Indian Institute of Technology Bombay

Department of Mechanical Engineering

ME 308: IEOR 1

Project Report

MID-MILE OPTIMISATION

Course Instructor: Prof. Avinash Bharadwaj & Prof. Makarand Kulkarni

Name	Roll Number
N V Sai Gangadhar	190100080
Rongali Sai Bhargav	190100102
Adesh Yadav	190100005
Kshitij Garg	190100067

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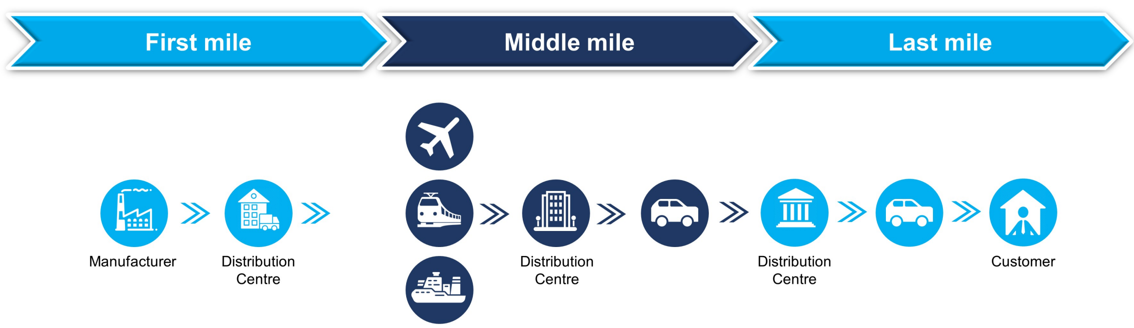
1 Introduction

1.1 What is middle mile delivery

- Middle mile delivery is delivery of goods from warehouse/manufacturing centre to the fulfillment centres.Fulfillment centres can be wholesale retail shops where customers move-in to get their products or may be another warehouse, from where it is dispatched to customers delivery address.

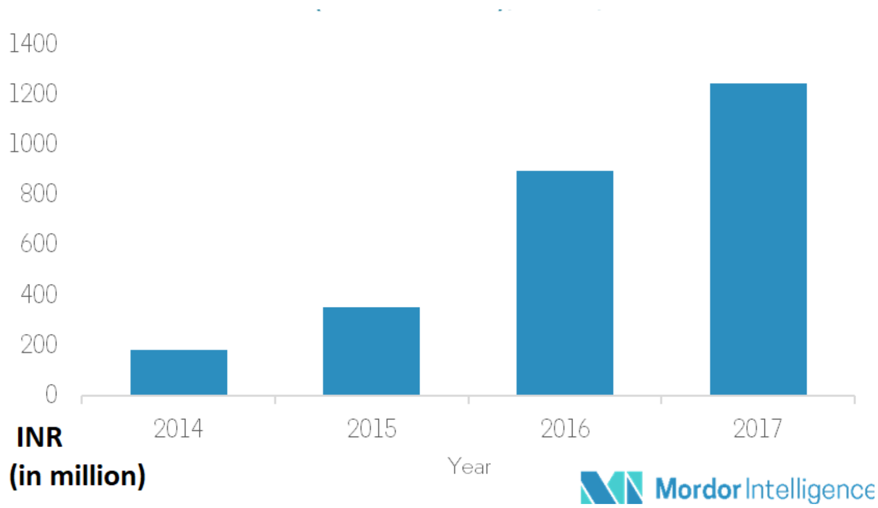
- Generally, middle mile problem deals with travelling to longer distances , so we have different modes of transportation as we have shown here, airlines, railways and seaways (Ofcourse this is one of our parameter) and uncertainty of having amount of inventory in different warehouses. Aramex is the company from where we got the data to do this project.



1.2 Motivation

1.2.1 Why optimization is required?

- Middle mile delivery can actually help tighten the supply chain and make businesses get ahead of the competition with competitive pricing and healthy margins.



- This graph depicts the increase in revenue by optimizing the middle mile logistics of company “Mordar Intelligence” during the financial period 2015-2107. This is released in their annual report

- This graph shows the increase in demand for the same from “Aramex”company and after dip due to covid there is increase by 150% and this shows how important it is to optimize the middkle mile to get a greater market share in this online era.

2 Linear Programming Formulation

2.1 Problem Statement

Set of warehouses their location, capacity, and estimated demand for any commodity from a third party company are given. Choosing origin and destination warehouses at a given point in time, we need to minimize the total cost and the route to be selected for each origin, and destination pair and find out the optimized frequency of vehicles. The total cost depends on the route taken and the number of loads that have been traversed from one warehouse to another. Given a demand V, objective is to get the route with minimum cost and find the frequency of the loads in that route.

2.2 Parameters

1. N - Total number of warehouses in the map
2. L - Total number of legs
3. M - Modes of transport available from one node to another
4. K - Total number of delivery targets to be met in a day (pairs of (O_k, D_k))
5. O_k, D_k - Origin and Destination pairs of Kth delivery pair
6. R_k - Routes that have O_k and D_k as the origin and destination locations
7. R - Total possible routes for the given number of nodes. R can be written as a summation of routes of each k = Rk
8. C_{rm} - Cost associated with each route. This is the variable cost , it can be a toll cost when a truck moves interstate or intermediate seaport with standing costs when seaways are considered.
9. $V[k]$ - Demand to be supplied from O_k to D_k node
10. A_{lm} - Cost of traversing through leg 'l' via mode 'm' . This corresponds to fixed costs like fuel costs and driver's wages, halt costs etc.
11. F_{lm} - Maximum number of loads of mode 'm' that can traverse through leg 'l' . This limitation is because of number of available trucks would be different at different warehouses, if roadway is considered
12. v_{lm} - Maximum number of shipments that can be carried in a load of mode 'm' in leg 'l'. This is directly proportional to the capacity of truck/ship/flight being used in that particular leg.
13. Information Graph: Graph[n1][n2][r] - This is a binary graph containing the information if the nodes (n1, n2) are present in the route 'r' or not. Its value is 1 if the nodes are present, else it is zero. Summation of all the legs (N_i, N_j) gives us the route 'r'.

2.3 Decision Variables

f_{lmrk} - Number of loads moving via mode 'm', in lane 'l' for the corresponding origin (O_k), destination (D_k) pair.
Note here that even though there is no specific importance for 'r' in the decision variable as it is the sum of all lanes, it is included because of making one of the constraints linear. If 'r' is not included in 'f', then there is a need for another variable 'ylmk' which is going to get activated when a particular leg 'l' is chosen to be in the route. Having to decision variables in the constraint makes it non-linear

x_{rk} {r is the route, k is the pair} =

$$\begin{cases} 1 & \texttt{Route r is selected for kth delivery pair} \\ 0 & \texttt{Route r is not selected for kth delivery pair} \end{cases}$$

2.4 Objective Function

At a given time, there will be multiple demands from multiple origins and destinations. The objective is to minimize the cost for total 'k' transfers using an optimized number of loads, mode, and in possible legs. The first term is the variable cost in a particular route and that is to be summed up over all 'k' transfers.

$$\texttt{Minimize } \Sigma_k \Sigma_r C_r \times x_{rk} + \Sigma_k \Sigma_r \Sigma_l \Sigma_m A_{lm} \times f_{lmrk}$$

2.5 Constraints

1. $$\Sigma_r x_{rk} = 1$$

   - Each delivery pair can take only one route. Multiple paths is not allowed.

2. $$\text{Inflow} = \Sigma_m f_{lmrk} \times v_{lm} \text{ for incoming leg 'l' in route 'r'}$$

   $$\text{Outflow} = \Sigma_m f_{lmrk} \times v_{lm} \text{ for outgoing leg 'l' in route 'r'}$$

   $$\text{Inflow} \leq \text{Outflow}$$

   - Outflow should be greater than or equal to Inflow at each node for the given kth pair and route. This is to ensure that demand is met at every node when reached through various modes of transport.

3. $$(\Sigma_m f_{lmrk} \times v_{lm}) - V[k] \times x_{rk} \geq 0 \forall \text{ k in } K \text{ and r in } R \text{ where 'l' is the first leg of route r}$$

   - Demand is given as an input from outside and initial flow should be equal to that demand.

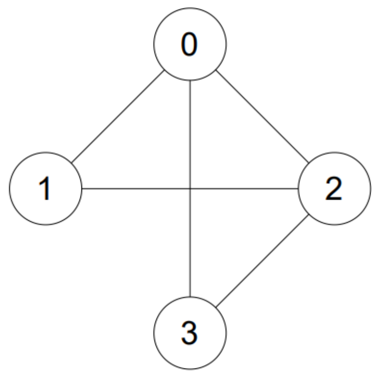
4. $$\Sigma_r \Sigma_k f_{lmrk} \leq F_{lm} \forall \text{ m in } M \text{ and } \forall \text{ l in } L$$

- Limit on the number of loads that can traverse in a given leg. 'Flm' is the maximum load as described in the parameter corresponding to the particular leg.

3 Sample Problem

To understand the problem more clearly and prove the correctness of the solution provided by our code, we are incorporating a trivial problem here. The solution can be calculated manually and can be cross-checked with the answer returned by the code.

3.1 Problem



Let the parameters be
k = 1
Origin Node - 0
Destination Node - 3
Demand - 2680 shipments / day
Possible routes - (0,3), (0,2,3), (0,1,2,3)

Let there be only two modes of transport, roadways and airways. Costs associated with these modes are different for different legs and hence it is represented as a matrix. The matrix is mentioned below.

A[0][3][air] = 100000 units per load
A[0][3][road] = 5000
A[0][2][air] = 10000
A[0][2][road] = 600
A[2][3][air] = 7500
A[2][3][road] = 100
A[0][1][air] = 7500
A[0][1][road] = 300
A[1][2][air] = 7500
A[1][2][road] = 100

Note: Since direct route Node 0 $\rightarrow$ Node 3 is available, the cost on that route is kept high to prove the correctness of our code that it doesn't directly choose the shortest path.

Maximum load frequency on each leg is given as a matrix as shown below.

F[0][3][air] = 5
F[0][3][road] = 20
F[0][2][road] = 15
F[0][2][air] = 0
F[2][3][air] = 2
F[2][3][road] = 18
F[0][1][air] = 3
F[0][1][road] = 15
F[1][2][road] = 20
F[1][2][air] = 0

Note: Frequency of loads between Node 0 → Node 2 is kept low enough so that it cannot fulfill the demand even at maximum capacity. Therefore, route Node 0 → Node 2 is not a feasible solution

Capacities (v_{lm}) of available airways and roadways are 400 and 150 respectively.

Cost associated with the routes (c_{rm}) are 50, 15, and 15 for both modes of transport.

3.2 Optimal Solution

Optimal Route - Node 0 → Node 1 → Node 2 → Node 3

Modes Chosen
Node 0 → Node 1 : Airlines , Roadway
Node 1 → Node 2 : Roadway
Node 2 → Node 3 : Roadway

Frequencies :
Node 0 → Node 1 : 3 flights , 10 trucks
Node 1 → Node 2 : 18 trucks
Node 2 → Node 3 : 18 trucks

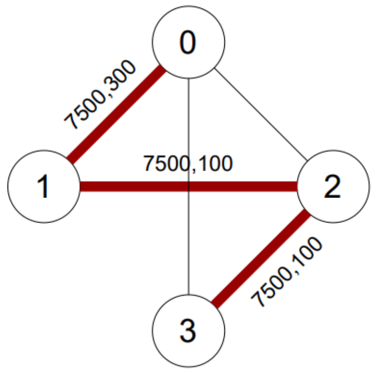
Node 0 → Node 1
Cost = 7500*3 + 300*10 = 25500
Capacity = 400*3 + 150*10 = 2700

Node 1 → Node 2
Cost = 150*10 = 1800
Capacity = 150*18 = 2700

Node 2 → Node 3

Cost = 100*18 = 1800
Capacity = 150*18 = 2700

Optimized total cost = 25500 + 1800 + 1800 + 15 = 29115 units



Optimal route showing respective costs on its edges

Result from our optimizer:

```
Restricted license - for non-production use only - expires 2023-10-25
Gurobi Optimizer version 9.5.1 build v9.5.1rc2 (linux64)
Thread count: 1 physical cores, 2 logical processors, using up to 2 threads
Optimize a model with 138 rows, 195 columns and 230 nonzeros
Model fingerprint: 0xf583d1fe
Variable types: 0 continuous, 195 integer (99 binary)
Coefficient statistics:
  Matrix range     [1e+00, 3e+03]
  Objective range  [1e+00, 1e+05]
  Bounds range     [1e+00, 1e+00]
  RHS range        [1e+00, 2e+01]
Presolve removed 134 rows and 187 columns
Presolve time: 0.00s
Presolved: 4 rows, 8 columns, 12 nonzeros
Variable types: 0 continuous, 8 integer (1 binary)
Found heuristic solution: objective 30015.000000

Root relaxation: objective 2.241500e+04, 2 iterations, 0.00 seconds (0.00 work units)

    Nodes    |    Current Node    |     Objective Bounds      |     Work
 Expl Unexpl |  Obj  Depth IntInf | Incumbent    BestBd   Gap | It/Node Time

     0     0 22415.0000    0    1 30015.0000 22415.0000  25.3%     -    0s
H    0     0                    29115.000000 22415.0000  23.0%     -    0s
     0     0 22415.0000    0    1 29115.0000 22415.0000  23.0%     -    0s

Explored 1 nodes (2 simplex iterations) in 0.04 seconds (0.00 work units)
Thread count was 2 (of 2 available processors)

Solution count 2: 29115 30015

Optimal solution found (tolerance 1.00e-04)
Best objective 2.911500000000e+04, best bound 2.911500000000e+04, gap 0.0000%
```

The output of the code shows ‘Best objective’ as 29115 which matches with the analytical solution

4 Optimised Results

4.1 Real World Data

- We have taken our data from the Aramex Delivery Company Logistics which is the delivery partner for Amazon India. They have 50 warehouses throughout India and their centres are divided as primary facility centres and secondary facility centres. Major cities like Chennai, Mumbai, Delhi, Kolkata are considered as the origins or the primary facility centres whereas cities like Jaipur, Ludhiana, Gurugram are considered as the destinations or the secondary facility centres.

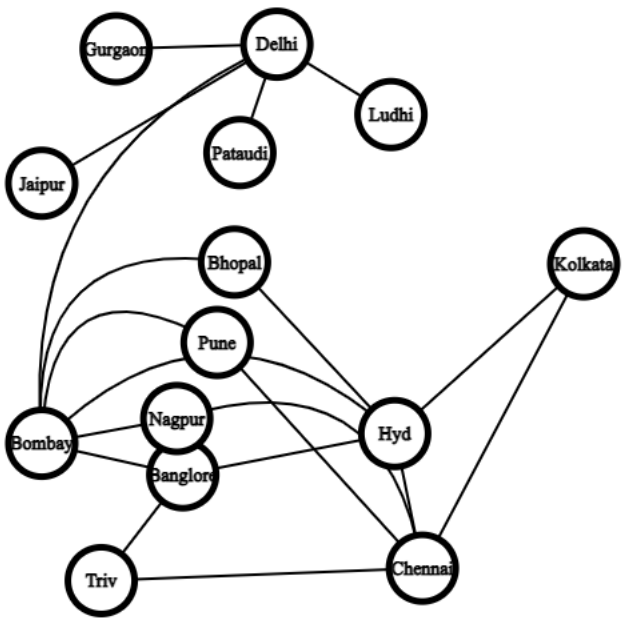
- For simplicity in our model we have assumed some of our own data and restricted total warehouses to only 13 because more of them exhausts our model. Also in our model each city can be origin and each city can also be destination.

- We have acquired the data like number of shipping and assumed the cost associated with each type of truck including the halt cost.Demand of the commodity and the max loads which can be shipped via each leg.

- These costs are based upon considering our mode is via road only and also we are considering only one commodity. Demand (number of shipments) is going to be different in different routes. So, we have assumed the demand for each of the legs as shown in this excel sheet then we got these results.

S.No	Ware House1	Ware House2	Distance (km)	Demand	Available modes	Optimized mode	6 tyre truck(10-12 km/l	8 tyre truck(6-10 km/l)	Working hrs	Drivers wage
1	Delhi	Ludhiana	308		Road		2566.666667	3080	3.82608695	454.3478261
2	Delhi	Jaipur	340		Road, Airways		2833.333333	3400	4.22360248	501.552795
3	Delhi	Bombay	1412		Airways, Road		14120	23533.33333	17.5403726	2082.919255
1	Delhi	Pataudi	380		Road		3166.666667	3800	4.72049689	560.5590062
1	Bombay	Pune	310		Road		2583.333333	3100	3.85093167	457.298136
1	Bombay	Nagpur	900		Road		9000	11250	11.1801242	1327.639752
1	Bombay	Banglore	1000		Road, Airways		10000	16666.66667	12.4223602	1475.15528
1	Bombay	Bhopal	780		Road, Airways		6500	9750	9.68944099	1150.621118
1	Chennai	Nagpur	1125		Road, Airways		11250	18750	13.9751552	1659.549689
1	Chennai	Hyderabad	700		Road,Airways		5833.333333	8750	8.69565217	1032.608696
1	Chennai	Pune	1200		Road, Airways		12000	20000	14.9068323	1770.186335
1	Chennai	Trivendram	725		Road, Airways		6041.666667	9062.5	9.00621118	1069.487578
1	Bhopal	Hyderabad	865		Road, Airways		7208.333333	10812.5	10.7453416	1276.009317
1	Kolkata	Hyderabad	1500		Road, Airways		15000	25000	18.6335403	2212.732919
1	Kolkata	Chennai	890 (Nautical miles)		Ship					
1	Bombay	Trivendram	740 (Nautical miles)		Ship					

6 tyre truck(10-12 km/l	8 tyre truck(6-10 km/l)	Working hrs	Drivers wage	6 tyre total cost	8 tyre total cost	Transit cost (0,300,500)	
2566.666667	3080	3.82608695	454.3478261	3021.014493	3534.347826	3021.014493	3534.347826
2833.333333	3400	4.22360248	501.552795	3334.886128	3901.552795	3334.886128	3901.552795
14120	23533.33333	17.5403726	2082.91925	16202.91925	25616.25259	16702.91925	26116.25259
3166.666667	3800	4.72049689	560.5590062	3727.225673	4360.559006	3727.225673	4360.559006
2583.333333	3100	3.85093167	457.298136	3040.63147	3557.298137	3040.63147	3557.298137
9000	11250	11.1801242	1327.639752	10327.63975	12577.63975	10627.63975	12877.63975
10000	16666.66667	12.4223602	1475.15528	11475.15528	18141.82195	11975.15528	18641.82195
6500	9750	9.68944099	1150.621118	7650.621118	10900.62112	7950.621118	11200.62112
11250	18750	13.9751552	1659.54968	12909.54969	20409.54969	13409.54969	20909.54969
5833.333333	8750	8.69565217	1032.60869	6865.942029	9782.608696	7165.942029	10082.6087
12000	20000	14.9068323	1770.18633	13770.18634	21770.18634	14270.18634	22270.18634
6041.666667	9062.5	9.00621118	1069.48757	7111.154244	10131.98758	7411.154244	10431.98758
7208.333333	10812.5	10.7453416	1276.009317	8484.34265	12088.50932	8784.34265	12388.50932
15000	25000	18.6335403	2212.73291	17212.73292	27212.73292	17712.73292	27712.73292

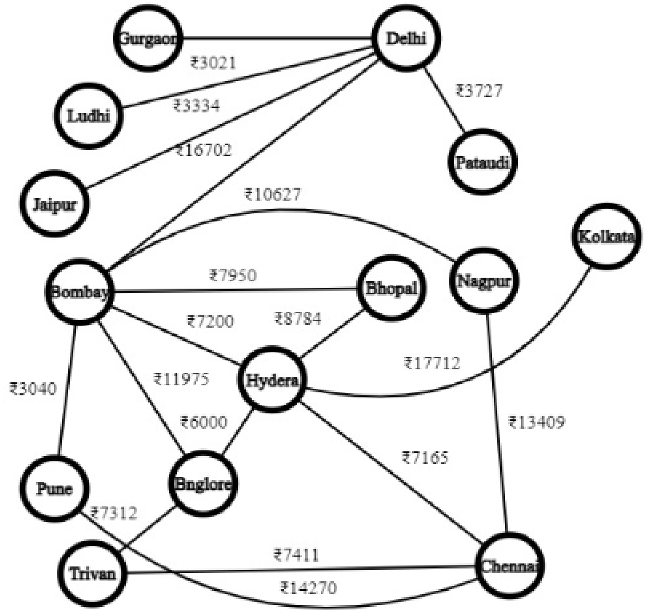


4.2 Modes of Transport

Two types of vehicles for road ways are assumed ,

- Truck 1 (6 wheeler) - 1400 lbs capacity
- Truck 2 (8 wheeler) - 1800 lbs capacity

4.2.1 Truck Type 1 : 6 Wheeler

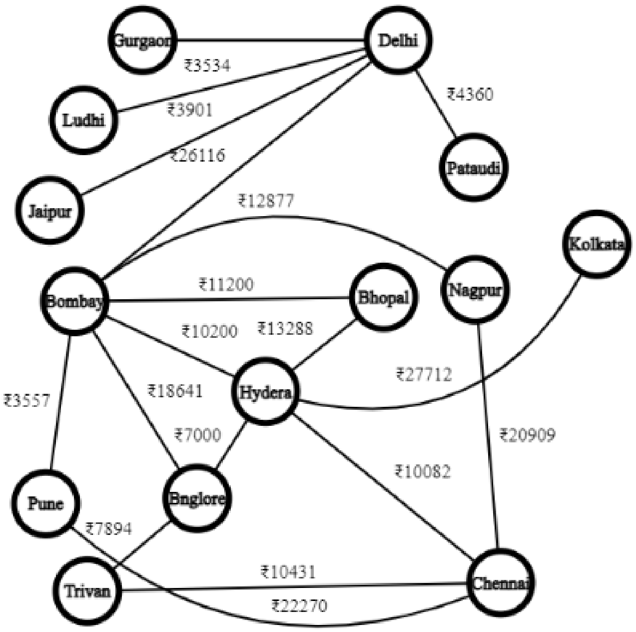


The chart here represents the cost function for each leg through which we perform our optimization. On performing our Optimization we have tabulated the results for these routes.Demand is tabulated in the above excel sheet. These are the results for 6 wheeler trucks

Warehouse1	Warehouse2	Optimized Cost(in INR)
Mumbai	Kolkata	87,918
Delhi	Bhopal	65,715
Mumbai	Trivendram	77,600
Nagpur	Chennai	52,650
Delhi	Chennai	96,415
Delhi	Hyderabad	73,650
Kolkata	Trivendram	81,100
Banglore	Bhopal	43,540
Jaipur	Nagpur	66,870
Bhopal	Chennai	58,930
Jaipur	Kolkata	96,350
Hyderabad	Pataudi	75,870
Hyderabad	Ludhiana	74,660

Table 1: Optimization results for 6 wheeler truck

4.2.2 Truck Type 2 : 8 Wheeler



Warehouse1	Warehouse2	Optimized Cost(in INR)
Mumbai	Kolkata	80,640
Delhi	kolkata	64,890
Mumbai	Trivendram	80,935
Nagpur	Chennai	56,900
Delhi	Chennai	1,05,200
Delhi	Hyderabad	69,900
Kolkata	Trivendram	70,400
Banglore	Bhopal	49,600
Jaipur	Nagpur	60,780
Bhopal	Chennai	61,480
Jaipur	Kolkata	87,540
Hyderabad	Pataudi	70,740
Hyderabad	Ludhiana	70,950

Table 2: Optimization results for 8 wheeler truck

It is observed that the optimized cost have changed for all our routes. It has increased for some routes whereas decreased for others, this is because there are two opposing factors. First the number of truck that can move in a route and second cost associated with each truck. The cost associated with the truck increases but here the number of shipments also increases simultaneously. These two factors attributed for the results we got. Our interpretation from these results is that the optimized cost reduces for Mumbai to Kolkata whereas there is a increase for Delhi to Chennai. Amount and cost of shipments that can travel in airways, seaways is not precisely available but our code chooses the mode as shown in sample problem.

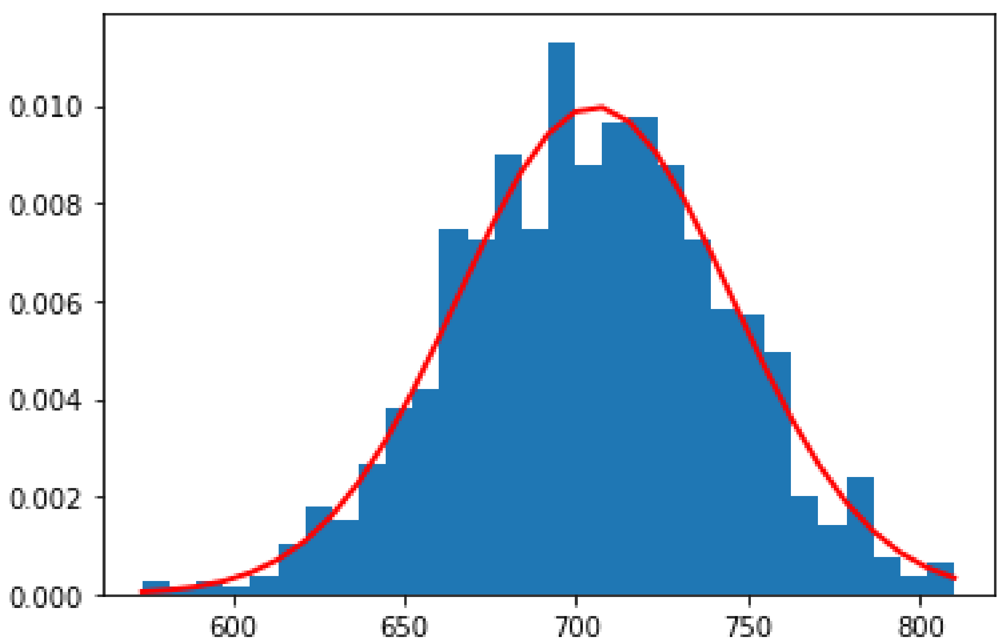
5 Uncertainty Analysis

There are different possible uncertainties in our model.

- The demand for commodities from origin to destination nodes is not always constant, hence it should be probabilistic rather than deterministic.
- Costs associated with the mode are also uncertain to some extent when petrol/diesel prices are taken into account and daily wages/salaries of labor are considered which our mode doesn't taken into account.
- Based on the demand for a particular route, the number of trucks that are actually available for transport and inventory costs are some of the uncertainties that can be listed.

5.1 Demand Uncertainity

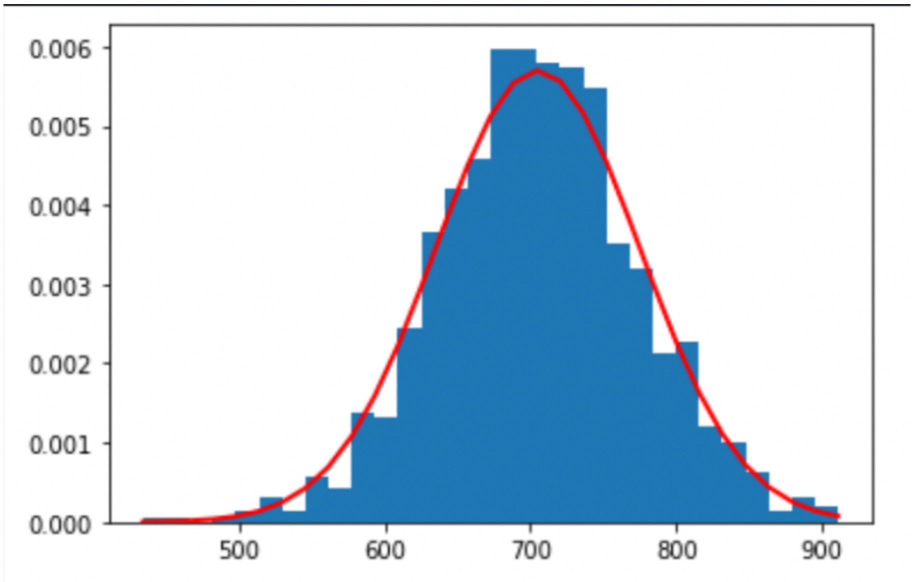
We have modeled demand uncertainty as normal distribution and sampled from it. Data from 'Aramex' has given $\mu = 705 shipments/day, \sigma = 70 shipments/day$.



$$\text{Demand} = \mu \pm 3 * \sigma$$
$$\text{Demand} = (705 \pm 210) shipments/day$$

Demand is sampled from the distribution and it is used as input to our model for the sample problem discussed in the before section. For one (Ok,Dk) pair the output frequencies are plotted against probability distribution functions.

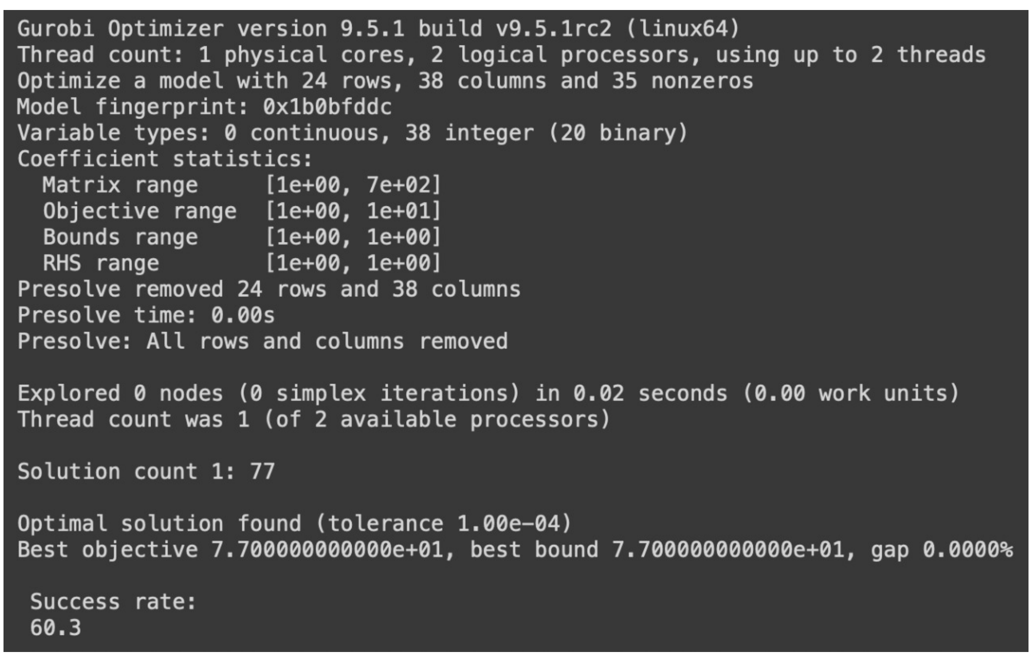
5.2 Output Of Sampling



Probability distribution function vs frequency

5.3 Simulation for multiple demands

For the stated inputs in the sample problem, the demand is sampled from the distribution 1000 times and 603 times the constraints are satisfied, and obtained optimal results. The code snippet of the simulation is shown below.



```
Gurobi Optimizer version 9.5.1 build v9.5.1rc2 (linux64)
Thread count: 1 physical cores, 2 logical processors, using up to 2 threads
Optimize a model with 24 rows, 38 columns and 35 nonzeros
Model fingerprint: 0x1b0bfddc
Variable types: 0 continuous, 38 integer (20 binary)
Coefficient statistics:
  Matrix range     [1e+00, 7e+02]
  Objective range  [1e+00, 1e+01]
  Bounds range     [1e+00, 1e+00]
  RHS range        [1e+00, 1e+00]
Presolve removed 24 rows and 38 columns
Presolve time: 0.00s
Presolve: All rows and columns removed

Explored 0 nodes (0 simplex iterations) in 0.02 seconds (0.00 work units)
Thread count was 1 (of 2 available processors)

Solution count 1: 77

Optimal solution found (tolerance 1.00e-04)
Best objective 7.700000000000e+01, best bound 7.700000000000e+01, gap 0.0000%

 Success rate:
 60.3
```

6 Assumptions

- The travelling speed was taken as constant throughout the journey
- Taken halt times but not exact traffic times to complete the journey
- Average wage of labour/driver is taken
- The price of fuel was assumed to be constant throughout the country
- Inventory costs of trucks was ignored
- No outsourcing of trucks in case excessive demand
- Total average weight of products per truck is considered which may not be true in all cases

7 Optimizer Used

- Gurobipy, version 9.5.1 is used for linear programming, coded in python programming language
- The Gurobi distribution includes a Python interpreter and a basic set of Python modules, it is faster when compared with other solvers like OR tools
- The query and modification method is associated with set of attribute values unlike other solvers which uses variables

8 References

https://www.hindawi.com/journals/mpe/2020/7248492/
https://support.route4me.com/faq/route-planning-glossary/middle-mile-delivery/
https://www.fleet-pulse.com/the-rise-of-middle-mile-delivery/
https://lessplatform.com/blog/distribution-management/what-is-a-middle-mile-logistics-and-how-to-optimize-it/
https://csacademy.com/app/graph$_{e}$*ditor*/