Modelling and Identification

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Institute for Automatic Control

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Organisation of this course

Chapter 1: Introduction

Chapter 2: **Theoretical modelling**

Chapter 3: Experimental modelling

Chapter 4: Least-Squares methods

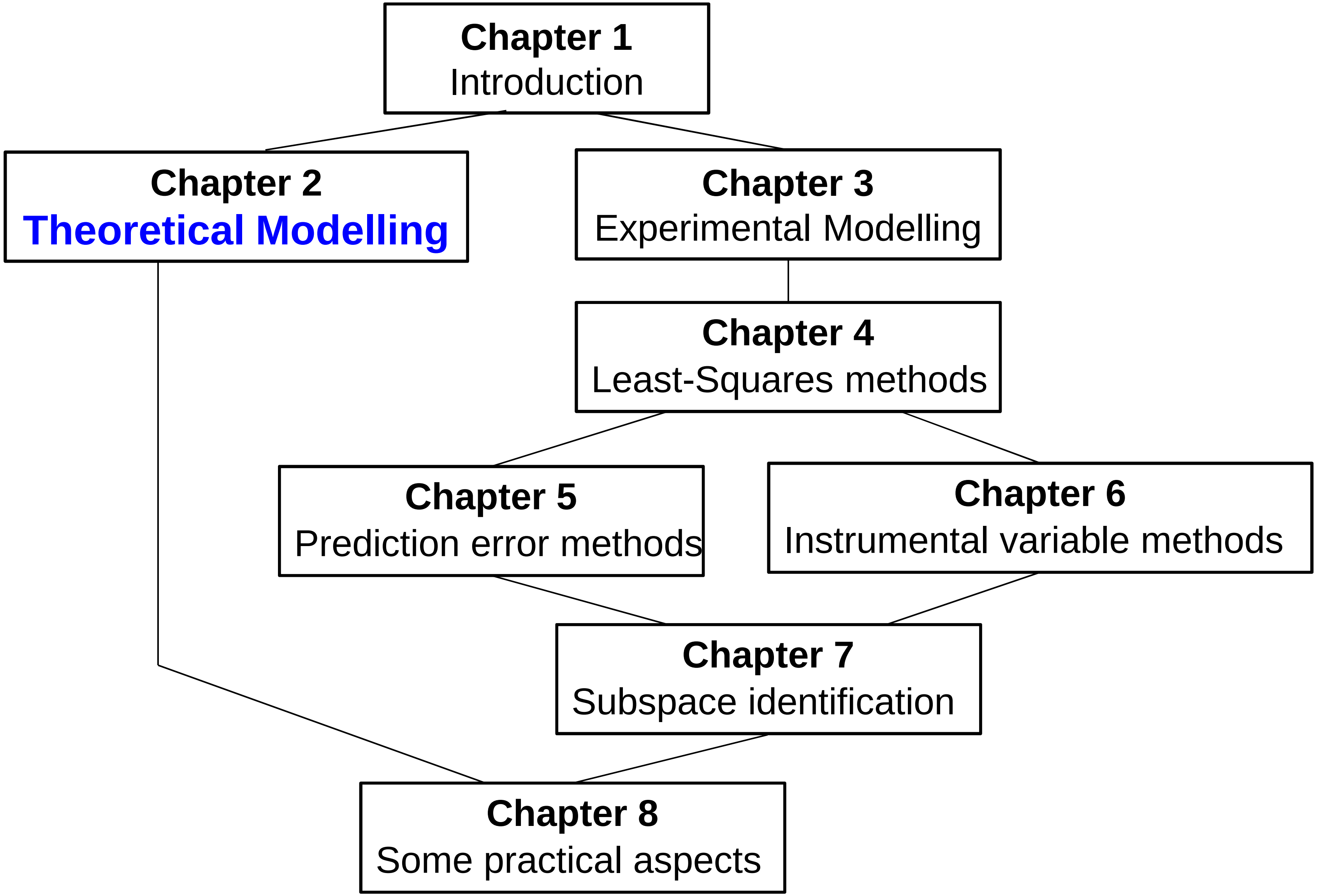
Chapter 5: Prediction error methods

Chapter 6: Instrumental variable methods

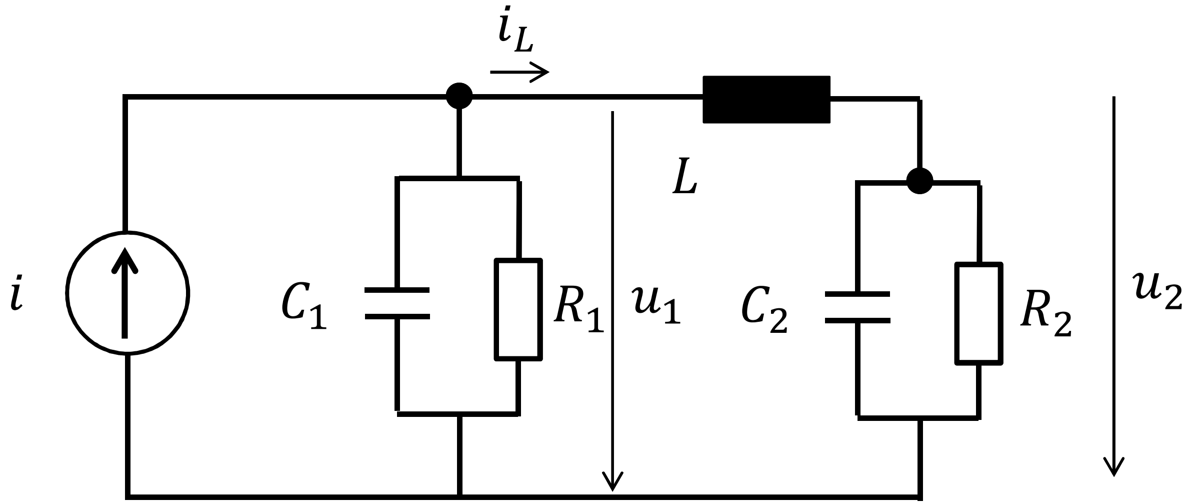
Chapter 7: Subspace identification methods (SS model!)

Chapter 8: Some practical aspects

Organisation of this course



Example: electrical system

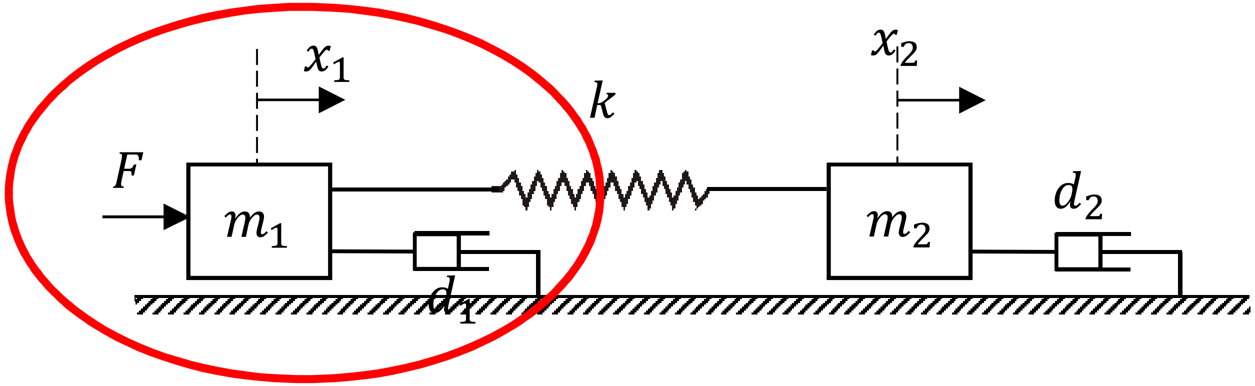


Kirchhoff current law: $$C_1 \frac{du_1}{dt} = i - i_L - \frac{u_1}{R_1}$$

Kirchhoff current law: $$C_2 \frac{du_2}{dt} = i_L - \frac{u_2}{R_2}$$

Kirchhoff voltage law: $$L \frac{di_L}{dt} = u_1 - u_2$$

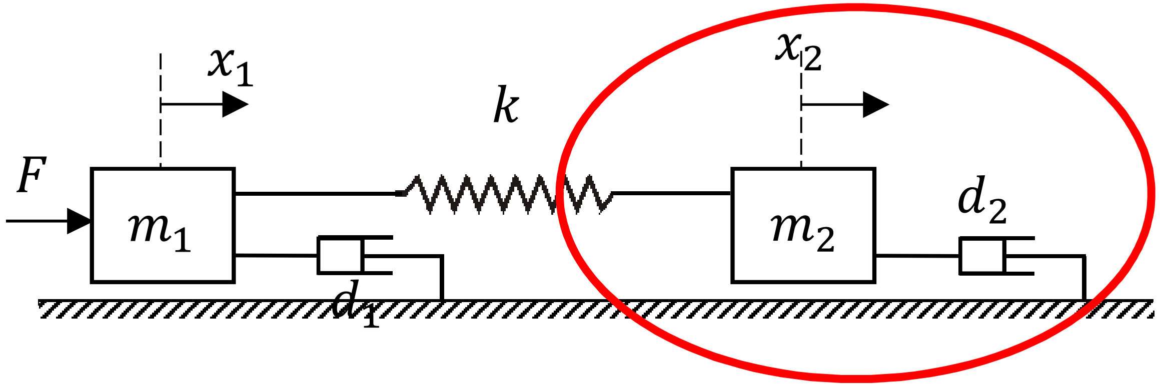
Example: mechanical system



Force balance for the first mass:

$$m\ddot{x}_1 = F - d_1\dot{x}_1 - f_s$$

Example: mechanical system



Force balance for the first mass:

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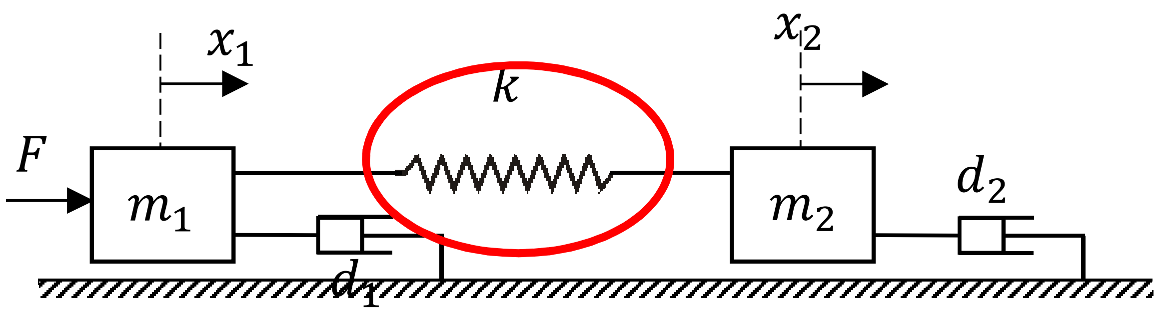
Force balance for the second mass:

$$m_2\ddot{x}_2 = f_s - d_2\dot{x}_2$$

Coupling between two subsystems?

Network analysis

Example: mechanical system



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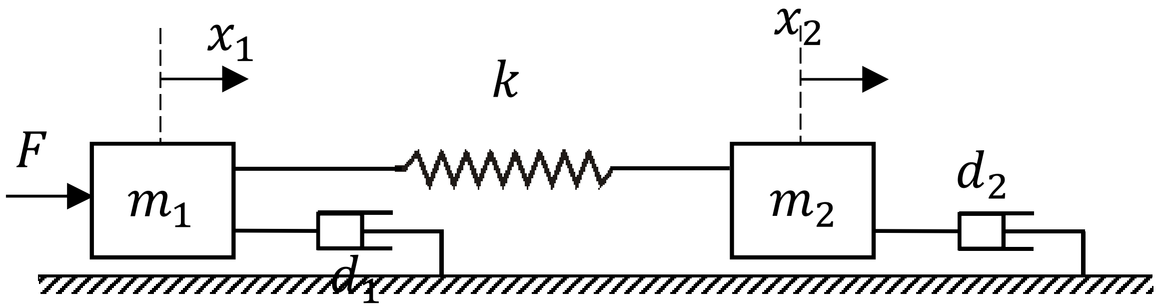
Coupling between two subsystems:

$$f_s = k(x_1 - x_2)$$

Example: mechanical system

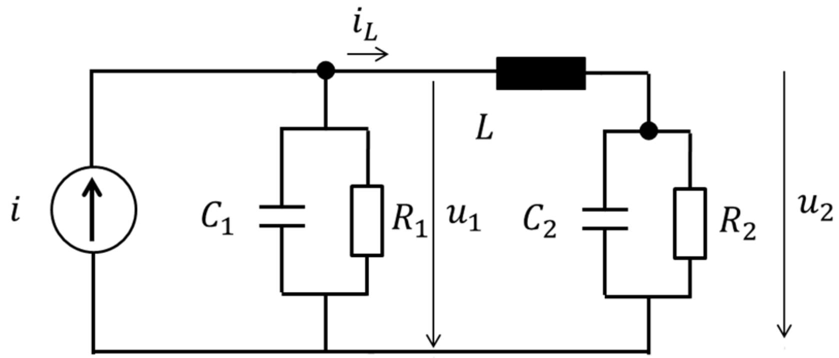


System model

$$\begin{cases} m_1\ddot{x}_1 = F - d_1\dot{x}_1 - k(x_1 - x_2) \\ m_2\ddot{x}_2 = k(x_1 - x_2) - d_2\dot{x}_2 \end{cases}$$

Network analysis

Electrical system

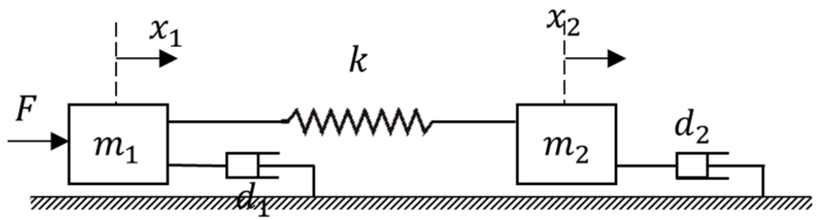


$$C_1 \frac{du_1}{dt} = i - i_L - \frac{u_1}{R_1}$$

$$C_2 \frac{du_2}{dt} = i_L - \frac{u_2}{R_2}$$

$$L \frac{di_L}{dt} = u_1 - u_2$$

Mechanical system



$$m_1 \ddot{x}_1 = F - d_1 \dot{x}_1 - f_s$$

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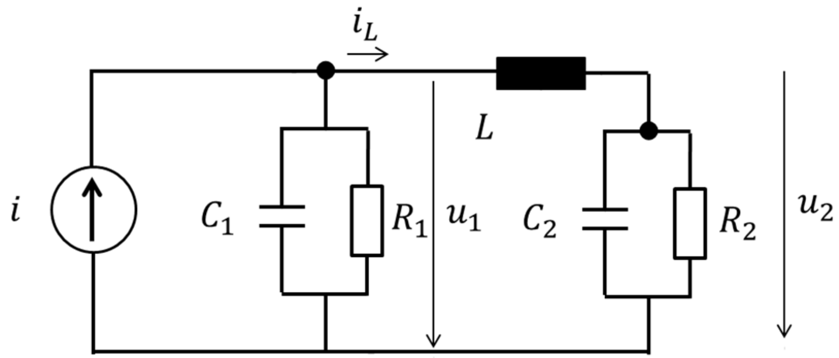
Network analysis

Electrical system



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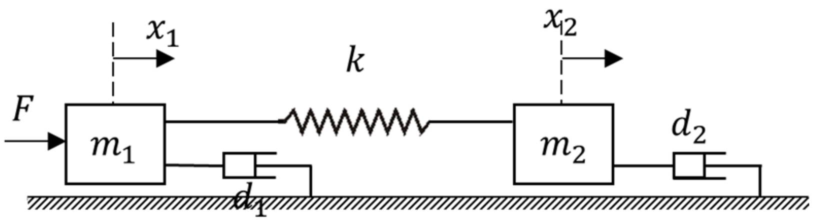
Mechanical system



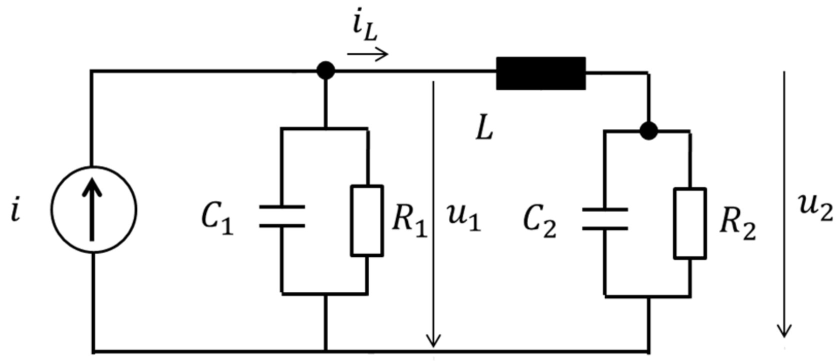
$$m_1 \frac{dv_1}{dt} = F - d_1 v_1 - f_s$$

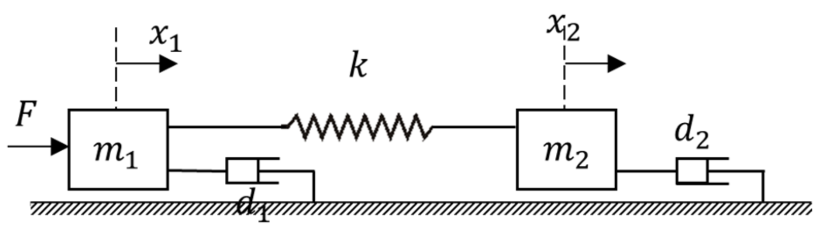
$$m_2 \frac{dv_2}{dt} = f_s - d_2 v_2$$

$$\frac{df_s}{dt} = k v_1 - k x_2$$

Electrical system

Mechanical system





$$C_1 \frac{du_1}{dt} = i - i_L - \frac{u_1}{R_1}$$

$$C_2 \frac{du_2}{dt} = i_L - \frac{u_2}{R_2}$$

$$L \frac{di_L}{dt} = u_1 - u_2$$

$u_1 <-> v_1$

$u_2 <-> v_2$

$i_L <-> f_s$

$i <-> F$

Voltage <-> velocity

Current <-> force

$$m_1 \frac{dv_1}{dt} = F - d_1 v_1 - f_s$$

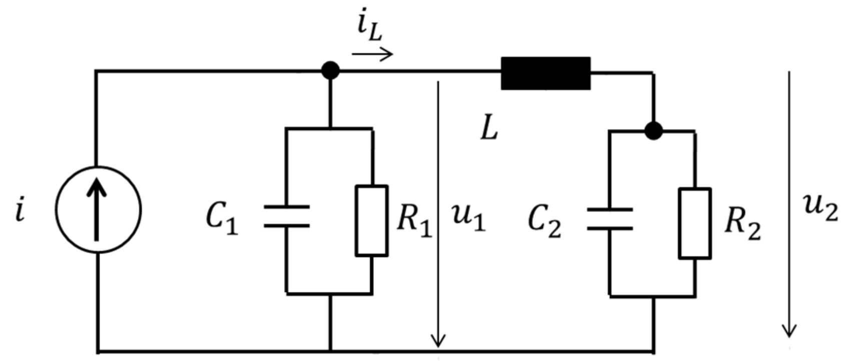
$$m_2 \frac{dv_2}{dt} = f_s - d_2 v_2$$

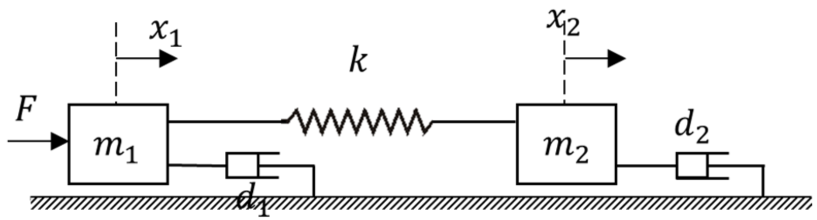
$$\frac{df_s}{dt} = k v_1 - k x_2$$

Network analysis

Electrical system

Mechanical system





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$$L \frac{di_L}{dt} = u_1 - u_2$$

L <-> $\frac{1}{k}$

C_1 <-> m_1

C_2 <-> m_2

R_1 <-> $\frac{1}{d_1}$

R_2 <-> $\frac{1}{d_2}$

$$m_1 \frac{dv_1}{dt} = F - d_1 v_1 - f_s$$

$$m_2 \frac{dv_2}{dt} = f_s - d_2 v_2$$

$$\frac{df_s}{dt} = k v_1 - k v_2$$

Generalized network analysis

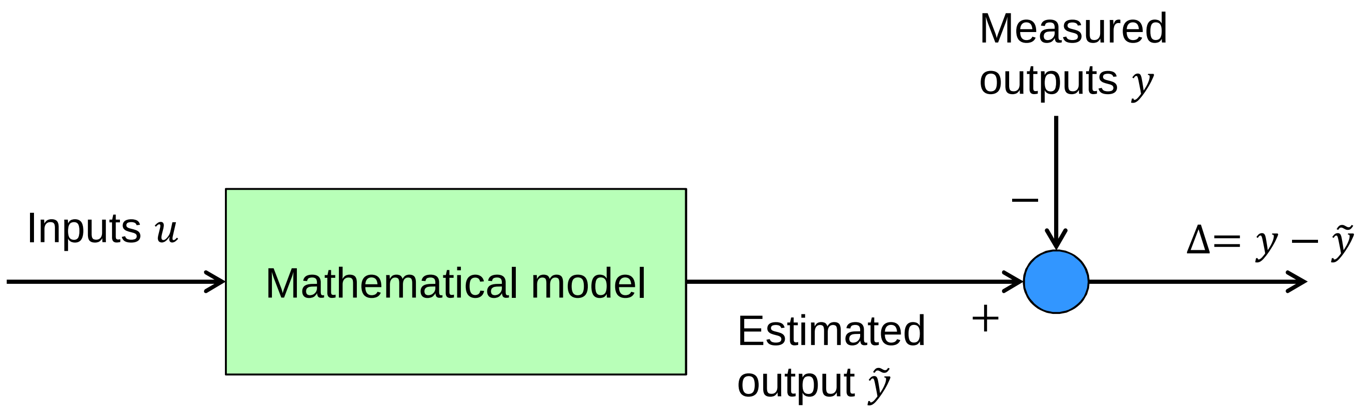
- **Basic idea:**
  - transform non-electrical system into equivalent electrical system
  - Analyse the resulting electrical system

Generalized network analysis

System	Variable 1	Variable 2
electrical	current	voltage
Translation mechanical	force	velocity
Rotational mechanical	torque	Angular velocity
hydraulic	volume flow rate	pressure
pneumatic	gas mass flow rate	pressure
thermal	heat flow rate	temperature

Validation of models

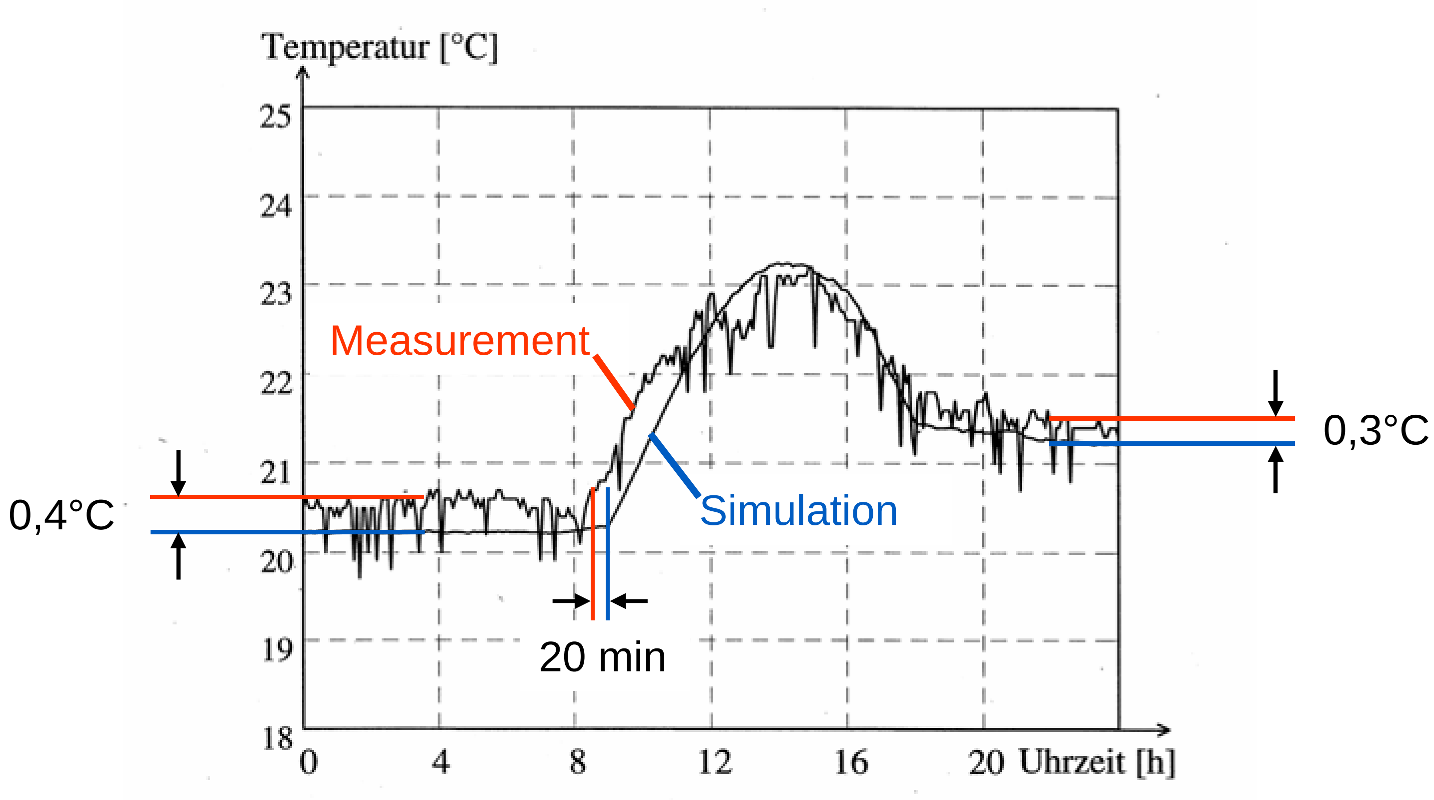
- **Validation:** Check the performance of the model
- The model validation can be carried out in the **time domain** or in the **frequency domain.**
- **What we need for validation:** measurements of inputs and outputs



Validation of models

Time domain analysis:

Compare the **signals calculated based on the model** with the **measured signals**

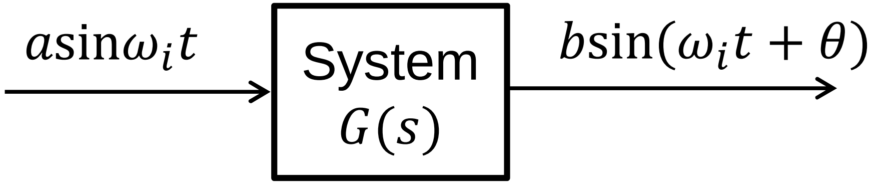


Validation of models

Frequency domain analysis:

Compare the **frequency response calculated based on the model** with the **measured frequency response**

The frequency response at some given frequency ω_i can be measured as follows:



Magnitude of $G(j\omega)$ at $\omega = \omega_i$: $\frac{b}{a}$

Phase angle of $G(j\omega)$ at $\omega = \omega_i$: θ

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Model transformation

Description in the time domain

- Differential equation
- State space model
- Step response
- Impulse response

Fourier transform & Laplace transform

Description in the frequency domain

- Transfer function
- Frequency response

Summary

- Theoretical modelling derives a mathematical model of dynamic systems based on physical and chemical principles of the components in the system.
- Some often used physical principles in different kinds of systems have been reviewed:
  - Mechanical systems
  - Electrical systems
  - Electromagnetic systems
  - Fluid systems
  - Thermal systems
- The models got by theoretical modelling have clear physical meaning. → white-box modelling
- The theoretical models can be validated with measurement data in the time domain or in the frequency domain.

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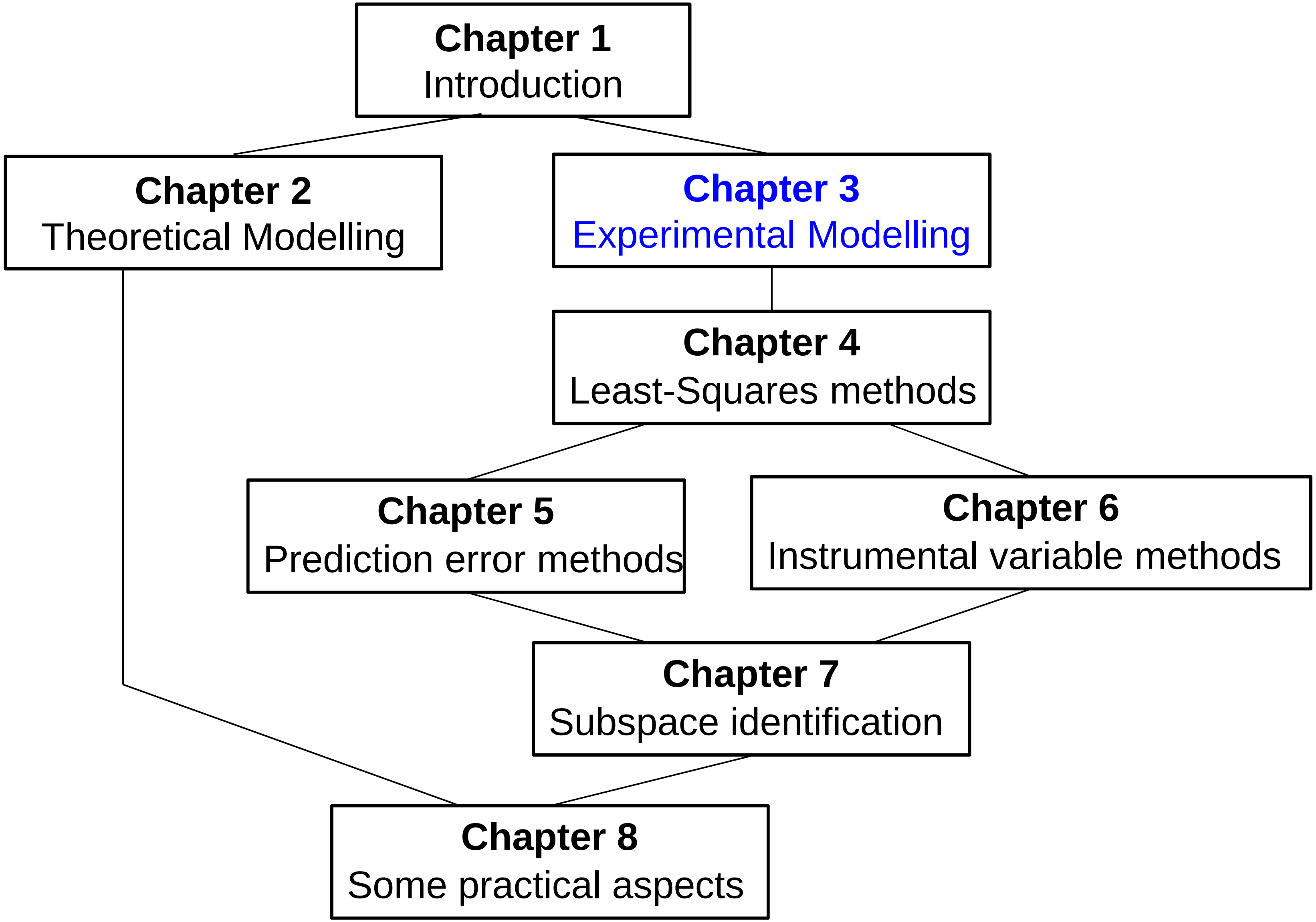
Chapter 5: Prediction error methods

Chapter 6: Instrumental variable methods

Chapter 7: Subspace identification methods (SS model!)

Chapter 8: Some practical aspects

Organisation of this course

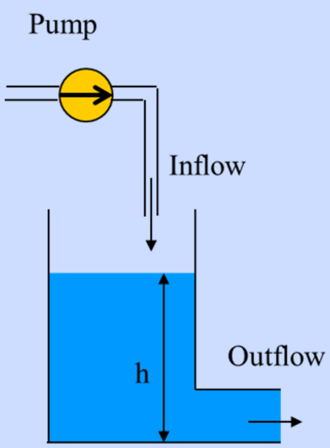
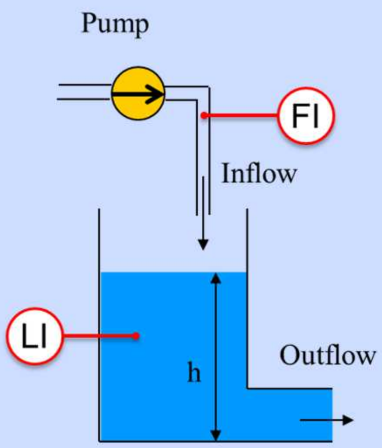
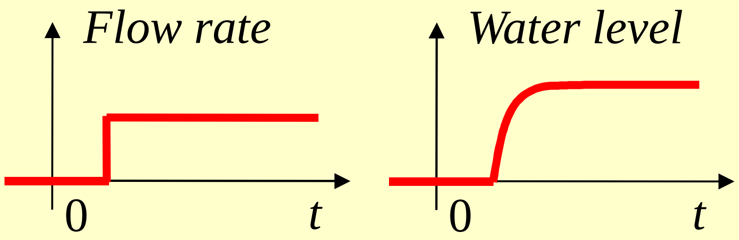


Chapter 3

Experimental Modelling

Review: Modelling strategies

Example: One-tank system

Theoretical modelling	Experimental modelling
 	
Mass balance: $A\frac{dh}{dt} = Q_{\text{inflow}} - Q_{\text{outflow}}$ Torricelli' law: $Q_{\text{outflow}} = aA_0\sqrt{2gh}$	
$A\frac{dh}{dt} = Q_{\text{inflow}} - aA_0\sqrt{2gh}$	$H(s) = \frac{K}{Ts+1} Q_{in\ flow}(s)$

General aspects

- **Basic idea of experimental modelling**:
  - Collect the system input and output data during experiments or during normal operation
  - Based on the data, derive a mathematical model of the system
- Experimental modelling is often called **system identification**.
- A number of **identification approaches** have been developed.
  - Time and frequency domain responses
  - Least squares methods
  - Prediction error methods
  - Instrumental variable methods
  - Subspace identification methods

General aspects

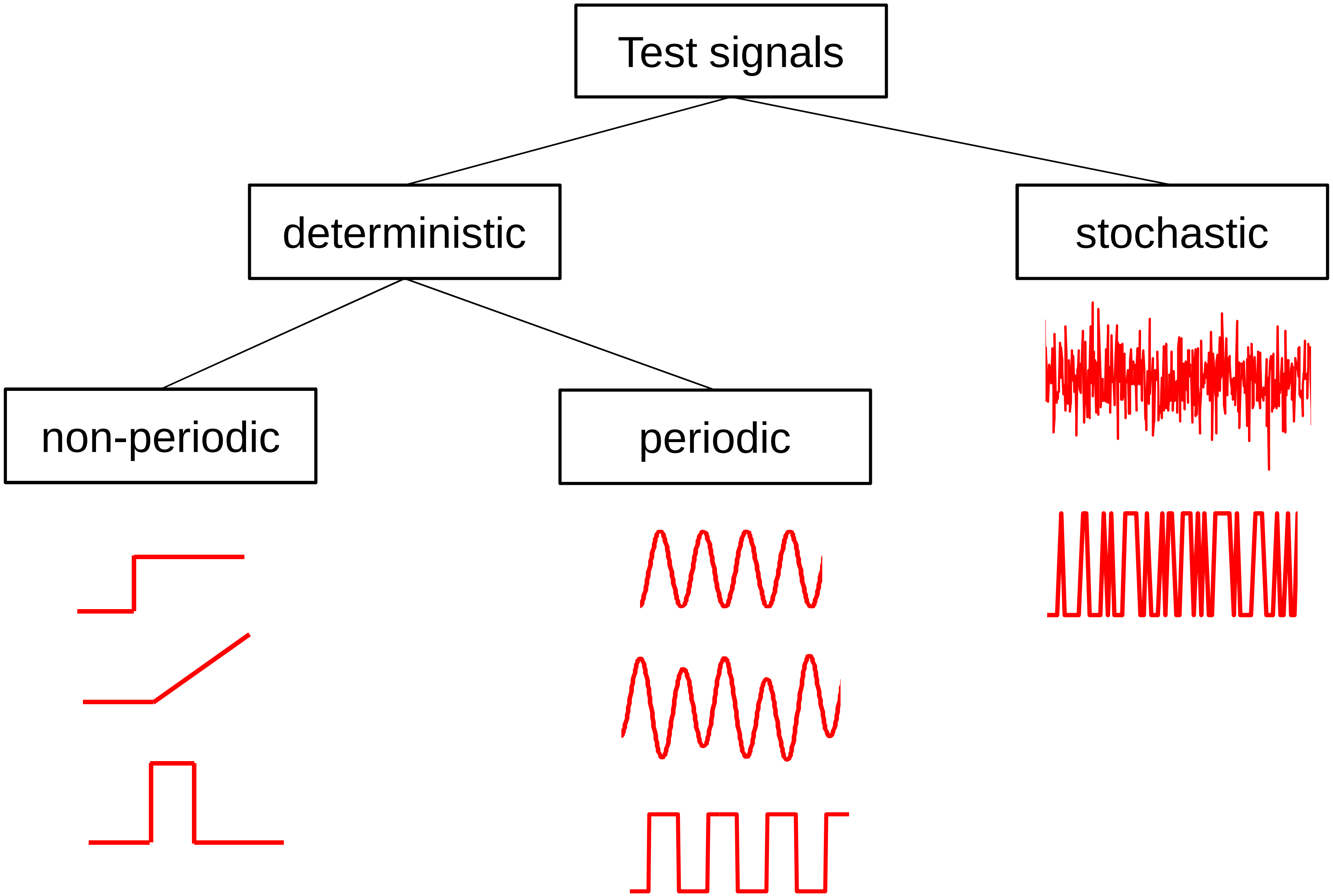
- **Model types**:
  - **Parametric models** (e.g. transfer functions, differential/difference equations, state space models)
  - **Nonparametric models** (e.g. step response, impulse response, frequency response)

- A non-parametric model can be approximated by a parametric model.

General aspects

- **Test signals** play an important role for getting a good model.
- **Requirements for test signals**:
  - The relevant frequency range has to be excited.
  - So large that the response is sufficiently large (larger than disturbances)
  - In case of a linear model, so small that the system remains approximately linear.

General aspects



Focus of today

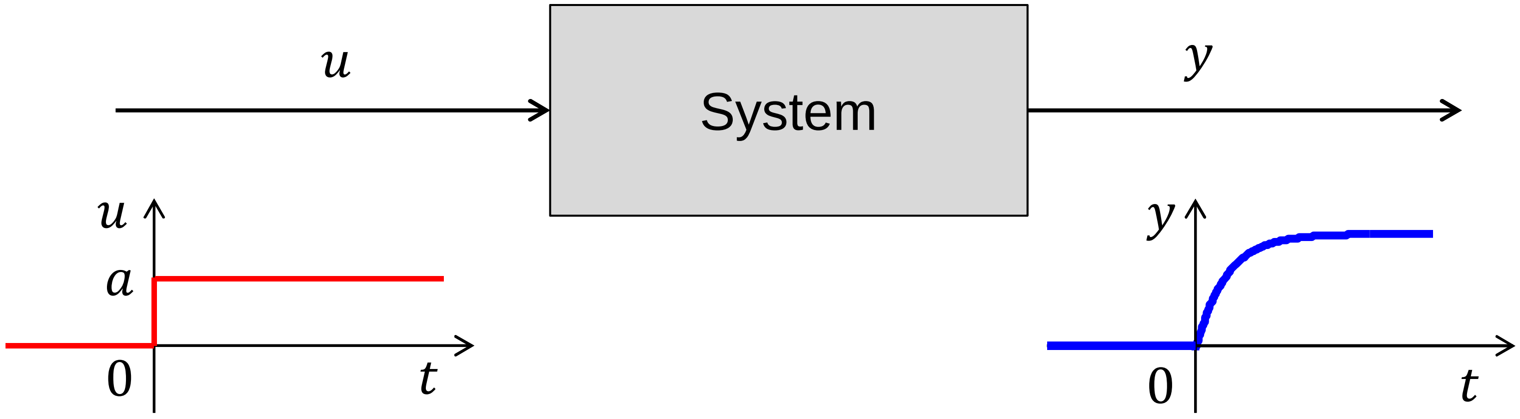
- **Measurement of non-parametric models**
  - Step response
  - Impulse response
  - Frequency response

- **Identification of parametric models (Part 1)**
  - Get parametric model from step response
  - Get parametric model from frequency response

Measurement of step response

Basic procedure:

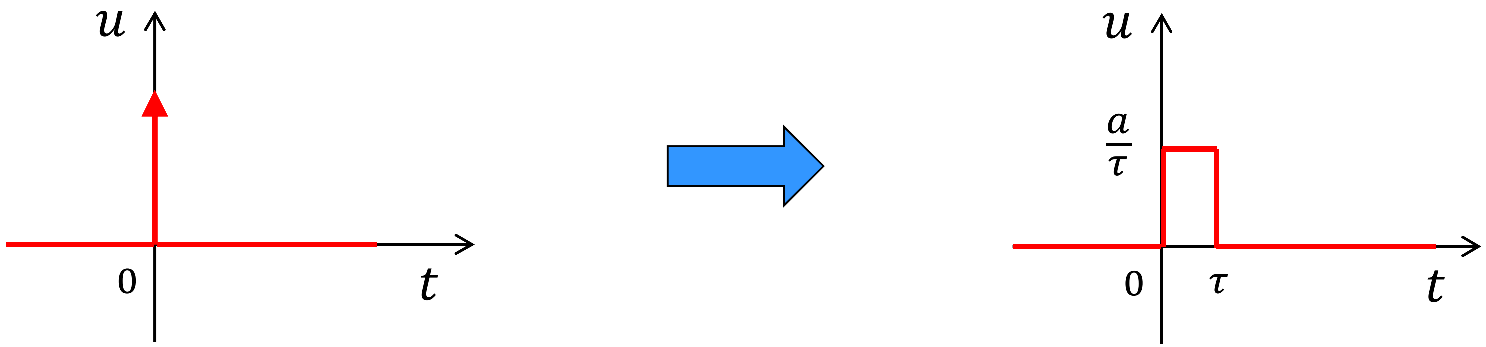
- Bring the system into steady state
- Add a step change of suitable amplitude ($u(t) = a\sigma(t)$) to the control input signal
- Record the system output $y(t)$
- Get the step response $h(t) = \frac{y(t)}{a}$.



$u(t) = a\sigma(t)$

Measurement of impulse response

- **Approach 1**: Measure step response and then take derivative
- **Approach 2**: Add an impulse signal as control input signal, get the output signal as impulse response $g(t)$



$u(t) = a\delta(t)$

duration of impulse $\ll$ time constant

Measurement of impulse response

- **Approach 3**:
  - Add an arbitrary signal as control input signal $u(t)$
  - Measure the response and then calculate for impulse response $\mathrm{g}(t)$

Recall

$$y(t) = \int_0^t g(\tau)u(t-\tau)d\tau$$

At discrete time instants $t = kT$,

$$y(kT) \approx T \sum_{j=0}^{k} g(jT)u\big((k-j)T\big)$$

Measurement of impulse response

At discrete time instants $t = kT$,

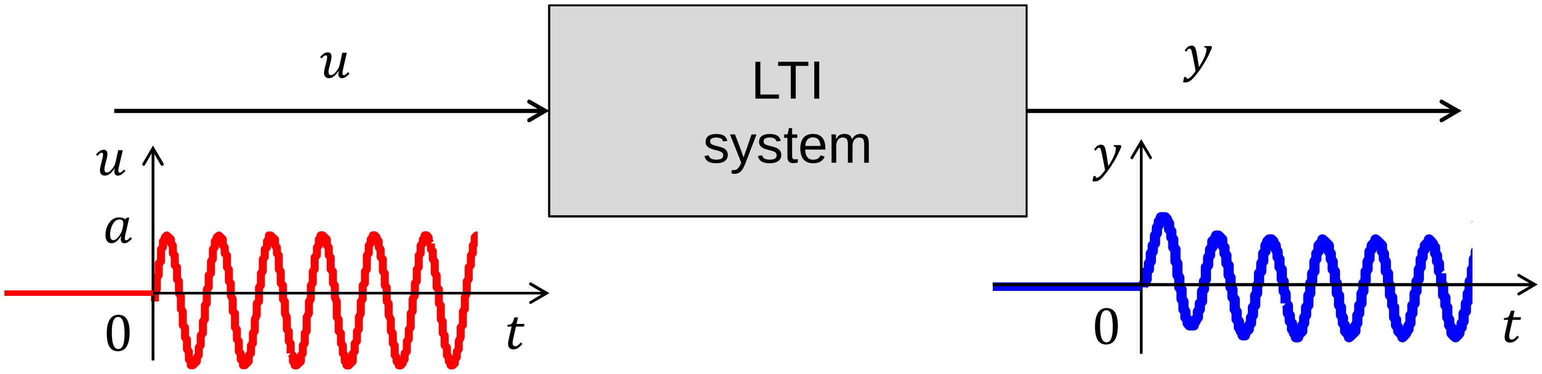
$$y(kT) \approx T\sum_{j=0}^{k} g(jT)u\big((k-j)T\big)$$

$$\begin{aligned}
y(0) &\approx Tg(0)u(0)\\
y(T) &\approx Tg(0)u(T) + Tg(T)u(0)\\
y(2T) &\approx Tg(0)u(2T) + Tg(T)u(T) + Tg(2T)u(0)
\end{aligned}$$

$$\begin{bmatrix} y(0)\\ y(T)\\ \vdots\\ y(kT)\end{bmatrix} = T\begin{bmatrix} u(0) & 0 & \cdots & 0\\ u(T) & u(0) & \ddots & \vdots\\ \vdots & & \ddots & 0\\ u(kT) & u((k-1)T) & \cdots & u(0)\end{bmatrix}\begin{bmatrix} g(0)\\ g(T)\\ \vdots\\ g(kT)\end{bmatrix}$$

Solve the equation for $g(0), g(T), \cdots, g(kT)$!

Review: Frequency response



If the system input is $u(t) = a sin\omega t$, then the output **in the steady state** is

$$y(t) = b\sin(\omega t + \varphi(\omega))$$

where

$$b = a|G(j\omega)|$$

with $|G(j\omega)|$ the modulus of $G(j\omega)$ and $\varphi(\omega)$ the phase angle of $G(j\omega)$.

Measurement of frequency response

Basic procedure:

1. Select the sinusoidal signal $u(t) = asin\omega t$ with frequency ω.
2. Record the system response $y(t)$.
3. Calculate the modulus and phase angle of $G(j\omega)$ at frequency ω as

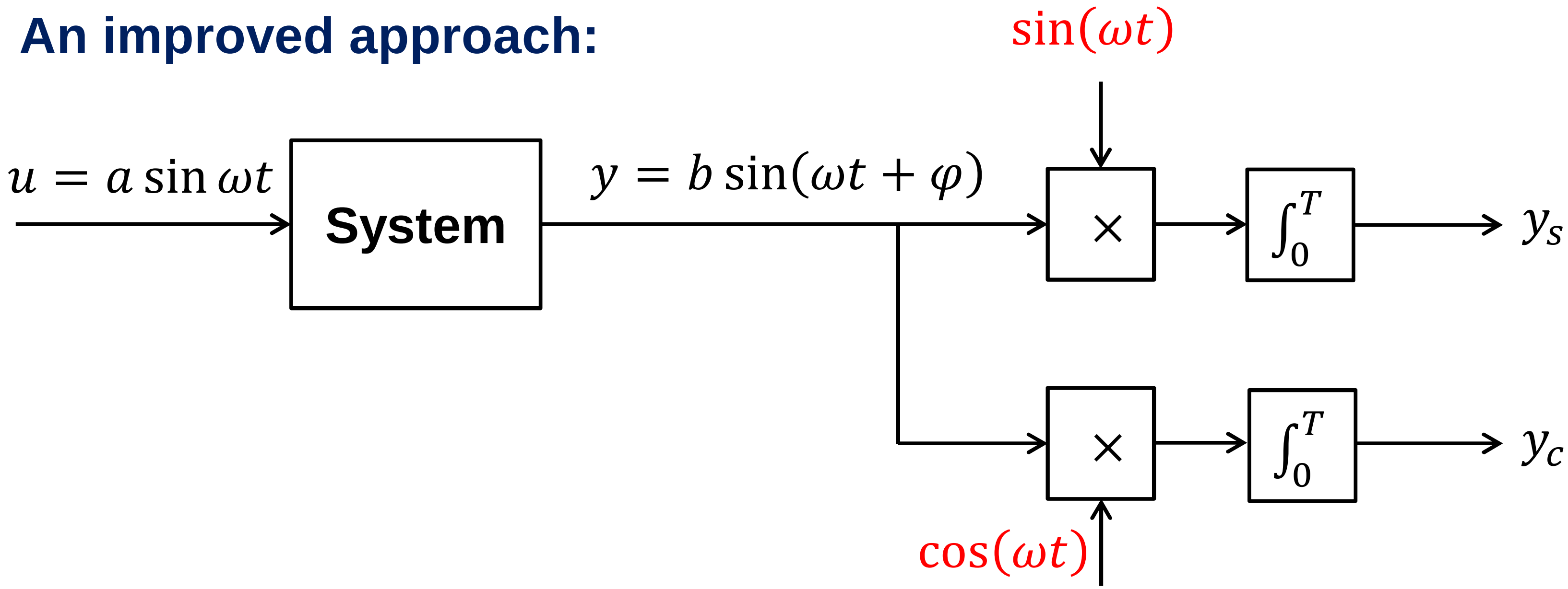
$$|G(j\omega)| = \frac{b}{a} = \frac{\text{Amplitude of the output sinusoid in the steady state}}{\text{Amplitude of the input sinusoid}}$$

$$\varphi(\omega) = \text{Phase shift of the output sinusoid with respect to the input sinusoid}$$

4. Repeat the above procedure for a number of frequencies $\omega_1, \omega_2, \cdots, \omega_N$.
5. The frequency response of the system at discrete points is obtained.

Measurement of frequency response

An improved approach:



If $T = k\frac{2\pi}{\omega}$, then

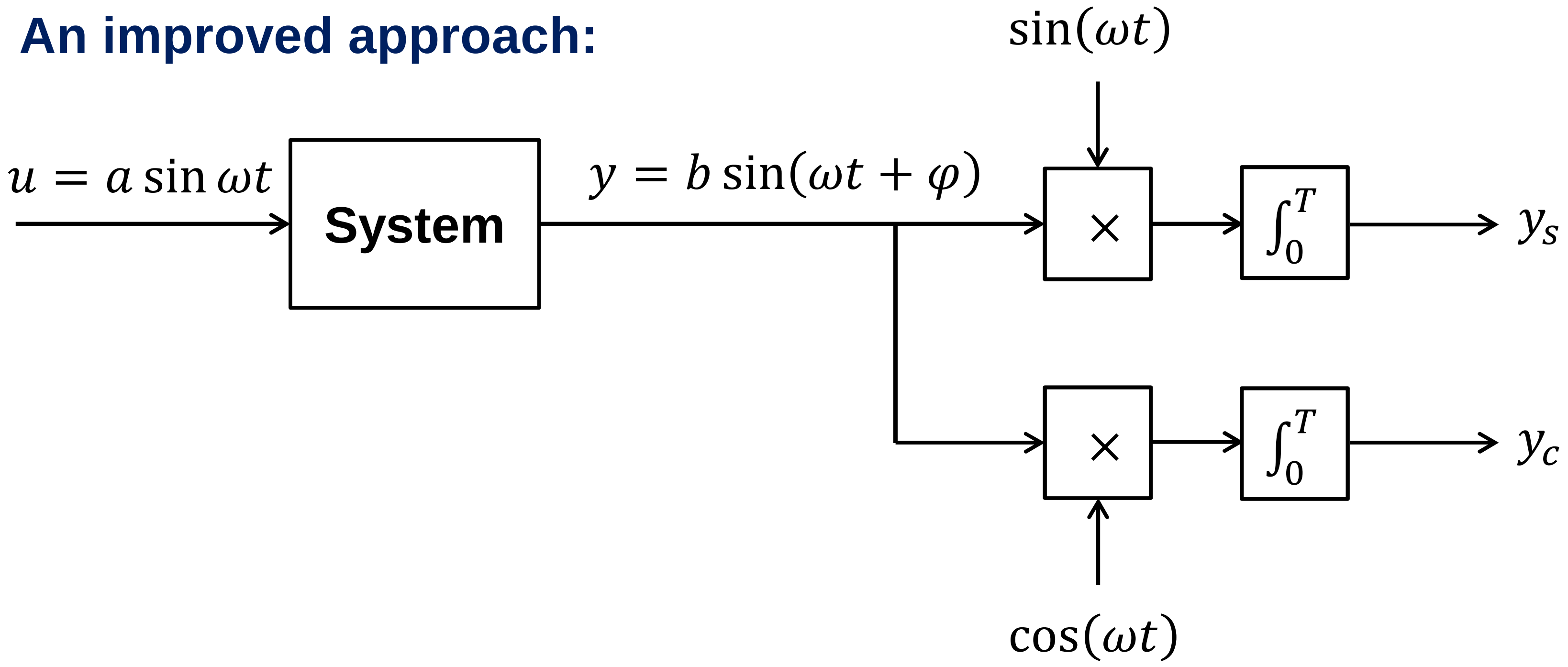
$$y_s = \int_0^T y(t)\sin(\omega t)dt \;= \int_0^T b\sin(\omega t+\varphi)\sin(\omega t)dt$$

$$= \int_0^T b\,\frac{-1}{2}[\cos(2\omega t+\varphi) - \cos(\varphi)]dt \;= \frac{bT}{2}\cos(\varphi)$$

$$= \frac{a|G(j\omega)|T}{2}\cos(\varphi) = \frac{aT}{2}\mathbf{Re}[G(j\omega)]$$

Measurement of frequency response

An improved approach:



If $T = k\frac{2\pi}{\omega}$, then

$$y_s = \int_0^T y(t)\sin(\omega t)dt \ = \frac{bT}{2}\cos(\varphi) = \frac{Ta|G(j\omega)|}{2}\cos\varphi = \frac{Ta}{2}\mathbf{Re}[G(j\omega)]$$

$$y_c = \int_0^T y(t)\cos(\omega t)dt \ = \frac{bT}{2}\sin(\varphi) = \frac{Ta|G(j\omega)|}{2}\sin\varphi = \frac{Ta}{2}\mathbf{Im}[G(j\omega)]$$

Measurement of frequency response

The direct measurement of freuency response using sinusoidal test signal

- Pointwise determination of the frequency response
- Good results but time-consuming for systems with slow dynamics

The improved approach

- Reduce the effect of noises
- Employed in many commercial frequency response measurement devices and software tools