Modelling and Identification

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Organisation of this course

Chapter 1: Introduction

Chapter 2: Theoretical Modelling

Chapter 3: **Experimental modelling**

Chapter 4: Least-Squares methods

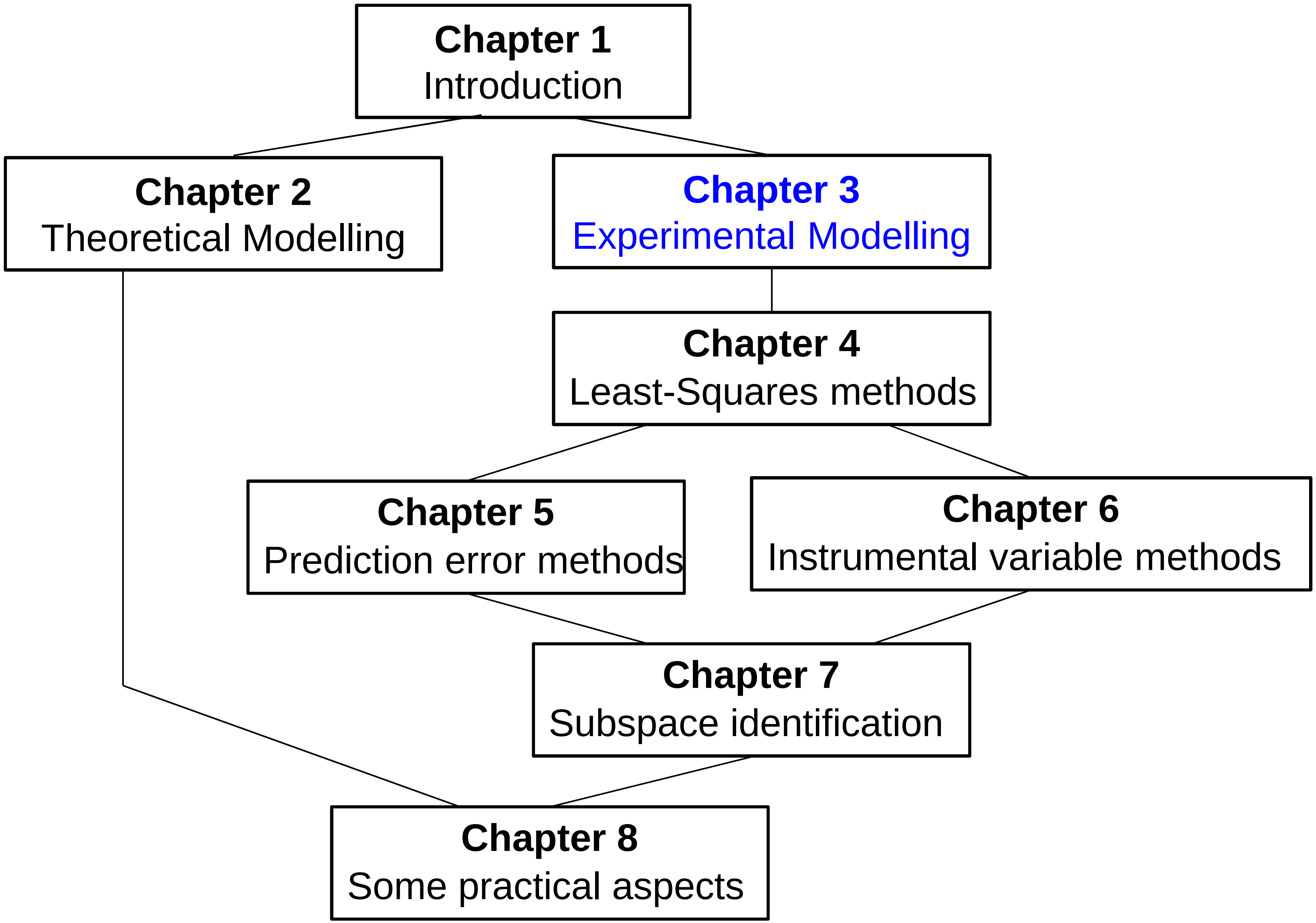
Chapter 5: Prediction error methods

Chapter 6: Instrumental variable methods

Chapter 7: Subspace identification methods (SS model!)

Chapter 8: Some practical aspects

Organisation of this course

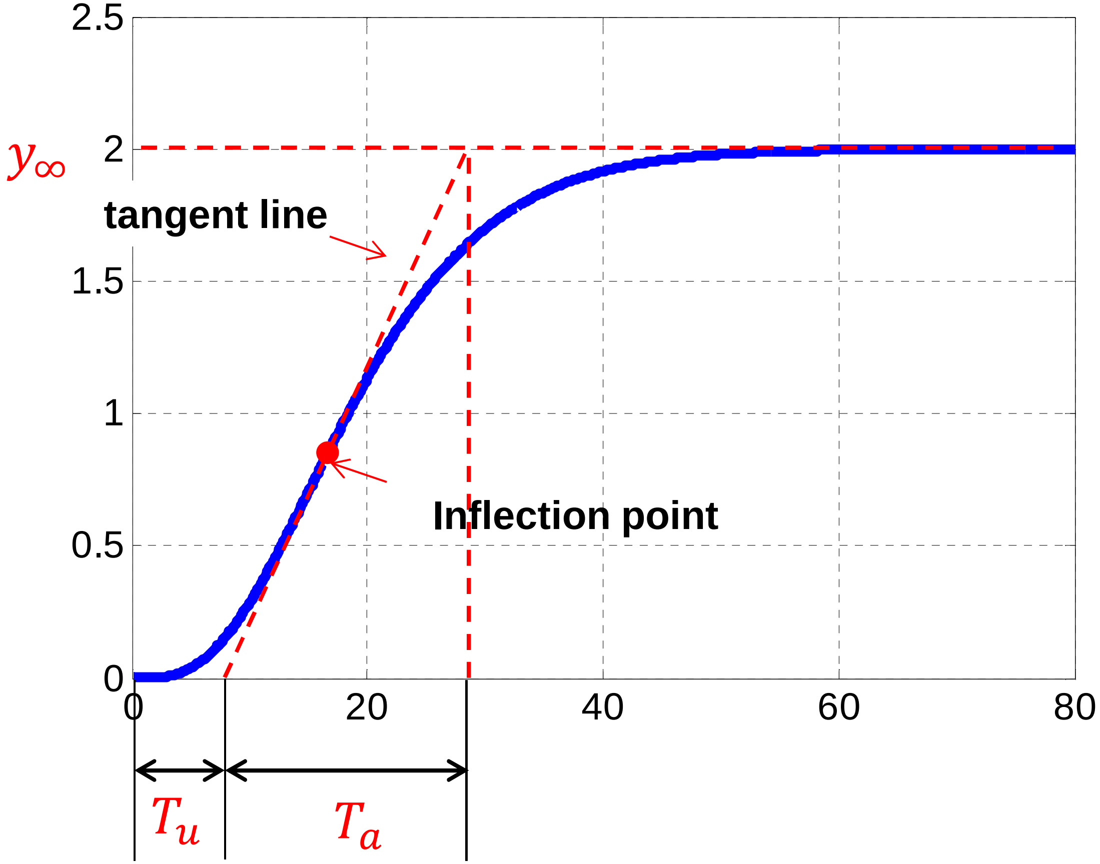


Küpfmüller approach

Approximation by a first order system with time delay

$$G(s) = \frac{K}{1+Ts}e^{-\tau s}$$

Step response ($u(t) = a\sigma(t)$)



Küpfmüller approach:

- Gain

$$K = \frac{y_\infty}{a}$$

- Time delay

$$\tau = \mathrm{T_u}$$

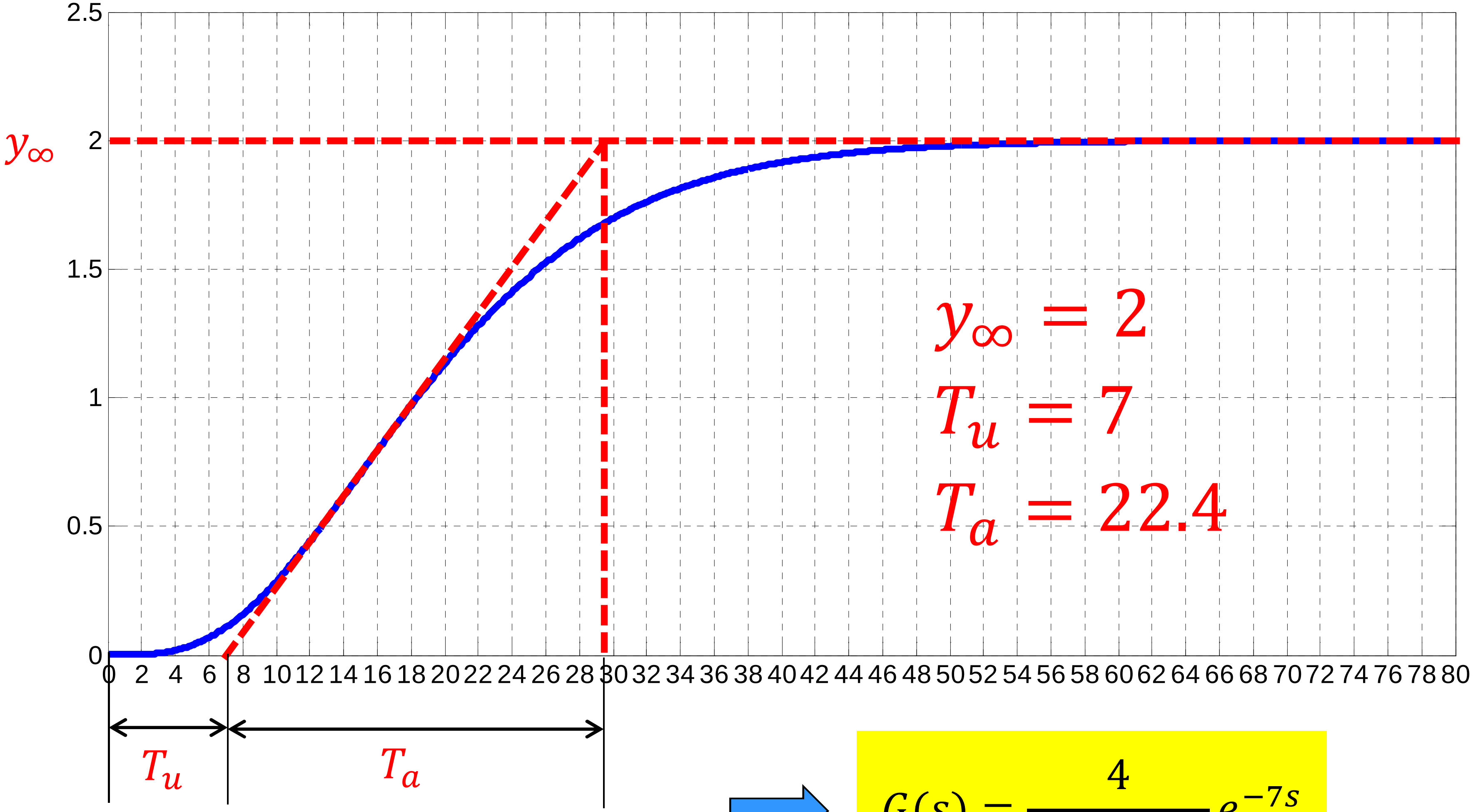
- Time constant

$$T = \mathrm{T_a}$$

Küpfmüller approach

Example

Step response ($u(t) = a\sigma(t), a = 0.5$)



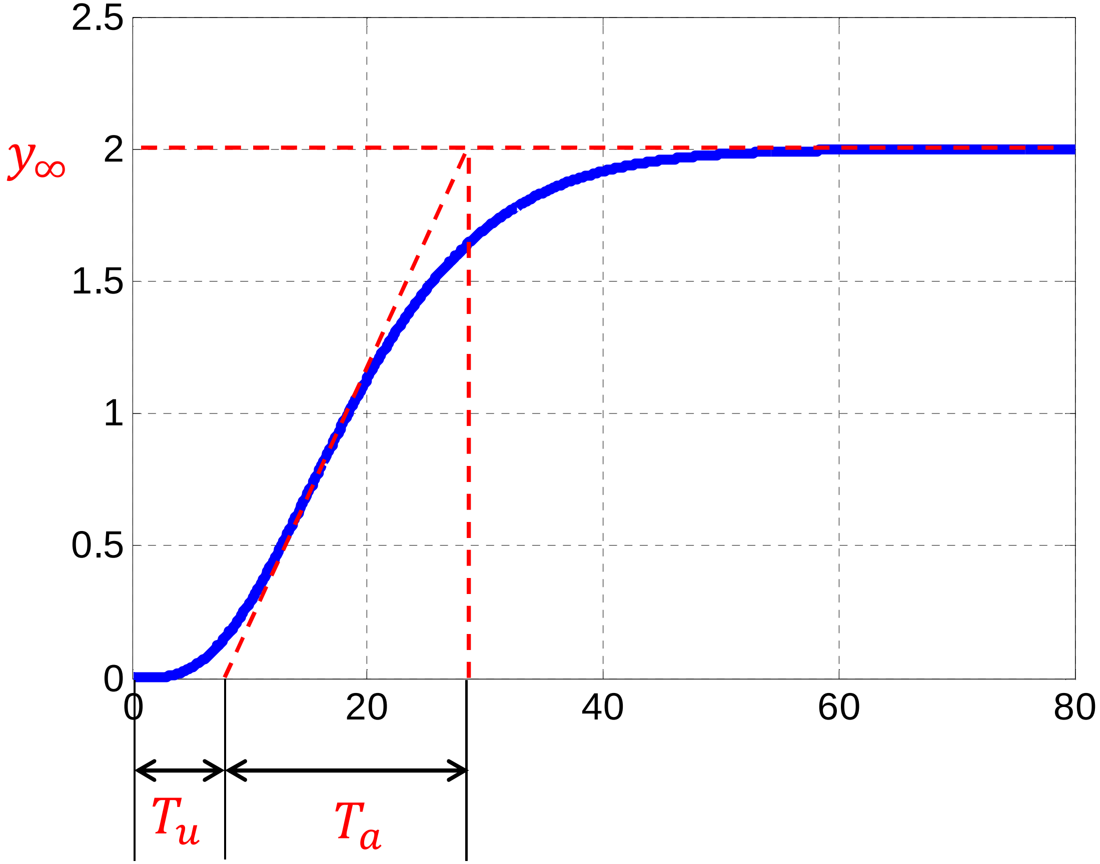
$$G(s) = \frac{4}{1 + 22.4s} e^{-7s}$$

Strejc approach

Approximation by a n-th order system with equal time constants

$$G(s) = \frac{K}{(1+Ts)^n}$$

Step response ($u(t) = a\sigma(t)$)



Read characteristic values:

- final value y_∞
- time T_u
- time T_a

Strejc approach

Identification procedure:

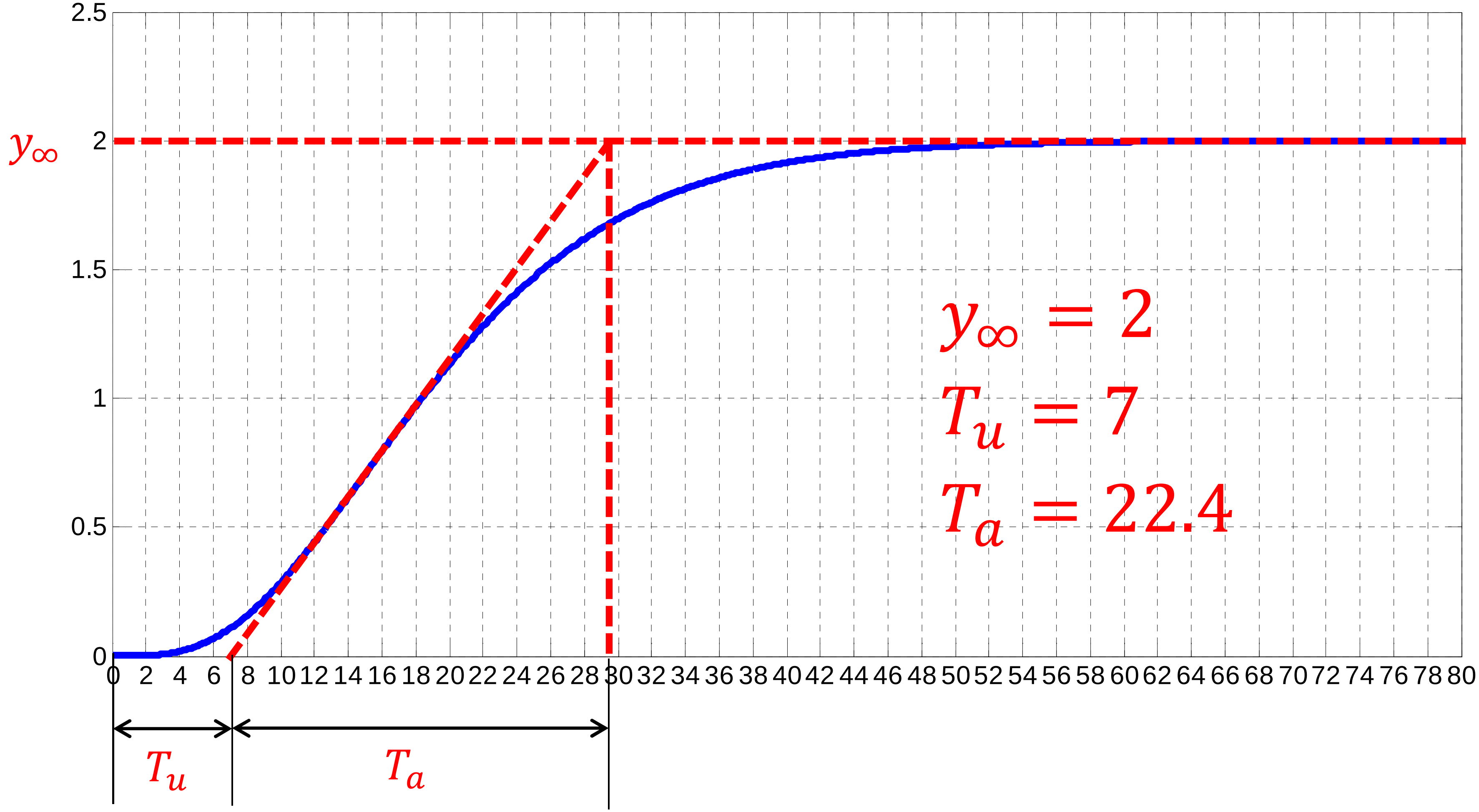
1. Calculate the gain $K = \frac{y_\infty}{a}$.
2. Draw the tangent at the inflection point
3. Read T_u and T_a
4. Based on $\frac{T_a}{T_u}$, read the order n from the following table. A rough estimate of n can also be got by $n \approx 10\frac{T_u}{T_a} + 1$.
5. Based on n, get the time constant T from the table.

n	2	3	4	5	6	7	8	9	10
T_a/T_u	9.65	4.58	3.13	2.44	2.03	1.75	1.56	1.41	1.29
T_a/T	2.72	3.69	4.46	5.12	5.70	6.23	6.71	7.16	7.59
T_u/T	0.28	0.80	1.42	2.10	2.81	3.55	4.30	5.08	5.87

Strejc approach

Example

Step response ($u(t) = a\sigma(t), a = 0.5$)



Strejc approach

Approximation by a n-th system with equal time constants

$$G(s) = \frac{K}{(1+Ts)^n}$$

$$y_\infty = 2 \;\Rightarrow\; K = \frac{y_\infty}{a} = 4$$

$$\begin{cases} T_u = 7 \\ T_a = 22.4 \end{cases}$$

$$\Downarrow$$

$$\frac{T_a}{T_u} = \frac{22.4}{7} = 3.2 \;\Longrightarrow n = 4$$

$$\Downarrow$$

$$\frac{T_a}{T} = 4.46 \Longrightarrow\; T = \frac{T_a}{4.46} = 5.02$$

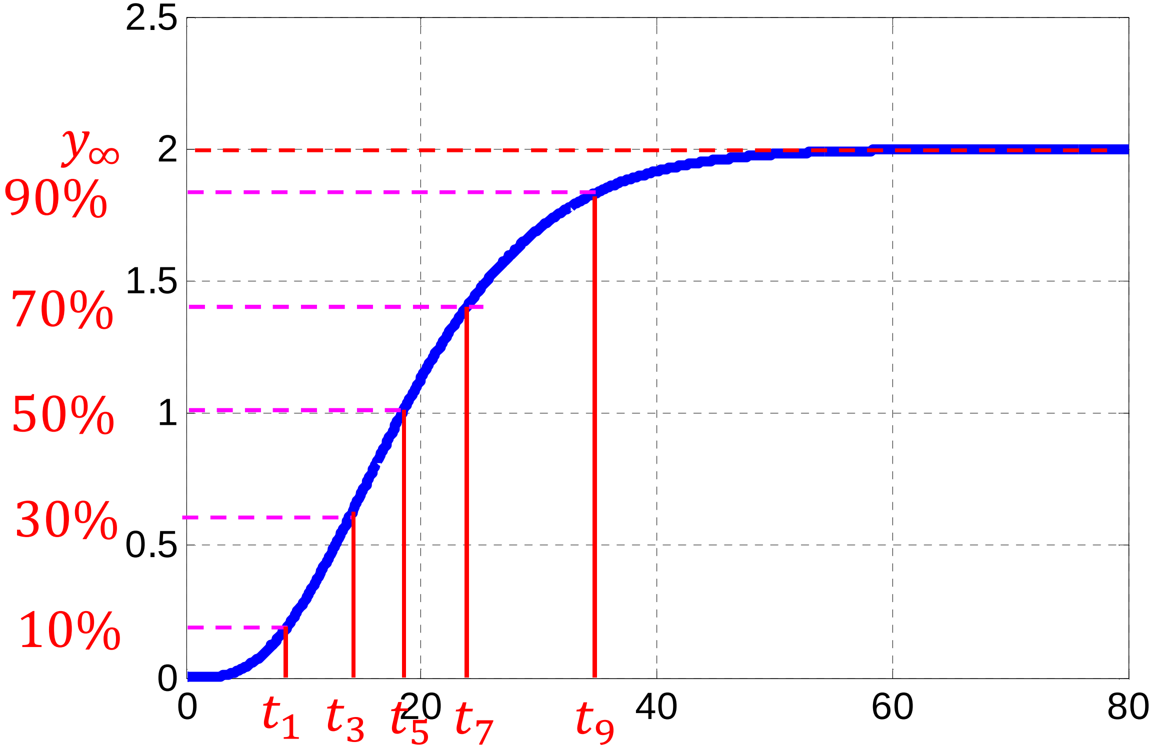
$$\boxed{G(s) = \frac{4}{(1+5.02s)^4}}$$

Schwarze approach

Approximation by a n-th system with equal time constants

$$G(s) = \frac{K}{(1+Ts)^n}$$

Step response ($u(t) = a\sigma(t)$)

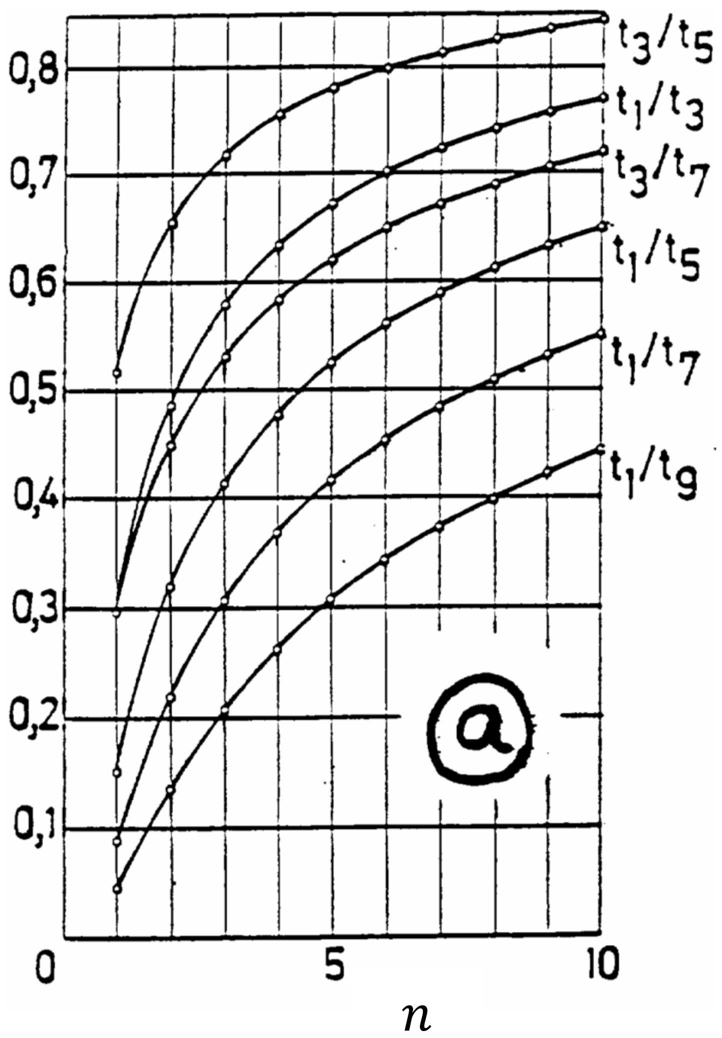


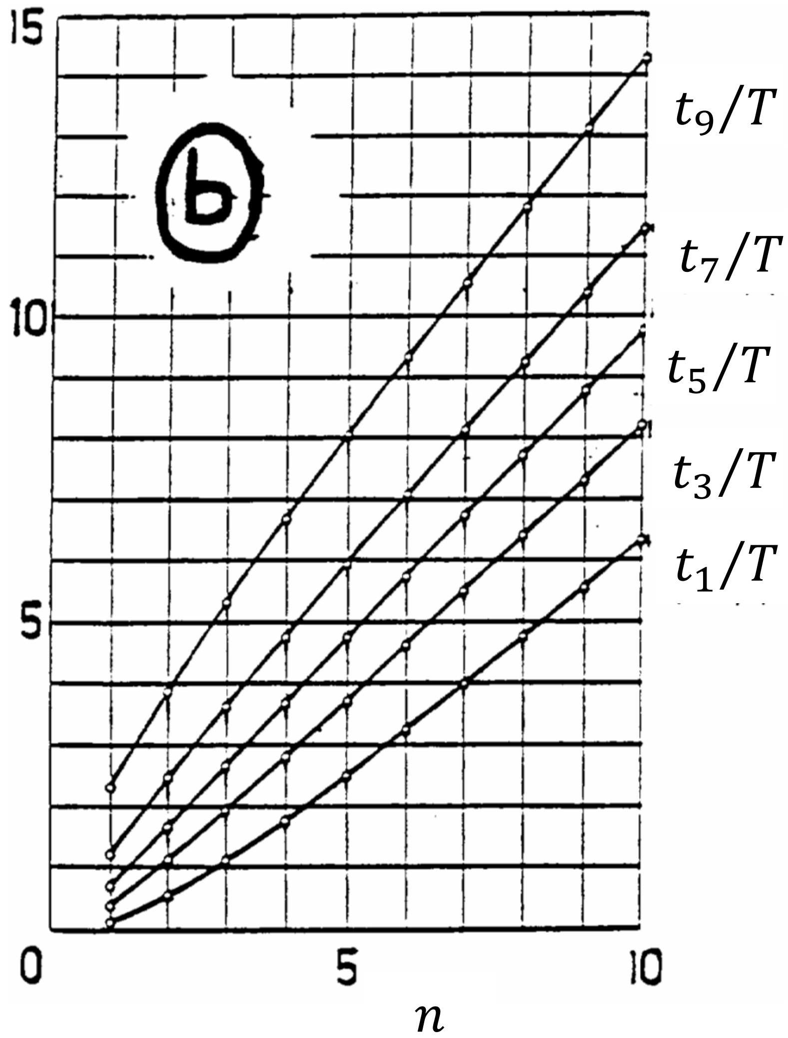
Schwarze approach make use of the characteristic values:

$y_\infty, t_1, t_3, t_5, t_7, t_9$

Schwarze approach

Key of identification:





Schwarze approach

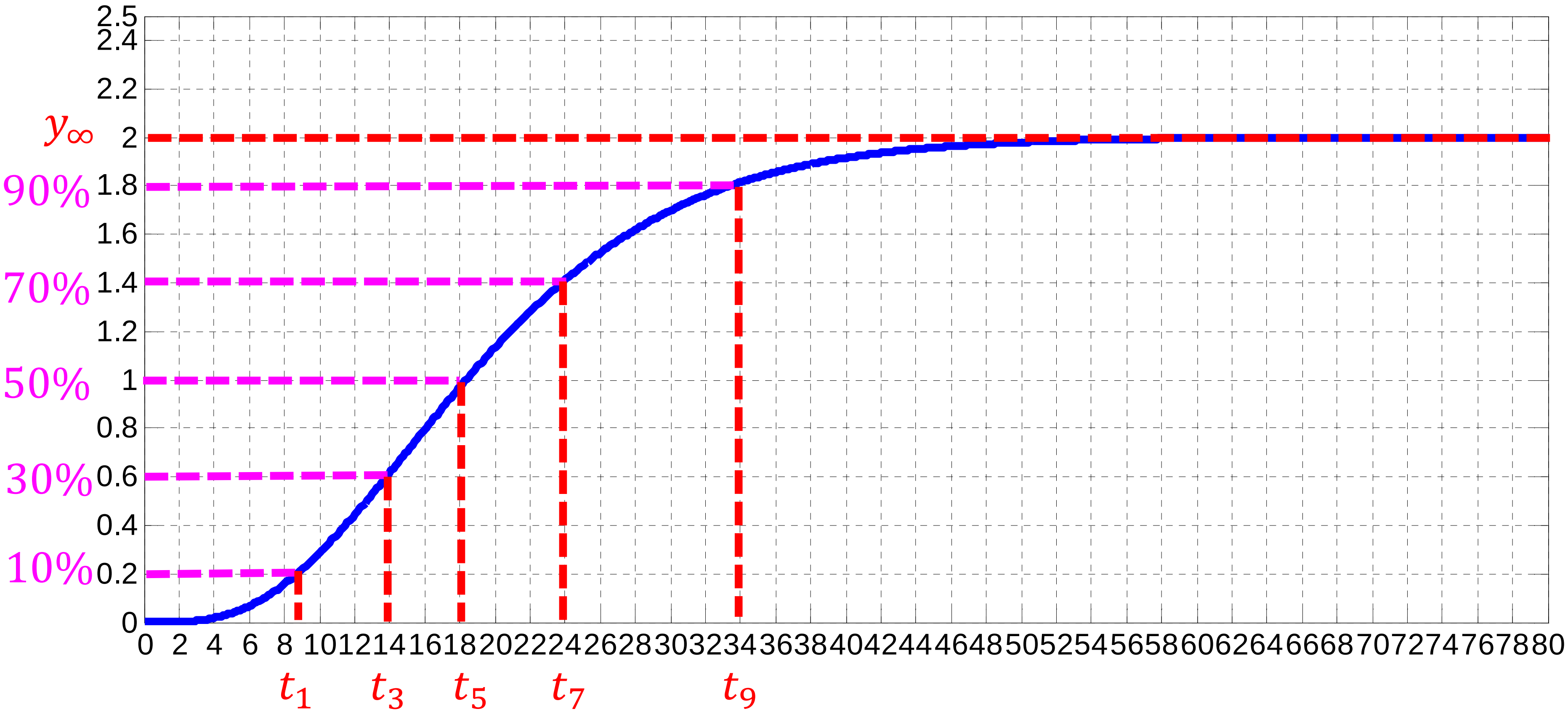
Identification procedure:

1. Get the characteristic values $y_\infty, t_1, t_3, t_5, t_7, t_9$ from the step response
2. The gain K is got by $K = \frac{y_\infty}{a}$.
3. Based on $\frac{t_i}{t_j}$, determine the order of the system n.
4. Based on n and one of the curves $\frac{t_l}{T}$, determine the time constant T.

Schwarze approach

Example:

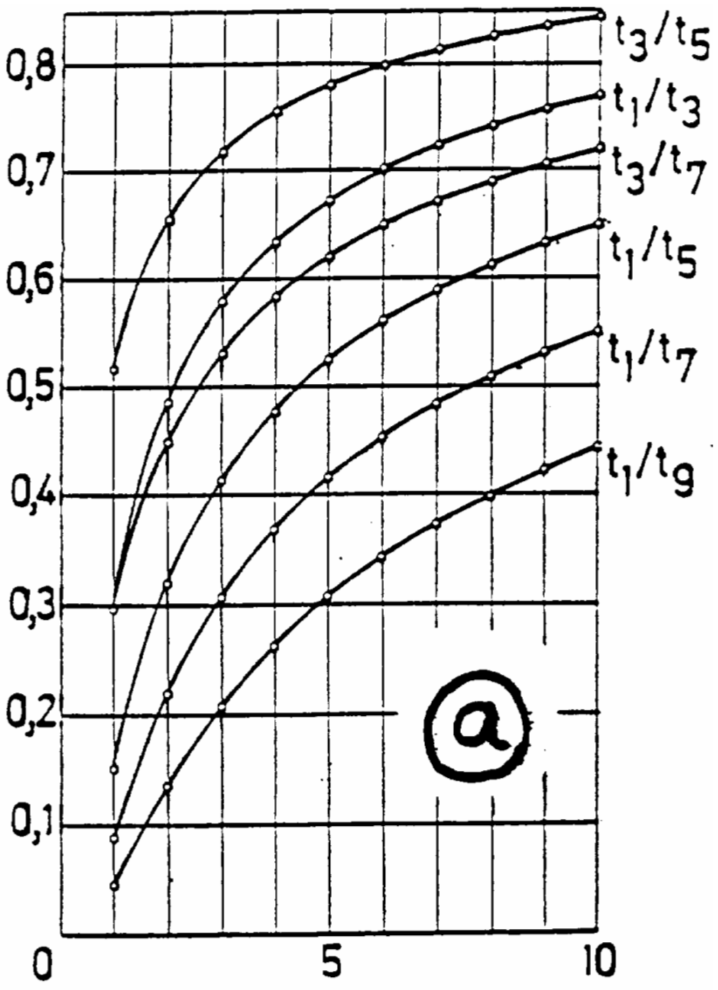
Step response ($u(t) = a\sigma(t), a = 0.5$)



$$y_\infty = 2, t_1 = 8.6,\ t_3 = 14, t_5 = 18, t_7 = 24, t_9 = 34$$

Schwarze approach

Example:



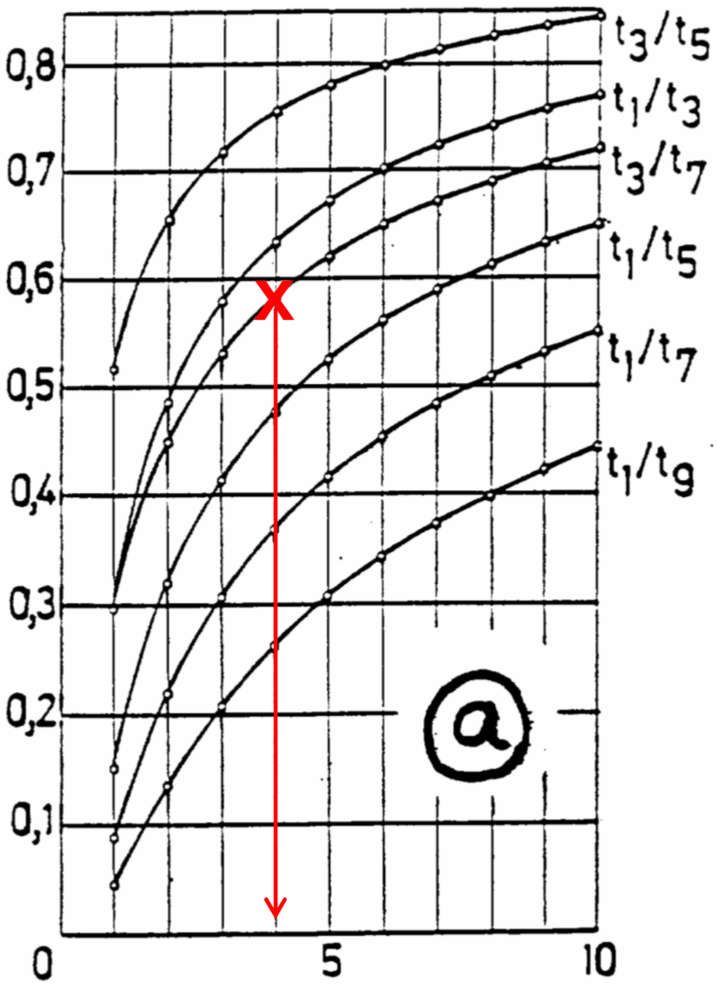
$t_1 = 8.6,\ t_3 = 14, t_5 = 18, t_7 = 24, t_9 = 34$

The gain of TF

$$K = \frac{y_\infty}{a} = \frac{2}{0.5} = 4$$

Schwarze approach

Example:



$t_1 = 8.6,\ t_3 = 14, t_5 = 18, t_7 = 24, t_9 = 34$

The gain of TF

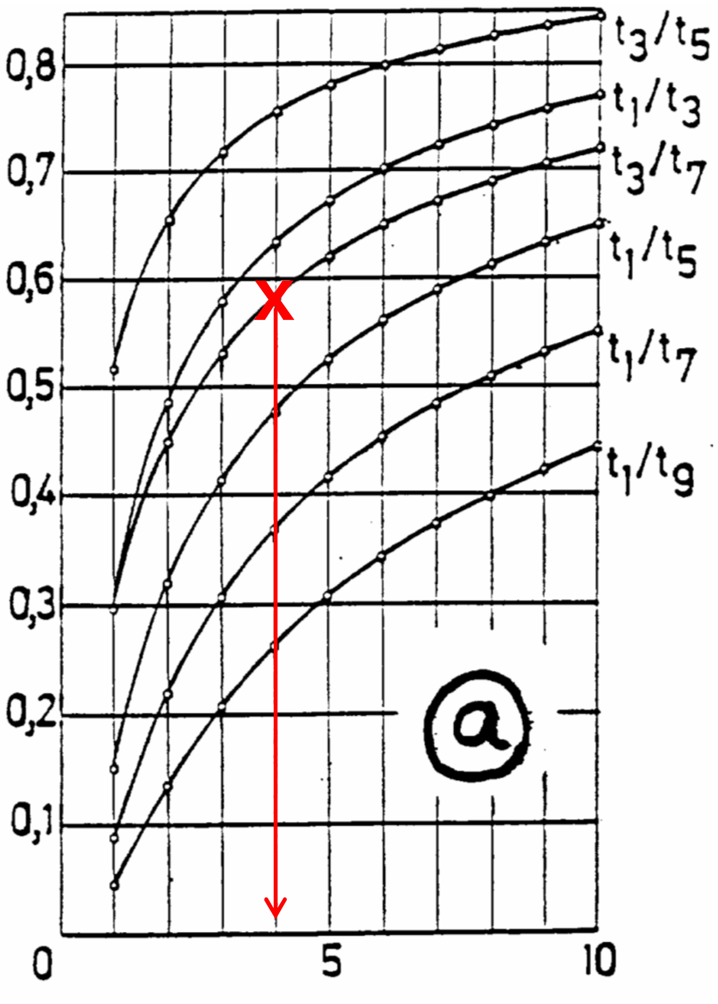
$$K = \frac{y_\infty}{a} = \frac{2}{0.5} = 4$$

As

$$\frac{t_3}{t_7} = \frac{14}{24} = 0.5833$$

Schwarze approach

Example:



$t_1 = 8.6,\ t_3 = 14, t_5 = 18, t_7 = 24, t_9 = 34$

The gain of TF

$$K = \frac{y_\infty}{a} = \frac{2}{0.5} = 4$$

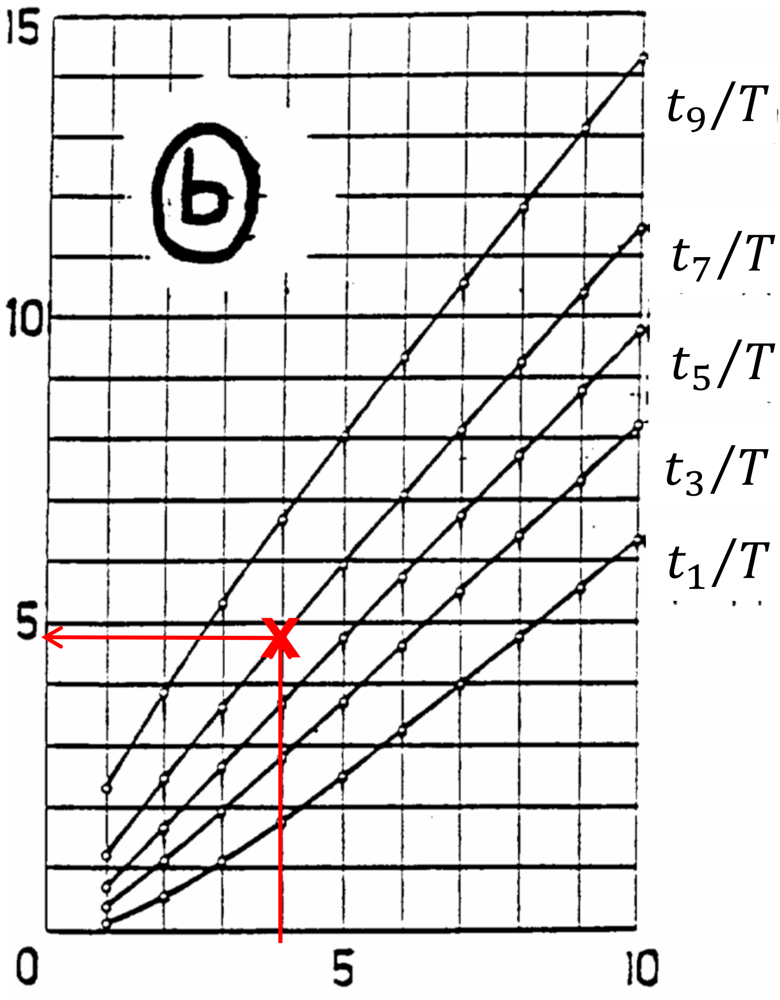
As

$$\frac{t_3}{t_7} = \frac{14}{24} = 0.5833$$

From **Figure a** it can be seen that

$$n = 4$$

Example:



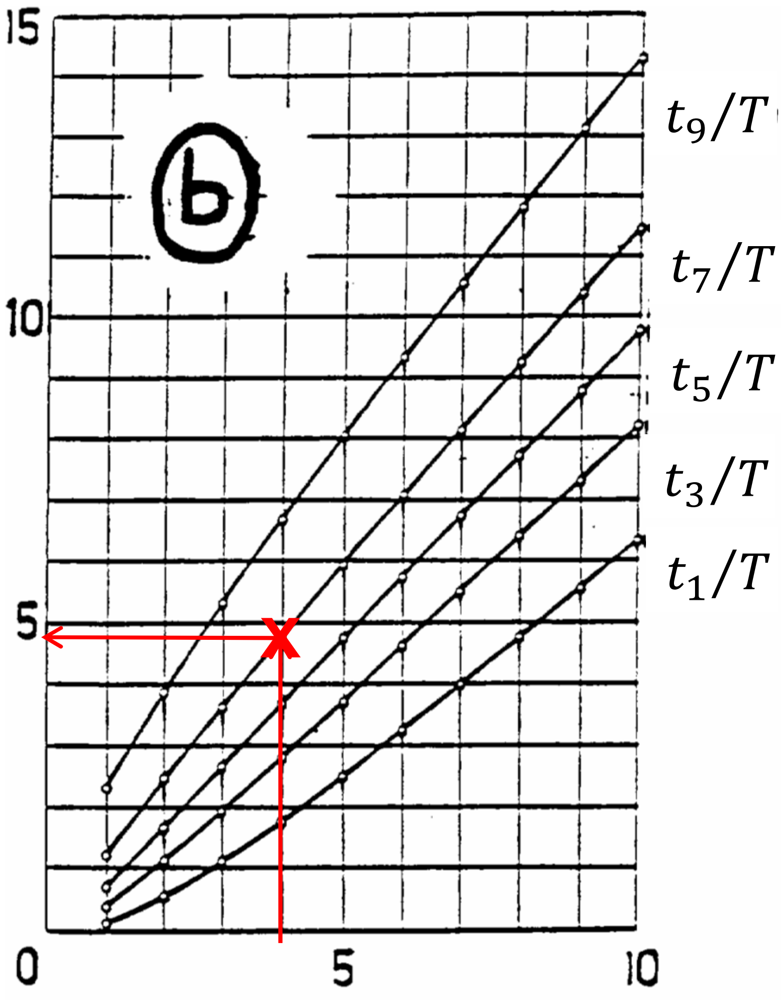
$t_1 = 8.6,\ t_3 = 14, t_5 = 18, t_7 = 24, t_9 = 34$

From **Figure b** it can be seen that

$$\frac{t_7}{T} \approx 4.8$$

Schwarze approach

Example:



$t_1 = 8.6,\ t_3 = 14, t_5 = 18, t_7 = 24, t_9 = 34$

From **Figure b** it can be seen that

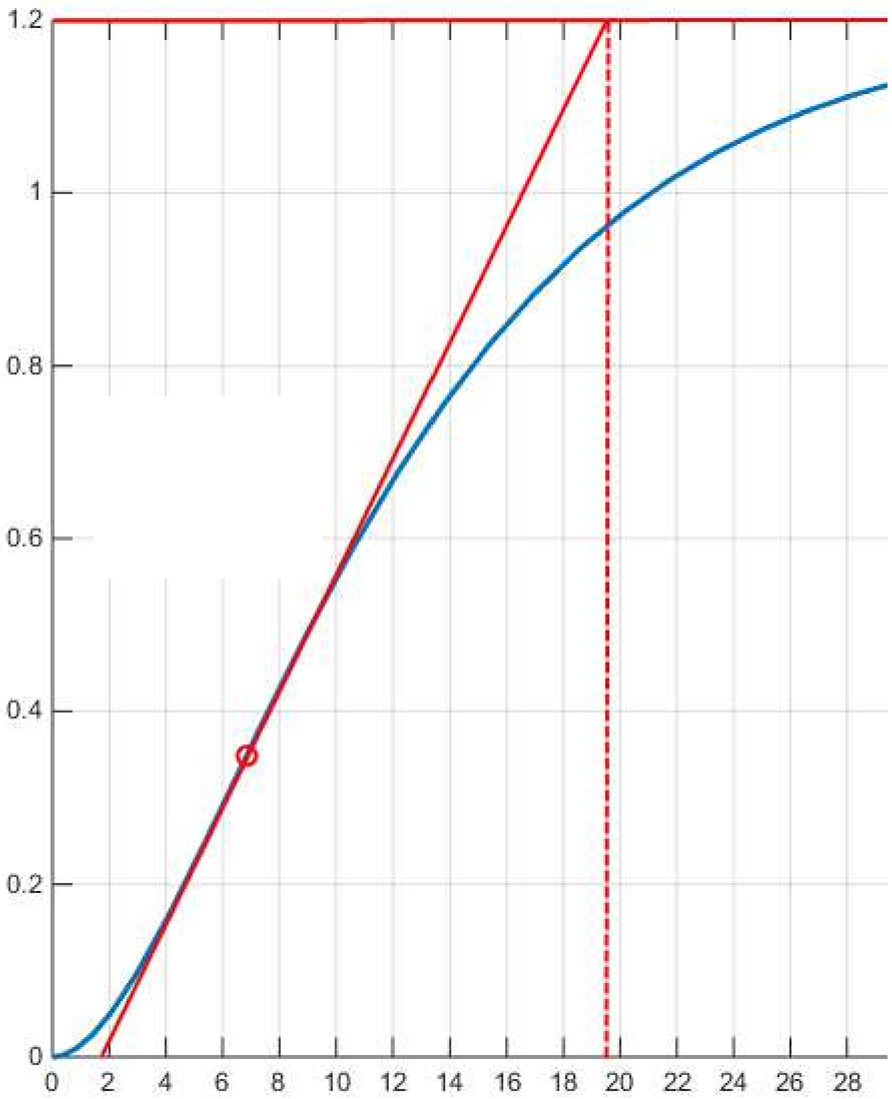
$$\frac{t_7}{T} \approx 4.8$$

Hence, the time constant is

$$T = \frac{t_7}{4.8} = \frac{24}{4.8} = 5$$

$$G(s) = \frac{4}{(1+5s)^4}$$

Comparison of approaches



Step response ($u(t) = a\sigma(t), a = 0.5$)

Available approaches:

- **Nomogram approach**

  Second order system with two real poles

- **Küpfmüller approach**

  First order system with time delay

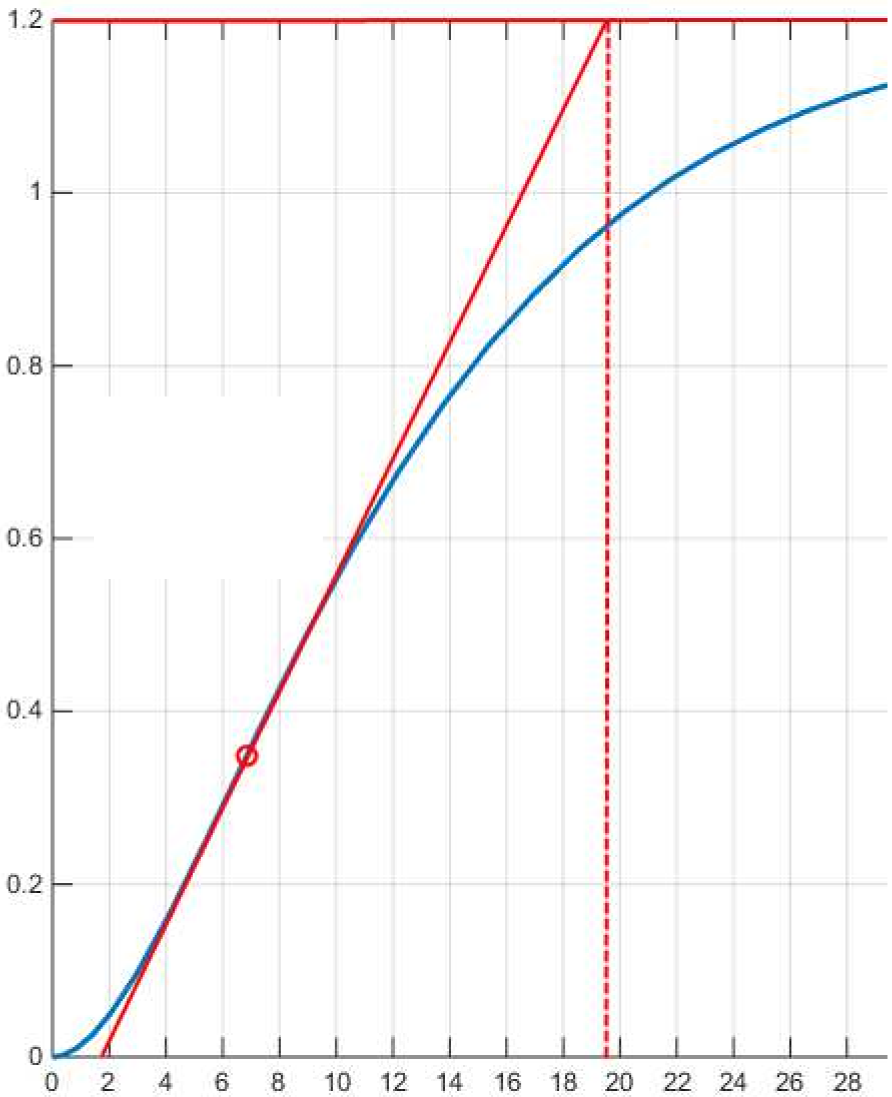
- **Strejc approach**

  N-th order system with the same time constants

- **Schwarze approach**

  N-th order system with the same time constants

Comparison of approaches



Step response ($u(t) = a\sigma(t), a = 0.5$)

Available approaches:

- **Nomogram approach**

  Tangent based approach

- **Küpfmüller approach**

  Tangent based approach

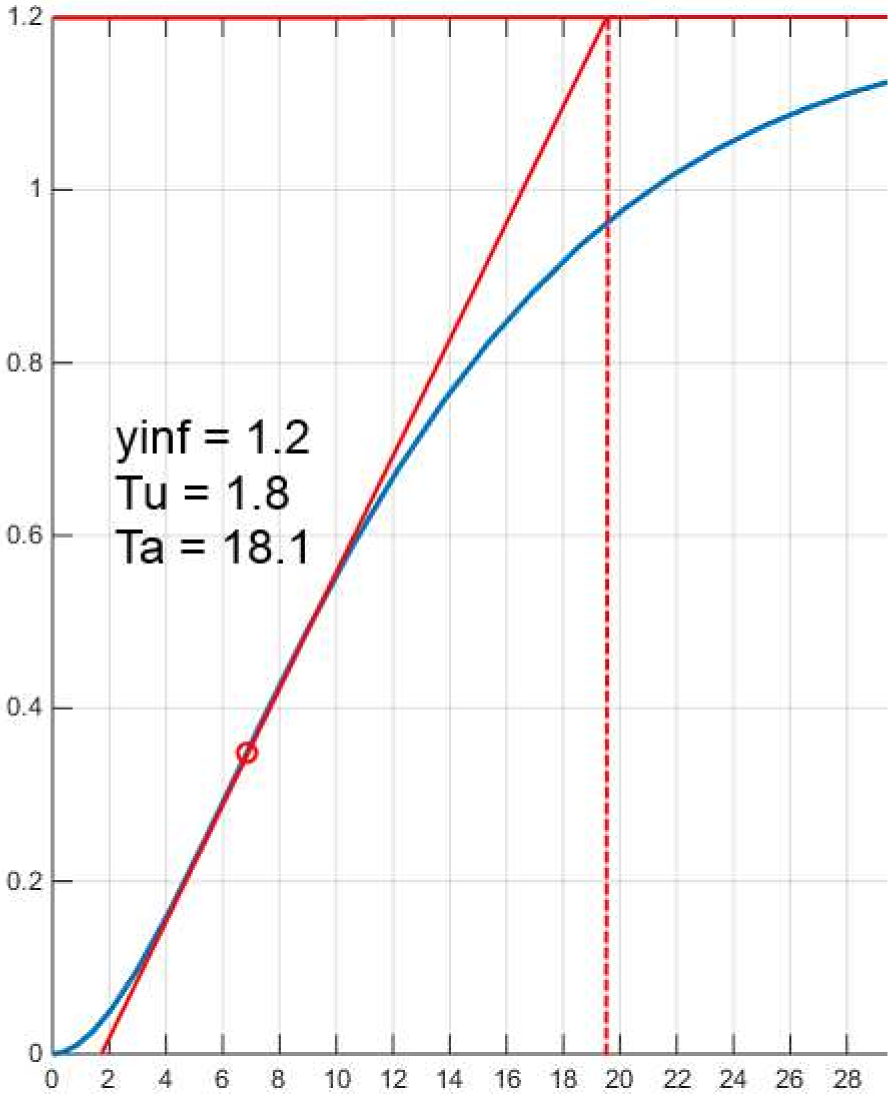
- **Strejc approach**

  Tangent based approach

- **Schwarze approach**

  Time-Percent based approach

Comparison of approaches



Step response ($u(t) = a\sigma(t), a = 0.5$)

Results of identification:

- **Nomogram approach**

$$G(s) = \frac{2.4}{(1 + 5.028s)(1 + 8.044s)}$$

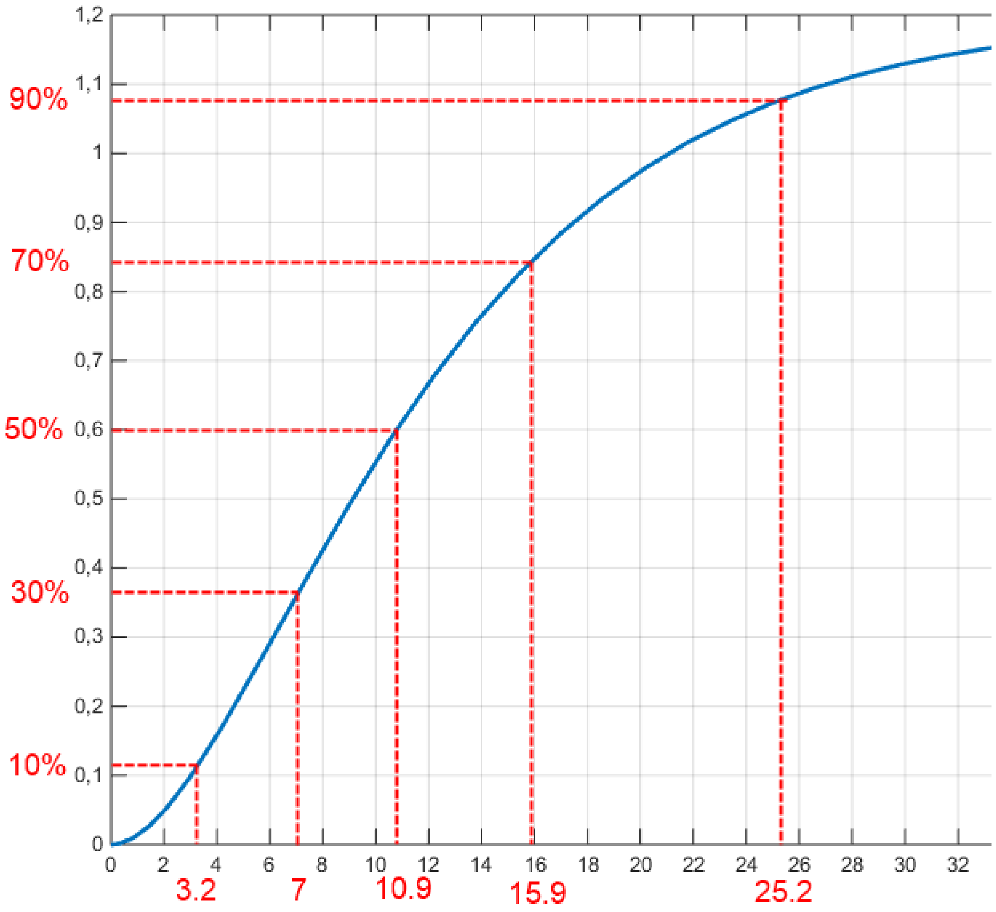
- **Küpfmüller approach**

$$G(s) = \frac{2.4}{1 + 18.1s} e^{-1.8s}$$

- **Strejc approach**

$$G(s) = \frac{2.4}{(1 + 6.54s)^2}$$

Comparison of approaches



Results of identification:

- **Schwarze approach**

$$G(s) = \frac{2.4}{(1+6.4s)^2}$$

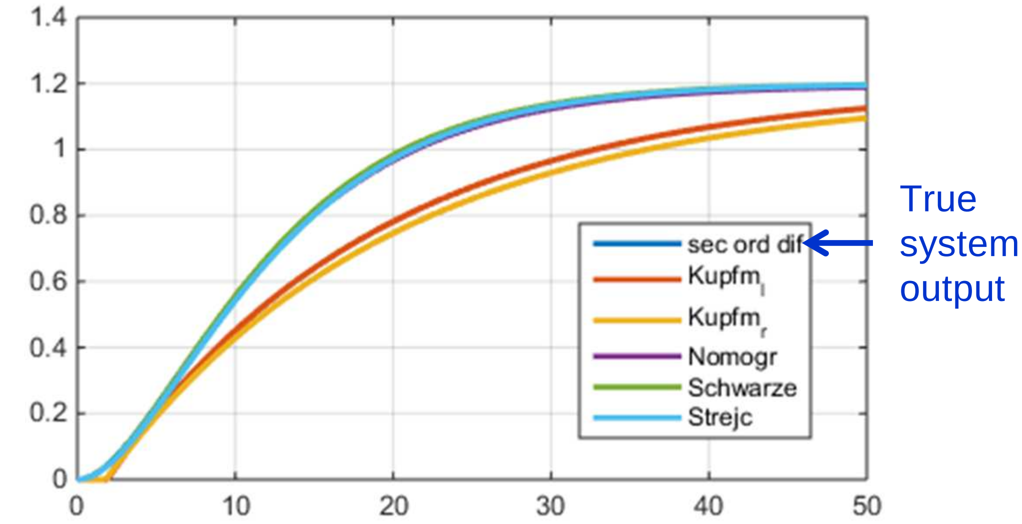
Comparison of approaches

Validation of model for the above example

Scenario 1:

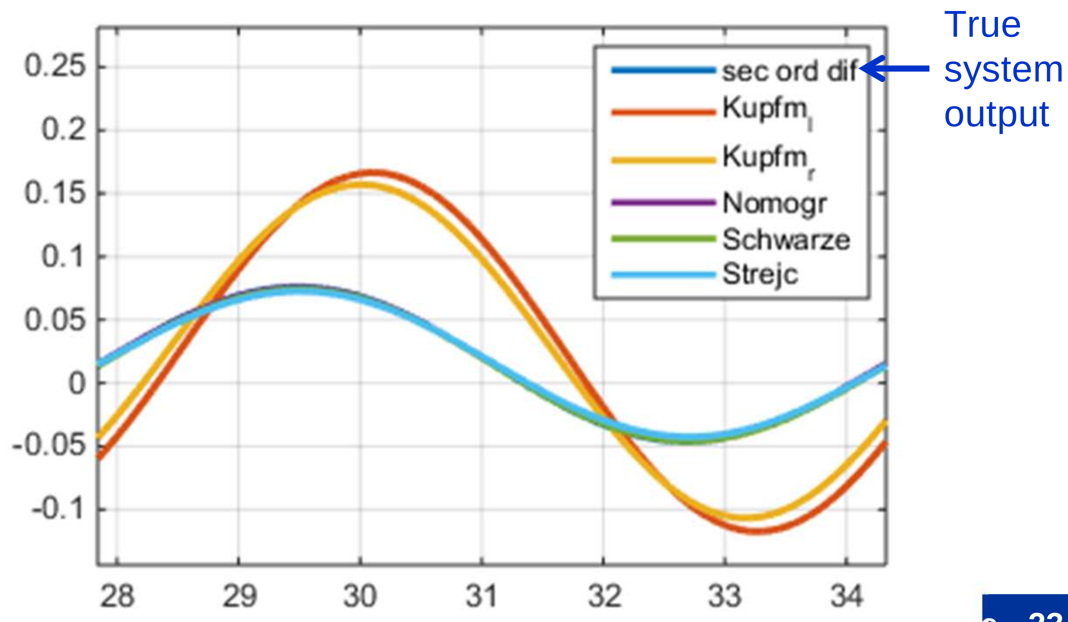
Step-response

(amplitude of the step input: 0.5)



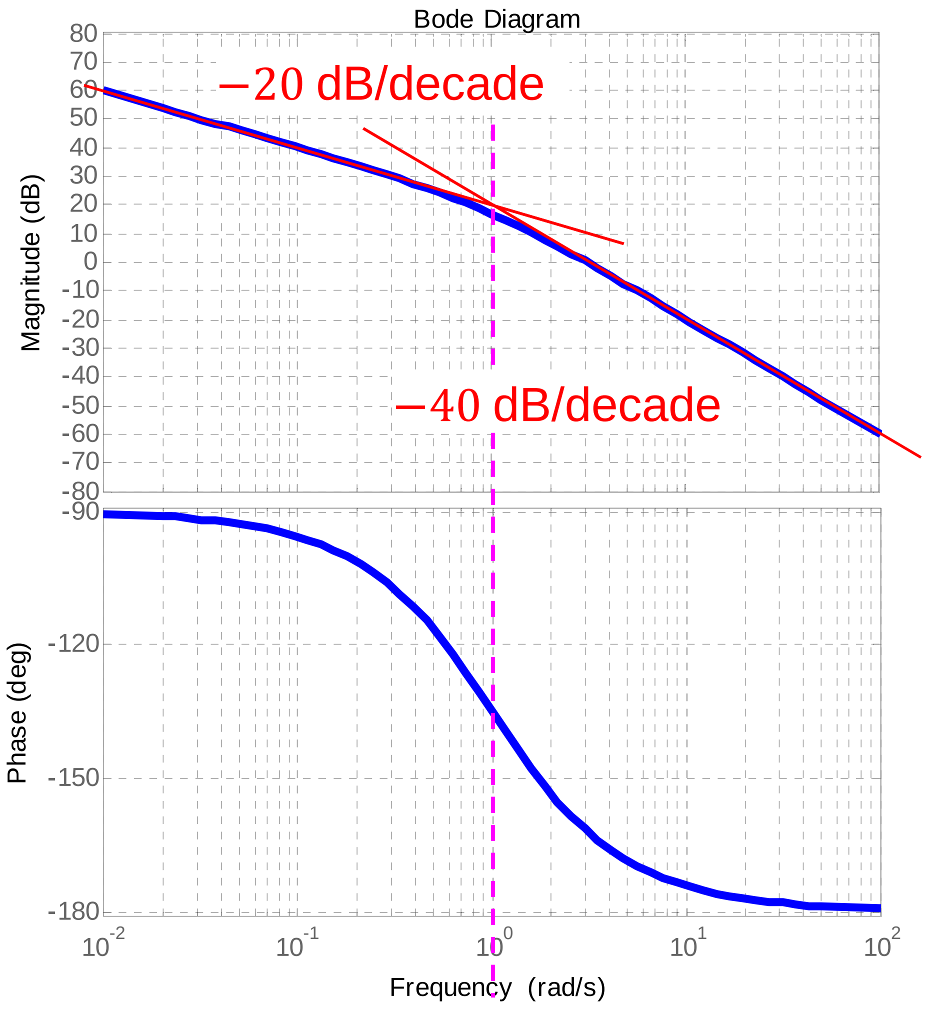
Scenario 2:

Response to sinusoidal input



Get transfer function from frequency response

Measured frequency response



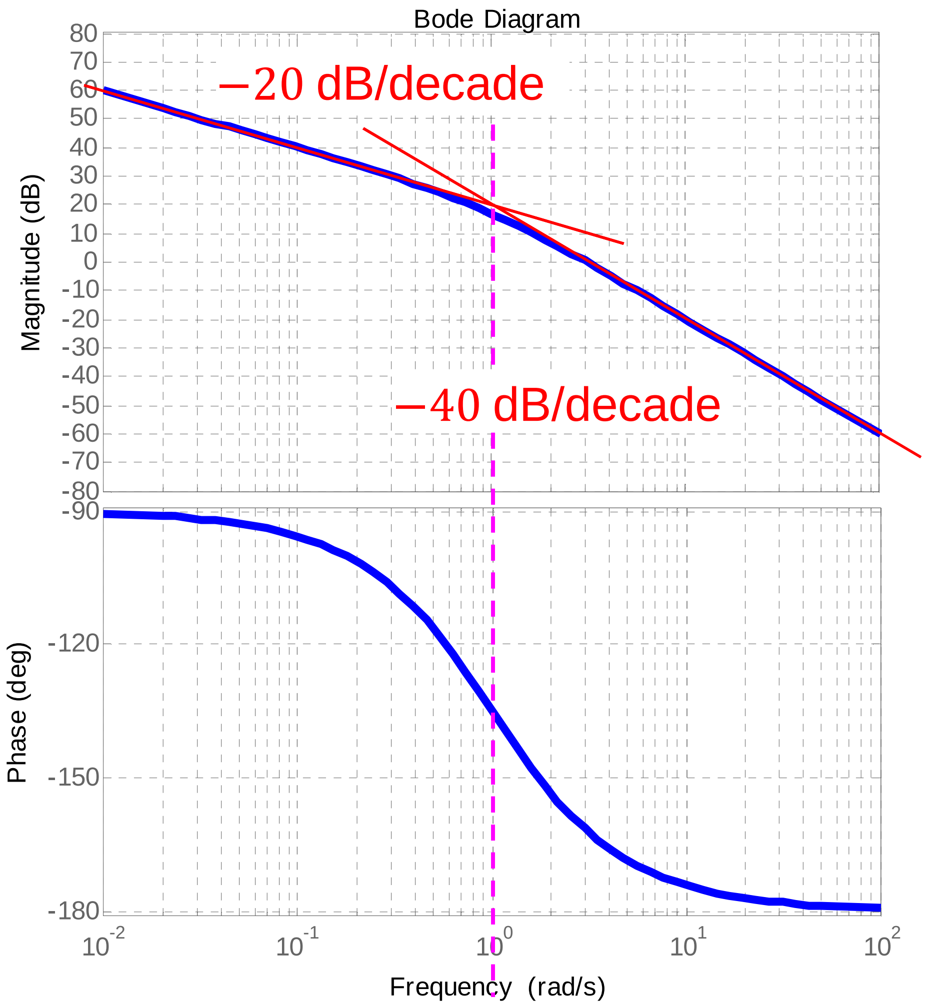
Corner frequency: ω_D

Determine the structure of the transfer function

$$G(s) = \frac{K}{s(Ts+1)}$$

Get transfer function from frequency response

Measured frequency response



Corner frequency: ω_D

Determine the structure of the transfer function

$$G(s) = \frac{K}{s(Ts+1)}$$

Read **corner frequency**

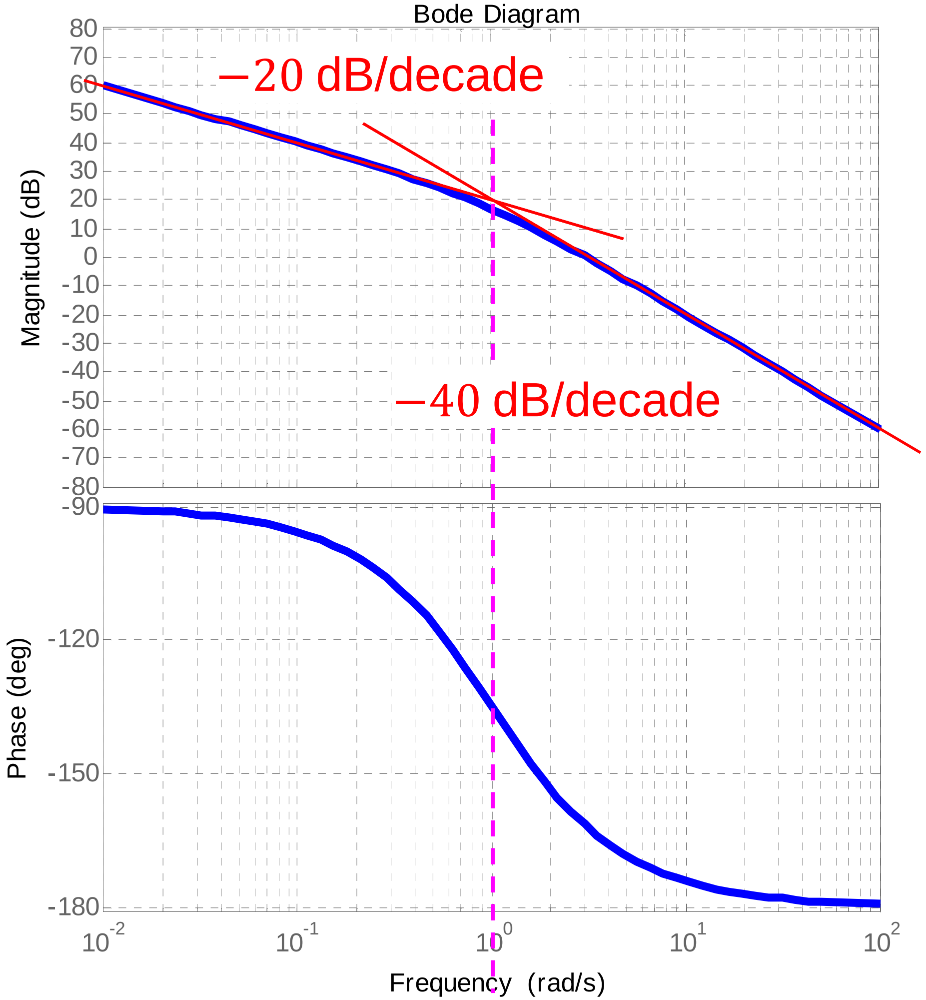
$$\omega_D = 1$$

Hence, the time constant is

$$T = \frac{1}{\omega_D} = 1$$

Get transfer function from frequency response

Measured frequency response



Corner frequency: ω_D

Read $20\log_{10}|G(j\omega)| = 60dB$ at frequency $\omega = 0.01$.

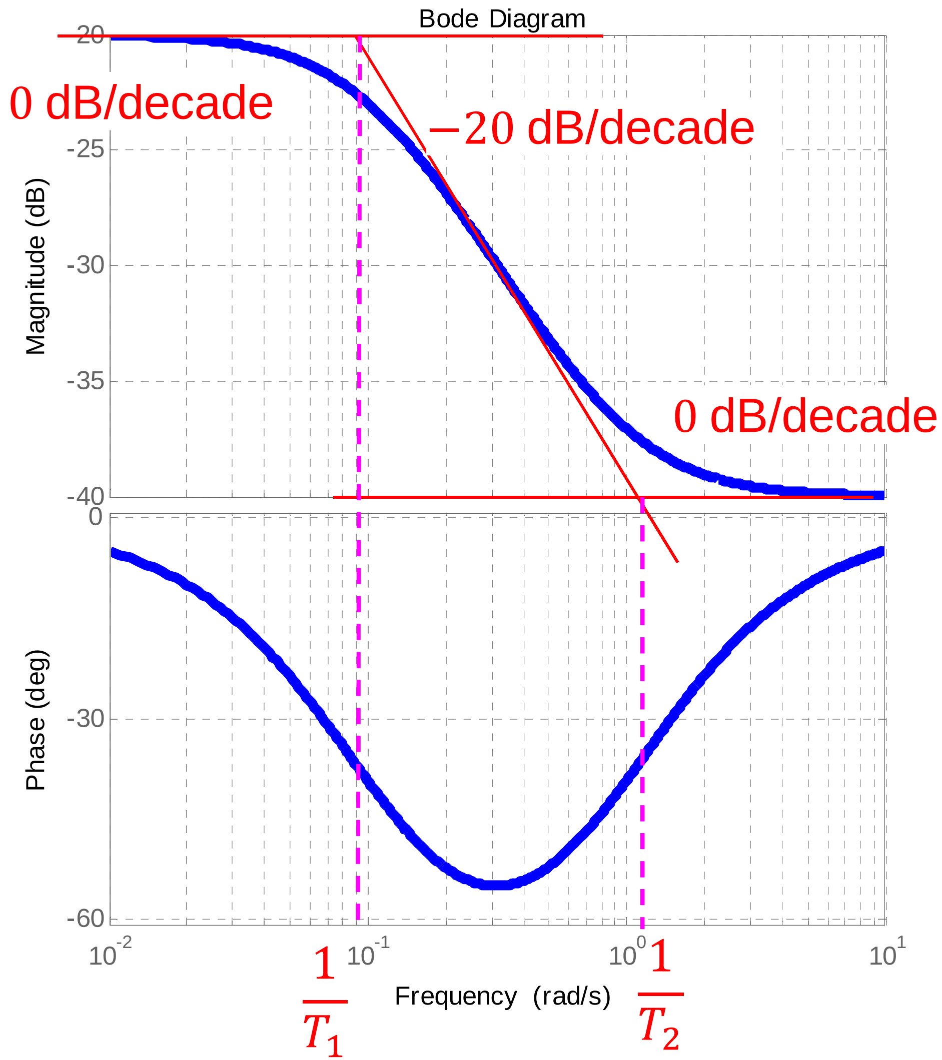
$$20\log_{10}|G(j\omega)|$$
$$= 20\log_{10} K - 20\log_{10} 0.01$$
$$= 60$$

$$K = 10$$

$$G(s) = \frac{10}{s(s+1)}$$

Get transfer function from frequency response

Example 2:



Determine the structure of the transfer function

$$G(s) = \frac{K(1 + T_2 s)}{(1 + T_1 s)}$$

$$20 \log_{10} K = -20$$

$$\Downarrow$$

$$K = 0.1$$

Get transfer function from frequency response

Assume that the transfer function of the system is

$$G(s) = \frac{b_m s^m + \cdots + b_1 s + b_0}{s^n + \cdots + a_1 s + a_0}$$

Given the measured frequency responses at $\omega_1, \omega_2, \cdots, \omega_N$.
Determine the tranfer function of the system.

Based on the frequency responses

$$G(j\omega_i) = \frac{b_m (j\omega_i)^m + \cdots + b_1 (j\omega_i) + b_0}{(j\omega_i)^n + \cdots + a_1 (j\omega_i) + a_0}, \quad i = 1,2,\cdots,N$$

A group of equations in the following form can be obtained

$$Q \begin{bmatrix} a_{n-1} \\ \vdots \\ a_0 \\ b_m \\ \vdots \\ b_0 \end{bmatrix} = P \quad \Rightarrow \quad \text{Least squares estimate} \begin{bmatrix} \hat{a}_{n-1} \\ \vdots \\ \hat{a}_0 \\ \hat{b}_m \\ \vdots \\ \hat{b}_0 \end{bmatrix}$$

Summary of Chapter 3

- Measurement of non-parametric models:
  - Step response
  - Impulse response
  - Frequency response
- Get parametric model from non-parametric model