Examination

Modelling and Identification /

Modellbildung und Identifikation

Date: 9th September, 2015

Duration of examination: 120 Minutes

Points: 100

Please write legibly!

You must document how you obtain the answer to qualify for full credits.

This examination consists of **6 problems**. First, **check that your copy contains all 6 problems**.

Admitted aids: indelible pens ruler and means of drawing, non-programmable calculator, one A4 page of handwritten notes. Having **non-admitted aids present** after the distribution of the examination tasks also constitutes as an attempt to cheat and leads to the non-approval of your examination.

Write your name and matriculation number on each page of the answer sheets including the cover sheet.

Use only the provided answer sheet for the answers. Only the answer sheets will be collected.

Write an explanation for all answers and give the approach for your calculations. The absence of an explanation or the approach has an influence on the assessment of the answer. Answers only consisting “Yes” or “No” will not gain points.

Problem 1: Theoretic Modelling (14+10 points)

Consider the hydraulic-mechanical system of Figure 1. A force F influences, via a massless piston and a hydraulic cylinder, a piston with mass m_1. The hydraulic fluid is **incompressible** and massless. The mass m_1 is then connected via a spring (with spring constant k_1) with a friction-free moving mass m_2. The mass m_2 is attached to another mass m_3 through a frictionless pully and a spring of spring constant k_2. The mass m_3 is connected to the surface through a linear damper with damping constant b. The cross section of the pistons are A_1 and A_2.

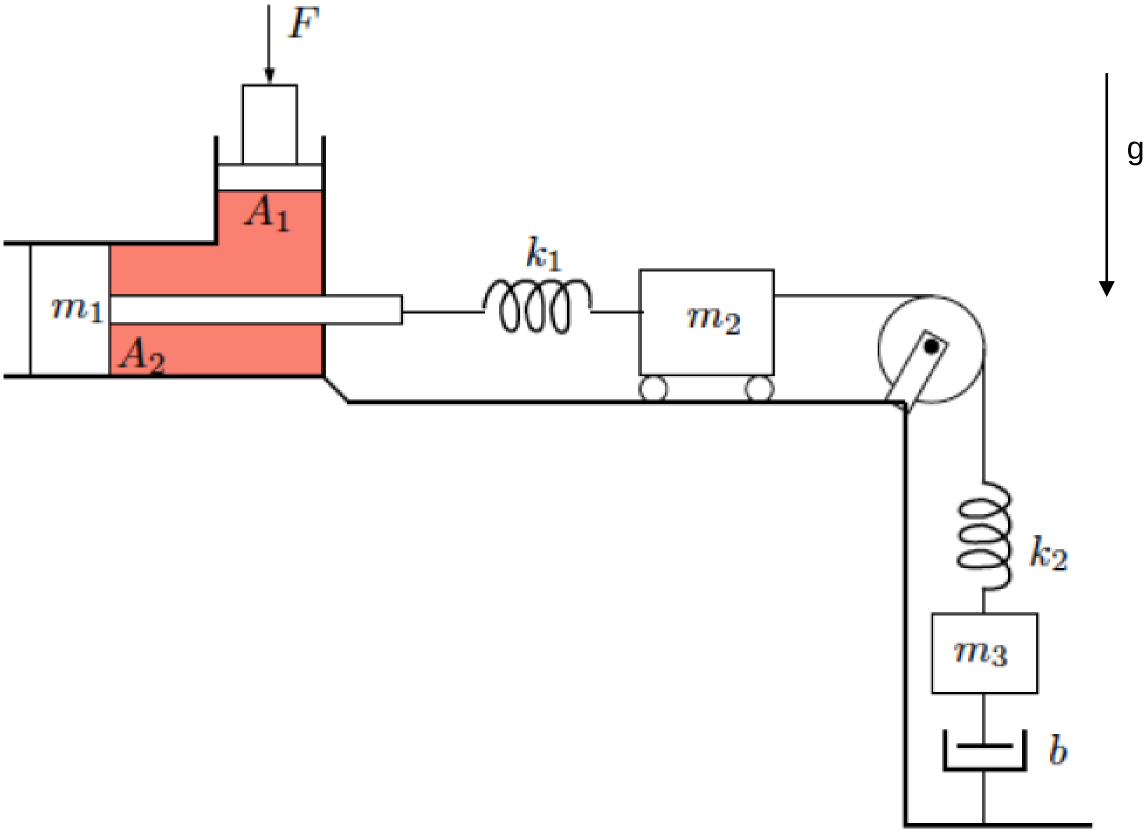

Figure 1: Hydraulic-mechanical system

a) Set up differential equations for the three masses.
b) Derive a state space model with the forces F and the gravitation g as inputs and the speed of mass m_3 as output.

Problem 2: Identification based on step response (4+8 points)

a) Given the result of a step response experiment in Figure 2, what is an appropriate mathematical model of the unknown system?
b) Determine the transfer function of the unknown system. The same figure is given also on the answer sheets.

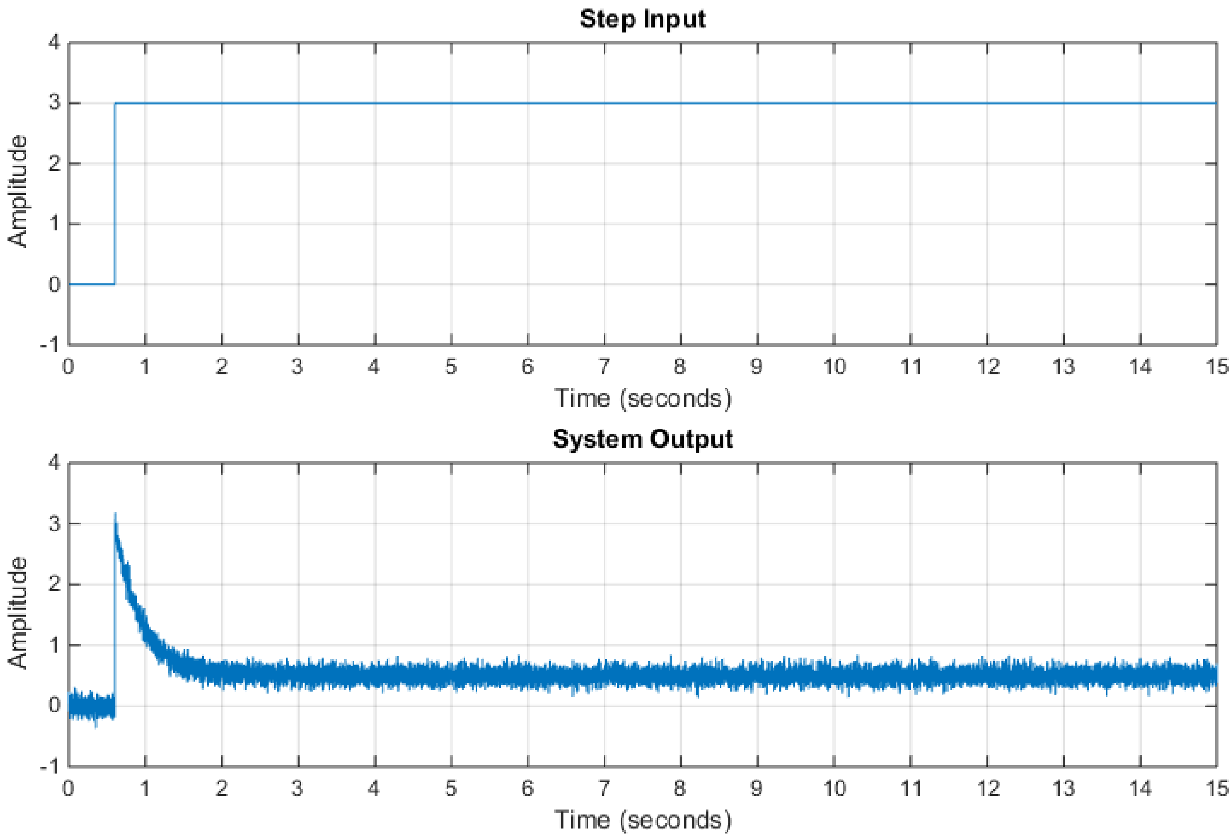

Figure 2: Step response experiment

Problem 3: Stochastic Processes (5+8 points)

a) For an unknown system a pseudo random binary signal (PRBS) is available. Figure 3 shows the autocorrelation of this signal. Up to which order is the signal persistent exiting?

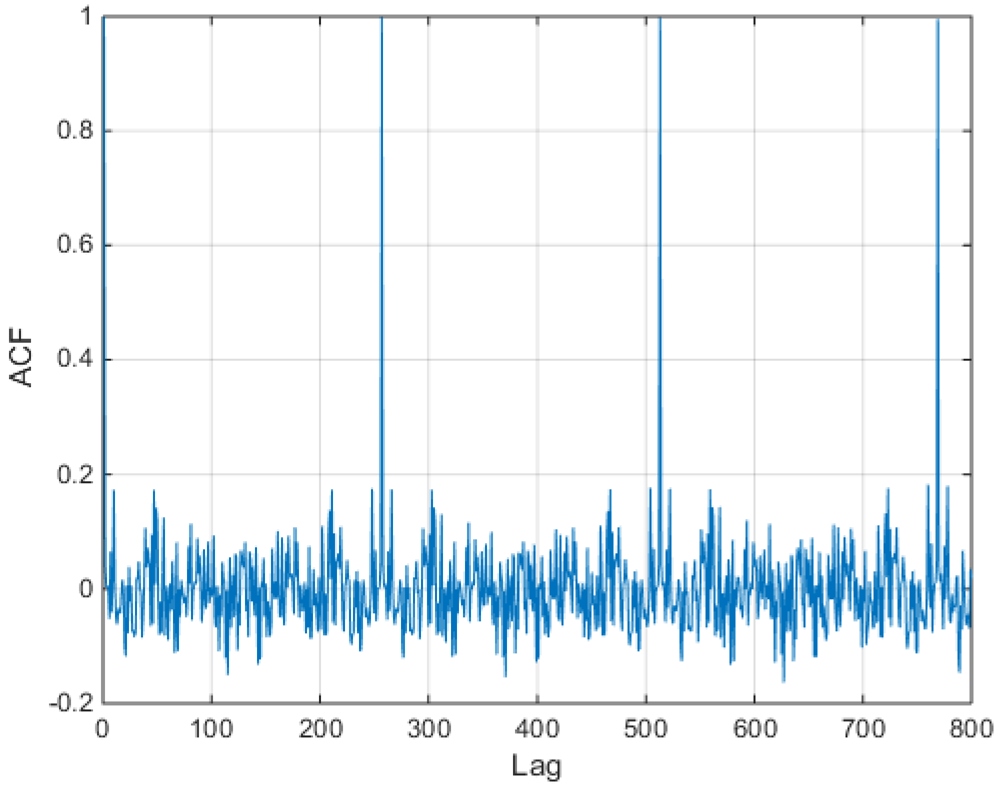

Figure 3: Autocorrelation of the PRBS signal

b) The measurement is subject to coloured noise with zero mean. Is a model identified with correlation analysis biased? (Prove your answer by calculation!)

Problem 4: Least Square Identification (8+4+5 points)

The unknown linear dynamic system is supposed to be approximated by a model in form of

$$y(k) = b_1 u(k-1) + b_2 u(k-2) + a_1 y(k-1) + v(k)$$

where y is the system output, u is the control input, $v(k) = 1 + w(k) + c_1 w(k-1)$, $w(k)$ is a white noise, a_1, b_1, b_2 and c_1 are unknown constants.

To identify the model parameters the system was excited and measurement data are given in Table 2.

k	1	2	3	4	5	6	7	8	9	10	11
u(k)	0	1	1	0	1	1	0	0	1	1	1
y(k)	1.11	1.22	1.82	-1.22	0.53	1.70	-1.12	0.90	1.23	1.87	-0.25

Table 1: Measurement data

a) Assume that the least-square approach is used to identify the parameters b_1, b_2, a_1 in the model. Show how these data can be used for least-square identification. For this purpose derive the equation and solve it for the parameters b_1, b_2, a_1 up to the point where you would need to invert the matrix.

b) What are the statistical properties of this least-square estimate?

c) A colleague suggests a model of higher order. Assume that the low order approximation is indeed correct. Which model will describe the input-output relation of the **identification data** with the lower error?

Problem 5: Closed Loop Identification (6+6 points)

An unstable system which is stabilized by an unknown controller should be identified

a) Which method could be used for direct identification and what are the problems which may arise?

b) Describe another approach which avoids these problems. Which characteristics of the signals are necessary assumptions for this approach?

Problem 6: Miscellaneous (2+6+6+8 points)

a) The unknown dynamic behaviour of a linear single-input single-output system (SISO) is supposed to be approximated by a state-space model (A, B, C, D). For this purpose the system was excited and measurement data were generated. The result of the singular value decomposition of $Y_N U_N^T$ was obtained as,

$$USV^T = U \cdot \begin{bmatrix} 86.63 & 0 & 0 & 0 & \cdots & 0 \\ 0 & 33.28 & 0 & 0 & \cdots & 0 \\ 0 & 0 & 6.701 \cdot 10^{-14} & 0 & \cdots & 0 \\ 0 & 0 & 0 & 5.831 \cdot 10^{-14} & \cdots & 0 \\ \vdots & \vdots & \vdots & \vdots & \ddots & 0 \\ 0 & 0 & 0 & 0 & 0 & 0 \end{bmatrix} \cdot V$$

where U and V are orthogonal matrices.

Determine the order of the unknown system.

b) Assume you use a prediction error based identification method and there is not enough data available to be split up into identification and validation data set. Describe a method how validation is still possible in this case!

c) Describe two approaches how to determine a time delay of a dynamic system.

d) Assume measurement noise is the realization of a stochastic process $A\sin(\omega t + \phi)$ where A and ϕ are random variables. A is uniformly distributed in the interval [-2V, 2V] and ϕ is uniformly distributed in the interval [0° ,180°]. State if the described process is wide sense stationary!

Hint: The parts of Problem 6 have no connection.