Assignment-Solutions
Hypothesis Testing

A textile factory consumes on average $1000m^3$ of water per day. If a sample of 100 days is drawn randomly to test is the mean daily water intake remains 1000 m^3 against the alternative that the mean water consumption has increased.

Assume that the sample mean equals ¯x (x-bar) = 1005 m^3

and the sample variance is $s^2 = 400\ m^6$.

a) Define the null and alternative hypotheses for this test
b) Perform the test at the significance level $\alpha = 0.05$.
c) What is the smallest value of α for which the null hypothesis can be rejected?
d) Calculate the power of the test in two scenarios: If the true mean water consumption equals $\mu 1 = 1000m^3$ and $\mu 1 = 1008m^3$.
e) Indicate which of the following statements are true/false and justify your answer:
    1. If we reject the hypothesis at the level $\alpha = 0.05$, we can also reject H_0 at the level $\alpha = 0.1$
    2. The Type-I error is the probability to reject the null hypothesis when H1 is true
    3. If the p-value equals 0.15, we can reject the null hypothesis at the level 10%

Solution:

a) Hypothesis stated as

H_0: $\mu \leq 1000$ m3 vs. H1: $\mu > 1000$ m3

b) Under H_0, T =

$$\frac{\bar{X}-\mu_0}{s/\sqrt{n}} \sim_{aprox.} \mathcal{N}(0,1).$$

t = 1005 – 1000/ √ 400/ √ 100 = 2.5

Rejection Region is given by

$RR_{0.05}$= ($z_{0.05}$, ∞) = (1.65, ∞)

$t \in \quad RR_{0.05}$, we reject H_0 for $\alpha = 0.05$

c) p-value = P (Z > 2.5) = 0.0062, where $Z \sim N(0, 1)$

d) The probability of rejecting H_0 when the true mean equals μ1.

Thus, power(μ1) = P (X¯ − μ0 /s/√ n > 1.65 | μ = μ1)

= P (X ¯ >1003.3 | μ = μ1)

≈ P (Z > 1003.3 − μ1/ 2).

For μ1 = 1000 m3, as μ1 = μ0, the power coincides with the significance level α.

That is, power of test (1000) = 0.05.

For μ1 = 1008 m3,

power of test (1008) ≈ P (Z > −2.35)

= 1 – P (Z > 2.35)

= 0.9906

e)

1) True (our p-value is less than 0.05, implying that it is also lower than 0.1)
2) False (it is the error made when we reject H_0 while H_0 is true),
3) False (we would reject for significance levels larger than 15%)