Math 432/532 Spring 2020 Exam 2
April 8, 2020

Name: ______________________ **Student Number:** ______________________

Signature: ____

Instructor: ______________________ **Section:** ______________________

Instructions: Answer all questions and show all work. Answers which are not justified with appropriate work will receive 0 points. Students who cheat will receive zero points on the exam and will be subject to the university's disciplinary procedure for academic dishonesty. Cheating includes, but is not limited to, looking at any other student's exam, looking at any books or notes during the exam, or using any electronic device other than your calculator for any purpose. For example, you may not use cell phones during this exam.

- Math 432 students are only required to solve the first four problems. Math 532 students should solve all five problems, including the last problem.
- Write your solutions to the other problems on blank paper.
- No piece of paper should include solutions or parts of solutions to more than one problem.
- Write your name at the top of every piece of paper you plan to turn in.
- Clearly write the problem number (and part) next to every solution you plan to turn in.
- When you complete the exam scan your solutions, in order, using an app like Adobe Scan, and upload your solutios to Gradescope.
- The exam is due by 10am on Thursday April 9th.

Problem	Points	Student's Score
1	15	
2	15	
3	35	
4	20	
5	10	
Total:	95	

DO NOT OPEN THE EXAM UNTIL YOU ARE TOLD TO START

1. Let Y_1 and Y_2 be compact subspaces of a topological space X.
   (a) (5 points) Prove that the union $Y_1 \cup Y_2$ is compact if both Y_1 and Y_2 are compact. Give an example to show that the converse is false.
   (b) (5 points) Does the result of part (a) together with the induction principle imply that a *finite* union of compact subspaces is compact? Explain why, or find a counterexample.
   (c) (5 points) Does the result of part (a) together with the induction principle imply that a *countable* union of compact subspaces is compact? Explain why, or find a counterexample.

2. Let $X_3 = \{a, b, c\}$ be a topological space containing exactly three points, and let $X_4 = \{a, b, c, d\}$ be a topological space containing exactly four points.
   (a) (5 points) Prove that if there is no open singleton in X_3, then X_3 is connected.
   (b) (5 points) Is the following true? If X_3 is connected, then there is no open singleton in X_3.
   (c) (5 points) Is the following true? If there is no open singleton in X_4, then X_4 is connected.

3. Let $X = \mathbb{R}^\omega$ with the product topology. Let $A \subseteq X$ be the subset

$$A = [0, 1] \times [0, 2] \times [0, 3] \times \cdots$$

   endowed with the subspace topology. Explain each answer below.
   (a) (5 points) Is X metrizable?
   (b) (5 points) Is X compact?
   (c) (5 points) Is X sequentially compact? If not, construct a sequence in X that has no convergent subsequence.
   (d) (5 points) Is X connected?
   (e) (5 points) Is A metrizable?
   (f) (5 points) A is compact by the Tychonoff theorem. Is A sequentially compact? If not, construct a sequence in X that has no convergent subsequence.
   (g) (5 points) Is A connected?

4. Let X be $\mathbb{R}$ with the countable complement topology. (A subset is open if and only if its complement is either countable or all of $\mathbb{R}$.)
   (a) (5 points) Find the closure and the interior of $Y = (0, \infty) \subseteq X$.
   (b) (5 points) Consider the map $f : X \to X$ defined by $f(x) = \cos(x)$. Is f continuous on X with the countable complement topology. (*Hint:* It may be helpful to know that the countable union of countable sets is countable.)
   (c) (5 points) Is X compact? (*Hint:* Consider the cover $\{U_k\}_{k \in \mathbb{Z}_+}$ where $U_k = (\mathbb{R} - \mathbb{Z}_+) \cup \{k\}$.)
   (d) (5 points) Is X connected?

5. (10 points) (Math 532 Extra Problem) Let X be a Hausdorff space, let C be a compact subset of X, and let a be a point of X which is not in C. Prove that there are disjoint open sets U and V with $C \subseteq U$ and $a \in V$.