Examination

Modelling and Identification /

Modellbildung und Identifikation

Date:	18th March, 2016
Duration of examination:	120 Minutes
Points:	100

Please write legibly!

You must document how you obtain the answer to qualify for full credits.

This examination consists of **6 problems**. First, **check that your copy contains all 6 problems**.

Admitted aids: indelible pens ruler and means of drawing, non-programmable calculator. Having **non-admitted aids present** after the distribution of the examination tasks also constitutes as an attempt to cheat and leads to the non-approval of your examination.

Write your name and matriculation number on each page of the answer sheets including the cover sheet.

Use only the provided answer sheet for the answers. Only the answer sheets will be collected.

Write an explanation for all answers and give the approach for your calculations. The absence of an explanation or the approach has an influence on the assessment of the answer. Answers only consisting “Yes” or “No” will not gain points.

Problem 1: Theoretic Modelling

Consider the two-tank system of Figure 1. There are two inflow rates $Q_{E1}$and Q_{E2} with the temperatures $T_{E1} = 20°C$ and $T_{E2} = 30°C$, respectively. The first tank has cross section $A_1 = 10m^2$ and second tank $A_2 = 5m^2$. The cross section of the tube of between the tanks and the outflow tube are $A_{12} = A_{1O} = 0.5m^2$, respectively. The liquids are incompressible, with heat capacity $c = 4200[J/(kg \cdot K)]$ and density $\rho = 1000\ [kg/m^3]$. In the second tank a heating facility is installed. It can be assumed, that the liquids in the tanks are mixed perfectly.

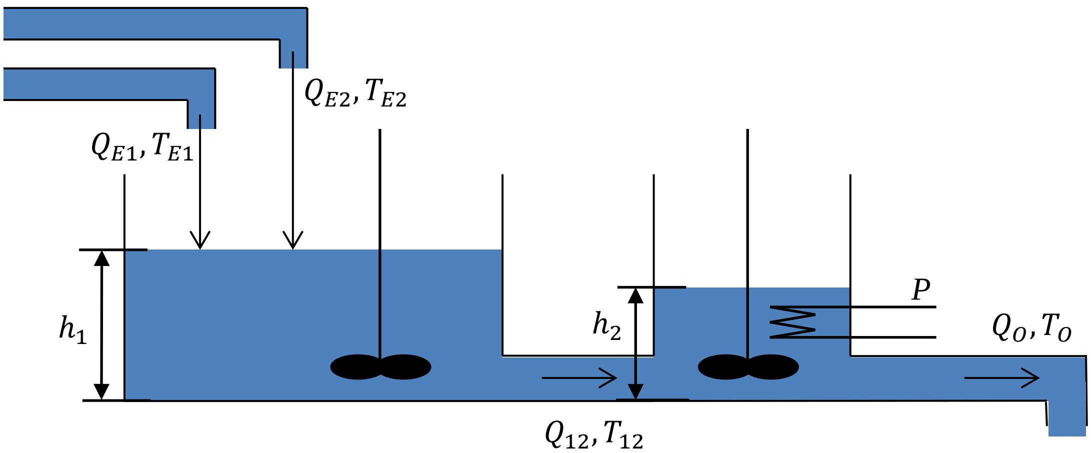

Figure 1: Two-Tank System

a) Set up differential equations for the level of the two tanks depending on the inflow rates.

b) Set up differential equations for the temperatures in each tank depending on the inflow rates and the heating power P.

c) Calculate the equilibrium points of the levels with $Q_{E1} = 1.5[m^3/s]$ and $Q_{E2} = 2{,}5[m^3/s]$ and linearize the equations of a) and b).

d) Derive a state-space model with inflow rates Q_{E1}, Q_{E2} and heating power P as inputs and levels h_1, h_2 and temperatures T_{12}, T_{2O} as outputs.

Torricelli's law: $\boldsymbol{Q_{ij} = a\, A_{ij}\, sgn(|h_i - h_j|)\sqrt{2g|h_i - h_j|}}$

with $\boldsymbol{a = 0.6}$ **and** $\boldsymbol{g = 9{,}81[N/kg]}$

Problem 2: Identification based on step response

a) Given the result of a step response experiment in Figure 2, what is an appropriate mathematical model of the unknown system?

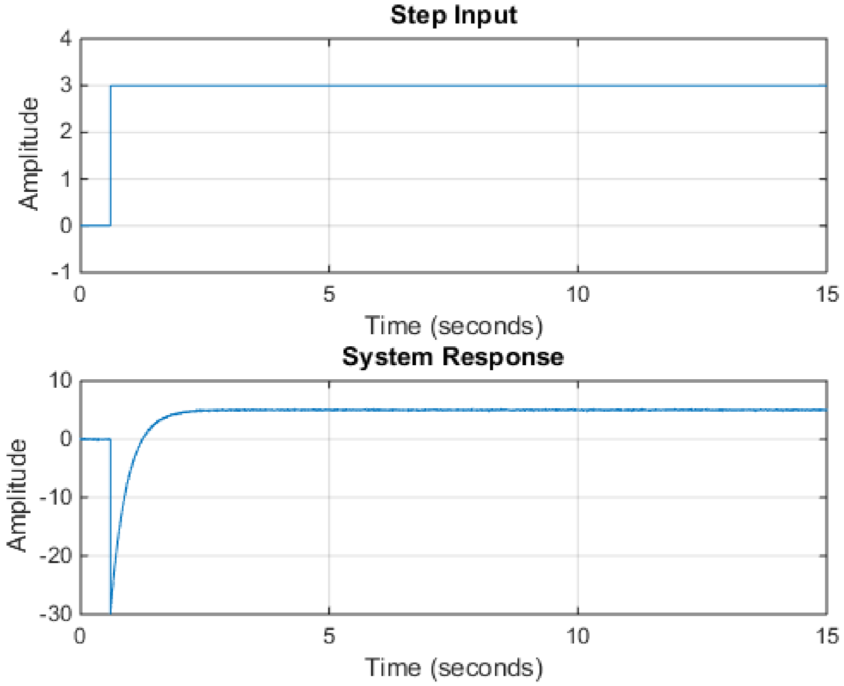

Figure 2: Step response experiment

b) Given the result of a step response experiment in Figure 3, determine the transfer function of the unknown system with the Schwarze approach. The same step response is given also on the answer sheets.

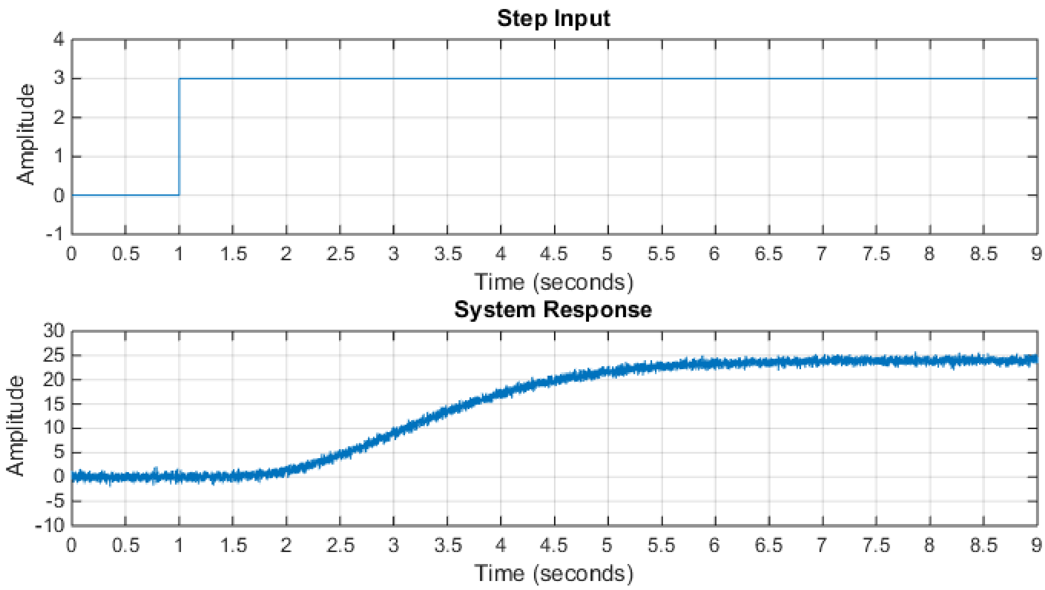

Figure 3: Step response experiment

Problem 3: Least Square Identification

The unknown linear dynamic system is supposed to be approximated by a model in form of

$$y(k) = b_1 u(k-1) + b_2 u(k-2) + a_1 y(k-1) + v(k)$$

where y is the system output, u is the control input, $v(k) = w(k) + c_1 w(k-1)$, $w(k)$ is a white noise, a_1, b_1, b_2 and c_1 are unknown constants.

To identify the model parameters the system was excited and measurement data are given in Table 2.

k	1	2	3	4	5	6	7	8	9	10
u(k)	0	1	1	0	1	1	0	0	1	1
y(k)	0.40	0.61	1.83	-0.82	1.22	1.97	-0.93	1.14	1.05	2.23

Table 1: Measurement data

a) Assume that the least-square approach is used to identify the parameters b_1, b_2, a_1 in the model. Show how these data can be used for least-square identification. For this purpose derive the equation and solve it for the parameters b_1, b_2, a_1 up to the point where you would need to invert the matrix.

b) Would the application of the Instrumental Variable Method be beneficial for this system?

c) What are in general the benefits and drawbacks of introducing a forgetting factor?

d) What happens if a Least Square Identification Method is applied to a system hold in its set point by a controller?

Problem 4: Prediction Error Method

The unknown linear dynamic behaviour of a system is supposed to be approximated as follows:

$$y(k) + a_2 y(k-2) = b_1 u(k-1) + b_2 u(k-2) + w(k) + c_1 w(k-1).$$

where y is the system output, u is the control input and $w(k)$ is white noise.

a) What is the optimal predictor for this system?

b) Formulate the optimisation problem for this system.

Problem 5: Miscellaneous

a) Assume you have the task of identification of a system and there is not enough data available to be split up into identification and validation data set. Describe a method how validation is still possible in this case!

b) A colleague suggests a model of higher order. Assume that the low order approximation is indeed correct. Which model will describe the input-output relation of the **identification data** with the lower error?

c) A system with unmeasured, stochastic input shall be identified. Which assumptions are necessary for identification of the system?

d) The influence of a stochastic Process on the product quality of a production plant is subject of interest. Only the spectral density of the stochastic process and the product quality are known. Describe a procedure to identify the transfer function. Which assumptions are necessary?

Hint: The parts of Problem 5 have no connection.