# Modelling and Identification


**Prof. Dr. Ping Zhang**

**Institute for Automatic Control**


**WS 2017/18**

# Organisation of this course

**Chapter 1**: Introduction

**Chapter 2**: Theoretical Modelling

**Chapter 3**: **Experimental modelling**

**Chapter 4**: Least-Squares methods

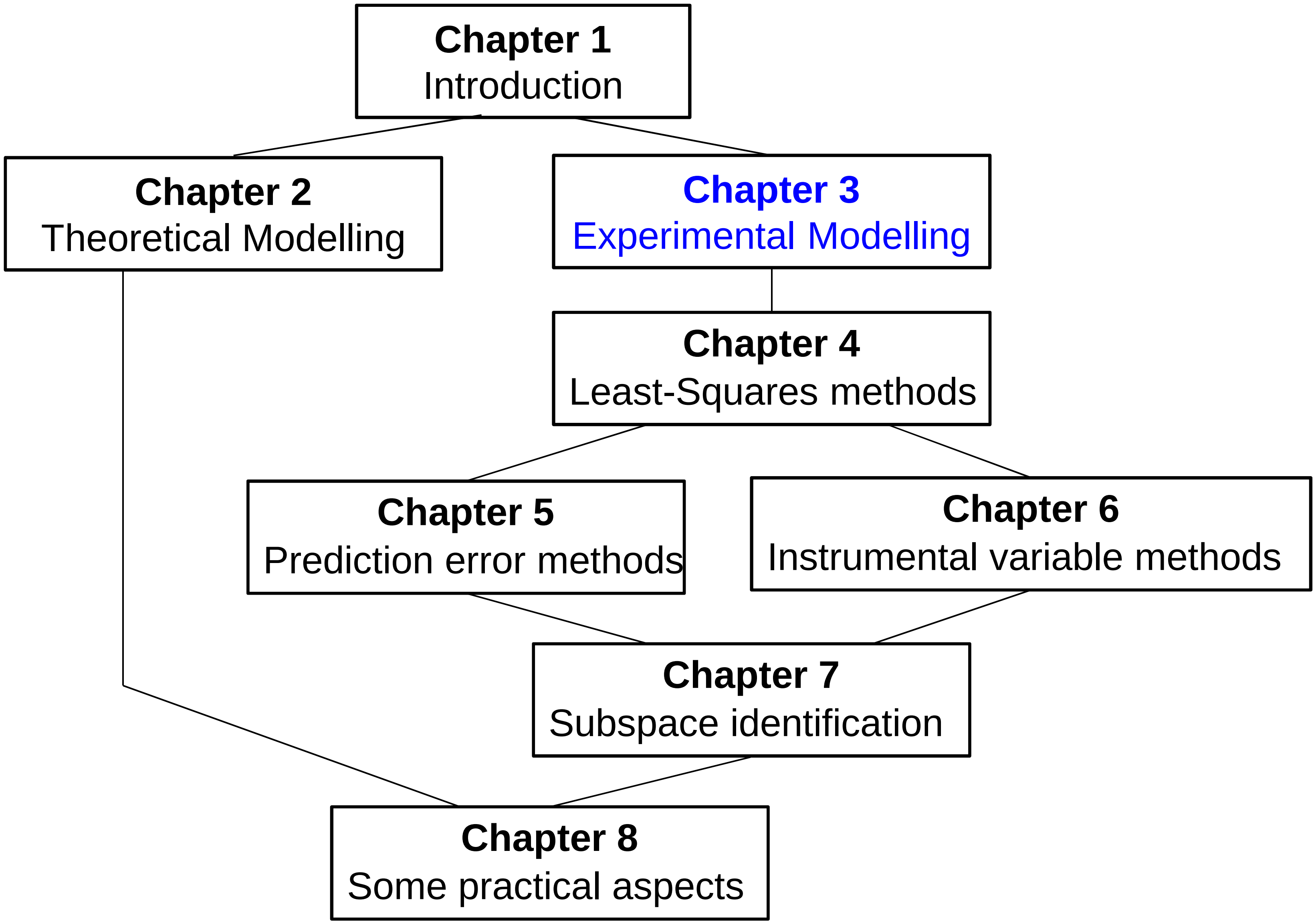
**Chapter 5**: Prediction error methods

**Chapter 6**: Instrumental variable methods

**Chapter 7**: Subspace identification methods (SS model!)

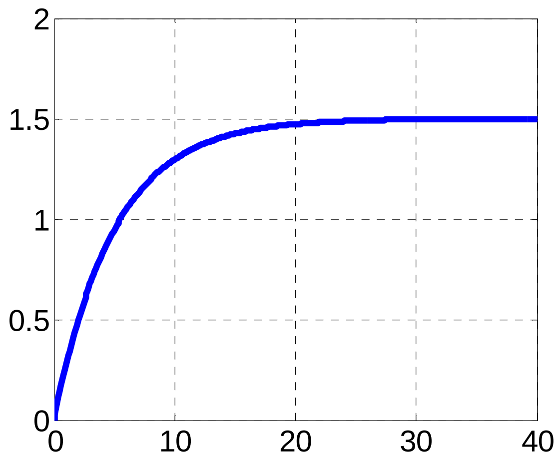
**Chapter 8**: Some practical aspects

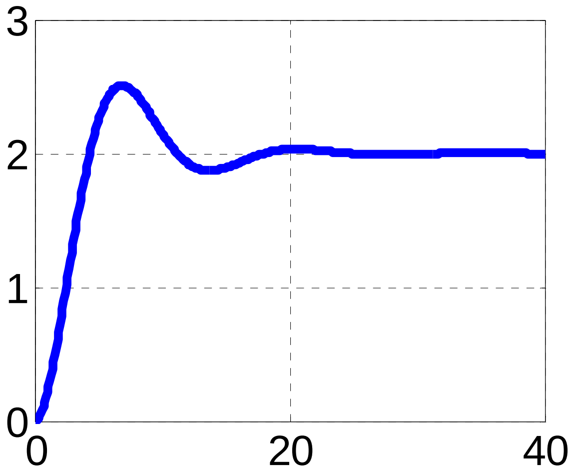
# Organisation of this course

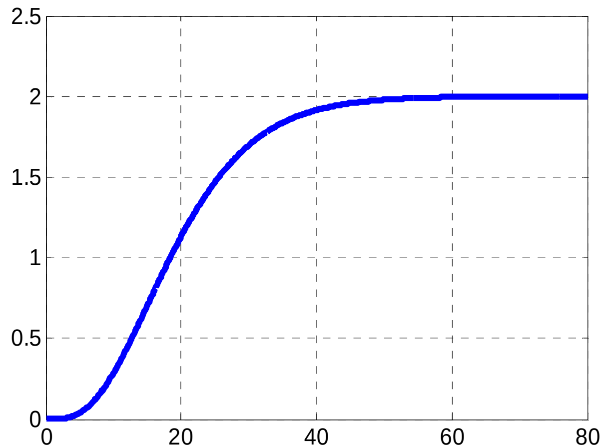


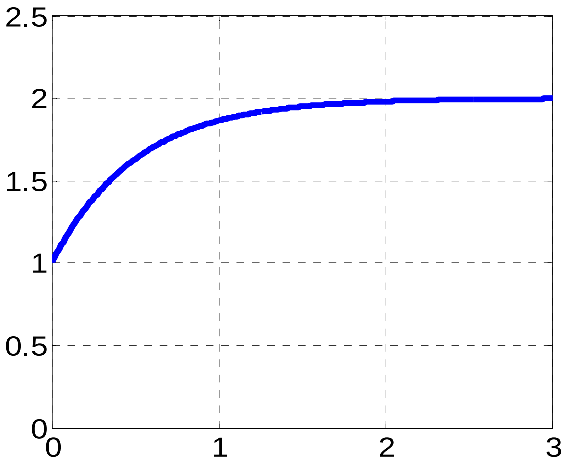
# Get transfer function from system response

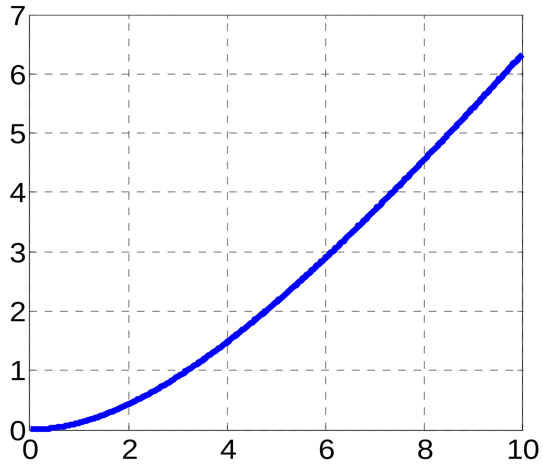
For **simple systems**, it is sometimes possible to read the parameters of transfer functions **directly** from system response to a test signal.

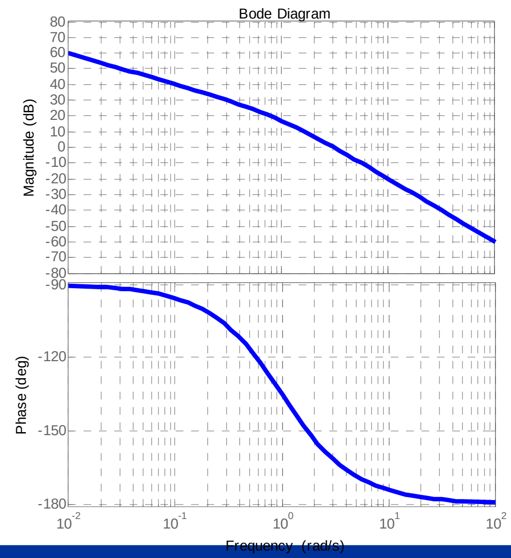


# Step response of 1. order system

Given a first order system described by

$$G(s) = \frac{K}{Ts+1}$$

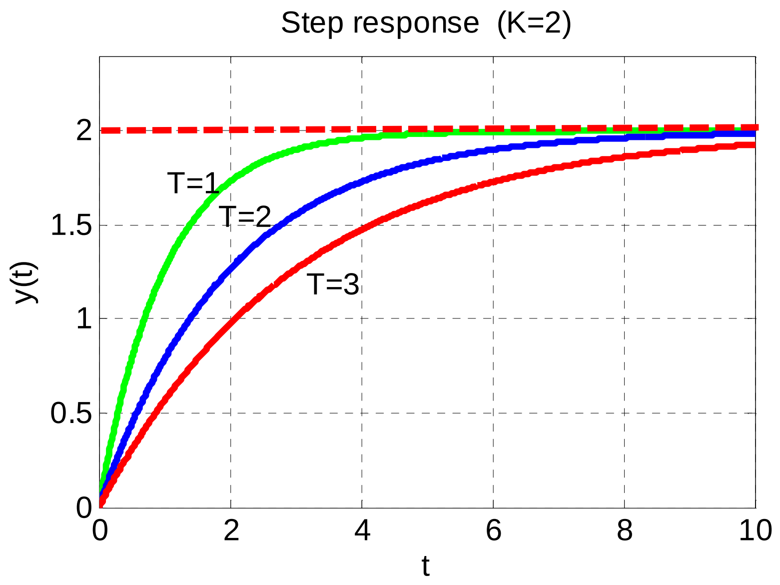
Under the control input signal $u(t) = a\sigma(t), a > 0$, the system output is

$$y(s) = \frac{K}{Ts+1}\frac{a}{s} = aK\left(\frac{1}{s} - \frac{1}{s+\frac{1}{T}}\right)$$

$$\Downarrow$$

$$y(t) = aK\left(1 - e^{-\frac{1}{T}t}\right)$$

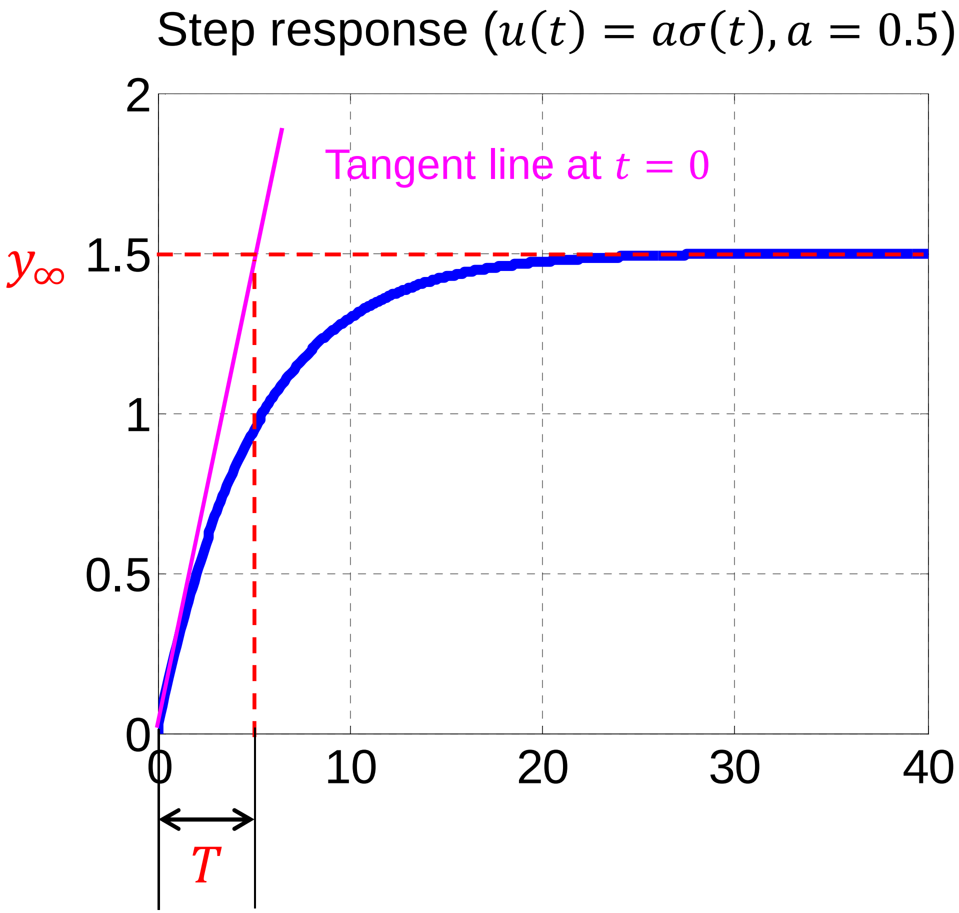
$$\frac{dy}{dt}|_{t=0} = \frac{aK}{T}$$



# Step response approach – 1. order system

**Assume that the system is approximated by a first order system**

$$G(s) = \frac{K}{Ts+1}$$




**Read characteristic values:**

- final value $y_\infty$
- time $T$

# Step response approach – 1. order system

## Identification procedure:

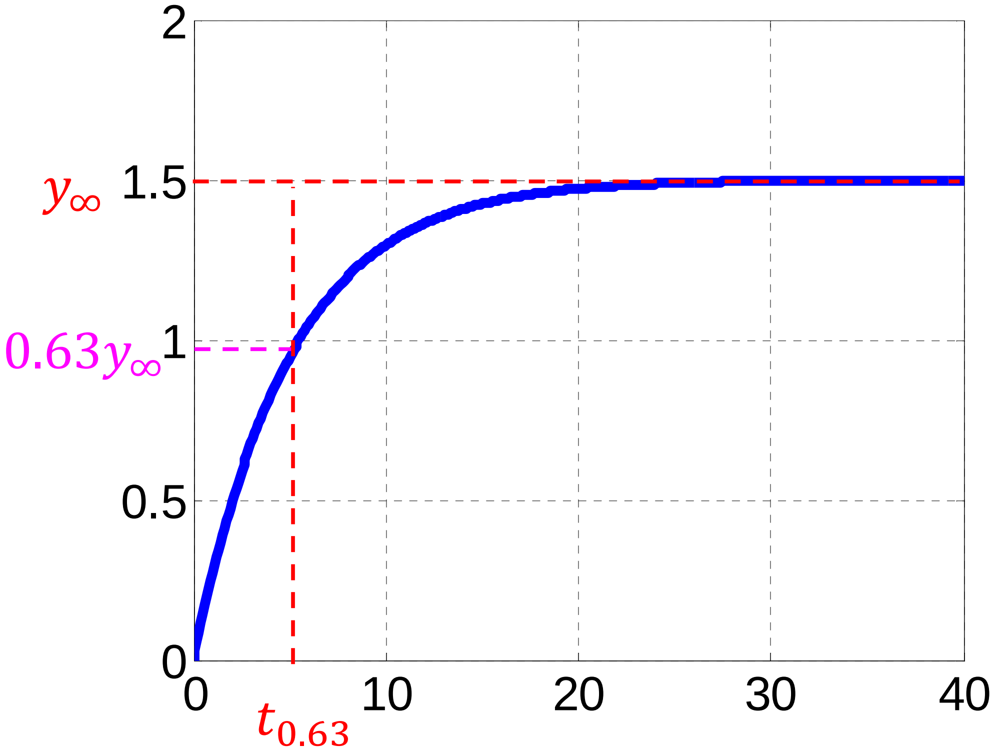
1. Calculate the gain $K = \frac{y_\infty}{a}$.
2. Draw the tangent line at $t = 0$.
3. Read $T$

# Step response approach – 1. order system

**Assume that the system is approximated by a first order system**

$$G(s) = \frac{K}{Ts+1}$$

Step response ($u(t) = a\sigma(t), a = 0.5$)




**Read characteristic values:**

- final value $y_\infty$
- time $t_{0.63}$: the time at which the output reaches 63% of the final value

# Step response approach – 1. order system

## Identification procedure:

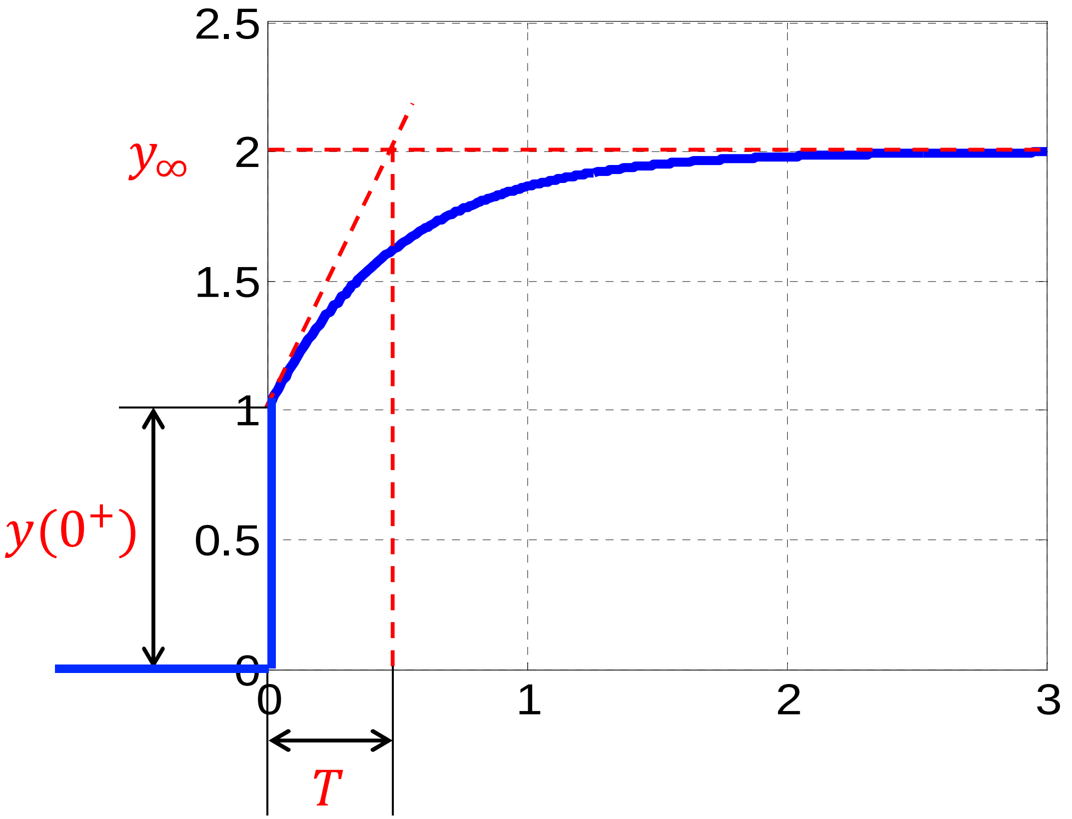
1. Calculate the gain $K = \frac{y_\infty}{a}$.
2. Read time $t_{0.63}$, i.e. the time at which the output reaches 63% of the final value
3. Get $T = t_{0.63}$.

# Step response approach – systems with derivative action

## Approximation by a first order system with derivative action

$$G(s) = \frac{K(1 + T_D s)}{1 + Ts}$$

Step response ($u(t) = a\sigma(t), a = 0.5$)




**Read characteristic values:**

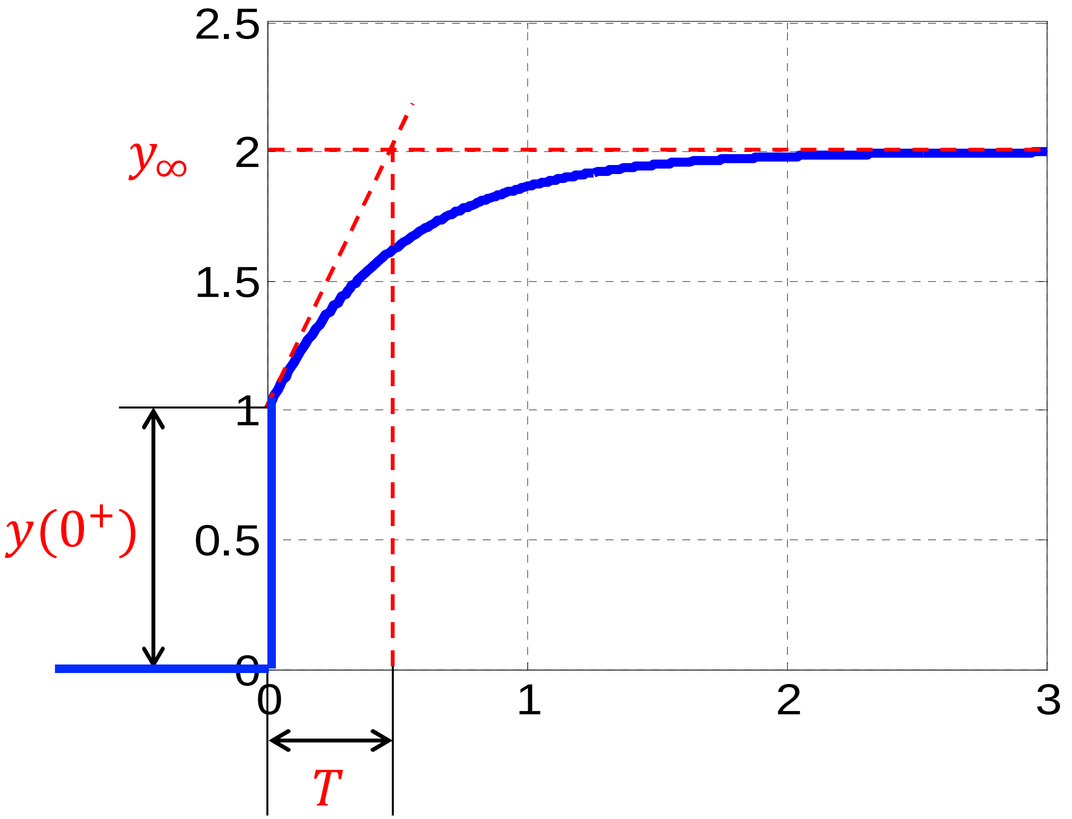
$y(\infty)$
$y(0^+)$
$T$

# Step response approach – systems with derivative action

## Approximation by a first order system with derivative action

$$G(s) = \frac{K(1 + T_D s)}{1 + Ts}$$

Step response ($u(t) = a\sigma(t), a = 0.5$)




According to the initial value theorem,

$$\lim_{t \to 0^+} y(t) = \lim_{s \to \infty} sY(s)$$
$$= \lim_{s \to \infty} K \frac{T_D}{T} a$$

$$\Rightarrow \quad T_D = \frac{y(0^+)T}{Ka}$$

# Step response approach – systems with derivative action

**Identification procedure:**

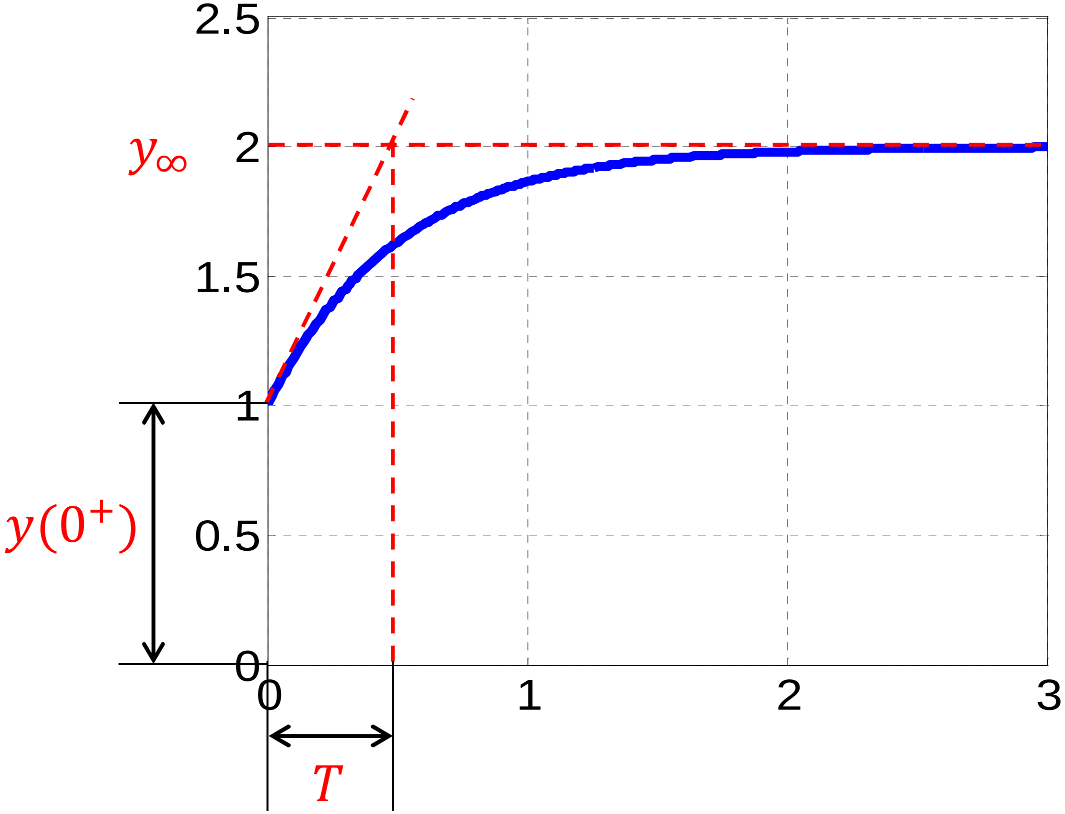
1. Calculate the gain $K = \frac{y_\infty}{a}$.
2. Draw the tangent at $t = 0$.
3. Read $T$ and $y(0^+)$.
4. Calculate $T_D = \frac{y(0^+)T}{Ka}$.

# Step response approach – systems with derivative action

**Example 1:** Approximate the system with the following step response by

$$G(s) = \frac{K(1 + T_D s)}{1 + Ts}$$

Step response ($u(t) = a\sigma(t), a = 0.5$)




$y_\infty = 2 \Rightarrow K = \frac{y_\infty}{a} = 4$

$T = 0.5$
$y(0^+) = 1$ $\Rightarrow$

$$T_D = \frac{y(0^+)T}{Ka} = 0.25$$

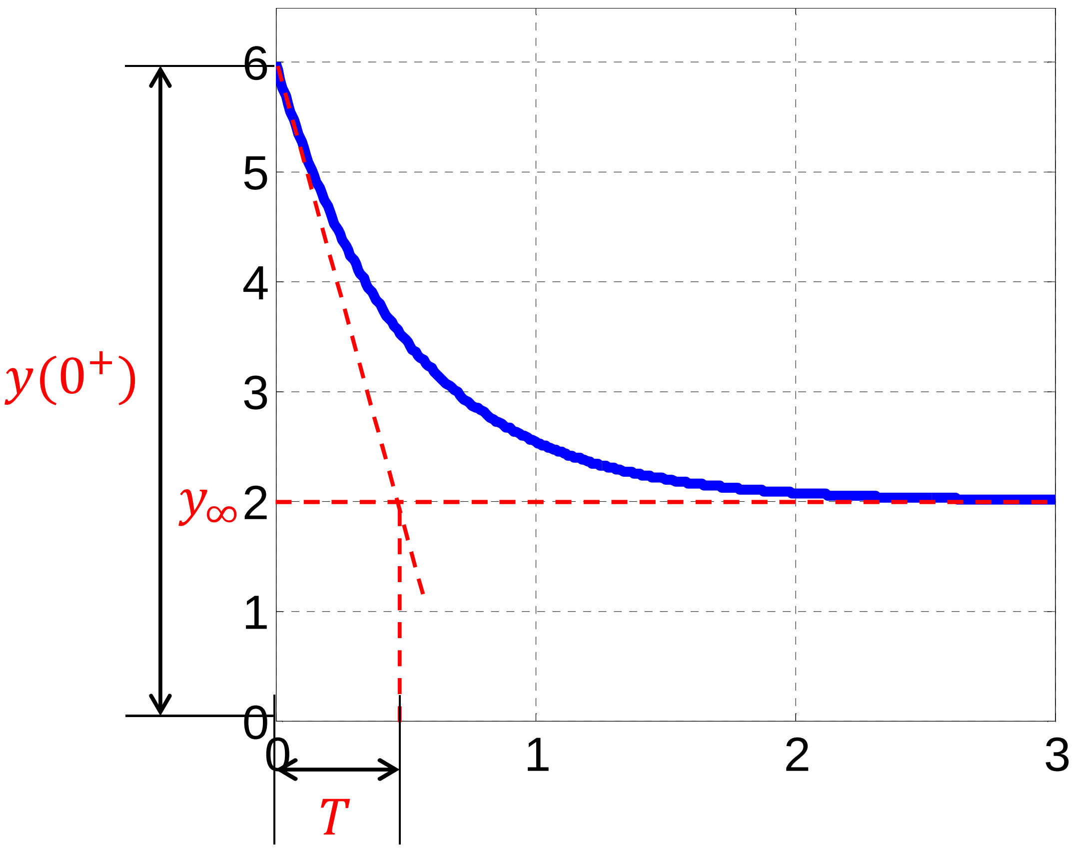
$$\boxed{G(s) = \frac{4(1 + 0.25s)}{1 + 0.5s}}$$

# Step response approach – systems with derivative action

**Example 2:** Approximate the system with the following step response by

$$G(s) = \frac{K(1 + T_D s)}{1 + Ts}$$

Step response ($u(t) = a\sigma(t), a = 0.5$)




$y_\infty = 2$ ⇨ $K = \frac{y_\infty}{a} = 4$

$T = 0.5$
$y(0^+) = 6$ ⇨

$$T_D = \frac{y(0^+)T}{Ka} = 1.5$$

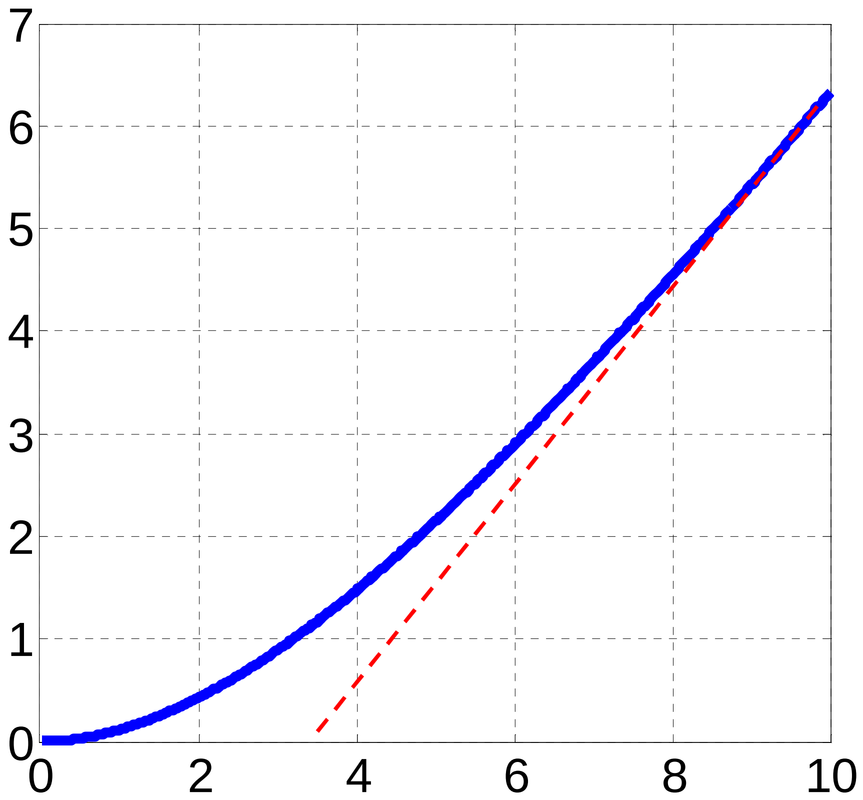
$$\boxed{G(s) = \frac{4(1 + 1.5s)}{1 + 0.5s}}$$

# Step response approach – systems with integral action

## Approximation by a $n$-th system with integral action

$$G(s) = \frac{K}{s(1+Ts)}$$

Step response ($u(t) = a\sigma(t), a = 0.5$)




$$\frac{dy}{dt} = Ka$$

$$\Downarrow$$

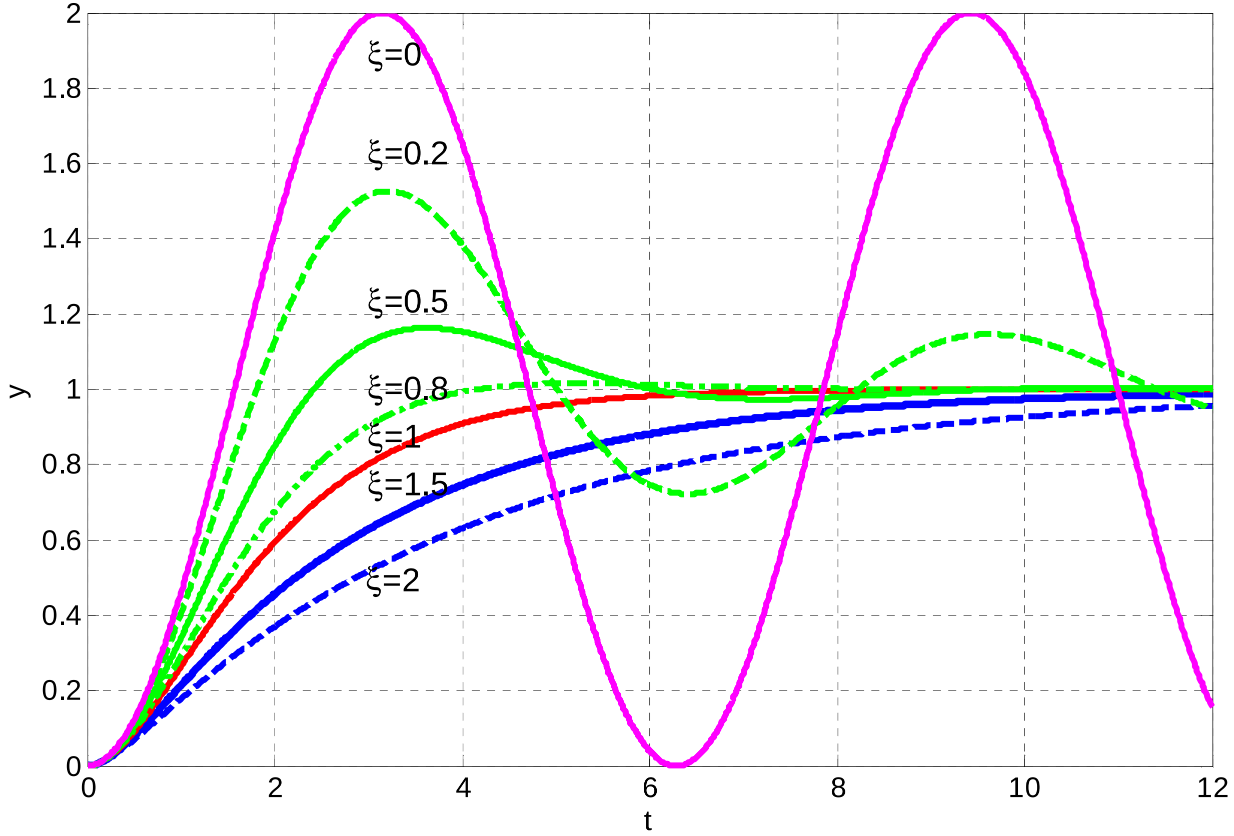
$$K = \frac{\frac{dy}{dt}}{a}$$

# Step response of 2. order system

Given a second order system described by

$$G(s) = \frac{K{\omega_n}^2}{s^2 + 2\xi\omega_n s + {\omega_n}^2}$$

Step response ($u(t) = a\sigma(t), a = 1, K = 2, \omega_n = 1$)



# Step response of 2. order system

Step response of the **underdamped system** $(\mathbf{0} < \xi < \mathbf{1})$

**Final value:**

$$y_\infty = Ka$$

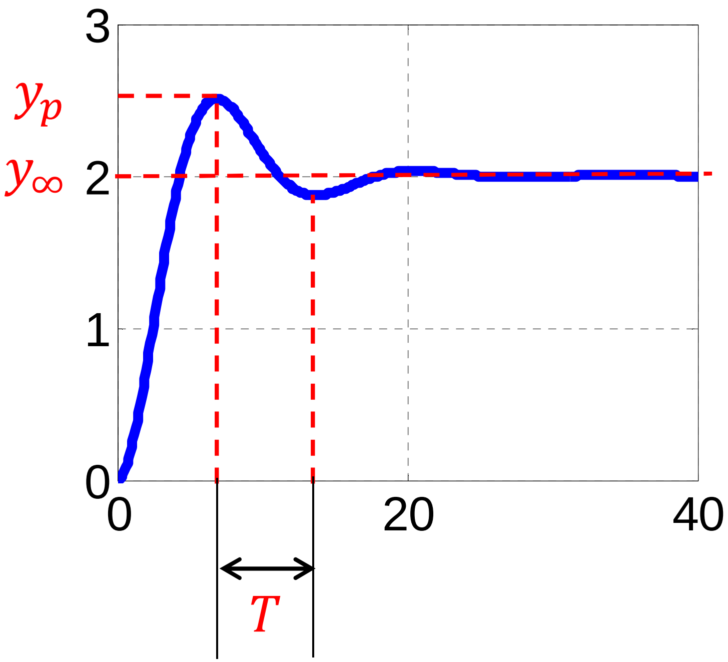
**Half period of the oscillation:**

$$T = \frac{\pi}{\omega_n\sqrt{1-\xi^2}}$$

**Overshoot:**

$$M_p = \frac{y_p - y_\infty}{y_\infty} = e^{\frac{-\xi\pi}{\sqrt{1-\xi^2}}} \times 100\%$$

Step response $(u(t) = a\sigma(t))$



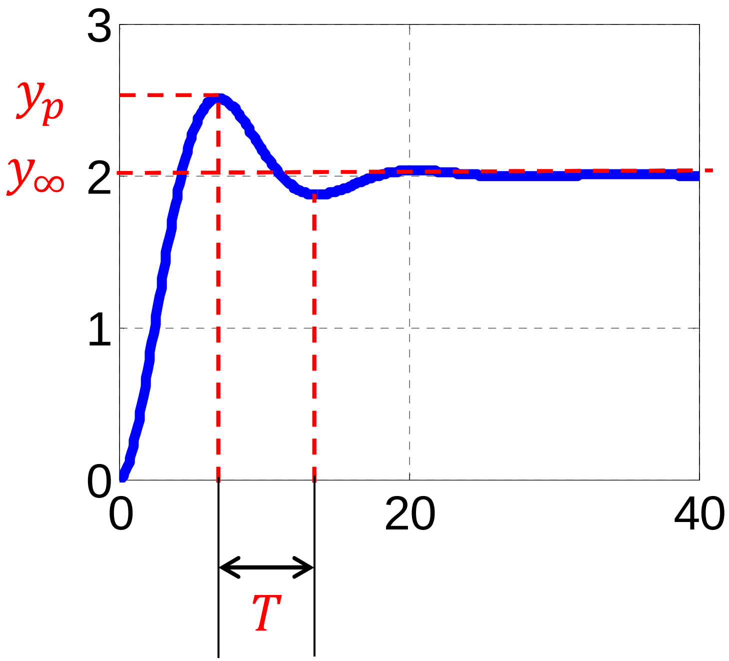

$$y(t) = aK\left(1 - \frac{e^{-\xi\omega_n t}}{\sqrt{1-\xi^2}}\sin(\sqrt{1-\xi^2}\,\omega_n t + \arccos\xi\right)$$

# Step response approach – 2. order system

**Assume that the system is approximated by a second order system**

$$G(s) = \frac{K{\omega_n}^2}{s^2 + 2\xi\omega_n s + {\omega_n}^2}$$

Step response ($u(t) = a\sigma(t)$)




**Read characteristic values:**

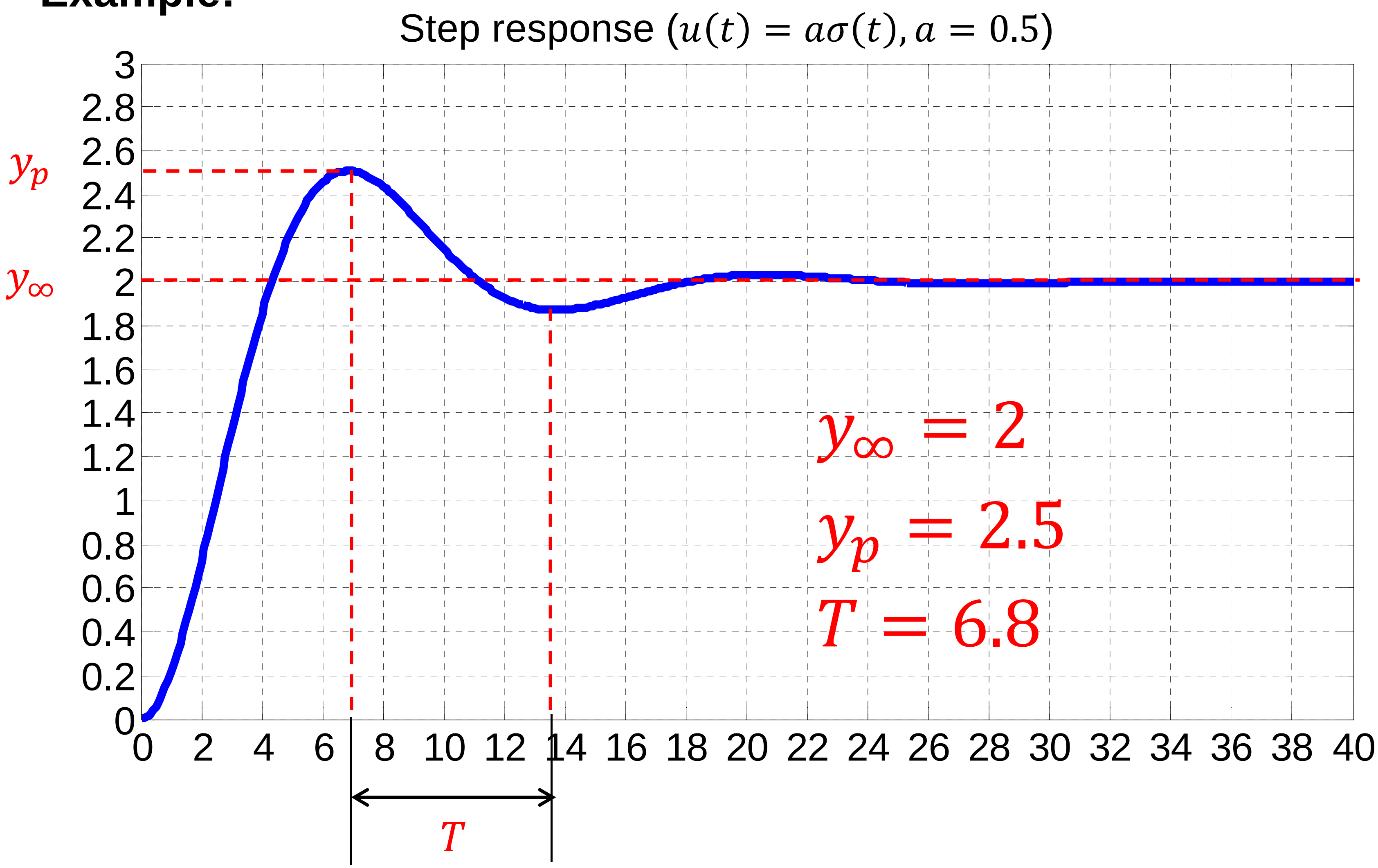
- final value $y_\infty$
- peak value $y_p$
- time $T$

# Step response approach – 2. order system

**Identification procedure:**

1. Calculate the gain $K = \frac{y_\infty}{a}$.
2. Read the overshoot $M_p$ and the period $T$ from the step response
3. Calculate the damping ratio $\xi$ based on $M_p$.
4. Calculate $\omega_n$ based on $T$ and $\xi$.

**Example:**



# Step response approach – 2. order system

$$y_\infty = 2 \Rightarrow K = \frac{y_\infty}{a} = 4$$

$$y_\infty = 2,\ y_p = 2.5 \Rightarrow M_p = \frac{y_p - y_\infty}{y_\infty} = 25\%$$

$$\Rightarrow \xi = \frac{1}{\sqrt{1 + \left(\frac{\pi}{\ln M_p}\right)^2}} = 0.4037$$

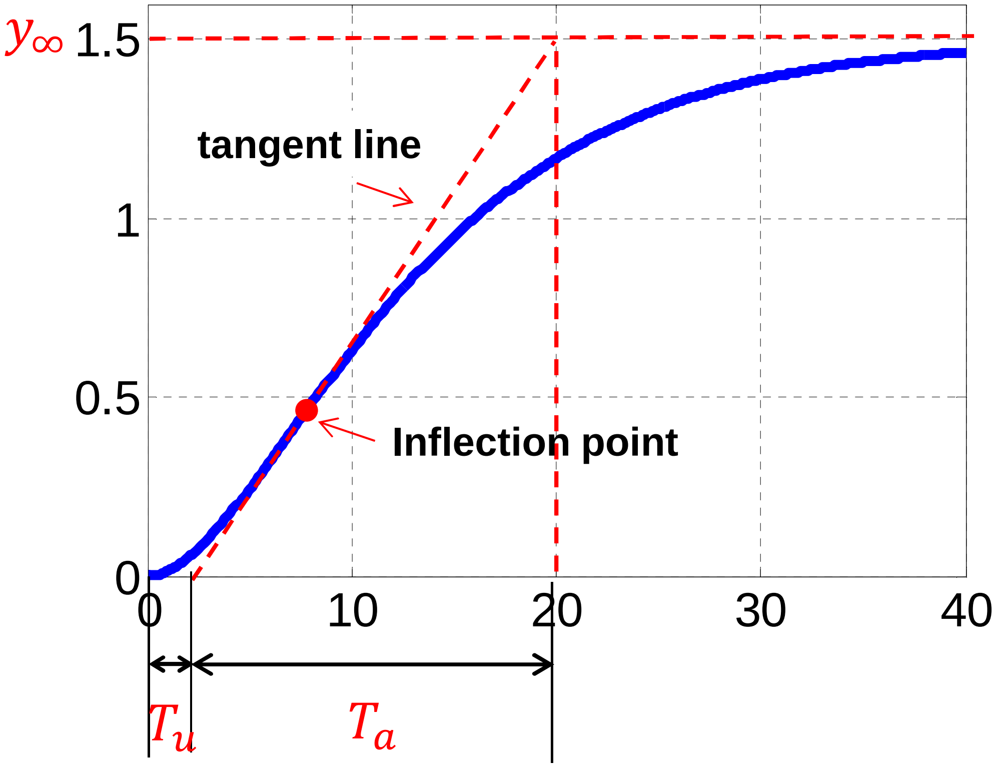
$$T = 6.8 \Rightarrow \omega_n = \frac{\pi}{T\sqrt{1-\xi^2}} = 0.5050$$

$$G(s) = \frac{4 \times 0.5050^2}{s^2 + 2 \times 0.4037 \times 0.5050 s + 0.5050^2}$$

# Review: Step response of 2. order system

Step response of the **overdamped system** ($\xi > 1$)

$$\rightarrow \boxed{\frac{K}{T_1 s + 1}} \rightarrow \boxed{\frac{1}{T_2 s + 1}} \rightarrow \qquad \xi > 1: \quad T_1 < T_2$$




$$T_a = T_1 \left( \frac{T_2}{T_1} \right)^{\frac{T_2}{T_2 - T_1}}$$

$$T_u = \frac{T_1 T_2}{T_2 - T_1} \ln \frac{T_2}{T_1} - T_a + T_1 + T_2$$

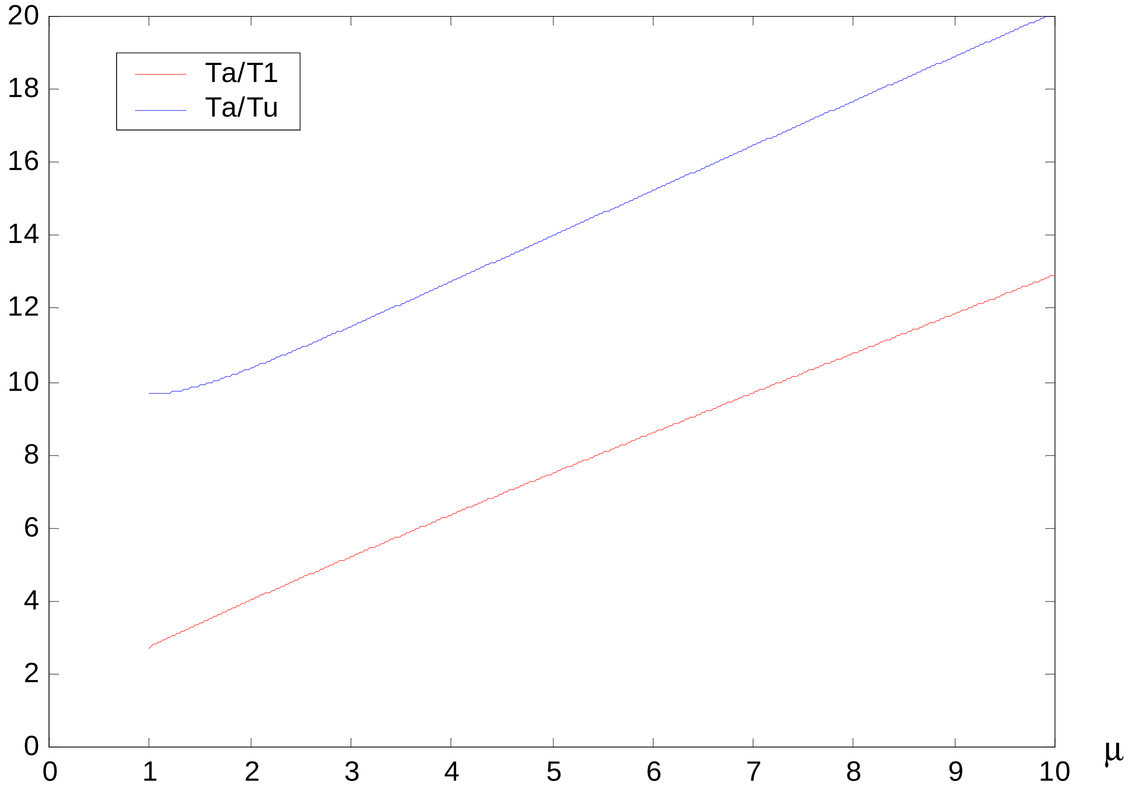
$$\Downarrow \qquad \mu = \frac{T_2}{T_1}$$

$$\frac{T_a}{T_1} = \mu^{\frac{\mu}{\mu - 1}}$$

$$\frac{T_a}{T_u} = \frac{1}{\mu^{\frac{-\mu}{\mu-1}} \left( 1 + \mu + \frac{\mu}{\mu - 1} \ln \mu \right) - 1}$$

# Step response approach – 2. order system

**Key of identification:** Nomogram for the calculation of $T_1, T_2$



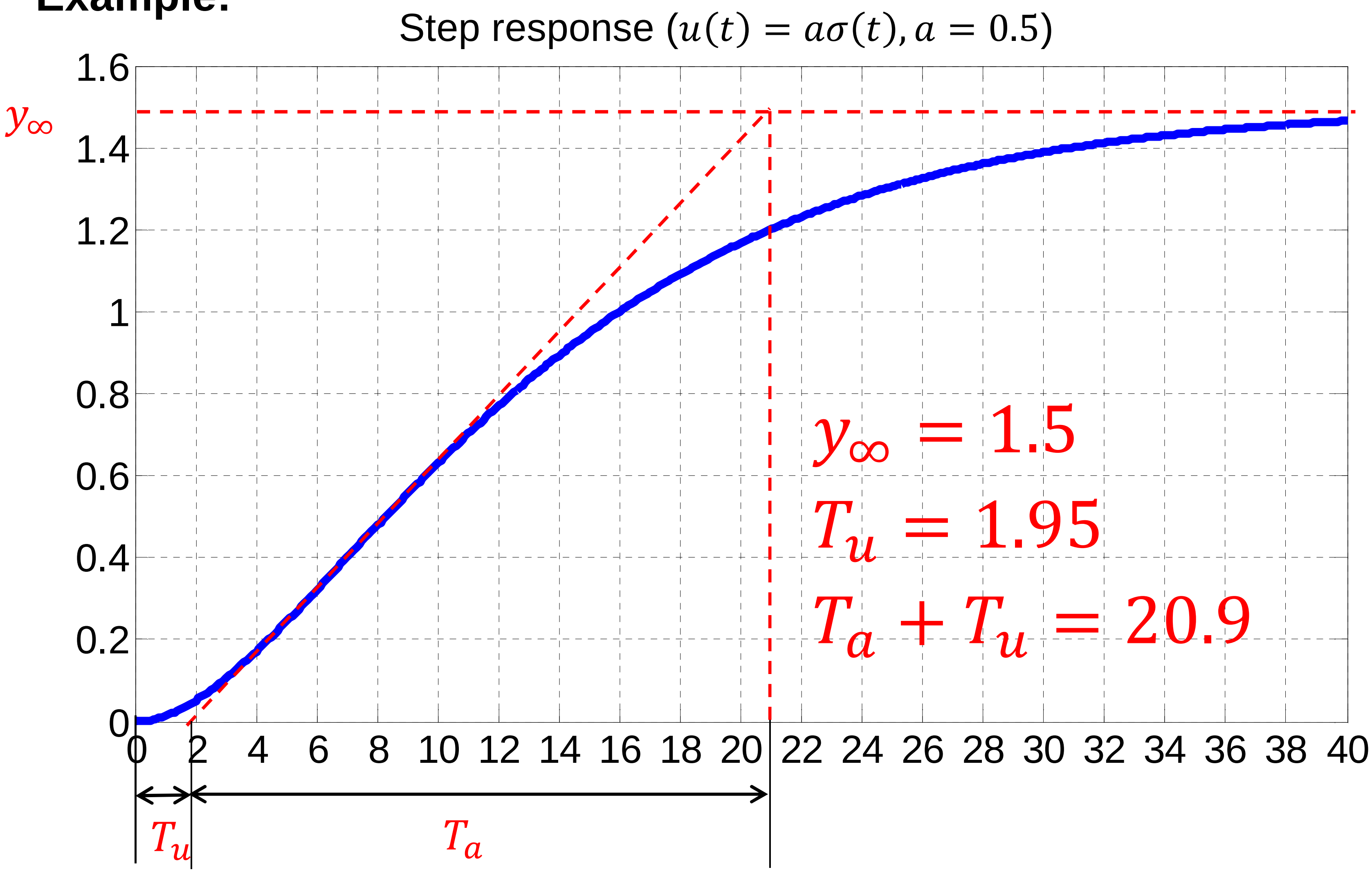
# Step response approach – 2. order system

**Identification procedure:**

1. Calculate the gain $K = \frac{y_\infty}{a}$.
2. Determine the **inflection point** and the **tangent line** in the step reponse
3. Read $T_u$ and $T_a$
4. Determine $\mu$ based on $\frac{T_a}{T_u}$ according to the Nomogram (see the blue curve)
5. Determine $T_1$ according to the Nomogram (see the red curve)
6. Calculate $T_2 = \mu T_1$.

# Step response approach – 2. order system

**Example:**



# Step response approach – 2. order system

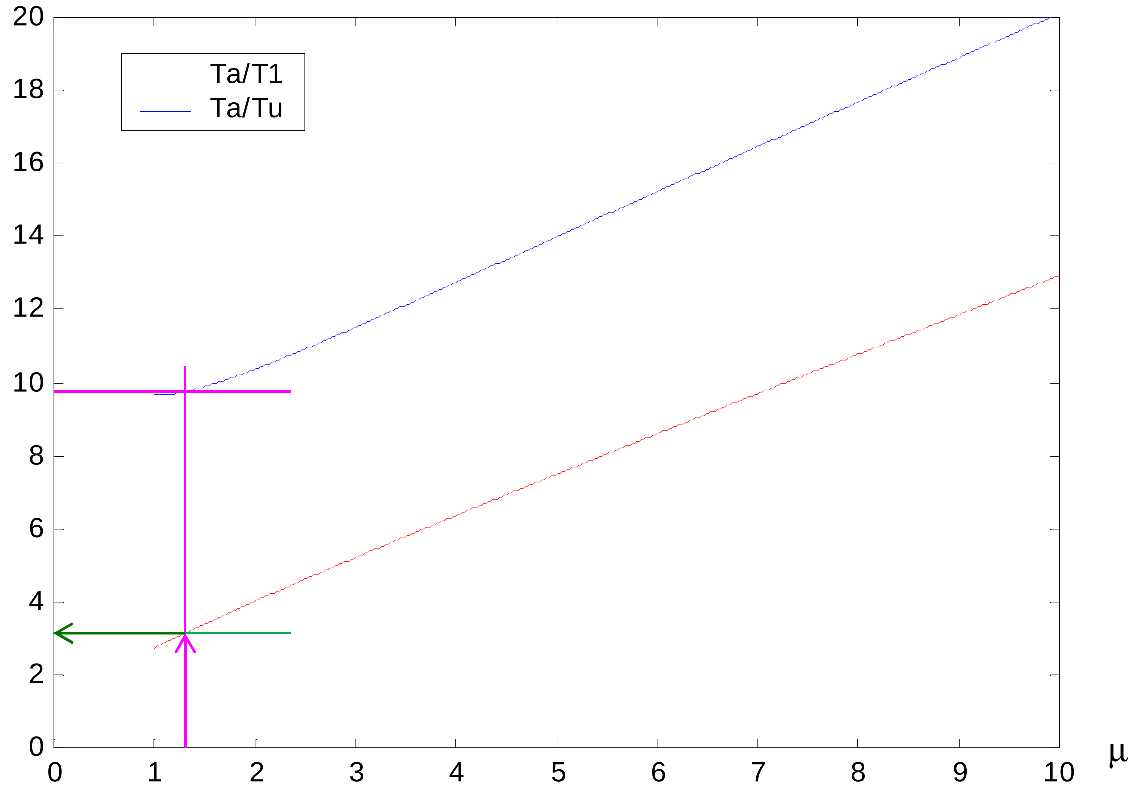
$y_\infty = 1.5 \Rightarrow K = \frac{y_\infty}{a} = 3$

$T_u = 1.95, \quad T_a = 18.95$

$$\frac{T_a}{T_u} = 9.72$$

$$\mu = 1.3$$

$$\frac{T_a}{T_1} = 3.2$$



# Step response approach – 2. order system

$y_\infty = 1.5 \Rightarrow K = \dfrac{y_\infty}{a} = 3$

$T_u = 1.95, \quad T_a = 18.95$

$\Downarrow$

$$\frac{T_a}{T_u} = 9.72$$

$\Downarrow$

$$\mu = 1.3$$

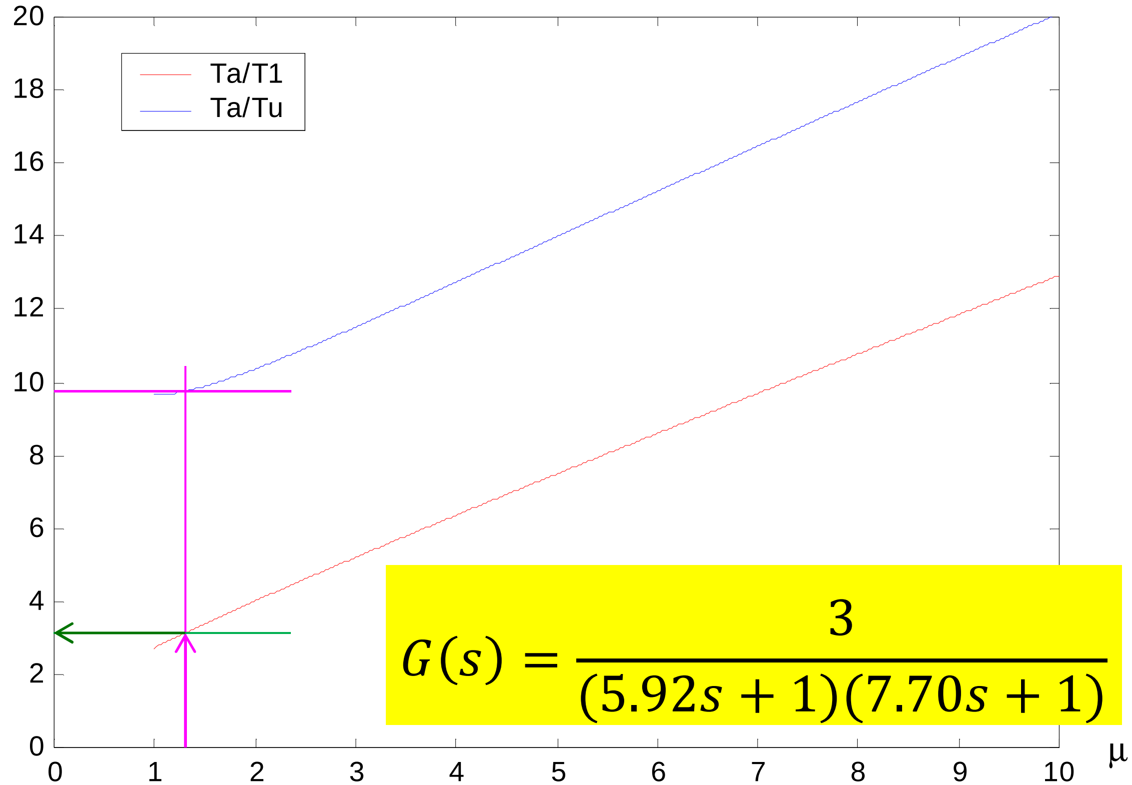
$\Downarrow$

$$\frac{T_a}{T_1} = 3.2$$

$\Downarrow$

$$T_1 = \frac{T_a}{3.2} = 5.92$$

$$T_2 = \mu T_1 = 7.70$$




$$G(s) = \frac{3}{(5.92s + 1)(7.70s + 1)}$$