Exercise 1 Theoretic Modelling

Task 1 Mechanic System

Given is a mechanic system with 2 gyrating masses, one rotational damper, one torsional spring, as well as one mass, one spring and one damper. The system transfers rotational movement into translational movement.

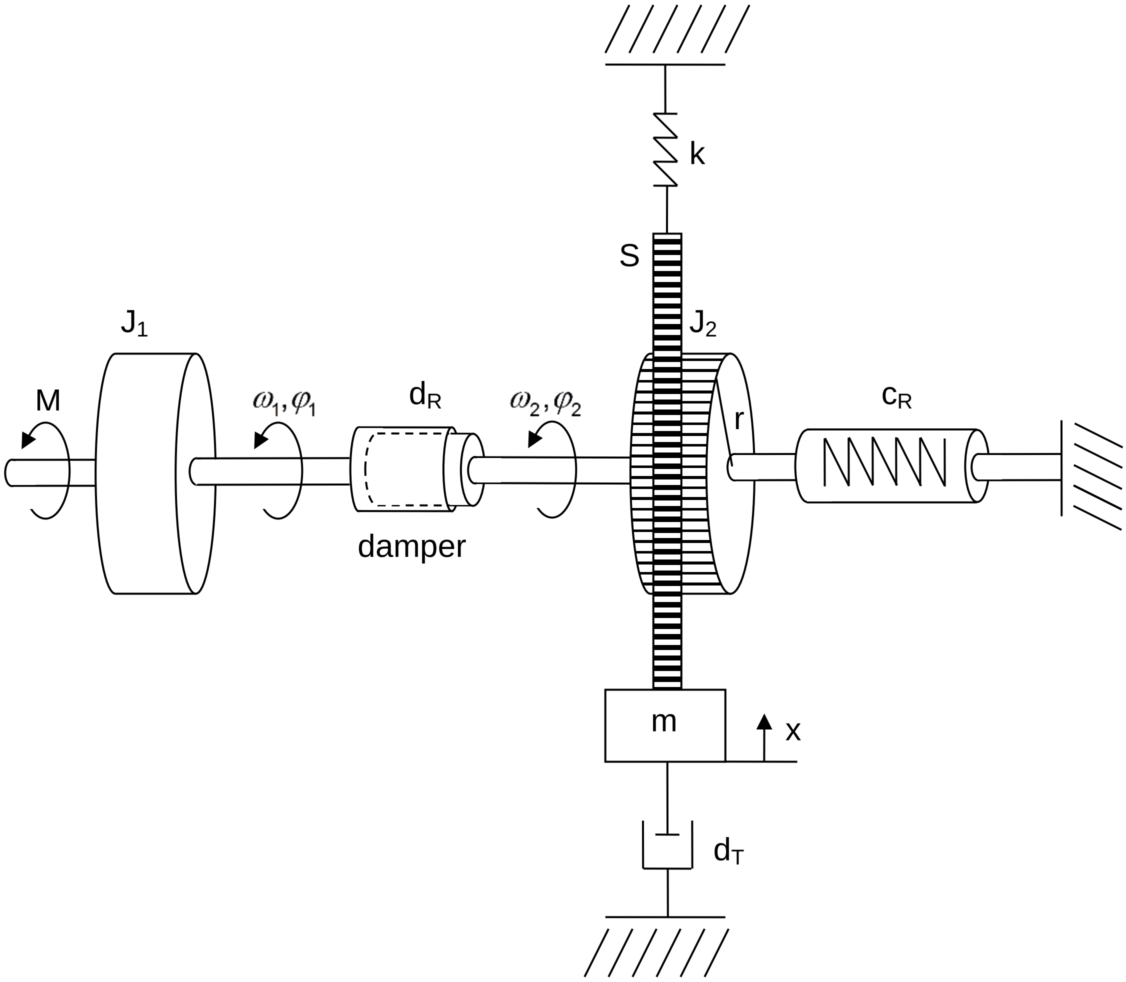


Image 1: *mechanic system*

Simplifying Assumptions:

- All axles are massless and inflexible.
- The damper transmits torque proportional to the difference in revolution speed.

The outer torque M is connected to the gyrating mass J_1. The gyrating mass J_2 has the radius r and a gear ring. S is a massless gear rod which can be moved in vertical direction. The movement of the mass m relative to the position of rest is x. In the position of rest $\varphi_1 = \varphi_2 = 0$ holds.

a) State the balance of angular momentum for gyrating mass J_1! Specify the time derivative $d\omega_1/dt$ as function of M, ω_1, ω_2 and correspondent constants.

b) State the balance of angular momentum for gyrating mass J_2! Introduce therefor the internal force F_S, which acts at the contact point between gear ring of J_2 and gear rod of S. With this state the time derivative $d\omega_2/dt$ as function of ω_1, ω_2, φ_2 and F_S plus correspondent constants.

c) State the balance of impulses for mass m. Introduce the speed $v = dx/dt$ and state the time derivative dv/dt as function of F_S, v, x and correspondent constants.

d) What are the relationship between the displacement x and the angle of rotation φ_2, respectively the speed v and the angular velocity ω_2?

e) State the time derivative $d\omega_2/dt$ as function of ω_1, ω_2, φ_2 and correspondent constants without F_S. Use for this the results of b), c) and d).

Task 2 Mechanical System

Given is following „inverted pendulum“

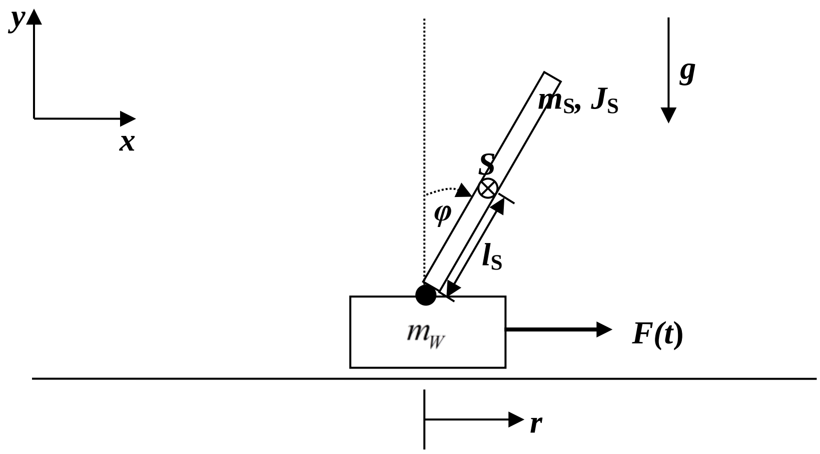


Image 2: *inverted pendulum*

$F(t)$	outer force
r	displacement of the cart out of the setting point
m_W	mass of the cart
l_S	distance between focus S to the bearing
m_S	mass of the bar
J_S	mass moment of inertia of the bar relative to the focus S of the bar
φ	angle of the pendulum relative to vertical

In the mathematical modelling the dynamic behaviour of the pendulum shall be replicated. Following assumptions hold:

- The cart only moves horizontal (x-direction).
- The bar is pivot-mounted at the cart.
- The bar is homogenous which means the focus is in the middle.
- There is no friction.

a) Separate the complete system into 2 subsystems "cart" and "bar". Draw all active forces. Separate the force which is active in the support point into two components. These forces F_X und F_Y, shall act in x- and y-direction.

b) State the balance of impulse for the subsystem „cart“ in x-direction.

c) State the balance of impulse for the subsystem „bar“ in x- and y-direction. Hint: Consider for the balance of impulse just the movement of the focus *S*. The speed of the focus in x-direction is v_X, in y-direction v_Y!

d) State the balance of angular momentum for the rotation around the focus *S* of the bar.

e) Derive the differential equation $\ddot{\varphi} = f(\varphi, \ddot{r})$ for the angle of the pendulum. Hint: Replace v_X and v_Y with functions of angle φ.

Task 3 Thermal System

q_A	*mass inflow material A*
$q_{A\,ab}$	*mass outflow material A*
q_B	*mass inflow material B*
$q_{B\,ab}$	*mass outflow material B*
q_{ab}	*total mass outflow*
c_A	*specific heat capacity of material A*
c_B	*specific heat capacity of material B*
T_A	*absolute temperature of material A*
T_B	*absolute temperature of material B*
m_A	*mass of A in tank*
m_B	*mass of B in tank*
m_{ges}	*total mass of A and B in tank*
E_A, R, k_R	*constants*
T	*mixing temperature in the tank*
g	*resulting free specific energy*

Given the tank with den material A and B:

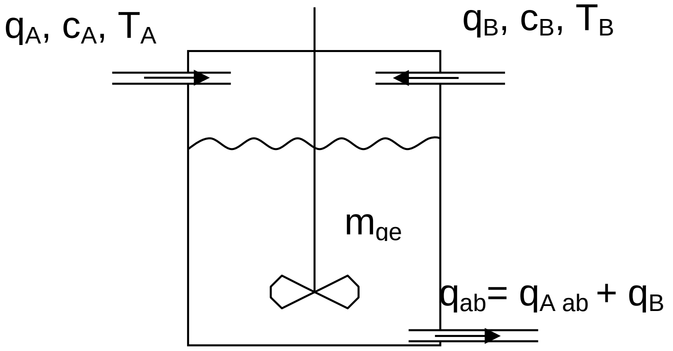


Image 3: *stirred tank reactor*

In the tank an exothermal reaction takes place. In this reaction material A is converted into material B. With

$$r_{AB} = \frac{m_A}{m_{ges}} e^{-\frac{E_A}{RT}} \frac{kg}{s}$$

as per time unit converted A from B.

Per converted mass of A following resulting free specific energy is produced:

$$g = k_R \left(1 - e^{-\frac{m_A}{m_{ges}}} \right).$$

a) Is the reaction from A into B with increasing temperature faster or slower? Justify your statement!

b) State the balance of mass separately for m_A and m_B.

c) State the total mass balance (m_{ges}) for the tank.

d) Determine with support of the balance of enthalpy

$$\dot{T} = f(T, q_A, q_B, c_A, m_A, m_B, q_B, c_B, T_A, T_B, g, r_{AB})$$

e) Under which conditions linearisation of a model is reasonable in control?

f) Linearise the function around the known working point T_0:

$$\dot{T} = T(a + be^{-\frac{1}{T}}) + ce^{-\frac{1}{T}} + d$$

a, b, c, d are constants.

Task 4 Thermal System

The heating of water in a kettle is to be modelled with theoretic analysis. The physical analogous model is build up by the steel bottom with mass M_S and specific heat capacity c_S. This steel body is heated by a constant heat j_P.

$M_S = 0{,}2 \text{ kg}$

$c_S = 500 \text{ Jkg}^{-1}\text{K}^{-1}$

$j_P = 2000 \text{ W}$

$M_W = 1 \text{ kg}$

$c_W = 4200 \text{ Jkg}^{-1}\text{K}^{-1}$

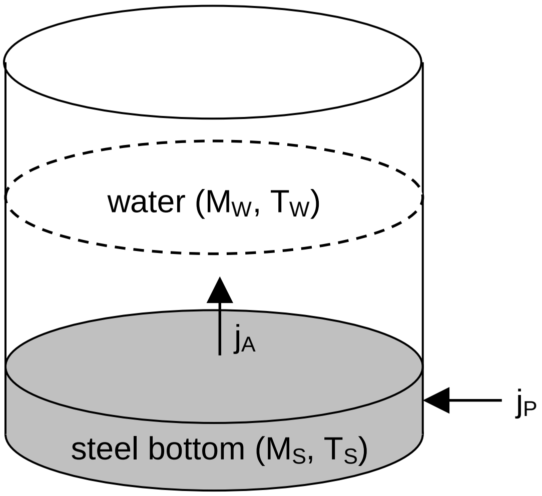


Image 4. *The physical analogous model oft he kettle*

The heating power j_P is 2000 watt, if the kettle is turned on *(u(t) = 1, u(t) = 0 else)*. The kettle is filled with one litre water with the mass M_W and the specific heat capacity c_W. The heat flow j_A goes from the steel bottom of the kettle into the water. The kettle is ideally isolated.

a) State the energy balances for the steel bottom of the kettle and the water inside. State the differential equations for the steel temperature T_S and the water temperature T_W.

b) Express a phenomenological law for the heat flow j_A under following assumptions:

The heat flow is proportional to the difference in temperature between steel bottom and water.
At the difference in temperature of 15° C between steel bottom and water the heat flow is 1,5 kW.

c) Build a state space model of the kettle with the results of a) and b). Use the state vector $\underline{x} = [T_S\ T_W]^T$ and the input $u(t)$. Output should be T_W.

d) Draw the generalised electrical network for the dynamic system kettle corresponding to image 1. Insert all potential and flow variables and state all component sizes.

Generalised Network Analysis

Eigenschaften:

E1: f ist die Ableitung einer **mengenartigen** Größe nach der Zeit

E2: e ist **nicht mengenartig**

E3: e•f ist eine Leistung

System	Variable		
	Strom f	Spannung e	
elektrisch	el. Strom $[f] = A$	el. Spannung $[e] = V$	
translatorisch mechanisch	Kraft $[f] = N$	Geschwindigkeit $[e] = \frac{m}{s}$	
rotatorisch mechanisch	Drehmoment $[f] = Nm$	Winkelgeschwindigkeit $[e] = \frac{rad}{s}$	
hydraulisch	Volumenstrom $[f] = \frac{m^3}{s}$	Druck $[e] = \frac{N}{m^2}$	
pneumatisch	Gasstrom $[f] = \frac{kg}{s}$	Druck $[e] = \frac{N}{m^2}$	
thermisch	Wärmestrom $[f] = \frac{Nm}{s} = W$	Temperatur $[e] = K$	

System	Variable		Bauelemente		
	Strom f	Spannung e	Widerstand R	Kapazität C	Induktivität L
elektrisch	el. Strom $[f] = A$	el. Spannung $[e] = V$	Ohm'scher Widerstand $[R] = \Omega = \frac{V}{A}$	Kondensator $[C] = F = \frac{As}{V}$	Spule $[L] = H = \frac{Vs}{A}$
translatorisch mechanisch	Kraft $[f] = N$	Geschwindig-keit $[e] = \frac{m}{s}$	Reziproke Reibkonstante $[R] = \frac{m}{Ns}$	Masse $[C] = kg$	Reziproke Federkonstante $[L] = \frac{m}{N}$
rotatorisch mechanisch	Drehmoment $[f] = Nm$	Winkelge-schwindigkeit $[e] = \frac{rad}{s}$	Reziproke Reibkonstante $[R] = \frac{1}{Nms}$	Trägheits-moment $[C] = kg\ m^2$	Reziproke Steifigkeit $[L] = \frac{1}{Nm}$
hydraulisch	Volumen-Strom $[f] = \frac{m^3}{s}$	Druck $[e] = \frac{N}{m^2}$	Laminar- oder Turbulenz-widerstand $[R] = \frac{Ns}{m^5}$	Volumen-speicher $[C] = \frac{m^5}{N}$	Trägheit $[L] = \frac{kg}{m^4}$
pneumatisch	Gasstrom $[f] = \frac{kg}{s}$	Druck $[e] = \frac{N}{m^2}$	pneumat. Widerstand $[R] = \frac{1}{ms}$	Masse-speicher $[C] = \frac{kg\ m^2}{N}$	-
thermisch	Wärmestrom $[f] = \frac{Nm}{s} = W$	Temperatur $[e] = K$	Wärme-widerstand $[R] = \frac{K}{W}$	Wärme-kapazität $[C] = \frac{Ws}{K}$	-

Appendix: Linerarisation

Why is linearization important?

- The most control approaches are based on linear system models.
- The real systems in practice are often nonlinear systems, which are difficult to handle.
- In many problems, the main task of the controller is to compensate the influence of the disturbances on the system behaviour at some given working points.
- The basic idea of linearization is to approximate the original nonlinear system by considering the deviations from the working point.

Linearization at some given working point

Nonlinear function:

$$y = f(u)$$

Working point:

$$(u_P, y_P)$$

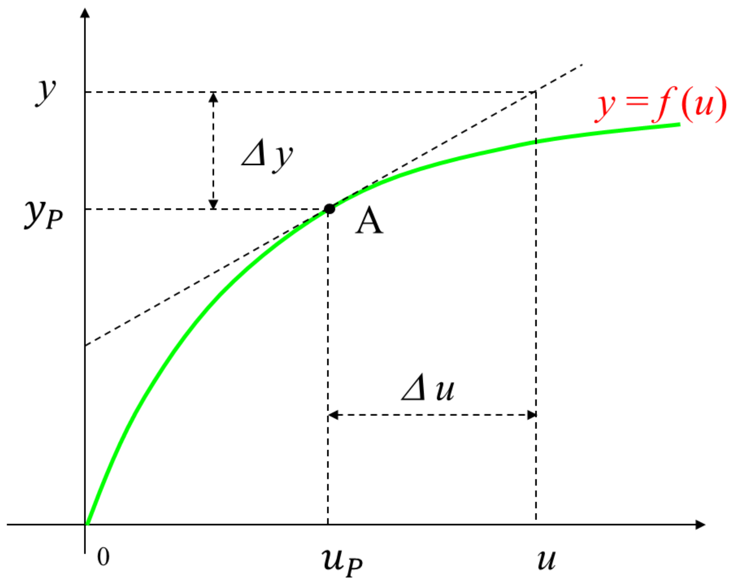
Deviation:

$$(\Delta u, \Delta y)$$

Approximation:

$$u = u_A + \Delta u$$

$$y = y_A + \Delta y$$



Given a dynamic system by nonlinear differential equations

$$\dot{x} = f(x, u)$$

At the equilibrium (x_e, u_e), the state vector doesn't change with time any more, i.e.

$$\dot{x} = f(x, u) = 0 \quad \text{at } (x_e, u_e)$$

Denote

$$x = x_e + \Delta x, \;\; u = u_e + \Delta u,$$

According to the Taylor expansion,

$$\Delta\dot{x} = f(x_e + \Delta x, u_e + \Delta u) = f(x_e, u_e) + \left.\frac{\partial f}{\partial x}\right|_{\substack{x = x_e, \\ u = u_e}} \Delta x + \left.\frac{\partial f}{\partial u}\right|_{\substack{x = x_e, \\ u = u_e}} \Delta u + \cdots$$

Hence, the nonlinear system can be approximated by

$$\Delta\dot{x} = A\Delta x + B\Delta u,$$

$$A = \left.\frac{\partial f}{\partial x}\right|_{\substack{x = x_e, \\ u = u_e}}, \quad B = \left.\frac{\partial f}{\partial u}\right|_{\substack{x = x_e, \\ u = u_e}},$$