# Orbit Determination via Topocentric Angular Observations

Jacob Bailey, Gustavo Lee, Michael Lesnewski

October 16, 2018

**Abstract**

In this work, a set of topocentric angular observations of a satellite's motion are used to determine the salient parameters of the satellie's orbit. Two different methods of orbit determination are herein examined: the methods of Gauss and Laplace. After discussion of the merits and pitfalls of these methods, we demonstrate the accuracy of the two by computing a best-fit orbit for the Tiangon-1 satellite.

## 1 Introduction

The determination of patterns of motion for celestial bodies is a surprisingly difficult problem, and one that has been oft studied through the history of celestial mechanics. Although the aim of the method is simple, it has been incredibly fruitful in its products that the rest of science has benefitted from. The struggle of rationalizing Tycho's observational data on the known planets led to Kepler's three laws, which are a fundamental piece of our understanding of the solar system. The determination of the orbit of Ceres as it passed the sun in 1801 led Gauss to develop the method of least squares regression, which has seen considerable use in the last century to fit models to observational data in all branches of science.

## 2 Theory of Orbit Determination

### 2.1 Gauss' Method

Gauss developed his method of orbit determination to solve a troubling problem: on January 1st 1801, Giuseppe Piazzi discovered Ceres and was able to track it for 40 days before it was lost in the glare of the sun. As it continued its solar orbit, the problem was to determine the orbital path, and predict the position at which it would again become observable. Gauss is credited with the predictions which allowed another astronomer, Franz Xaver von Zach, to observe the minor planet again on December 31st of the same year.

### 2.1.1 Observational Quantities

Gauss' method centers on two key quantities: the observer's position vector in the equatorial coordinate system, and unit vectors along the direction of the observation. The latter is a result of the observational technology at the time: ranging methods such as radar were not available at the time, and thus the best observation one could achieve with the technology of the time (telescopes) was simply two angles describing the orientation of the observed object's line of sight vector.

The equatorial position vector of the observer can be found as

$$\begin{aligned} R_n = &\left[\frac{R_e}{\sqrt{(1-(2f-f^2)sin^2\phi)}} + H_n\right] cos\phi_n(cos\theta_n\hat{I} + sin\theta_n\hat{J}) \\ &+ \left[\frac{R_e(1-f)^2}{\sqrt{(1-(2f-f^2)sin^2\phi)}} + H_n\right] sin\phi_n\hat{K} \end{aligned} \tag{1}$$

$R_n$ is the observer's position vector (in Equatorial Coordinate System)
$R_e$ is the equatorial radius of the body (e.g., Earth's Re is 6,378 km)
$f$ is the oblateness (or flattening) of the body (e.g., Earth's f is 0.003353)
$\phi_n$ is the respective geodetic latitude
$\phi'_n$ is the respective geocentric latitude
$H_n$ is the respective altitude
$\theta_n$ is the respective local sidereal time
The observation unit vector can also be found (in the topocentric coordinate system) via the following:

$$\hat{\rho}_n = cos_n cos\alpha_n\hat{I} + cos_n sin\alpha_n\hat{J} + sin_n\hat{K}$$