Through Inspiration, Discovery
King Abdullah University of Science and Technology

Approximate String Matching: Bit-Parallel Approach

Midterm Presentation

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Background

Approximate string matching is one of the main problems in classical algorithms, with applications to text searching, computational biology, pattern recognition, text processing, spell checking, spam filtering, etc.

Background

For the approximate string matching problem we look for a substring that is similar to pattern P in text T. The word similar here refers to a string that needs a minimum number of operations (insertion, deletion and substitution) to be converted to P. This minimum number of operations is what we refer to as the edit distance.

Example:

Insertion: cat → c**a**s**t**

Deletion: **c**at → at

Substitution: ca**t** → ca**r**

Background

Our approximate string matching problem is defined as follows:

We are given a text $T_{1,n}$, a pattern $P_{1,m}$ and an error bound k.

We want to find all locations T_i when $1 \leqq i \leqq n$ such that there exists a suffix A of $T_{1,i}$ and $\mathrm{d}(A,P) \leqq k$ where $\mathrm{d}(x,y)$ is the edit distance (or **difference**) between x and y.

To solve our approximate string matching problem, we use a table, called $R^k[n,m]$.

$$R^k(i,j) \begin{cases} = 1 \text{ if there exists a suffix } A \text{ of } T_{1,i} \text{ such that } d(A,P_{1,j}) \leqq k. \\ = 0 \text{ otherwise.} \end{cases}$$

where $1 \leqq i \leqq n$ and $1 \leqq j \leqq m$.

It will be shown later that R^k is obtained from R^{k-1}. Thus, it is important for us to know how to find R^0. R^0 will be obtained by Shift-And algorithm later under bit-parallel. But now, we will explain it in dynamic programming [S80](String Matching with Errors).

$R^0(i,j)$ can be found by dynamic programming.

$R^0(i,0) = 1$ for all i. $R^0(0,j) = 0$ for all j.

$$R^0(i,j) \begin{cases} = 1 \text{ if } R^0(i-1,j-1) \text{ and } t_i = p_j. \\ = 0 \text{ if otherwise.} \end{cases}$$

		1	2	3	4	5	6	7	8	9
		a	a	b	a	a	c	a	a	b
	1	1	1	1	1	1	1	1	1	1
1	a	0								
2	a	0								
3	b	0								
4	a	0								
5	c	0								

Given an alphabet α and a pattern *P*, we need to know where α appears in *P*.

This information is contained in a vector Σ(α) which is defined as follows:
Σ(α)[*i*]=1 if p_i=α .
Σ(α)[*i*]=0 if otherwise.

Example: *P* : aabac
Σ(a) =(1,1,0,1,0)
Σ(b) =(0,0,1,0,0)
Σ(c) =(0,0,0,0,1)

Shift-And

$$R_i^0 = ((R_{i-1}^0 >> 1) \vee 10^{m-1}) \,\&\, \Sigma(t_i)$$

Example:

T : aabaacaabacab,

P : aabac and k=0.

$R^0[13,5]$

		1	2	3	4	5	6	7	8	9	10	11	12	13
		a	a	b	a	a	c	a	a	b	a	c	a	b
1	a	1	1	0	1	1	0	1	1	0	1	0	1	0
2	a	0	1	0	0	1	0	0	1	0	0	0	0	0
3	b	0	0	1	0	0	0	0	0	1	0	0	0	0
4	a	0	0	0	1	0	0	0	0	0	1	0	0	0
5	c	0	0	0	0	0	0	0	0	0	0	1	0	0

Σ(a)	11010
Σ(b)	00100
Σ(c)	00001
*	00000

$$R_{10}^0 = ((R_9^0 >> 1) \vee 10^4) \,\&\, \Sigma[t_{10}]$$

$$= ((0,0,0,1,0) \text{v} (1,0,0,0,0)) \& (1,1,0,1,0)$$

$$= (1,0,0,1,0)$$

Similarly, we can conclude that $R^0(10,1) = 1$.

The Shift-And formula $R^0 = ((R^0_{i-1} >> 1) \vee 10^{m-1}) \,\&\, \Sigma(t_i)$

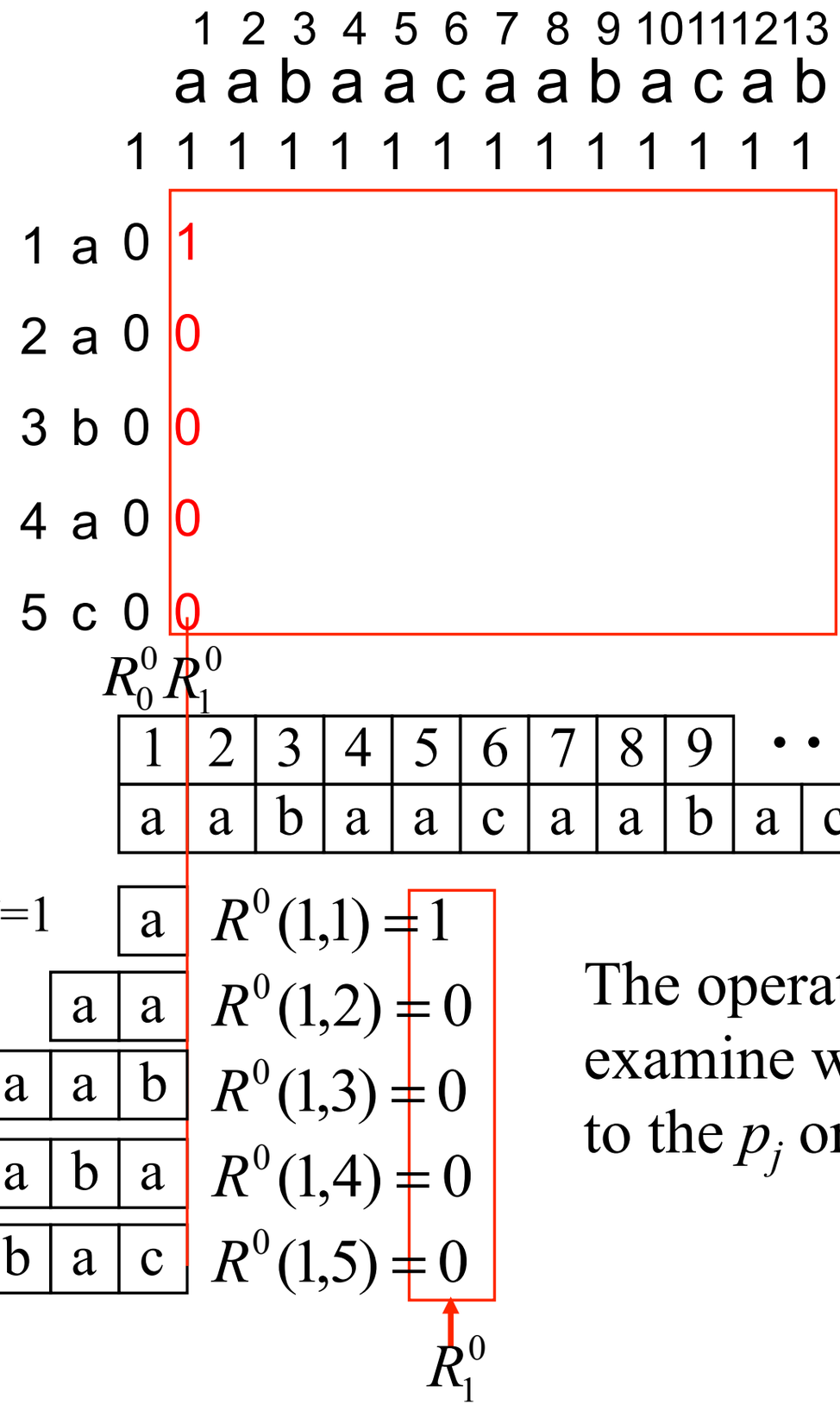
Example:

Text = aabaacaabacab

pattern = aabac

when i=1

Σ(a)	11010
Σ(b)	00100
Σ(c)	00001
*	00000



$$\begin{array}{rl} & 10000 \quad (R^0_0 >> 1) \vee 10^{m-1} \\ \& & 11010 \quad \Sigma(a) \\ \hline R^0_1 & 10000 \end{array}$$

The operation "&" will examine whether the $\Sigma(t_i)$ equals to the p_j or not currently.

The Shift-And formula $R^0 = ((R^0_{i-1} >> 1) \vee 10^{m-1}) \,\&\, \Sigma(t_i)$

Example:

Text = aabaacaabacab

pattern = aabac

when i=2

Σ(a)	11010
Σ(b)	00100
Σ(c)	00001
*	00000

			1	2	3	4	5	6	7	8	9	10	11	12	13
			a	a	b	a	a	c	a	a	b	a	c	a	b
			1	1	1	1	1	1	1	1	1	1	1	1	1
1	a	0	1	1											
2	a	0	0	1											
3	b	0	0	0											
4	a	0	0	0											
5	c	0	0	0											

$R^0_1 R^0_2$

1	2	3	4	5	6	7	8	9	. . .
a	a	b	a	a	c	a	a	b	a	c	a	b

j=1 a $R^0(2,1) = 1$

j=2 a a $R^0(2,2) = 1$

j=3 a a b $R^0(2,3) = 0$

j=4 a a b a $R^0(2,4) = 0$

j=5 a a b a c $R^0(2,5) = 0$

R^0_2

$$\begin{array}{rl} 11000 & (R^0_1 >> 1) \vee 10^{m-1} \\ \underline{\&\ \ 11010} & \Sigma(\mathrm{a}) \\ R^0_2\ 11000 & \end{array}$$

Example:

Text = aabaacaabacab

pattern = aabac

Σ(a)	11010
Σ(b)	00100
Σ(c)	00001
*	00000

			1	2	3	4	5	6	7	8	9	10	11	12	13
			a	a	b	a	a	c	a	a	b	a	c	a	b
		1	1	1	1	1	1	1	1	1	1	1	1	1	1
1	a	0	1	1	0	1	1	0	1	1	0	1	0	1	0
2	a	0	0	1	0	0	1	0	0	1	0	0	0	0	0
3	b	0	0	0	1	0	0	0	0	0	1	0	0	0	0
4	a	0	0	0	0	1	0	0	0	0	0	1	0	0	0
5	c	0	0	0	0	0	0	0	0	0	0	0	**1**	0	0

aabaacaabacab
aabac

Approximate String Matching

The approximate string matching problem is exact string matching problem with errors. We can use like Shift-And to achieve the $R^k[n,m]$ table. Here, we will introduce three operation which are insertion, deletion and substitution in approximate string matching.

$$R^k(n,m)\begin{cases} = 1 \text{ if there exists a suffix } A \text{ of } T_{1,i} \text{ such that } \mathrm{d}(A,P_{1,j}) \leqq k. \\ = 0 \text{ otherwise.} \end{cases}$$

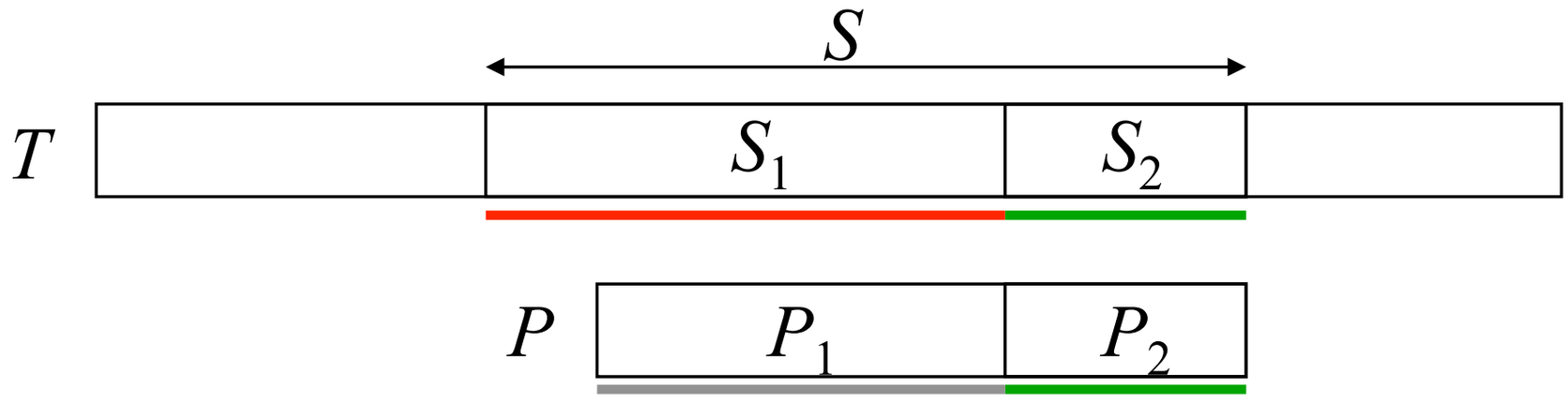
where $1 \leqq i \leqq n$ and $1 \leqq j \leqq m$.

Let $R_I^k(i,j)$, $R_D^k(i,j)$ and $R_S^k(i,j)$ denote the $R^k(i,j)$ related to insertion, deletion and substitution respectively.

$R_I^k(i,j)=1$, $R_D^k(i,j)=1$ and $R_S^k(i,j)=1$ indicate that we can perform an insertion, deletion and substitution respectively without violating the error bound which is k.

Our approach is based upon the following observation:

Case 1 :



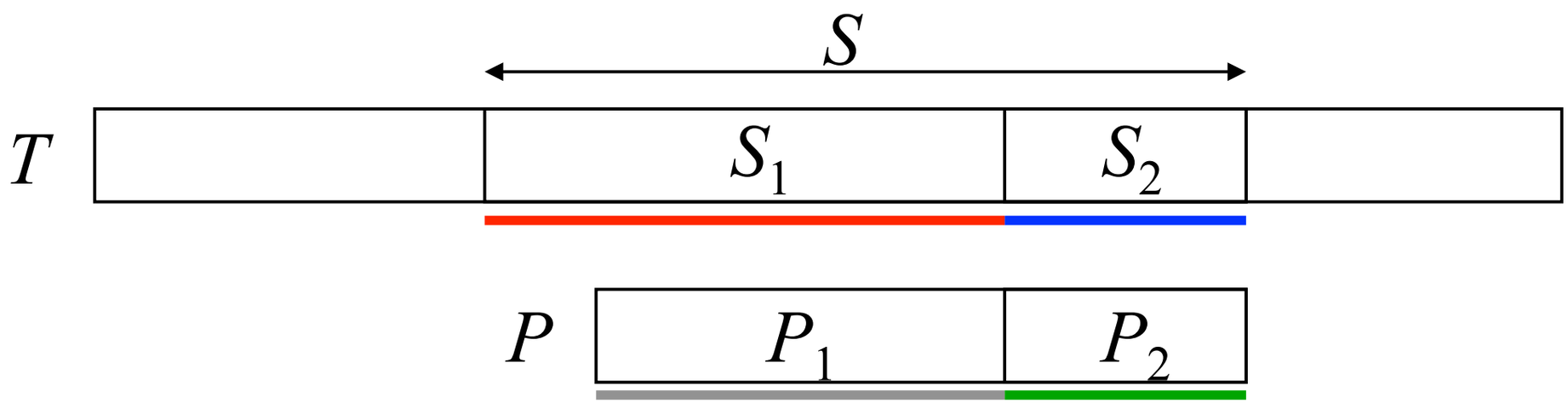
Let S be a substring of T.

If there exists a suffix S_2 of S and a suffix P_2 of P such that

$d(S_2, P_2) = 0$, and $d(S_1, P_1) \leqq k$,

we have $d(S, P) \leqq k$.

Case 2 :



Let S be a substring of T.

If there exists a suffix S_2 of S and a suffix P_2 of P such that

$d(S_2, P_2) = 1$, and $d(S_1, P_1) \leqq k\text{-}1$,

we have $d(S, P) \leqq k$.

If $t_i = p_j$, and $R^k(i-1, j-1) = 1$, $R^k(i, j) = 1$.

Otherwise, $R^k(i, j) = R_I^k(i, j) \vee R_D^k(i, j) \vee R_S^k(i, j)$.

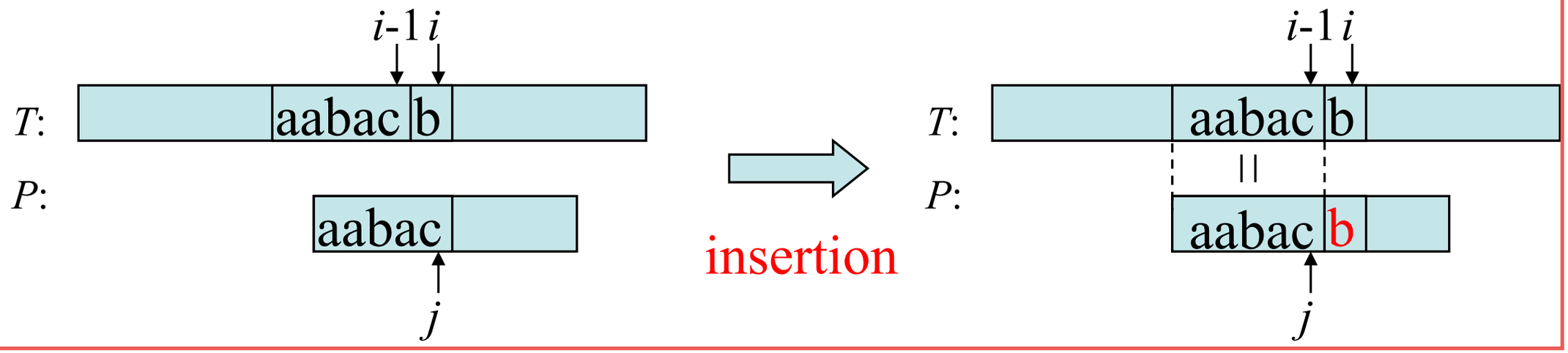
Insertion

$R_I^k(i, j) = 1$ if $t_i \neq p_j$ and $R^{k-1}(i-1, j) = 1$.

$R_I^k(i, j) = 0$ otherwise.

The insertion formula: $R_i^k = (((R_{i-1}^k >> 1) \vee 10^{m-1}) \,\&\, \sum(t_i)) \vee R_{i-1}^{k-1}$

Example: when k=1.



If t_i=p_j, and $R^k(i-1, j-1)=1$, $R^k(i, j)=1$.

Otherwise, $R^k(i, j)=R_I^k(i, j)\vee R_D^k(i, j)\vee R_S^k(i, j)$.

Deletion

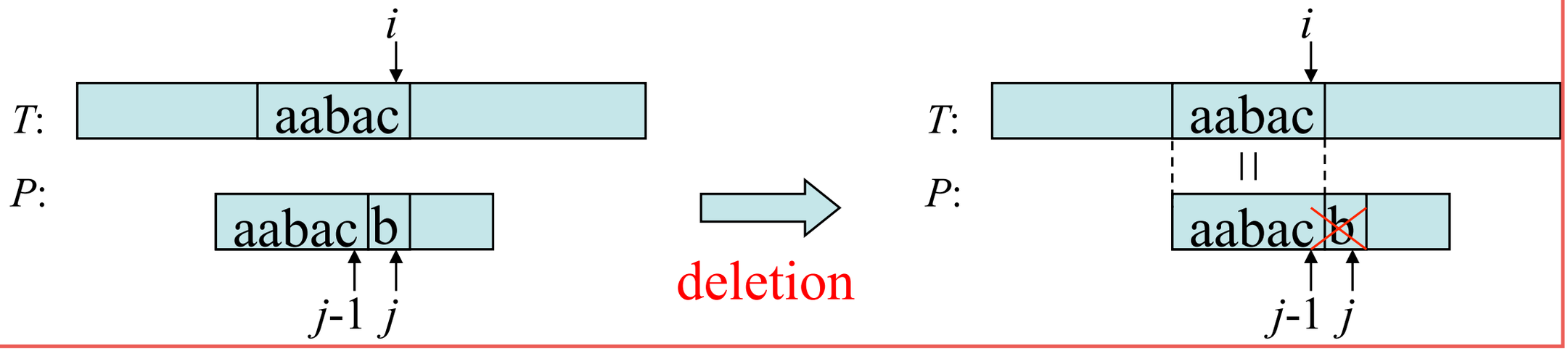
$R_D^k(i, j)=1$ if t_i≠p_j and $R^{k-1}(i, j-1)=1$.

$R_D^k(i, j)=0$ otherwise.

The deletion formula:

$$R_i^k=(((R_{i-1}^k>>1)\vee 10^{m-1})\,\&\,\sum(t_i))\vee((R_i^{k-1}>>1)\vee(10^{m-1}))$$

Example: when k=1.



If $t_i = p_j$, and $R^k(i-1, j-1) = 1$, $R^k(i, j) = 1$.

Otherwise, $R^k(i, j) = R_I^k(i, j) \vee R_D^k(i, j) \vee R_S^k(i, j)$.

Substitution

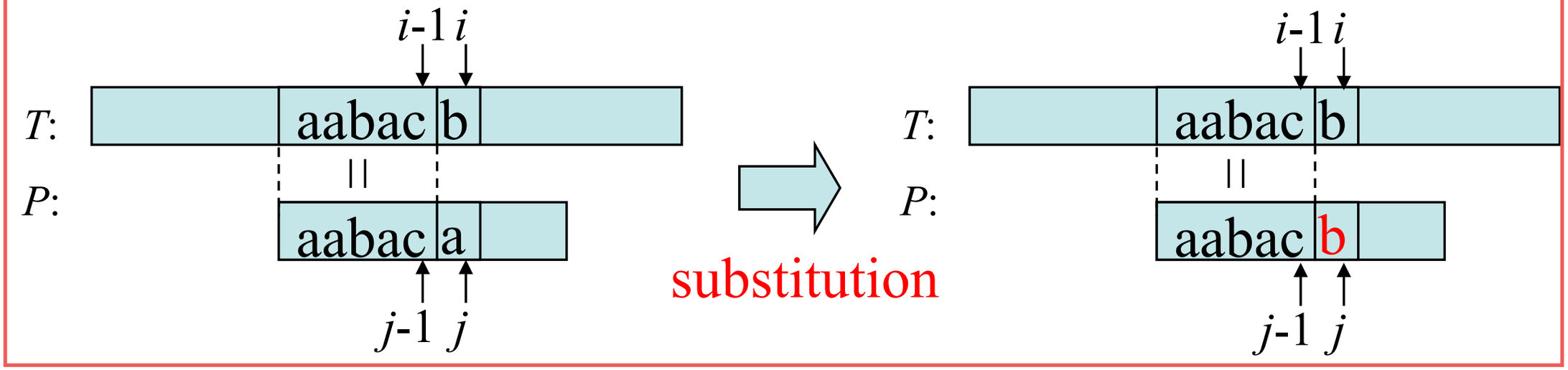
$R_S^k(i, j) = 1$ if $t_i \neq p_j$ and $R^{k-1}(i-1, j-1) = 1$.

$R_S^k(i, j) = 0$ otherwise.

The substitution formula:

$$R_i^k = (((R_{i-1}^k >> 1) \vee 10^{m-1}) \,\&\, \Sigma(t_i)) \vee ((R_{i-1}^{k-1} >> 1) \vee (10^{m-1}))$$

Example: when k=1.



The algorithm which contains insertion, deletion and substitution allows *k* error. In the physical meaning, we just combine the formulas of insertion, deletion and substitution. So now, we combine the three formula using the "or" operation as follows:

The insertion : $R_i^k = (((R_{i-1}^k >> 1) \vee 10^{m-1}) \& \sum(t_i)) \vee R_{i-1}^{k-1}$

The deletion : $R_i^k = (((R_{i-1}^k >> 1) \vee 10^{m-1}) \& \sum(t_i)) \vee ((R_i^{k-1} >> 1) \vee (10^{m-1}))$

The substitution : $R_i^k = (((R_{i-1}^k >> 1) \vee 10^{m-1}) \& \sum(T_i)) \vee ((R_{i-1}^{k-1} >> 1) \vee (10^{m-1}))$

Initially, $R_0^k = 1^k 0^{m-k}$

$$R_i^k = ((R_{i-1}^k >> 1) \& \sum(t_i)) \vee (R_{i-1}^{k-1}) \vee (R_i^{k-1} >> 1) \vee (R_{i-1}^{k-1} >> 1) \vee 10^{m-1}$$

$t_i = p_j$ insertion deletion substitution

and $R^k(i-1, j-1) = 1$

Time complexity

$$O(k\left\lceil \frac{m}{w} \right\rceil n)$$

https://github.com/GangLiao/Approximate-String-Matching

k: error value

m: the length of pattern

n : the length of text

w : the size of word of computer

The Second Algorithm

Our team member, Fatima will present 2nd algorithm.

Very Fast and Simple Approximate String Matching

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Abstract

We improve the fastest known algorithm for approximate string matching. This algorithm can only be used for low error levels. By using a new algorithm to verify potential matches and a new optimization technique for biased texts (such as English), the algorithm becomes the fastest one for medium error levels too. This includes most of the interesting cases in this area.

Key words: Approximate Matching, Text Searching, Information Retrieval.

1 Introduction

Approximate string matching is one of the main problems in classical string algorithms, with applications to text searching, computational biology, pattern recognition, etc.

The problem can be formally stated as follows: given a (long) text of length n, a (short) pattern of length m, and a maximal number of errors allowed k, find all text positions that match the pattern with up to k errors. The allowed errors are character insertions, deletions and substitutions. Text and pattern are sequences of characters from an alphabet of size σ. We call $\alpha = k/m$ the *error ratio* or *error level*.

In this work we focus on on-line algorithms, where the classical solution, involving dynamic programming, is $O(mn)$ time [8]. In the last years many

⋆ This work has been supported in part by FONDECYT grant 1950622.

2nd Algorithm

- Simple algorithm based on the following lemma:

Let *T* and *P* be two strings. Let *P* be divided into *j* pieces $p_1, p_2, \ldots, p_j$. If ed(*T*,*P*) ≤ *k*, then there exists at least one p_i and a substring *S* in *T* such that ed(*S*,p_i) ≤ $\lfloor k / j \rfloor$

-> What if we set *j=k+1* ?

2nd Algorithm

If we let $j=k+1$, then .

In this case, if $ed(T,P) \leq k$, then at least one p_i occurs in T exactly.

2nd Algorithm

- Idea of the algorithm:

In each window in T, look for each *pi*:

- If we find an exact matching of a p_i. we use the dynamic programming approach to determine whether there exists an approximate matching of *P* allowing *k* errors in this window.

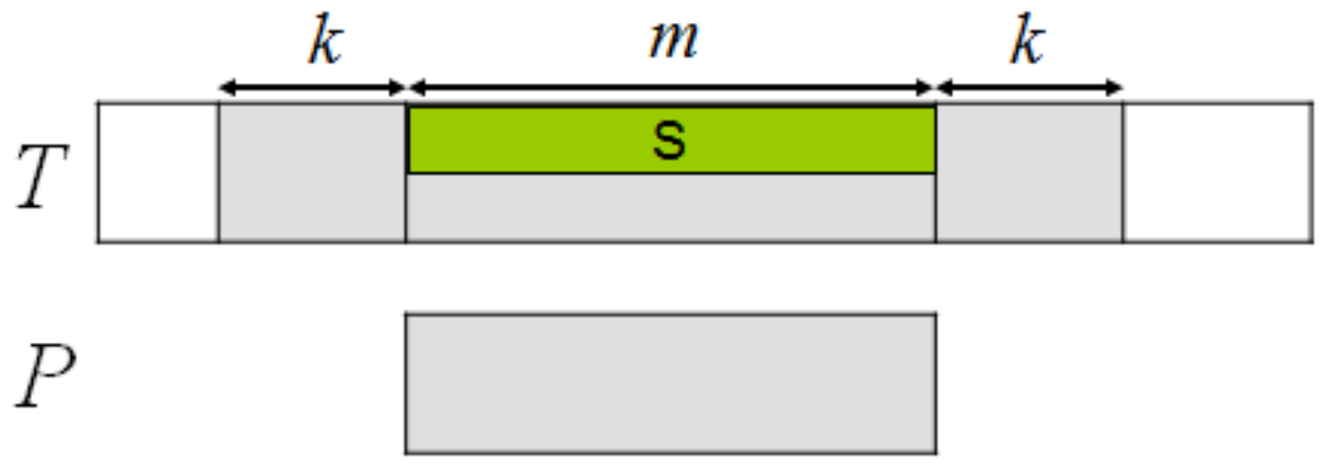
- If not we ignore the window

2nd Algorithm

- How large can a window be ?

Maximum size is *m+2k*

Why ?



Suppose we have P = ATCCTC with k = 2.

We divide P into three pieces : p_1 = AT, p_2 = CC and p_3 = TC.

To search for exact matching, we actually perform an exhaustive search. Let us assume that we search for AT.

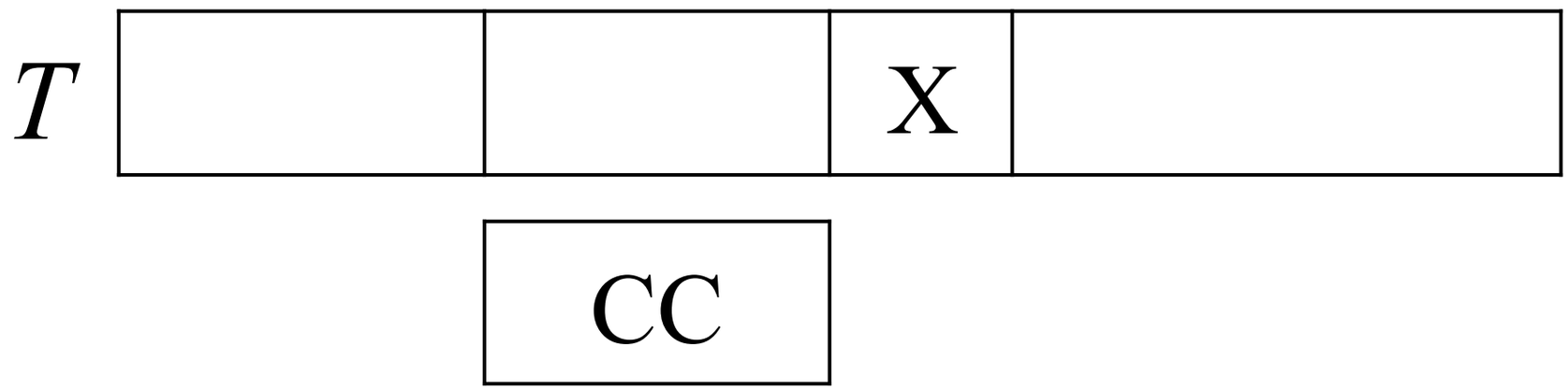
Note that there are three cases:

Case 1 : X = A. We move AT 2 steps.

Case 2 : X = T. We move AT 1 steps.

Case 3 : X≠A and X≠T, we move AT 3 steps.

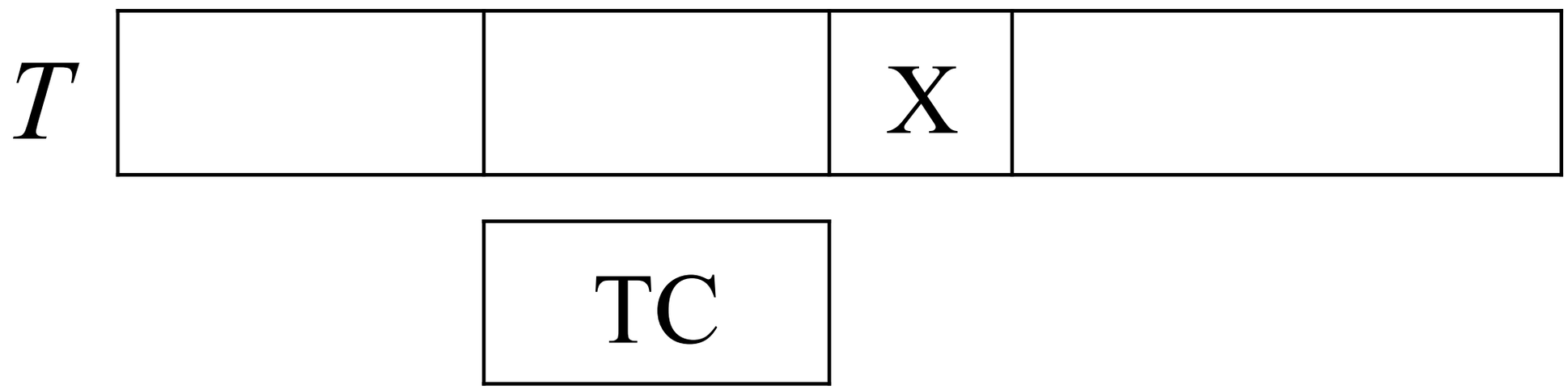
Let us assume that we search for CC.



Case 1 : X = C. We move CC 1 step.

Case 2 : X ≠ C. We move CC 3 steps.

Let us assume that we search for TC.



Case 1 : X = T. We move TC 2 steps.

Case 2 : X = C. We move TC 1 step.

Case 3 : X≠T and X≠C, we move TC 3 steps.

T = TCCAAGTTATAGCTC

p_1 = AT, p_2 = CC , p_3 = TC

shift table

A	T	C	*
2	1	1	3

First step: T C C A A G T T A T A G C T C

We open a window with length 2 to compare with AT, CC and TC. We found that it has a exact matching with p_3. Then shift the window according to shift table value of next position.

Second step: T C C A A G T T A T A G C T C

We found CC has a exact matching with p_2. Then we shift the window 2 positions.

Third step: T C C A A G T T A T A G C T C

We cannot find AA among p_1, p_2 and p_3.

Then shift the window 3 positions and continue to compare.

T = TCCAAGTTATAGCTC
P = ATCCTC

Using this *shift table*, we may have the following.
We will find AT occurring at 9 in T, CC occurring at 2 in T and TC occurring at 1 and 14 in T. Table *d* contains all text positions of P's pieces.

shift table

A	T	C	*
2	1	1	3

Table *d*

AT	9
CC	2
TC	1,14

Theoretical Time complexity of 2nd Algorithm

- Time Complexity is $\boldsymbol{O}(kn/m)$ where n is length of the text T and m is the length of the pattern P

Thank You!