Chapter 9

Reverberation

In this chapter the reverberation effect is designed. The chapter begins with a description of what reverberation is and what it is typically used for. Afterwards, some simple reverberation algorithms are examined. This comprises the simple plain reverberator and the simple allpass reverberator which are building blocks in more advanced reverberators as Scroeder's and Moorer's reverberator. These reverberators will also be examined in some detail. The final reverberator is a mix between Scroeder's and Moorer's reverberator and the rest of this chapter will focus on the implementation of it on the DSP-system. The implementation comprises scaling and SNR calculations.

9.1 Real and Simulated Reverberation

Reverberation is the acoustic effect of a room on an acoustic signal. Figure 9.1 shows an example of reverberation. A loudspeaker generates acoustic waves which travel inside a room. The direct sound is the part of the sound that travels directly from the loudspeaker to the listener and is indicated by a solid line in figure 9.1. The early reflections correspond to the first couple of reflections off the walls and are indicated by long dotted lines in the figure. The acoustic waves perceived by the listener increase in density as the acoustic wave reflects more and more times off the walls. This is known as the late or subsequent reflections and is indicated by short dotted lines in the figure.

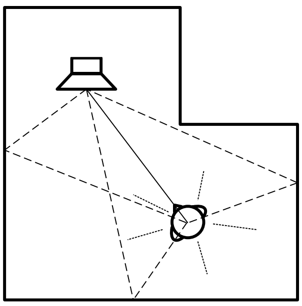

Figure 9.1: The influence of the room on the sound. The direct sound, early and late reflections are shown.

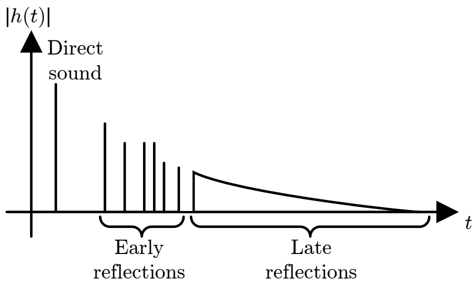

Figure 9.2: Partitioning of reverberation in direct sound, early and late reflections.

The influence of the room on the acoustic signal perceived by the listener can be characterized by the impulse response of the room. An example of the impulse response of a room is shown in figure 9.2. The impulse response is dependent upon the placement of loudspeaker and the listener. Especially the early reflections vary with placement of the loudspeaker and the listener while the late reflections are relatively independent of the location in the room [Kahrs, 1998]. The amplitude level is approximately decaying exponentially and the density of the acoustic signal perceived by the listener increases quadratic with time. The time it takes for the sound pressure to decrease by 60 dB is defined as the reverberation time [Zölzer, 1997].

Artificial reverberation is mainly used for post-processing [Zölzer, 1997]. Music is often recorded in a studio with a microphone in the vicinity of a voice or an instrument for which reason mainly the direct signal is recorded. To make the music sound more natural, artificial reverberation is often applied to the direct signal which maps it to an acoustical room. In fact, almost all the audio from the radio, movies and recordings have had artificial reverberation added [Kahrs, 1998]. In that way classical music can sound like it is played in a concert hall even though it is recorded in a studio.

There exist several ways to implement reverberation in a digital computer. One way is to measure an actual typical impulse response of a room and implement it as a FIR-filter. This approach works only for impulse responses with a very short reverberation time since the FIR-filter length otherwise will be very long and leads to long computation time. The method can be speeded up by FFT-block convolution, but this approach suffers from processing delay whose length is often unacceptable in real time reverberation [Zölzer, 2005]. There exist algorithms that speed up the FFT-block convolution considerably, but those methods are advanced and will not be investigated further in this text [Zölzer, 2005]. Instead several artificial reverberation algorithms without significant processing delay or computation time, have been developed to simulate real reverberation. This approach has in addition the advance of adjustability. Hence, it is possible to adjust several parameters of the algorithm which allows the same algorithm to simulate various types of reverberation. The parameters could for instance be the reflection coefficients of the walls, reverberation time and the initial delay between the direct sound and the first reflection.

A lot of reverberation algorithms have been developed spanning from extremely simple ones to advanced ones. The rest of the content of this chapter will focus on a few simple reverberation algorithms which form the basis of more advanced ones. It is, however, beyond the scope of this project to cover more advanced reverberation algorithms.

9.2 Reverberation Algorithms

The first digital artificial reverberator was invented by Scroeder in 1961 and is known as Schroeder's reverberator [Zölzer, 1997]. It consists of two basic building blocks; the plain reverberator and the allpass reverberator which will be examined in the beginning of this section. After that, Schroeder's reverberator will be examined. Schroeder's reverberator has some disadvantages that will be discussed in the conclusion of this section in the context of Moorer who proposed some improvements of Schroeder's reverberator.

9.2.1 Plain Reverberator

The plain reverberator is the simplest reverberation tool. It simulates the exponential decay of the reverberation and it is shown in figure 9.3. It consists of a delay unit z^{-m}, a gain g in the feedback path and a summer which add the input signal to the feedback signal. The Z-transform can be derived from the figure by

$$X(z) + gY(z)z^{-m} = Y(z) \tag{9.1}$$

which can be rearranged to yield the transfer function

$$H(z) = \frac{Y(z)}{X(z)} = \frac{1}{1 - gz^{-m}} \; . \tag{9.2}$$

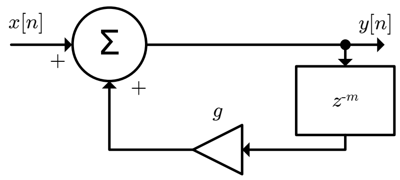

Figure 9.3: Plain reverberator.

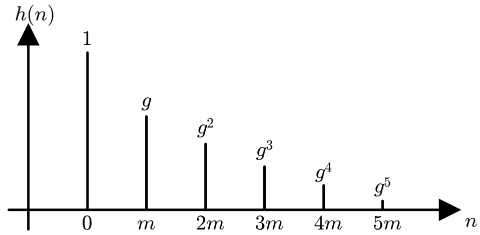

Figure 9.4: Impulse response of plain reverberator.

From the transfer function given in equation 9.2 several important properties of the plain reverberator can be found. One important property can be derived from the impulse response. The difference equation can be found to

$$y[n] = x[n] + gy[n - m] \tag{9.3}$$

which can be used to plot the impulse response of the plain reverberator recursively. If the input signal, $x[n]$, is Dirac's delta function, $\delta[n]$, the output signal, $y[n]$, will be the impulse response, $h[n]$. Figure 9.4 shows the first six samples of the impulse response of the plain reverberator. The figure shows that the spacing between two successive nonzero samples is the delay m and that the amplitude of the nonzero samples decreases exponentially.

Another property that can be derived from the transfer function in equation 9.2, is the frequency response. By setting $z = e^{j\omega}$ the frequency response can be found from the transfer function to

$$H(e^{j\omega}) = \frac{1}{1 - ge^{-j\omega m}} \; , \tag{9.4}$$

whose amplitude response is given by

$$|H(e^{j\omega})| = \frac{1}{|1 - g[\cos(\omega m) - j\sin(\omega m)]|} = \frac{1}{\sqrt{(1 - g\cos(\omega m))^2 + (g\sin(\omega m))^2}}$$
$$= \frac{1}{\sqrt{1 + g^2 - 2g\cos(\omega m)}} \ . \tag{9.5}$$

The maximum and minimum values of the amplitude response can be found to be

$$|H(e^{j\omega})|_{\text{MAX}} = \frac{1}{\sqrt{1 + g^2 - 2g}} = \frac{1}{1-g} \quad \text{for } \omega = p\frac{2\pi}{m} \ , \quad p = 0, 1, 2, \cdots, m-1 \tag{9.6}$$

and

$$|H(e^{j\omega})|_{\text{MIN}} = \frac{1}{\sqrt{1 + g^2 + 2g}} = \frac{1}{1+g} \quad \text{for } \omega = (2p+1)\frac{\pi}{m} \ , \quad p = 0, 1, 2, \cdots, m-1 \ . \tag{9.7}$$

Figure 9.5 shows an example of the amplitude response of the plain reverberator in the frequency interval spanning from 0 to 2π. In the example the delay, m, is set to 8 and the feedback gain, g, is set to 0.5. The amplitude has $m = 8$ uniformly spaced peaks with a value of $|H(e^{j\omega})|_{\text{MAX}} = 2$ as suggested by equation 9.6. The minimum value is $|H(e^{j\omega})|_{\text{MIN}} = 2/3$ as suggested by equation 9.7. Due to the shape of the amplitude response it is often referred to as a comb filter.

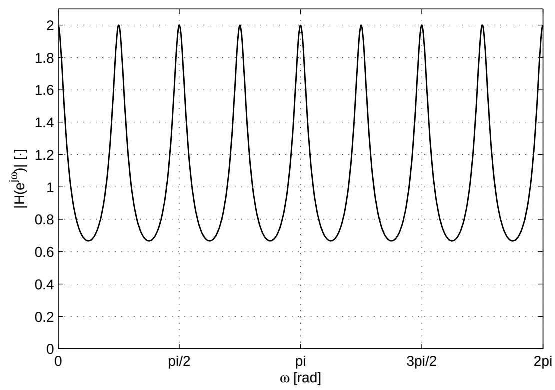

Figure 9.5: Frequency response of comb filter with $m = 8$ and $g = 0.5$.

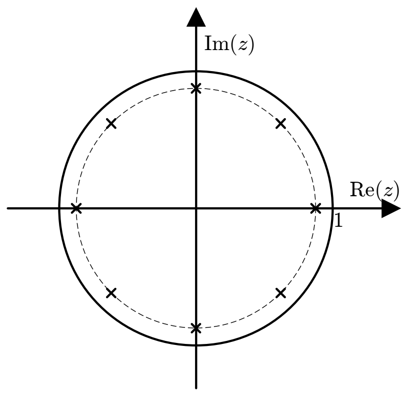

Figure 9.6: Poles of the comb filter transfer function.

The poles of the transfer function are closely related to the frequency response. The roots of the denominator of the transfer function in equation 9.2 can be found to

$$0 = 1 - gz^{-m} \Leftrightarrow z^m = g \ . \tag{9.8}$$

If z is written in polar form, the poles can be found to

$$r^m e^{j\omega m} = g \ , \tag{9.9}$$

which yields a pole radius of

$$r = g^{1/m} \tag{9.10}$$

and a pole angle of

$$\omega = p\frac{2\pi}{m} \ , \quad p = 0, 1, 2, \cdots, m-1 \ . \tag{9.11}$$

The poles are plotted in figure 9.6 with $m = 8$ and $g = 0.5$.

The reverberation time of the plain reverberator can be found from the impulse response shown in figure 9.4. The amplitude of the kth nonzero sample is g^k. The amplitude has therefore decreased 60 dB at

$$-60 \text{ dB} = 20\log_{10}\left(\frac{g^{k_\mathrm{r}}}{g^0}\right) = 20\log_{10}\left(g^{k_\mathrm{r}}\right) \tag{9.12}$$

where k_r is the nonzero sample number corresponding to the reverberation time t_r. Since $n = k \cdot m$ and $n = tf_\mathrm{s}$, k_r can be replaced with $\frac{t_\mathrm{r} f_\mathrm{s}}{m}$ which yields a reverberation time of

$$-60 \text{ dB} = 20\log_{10}\left(g^{\frac{t_\mathrm{r} f_\mathrm{s}}{m}}\right) \tag{9.13}$$

$$\Updownarrow$$

$$t_\mathrm{r} = \frac{-3m}{\log_{10}(g) f_\mathrm{s}} \ . \tag{9.14}$$

Thus, given the sampling frequency, the reverberation time can be determined by the delay length, m, and the feedback gain, g, of the plain reverberator.

The shape of the amplitude response is often undesirable since the frequencies between the peaks of the spectrum will be attenuated. The density of the peaks in the frequency can be increased by increasing the delay m, but this leads to a decreasing echo density in the time domain. One way to obtain high frequency density and high echo density is to use several comb filters with different delays and gains in parallel. This design is used in Schoeder's reverberator.

9.2.2 Allpass Reverberator

Since the peaks in the frequency response of the comb filter are undesirable, Schroeder modified the comb filter so that the amplitude response was flat [Kahrs, 1998]. The resulting filter is called an allpass filter and its block diagram is shown in figure 9.7.

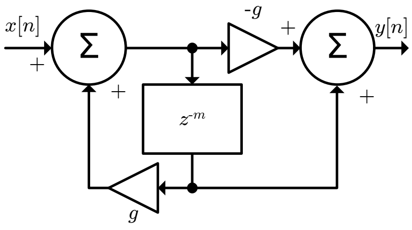

Figure 9.7: Allpass reverberator.

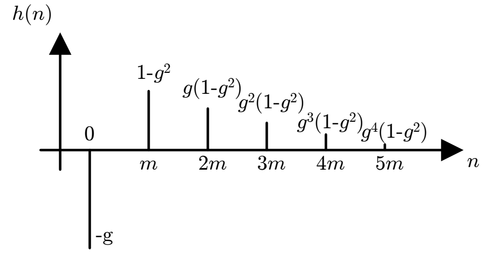

Figure 9.8: Impulse response of allpass reverberator.

From the block diagram the transfer function can be found to

$$H(z) = \frac{Y(z)}{X(z)} = \frac{z^{-m} - g}{1 - gz^{-m}} \ , \tag{9.15}$$

from which the difference equation is

$$y[n] = x[n-m] - gx[n] + gy[n-m] \ . \tag{9.16}$$

The impulse response can now be plotted recursively by setting $x[n] = \delta[n]$, and the first few samples of the impulse response are shown in figure 9.8. Like the impulse response of the plain reverberator, the amplitude of the impulse response of the allpass filter decreases exponentially.

The amplitude response of the allpass filter is constant since

$$|H(e^{j\omega}| = \frac{|e^{-j\omega m} - g|}{|1 - ge^{-j\omega m}|} \tag{9.17}$$

$$= |e^{-j\omega m}| \frac{|1 - ge^{j\omega m}|}{|1 - ge^{-j\omega m}|} \tag{9.18}$$

$$= 1 \frac{\sqrt{1 + g^2 - 2g\cos(\omega m)}}{\sqrt{1 + g^2 - 2g\cos(\omega m)}} = 1 \ . \tag{9.19}$$

Even though the amplitude response of the allpass reverberator is flat, it sounds quite like the plain reverberator [Kahrs, 1998]. This is due to the physics of the human ear which performs a short-time frequency analysis in contrast to the Fourier transform that is defined for infinity time integration [Kahrs, 1998]. Figure 9.9 illustrates the spectrum of the allpass reverberator and has been computed using an FFT and window length of 50 ms which corresponds to the integration time of the human ear [Zwicker and Fastl, 1999]. The window is a Hanning window which has been shifted 1 ms for each FFT computation. The spectrum has the form of a comb filter for a fixed time which explains, that it sounds like the plain reverberator.

The denominator of the transfer function of the allpass filter in equation 9.15 is equal to the denominator of the transfer function of the comb filter in equation 9.2. Hence, they

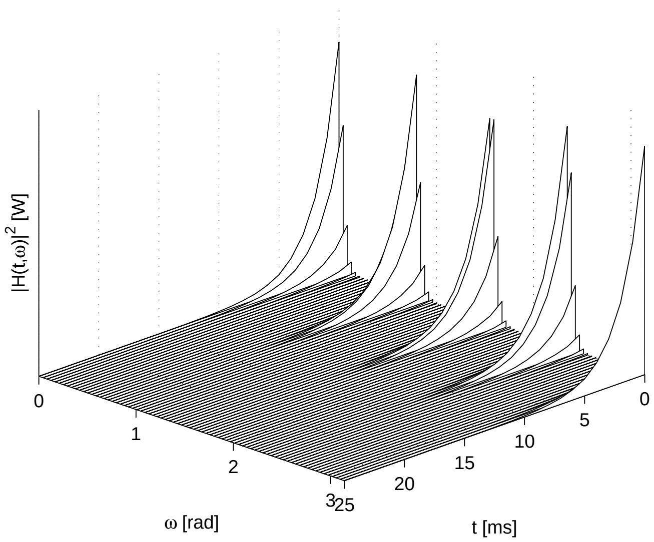

Figure 9.9: Spectrum of the allpass reverberator with $m = 8$ and $g = 0.5$.

have the same poles. The zeros of the allpass filter transfer function are

$$0 = z^{-m} - g \Leftrightarrow z^{-m} = g \ . \tag{9.20}$$

If z is written in polar form, the zeros can be found to

$$r^{-m} e^{-j\omega m} = g \ , \tag{9.21}$$

which yields a zero radius of

$$r = g^{-1/m} \tag{9.22}$$

and a zero angle of

$$\omega = -p\frac{2\pi}{m} \ , \quad p = 0, 1, 2, \cdots, m-1 \ . \tag{9.23}$$

An example of the pole-zero plot of the allpass reverberator is shown in figure 9.10 with $m = 8$ and $g = 0.5$.

Like the plain reverberator, the allpass reverberator is not suitable to act as a standalone reverberator since it sounds quite like the plain reverberator. It is, however, very useful as a building block in a reverberator and is widely used to increase echo density without affecting the frequency response of the reverberator [Zölzer, 1997].

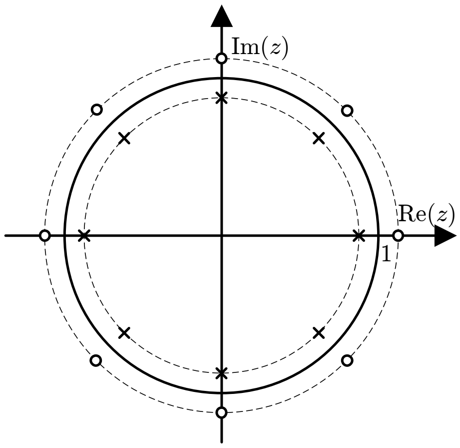

Figure 9.10: Pole-zero plot of allpass filter.

9.2.3 Schroeder's Reverberator

As discussed previously, neither the plain reverberator nor the allpas reverberator can be used as a standalone reverberator, because neither of them can simulate the required frequency and echo density at the same time. Schroeder found that the frequency density should be at least 0.15 Hz^{-1} and the echo density should be at least 1000 s^{-1} in order to make the reverberator sound realistic [Zölzer, 1997]. This means that the distance between two successive peaks in the amplitude response of the reverberator should be less than 6.67 Hz and the number of echoes per second that a listener perceives should be greater than 1000.

One way to achieve the desired frequency and echo density is to use several comb filters in parallel. If the delays of the different comb filters are chosen in such a way that they are prime relatively to each other, the frequency and echo density can be increased. Schroeder showed that at least 12 comb filters in parallel was required in order to meet the required density specifications [Kahrs, 1998]. He also showed that if two allpass filters in series were combined with the comb filters in parallel only four comb filters were required [Kahrs, 1998]. Figure 9.11 shows this basic reverberator which is known as Schroeder's reverberator. The input signal is fed into the four comb filters whose outputs are summed. The summed signal is then fed into two allpass filters in series. The four comb filters increase the frequency density while the allpass filters increase the echo density.

It is half an art half a science to find the optimal gain and delay factors involved in the reverberator. Schroeder suggested that the delays in the comb filters should be in the interval of 30 ms to 45 ms [Kahrs, 1998]. The gains of the comb filters are determined by the desired reverberation time which from equation 9.14 is given by

$$g_i = 10^{\frac{-3m_i}{t_\mathrm{r} f_\mathrm{s}}} \ . \tag{9.24}$$

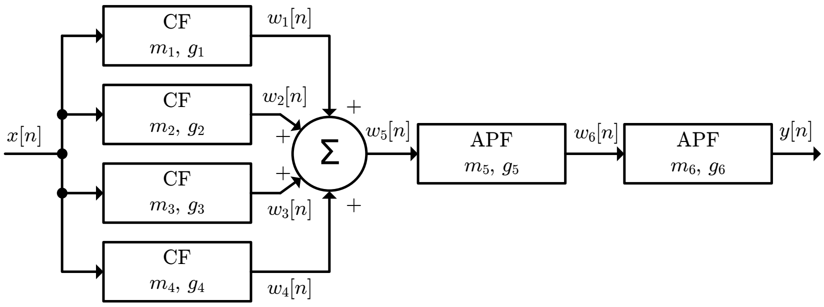

Figure 9.11: The basic Scroeder reverberator.

The delays of the allpass filter are chosen to be small with respect to the delays of the comb filters. This ensures that the allpass filters increase the echo density significantly. Additionally, the overall reverberation time only depends on the reverberation time set by the comb filters. The gains of the allpass filter are typically set to around 0.7 [Kahrs, 1998].

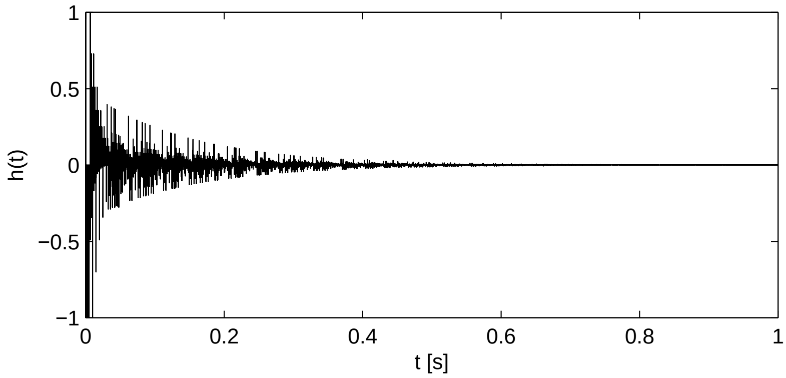

Figure 9.12: Impulse response of Scroeder's reverberator with reverberation time set to 1 s.

Delays [ms]	τ_1	τ_2	τ_3	τ_4	τ_5	τ_6
	31	37	41	43	5	1.7
Gains [·]	g_1	g_2	g_3	g_4	g_5	g_6
	0.81	0.77	0.75	0.74	0.70	0.70

Table 9.1: Selected delays and gains in Schroeder's reverberator.

Figure 9.12 shows the impulse response of Schroeder's reverberator with the values of the various gains and delays as given in table 9.1. The gains of the comb filters are calculated from equation 9.24 with a reverberation time of 1 s and $m_i = \tau_i f_s$. The envelope of the impulse response is clearly decreasing exponentially. Figure 9.13 shows the spectrum of Schroeder's reverberator. The spectrum has been created using a short-time Fourier transform again with an FFT and window length of 50 ms. The window is a Hanning window which has been shifted 25 ms for each FFT computation. The spectrum indicates that every frequency component has almost the same reverberation time.

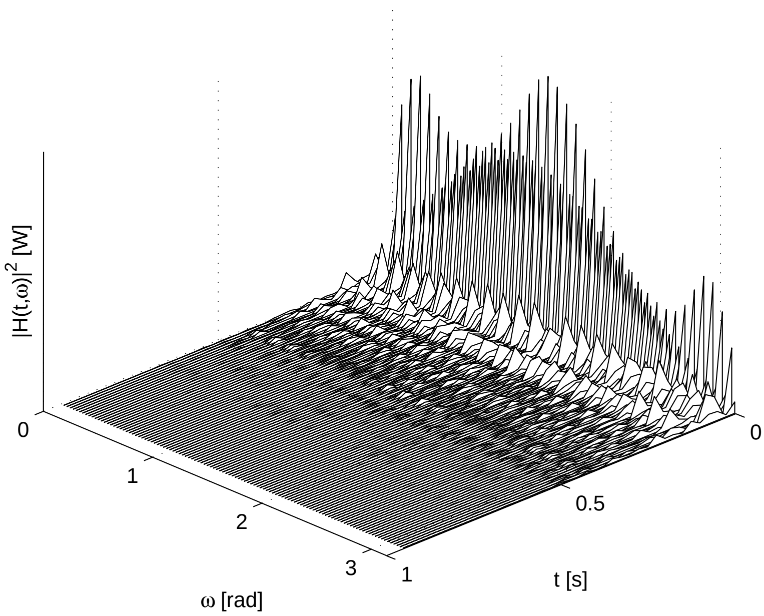

Figure 9.13: Spectrum of Scroeder's reverberator with reverberation time set to 1 s.

A sound test has been performed with Schroeder's reverberator. In the test a source signal is convolved with the impulse response of Schroeder's reverberator to create a reverberated signal. Different impulse responses with various reverberation times have been generated and used, and it is easy to distinguish between the different reverberation times when the reverberated signals have been played. The reverberated signals suffer, however, from one major disadvantage which make the reverberated signals sound unnatural. They sound metallic, especially for longer reverberation times. This observation was also done by Moorer for which reason, he made some small modification on Schroeder's revereberator [Kahrs, 1998]. These modifications will be described next.

9.2.4 Moorer's Reverberator

In 1979 Moorer proposed two improvements of Schroeder's original reverberator [Kahrs, 1998]. The first improvement was to increase the number of comb filters from four to six which increased both the frequency and echo density. The second improvement was to add a one-pole lowpass filter in the feedback path of the comb filters. This was based on the physical properties of sound wave propagation inside a room where higher frequency components tend to get attenuated more than lower frequency components.

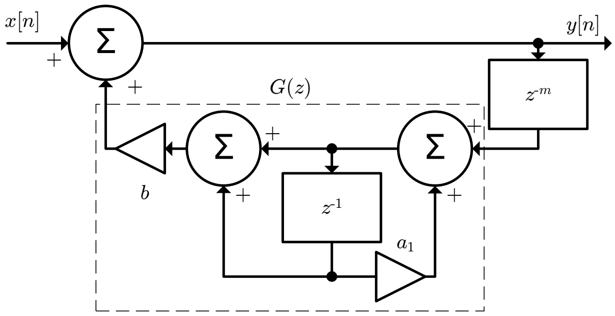

Figure 9.14: The plain reverberator with a one-pole lowpass filter in the feedback path.

Figure 9.14 shows the plain reverberator with a one-pole lowpass filter in the feedback path. The transfer function of the lowpass filter, $G(z)$, is

$$G(z) = b\frac{1 + z^{-1}}{1 - a_1 z^{-1}} \ . \tag{9.25}$$

The coefficients b and a_1 are found from filter design based on the bilinear transformation method described in appendix D. The steps shown in figure D.1 on page 124 gives the following procedure for the design:

1. **Digital filter specifications**
   The digital filter must be a first order Butterworth lowpass filter with a cutoff frequency of ω_c .

2. **Analog filter specifications**
   The analog specifications are the same as the digital except for the pre-warping of the cutoff frequency. The pre-warped analog cutoff frequency is according to equation D.9 page 126
   $$\Omega_c = \frac{2}{T_\mathrm{s}} \tan(\omega_c/2) \tag{9.26}$$
   where T_s is the sampling time.

3. **Analog filter design**
   The Butterworth filter design leads to a first order lowpass filter with the transfer function
   $$G_a(s) = c\frac{\Omega_c}{s + \Omega_c} \tag{9.27}$$
   where c is the DC gain.

4. **Digital filter design**
   The digital filter is obtained from the analog filter through the bilinear transformation. Specifically equation D.4 page 125 is applied which gives the transfer function

of the digital first order lowpass filter to

$$
\begin{aligned}
G_d(z) = G_a(s)\Big|_{s=\frac{2}{T_s}\frac{1-z^{-1}}{1+z^{-1}}} &= c\frac{\Omega_c}{2/T_s+\Omega_c}\frac{1+z^{-1}}{1-\dfrac{2/T_s-\Omega_c}{2/T_s+\Omega_c}z^{-1}} \\
&= c\frac{\tan(\omega_c/2)}{1+\tan(\omega_c/2)}\frac{1+z^{-1}}{1-\dfrac{1-\tan(\omega_c/2)}{1+\tan(\omega_c/2)}z^{-1}} \ . \qquad (9.28)
\end{aligned}
$$

Comparing equation 9.28 with equation 9.25 gives expressions for the coefficients b and a_1 to

$$
b = c\frac{\tan(\omega_c/2)}{1+\tan(\omega_c/2)} \qquad (9.29)
$$

and

$$
a_1 = \frac{1-\tan(\omega_c/2)}{1+\tan(\omega_c/2)} \ . \qquad (9.30)
$$

The DC gain of the digital lowpass filter must equal the gain, g, from the original plain reverberator to ensure the same reverberation time in the passband. This means that

$$
g = G(z=1) = \frac{2b}{1-a_1} \Leftrightarrow b = \frac{g(1-a_1)}{2} \qquad (9.31)
$$

which can used to compute the b coefficient.

Figure 9.15 shows the spectrum of the lowpass filter version of Schroeder's reverberator (see figure 9.11) with the delays and gains as in table 9.1 and the corner frequency set to 0.25π. From the figure it is seen that the reverberation time is frequency dependent. The reverberation time decreases with increasing frequency. A sound test has revealed that the metallic sound has been reduced significantly with the addition of the lowpass filter into the feedback path of the plain reverberator.

9.3 Design of Reverberator

Specification 8 in the requirement specification specifies that

- the reverberation time should be adjustable in the range from 0.1 s to 4 s.
- the reverberation level should be adjustable in the range from 10 % to 100 %.
- the pre-delay should be adjustable in the range from 0 ms to 100 ms.

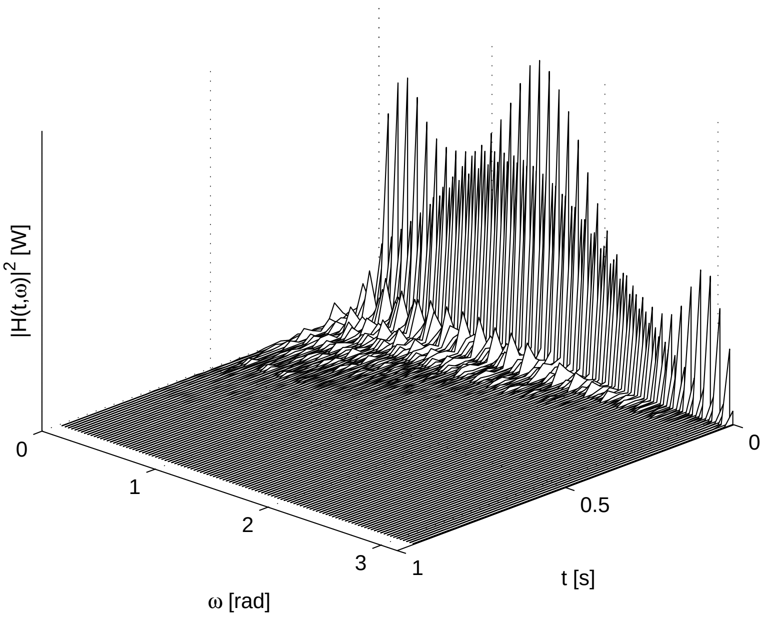

Figure 9.15: Spectrum of the lowpass filters version of Scroeder's reverberator with reverberation time set to 1 s.

A reverberator that enables adjustability of these factors is shown in figure 9.16. The pre-delay is a simple delay block with adjustable delay length d. The reverberation level g_{r} is controlled with a simple gain block. The reverberation time is controlled with the b coefficients that can be calculated from equation 9.31 and 9.24. The coefficient a_1 is calculated from equation 9.30 to -0.0189 which corresponds to a corner frequency of 5 kHz, and the rest of the coefficients are set according to table 9.1.

9.4 Implementation of Reverberator

Before the reverberator is implemented it must be chosen whether the filters must be implemented as direct form type I or direct form type II. As shown later in this section, type I has the best SNR for this application, but requires more memory. Type II has a worse SNR but has the advantage of a lesser buffer size. Type I is chosen due to the better SNR.

First, the scale factor and the SNR of the reverberator are calculated.

9.4.1 Scaling and Signal to Noise Ratio

The DSP system is a fixed point arithmetic system, meaning that values like 2.3 and 23 are represented alike. So it is the user that must take care of handling where the point is to be. If numbers are specified to be in the Q_{15} format, 15 bit are used to represent

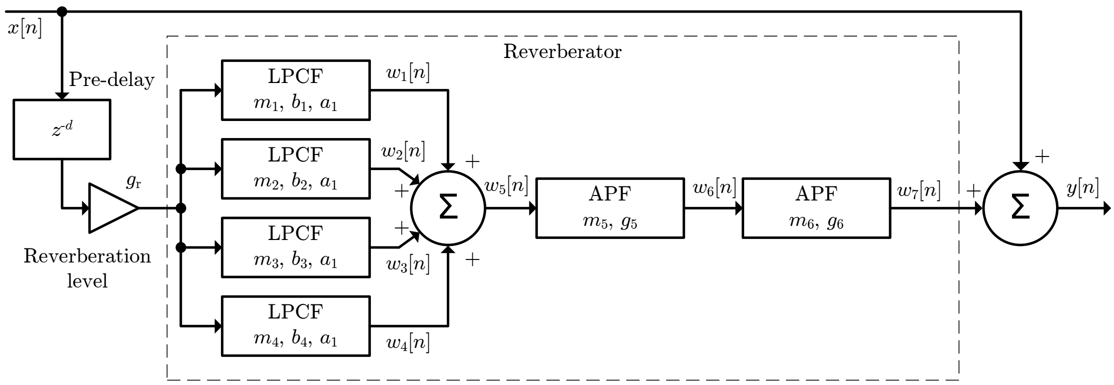

Figure 9.16: Scroeder's reverberator with lowpass filter version of comb filters, pre-delay and reverberation level.

the numbers before the point, meaning that the values between -1 and 1-LSB can be represented with a resolution of 15 bits. This can be problematic if the implemented filters increases the values of the input sample, causing overflow and thereby corrupting the output sample. To prevent this from happening the filter coefficients or the input sample must be scaled. To determine whether scaling is necessary, the filters must be analyzed.

Filter Analysis to Determine the Scale Factor

Filters consist of multiply and addition operands. As all filter coefficients for the reverberator are between -1 and 1-LSB and Q_{15} is used, a multiplication can never lead to overflow. Hence, the analysis must be focused on the summing point at the output of the respective filters. If some points result in overflow, the input signal must be scaled accordingly.

The analysis for the allpass filter can be done finding the filter impulse response. When this is found a variance scaling can be performed. Using equation [Oppenheim and Schafer, 1999, page 401]

$$s \leq \frac{1}{\sqrt{\sum_{n=-\infty}^{\infty} |h[n]|^2}} \tag{9.32}$$

the scaling factor s is then the factor that the input must be scaled with. If the scale factor is multiplied with the input before it is fed to the filter, the output will be scaled so that it in *most* cases will not overflow. If overflow should be totally avoided, variance scaling is not the approach to take. This is because variance scaling, scale on basis of the energy in the signal and not the signal itself [Oppenheim and Schafer, 1999, page 401].

The impulse response for one of the allpass filters is calculated here. For analysis of the lowpass filters, see appendix E page 127.

To find the impulse response for the allpass filter, the transfer function 9.15 is rewritten into the form

$$H(z) = A + \frac{B}{1 - gz^{-m}} \ , \tag{9.33}$$

where $A = \frac{-1}{g}$ and $B = \frac{(1-g^2)}{g}$. The last term of equation 9.33 is a geometric series, and equation 9.33 is equal to [Orfanidis, 1996, page 369]

$$H(z) = (A+B) + B\sum_{k=1}^{\infty} g^k z^{-n\cdot m} \ . \tag{9.34}$$

The inverse Z-transform of equation 9.34 is

$$h[n] = (A+B)\delta[n] + B(g\delta[n-m] + g^{2m}\delta[n-2m] + g^{3m}\delta[n-3m] + g^{4m}\delta[n-4m] + ...) \ , \tag{9.35}$$

where $g = 0.7$, $A = \frac{-1}{0.7} = -1.43$ and $B = \frac{(1-0.7^2)}{0.7} = 0.729$. Using equation 9.35, $\sum_{n=-\infty}^{\infty} |h[n]|^2$ can be calculated to

$$\begin{aligned}\sum_{n=-\infty}^{\infty} |h[n]|^2 &= \sum_{n=-\infty}^{\infty} \left|(A+B)\delta[n] + B(g\delta[n-m] + g^{2m}\delta[n-2m] + g^{3m}\delta[n-3m] + ...)\right|^2 \\ &= -0.7^2 + 0.729(0.7^2 + 0.49^2 + 0.343^2 + 0.240^2 + ...) \\ &= 1. \end{aligned} \tag{9.36}$$

Using this result with equation 9.32, the scale factor can be found

$$s \leq \frac{1}{\sqrt{\sum_{n=-\infty}^{\infty} |h[n]|^2}} = \frac{1}{\sqrt{1}} = 1 \ . \tag{9.37}$$

The scale factors for the lowpass comb filters are calculated in appendix E page 127, and the results are repeated in table 9.2.

Filter:	Scale factor:
LPCF1	0.5590
LPCF2	0.5832
LPCF3	0.5976
LPCF4	0.6042
APF5	1
APF6	1

Table 9.2: Simulated scale factors using the derivations of appendix E.

Since LPCF1 to LPCF4 are coupled in parallel the filters must be scaled equally. Choosing the lowest scale factor ensures the best protection against overflow. Furthermore since all parallel filters are summed after filtering, each scale factor must be divided by four, so the result of the summation is not greater than one.

From figure 9.16 it is seen that the last thing the reverberator does is adding the output of the filters with the input sample, meaning that the input must be scaled by $\frac{1}{2}$. To avoid the scaling from interfering with the sound of the reverberator, all scaling must be done prior to filtering. All in all the input must be scaled with a factor $s = \frac{1}{2} \cdot \frac{1}{4} \cdot 0.5602 = 0.0699$.

Allpass Filters Analysis to Determine SNR

Scaling the input signal to the filters have the advantage of making the system output more resistant to overflow. But it also has the disadvantage of decreasing the SNR. This can be seen from the equation [Oppenheim and Schafer, 1999, page 403]

$$\mathrm{SNR} = s^2 \frac{\sigma_y^2}{\sigma_f^2} \tag{9.38}$$

which defines SNR. σ_y^2 is the variance of the output signal and σ_f^2 is the noise variance. It can be seen from the equation 9.38, that the SNR is dependent on the scale factor squared.

To calculate SNR on the reverberator effect, first the noise variance σ_f^2 of the individual filters must be found. Since this is quite extensive only the principle is demonstrated for the allpass filter and the principles for the lowpass comb filters can be found in appendix E, page 127. Figure 9.17 show the linear noise model for one of the reverberators allpass filters [Hayes, 1999, page 350].

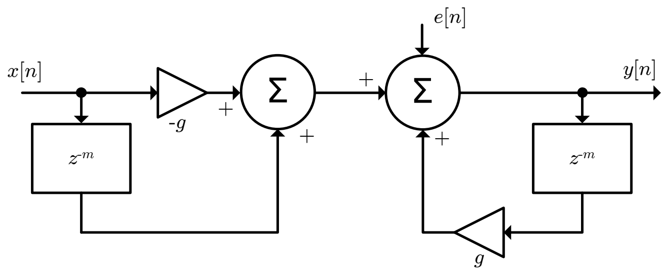

Figure 9.17: The principle of calculating noise variance on a direct form I allpass filter.

The variance of the noise $e[n]$ is $\sigma_e^2 = \frac{1}{12} 2^{-2B}$ [Hayes, 1999, page 349]. This represent the quantization noise where $B = 15$ is the numbers of bit representing the signal values. This amount of quantization noise is added for every truncation in the filter. The DSP has a 32 bit accumulator, which has the advantage that the quantization noise only happens during truncation when data is stored as a 16 bit word.

The variance of the output noise defined as [Hayes, 1999, page 349, 350]

$$\sigma_f^2 = \sigma_e^2 \cdot \sum_{n=-\infty}^{\infty} |h_{ef}[n]|^2 \tag{9.39}$$

Before this can be calculated the impulse response has to be found. From figure 9.17 it is seen that the quantization noise only passes through the pole of the filter. The effect of this is that only the impulse response from where $e[n]$ is added to $y[n]$ must be found. This is done using the same method as for the scale factor, as the transfer function is equivalent to equation 9.33, where $A = 0$ and $B = 1$. Using the same procedure $\sum_{n=-\infty}^{\infty} |h_{ef}[n]|^2$ is found to 1.961. The output noise variance can then be calculated using equation 9.39 to

$$\sigma_f^2 = \frac{1}{12} 2^{-2\cdot 15} \cdot 1.961 = 152.18 \cdot 10^{-12} \ . \tag{9.40}$$

The signal variance can be calculated setting the input to white noise with variance $\sigma_{\text{input}}^2 = \frac{1}{3}$ [Oppenheim and Schafer, 1999, page 402] and the SNR for the allpass filter can be found using equation 9.38 and setting the scale factor $s = 1$

$$\text{SNR} = s^2 \frac{\sigma_y^2}{\sigma_f^2} = 1^2 \frac{\frac{1}{3}}{152.18 \cdot 10^{-12}} = 2.19 \cdot 10^9. \tag{9.41}$$

In decibel this gives $10 \cdot \log(2.19 \cdot 10^9) = 93.41$ dB.

A similar analysis have been made for the lowpass comb filters. This can be found in appendix E on page 127.

SNR for Reverberator Effect

In this section the SNR for the entire reverberator will be elaborated. Figure 9.18 shows the different variance amplification factors that are needed for the SNR calculations.

It can be seen from the figure that for the reverberator a type I filter structure will have a superior SNR over a type II filter. This is due to the noise variance for a type I filters is $\sigma_f^2 = \sigma_e^2 \cdot \sum_{n=-\infty}^{\infty} |h_{ef}[n]|^2$ and for a type II $\sigma_f^2 = \sigma_e^2 \left(1 + \sum_{n=-\infty}^{\infty} |h_{xy}[n]|^2\right)$. From the values on the figure it is seen that $1 + \sum_{n=-\infty}^{\infty} |h_{xy}[n]|^2 > \sum_{n=-\infty}^{\infty} |h_{ef}[n]|^2$. Hence, type I has the better SNR.

During calculations one must notice that the noise for one filter is both dependent on the noise from the filter itself and the noise on the input that was made in the previous filter. This implies that the noise is accumulated through the filters of the reverberator. So the output noise of the reverberator can be found by identifying each noise source, calculating the noise on the reverberator output from each of them and finally adding

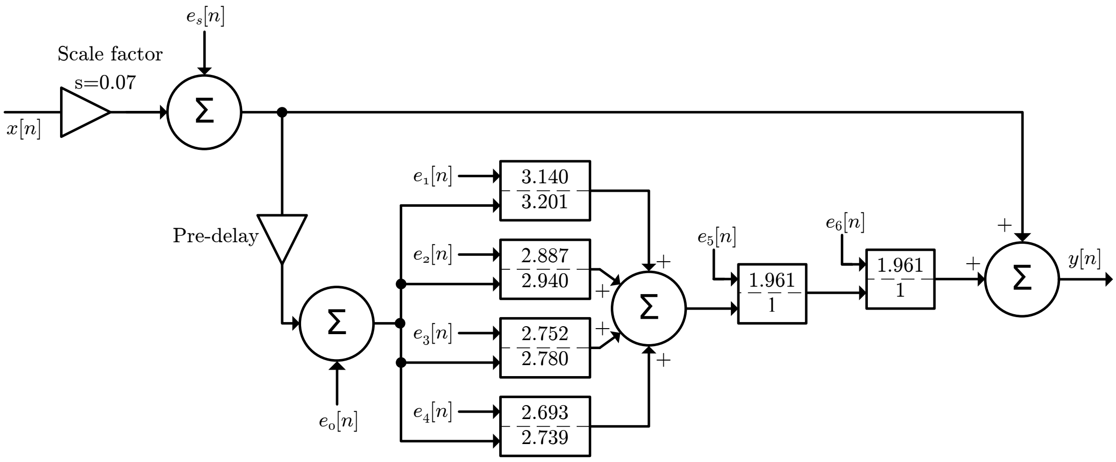

Figure 9.18: The variance amplification factors for calculating the SNR on the reverberator effect for reverberation time of 4 seconds.

them all together.

When measuring SNR, the noise variance on the system is found when the input is set $x[n] = 0$, and when the input is set to white noise.

First the noise variance is derived using figure 9.18. The figure shows the variance amplification from signal input to output and noise input to output of each filter block respectively. As a block in the reverberator has the previous block as input, the noise variance can be derived to

$$\begin{aligned}\sigma_f^2 = \Big(\big((\sigma_{e_s}^2 + \sigma_{e0}^2)(\sigma_{1xy}^2 + \sigma_{2xy}^2 + \sigma_{3xy}^2 + \sigma_{4xy}^2) \\ + \sigma_{e_1}^2(\sigma_{1ef}^2 + \sigma_{2ef}^2 + \sigma_{3ef}^2 + \sigma_{4ef}^2) \big) \cdot \sigma_{5xy}^2 + \sigma_{e_5}^2 \sigma_{5ef}^2 \Big) \cdot \sigma_{6xy}^2 + \sigma_{e_6}^2 \cdot \sigma_{6ef}^2 + \sigma_{e_s}^2 \ ,\end{aligned} \tag{9.42}$$

where each of the quantization noises $\sigma_{e_x}^2 = \frac{1}{12} 2^{-2 \cdot 15}$. If equation 9.42 is read backwards it is like looking on figure 9.18, from $y[n]$ towards $x[n]$. It is seen that each filter takes all the noise from the previous filters as input. The noise variance for a 4 second reverberation time can be calculated to

$$\begin{aligned}\sigma_f^2 = &\Bigg(\Bigg((\frac{1}{12} 2^{-2 \cdot 15} + \frac{1}{12} 2^{-2 \cdot 15})(3.201 + 2.940 + 2.780 + 2.739) \\ &+ \frac{1}{12} 2^{-2 \cdot 15}(3.140 + 2.887 + 2.752 + 2.693) \Bigg) \cdot 1 + \frac{1}{12} 2^{-2 \cdot 15} \cdot 1.961 \Bigg) \cdot 1 + \frac{1}{12} 2^{-2 \cdot 15} \cdot 1.961 \\ = &3.008 \cdot 10^{-9} \ .\end{aligned} \tag{9.43}$$

Next the variance for the signal must be found. This can be done using the already calculated impulse responses for filters input to output σ_{xy}^2. See figure 9.18 for values.

The signal variance can be calculated setting the input to white noise with variance $\sigma^2_{\text{input}} = \frac{1}{3}$ [Oppenheim and Schafer, 1999, page 402]. Then the signal variance is

$$\begin{aligned}\sigma^2_{\text{signal}} =& \sigma^2_{input} \cdot s^2(\sigma^2_{1xy} + \sigma^2_{2xy} + \sigma^2_{3xy} + \sigma^2_{4xy}) \cdot \sigma^2_{5xy} \cdot \sigma^2_{6xy} + s^2 \cdot \sigma^2_{\text{input}} \\ =& \frac{1}{3} \cdot (69.87 \cdot 10^{-3})^2(3.201 + 2.940 + 2.780 + 2.739) \cdot 1 \cdot 1 + (69.87 \cdot 10^{-3})^2 \cdot \frac{1}{3} \\ =& 20.633 \cdot 10^{-3} \ . \end{aligned} \tag{9.44}$$

Now the scaling factor and quantization noise is known the SNR can be calculated

$$\text{SNR} = \frac{\sigma^2_{\text{signal}}}{\sigma^2_f} = \frac{20.633 \cdot 10^{-3}}{3.008 \cdot 10^{-9}} = 6.860 \cdot 10^6 \tag{9.45}$$

In decibel this gives $10 \cdot \log(6.860 \cdot 10^6) = 68.36$ dB. Figure 9.19 show the SNR depending on the reverberation time.

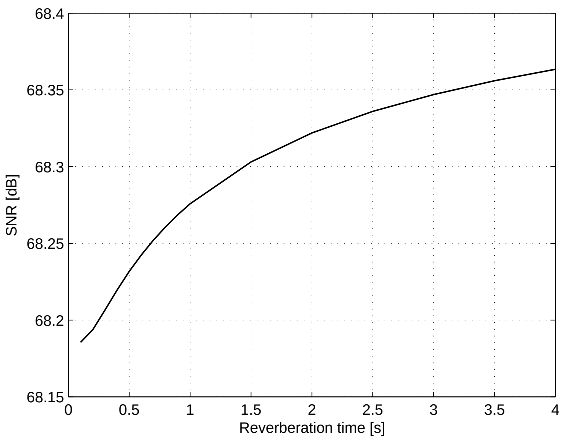

Figure 9.19: SNR of the reverberation effect based on derivation and calculations in MATLAB.

It is seen on the figure that the noise in the filters is almost unaffected over time. Since both the signal variance and the noise variance raises over time, making the SNR almost unaffected. This results in a difference of 1.2 dB from the shortest reverberation time to the longest.

9.5 Reverberation Memory Requirements

The reverberation effect are dependent on several ring buffers due to the different filters. To give an overview of the memory requirements this section describes the structure of the memory. The different ring buffers are listed in table 9.3

From the table the memory requirements can be calculated. As described in the chapter, the filters are implemented as type I filters. Each type I filter has an input buffer and an

Ring buffer:	Maximal length: [ms]	Maximal length: [words]
Pre-delay	100	1953
LPCF1	31	606
LPCF2	37	723
LPCF3	41	801
LPCF4	43	840
APF5	5	98
APF6	1.7	33

Table 9.3: Ring buffers for the reverberation effect.

output buffer. This means that the total memory requirements are the double of what table 9.3 lists, since every sample must be stored in both an input and an output buffer. For two successive filters, the output buffer of the first filter will be identical to the input buffer of the second filter. Using this fact the pre-delay buffer can be used as input buffer for the four lowpass comb filters and the output buffer of the first allpass filter can be used as input buffer for the second allpass filter. For the software implementation the total memory requirement can now be calulated. In total, eight ring buffers are needed:

1. Pre-delay buffer (also used for input buffer for all LPCF filters) length of 1953 words.
2. LPCF1 output buffer length of 606 words.
3. LPCF2 output buffer length of 723 words.
4. LPCF3 output buffer length of 801 words.
5. LPCF4 output buffer length of 840 words.
6. APF5 input buffer length 98 words.
7. APF5 output buffer (also used for input buffer for APF6 filter) length 98 words.
8. APF6 output buffer length 33 words.

This requires a total memory size of $1953+606+723+801+840+98+33 = 5054$ words in the use for ring buffers, which is about $\frac{1}{6}$ of the total system memory.

9.6 Conclusion on Reverberator

This chapter was begun with review of different reverberation algorithm. A mix between the Shroeder's and Moorer's reverberator was chosen for implementation since it performed quite well in MATLAB simulation and listening tests. The best implementation structure was found to be a direct form I structure since the noise caused by rounding is lower and the memory consumption can be reduced by recycling the buffers. Due to time limitations, the acceptance test specified has not been carried out.