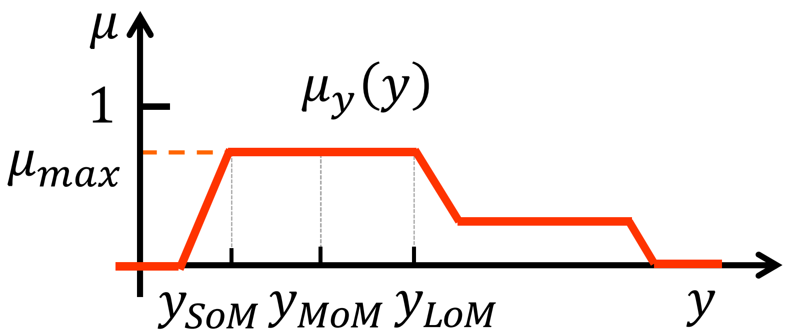
$x = 3000\ rpm$

Defuzzification

Defuzzification aims to find the crisp value y corresponding to the membership function $\mu(y)$.

Maximum methods:

- Mean of maximum (MoM)
- Smallest of maximum (SoM)
- Largest of maximum (LoM)



Let Y_m denote the set of y that correpsond to the maximum of the membership function, i.e. $Y_m = \{y | \mu_y(y) = \mu_{max}\}$.

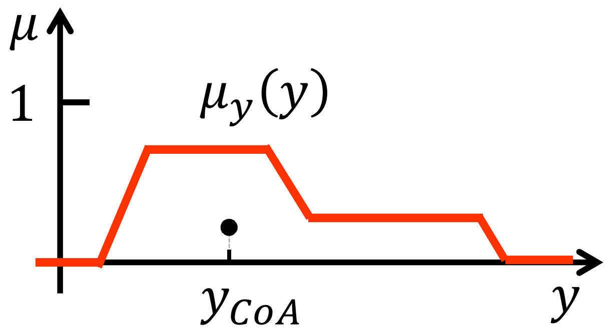
$$y_{MoM} = \underset{y \in Y_m}{\mathrm{mean}}\ y$$

$$y_{SoM} = \min_{y \in Y_m} y$$

$$y_{LoM} = \max_{y \in Y_m} y$$

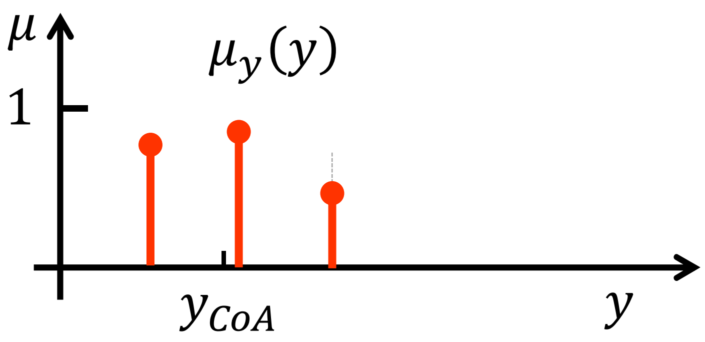
Defuzzification

Center of area (CoA) method: The crisp value is given by the y-coordinate of the center of area of the membership function $\mu_y(y)$.



$$y_{CoA} = \frac{\int y\mu_y(y)dy}{\int \mu_y(y)dy}$$

Center of average for Singletons: The crisp value is given by the weighted sum of the y-coordinates.



$$y_{COA} = \frac{\sum_{j=1}^{m} y_i\mu_y(y_i)}{\sum_{j=1}^{m} \mu_y(y_i)}$$