Sensor Signal Processing

Prof. Dr.-Ing. Andreas König
Lehrstuhl Integrierte Sensorsysteme

FB Elektrotechnik und Informationstechnik
Technische Universität Kaiserslautern

Fall Semester 2005

Course Contents

Chapter Contents

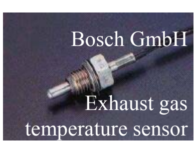
Sensor Organization

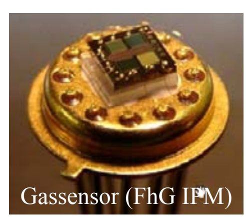
- Sensors can be grouped according to their organization

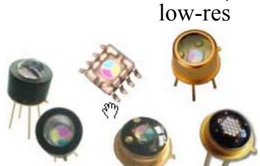
single | multiple

multiple: homogeneous | heterogeneous

homogeneous: low-res | highres/1D | highres/2D



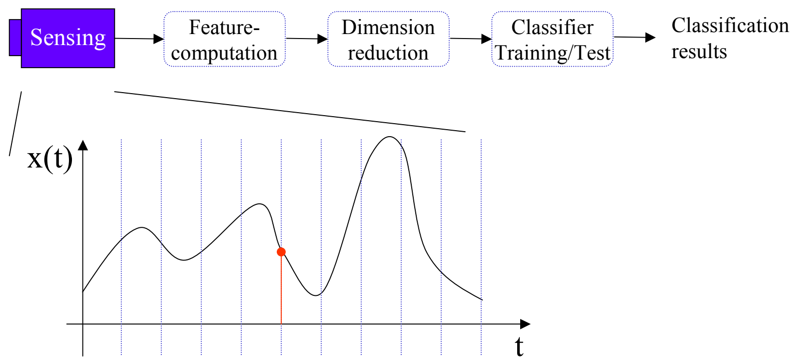




MAZeT JENCOLOUR ICs

Sensor Organization

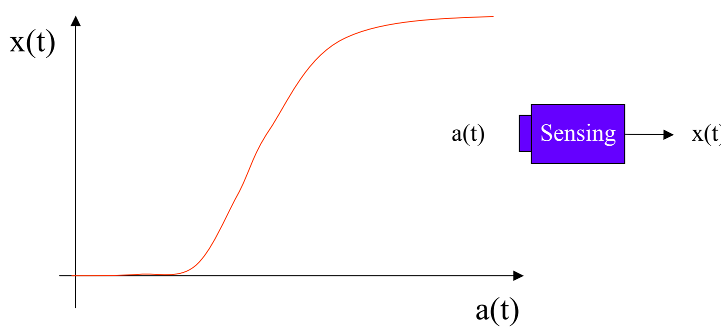
- Single sensor delivers a time dependent signal x(t):



- Instanteneous value, time- and value continuous
- Typically sampling of sensor registration by cyclic readout
- Returns scalar value at each observation or measurement time

Sensor Organization

- Sensor converts arbitrary quantity a(t) to an electrical representation x(t)
- The underlying operating principle and physical realization determine the sensors tranfer characteristics
- Arbitrary non-linear relation (no analytical model) often results



- Commonly, only discrete value pairs available to describe relationship
- Application of function approximation to obtain model for transfer function
- Correction and linearization for optimized transfer function

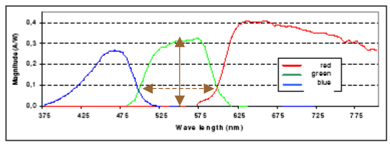
Sensor Organization

- Key terms for sensor capability:

  Selectivity Sensitivity Stability

- A sensor's sensitivity characterizes the change of the sensor output with regard to the change of the measured quantity

- Selectivity characterizes the sensor's vulnerability or sensitivities to other than the currently interesting quantity, e.g., influence of moisture

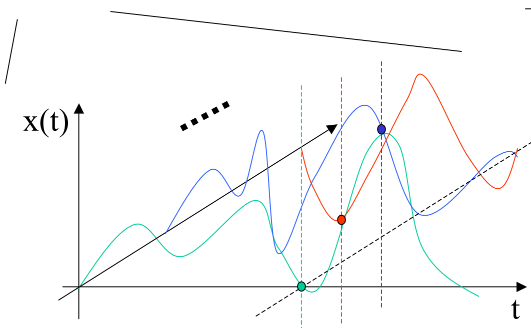


- Stability characterizes the sensor's susceptibility to aging, drift, poisoning

Sensor Organization

- Multiple sensors deliver a group of time dependent signal x(t):

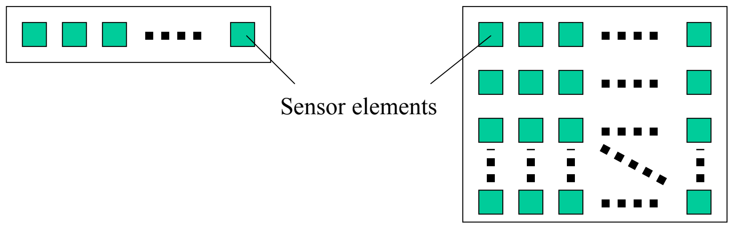
Sensing → Feature-computation → Dimension reduction → Classifier Training/Test → Classification results



- Instantaneous value, time- and value continuous
- Typically sampling of sensor registration by cyclic readout (age of data !)
- Returns value vector at each observation or measurement time

Sensor Organization

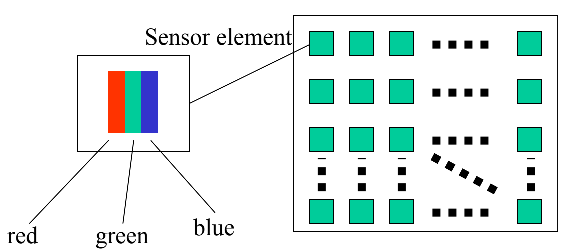
- Multiple sensors deliver a group of time dependent signal x(t):



- Image and infrared sensors typically are organized in row or column acrchitecture
- Sensor pitch relates to spatial sampling
- Typically time sampling of sensor registration by cyclic readout
- Latch mechanisms sample data at same time instance age of data
- Returns matrix value at each observation or measurement time
- Spatio-temporal data requires appropriate data structures

Sensor Organization

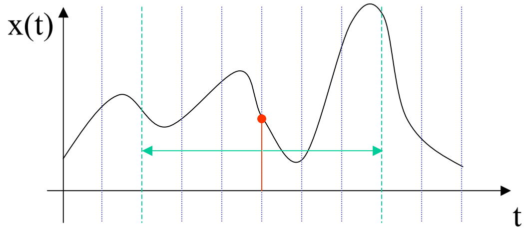
- Sensor data representation requires three dimensions for spatio-temporal registration
- Images most lucent example
- Color image sensors add one more dimension:



- In the processing of sensor data, this additional information is organized as a fourth dimension (planes)
- A collection of examples can be organized as a fifth dimension
- Number of dimensions/organization implementation dependent

Sensor Organization

- In particular for recognition tasks, spatial and/or temporal context is required to process sensor registration at a certain instant

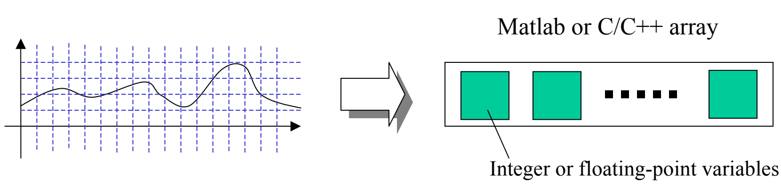


observation window

- Data in a window, encompassing neighboring registrations will be collectively processed
- The window moves to the next sampling position (sliding window)
- Corresponding concept in the spatial domain for 1D or 2D

Data Acquisition

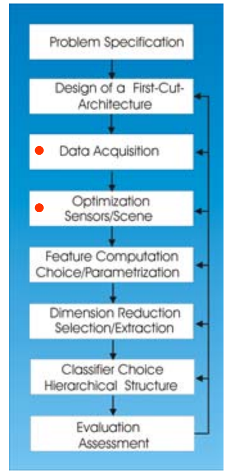
- Sensor signal processing can be carried out in the analog domain, e.g., by analog filtering, in the digital domain, or in both domains
- Signal transduction and conditioning can be implicit signal processing
- It must be assured, that all relevant information is conveyed to the final domain for signal processing
- Digital domain representation of time-signal:



- Adequate sampling and discretization must be assured
- In analysis mode, data acquisited in one or several measurement campaigns is analysed off-line without real-time constraints
- In operating mode, data is acquisited and processed on-line under (tight) real-time constraints

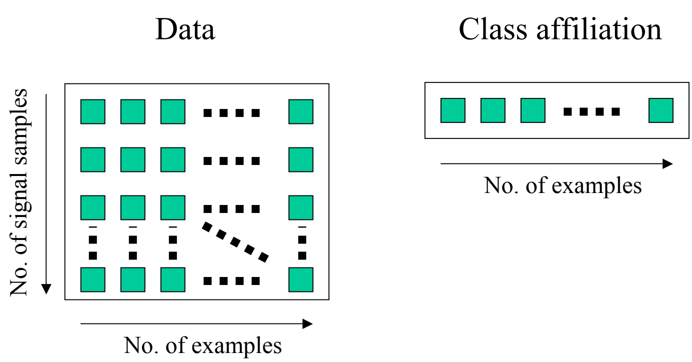
Data Acquisition

- Design of an application-specific sensor system requires data acquisition:
- The data must be representative and cover all the application's requirements
- Manual evaluation of operators for the available data can take place
- In particular for recognition systems the paradigm *learning-from-examples* is employed
- Representative sensory stimuli for all regarded categories or classes to be discerned by the recognition system under design must be provided !
- Scene and sensors optimization !



Data Acquisition

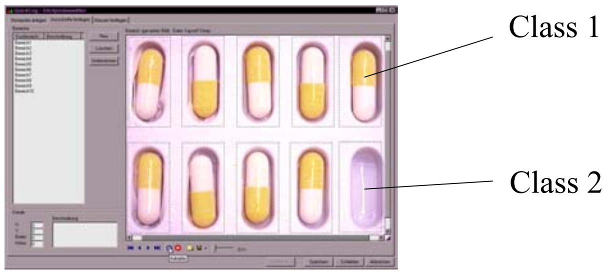
- The paradigm *learning-from-examples* implies an additional data structure, where the supervisor or teacher gives his or her opinion
- Each data sample is affiliated to a category or class of the decision problem according to the *ground truth* or an appropriate expert opinion



- Commonly, multiple such data sets for training, testing, and validation purposes during sensor system design

Data Acquisition

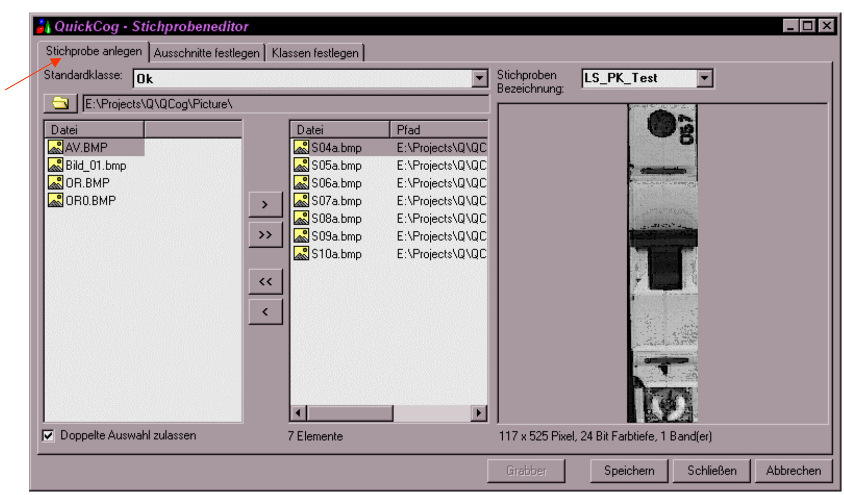
- Similar to attention mechanisms to be observed in living beings perception, technical systems for machine vision, olfaction or other sensing modality process sensory data in a hierarchical fashion
- Subregions, denoted as region-of-interest (ROI) are identified and extracted from signals or images:



- ROI data is affiliated to classes data is acquired for each subregion separately and corresponding data sets are established

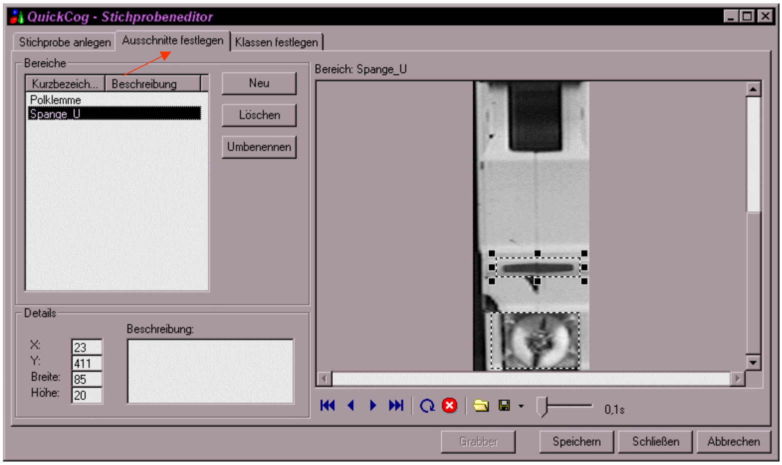
Data Acquisition

- This can be a tedious and error prone process
- The sample set editor is a simple three-stage tool to optimize this task



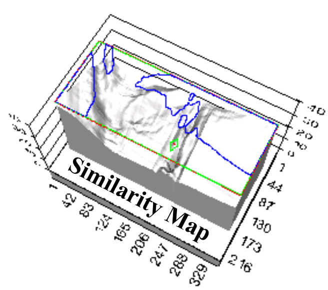
Data Acquisition

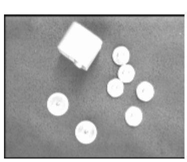
- After definition of the logical sample set structure, one or several ROIs can be defined for later data generation and class affiliation
- By preprocessing, aligned object presentation must be assured



Data Acquisition

- Single object is separated from background using, e.g., template correlation





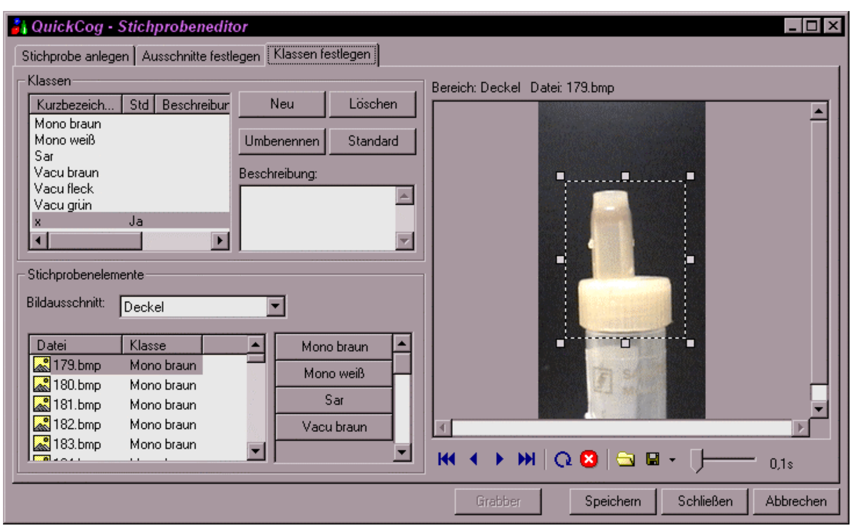
- Segmentation of objects by homogeneity assumptions
- Selective Attention

Data Acquisition

- In the final step, for each ROI, the available classes or categories are defined and object ROIs are assigned to those

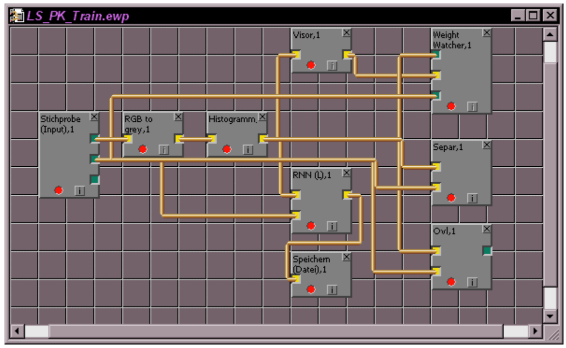
Data Acquisition

- Same procedure for alternative task (medical object recognition)



Data Acquisition

- This approach returns an organized data base of selected examples ready for signal processing, feature computation, and classification

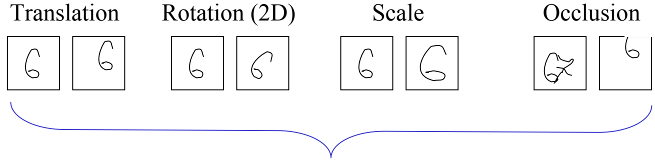


- The prequisite is an appropriate choice of sensors and scene/environment
- In the visual case, a feature denoted life-image allows to interactively optimize sensor and scene parameters, e.g., illumination

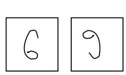
Invariance Issues

- The aspired recognition system must cope with variations in object appearance and retain discriminance abilities
- Invariances are due to: Illumination, translation, rotation, scale, non-rigidity, occlusion

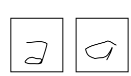
Translation Rotation (2D) Scale Occlusion



Desired invariance property: recognition of number six !



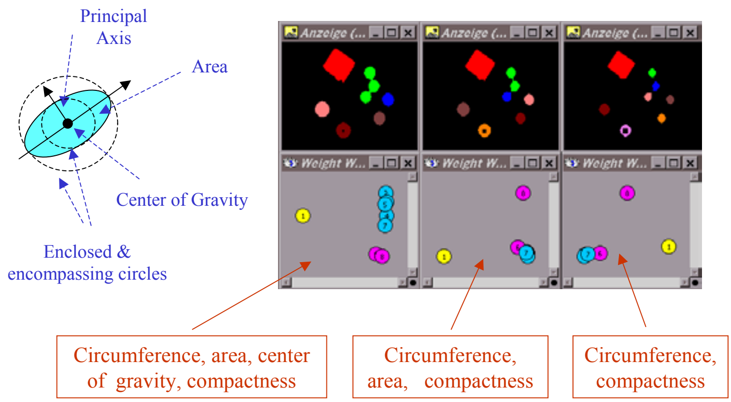
Undesirable rotation invariance:
Nine is indistinguishable from six



Two phenotypes,
one meaning: **a**

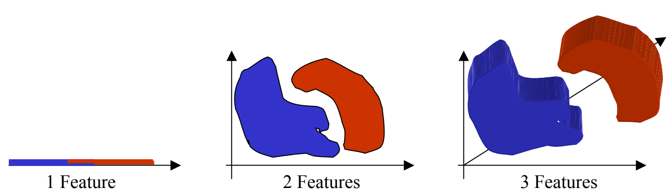
Invariance Issues

- Example of geometric feature computation & feature elimination issue approach



Invariance Issues

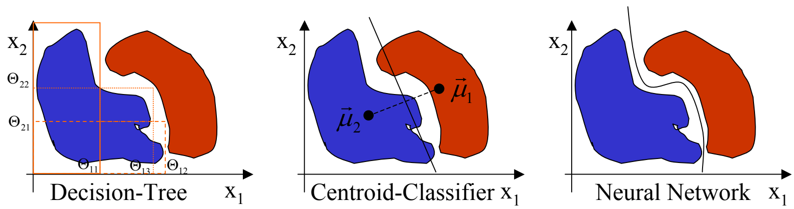
- Computed features shall provide a compact & invariant description
- Sufficient features must be computed from one or combination of several methods with optimum parameter settings



- Redundance & irrelevance in the feature set must be eliminated (*Curse of dimensionality, computational complexity*)
- Dimensionality reduction: (un)supervised, (non)linear methods
- Application-specific compression or elimination of features from the initial set
- Backtracking of DR decisions can simplify the architecture !

Invariance Issues

- Appropriate classifier model selection (Simplicity, separability)
- Classifier configuration:
  - ✓ Learning from examples
  - ✓ Exploitation of available rules



$$
\begin{aligned}
C_1 &= (x_1 < \Theta_{11}) \\
&\quad \vee ((x_1 < \Theta_{12}) \wedge (x_2 < \Theta_{21})) \\
&\quad \vee ((x_1 < \Theta_{13}) \wedge (x_2 < \Theta_{22})) \\
C_2 &= \overline{C_1}
\end{aligned}
$$

$$
C_l = \begin{cases} 1 & if \quad d_l \min_j d_j \\ 0 & else \end{cases}
$$

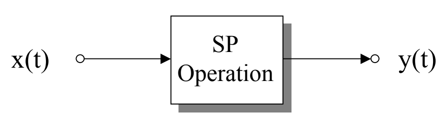
$$
d_j = (\sum_{i=1}^{M} (x_i - \mu_{ij})^2
$$

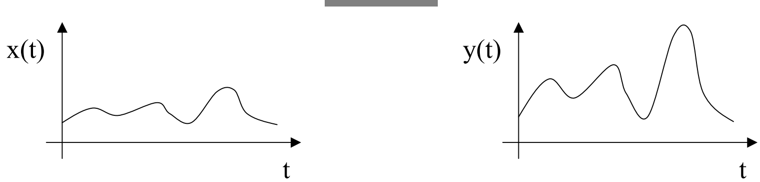
$$
C_l = \begin{cases} 1 & if \quad y_l = \max_i y_i \\ 0 & else \end{cases}
$$

$$
y_i = f(\sum_{j=1}^{HN} w_{ij} h_j); h_j = f(\sum_{k=1}^{M} w_{jk} x_k)
$$

Signal Arithmetics

- Signal processing in contrast to feature computation is understood in this course as a transformation, that maps one signal representation to another:

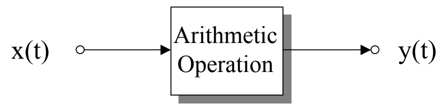


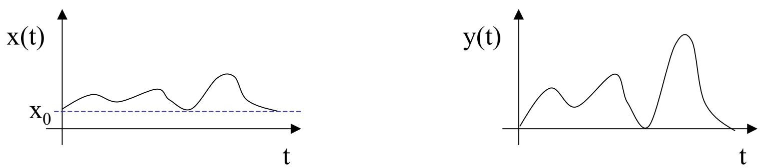


- Feature computation condenses the signal information in characteristic descriptors or features
- Commonly denoted as epresentation change from iconic to symbolic level in image representation

Signal Arithmetics

- The simplest, but practically relevant case are arithmetic operations on signals
- Common operations: Addition, subtraction, multiplication, division

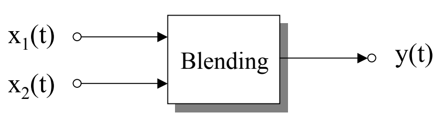




- Simple example of offset removal and amplification or scaling (multiplication) of the input signal
- Arithmetic operators can involve a scalar and a signal (monadic operators) or pairs of signals (dyadic operators)

Signal Arithmetics

- Mixing or blending of signals is an easy example for this kind of arithmetic operation:



$$y(t) = a_1 \cdot x_1(t) + a_2 \cdot x_2(t) \tag{2.1}$$

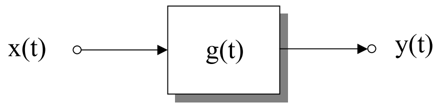
- For instance, noise signals (images) can be added to training samples for perturbation to create larger richer diversity of examples
- Nonlinear transformations can be applied, e.g., for dynamic range compression:

$$y(t) = \log(x_1(t)) \tag{2.2}$$

- Squaring, absolute value computation, (multi-level)-thresholding, clipping, saturation are further common operators

Convolution

- Convolution is an operator well known from systems theory
- In particular, for linear, time invariant systems, time-domain (or spatial domain) filtering is supported by convolution operations
- The impulse response g for a Dirac impulse input characterizes such a system



- Assuming an input signal to be a superposition of weighted Dirac impulses gives [1]

$$x(t) = \int_{-\infty}^{\infty} x(\tau) \cdot \delta(t-\tau) d\tau \qquad (2.3)$$

- For time-continuous processing this gives for the output signal

$$y(t) = \int_{-\infty}^{\infty} x(\tau) \cdot g(t-\tau) d\tau \qquad (2.4)$$

Convolution

- This operation is denoted as convolution

$$y(t) = (x * g)(t) \qquad (2.5)$$

- There is a correspondence with frequency domain processing; multiplication in the frequency domain (DFT/IDFT) corresponds to convolution in the time/spatial domain
- In the discrete case for a time domain signal convolution is commonly given as [1]

$$y(n \cdot \Delta t) = \sum_{k=-\infty}^{\infty} x(k \cdot \Delta t) g((n-k) \cdot \Delta t) \qquad (2.6)$$

- For finite series of sampling values of length M this reduces to

$$y_n = x_n * g_n = \sum_{k=0}^{M-1} x_n \cdot g_{n-k} = \sum_{k=0}^{M-1} x_{n-k} \cdot g_n \qquad (2.7)$$

Convolution

- It becomes obvious, that convolution bases on the mirroring of one of the operands,
- This is followed by the multiplication of corresponding value pairs and accumulation of the products for each shift position
- A cyclic processing is commonly assumed for operands of equal length
- In practical applications, one of the operands is regarded as a mask while the other is the signal to be processed
- Further, significantly *smaller* masks than signal vectors are assumed

0	0	0	0	1	1	1	1	1	0	0	0	0

0.0	0.1	0.2	0.3	0.35	0.3	0.2	0.1	0.0	-0.1	-0.2	-0.3	-0.35

- Only non-zero elements of the mask vectors must be regarded in convolution !

Convolution

- For convolution operation, the mask is mirrored at the origin and placed at every signal value position
- The weighted sum of the regarded value and ist neighbors is computed and represents the resulting value of y_n

0	0	0	0	1	1	1	1	1	0	0	0	0

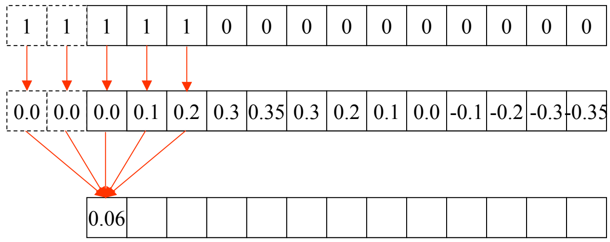
0.0	0.1	0.2	0.3	0.35	0.3	0.2	0.1	0.0	-0.1	-0.2	-0.3	-0.35

						0.95						

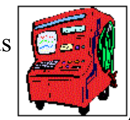
- If the mask is normalized to unity, the accumulation result must be normalized by the sum of mask coefficients, i.e., y_6= 0.95/5=0.19
- This repeated at every index position of the signal

Convolution

- The assumption of periodicity for general real-world signals will not hold
- Consequently, border values require particular attention, either leaving them out of computation (reducing signal size) or by padding the borders by zeros or interpolated values corresponding to mask size



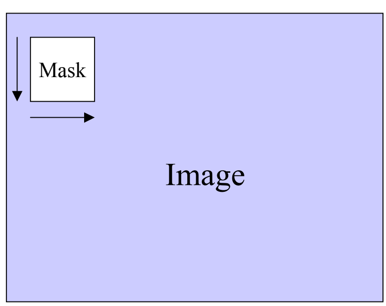
- The convolution mask is computed according to application needs
- Large number of coefficient sets available
- Properties of convolution allow superposition of multiple masks in a single mask, e.g., smoothing and derivation



Sensor Signal Processing
Signal Processing

Convolution

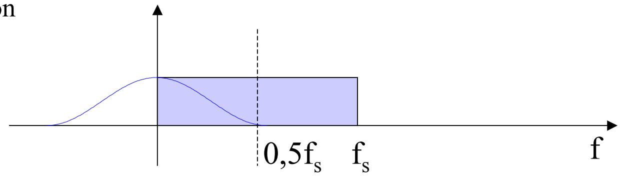
- The concept of convolution can be extended to dimensions larger than one and from temporal to spatial representation
- The most common example is 2D image processing



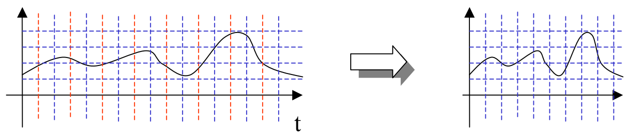
- Numerous operators for smoothing or edge enhancement have been introduced in image processing
- Convolution can be applied to spatio-temporal data, e.g., image sequences, by smoothing with a 3D mask or kernel
- True voxel data can be addressed in a similar way

Convolution

- Convolution can be employed for smoothing or low-pass filtering of images and signals, e.g, for noise removal
- This time (spatial) domain filtering corresponds to a bandwidth reduction



- Sufficient bandwidth reduction allows to subsample the signal by an appropriate factor, e.g., by a factor of two:



- Storage requirement is halved and feature computation or even classification can base on that more compact representation

DFT and FFT

- The straightforward analysis of temporal and/or spatial signal can be difficult and tedious in many cases
- Instead of time domain analysis, frequency domain analysis based on the well-known Fourier-Analysis can be more efficiently be employed
- For spatial signals, spatial frequencies are computed in, e.g., 2D !
- The spectral representation in magnitude and phase allows the extraction of frequencies of interest (spectral lines) or ranges
- From the spectral repesentation for applications and evalutions key characterisitics can be computed, such as, e.g., *Total-Harmonic-Distortion (*THD), denoted as k, or other features

$$k = \sqrt{\frac{U_{2eff}^2 + U_{3eff}^2 + \ldots + U_{neff}^2}{U_{1eff}^2 + U_{2eff}^2 + U_{3eff}^2 + \ldots + U_{neff}^2}} 100\%$$

$$k = \sqrt{\frac{\hat{u}_2^2 + \hat{u}_3^2 + \ldots + \hat{u}_n^2}{\hat{u}_1^2 + \hat{u}_2^2 + \hat{u}_3^2 + \ldots + \hat{u}_n^2}} 100\% \qquad (2.8)$$

- **Overview of signal representations and Fourier-Analysis-Methods [1]:**

	Time-continuous signal	Time-discrete signal
periodic or periodically repeated signal	**Fourier-Series** Aperiodic Discrete spectrum in [V]	**Discrete Fourier-Transformation (DFT)** periodic discrete spectrum in [V]
aperiodic signal	**Fourier-Transformation** Aperiodic contnuous spectrum in [V/Hz] or [Vs] (spectral density)	**Discrete-Time-Fourier-Transformation (DTFT)** periodic continuous spectrum in [V/Hz] or [Vs] (spectral density)

- The Discrete-Fourier-Transformation (DFT) is applicable for digital sensor signal processing
- Following sample&hold and AD-conversion, a sequence of N sample values x(k) in memory represent the signal
- Signal analysis takes place by digitale implementation of DFT-algorithm or an accelerated variation (FFT)
- DFT Forward- und Inverse Transformation (DFT, IDFT) [1]:

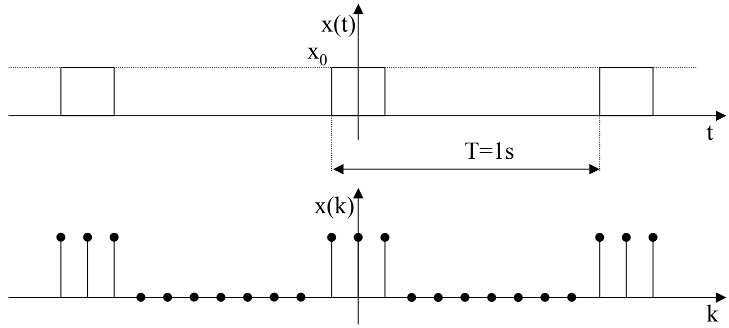
$$\underline{X}(n) = \frac{1}{N}\sum_{k=0}^{N-1} x(k)\cdot e^{-j\frac{2\pi nk}{N}} \tag{2.9}$$

$$x(k) = \sum_{k=0}^{N-1} \underline{X}(n)\cdot e^{j\frac{2\pi nk}{N}} \tag{2.10}$$

- Simplifications for even or odd functions of the signal apply
- *DFT requires scaling with sample time to equal FT in value and unit !*

DFT and FFT

- Elucidation of the procedure employing a peridocial asymmetric square wave as a time signal



- Equidistant sampling with N=10 samples per signal period:

$$\Rightarrow \Delta t = t_{Abt} = \frac{T}{N} = 0.1s$$

- Obviously, the regarded function is even with just three sample values per period with values not equal to zero !

DFT and FFT

- Spectral coefficients are computed exploiting that

$$x(0) = x(1) = x(9) = x_0; \quad x(2) = x(3) = \ldots x(8) = 0$$

- This allows simplification of (2.9) to:

$$\underline{X}(n) = \frac{1}{10}\left(x(0)\cdot\cos\left(\frac{2\pi n\cdot 0}{10}\right) + x(1)\cdot\cos\left(\frac{2\pi n}{10}\right) + x(9)\cdot\cos\left(\frac{2\pi n\cdot 9}{10}\right)\right)$$

symmetry considerations

$$\underline{X}(n) = \frac{x_0}{10}\left(1+\cos\left(\frac{2\pi n}{10}\right)\right) \qquad (2.11)$$

- With (2.11) spectral coefficients are computed as

$$\underline{X}(0) = \frac{3}{10}x_0 = 0.3x_0; \quad \underline{X}(1) = \underline{X}(1) = \frac{2.618}{10}x_0 \cong 0.262x_0$$

DFT and FFT

$$\underline{X}(2)=\underline{X}(8)=\frac{1.618}{10}x_0\cong 0.162x_0$$

$$\underline{X}(3)=\underline{X}(7)=\frac{0.3819}{10}x_0\cong 0.382x_0$$

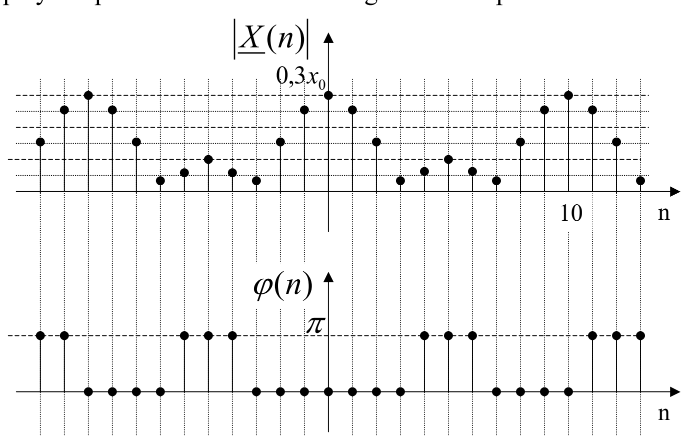
$$\underline{X}(4)=\underline{X}(6)=\frac{-0.618}{10}x_0\cong -0.0618x_0$$

$$\underline{X}(5)=-\frac{1}{10}x_0=-0.1x_0$$

- Obviously, for this particular case only phase values of 0 and 180 degrees are assumed
- Commonly, spectral coefficients will be stored and displayed in magnitude and phase representation
- Particular advantage: Shift in time/spatial domain corresponds to phase shift in frequency domain, leaving the magnitude untouched
- Shift invariance by DFT !

DFT and FFT

- Display of spectral coefficients in magnitude and phase:



- Symmetrical and periodic spectrum
- However, shape deviates from expected (si-function)
- Obviously, N is too low (Subsampling, Aliasing)

DFT and FFT

- (Well-known) relation of signal bandwidth and number of sample values N (sampling frequency):

$$f_{SA} \geq 2 f_g \quad with \quad f_{SA} = \frac{1}{t_{SA}} \frac{1}{\frac{T}{N}}$$

$$\Rightarrow \frac{N}{T} \geq 2 f_g$$

$$\Rightarrow N \geq 2 f_g T = \frac{\omega_g T}{\pi} \tag{2.12}$$

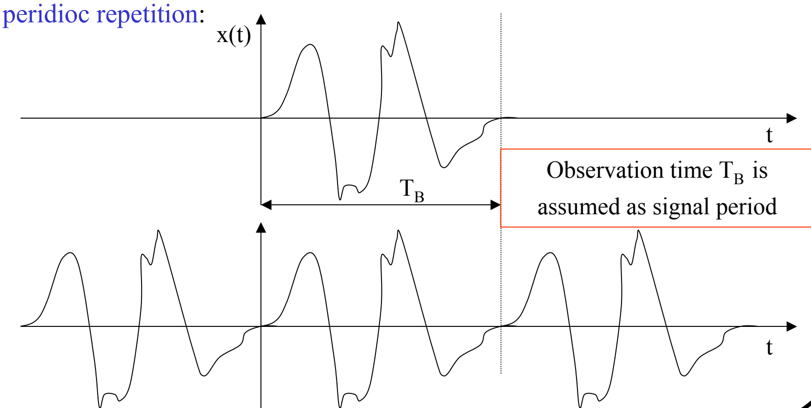
- Bandlimited signal (by AAF) required
- The resulting spectrum is periodic in ω or f, respectively with

$$\frac{2\pi}{t_{SA}} = \omega_{SA} \quad \text{or} \quad \frac{1}{t_{SA}} = f_{SA}$$

- DFT can only be applied to compute discrete values of ω_n

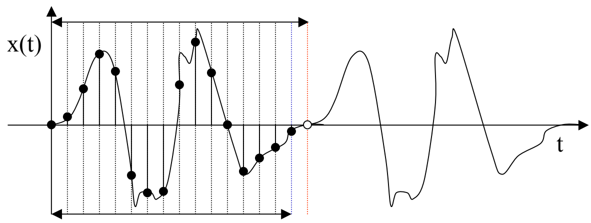
DFT and FFT

- **Practical application of FFT for signal processing:** Consideration of transient signals, i.e. limited duration and aperiodic signals !
- This actually represents the DTFT case given in slide (2.37) for aperiodic time-discrete signal
- To maintain the applicability of DFT the regarded signal is subject to peridioc repetition:



- However, only samples for the onbservation time T_B are kept in memory
- For practical reasons, also periodic signals will be sampled only for a limited duration and are subject to the same periodic repetition
- Case of coherent sampling:

$$T = N\Delta t = Nt_{SA}$$



- The onservation time or duration T_B is in this case an integer multiple of the sampling time or interval **and** the signal period:

$$k' \cdot N \cdot t_{SA} = k' \cdot T = T_B \qquad (2.13)$$

- As previously discussed, the number of samples N is determined by the minimum requirement given in (2.12)
- The spectral resolution, i.e., the frequency increment between the discrete spectral coefficients is given by:

$$\Delta\omega = \frac{2\pi}{T} = \frac{2\pi}{N \cdot t_{SA}}$$

$$\Delta f = \frac{1}{T} = \frac{1}{N \cdot t_{SA}} \qquad (2.14)$$

- Depending on application demands, a higher spectral resolution can be required. Potential solution to achieve a resolution increase ?
- Increase of the number of samples N: Resulting decrease of the sampling time t_{SA} and, thus, unchanged spectral resolution:

$$\Delta f' = \frac{1}{N' \cdot t'_{SA}} = \frac{1}{k' \cdot N \cdot \frac{t_{SA}}{k'}} = \Delta f = const \qquad (2.15)$$

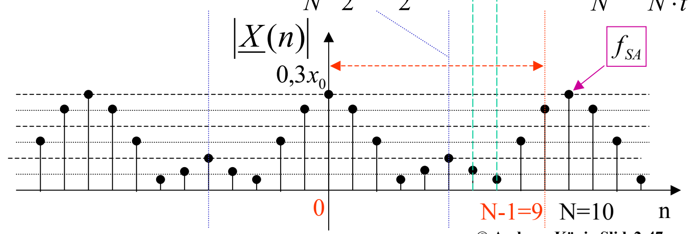
DFT und FFT

- The increase of the number of sample values by increasing the sample frequency leads to a corresponding increase of the represented frequency range at constant spectral resolution
- Increase of the observation time T_B:

$$\Delta f' = \frac{1}{T_B} = \frac{1}{k' \cdot T} = \frac{1}{k' \cdot N \cdot t_{SA}} = \frac{\Delta f}{k'} \qquad (2.16)$$

- The spectral resolution increases proportional to T_B while the highest represented frequency, and, thus, the frequency range is maintained

$$f = \frac{f_{SA}}{N}\frac{N}{2} = \frac{f_{SA}}{2} \qquad \Delta f = \frac{f_{SA}}{N} = \frac{1}{N \cdot t_{SA}}$$



DFT and FFT

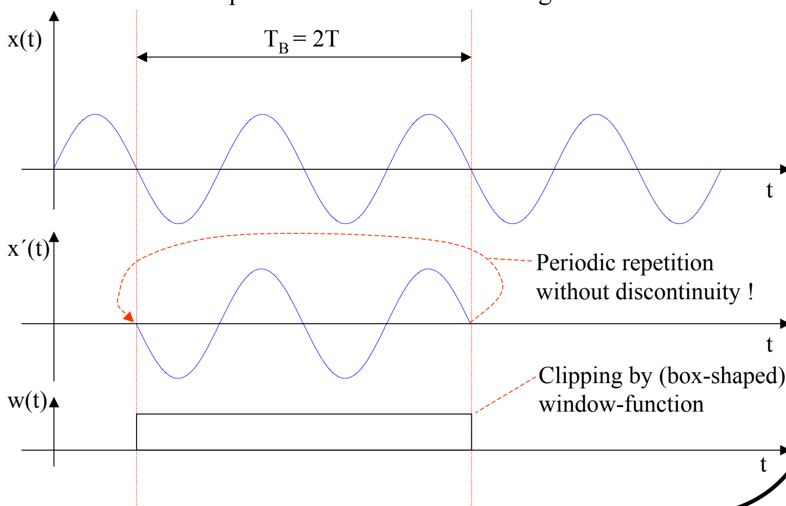
- Due to symmetry properties of the resulting spectrum only half of the resulting coefficients have to be stored
- However, due to the resulting complex values of the DFT the storage requirement in time/spatial domain equals the frequency/spatial frequency representation
- Commonly, odd numbers of spectral coefficients are considered:

$$-\frac{N-1}{2} \leq \frac{\omega_n T}{2\pi} \leq \frac{N-1}{2}$$

- This implies a corresponding odd number of samples N
- This limitation is not necessary for DFT (N-dimensional linear transfor-m, treatment of even and odd N in [1])
- The assumption of coherent, signal synchronous sampling in practical cases commonly not met
- Reasons are excessive effort or unknown T
- Individual choice of N per signal registration leads to individual Δf in resulting spectrum and aggravates result evaluation

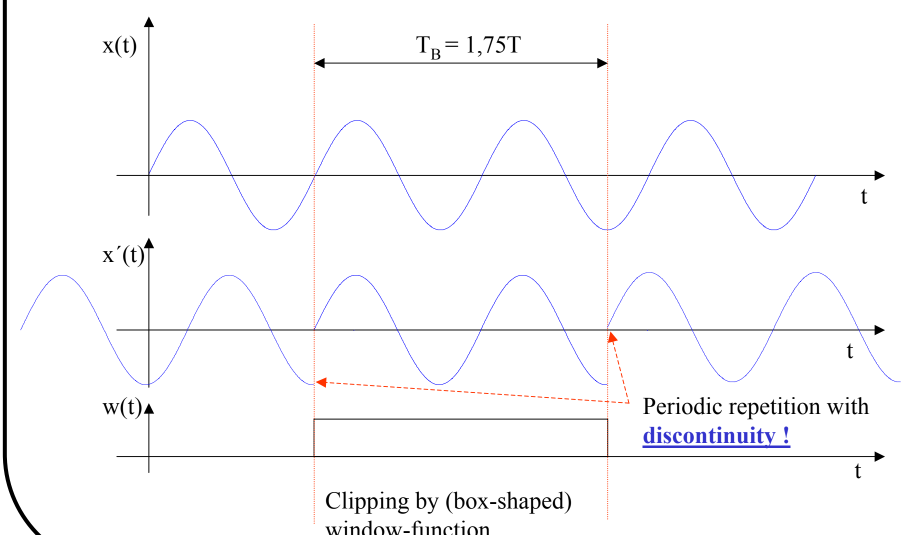
DFT and FFT

- Typical procedure is the asynchronous clipping of a signal section from a lolnger or even infinitively extended signal
- Elucidation of the consequences for sinusoidal time signal:



DFT and FFT

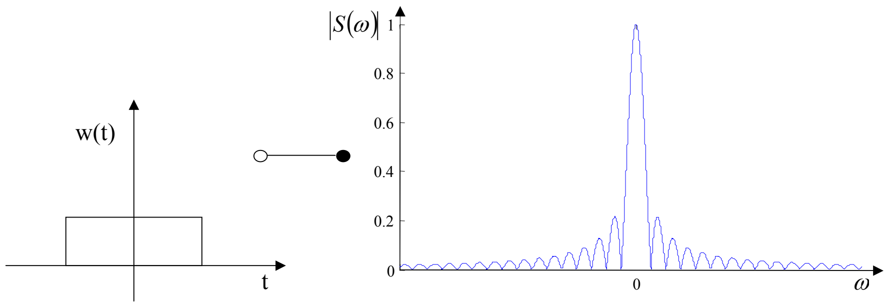
- Unfortunate case of measurement time choice:



DFT and FFT

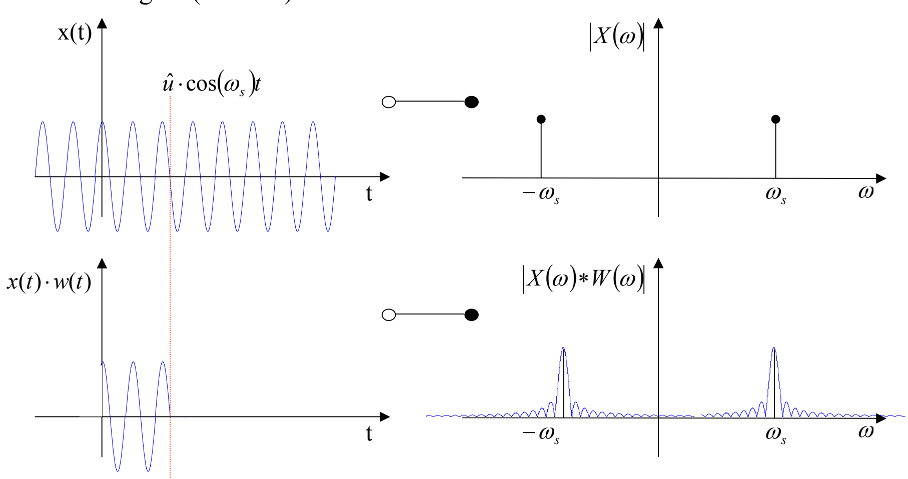
- Alleviation of the discontinuity effect can be achieved by extension of the observation or measurement time
- In the second, unfortunate case the chosen measurement duration in the domain has the effect of the multiplication with a (box-shaped) window function:

$$x'(t) = x(t) \cdot w(t) \quad \circ\!\!-\!\!\bullet \quad \underline{X}'(n) = \underline{X}(n) * W(\omega) \qquad (2.17)$$



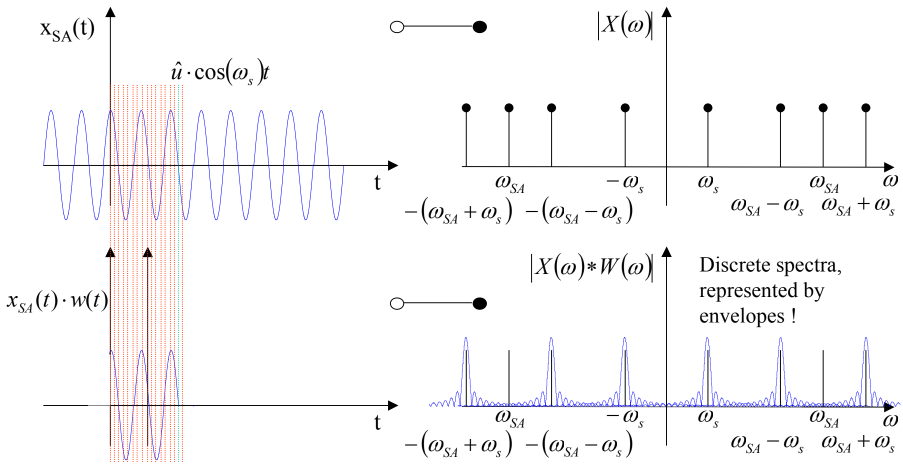
DFT and FFT

- Conceptual analysis of the effect of the window function for cosinusoidal time signal (sketch !):



DFT and FFT

- Extension of the analysis to the sampled signal case (sketch):



$$F_d(j\omega)=\frac{1}{2}\left[\frac{\sin N\frac{(\omega-\omega_s)t_{SA}}{2}}{\sin\frac{(\omega-\omega_s)t_{SA}}{2}}+\frac{\sin N\frac{(\omega+\omega_s)t_{SA}}{2}}{\sin\frac{(\omega+\omega_s)t_{SA}}{2}}\right]$$

DFT and FFT

- Smearing of the spectral lines by envelope of the window function:

Leakage-Effect

- The occuring spectral extension can cause Aliasing effect
- Zero-crossing of the envelope are at:

$$\pm i\cdot\frac{1}{N\cdot t_{SA}};\quad i=0,1,2,\dots$$

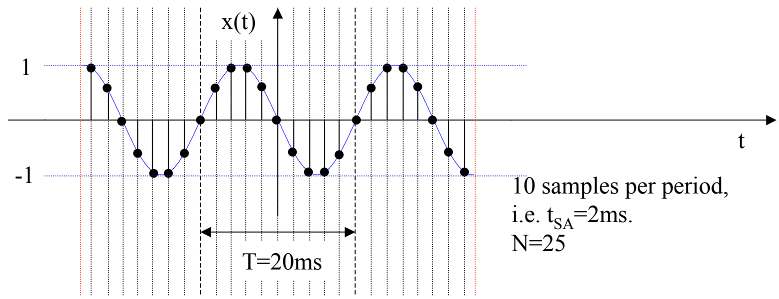
- The DFT return spectral lines only at the following discrete frequencies:

$$\Delta f=\frac{1}{T_B}=\frac{1}{k'\cdot T}=\frac{1}{k'\cdot N\cdot t_{SA}};\quad k\in\Re \qquad (2.18)$$

- For an integer k′, i.e. T_B is an integer multiple of T, the zero-crossings of the envelope are located at the discrete (visible) frequencies of the DFT
- If this condition is not met, additive contributions of the corresponding envelope values at the discrete DFT frequencies will occur

DFT and FFT

- Visualization for the case of a sinusoidal wave form:

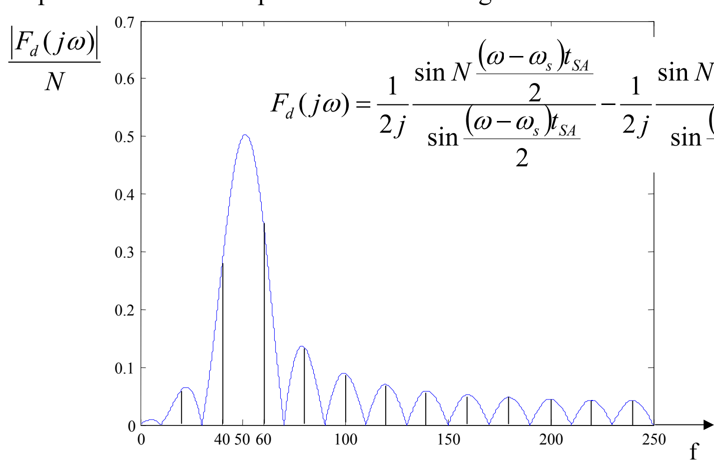


$$\Delta f = \frac{1}{T_B} = \frac{1}{2.5 \cdot T} = \frac{1}{2.5 \cdot 10 \cdot 2ms} = \frac{1}{0.05s} = 20Hz$$

- The frequency of the sinusoidal Signal is f=50Hz
- The sampling frequency is f_{SA}=500 Hz

DFT and FFT

- Depiction of the envelope for a sinusoidal signal and a box-window:

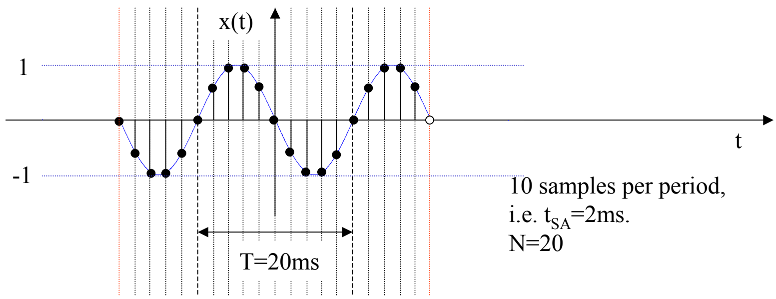


$$F_d(j\omega) = \frac{1}{2j} \frac{\sin N \frac{(\omega - \omega_s) t_{SA}}{2}}{\sin \frac{(\omega - \omega_s) t_{SA}}{2}} - \frac{1}{2j} \frac{\sin N \frac{(\omega + \omega_s) t_{SA}}{2}}{\sin \frac{(\omega + \omega_s) t_{SA}}{2}} \qquad (2.19)$$

- Zero-crossings of the envelope occur in intervalls of 20 Hz
- The actual signal frequency can not be represented in this case !
- ´´Smearing´´ to neighboring, represented frequencies

DFT and FFT

- Modification of the measurement time to an integer multiple of the signal period :

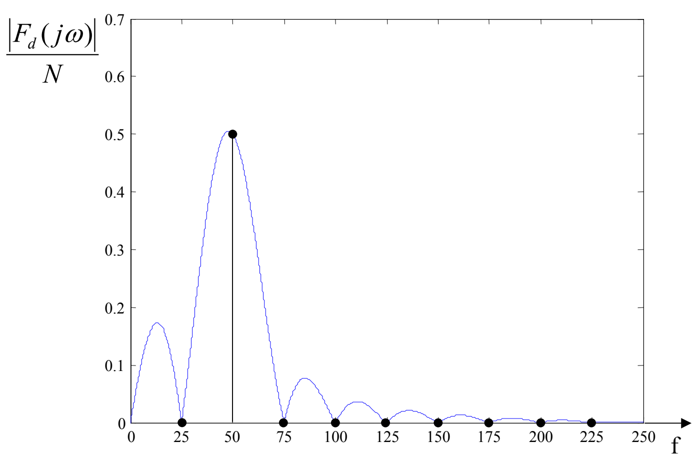


$$\Delta f = \frac{1}{T_B} = \frac{1}{2 \cdot T} = \frac{1}{2 \cdot 10 \cdot 2ms} = \frac{1}{0.04s} = 25Hz$$

- The frequency of the sinusoidal signal continues to be f=50Hz
- The sampling frequency remains unchanged at f_{SA}=500 Hz

DFT and FFT

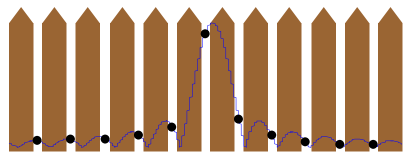
- Depiction of the envelope for a sinusoidal signal and a box-window:



- Zero-crossings of the envelope occur in intervalls of 25 Hz
- The actual signal frequency can now be represented !
- Elimination of contributions at neighboring, represented frequencies

DFT and FFT

- The fact, that the interesting spectrum is only visible and evaluable at discrete points, is denoted as *Picket-Fence*-effect [1]:



Sketch to illustrate the picket-fence-effect

- Potentially, due to an unfortunate choice of the frequency resolution, the frequency of interest can not be appropriately visible and evaluable

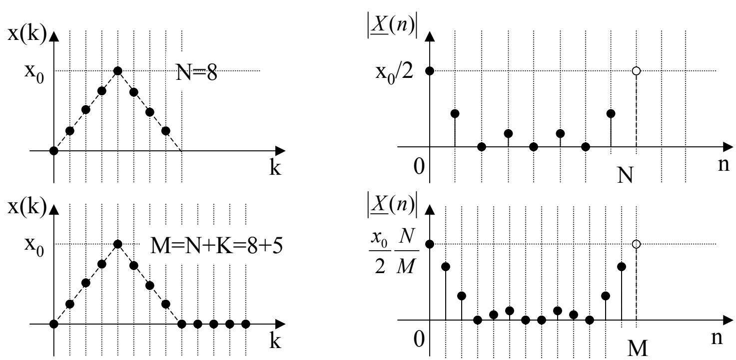
- Control of visible frequencies or spectral lines by appropriate choice of measurement duration or adaptation of the number of sample values or the signal length

DFT and FFT

- Zero-Padding of the sample data, to modify from application-specific fixed N by adding K zero-valued samples to M=N+K samples, and, thus, achieve a fixed spectral resolution of:

$$\Delta f^{ZP} = \frac{1}{M \cdot t_{SA}} = \frac{1}{(N+K) \cdot t_{SA}} < \frac{1}{M \cdot t_{SA}}$$

Sketch



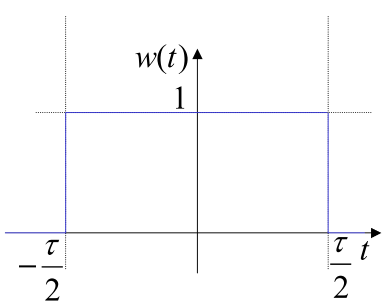
- **However:** No novel information, i.e., higher frequencies, are gained !

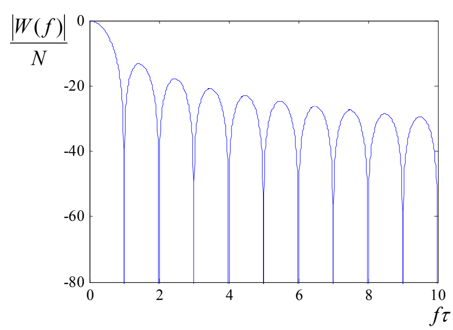
- Alleviation of the cosequences of asynchronous extraction of signals (Leakage-Effect) by appropriate window

- In addition to the presented box-window a collection of window functions have established themselves for signal processing [1]
- Attenuation of border elements will have different consequences for resulting signal spectra
- In the following, window functions will be represented as time-continuous, transient functions with resulting continuous spectra
- Discretization will be achieved by multiplication with the sampled signal in the time domain

- For true transient signals, i.e., time decaying signals of finite duration, the application of the box-window is compulsory to avoid spurious representation of signal shape and spectrum

- **Box- or Rectangular-Window:**





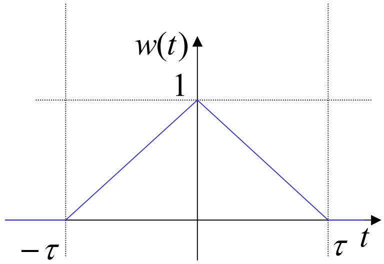
$$w(t) = \begin{cases} 1 & for \quad -\frac{\tau}{2} \le t \le \frac{\tau}{2} \\ 0 & \quad else \end{cases}$$

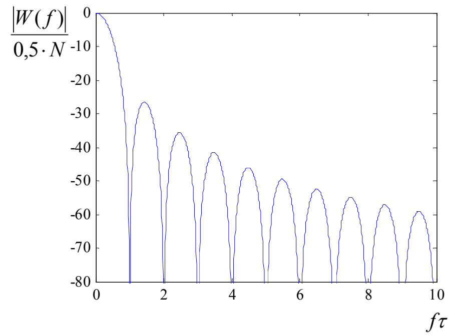
$$W(f) = \tau \cdot si(\pi f \tau) \qquad (2.20)$$

- The window width corresponds with the observation time or measurement campaign duration T_B

DFT and FFT

- **Triangle- or Bartlett-Window:**



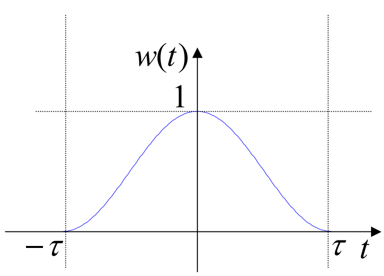


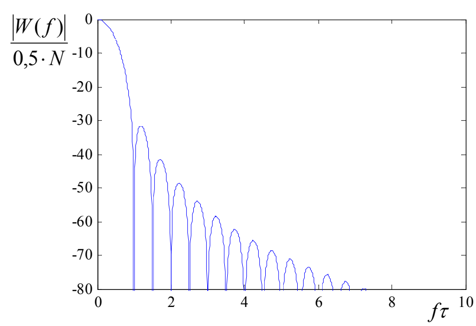
$$w(t)=\begin{cases}1+\dfrac{t}{\tau} & for \quad -\tau\le t\le 0\\ 1-\dfrac{t}{\tau} & for \quad 0\le t\le \tau\\ 0 & else\end{cases}$$

$$W(f)=\tau\cdot si^2(\pi f\tau) \qquad (2.21)$$

DFT and FFT

- **Von Hann-Window** (Hanning):

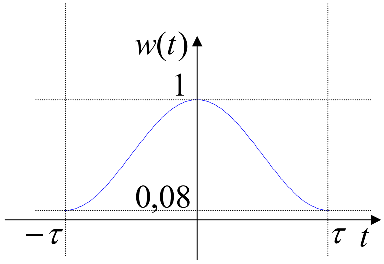


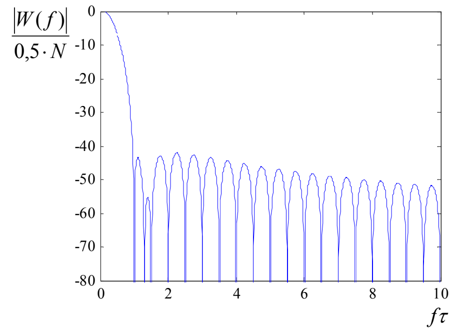


$$w(t)=\begin{cases}\cos^2\left(\dfrac{\pi t}{2\tau}\right) & for \quad -\tau\le t\le \tau\\ 0 & else\end{cases}$$

$$W(f)=\tau\cdot\frac{si(2\pi f\tau)}{1-(2f\tau)^2} \qquad (2.22)$$

- **Hamming-Window:**





$$w(t)=\begin{cases}0.54+0.46\cos\left(\frac{\pi t}{\tau}\right) & for \quad -\tau\le t\le\tau\\ 0 & else\end{cases}$$

$$W(f)=2\tau\cdot si(2\pi f\tau)\cdot\frac{0.54-0.08\cdot(2f\tau)^2}{1-(2f\tau)^2} \qquad (2.23)$$

- Criteria for window function assessment and selection:

- Attenuation of the first maximum
- Ratio of the amplitudes of the first - and second order maximum

- Maximum sample error: the spectral lines and the zero-crossing of the envelope only coincide in the best case. The maximum sample error for deviations from the best case is defined as:

$$f_{\max\_SA}=\frac{|W(f=0.5\cdot\Delta f)|}{|W(f=0)|} \qquad (2.24)$$

- Width of the first maximum: window functions with strongly attenuated higher order maxima have a wide first order maximum which causes a widening of the spectral lines. Determination by 3dB-corner frequency and is scaled by a real-valued factor

- **Qualitative comparison of regarded window properties:**

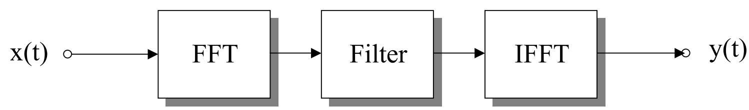
window	attenuation first order	attentuation ratio	width first order max.	Maximum sample error
Box	none	13.26dB	$0.45\Delta f_{3dB}$	0,64
Triangle	50% (6,02 dB)	26.52dB	$0.64\Delta f_{3dB}$	0,81
von Hann	50%	31.47dB	$0.72\Delta f_{3dB}$	0,85
Hamming	54%	42.67dB	$0.65\Delta f_{3dB}$	0,82
Blackmann	42%	58.11dB	$0.84\Delta f_{3dB}$	0,88

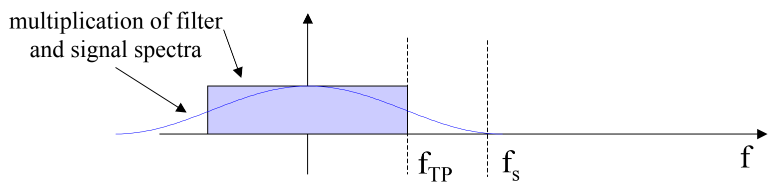
- Further windows are common in signal processing (see e.g. Matlab *Signal-Processing-Toolbox*, Gaussian- or Chebychev-window ...)
- **Trade-off for window selection in application :** Attenuation of signal information vs. suppression of Leakage-Effect

- The *Fast-Fourier-Transformation* (FFT):
- Applications of signal processing on PC/DSP/uP and in devices of digital measurement (DSO, MSO, SA) employ the Fourier-Analysis
- The DFT is computationally demanding and has a complexity of $O(N^2)$ (N^2 complex multiplications, N(N-1) kompl. Add.)
- For obvious reasons of computation time and real-time requirements effective alternatives have bee subject of study and research
- The FFT introduced by Cooley & Tukey 1965 exploits existing symmetry properties of the DFT for substantial savings
- Recursive decomposition of sample values in groups of two: *Decimation-in-Time*, DIT; *Decimation-in Frequency*, DIF (see e.g., [1])
- Prerequisite and limitation: $\boxed{N = 2^p}$
- Sample values must be available in powers of two or have to be replenished to the next power of two !
- The complexity of the FFT reduces to O(N ld N) if this condition is met

DFT and FFT

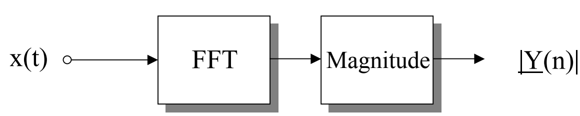
- Application of the Fourier-Transform in signal processing and analysis
- Alternatively to convolution based-filtering, frequency domain filtering can be carried out:

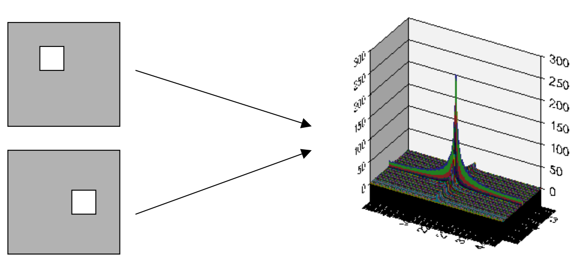




DFT and FFT

- Invariant representation can be achieved with regard to translation or shift by computing the magnitude of the FFT

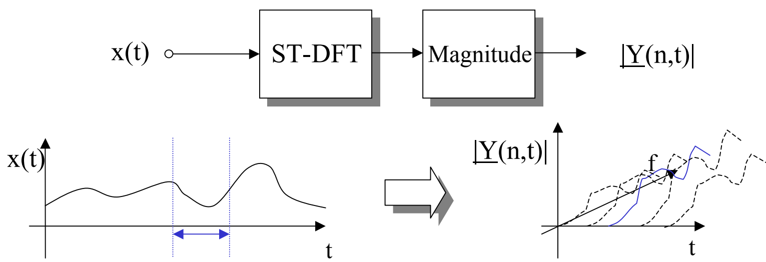




- Extension of DFT/FFT to higher dimension, e.g., 2D, straightforward due to separability property
- Transformation of all rows is followed by transformation of all columns

DFT and FFT

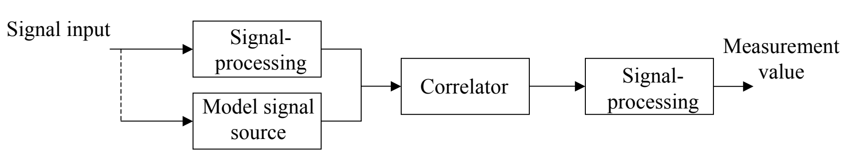
- Stationarity cannot be assumed for general signals
- The spectral composition has to be analyzed repeatedly in a restricted (short) time
- The corresponding transform is denoted as short-time FT and implies a two-dimensional spectrum indexed by time



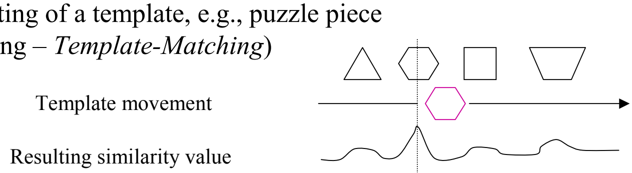
- The magnitude is commonly denoted as spectrogram (pseudo-image)
- Distance or spacing of ST-DFT windows obey an adapted sampling theorem
- Choice of appropriate window function mandatory

Correlation

- Correlation in addition to direct and indirect measurement is the third common measurement method applied
- In this approach, the correlation of a model signal, obtained by measurement itself or by analytical means with the input signal is computed
- Concept of the procedure:

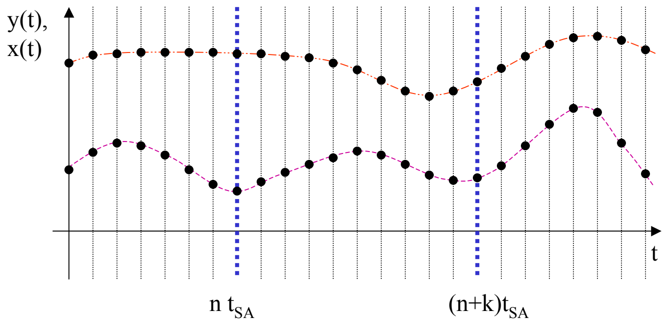


- Elucidation: Fitting of a template, e.g., puzzle piece (Image processing – *Template-Matching*)



Correlation

- Correlation techniques allow signal processing with increased immunity to noise
- Application to the sampled values x_n and y_n of two signals:



- Similar to convolution, the two signals are multiplied and a result value is accumulated
- The temporal shift allows the investigation of the correlation of current and previous signal values

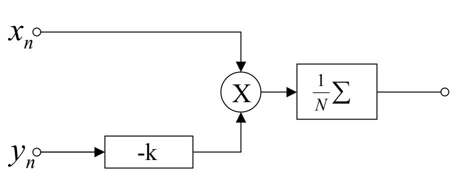
Correlation

- The corresponding function in dependance of the displacement in sample values (corresponds to displacement time !) is denoted as Cross-correlation-function CCF:

$$\Phi_{xy}(kt_{SA}) = \frac{1}{N}\sum_{n=1}^{N} x(nt_{SA}) \cdot y((n+k)t_{SA}) \quad (2.25)$$

$$\Phi_{xy}(kt_{SA}) = \frac{1}{N}\sum_{n=1}^{N} x_n \cdot y_{n+k} \quad (2.26)$$

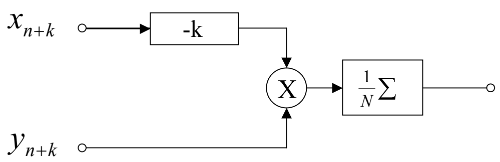
- Computational scheme for certain displacement:



$$\Phi_{xy}(kt_{SA}) = \frac{1}{N}\sum_{n=1}^{N} x_n \cdot y_{n-k}$$

Correlation

- Computational scheme for certain displacement (continued):



$$\Phi_{xy}(kt_{SA}) = \frac{1}{N}\sum_{n=1}^{N} x_n \cdot y_{n+k}$$

- Computational requirements are the operators delay, multiplication, addition and *division*
- The following combinations are possible

$$\sum x_n \cdot y_{n-k}; \quad \sum x_n \cdot y_{n+k}; \quad \sum x_{n-k} \cdot y_n; \quad \sum x_{n+k} \cdot y_n \qquad (2.27)$$

with

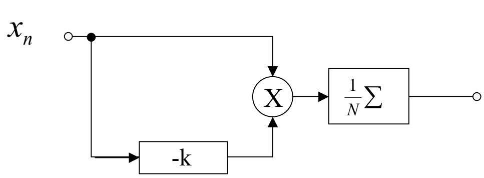
$$\sum x_n \cdot y_{n-k} = \sum x_{n+k} \cdot y_n$$
$$\sum x_n \cdot y_{n+k} = \sum x_{n-k} \cdot y_n \qquad (2.28)$$

Correlation

- This implies a mirroring of the resulting CCF at the ordinate, if the x_n will be delayed instead of the y_n :

$$\Phi_{xy}(kt_{Abt}) = \Phi_{xy}(-kt_{Abt}) \qquad (2.29)$$

- Displacement of the y_n to the right returns the identical CCF than for a displacement of the x_n to the left
- In identical fashion a single signal can be investigated for correlation between ist signal values for increasing temporal shift or displacement
- The resulting function is denoted as auto-correlation-function ACF:

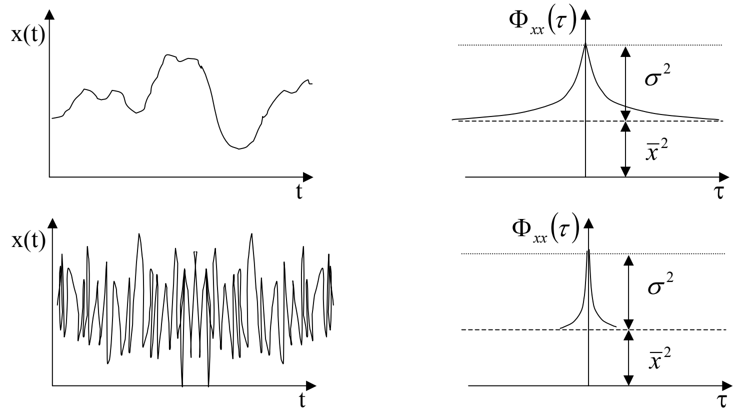


$$\Phi_{xx}(kt_{SA}) = \frac{1}{N}\sum_{n=1}^{N} x_n \cdot x_{n-k}$$

$$\Phi_{xx}(kt_{SA}) = \frac{1}{N}\sum_{n=1}^{N} x_n \cdot x_{n+k}; \quad \Phi_{yy}(kt_{SA}) = \frac{1}{N}\sum_{n=1}^{N} y_n \cdot y_{n+k} \qquad (2.30)$$

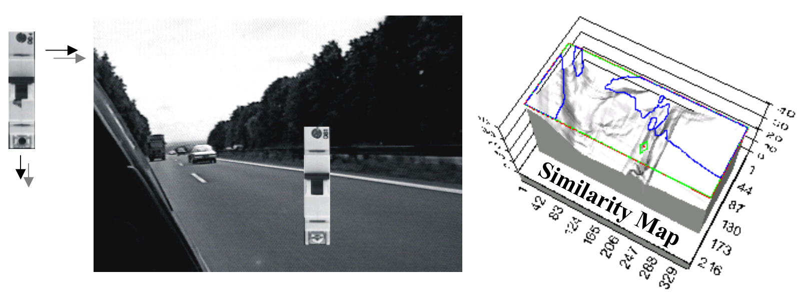
Correlation

- Practical application of ACF/CCF for finite application-dependent duration 2T
- ACF computation implies loss of phase information (reconstr. infeasible)
- ACF give information on signal coherence (self-similarity)
- Random signals show low coherence, i.e. a strong, focused maximum:



Correlation

- Single object is located/separated from background using, e.g., template correlation (CCF, translation invariance)



- Rotation invariance (2D) can be obtained carrying out a CCF at each translational position for a desired angular resolution
- Computationally prohibitive, combined with multiscale approach (e.g., Gaussian pyramid !)

Summary

- The chapter gave a survey of sensor organisation principles and basic, common signal (pre)processing techniques
- Arithmetic operations and sampling techniques allow basic signal manipulation
- Convolution allows the filtering of the signal by appropriate filter functions masks; decomposition of signal in hierarchical approach (pyramid)
- DFT/FFT allow the computation of discrete signal spectra for analysis and manipulation in the frequency domain and provides shift invariance (Mag)
- For non-stationary time-signals, the short-time FT allows the analysis for time and frequency (spectrogram)
- Correlation techniques (ACF/CCF) provide special properties with regard to signal coherence and noise; practical application for location invariance
- These signal-to-signal operations provide the basics for further processing and condensation in the hierarchy of an classification system