Math 432/532 Spring 2020 Homework 4
Due Wednesday, February 19, 2020

I strongly encourage you to come to my office hours early and often for help on the homework. However, please be aware that I will expect you to have given a problem serious effort before asking about it.

- Dr. Aubrey's office hours are: Monday from 2-3pm in Math 220 and Tuesday and Wednesday from 2-3pm in Math 219, and by appointment. You can contact Dr. Aubrey by email at jaubrey@math.arizona.edu.
- And, undergraduate students can get help from other faculty in the Math 220 Tutor Lab. Here is the schedule: `http://math.arizona.edu/academics/tutoring/math310`

To be collected on Wednesday, February 19, 2020 for all students in Math 432/532

1. (Munkres, pg. 101 #13) Show that X is Hausdorff if and only if the diagonal $\Delta = \{x \times x \mid x \in X\}$ is closed in $X \times X$.
2. (Munkres, pg. 101 # 15) Show that the T_1 axiom is equivalent to the condition that for each pair of points of X, each has a neighborhood not containing the other.
3. (Munkres, pg. 102 #19) If $A \subseteq X$, we define the **boundary** of A by the equation
$$\text{Bd } A = \overline{A} \cap \overline{(X - A)}.$$
   (a) Show that Int A and Bd A are disjoint and that $\overline{A} = \text{Int } A \cup \text{Bd } A$.

   (b) Show that Bd $A = \varnothing$ if and only if A is both open and closed.

   (c) Show that U is open if and only if Bd $U = \overline{U} \backslash U$.

   (d) If U is open, is it true that $U = \text{Int } \overline{U}$? Justify your answer.
4. (Munkres, pg. 111, #1) Prove that for functions $f : \mathbb{R} \to \mathbb{R}$ the $\epsilon - \delta$ definition of continuity implies the open set defintion.

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5. Let a, b, and c be real numbers with $a \leqslant b \leqslant c$, and $a < c$. Let X denote the set $[a, c] \cup \{b'\}$, where $[a, c]$ denotes a closed interval in the real line and b' is a point not in $[a, c]$ Let F be the family of subsets of X consisting of all open subsets of $[a, c]$ together with all subsets of the form $(U - \{b\}) \cup \{b'\}$, where U is an open subset of $[a, c]$ which contains b.

   (a) Show that F is a basis for a topology on X.

   (b) Show that the map which interchanges b and b' and is the identity otherwise is a homeomorphism.

   (c) Show that this topology on X is not Hausdorff.

   (d) Show that if $f : X \to \mathbb{R}$ is continuous, then $f(b) = f(b')$.

Reminders

- Exam 1 February 19th and 21st.
- Exam 2 April 8th and 10th.
- Final Exam: Thursday, May 14, 10:30am-12:30pm.