

CS 5/6110, Software Correctness Analysis, Spring 2022

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Lecture 14 : First-Order Logic aka predicate logic

- Read from Bradley and Manna, Chapter 2
- Practice using the files checked into this directory
- Having good implementations of SAT-checking for decidable fragments of FOL is why formal methods and rigorous software testing have suddenly become practical!
- Alloy introduces a relational logic
 - FOL expression allowed, maybe even HOL
 - But all “proofs” are finite-model proofs
- Moral: Know relations, functions, FOL, proofs
 - To do well in software testing, specification, and correctness verification
 - “Noble bugs” a direct consequence of computability theory
 - “Stupid bugs” arise due to the Towers of the Babel

An example of FOL (“classical”)

- Some patients like every doctor
- No patient likes a quack
- Prove that no doctor is a quack

An example of FOL (“classical”)

- Let “P” be people containing x, y, z, \dots
- Let “p” be a patient-predicate - e.g. $p(x)$
- Let “d” be a doctor-predicate - e.g. $d(y)$
- Let “q” be a quack-predicate - e.g. $q(z)$
- Let “l” be the likes-predicate - e.g. $l(x, y)$ [T/F], $\neg l(x, z)$ [T/F] etc..
- Let's now see how to model this problem
- Forall = “all” and ThereExists = “exists”
- In Alloy it is all / some

An example of FOL (“classical”)

- Reminder: P, p, d, q, l are the symbols we have so far
- All quantification is over “P” or people
- Some patients like every doctor
 - $\exists x : p(x) \text{ and } [\forall y : d(y) \rightarrow l(x,y)]$
- No patient likes a quack
 - ... give it a shot ...
- Prove that no doctor is a quack
 - ... state the proof-goal in FOL – give it a shot ...

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- No patient likes a quack
 - $\forall x, y : [p(x) \& q(y) \Rightarrow \neg l(x,y)]$
- Prove that no doctor is a quack
 - Goal : prove that $\forall x : d(x) \Rightarrow \neg q(x)$

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 - $\exists x : p(x) \text{ and } [\forall y : d(y) \rightarrow l(x,y)]$
 - question: Where else can we put “all y” ?
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An example of FOL (“classical”)

- Reminder: P, p, d, q, l are the symbols we have so far
- All quantification is over “P” or people
- Some patients like every doctor
 - $\exists x : p(x) \text{ and } [\forall y : d(y) \rightarrow l(x,y)]$
 - question: Where else can we put “all y” ?
 - Which of these “makes sense”
 - $\exists x : \forall y : [p(x) \text{ and } [d(y) \rightarrow l(x,y)]]$
 - $\forall y : \exists x : [p(x) \text{ and } [d(y) \rightarrow l(x,y)]]$
 - No patient likes a quack
 - $\forall x,y : [p(x) \& q(y) \Rightarrow \neg l(x,y)]$
 - Prove that no doctor is a quack
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Example of FOL (“classical”): Proof by contra

- A1: exists $x : p(x)$ and [all $y : d(y) \rightarrow l(x,y)$]
 - A2: all $x,y : [p(x) \& q(y) \Rightarrow !l(x,y)]$
 - G: all $x : d(x) \Rightarrow !q(x)$
 - NG: ...give it a shot...
-
- A1 skolemize x with $p0$
 - A1sk: $p(p0)$ and [all $y : d(y) \rightarrow l(p0,y)$]

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-
- A1 skolemize x with $p0$
 - A1sk: $p(p0)$ and [all $y : d(y) \rightarrow l(p0,y)$]
 - NGsk: Skolemize NG : $d(d0) \& q(d0)$

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- A1sk: $p(p0)$ and [all $y : d(y) \rightarrow l(p0,y)$]
- NGsk: $d(d0) \& q(d0)$
- A1sk with d0 specialization: $p(p0) \& d(d0) \Rightarrow l(p0,d0)$
- Now what?
 - Hint : we have not used A2

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- Now what?
 - Hint : we have not used A2
- A2sp with p0 and d0: $p(p0) \& q(d0) \Rightarrow !l(p0,d0)$
 - Do you smell the contradiction?
 - How to finish?

Example of FOL (“classical”): Proof by contra

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- NGsk: $d(d0) \& q(d0)$
- A1sk with d0 specialization: (A1sk'): $p(p0) \& [d(d0) \Rightarrow l(p0,d0)]$
- Now what?
- A2sp with $p0$ and $d0$: $p(p0) \& q(d0) \Rightarrow !l(p0,d0)$
- MP of A1sk' and NGsk : $l(p0,d0)$
- Contra of $!l(p0,d0)$ and $l(p0,d0)$

Lecture 14 : First-Order Logic aka predicate logic : Basics

- There exist consistent axiomatizations of FOL
- There exist complete axiomatizations of FOL
- FOL is only semi-decidable
 - Only procedures exist for deciding validity
 - No algorithms
 - No TMs that don't loop
 - Semi-algorithms exist because of completeness
 - If indeed valid (true), there is a proof
 - And we can discover the proof
 - via systematic enumeration of all legal inference sequences

Encoding the “doctor/quack theorem” in Alloy

- What all FOL concepts are involved?
- How to encode each of them in Alloy?

Encoding the “doctor/quack theorem” in Alloy

- What all FOL concepts are involved?
 - Propositional variables
 - Zero-ary predicates
 - Predicates of higher arity
 - One-ary predicates (or subsets of the universe in most cases)
 - Two-ary predicates (pairs over U)
 - Zero-ary functions
 - Or constants in various value domains
 - One-ary and higher-arity functions
- How to encode each of them in Alloy?
 - We will study how the encoding goes for the doctor/quack theorem
 - Then we will study how to encode general FOL concepts using Alloy
 - Again, for bounded-model proofs
 - Look at Alloy-Constructs-and-Usage.txt for a quick overview

Alloy doctors/quacks encoding

```
some sig P          -- our little universe of people
      { l : set P }  -- the "likes" binary relation
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sig doc in P {}    -- doctor unary relation ; could overlap with pat
sig quk in P {}    -- think of quacks as any subset of people
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pred p[x: P] { x in pat } -- patient predicate denotes unary reln pat
pred d[x: P] { x in doc } -- doctor predicate  denotes unary reln doc
pred q[x: P] { x in quk } -- quack predicate
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-- some patients like every doctor
fact { some x : P | p[x] and (all y : P | d[y] => x->y in l) }
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fact { all x, y : P | (p[x] and q[y]) => x->y not in l }
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fact { all x, y : P | (p[x] and q[y]) => x->y not in l }

-- check that no doctor is a quack
assert docthm { all x : P | d[x] => !q[x] }
```

Alloy checks asserts for validity
by negating them
and checking whether they are sat or not
If Sat then ... ?
If Unsat then ... ?

Remember that any Boolean formula is

- * Valid
- * A contradiction (its negation is valid)
- * Neither - it and its negation are satisfiable

Alloy checks asserts for validity
by negating them
and checking whether they are sat or not
If Sat then the original assertion is invalid
If Unsat then the original assertion is valid
Just to be thorough, we will (later) do this:

- * Write an assertion Assn
- * Write negAssn = !Assn
- * We will check both
- * If Assn's "check" says "no counterexample, "Assn may be valid",
then ensure that !Assn is satisfiable (there is a counterexample for it)

BUT if both Assn and !Assn generate counterexamples, then they are
merely satisfiable but not valid AND also not contradictions !!

Alloy doctors/quacks encoding

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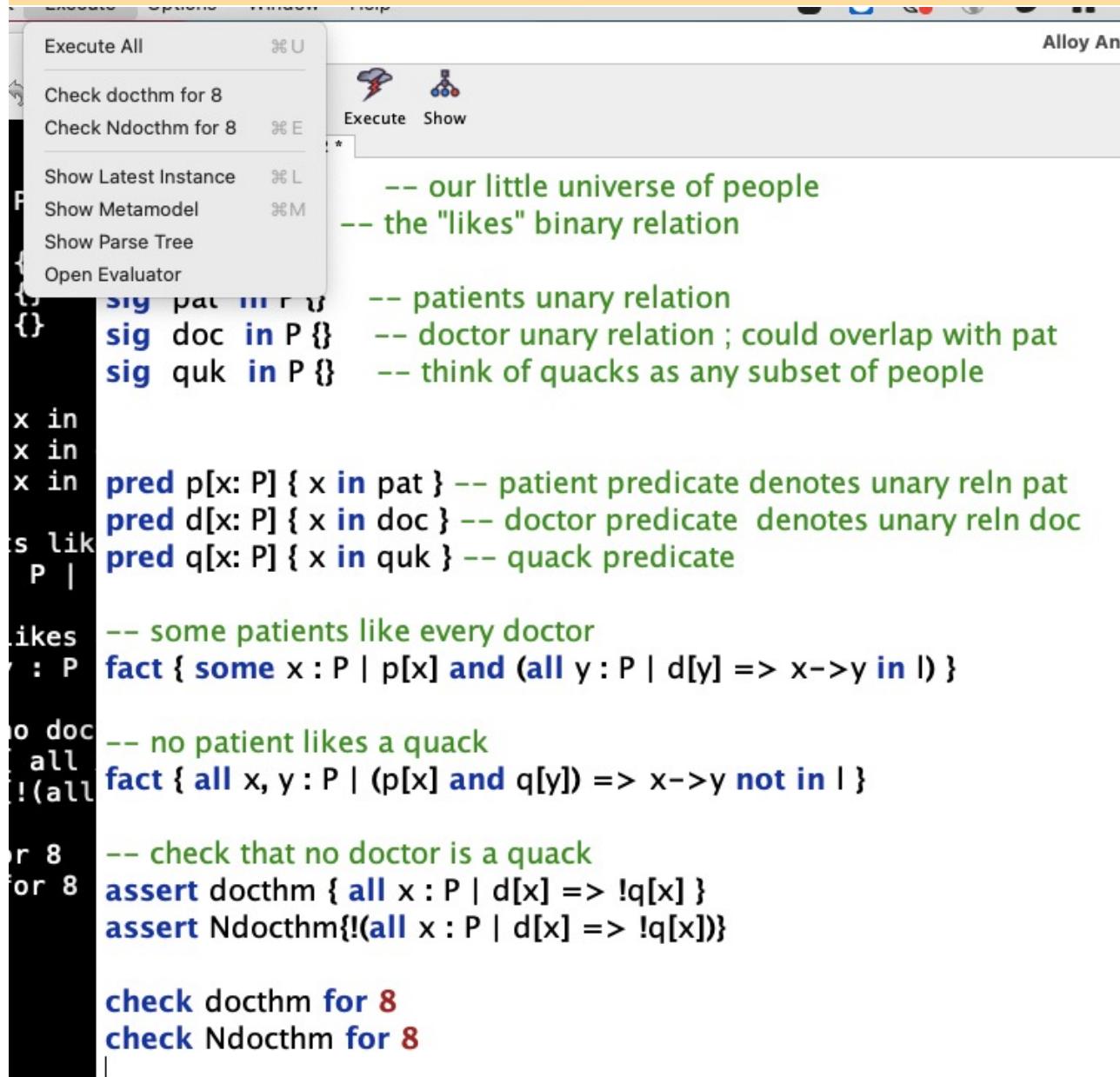
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assert docthm { all x : P | d[x] => !q[x] }
assert Ndocthm{!(all x : P | d[x] => !q[x])}

check docthm for 8
check Ndocthm for 8
```

Alloy doctors/quacks encoding



The screenshot shows the Alloy Analyzer interface with a model named "docthm". The code defines a set of people (P), relations for patients (pat), doctors (doc), and quacks (quk), and predicates for each. It includes assertions that no patient likes a quack and that no doctor is a quack. The interface has a menu bar with options like Execute All, Check docthm for 8, and Show Latest Instance.

```
-- our little universe of people
-- the "likes" binary relation
sig pat in P
sig doc in P
sig quk in P
x in
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x in pred p[x: P] { x in pat } -- patient predicate denotes unary reln pat
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s lik P |
ikes : P fact { some x : P | p[x] and (all y : P | d[y] => x->y in I) }
no doc all !(all !(all x, y : P | (p[x] and q[y]) => x->y not in I })
for 8 -- check that no doctor is a quack
for 8 assert docthm { all x : P | d[x] => !q[x] }
assert Ndocthm{!(all x : P | d[x] => !q[x])}

check docthm for 8
check Ndocthm for 8
```

Observe that
Via the pulldown

You can run any of the asserts
You've planted !!

Excerpts from Bradley and Manna

KLEE: Unassisted and Automatic Generation of High-Coverage Tests for Complex Systems Programs

Cristian Cadar, Daniel Dunbar, Dawson Engler *
Stanford University

```

1 : void expand(char *arg, unsigned char *buffer) { 8
2 :     int i, ac; 9
3 :     while (*arg) { 10*
4 :         if (*arg == '\\') { 11*
5 :             arg++;
6 :             i = ac = 0;
7 :             if (*arg >= '0' && *arg <= '7') {
8 :                 do {
9 :                     ac = (ac << 3) + *arg++ - '0';
10:                    i++;
11:                } while (i<4 && *arg>='0' && *arg<='7');
12:                *buffer++ = ac;
13:            } else if (*arg != '\\0')
14:                *buffer++ = *arg++;
15:            } else if (*arg == '[') { 12*
16:                arg++;
17:                i = *arg++;
18:                if (*arg++ != '-' ) { 13
19:                    *buffer++ = '[';
20:                    arg -= 2;
21:                    continue;
22:                }
23:                ac = *arg++;
24:                while (i <= ac) *buffer++ = i++; 14
25:                arg++; /* Skip ']' */
26:            } else { 15!
27:                *buffer++ = *arg++;
28:            }
29:        }
30:    ...
31:    int main(int argc, char* argv[]) { 1
32:        int index = 1; 2
33:        if (argc > 1 && argv[index][0] == '-') { 3*
34:            ...
35:        } 4
36:        ...
37:        expand(argv[index++], index); 5
38:        ... 6
39:    } 7

```

Figure 1: Code snippet from MINIX’s `tr`, representative of the programs checked in this paper: tricky, non-obvious, difficult to verify by inspection or testing. The order of the statements on the path to the error at line 18 are numbered on the right hand side.

1 KLEE constructs symbolic command line string arguments whose contents have no constraints other than zero-termination. It then constrains the number of arguments to be between 0 and 3, and their sizes to be 1, 10 and 10 respectively. It then calls `main` with these initial path constraints.

2 When KLEE hits the branch `argc > 1` at line 33, it uses its constraint solver STP [23] to see which directions can execute given the current path condition. For this branch, both directions are possible; KLEE forks execution and follows both paths, adding the constraint $\text{argc} > 1$ on the false path and $\text{argc} \leq 1$ on the true path.

3 Given more than one active path, KLEE must pick which one to execute first. We describe its algorithm in Section 3.4. For now assume it follows the path that reaches the bug. As it does so, KLEE adds further constraints to the contents of `arg`, and forks for a total of five times (lines denoted with a “*”): twice on line 33, and then on lines 3, 4, and 15 in `expand`.

4 At each dangerous operation (e.g., pointer dereference), KLEE checks if any possible value allowed by the current path condition would cause an error. On the annotated path, KLEE detects no errors before line 18. At that point, however, it determines that input values exist that allow the read of `arg` to go out of bounds: after taking the true branch at line 15, the code increments `arg` twice without checking if the string has ended. If it has, this increment skips the terminating '`\0`' and points to invalid memory.

5 KLEE generates concrete values for `argc` and `argv` (i.e., `tr [" " "`) that when rerun on a raw version of `tr` will hit this bug. It then continues following the current path, adding the constraint that the error does not occur (in order to find other errors).

Why can't we crank through all inputs/states?

Your answer here for a C program that has exactly one 32-bit register

How about a 64-bit register?

Your answer here for a C program with 1k bytes of state and 64 bits of input

How about a C program with 5 threads with 5 sequential steps? How many interleavings?

How about something larger?

The main technology!

- Progress in Boolean Satisfiability is central to the magic I'm going to demo
 - SAT-solving increased by 1000x or more (in performance)
- SAT alone is not sufficient
 - SMT-solving was essential
 - Think of the SEND + MORE = MONEY problem!
- Progress in SMT-solving happened this way
 - Nelson/Oppen methods of the 1980's and Shostak's method
 - SVC and CVC and Yices
 - Then Z3, and now CVC4 and a whole array of others!
- Rides on the power of SAT-solving but their “theory-solving” is key
- KLEE uses “STP”, an SMT solver that can handle bit-arrays and uninterpreted functions well
- This is why we must study mathematical logic
 - took 1000s of years since Euclid, 170 years since Boole, 100 years since Shannon, and 120 years since Russell, Peano and others to get here!
 - in addition to all those other “cool topics” to which students are flocking these days!
 - (What are some of those?!)

Demo-1
: a
hard-
to-test
progra
m
under
convent
ional
testing
and
then
KLEE

```
/*
 * An error that is hard to hit
 */

#include <stdio.h>
#include <assert.h>
#include <stdlib.h>
#include "klee/klee.h"

int rare(int x) {
    if (x < RAND_MAX/2) {
        if (x > RAND_MAX/2 - 3) {
            assert(0);
        }
    }
    else
        return(1);
}

int main() {
    int a, i;

#ifdef KLEEON
    klee_make_symbolic(&a, sizeof(a), "a");
    rare(a);
#endif

#ifdef PRINTON
#ifndef KLEEON
    printf("randmax = %d\n", RAND_MAX);
    for (i=0; i < 10000000; i++) {
        a = rand(); //% 100000000;
        rare(a);
    }
#endif
#endif
}
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```

Enter

KLEE !!

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    for (i=0; i < 10000000; i++) {
        a = rand(); /* 100000000;
        rare(a);
    }
#endif
#endif
}
```

```
klee@f88364af576a:~/klee_src/examples/hardToHit$ clang -DKLEEON -I ../../include -emit-llvm -g -c -O0 -Xclang -disable-O0-optnone hardToHit.c
hardToHit.c:18:1: warning: control may reach end of non-void function [-Wreturn-type]
}
^
1 warning generated.
klee@f88364af576a:~/klee_src/examples/hardToHit$ klee hardToHit.bc
KLEE: output directory is "/home/klee/klee_src/examples/hardToHit/klee-out-3"
KLEE: Using STP solver backend
[KLEE: ERROR: hardToHit.c:13: ASSERTION FAIL: 0
[KLEE: NOTE: now ignoring this error at this location

KLEE: done: total instructions = 27
KLEE: done: completed paths = 3
KLEE: done: generated tests = 3
klee@f88364af576a:~/klee_src/examples/hardToHit$ echo "see we already hit the assert in concolic"
see we already hit the assert in concolic
klee@f88364af576a:~/klee_src/examples/hardToHit$ echo "now to see the tests synthesized"
[now to see the tests synthesized
klee@f88364af576a:~/klee_src/examples/hardToHit$ ls -ld klee-last
[lrwxrwxrwx 1 klee klee 49 Feb 10 18:34 klee-last -> /home/klee/klee_src/examples/hardToHit/klee-out-3
klee@f88364af576a:~/klee_src/examples/hardToHit$ echo "this directory has the tests"
>this directory has the tests
klee@f88364af576a:~/klee_src/examples/hardToHit$ ktest-tool klee-last/test000002.ktest
ktest file : 'klee-last/test000002.ktest'
args       : ['hardToHit.bc']
num objects: 1
object 0: name: 'a'
object 0: size: 4
object 0: data: b'\x00\x00\x00\x00'
object 0: hex : 0x00000000
object 0: int : 0
object 0: uint: 0
object 0: text: ....
klee@f88364af576a:~/klee_src/examples/hardToHit$ ktest-tool klee-last/test000001.ktest
ktest file : 'klee-last/test000001.ktest'
args       : ['hardToHit.bc']
num objects: 1
object 0: name: 'a'
object 0: size: 4
object 0: data: b'\xff\xff\xff?'
object 0: hex : 0xfffffff3
object 0: int : 1073741821
object 0: uint: 1073741821
object 0: text: ...?
klee@f88364af576a:~/klee_src/examples/hardToHit$ ls klee-last
assembly.ll messages.txt run.stats          test000001.kquery test000002.ktest warnings.txt
info      run.istats  test000001.assert.err test000001.ktest test000003.ktest
klee@f88364af576a:~/klee_src/examples/hardToHit$ echo "ah ha , test 000001 is the one that hit assert"
[ah ha , test 000001 is the one that hit assert
```

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#endif
#endif
}
```

Enter

KLEE !!

```
klee@f88364af576a:~/klee_src/examples/hardToHit$ gcc -DKLEEON -DPRINTON -I ../../include -L /home/klee/klee_build/lib hardToHit.c -lkleeRuntst
klee@f88364af576a:~/klee_src/examples/hardToHit$ KTEST_FILE=klee-last/test00002.ktest ./a.out
klee@f88364af576a:~/klee_src/examples/hardToHit$ KTEST_FILE=klee-last/test00001.ktest ./a.out
[a.out: hardToHit.c:13: rare: Assertion `0' failed.
Aborted
klee@f88364af576a:~/klee_src/examples/hardToHit$ echo "BOOM - we hit the assert "
BOOM - we hit the assert
klee@f88364af576a:~/klee_src/examples/hardToHit$
```

Demo-2
Binary
Search

Looks
OK?

Ship it!?

```
#include <klee/klee.h>
#include <stdio.h>
#include <assert.h>
#include <stdlib.h>

void print_data(int arr[], int size, int target) {
    printf("searching for %d in:\n", target);
    for (int i=0; i < size-1; i++) {
        printf("%d, ", arr[i]);
    }
    printf("%d]\n", arr[size-1]);
}

int binary_search(int arr[], int size, int target) {
    #ifdef PRINTON
        print_data(arr, size, target);
    #endif
    int low = 0;
    int high = size - 1;
    int mid;
    while (low <= high) {
        mid = (low + high)/2;
        if (arr[mid] == target) {
            return mid;
        }
        if (arr[mid] < target) {
            low = mid + 1;
        }
        if (arr[mid] > target) {
            high = mid - 1;
        }
    }
    return -1;
}
```

```
int main() {

    int a[4]; // was a[10]

    #ifdef KLEEON
        klee_make_symbolic(&a, sizeof(a), "a");
        klee_assume(a[0] <= a[1]);
        klee_assume(a[1] <= a[2]);
        klee_assume(a[2] <= a[3]);

        klee_make_symbolic(&x, sizeof(x), "x");
    #endif

    #ifdef INITON
        a[0]=rand()%30;
        a[1]=a[0]+rand()%30;
        a[2]=a[1]+rand()%30;
        a[3]=a[2]+rand()%30;
    #endif

    int result = binary_search(a, 4, x); // was 10

    #ifdef PRINTON
        printf("result = %d\n", result);
    #endif
    // check correctness

    if (result != -1) {
        assert(a[result] == x);
    } else {
        // if result == -1, then we didn't find it.
        for (int i = 0; i < 3; i++) { // was 10
            assert(a[i] != x);
        }
    }
    return 1;
}
```

Demo-2 Binary Search

Looks
OK?

```
#include <klee/klee.h>
#include <stdio.h>
#include <assert.h>
#include <stdlib.h>

void print_data(int arr[], int size, int target) {
    printf("searching for %d in:\n", target);
    for (int i=0; i < size-1; i++) {
        printf("%d, ", arr[i]);
    }
    printf("%d\n", arr[size-1]);
}

int binary_search(int arr[], int size, int target) {
    #ifdef PRINTON
    print_data(arr, size, target);
    #endif
    int low = 0;
    int high = size - 1;
    int mid;
    while (low <= high) {
        mid = (low + high)/2;
        if (arr[mid] == target) {
            return mid;
        }
        if (arr[mid] < target) {
            low = mid + 1;
        }
        if (arr[mid] > target) {
            high = mid - 1;
        }
    }
    return -1;
}

int main() {
    int a[4]; // was a[10]
    #ifdef KLEEON
    klee_make_symbolic(&a, sizeof(a), "a");
    #endif
}
```

```
klee@f88364af576a:~/klee_src/examples/binsrch$ gcc -DINITON -DPRINTON -I ../../include binsrch.c
```

```
klee@f88364af576a:~/klee_src/examples/binsrch$ ./a.out
```

```
searching for -336381299 in:
```

```
[13, 29, 56, 81]
```

```
result = -1
```

```
klee@f88364af576a:~/klee_src/examples/binsrch$ clang -DKLEEON -I ../../include -emit-llvm -g -c -O0 -Xclang -disable-O0-optnone binsrch.c
```

```
klee@f88364af576a:~/klee_src/examples/binsrch$ klee binsrch.bc
```

```
KLEE: output directory is "/home/klee/klee_src/examples/binsrch/klee-out-18"
```

```
KLEE: Using STP solver backend
```

```
KLEE: WARNING: undefined reference to function: printf
```

```
KLEE: done: total instructions = 638
```

```
KLEE: done: completed paths = 9
```

```
KLEE: done: generated tests = 9
```

Demo-2 Binary Search

Looks OK?

Ship it?!

```
#include <Klee/klee.h>
#include <stdio.h>
#include <assert.h>
#include <stdlib.h>

void print_data(int arr[], int size, int target) {
    printf("searching for %d in:\n[%n, target);
    for (int i=0; i < size-1; i++) {
        printf("%d, ", arr[i]);
    }
    printf("%d]\n", arr[size-1]);
}

int binary_search(int arr[], int size, int target)
#ifdef PRINTON
    print_data(arr, size, target);
#endif
    int low = 0;
    int high = size - 1;
    int mid;
    while (low <= high) {
        mid = (low + high)/2;
        if (arr[mid] == target) {
            return mid;
        }
        if (arr[mid] < target) {
            low = mid + 1;
        }
        if (arr[mid] > target) {
            high = mid - 1;
        }
    }
    return -1;
}

int main() {
    int a[4]; // was a[10]

#ifdef KLEEOON
    klee_make_symbolic(&a, sizeof(a), "a");
    klee_assume(a[0] <= a[1]);
    klee_assume(a[1] <= a[2]);
    klee_assume(a[2] <= a[3]);

    klee_make_symbolic(&x, sizeof(x), "x");
#endif

#ifdef INITON
    a[0]=rand()%30;
    a[1]=a[0]+rand()%30;
    a[2]=a[1]+rand()%30;
    a[3]=a[2]+rand()%30;
#endif

    int result = binary_search(a, 4, x); // was 10

#ifdef PRINTON
    printf("result = %d\n", result);
#endif
    // check correctness

    if (result != -1) {
        assert(a[result] == x);
    } else {
        // if result == -1, then we didn't find it.
    }
    for (int i = 0; i < 3; i++) { // was 10

        assert(a[i] != x);
    }
}
return 1;
}
```

Demo-2
Binary
Search
Looks
OK.
Closer to
shipping.

What was
achieved?
* tests
that
cover
every
path

* default
C-level
checks
such as
array out
of
bounds,
null de
reference

i.e.
FEWER
BUGS

```
#include <klee/klee.h>
#include <stdio.h>
#include <assert.h>
#include <stdlib.h>

void print_data(int arr[], int size, int target) {
    printf("searching for %d in:\n", target);
    for (int i = 0; i < size; i++) {
        printf("%d ", arr[i]);
    }
}

klee@f88364af576a:~/klee_src/examples/binsrch$ ktest-tool klee-last/test00009.ktest
ktest file : 'klee-last/test00009.ktest'
args      : ['binsrch.bc']
num objects: 2
object 0: name: 'a'
object 0: size: 16
object 0: data: b'\x00\x00\x00\x06\x00\x00\x00\xfe\x00\x00\x00\x00\x00\x00\x03'
object 0: hex : 0x00000003600000fe00000000000003
object 0: text: ....6.....
object 1: name: 'x'
object 1: size: 4
object 1: data: b'\x00\x01\x00\x00'
object 1: hex : 0x00010000
object 1: int : 256
object 1: uint: 256
object 1: text: ....
klee@f88364af576a:~/klee_src/examples/binsrch$ gcc -DKLEEON -DPRINTON -I ../../include -L /home/klee/klee_build/lib binsrch.c -lkleeRuntst
klee@f88364af576a:~/klee_src/examples/binsrch$ KTEST_FILE=klee-last/test00001.ktest ./a.out
searching for 0 in:
[0, 0, 0, 0]
result = 1
klee@f88364af576a:~/klee_src/examples/binsrch$ KTEST_FILE=klee-last/test00002.ktest ./a.out
searching for 0 in:
[0, 16777216, 16777216, 16777216]
result = 0
klee@f88364af576a:~/klee_src/examples/binsrch$ KTEST_FILE=klee-last/test00009.ktest ./a.out
searching for 256 in:
[0, 54, 254, 50331648]
result = -1
klee@f88364af576a:~/klee_src/examples/binsrch$
```

What does KLEE do? (Cadar's slides)

KLEE

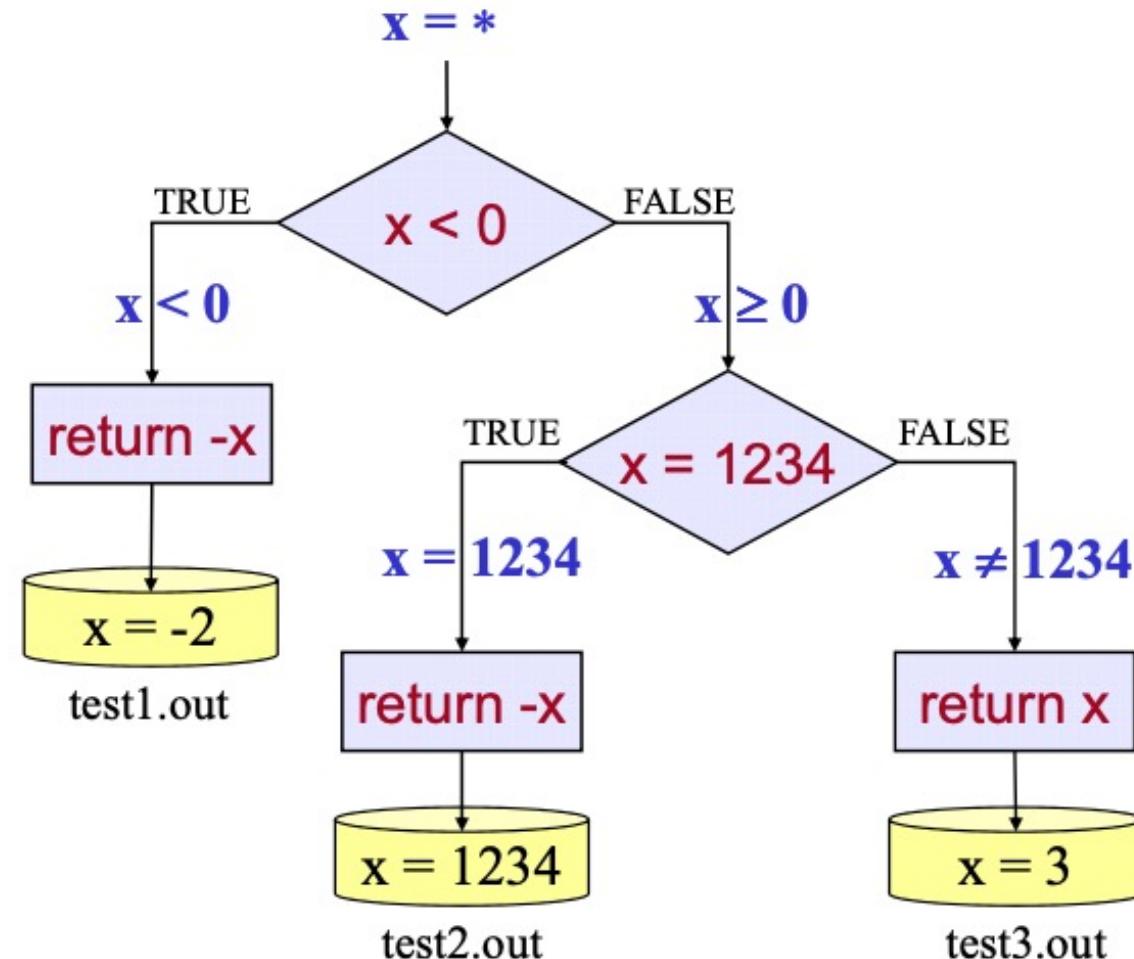
[OSDI 2008, Best Paper Award]

- Based on symbolic execution and constraint solving techniques
- Automatically generates high coverage test suites
 - Over 90% on average on ~160 user-level apps
- Finds deep bugs in complex systems programs
 - Including higher-level correctness ones

How does KLEE work? (Cadar's slides)

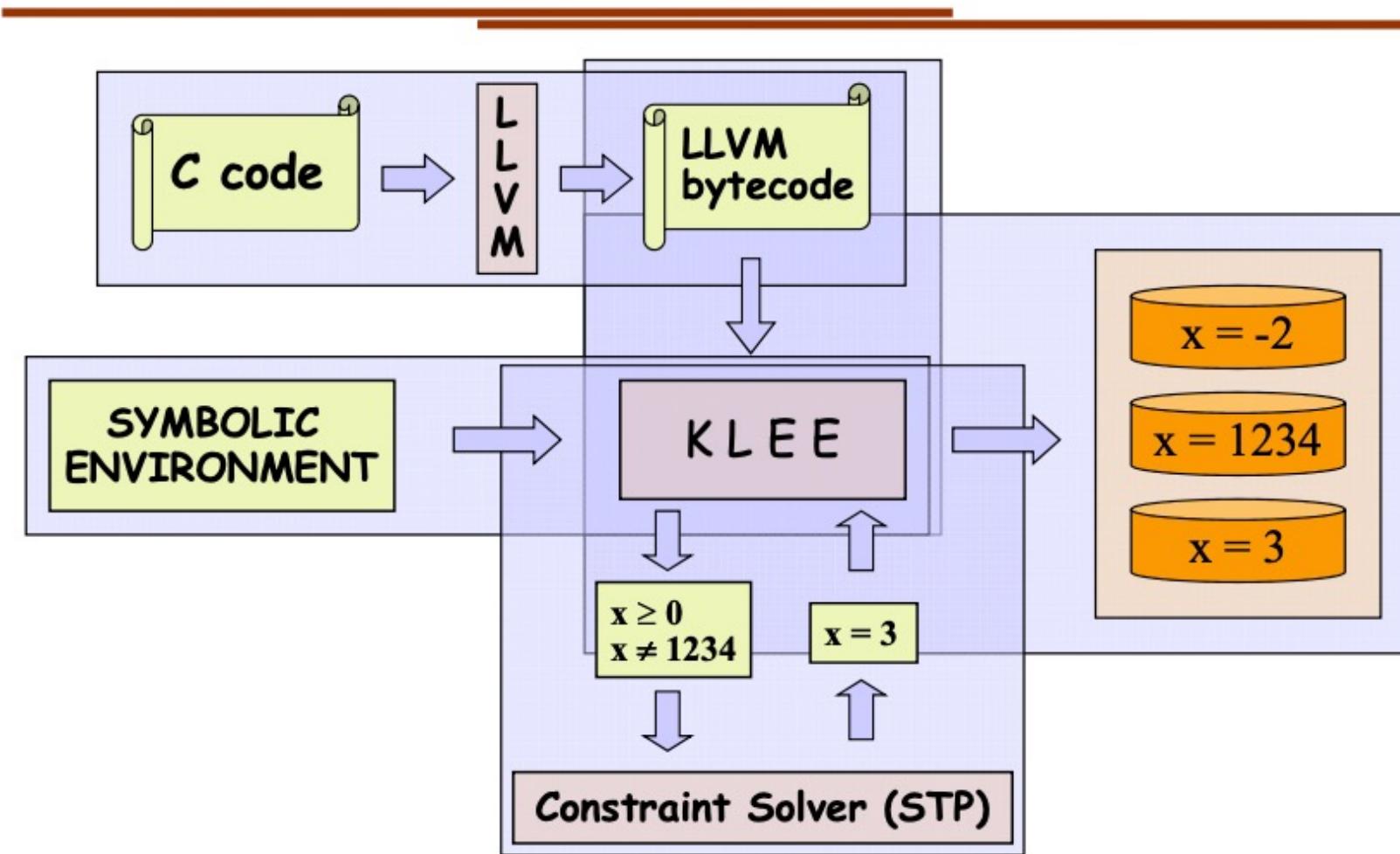
Toy Example

```
int bad_abs(int x)
{
    if (x < 0)
        return -x;
    if (x == 1234)
        return -x;
    return x;
}
```



How does KLEE work? (Cadar's slides)

KLEE Architecture



Engineering KLEE to be efficient (Cadar)

Three Big Challenges

- Motivation
- Example and Basic Architecture
- ➡ • **Scalability Challenges**
 - Exponential number of paths
 - Expensive constraint solving
 - Interaction with environment
- Experimental Evaluation

ConcFuzzer: a sanitizer guided hybrid fuzzing framework leveraging greybox fuzzing and concolic execution

Peng Li, Rundong Zhou, Yaohui Chen,

Yulong Zhang, Tao (Lenx) Wei

lipeng28@baidu.com



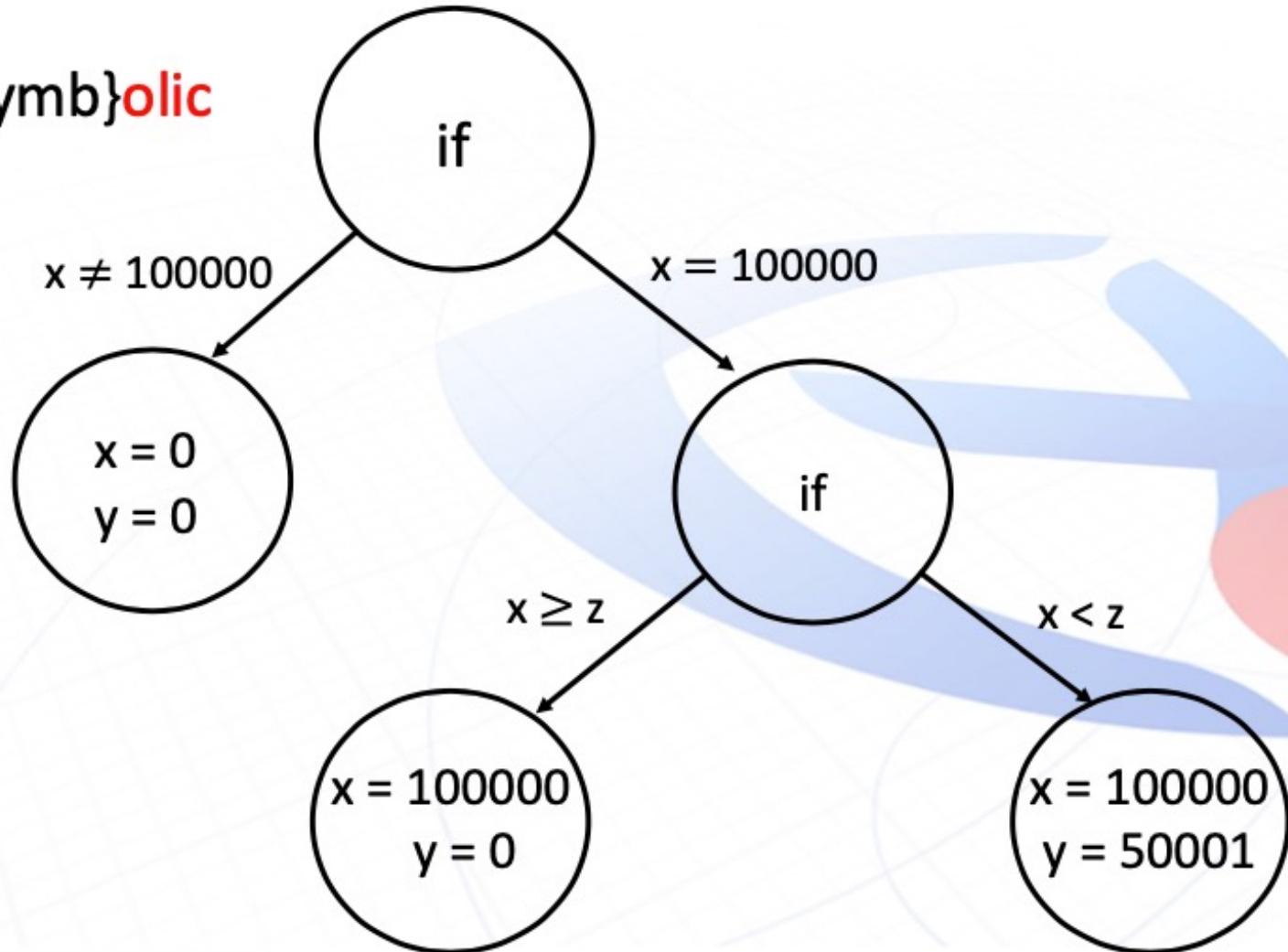
Concolic Execution

How
good is
KLEE
(Dr.
Peng
Li's
slides)

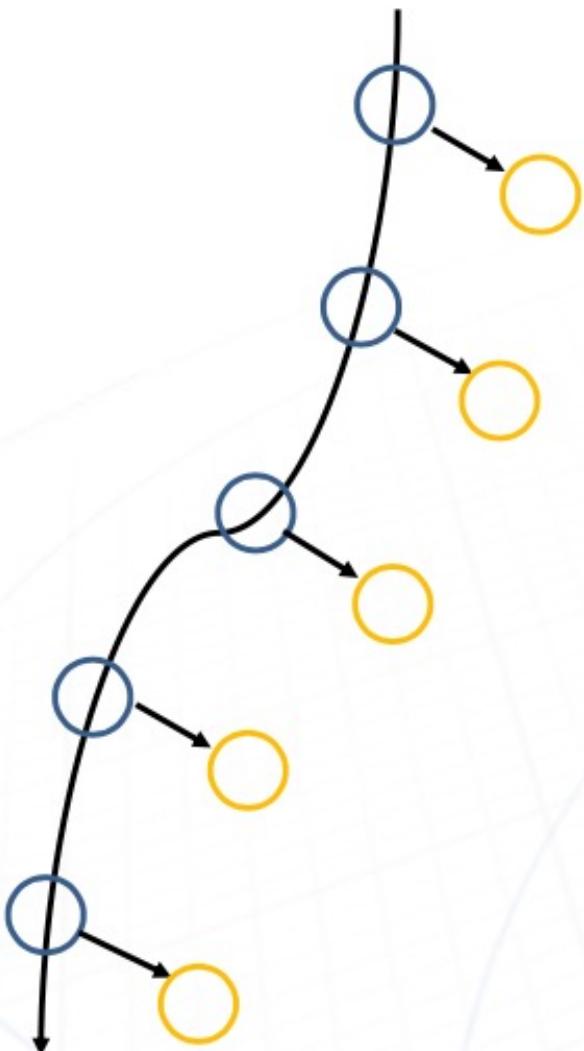
- Concolic = **Conc**rete + Symb**olic**

```
void foo (int x, int y) {  
    int z = 2*y;  
    if (x == 100000) {  
        if (x < z) {  
            /* error */  
            assert(0);  
        }  
    }  
}
```

Snippet of C code from wikipedia



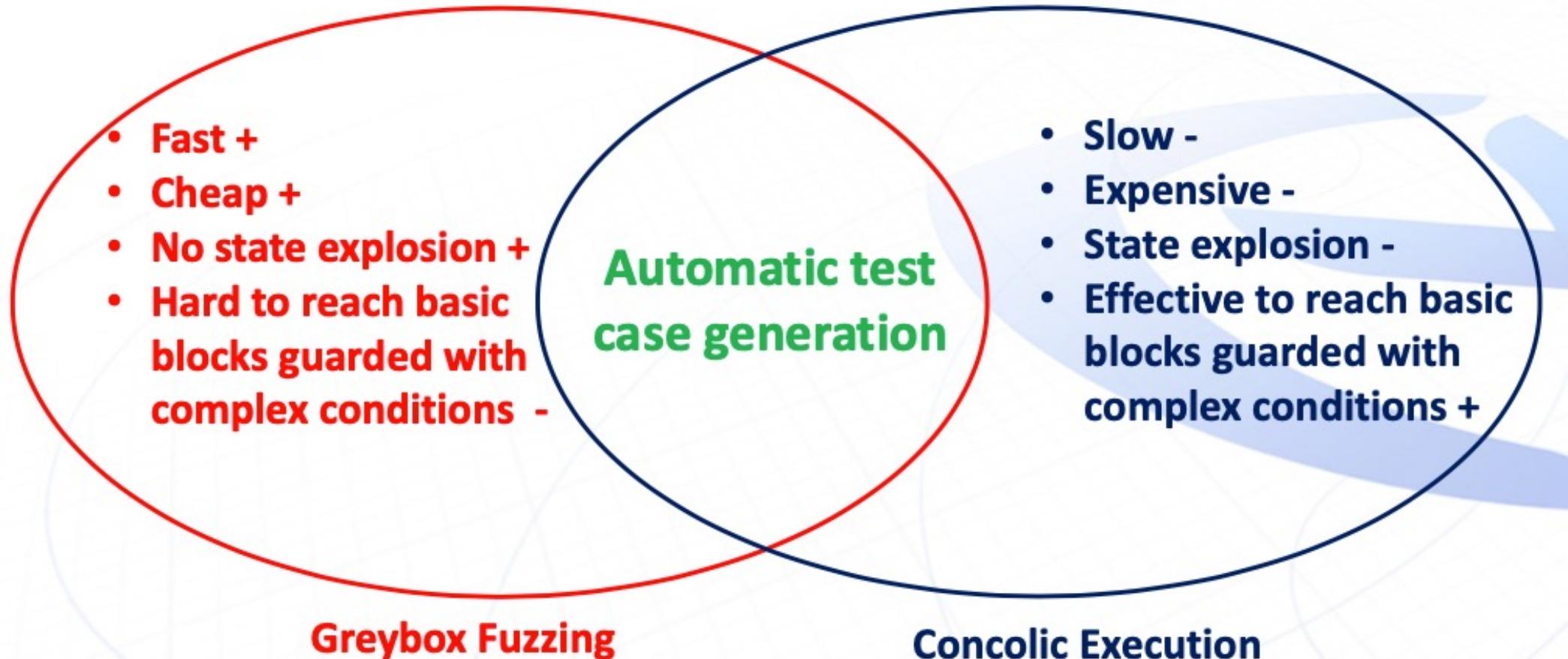
Concolic Execution on top of KLEE



1. Delay constraint solving until the state get scheduled
2. If the state's path constraint is satisfiable, compute the input
3. Otherwise, discard the state

Concolic execution

How
good is
KLEE
(Dr.
Peng
Li's
slides)



MbedTLS X509 Certificate Parser

How
good is
KLEE
(Dr.
Peng
Li's
slides)

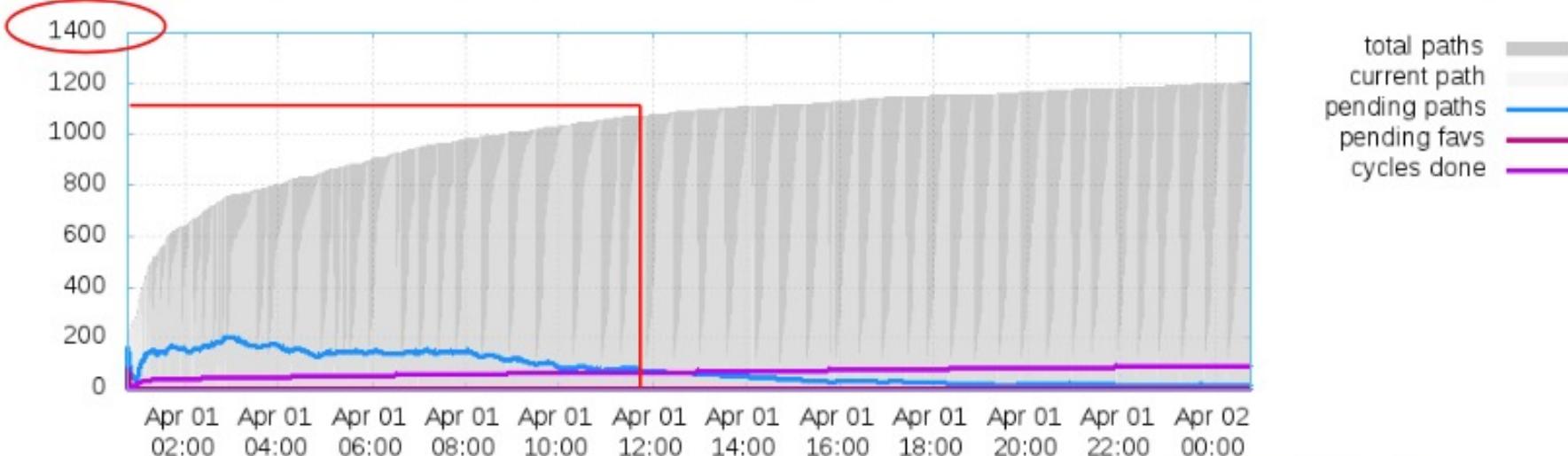


Fig 5. concfuzzer's 24 hours running results for mbedtls x509 certificate parser

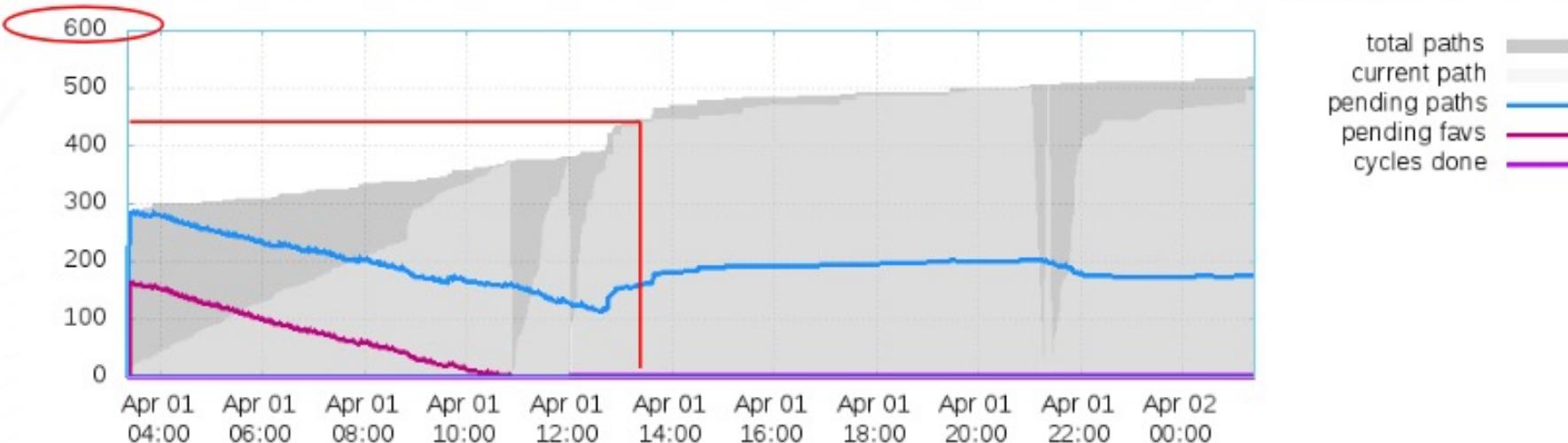


Fig 6. AFL's 24 hours running results for mbedtls x509 certificate parser

djpeg (Libjpeg-9b)

How
good is
KLEE
(Dr.
Peng
Li's
slides)

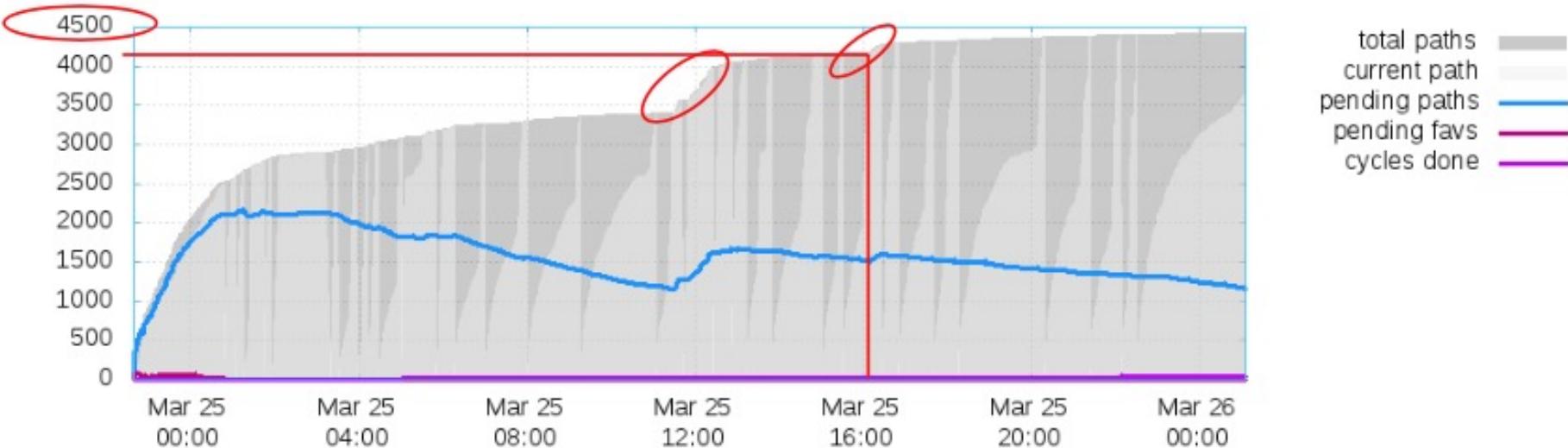


Fig 1. ConcFuzzer's 24 hours running results for djpeg

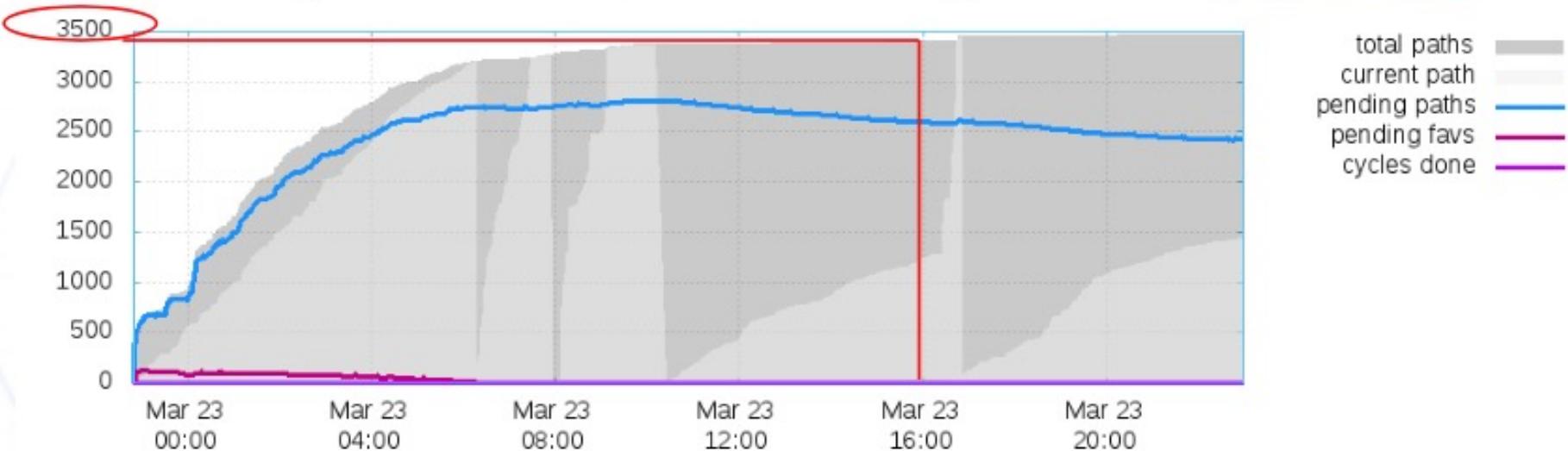


Fig 2. AFL's 24 hours running results for djpeg

Dr. Guodong Li's and Dr. Peng Li's FANTASTIC work on GKLEE follows

See also Guodong's work on PUG on the FSE 2010 publication site!

It was followed by GPUVerify, another fantastic verifier from Alastair Donaldson's groups at Imperial College, London

Summary

This is a huge field now - symbolic execution

You get to study this in projects

Even the MSR python Dyn Symb Executor - good project!

Correctness is central to Energy-Efficient Computing

The more performance per watt we can obtain (e.g. TeraFlop in 30W, Exaflop in 25 MW) ... **that is \$25 Million In electricity costs alone !!**

... the less we need to depend on dirty fuels,
... the faster we can innovate in science and engineering



Intrepid supercomputer (Image courtesy of Argonne)



pogoprinciple.wordpress.com

Emerging Heterogeneous Parallel Systems, and Role of Formal Methods in them



Problem Solving
Environment based
User Applications

Problem-Solving
Environments
e.g. Uintah, Charm++,
ADLB

(1) Formal
Methods at
the User
Application
Level

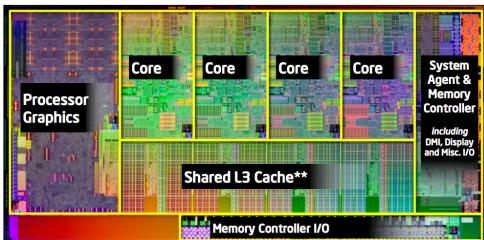


Infiniband style interconnect

High Performance
MPI Libraries

Concurrent
Data Structures

(2) Formal
Dynamic
Methods for
MPI



Sandybridge (courtesy anandtech.com)



AMD Fusion APU

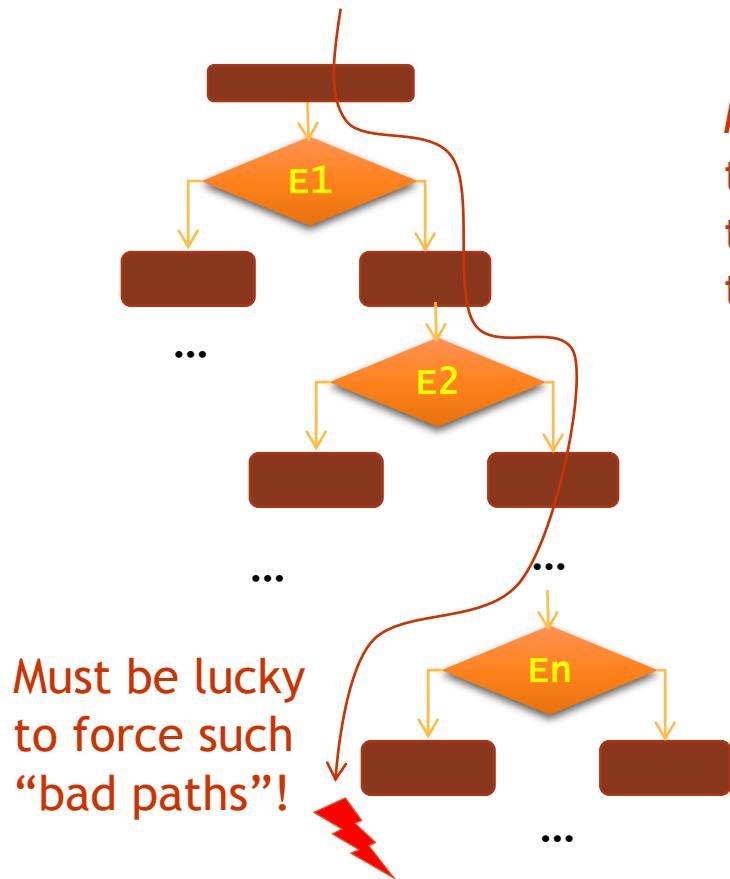


Geoforce GTX 480 (Nvidia)

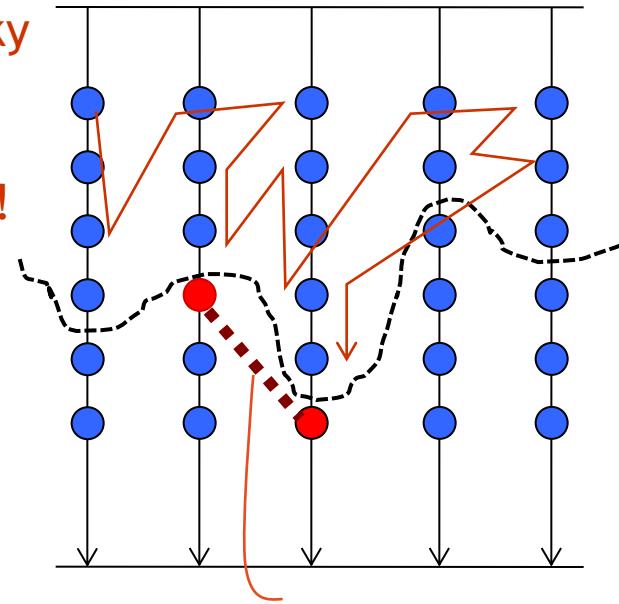
(3) Formal
Analysis for
CUDA
Programs

Why conventional testing is inadequate

Exponential number of paths;
the bug may be deeply hidden



Exponential number of schedules;
the bug may be triggered only by
one particular interleaving



Must be lucky
to schedule
threads in
this manner!

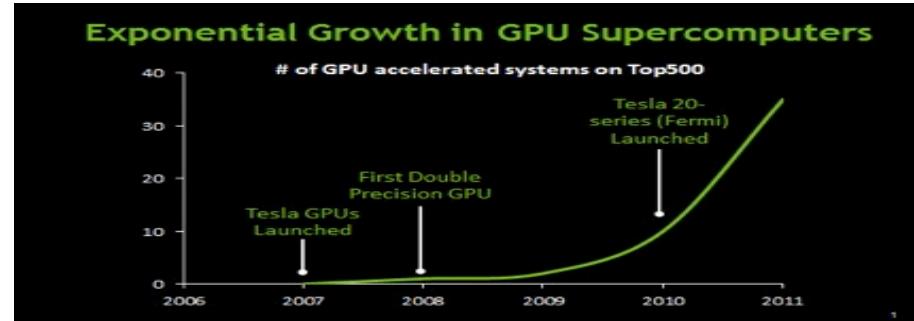
Some bad interaction, such as
a data race may depend on
the schedule...

GPU-based Computing

- About 40 of the top 500 machines were (in 2012) GPU-based



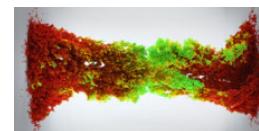
(courtesy of Nvidia,
www.engadget.com)



- Personal supercomputers used for scientific research (biology, physics, ...) increasingly based on GPUs



(courtesy of AMD)



(courtesy of Nvidia)

GKLEE (see PPoPP 2012)

```
__input__ int *values = (int *)malloc(sizeof(int) * NUM);

klee_make_symbolic(values, sizeof(int)*NUM, "values");

int *dvalues;
cudaMalloc((void **)&dvalues, sizeof(int) * NUM);
cudaMemcpy(dvalues, values, sizeof(int) * NUM, cudaMemcpyHostToDevice);

BitonicKernel <<< ... >>> (dvalues);
```

C++ CUDA Programs
with Symbolic Variable
Declarations

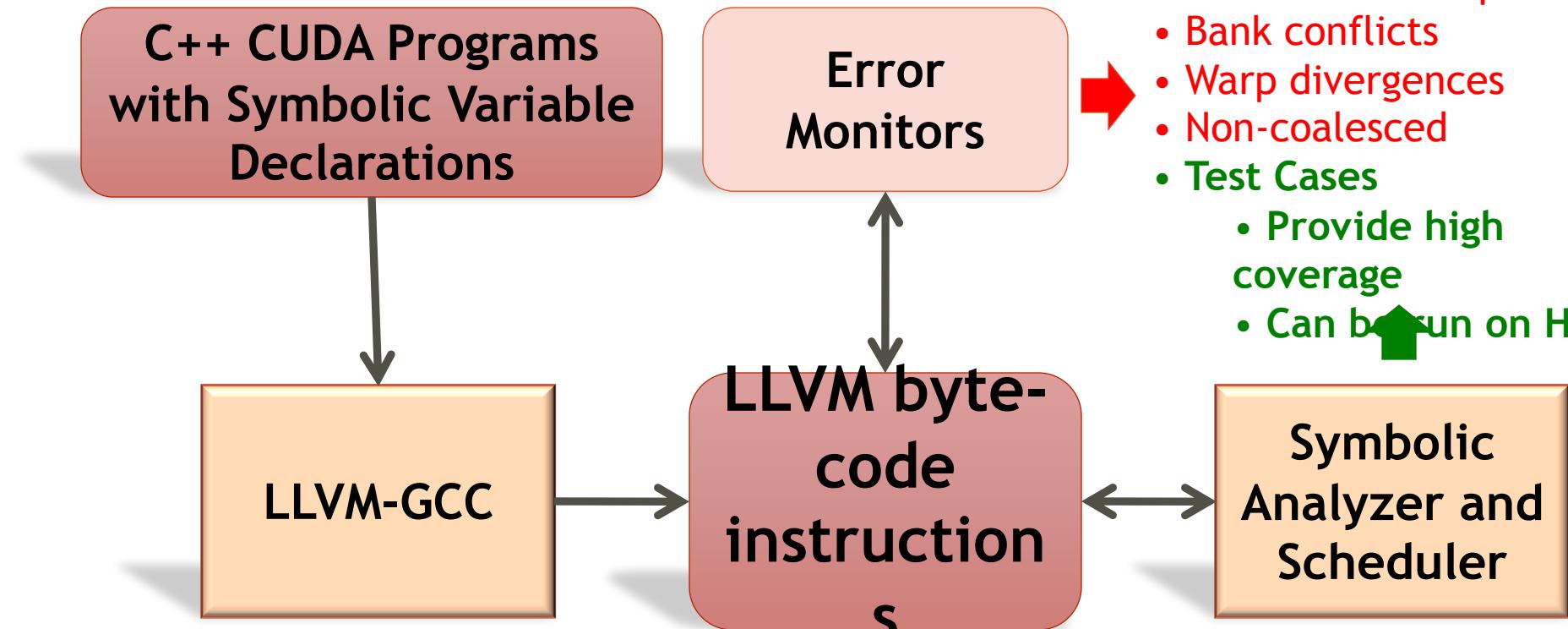
Error
Monitors

- Deadlocks
- Data races
- Concrete test inputs
- Bank conflicts
- Warp divergences
- Non-coalesced
- Test Cases
 - Provide high coverage
 - Can be run on HW

LLVM-GCC

LLVM byte-
code
instruction s

Symbolic
Analyzer and
Scheduler



Our Formal Verification of CUDA builds on top of Sequential Program Verification by KLEE [OSDI '08]

```
#include "stdio.h"
#include "cutil.h"
#include "string.h"
#include "klee.h"

#define STRLEN 5
int main(){
    char item;
    char str[STRLEN];
    int lo=0;
    int hi;

    int mid, found=0;
    // printf("Please give char to be searched: \n");
    // scanf("%c", &item);
    klee_make_symbolic(&item, sizeof(item), "item");
    // printf("Please give string within which to search: \n");
    // scanf("%s", str);
    klee_make_symbolic(str, sizeof(str), "str");
    klee_assume(str[0] <= str[1]);
    klee_assume(str[1] <= str[2]);
    klee_assume(str[2] <= str[3]);
    klee_assume(str[3] <= str[4]);
    str[4] = '\0';
    hi= 4; //strlen(str)-1; // strlen ignores null at the end of string
           // hi points to last char in a 0 based array

    while(!found && lo <= hi) {
        mid = (lo+hi)/2;

        if (item == str[mid])
            { printf("*");
             found = 1; }
        else
            if (item < str[mid])
                { hi = mid-1;
                  printf("L"); }
            else
                { lo = mid+1;
                  printf("H"); }

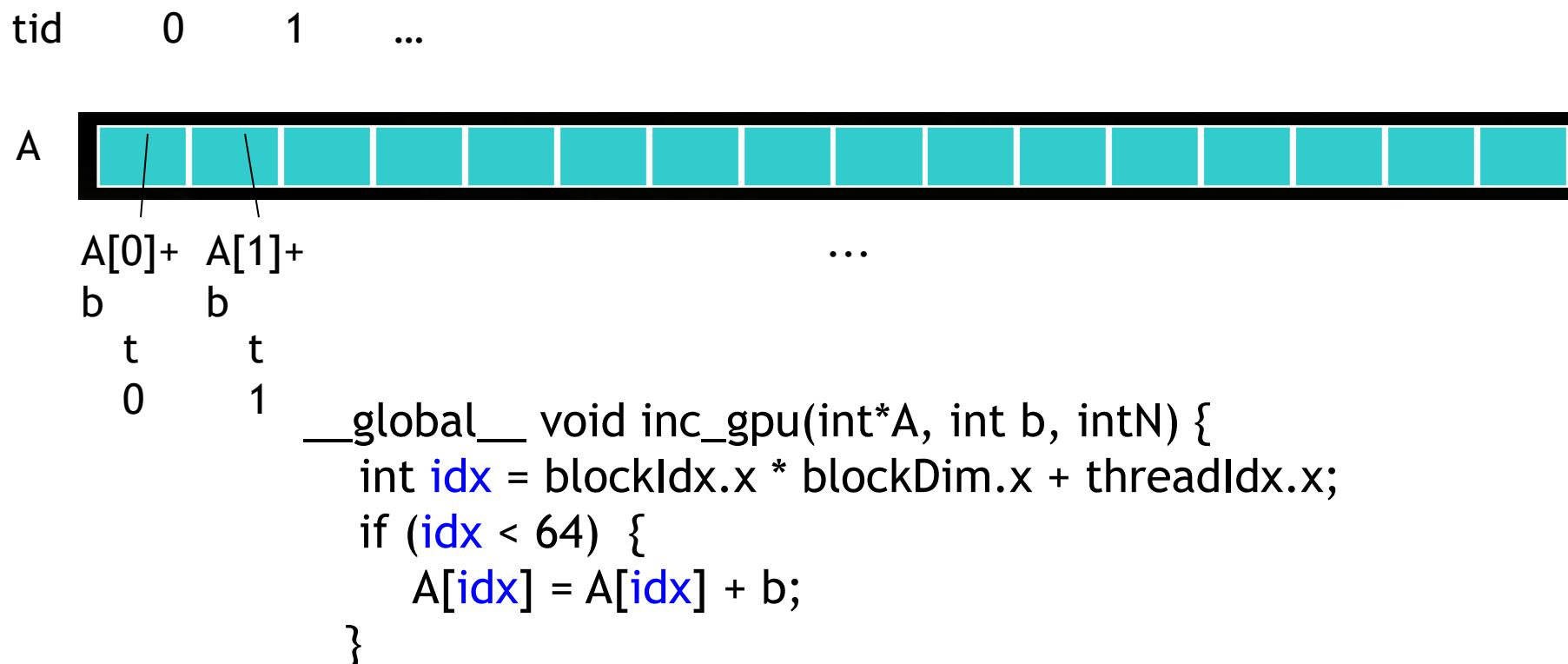
        if (found) printf("found\n"); //printf("\nitem %c found at posn %d\n", item, mid);
        else printf("not found\n"); //printf("\nitem %c not in str %s\n", item, str);
    }
    return found;
}
```

A simple Binary Search being prepared for formal symbolic analysis
This will force path coverage !!

Concurrency errors

Example: Increment Array Elements

Increment N-element array A by scalar b



Data Races in GPU Programs

The usual definition of a race:

“Two accesses, one of which is a write, that occur without a happens-before edge between them.”

In GPUs, the happens-before is provided by

- Barriers (`_syncthreads()`)
- Atomics

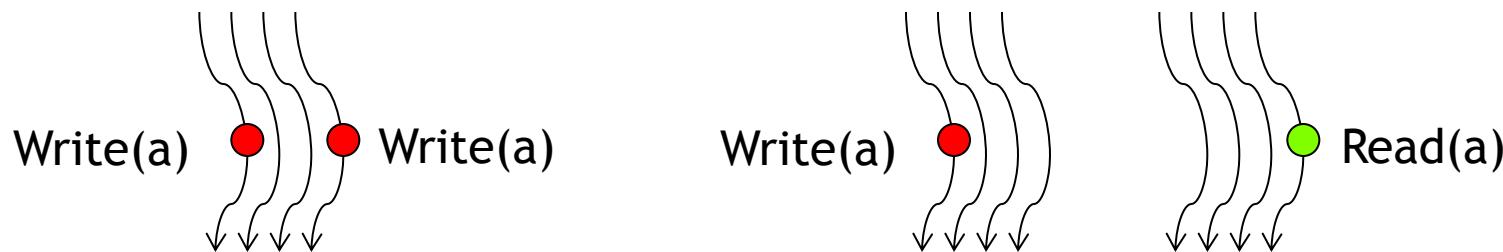


Illustration of Race

Increment N-element vector A by scalar b

tid 0 1



...

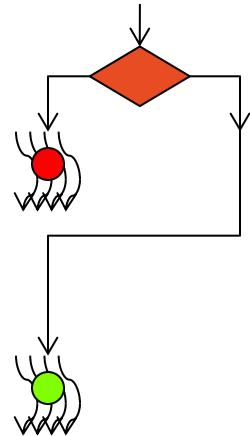
```
__global__ void inc_gpu(int*A, int b, int N) {  
    int idx = blockIdx.x * blockDim.x +  
    threadIdx.x;
```

```
        if (idx < 64) {  
            A[idx] = A[(idx - 1 + 64) % 64] + b;  
        }  
    }  
    RACE!  
    t63 t0
```

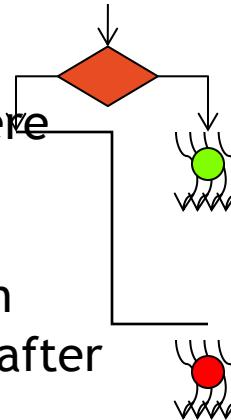
t0: read A[63]
t63: write A[63]

Porting Race: A Race-type Identified by us

- Some hardware platforms may evaluate “then” before “else” – others may reverse this order



If we have a divergent warp where a variable is **READ** in one path and **WRITTEN** in another path, then depending on the execution order, the **READ** may be before/after the **WRITE**



Example of Porting Race

```
#include <stdio.h>

#define NUM 32

__shared__ int v[NUM];

__global__ void PortingRace() {
    if (threadIdx.x % 2) {
        v[threadIdx.x] = v[ (NUM + threadIdx.x - 1) % NUM ] + 1; // W:even;
R:odd
    }
    else {
        v[threadIdx.x] = v[ (NUM + threadIdx.x + 1) % NUM ] - 1; // W:odd;
R:even
    }
}

int main() {
    PortingRace<<<1,NUM>>>();
    return 0;
}
```

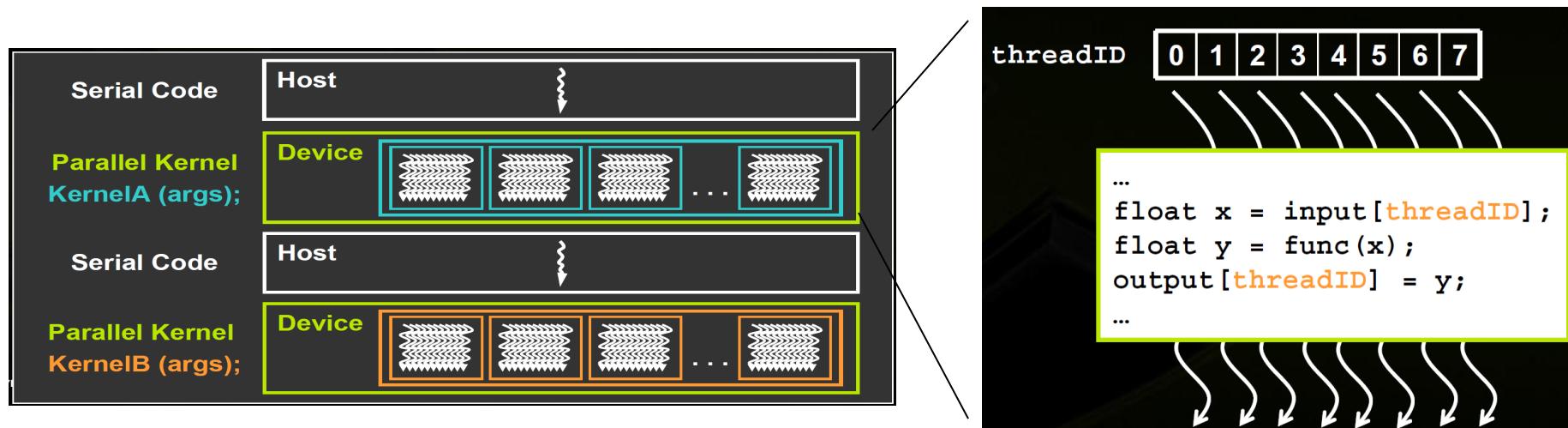
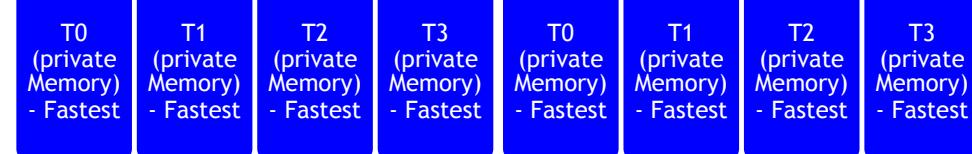
CUDA-based GPU Programming Basics

- A simple dialect of C++ with CUDA directives
- Synchronization within Block through Barriers
- `(__syncthread())`
- Threads within block scheduled in *warps*
- **Don't rely on warps for correctness!**

Device Global Memory (shared between blocks) - Slowest

Block (Shared Memory) - Faster

Block (Shared Memory) - Faster



Conventional GPU Debuggers are Inadequate

- Based on platform testing
- Full generality of executions not reflected
- Hard to cause races (input selection)
- Hard to observe races
 - vector-clock based solutions that run on GPUs exist
 - (last time we checked) only for shared memory races

Some of the GPU Program Bugs

- Data races
 - Cause unpredictable outputs
 - Cause compilers to misbehave
- Incorrectly used synchronizations
 - Syncthreads not used with textual barriers
 - Syncthreads causing deadlocks
- Misunderstood memory consistency models
 - When updates are visible across threads
- Incorrectly used atomics (synchronization)
 - Due to incorrect synchronization, invariants get broken
- Erroneous computational results - esp. with floating-point
 - Due to unpredictable computational order, results may diverge

Why are Data Races Highly Problematic?

- Testing is seldom conclusive
 - Must infer a race indirectly (e.g. corrupted results)
- Races manifest when code is ported or optimized
 - Scheduling can change
- Data races make compiled results suspect
 - Compilers optimize assuming the absence of races in the most general setting

Synchronization Primitives in CUDA

Device Global Memory (shared between blocks) - Slowest

Block (Shared Memory) - Faster

Block (Shared Memory) - Faster

T0
(private Memory)
- Fastest

T1
(private Memory)
- Fastest

T2
(private Memory)
- Fastest

T3
(private Memory)
- Fastest

T0
(private Memory)
- Fastest

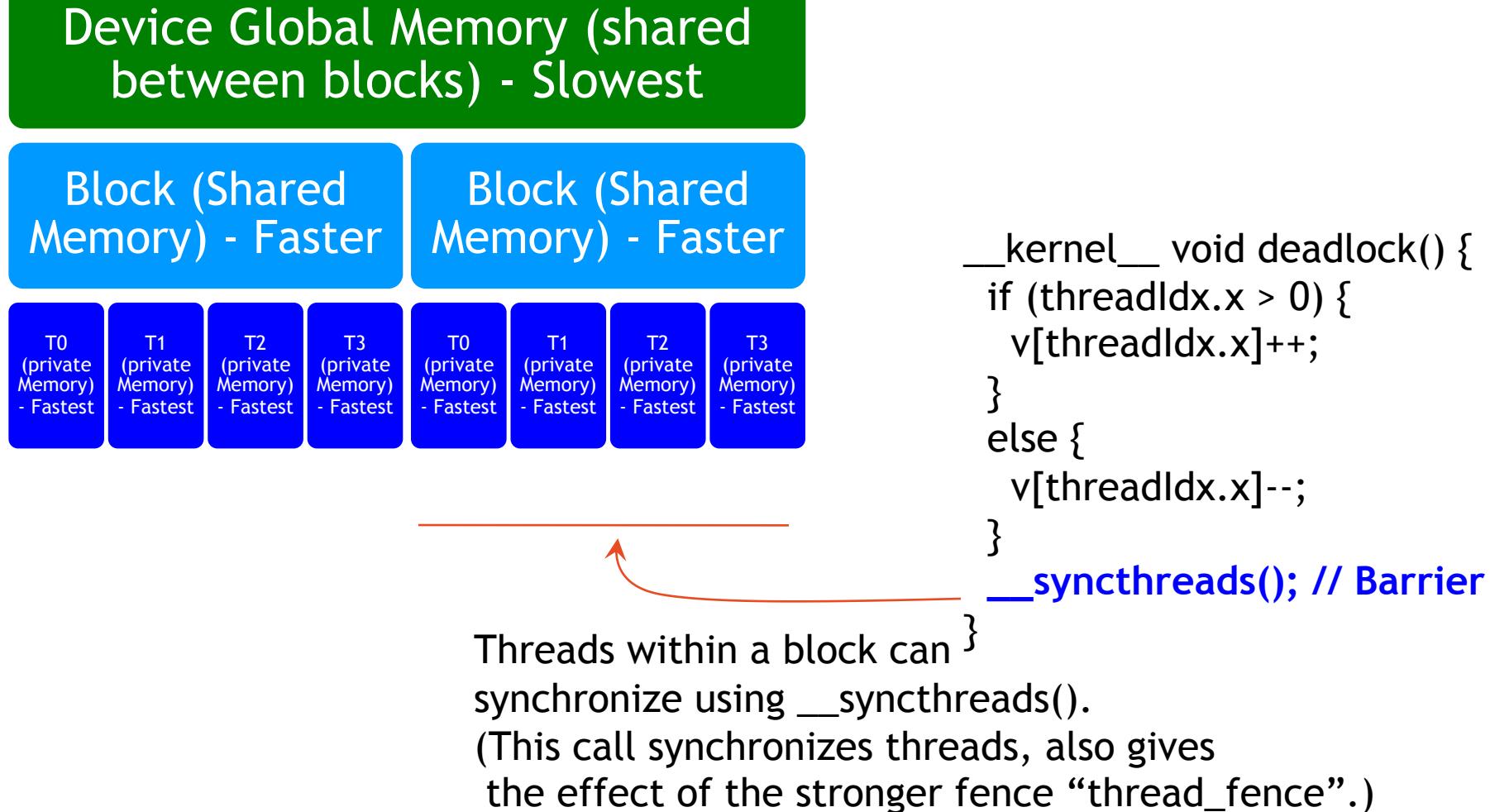
T1
(private Memory)
- Fastest

T2
(private Memory)
- Fastest

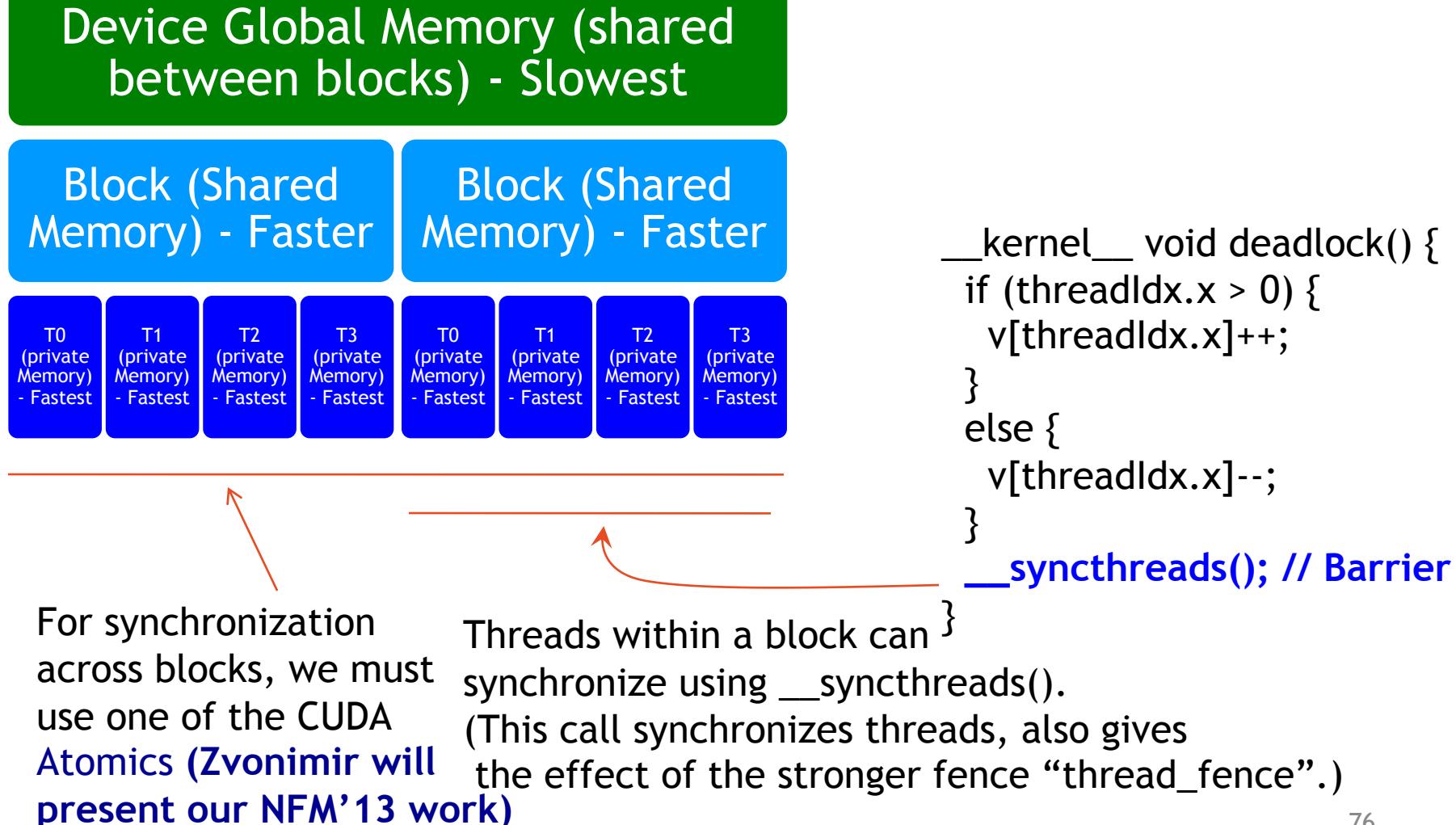
T3
(private Memory)
- Fastest

```
__kernel__ void deadlock() {
    if (threadIdx.x > 0) {
        v[threadIdx.x]++;
    }
    else {
        v[threadIdx.x]--;
    }
    __syncthreads(); // Barrier
}
```

Synchronization Primitives in CUDA



Synchronization Primitives in CUDA



Deadlocks caused by misused `__syncthreads()`

- Cause a “hang” or unspecified behavior

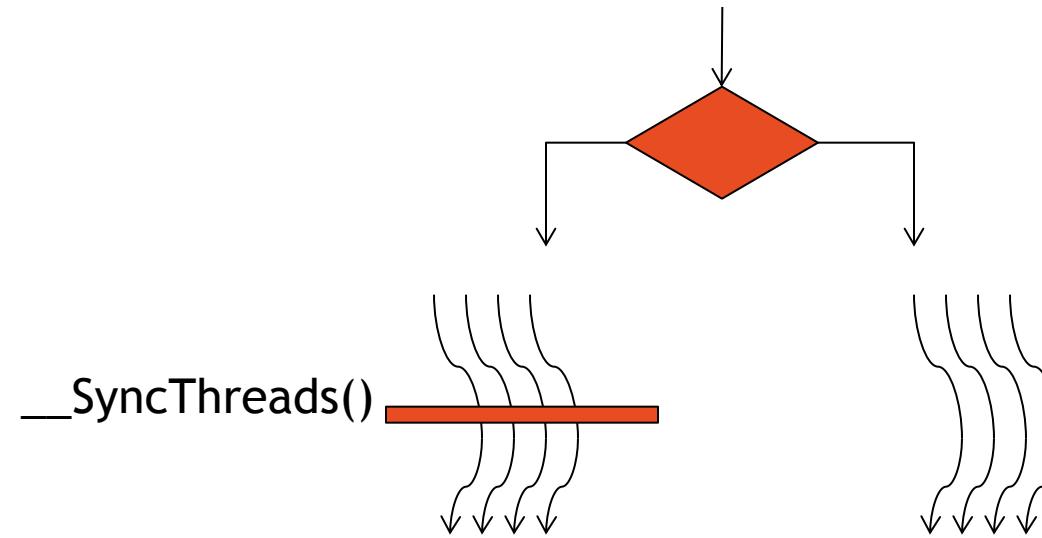
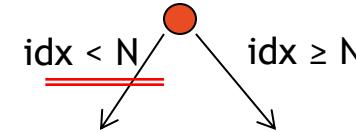


Illustration of “Deadlock” (conceptual deadlock; behavior is really undefined)

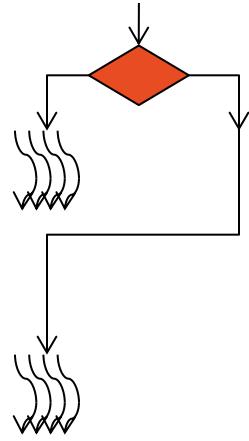
```
__global__ void kernelName (args) {  
  
    int idx = blockIdx.x * blockDim.x +  
    threadIdx.x;  
  
    if (idx == 3) {  
        ....  
        __syncthreads();  
    }  
}
```

Suffers from Textually
Non-aligned Barriers
(example Deadlock1.C)



```
__kernel__ void deadlock() {  
    if (threadIdx.x > 0) {  
        v[threadIdx.x]++;  
        __syncthreads();  
    }  
    else {  
        v[threadIdx.x]--;  
        __syncthreads();  
    }  
}
```

Warp Divergence



```
__global__ void kernelName (args) {
```

```
    int idx = blockIdx.x * blockDim.x +  
    threadIdx.x;
```

```
    if ( odd(idx) ) {
```

<Statements1>

```
} else {
```

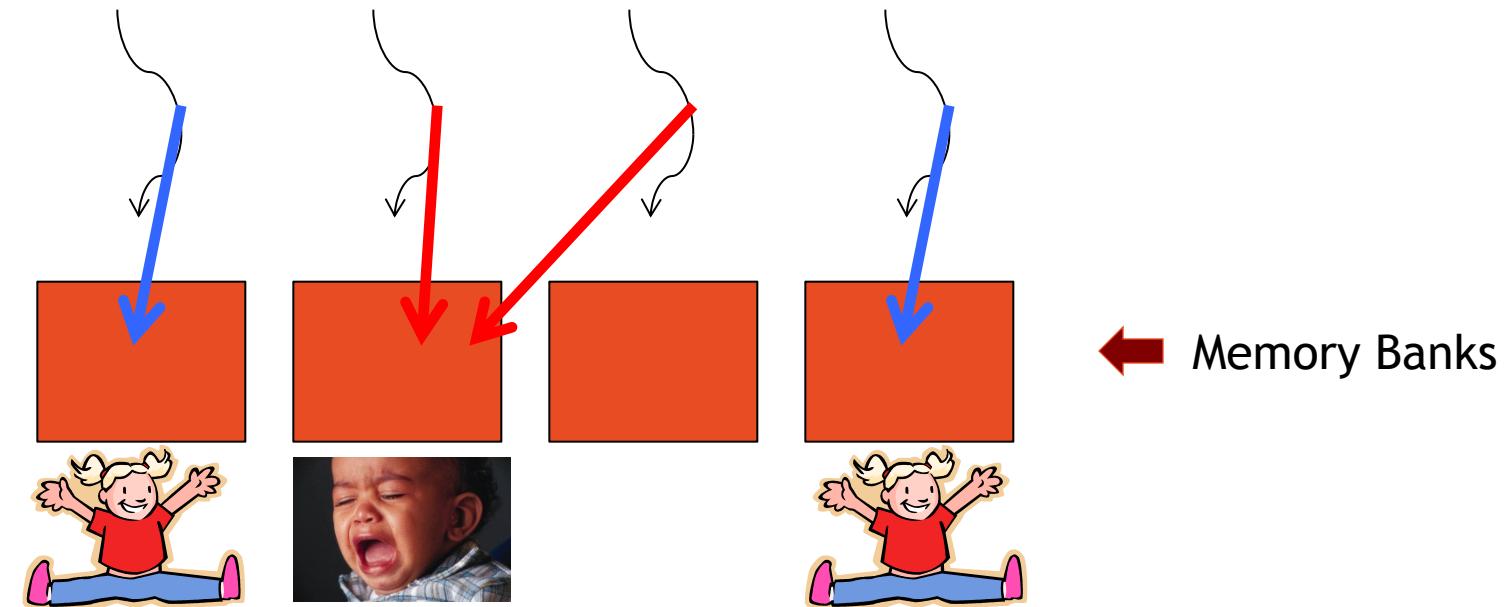
<Statements2>

}

This is a performance bug - but can be detected through formal symbolic analysis

Memory-bank Conflicts

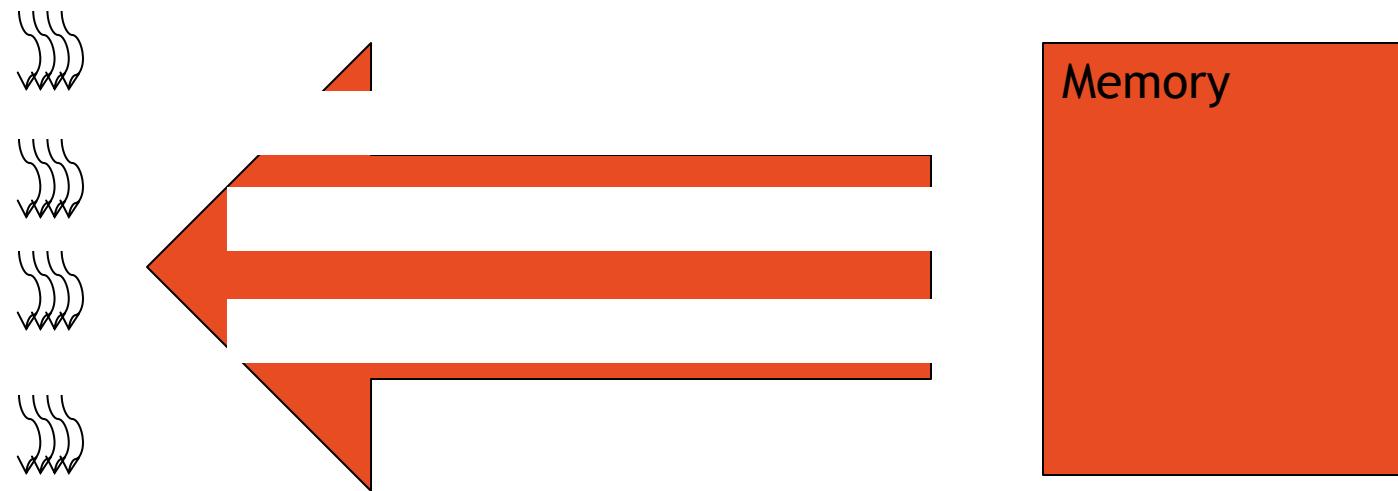
Happen when two threads generate addresses that fall into the same memory bank



This is a performance bug - but can be detected through formal symbolic analysis

Non-coalesced Memory Fetches

Happen when threads generate global memory addresses that do not fully utilize the bus



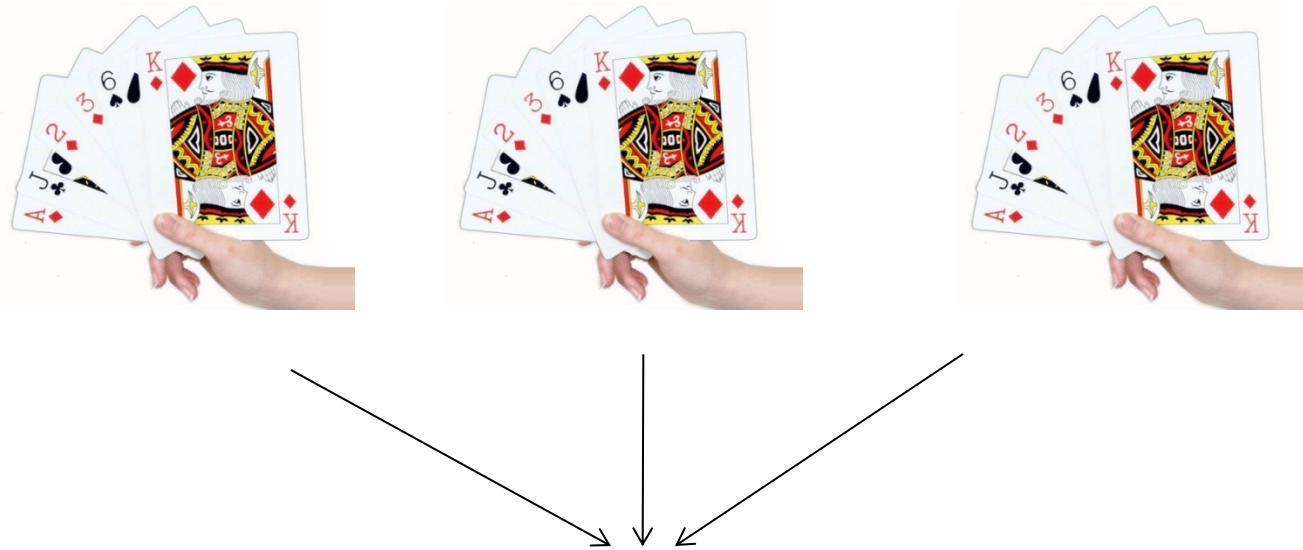
This is a performance bug - but can be detected through formal symbolic analysis
(10 x more severe than bank conflicts)

Two approaches to modeling

- Through interleaving
 - Followed in most works
- Through lock-step synchronous execution
 - Pioneered in GPUVerify

How bad is the interleaving problem?

It is like shuffling decks of cards



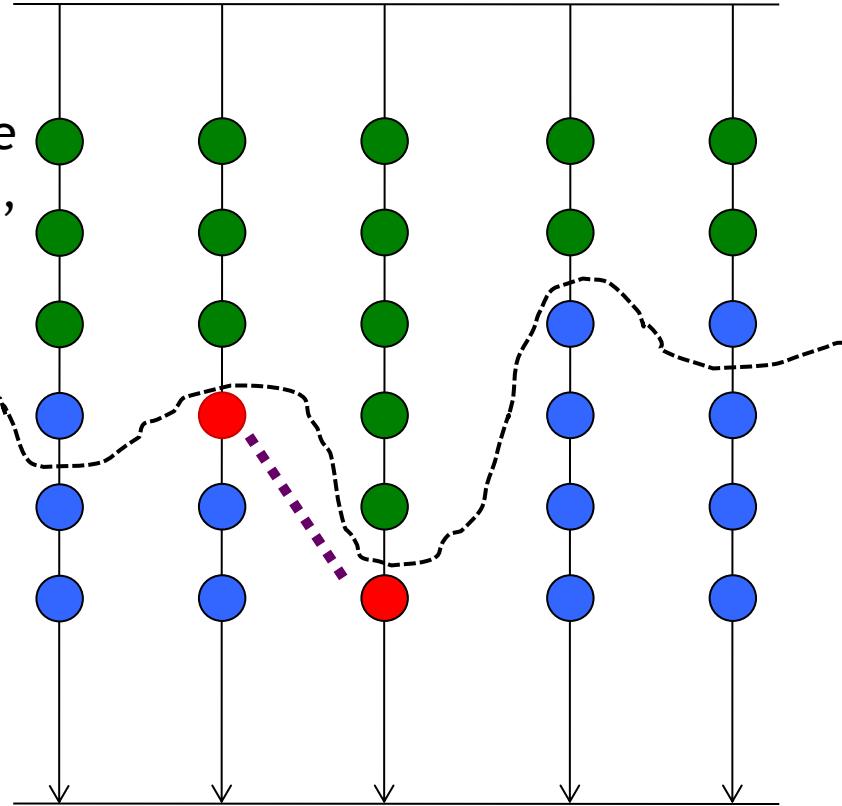
> $6 * 10^{14}$ interleavings for 5 threads with 5 instructions

Avoiding Interleaving Explosion

Solution to the interleaving problem: Find *representative* interleavings

For Example:

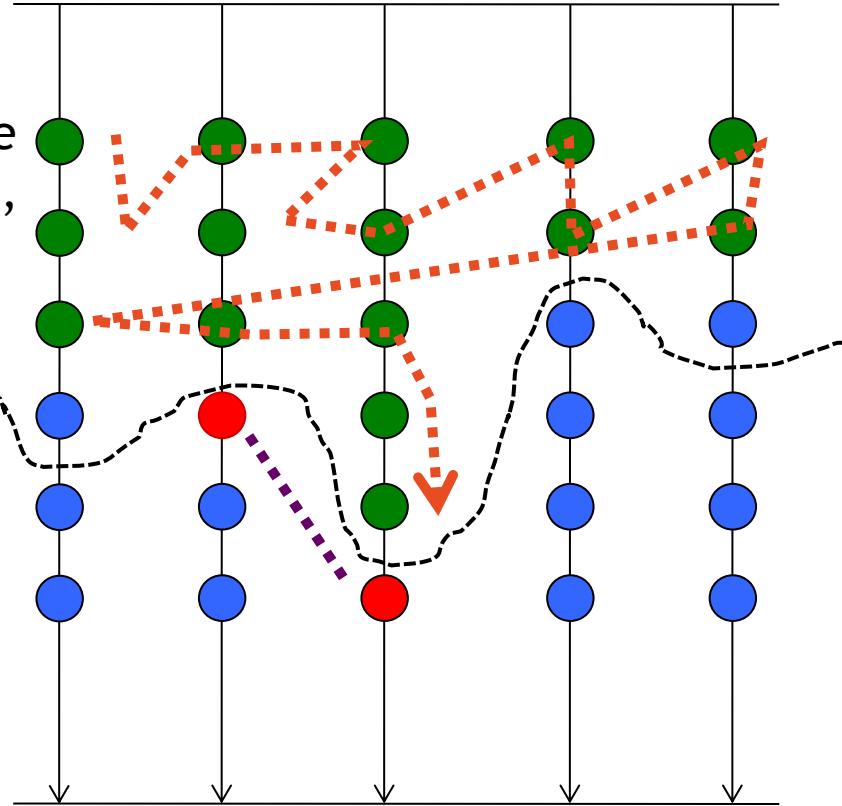
If the green dots are local thread actions,
then
all schedules
that arrive
at the “cut line”
are equivalent!



Solution to the interleaving problem: Find *representative* interleavings

For Example:

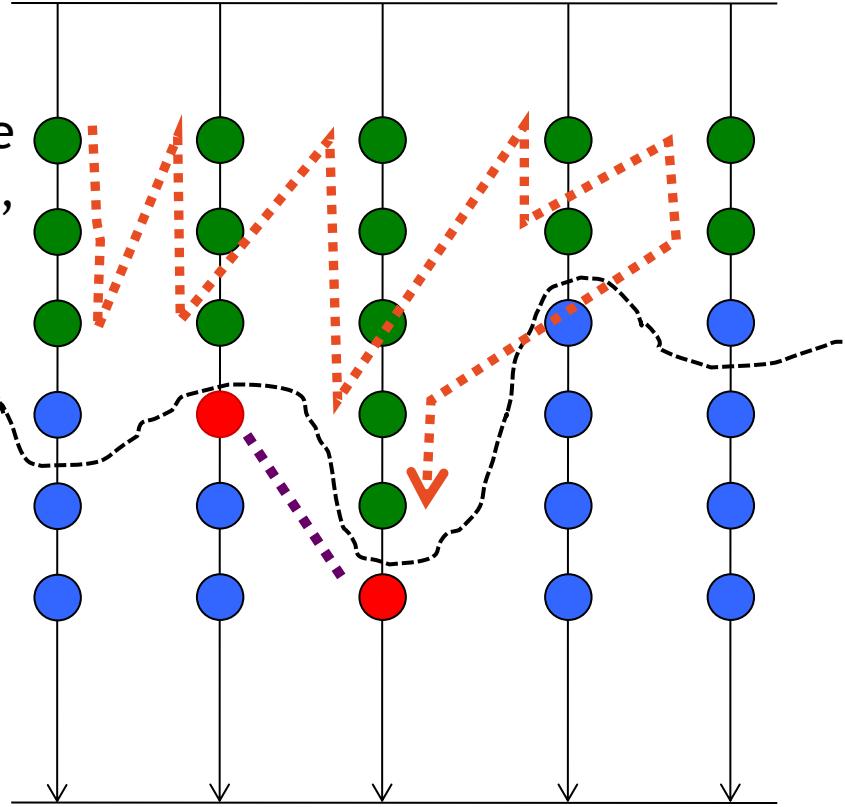
If the green dots are local thread actions,
then
all schedules
that arrive
at the “cut line”
are equivalent!



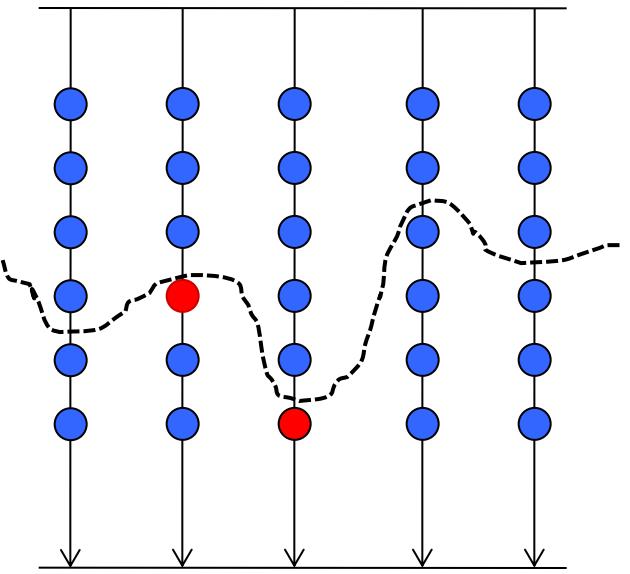
Solution to the interleaving problem: Find *representative* interleavings

For Example:

If the green dots are local thread actions,
then
all schedules
that arrive
at the “cut line”
are equivalent!

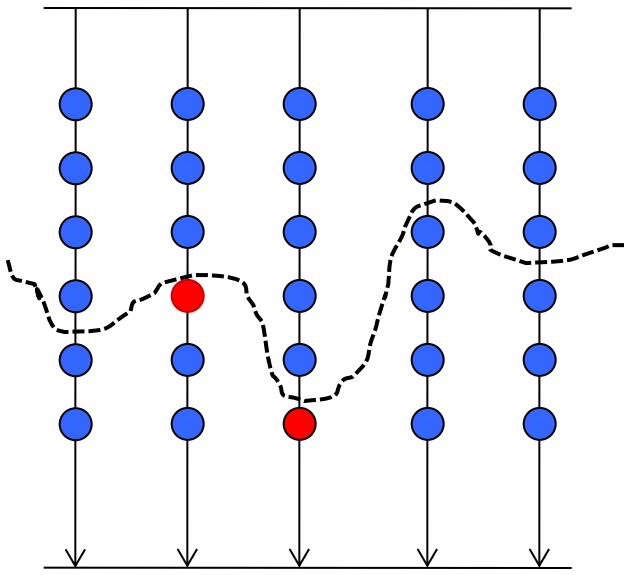


GKLEE Avoids Examining Exp. Schedules !!

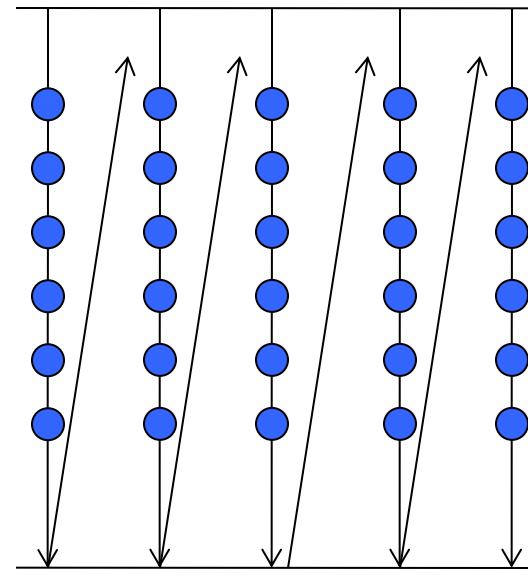


Instead of considering all
Schedules and
All Potential Races...

GKLEE Avoids Examining Exp. Schedules !!



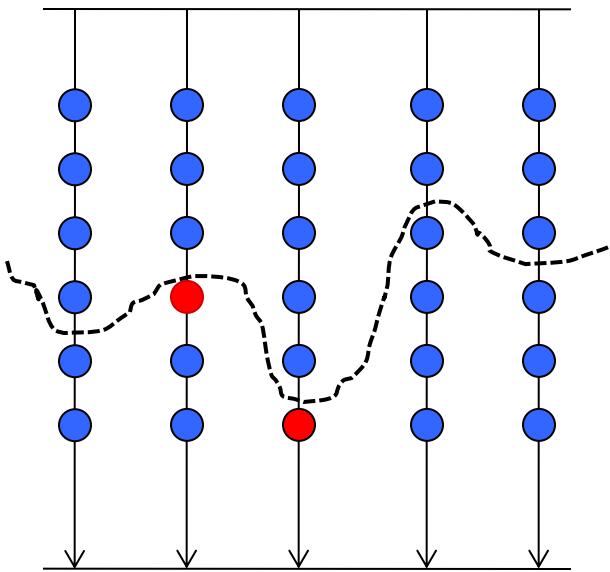
Instead of considering all
Schedules and
All Potential Races...



Consider JUST THIS SINGLE
CANONICAL SCHEDULE !!

Folk Theorem (proved in our paper):
“We will find **A RACE**
If there is ANY race” !!

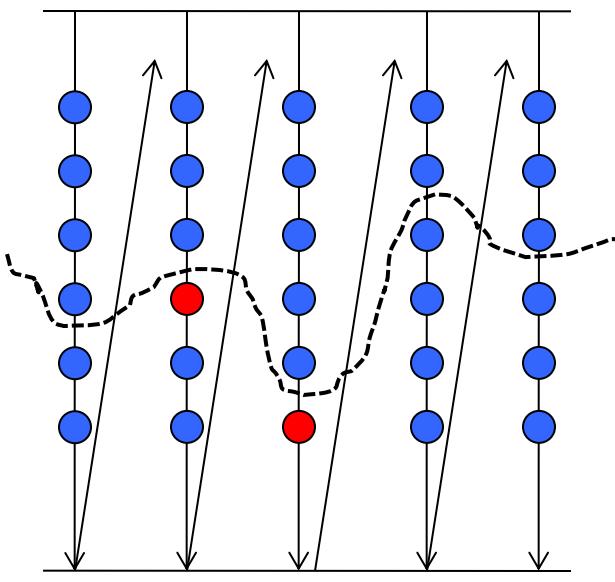
Why does a sequential run suffice?



If the red pair is the only race, then it occurs after the cut-line

So all the ways to reach the cut-line are EQUIVALENT !!

So this sequential run is equivalent too.



You my complain saying:
“But, the sequential run
Inches Past the Cutline !!”

But hey, you told me that
this is the ONLY race!

If there were some other
race encountered before,
then that will be caught.

... there is a “first race”
and that will be caught !

Concolic execution helps find witnesses to data races

```
__global__ void histogram64Kernel(unsigned *d_Result, unsigned *d_Data, int  
dataN) {  
    const int threadPos = ((threadIdx.x & (~63)) >> 0)  
                      | ((threadIdx.x & 15) << 2)  
                      | ((threadIdx.x & 48) >> 4);  
    ...  
    __syncthreads();  
    for (int pos = IMUL(blockIdx.x, blockDim.x) + threadIdx.x; pos < dataN;  
         pos += IMUL(blockDim.x, gridDim.x)) {  
        unsigned data4 = d_Data[pos];  
        ...  
        addData64(s_Hist, threadPos, (data4 >> 26) & 0x3FU); }  
        __syncthreads(); ...  
    }  
    inline void addData64(unsigned char *s_Hist, int threadPos, unsigned int data)  
    { s_Hist[ threadPos + IMUL(data, THREAD_N) ]++; }
```

“GKLEE: Is there a Race ?”

Concolic execution helps find witnesses to data races

```
__global__ void histogram64Kernel(unsigned *d_Result, unsigned *d_Data, int  
dataN) {  
    const int threadPos = ((threadIdx.x & (~63)) >> 0)  
                      | ((threadIdx.x & 15) << 2)  
                      | ((threadIdx.x & 48) >> 4);  
    ...  
    __syncthreads();  
    for (int pos = IMUL(blockIdx.x, blockDim.x) + threadIdx.x; pos < dataN;  
         pos += IMUL(blockDim.x, gridDim.x)) {  
        unsigned data4 = d_Data[pos];  
        ...  
        addData64(s_Hist, threadPos, (data4 >> 26) & 0x3FU); }  
        __syncthreads(); ...  
    }  
    inline void addData64(unsigned char *s_Hist, int threadPos, unsigned int data)  
    { s_Hist[ threadPos + IMUL(data, THREAD_N) ]++; }
```

Threads 5 and and 13 have a WW race

when d_Data[5] = 0x04040404 and d_Data[13] =

Summary

This is a huge field now - symbolic execution

You get to study this in projects

Even the MSR python Dyn Symb Executor - good project!