Assignment-Solutions
Probability Distributions

1. Simultaneously 10 coins were tossed, the probability of getting head for each coin is 0.6. What is the probability of getting 4 heads?

   Solution: Probability of getting head p = 0.6

   Probability of getting head q = 1 – p = 1 - 0.6 = 0.4
   Probability of getting 4 heads out of 10 =

   P(X=4) = $C_4^{10}\ 0.6^4\ \ 0.4^6$ = 0.111

2. Three employees are randomly selected. The probability a single employee is between 180 and 185 cm is 0.1157.

   a) What is the probability that all three are between 180 and 185 cm height?
   b) What is the probability that none are between 180 and 185 cm?

   Solution:

   a) The probability that all 3 are between 180 and 185

      = 0.1157 * 0.1157 * 0.1157 = 0.0015

   b) The probability that none are between 180 and 185

      = (1 - 0.1157) ^ 3 = 0.69

3. Let's do a "shock" study and select 4 students. Let's name them as
   a) Carol
   b) Zack
   c) Sarah
   d) James

   What exactly be the chance one of them will be success? Assume that 35% of the students a success.

Verify the situation where Carol is the only one to refuse to give the most severe shock has probability (0. 3) * (0.65) ^ 3

Solution: a) Let us assume a scenario where one person refuses:

P (A = refuse , B = shock , C = shock ,D = shock)

P (A = refuse) P (B = shock) P(C= shock) P (D = shock)

= (0.35) * (0.65) * (0.65) * (0.65) = (0.35) ^ 1 (0.65) ^ 3

= 0.096

But there are 3 other scenarios where: ZACK, SARAH AND JAMES could have been the one to refuse. Considering each case, the Probability is again (0.35) ^ 1 (0.65) ^ 3

These 4 scenarios exhaust all the possible ways that exactly one of these 4 students refuse to administer the most severe shock.

Hence the total probability = 4 * (0.35) ^ 1 (0.65 ^ 3 = =0.38

b) P (A = shock, B = refuse , C = shock ,D = shock)

= (0.65) * (0.35) * (0.65) * (0.65) = (0.35) ^ 1 (0.65) ^ 3

4. On observing 59 Passengers or fewer at station who are constantly using mobile in a sample of 400. If the true portion of passengers on using mobile is p = 0.20. Use the normal approximation to estimate the probability of passengers using mobile 59 or fewer?

Solution: Here np and n (1 – p) are at least 10

np = 400 * 0.20 = 80 n(1-p) = 400 * 0.8 = 320.

Normal approximation using mean and standard deviation =

μ = np = 80

σ = $\sqrt{np(1-p)}$ = 8

The probability of passengers using mobile fewer than 59 is required.

5. A survey indicates that for each trip Zack goes on for shopping in mall and spends an average μ=45 minutes with a standard deviation of σ=12 minutes. The length of time spent in the mall is normally distributed and is represented by the variable x.

   If 200 shoppers enter the mall, how many shoppers would you expect to be in the mall for each interval of time listed below?

   a) Between 24 and 54 minutes
   b) More than 39 minutes

Solution: a) The z-scores corresponding to x=24 and x=54 are

: Z1= (24-45)/12= -1.75, Z2

= (54-45)/12= .75

Thus, probability that a shopper will be in the mall between 24 and 54 minutes is

P(-1.75<=Z<=.75) = F(.75)- F(-1.75) =

a. F(.75)- [1-F(1.75)] = F(.75)+F(1.75)-1

=.7333 (from the standard normal table)

Interpretation is that 73.33 % probability that shopper will be in the mall between 24 and 54 minutes after entering.

So if 200 shoppers enter the stop, we expect

(200*.7333) =146.66 or 147 shoppers to stay between 24 and 54 minutes.

b)

The z-score corresponding to 39 mins is

Z= (39-45)/12 = -.5 P(Z> (-.5)) = 1- P(Z<=.5) = 1-.3085= .6915

If 200 shoppers enter the mall, you would expect 200* (.6915) = =138.3 shoppers to stay in the mall for more than 39 minutes.

6. A truck from Gati has to carry load from Mumbai to Chennai. The amount of diesel consumed is normally distributed random variable X, with μ = 5.7, σ = 0.5. Freight management wants to find the amount of fuel to fill so that there will be 0.99 probability that truck reaches Chennai on time.

Solution: First Step find the value of Z such that P(Z<= z)= 0 .99.

From the standard normal table, the value of z corresponding to 0 0.99 is 2.33

Transforming the z value to an x value, we get

x = μ+σ *z =5.7 + (0.5) *(2.33) = 6.865.

Thus, the truck should be loaded with 6.865 tons of diesel to give a 0.99 probability that the fuel will last throughout the travel.

7. Consider 80% of all business start-ups in the IT industry in the city report that they generate a profit in their first year. If a sample of 10 new IT business start-ups are selected, find the probability that exactly seven will generate a profit in their first year.

Solution: n = 10, p=0.80, q=0.20, x=7

$$P(x=7) = \frac{10!}{7!(10-7)!} 0.80^7 (1-0.80)^{10-7}$$

$$P(x=7) = \frac{10(9)(8)(7)(6)(5)(4)(3)(2)(1)}{[7(6)(5)(4)(3)(2)(1)]\ [(3)(2)(1]} 0.80^7 (1-0.80)^{10-7} = 0.2013$$

When the probability of profit in the first year for each start up is 80% there is a 20.13% probability that exactly 7 of 10 IT start ups will generate a profit in their first year

8. A roulette wheel consisting of 38 numbers 1 through 36, 0, and double 0. If Sam always bets that the outcome will be one of the numbers 1 through 12, what is the probability that Sam will lose his first 5 bets?

Solution: Sam always bets on the numbers 1 through 12, which occupy 12 spaces on the wheel

The probability of success (winning) is p = 12/38.

The first five bets form a finite set of n = 5 trials.

Each spin of the roulette wheel is independent, and the probability of success p is constant hence the binomial model is used.

'W' be the event that Sam wins bet. Using independence, the probability that Sam loses his first bet is

P(W'W'W'W'W') = (26 /38) ^ 5 = 0.15 app

Using Binomial Let X be the number of bets Sam wins, then,

$$P(X=0) = \binom{5}{0}\left(\frac{12}{38}\right)^0\left(\frac{26}{38}\right)^5 = \left(\frac{26}{38}\right)^5 \approx \boxed{0.15}$$

9. Ram got fever and has gone for diagnostic centre for test. A diagnostic test has a 0.95 probability of giving a positive result when tested on person affected with typhoid and 0.10 probability of giving as (false) positive when applied to non-typhoid.

It is estimated that 0.5% of the population are suffering from typhoid. If a person is selected from the population and suppose a test is conducted on him where we have no relevant information relating to fever.

Calculate the following probabilities:

A. that the test result will be positive;
B. that, given a positive result, the person is a suffering from typhoid;
C. that, given a negative result, the person is a non-typhoid;
D. that the person will be misclassified.

Solution: Let

T ≡ "Test positive",
F ≡ "Fever",
M ≡ "Misclassified"

Then $P(T|F) = 0.95$,
$P(T|S') = 0.10$,
$P(S)=0.005$.

Hence

a) $P(T) = P(T|F)\,P(F) + P(T|F')\,P(F') =$
$= (0.95 \times 0.005) + (0.1 \times 0.995)$
$= 0.10425$.

b) $P(F|T) = P(T|F)\,P(F)\;P(T|F)\,P(F) + P(T|F')\,P(F')$
$= 0.95 \times 0.005\;(0.95 \times 0.005) + (0.1 \times 0.995)$
$= 0.0455$.

c) $P(F'|T') = P(T'|F')P(F')\;P(T')$
$= 0.9 \times 0.995\;1 - 0.10425$
$= 0.9997$.

d) $P(M) = P(T \cap F') + P(T' \cap F)$
$= P(T|F')P(F') + P(T'|F)\,P(F)$
$= 0.09975$.

10. Brooklyn High School's A level is normally distributed. The total data which is 95% is between the age 15.6 and 18.4. Can you find the mean and standard deviation?

Solution: From Empirical rule 68 -95-99, normal distribution 95 % data comes under 2-standard deviation

Mean = 15.6 + 18.4 / 2 = 17

From the mean 17, to one end 18.4, there are 2 standard deviations.

Standard deviation = 18.4 – 17 / 2 =0.7

11. For a job interview a test has been taken and the final results have a mean of 70 and a standard deviation of 10. If normally distributed what percent of the job seekers.

    a) Test score higher than 80?
    b) Should pass the test (Test score ≥ 60)?
    c) Should fail the test (Test score <60)

Solution:

a) For x = 80, z = 1

Area to the right (higher than) z = 1 = 0.1586 = 15.87% scored more than 80.

b) For x = 60, z = -1

Area to the right of z = -1 = 0.8413 = 84.13% should pass the test.

c)100% - 84.13% = 15.87% should fail the test.

12. The longevity of musical instruments used by musicians has a normal distribution with mean of 12 months and standard deviation of 2 months. Find the probability that an instrument used by musicians will last.

    a) less than 7 months
    b) between 7 and 12 months.

Solution:

a) $P(x < 7) = P(z < -2.5) = 0.0062$

b) $P(7 < x < 12) = P(-2.5 < z < 0) = 0.4938$