Chapter 3

Linear Modulation Techniques

Contents

- We are typically interested in locating a *message* signal to some new frequency location, where it can be efficiently transmitted
- The carrier of the message signal is usually sinusoidal
- A modulated carrier can be represented as

$$x_c(t) = A(t)\cos\left[2\pi f_c t + \phi(t)\right]$$

where $A(t)$ is linear modulation, f_c the carrier frequency, and $\phi(t)$ is phase modulation

3.1 Linear Modulation

- For linear modulation schemes, we may set $\phi(t) = 0$ without loss of generality

$$x_c(t) = A(t)\cos(2\pi f_c t)$$

with $A(t)$ placed in one-to-one correspondence with the message signal

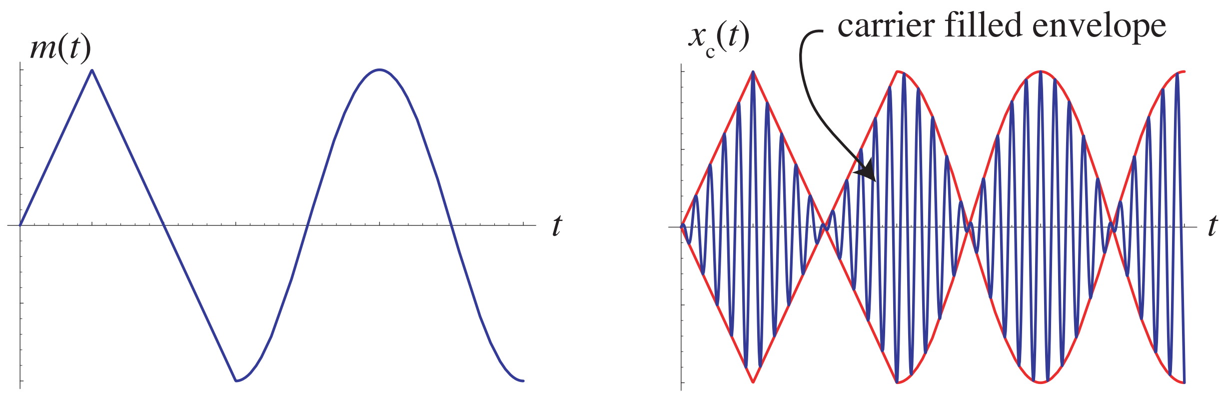
3.1.1 Double-Sideband Modulation (DSB)

- Let $A(t) \propto m(t)$, the message signal, thus

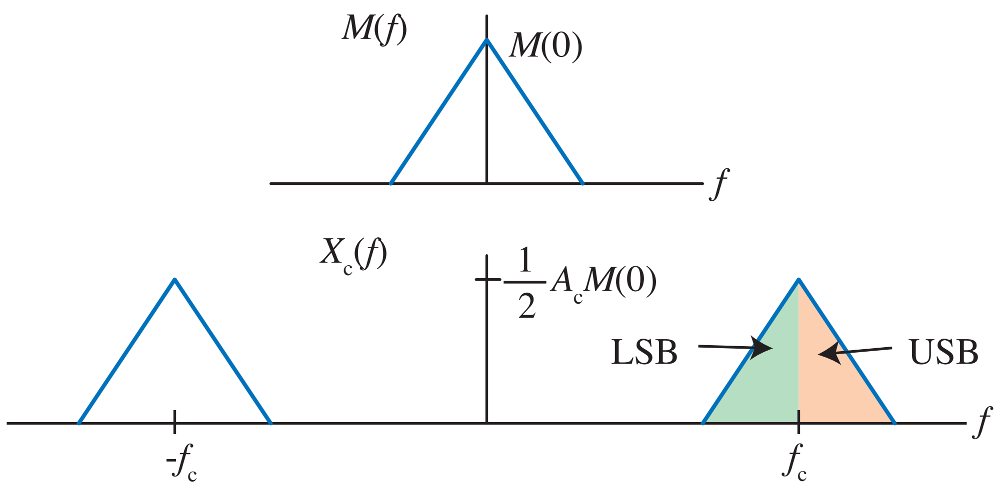
$$x_c(t) = A_c m(t)\cos(2\pi f_c t)$$

- From the modulation theorem it follows that

$$X_c(f) = \frac{1}{2}A_c M(f - f_c) + \frac{1}{2}A_c M(f + f_c)$$



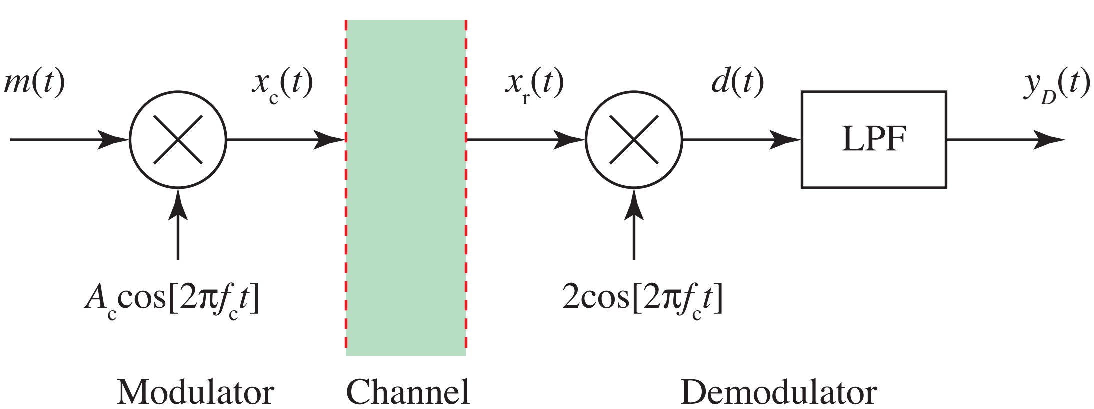
DSB time domain waveforms



DSB spectra

Coherent Demodulation

- The received signal is multiplied by the signal $2\cos(2\pi f_c t)$, which is synchronous with the transmitter carrier

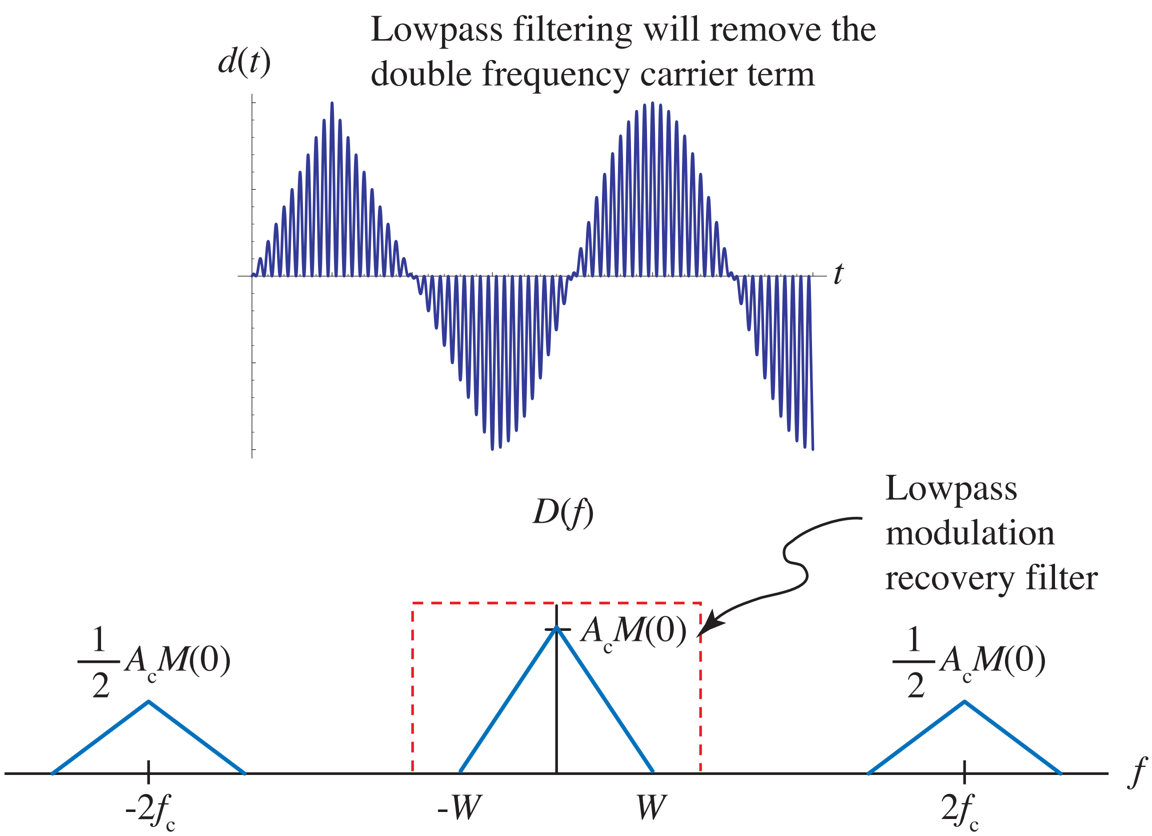


- For an ideal channel $x_r(t) = x_c(t)$, so

$$\begin{aligned} d(t) &= \left[A_c m(t)\cos(2\pi f_c t)\right] 2\cos(2\pi f_c t) \\ &= A_c m(t) + A_c m(t)\cos(2\pi(2f_c)t) \end{aligned}$$

  where we have used the trig identity $2\cos^2 x = 1 + \cos 2x$

- The waveform and spectra of $d(t)$ is shown below (assuming $m(t)$ has a triangular spectrum in $D(f)$)



Waveform and spectrum of $d(t)$

- Typically the carrier frequency is much greater than the message bandwidth W, so $m(t)$ can be recovered via lowpass filtering

- The scale factor A_c can be dealt with in downstream signal processing, e.g., an automatic gain control (AGC) amplifier

- Assuming an ideal lowpass filter, the only requirement is that the cutoff frequency be greater than W and less than $2f_c - W$
- The difficulty with this demodulator is the need for a coherent carrier reference
- To see how critical this is to demodulation of $m(t)$ suppose that the reference signal is of the form

$$c(t) = 2\cos[2\pi f_c t + \theta(t)]$$

  where $\theta(t)$ is a time-varying phase error

- With the imperfect carrier reference signal

$$\begin{aligned} d(t) &= A_c m(t)\cos\theta(t) + A_c m(t)\cos[2\pi f_c t + \theta(t)] \\ y_D(t) &= m(t)\cos\theta(t) \end{aligned}$$

- Suppose that $\theta(t)$ is a constant or slowly varying, then the $\cos\theta(t)$ appears as a fixed or time varying attenuation factor
- Even a slowly varying attenuation can be very detrimental from a distortion standpoint
  - If say $\theta(t) = \Delta f t$ and $m(t) = \cos(2\pi f_m t)$, then

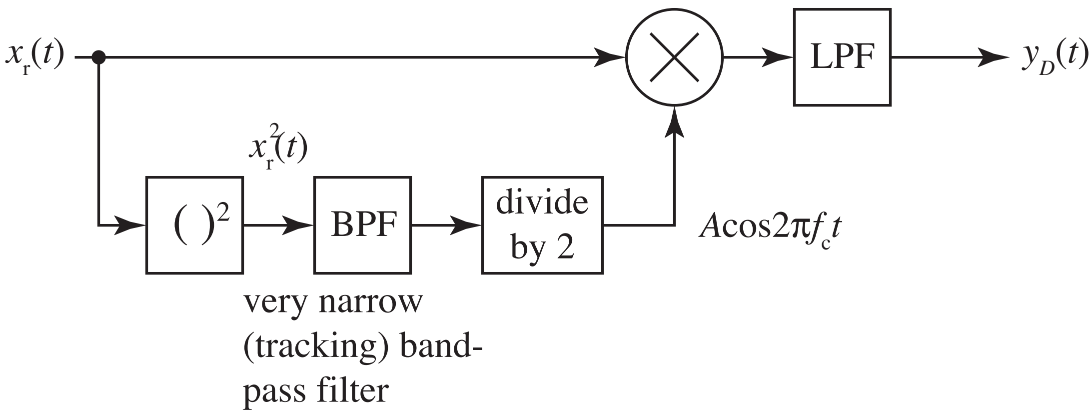
$$y_D(t) = \frac{1}{2}[\cos[2\pi(f_m - \Delta f)t] + \cos[2\pi(f_m + \Delta f)t]]$$

    which is the sum of two tones

- Being able to generate a coherent local reference is also a practical manner

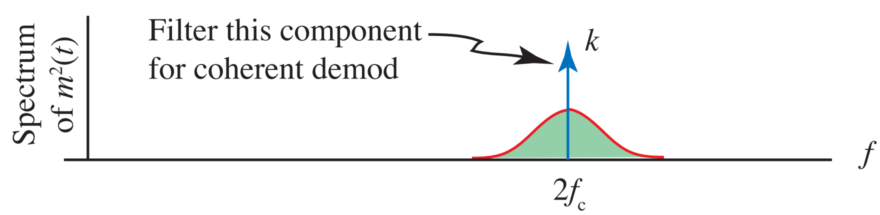
- One scheme is to simply square the received DSB signal

$$\begin{aligned} x_r^2(t) &= A_c^2 m^2(t)\cos^2(2\pi f_c t) \\ &= \frac{1}{2}A_c^2 m^2(t) + \frac{1}{2}A_c^2 m^2(t)\cos[2\pi(2f_c)t] \end{aligned}$$



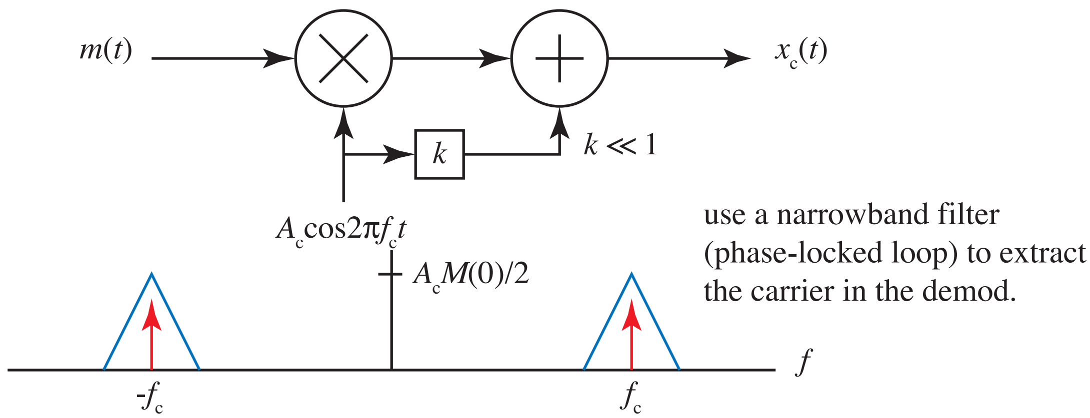
Carrier recovery concept using signal squaring

- Assuming that $m^2(t)$ has a nonzero DC value, then the double frequency term will have a spectral line at $2f_c$ which can be divided by two following filtering by a narrowband bandpass filter, i.e., $\mathcal{F}\{m^2(t)\} = k\delta(f) + \cdots$



- Note that unless $m(t)$ has a DC component, $X_c(f)$ will not contain a carrier term (read $\delta(f \pm f_c)$), thus DSB is also called a *suppressed carrier* scheme

- Consider transmitting a small amount of unmodulated carrier



3.1.2 Amplitude Modulation

- Amplitude modulation (AM) can be created by simply adding a DC bias to the message signal

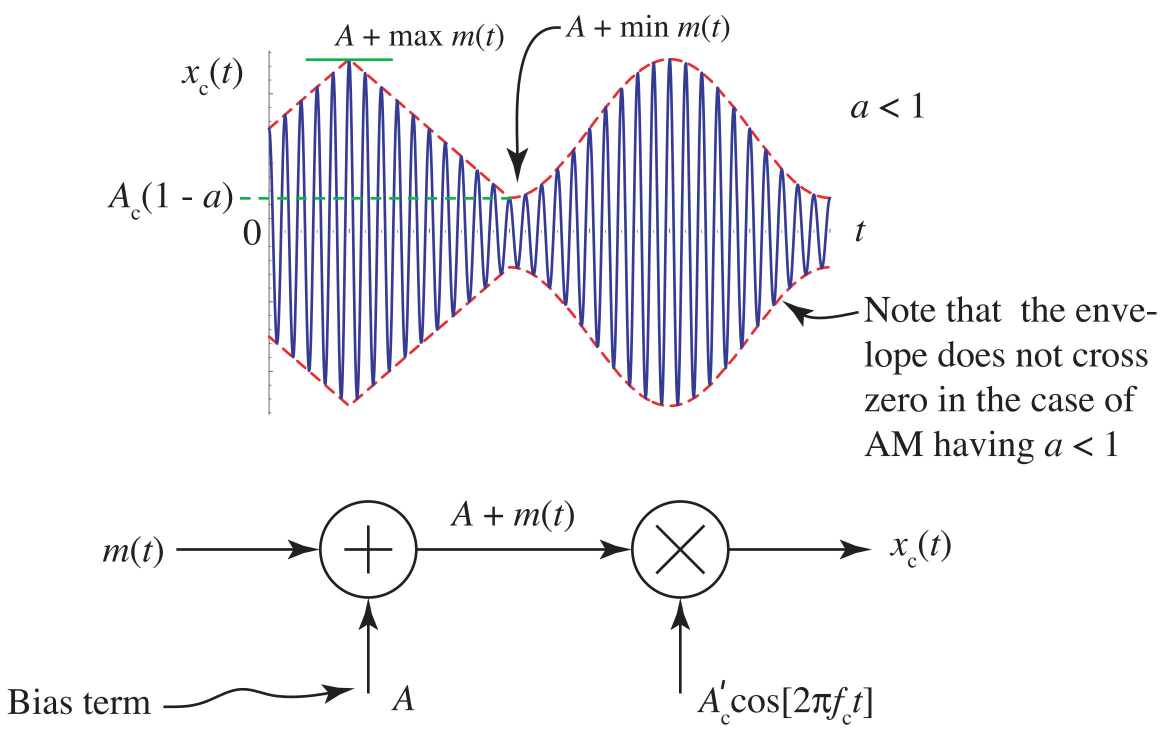
$$\begin{aligned} x_c(t) &= \big[A + m(t)\big]A'_c \cos(2\pi f_c t) \\ &= A_c\big[1 + a m_n(t)\big]\cos(2\pi f_c t) \end{aligned}$$

where $A_c = AA'_c$, $m_n(t)$ is the *normalized message* such that $\min m_n(t) = -1$,

$$\boxed{m_n(t) = \frac{m(t)}{|\min m(t)|}}$$

and a is the *modulation index*

$$\boxed{a = \frac{|\min m(t)|}{A}}$$



Generation of AM and a sample wavefrom

- Note that if $m(t)$ is symmetrical about zero and we define d_1 as the peak-to-peak value of $x_c(t)$ and d_2 as the valley-to-valley value of $x_c(t)$, it follows that

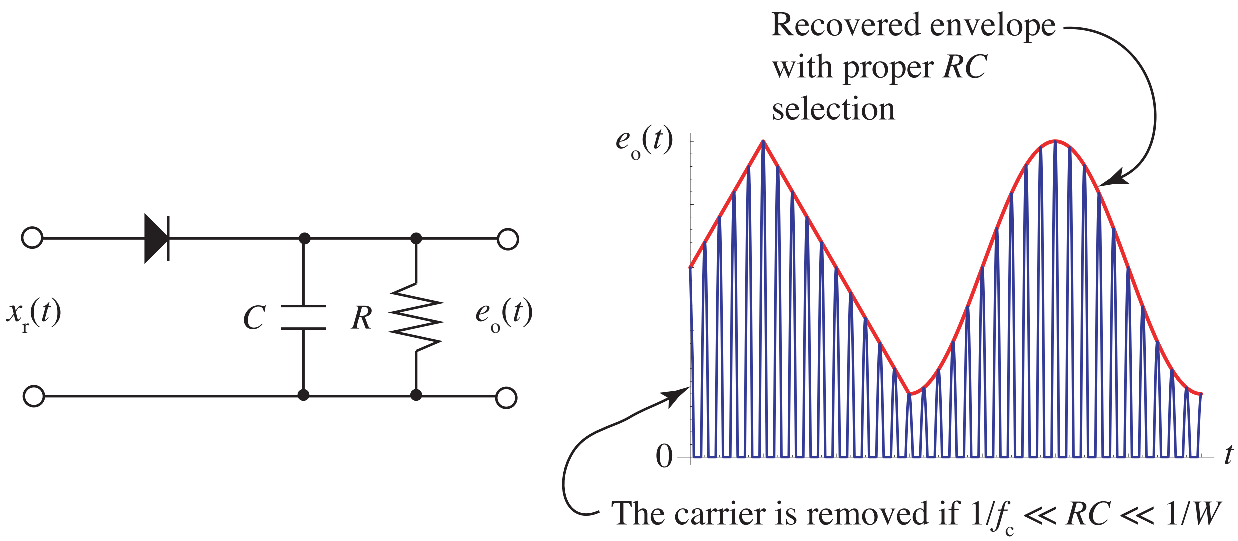
$$a = \frac{d_1 - d_2}{d_1 + d_2}$$

proof: $\max m(t) = -\min m(t) = |\min m(t)|$, so

$$\begin{aligned} \frac{d_1 - d_2}{d_1 + d_2} &= \frac{2[(A + |\min m(t)|) - (A - |\min m(t)|)]}{2[(A + |\min m(t)|) + (A - |\min m(t)|)]} \\ &= \frac{|\min m(t)|}{A} = a \end{aligned}$$

- The message signal can be recovered from $x_c(t)$ using a technique known as *envelope detection*

- A diode, resistor, and capacitor is all that is needed to construct and envelope detector

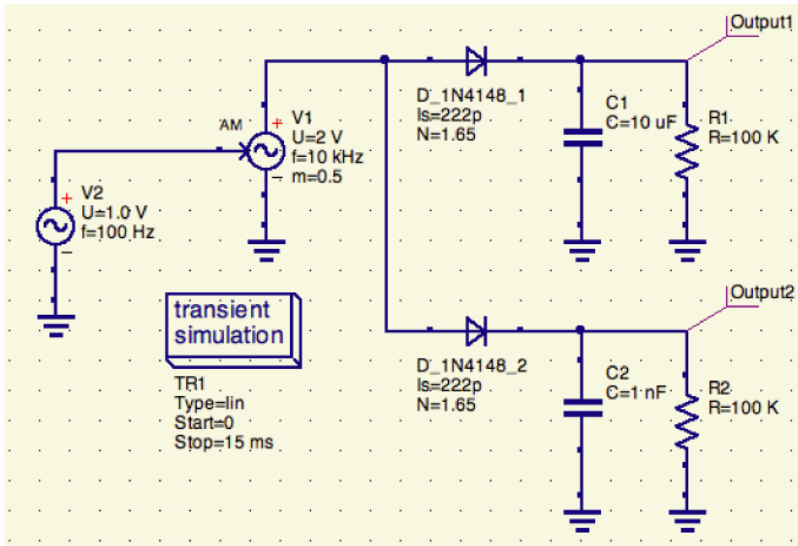


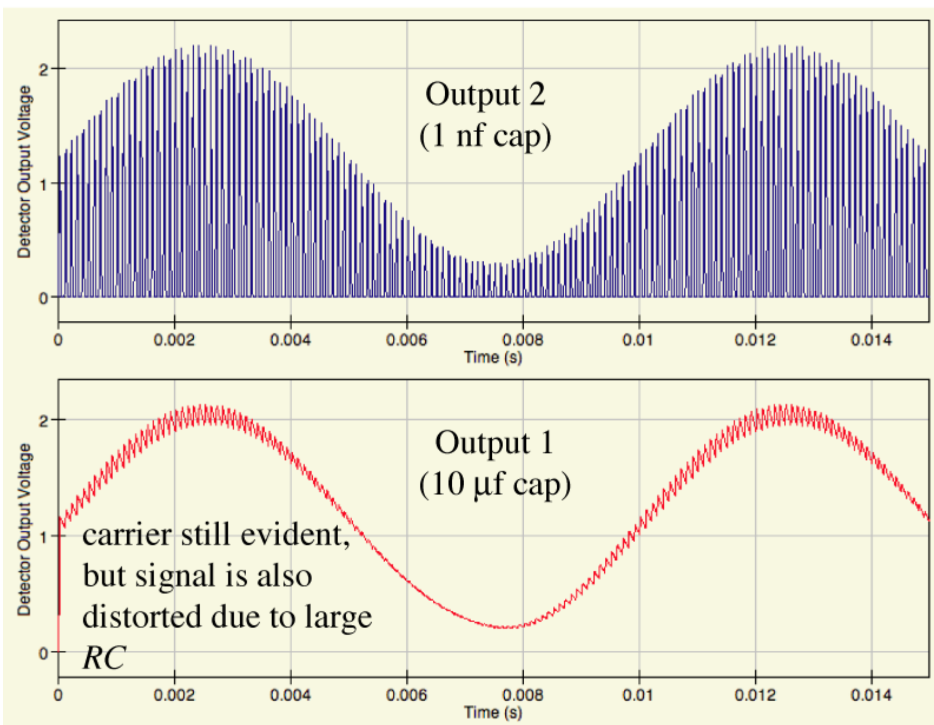
Envelope detector

- The circuit shown above is actually a combination of a nonlinearity and filter (system with memory)

- A detailed analysis of this circuit is more difficult than you might think

- A SPICE circuit simulation is relatively straight forward, but it can be time consuming if $W \ll f_c$





- The simple envelope detector fails if $A_c[1 + am_n(t)] < 0$
  - In the circuit shown above, the diode is not ideal and hence there is a turn-on voltage which further limits the maximum value of a
- The RC time constant cutoff frequency must lie between both W and f_c, hence good operation also requires that $f_c \gg W$

- Digital signal processing based envelope detectors are also possible
- Historically the envelope detector has provided a very low-cost means to recover the message signal on AM carrier
- The spectrum of an AM signal is

$$X_c(f) = \underbrace{\frac{A_c}{2}\big[\delta(f - f_c) + \delta(f + f_c)\big]}_{\text{pure carrier spectrum}} + \underbrace{\frac{aA_c}{2}\big[M_n(f - f_c) + M_n(f + f_c)\big]}_{\text{DSB spectrum}}$$

AM Power Efficiency

- Low-cost and easy to implement demodulators is a plus for AM, but what is the downside?
- Adding the bias term to $m(t)$ means that a fraction of the total transmitted power is dedicated to a pure carrier
- The total power in $x_c(t)$ is can be written in terms of the time average operator introduced in Chapter 2

$$\begin{aligned}\langle x_c^2(t)\rangle &= \langle A_c^2[1 + am_n(t)]^2 \cos^2(2\pi f_c t)\rangle \\ &= \frac{A_c^2}{2}\langle[1 + 2am_n(t) + a^2m_n^2(t)][1 + \cos(2\pi(2f_c)t]\rangle\end{aligned}$$

- If $m(t)$ is *slowly varying* with respect to $\cos(2\pi f_c t)$, i.e.,

$$\langle m(t)\cos\omega_c t\rangle \simeq 0,$$

then

$$\begin{aligned}\langle x_c^2(t)\rangle &= \frac{A_c^2}{2}\big[1 + 2a\langle m_n(t)\rangle + a^2\langle m_n^2(t)\rangle\big]\\ &= \frac{A_c^2}{2}\big[1 + a^2\langle m^2(t)\rangle\big] = \underbrace{\frac{A_c^2}{2}}_{P_{\text{carrier}}} + \underbrace{\frac{a^2A_c^2}{2}\langle m_n^2(t)\rangle}_{P_{\text{sidebands}}}\end{aligned}$$

where the last line resulted from the assumption $\langle m(t)\rangle = 0$ (the DC or average value of $m(t)$ is zero)

- Definition: AM Efficiency

$$\boxed{E_{ff} \triangleq \frac{a^2\langle m_n^2(t)\rangle}{1 + a^2\langle m_n^2(t)\rangle} \stackrel{\text{also}}{=} \frac{\langle m^2(t)\rangle}{A^2 + \langle m^2(t)\rangle}}$$

Example 3.1: **Single Sinusoid AM**

- An AM signal of the form

$$x_c(t) = A_c[1 + a\cos(2\pi f_m t + \pi/3)]\cos(2\pi f_c t)$$

contains a total power of 1000 W

- The modulation index is 0.8
- Find the power contained in the carrier and the sidebands, also find the efficiency
- The total power is

$$1000 = \langle x_c^2(t)\rangle = \frac{A_c^2}{2} + \frac{a^2A_c^2}{2}\cdot\langle m_n^2(t)\rangle$$

- It should be clear that in this problem $m_n(t) = \cos(2\pi f_m t)$, so $\langle m_n^2(t)\rangle = 1/2$ and

$$1000 = A_c^2\left[\frac{1}{2} + \frac{1}{4}0.64\right] = \frac{33}{50}A_c^2$$

- Thus we see that

$$A_c^2 = 1000 \cdot \frac{50}{33} = 1515.15$$

and

$$P_{\text{carrier}} = \frac{1}{2}A_c^2 = \frac{1515}{2} = 757.6 \text{ W}$$

and thus

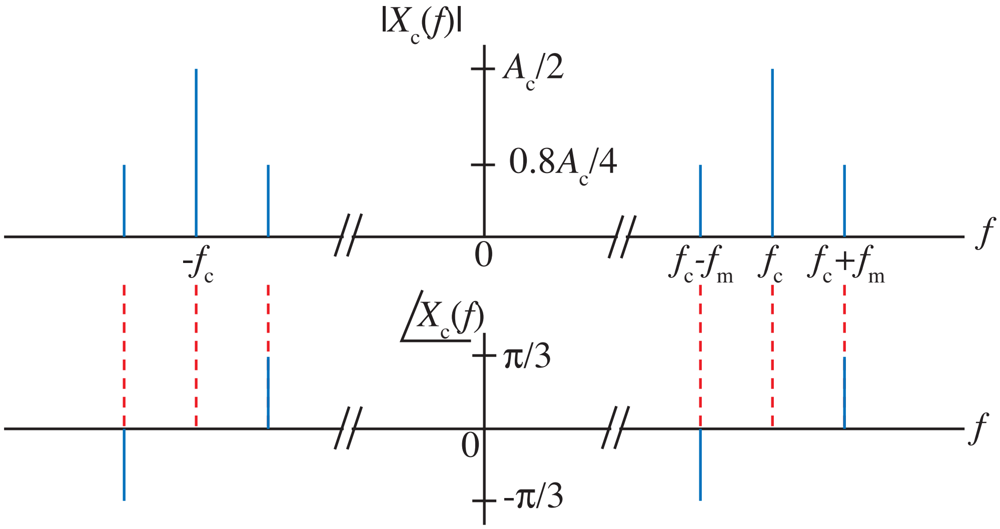
$$P_{\text{sidebands}} = 1000 - P_c = 242.4 \text{ W}$$

- The efficiency is

$$E_{ff} = \frac{242.4}{1000} = 0.242 \text{ or } 24.2\%$$

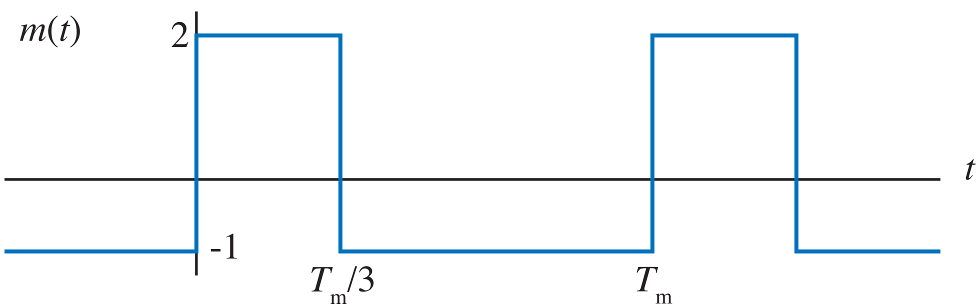
- The magnitude and phase spectra can be plotted by first expanding out $x_c(t)$

$$\begin{aligned} x_c(t) &= A_c\cos(2\pi f_c t) + aA_c\cos(2\pi f_m t + \pi/3)\cos(2\pi f_c t) \\ &= A_c\cos(2\pi f_c t) \\ &\quad + \frac{aA_c}{2}\cos[2\pi(f_c + f_m)t + \pi/3] \\ &\quad + \frac{aA_c}{2}\cos[2\pi(f_c - f_m)t - \pi/3] \end{aligned}$$



Amplitude and phase spectra for one tone AM

Example 3.2: **Pulse Train with DC Offset**



- Find $m_n(t)$ and the efficiency E
- From the definition of $m_n(t)$

$$m_n(t) = \frac{m(t)}{|\min m(t)|} = \frac{m(t)}{|-1|} = m(t)$$

- The efficiency is

$$E = \frac{a^2\langle m_n^2(t)\rangle}{1 + a^2\langle m_n^2(t)\rangle}$$

- To obtain $\langle m_n^2(t)\rangle$ we form the time average

$$\begin{aligned}\langle m_n^2(t)\rangle &= \frac{1}{T_m}\left[\int_0^{T_m/3}(2)^2\,dt + \int_{T_m/3}^{T_m}(-1)^2\,dt\right]\\ &= \frac{1}{T_m}\left[\frac{T_m}{3}\cdot 4 + \frac{2T_m}{3}\cdot 1\right] = \frac{4}{3}+\frac{2}{3}=\frac{6}{3}=2\end{aligned}$$

thus

$$E = \frac{2a^2}{1+2a^2}$$

- The best AM efficiency we can achieve with this waveform is when $a = 1$

$$E_{ff}\Big|_{a=1} = \frac{2}{3} = 0.67 \text{ or } 67\%$$

- Suppose that the message signal is $-m(t)$ as given here
- Now $\min m(t) = -2$ and $m_n(t) = m(t)/2$ and

$$\langle m_n^2(t)\rangle = \frac{1}{3}\cdot(-1)^2 + \frac{2}{3}\cdot(1/2)^2 = \frac{1}{2}$$

- The efficiency in this case is

$$E_{ff} = \frac{(1/2)a^2}{1+(1/2)a^2} = \frac{a^2}{2+a^2}$$

- Now when $a = 1$ we have $E_{ff} = 1/3$ or just 33.3%
- Note that for 50% duty cycle squarewave the efficiency maximum is just 50%

Example 3.3: Multiple Sinusoids

- Suppose that $m(t)$ is a sum of multiple sinusoids (multi-tone AM)
$$m(t) = \sum_{k=1}^{M} A_k \cos(2\pi f_k t + \phi_k)$$
where M is the number of sinusoids, f_k values might be constrained over some band of frequencies W, e.g., $f_k \leq W$, and the phase values ϕ_k can be any value on $[0, 2\pi]$

- To find $m_n(t)$ we need to find $\min m(t)$

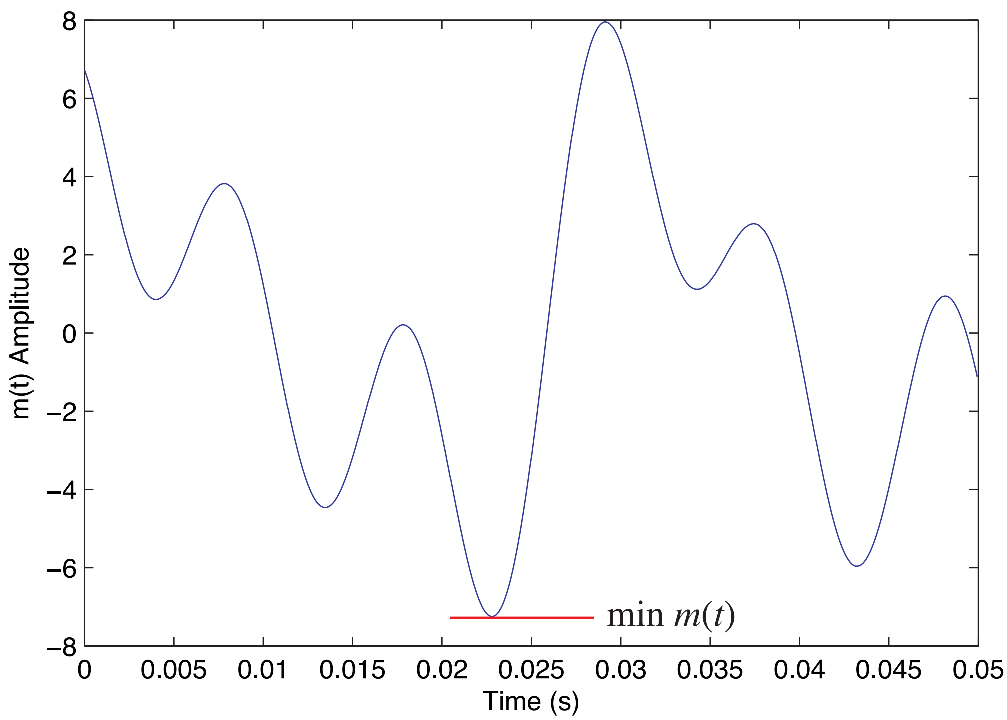
- A lower bound on $\min m(t)$ is $-\sum_{k=1}^{M} A_k$; why?

- The worst case value may not occur in practice depending upon the phase and frequency values, so we may have to resort to a numerical search or a plot of the waveform

- Suppose that $M = 3$ with $f_k = \{65, 100, 35\}$ Hz, $A_k = \{2, 3.5, 4.2\}$, and $\phi_k = \{0, \pi/3, -\pi/4\}$ rad.

```
>> [m,t] = M_sinusoids(1000,[65 100 35],[2 3.5 4.2],...
                [0 pi/3 -pi/4], 20000);>> plot(t,m)
>> min(m)

     ans =  -7.2462e+00

>> -sum([2 3.5 4.2]) % worst case minimum value

     ans =  -9.7000e+00

>> subplot(311)
>> plot(t,(1 + 0.25*m/abs(min(m))).*cos(2*pi*1000*t))
>> hold
```

```
Current plot held
>> plot(t,1 + 0.25*m/abs(min(m)),'r')
>> subplot(312)
>> plot(t,(1 + 0.5*m/abs(min(m))).*cos(2*pi*1000*t))
>> hold
Current plot held
>> plot(t,1 + 0.5*m/abs(min(m)),'r')
>> subplot(313)
>> plot(t,(1 + 1.0*m/abs(min(m))).*cos(2*pi*1000*t))
>> hold
Current plot held
>> plot(t,1 + 1.0*m/abs(min(m)),'r')
```

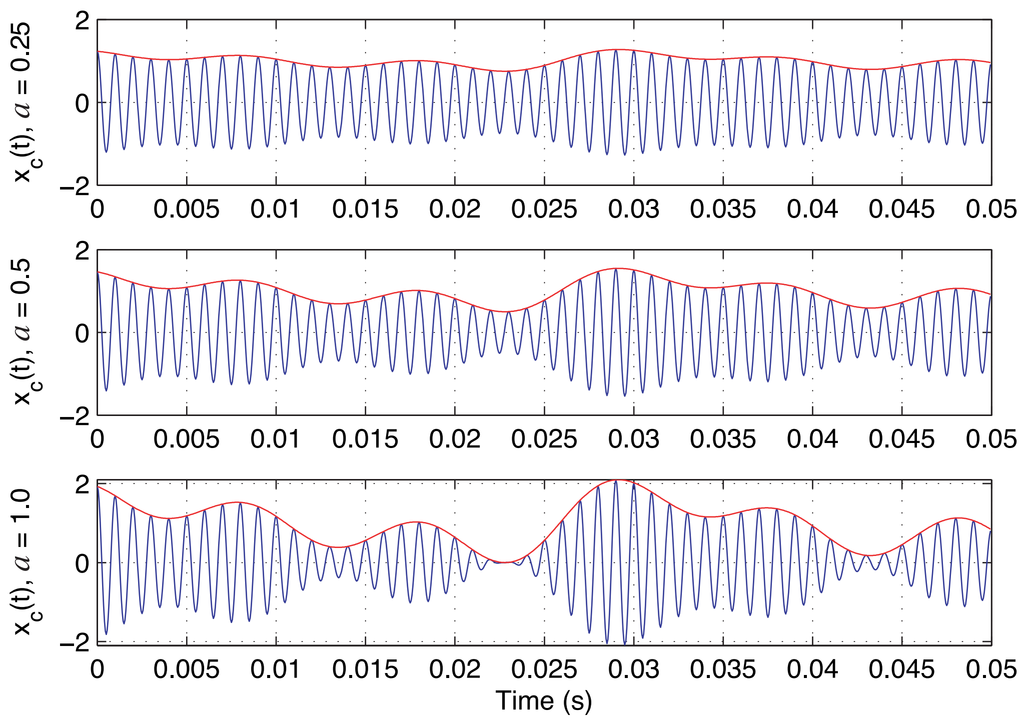


Finding $\min m(t)$ graphically

- The normalization factor is approximately given by 7.246, that is

$$m_n(t) = \frac{m(t)}{7.246}$$

- Shown below are plots of $x_c(t)$ for $a = 0.25, 0.5$ and 1 using $f_c = 1000$ Hz



Modulation index comparison ($f_c = 1000$ Hz)

- To obtain the efficiency of multi-tone AM we first calculate $\langle m_n^2(t)\rangle$ assuming unique frequencies

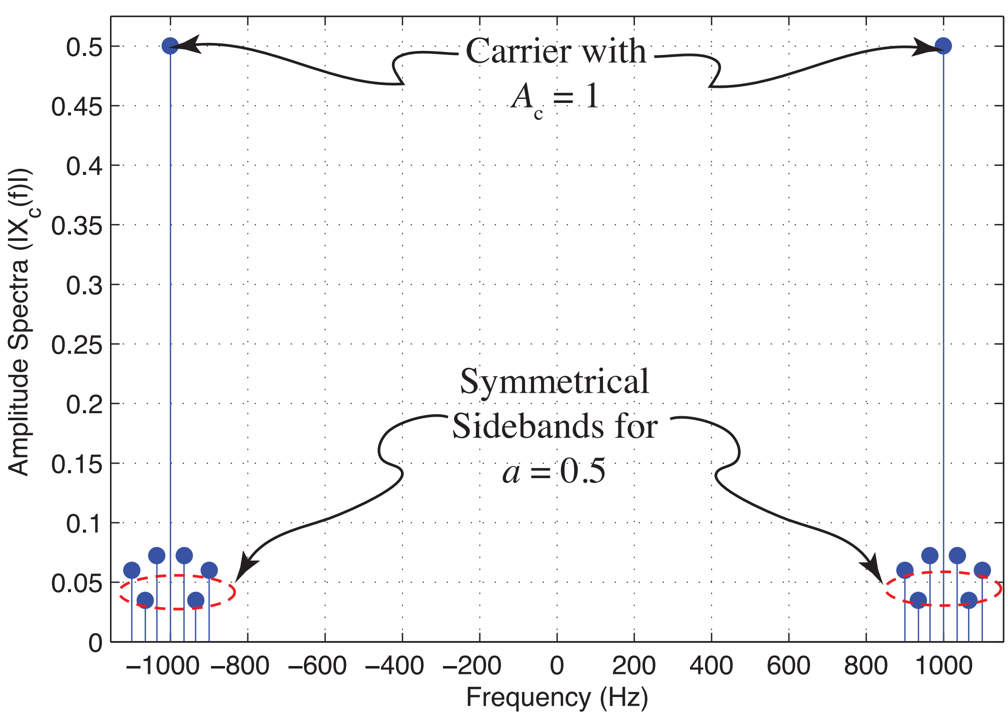
$$\begin{aligned}\langle m_n^2(t)\rangle &= \sum_{k=1}^{M} \frac{A_k^2}{2|\min m(t)|^2}\\ &= \frac{2^2 + 3.5^2 + 4.2^2}{2 \times 7.246^2} = 0.3227\end{aligned}$$

- The maximum efficiency is just

$$E_{ff}\Big|_{a=1} = \frac{0.3227}{1 + 0.3227} = 0.244 \text{ or } 24.4\%$$

- A remaining interest is the spectrum of $x_c(t)$

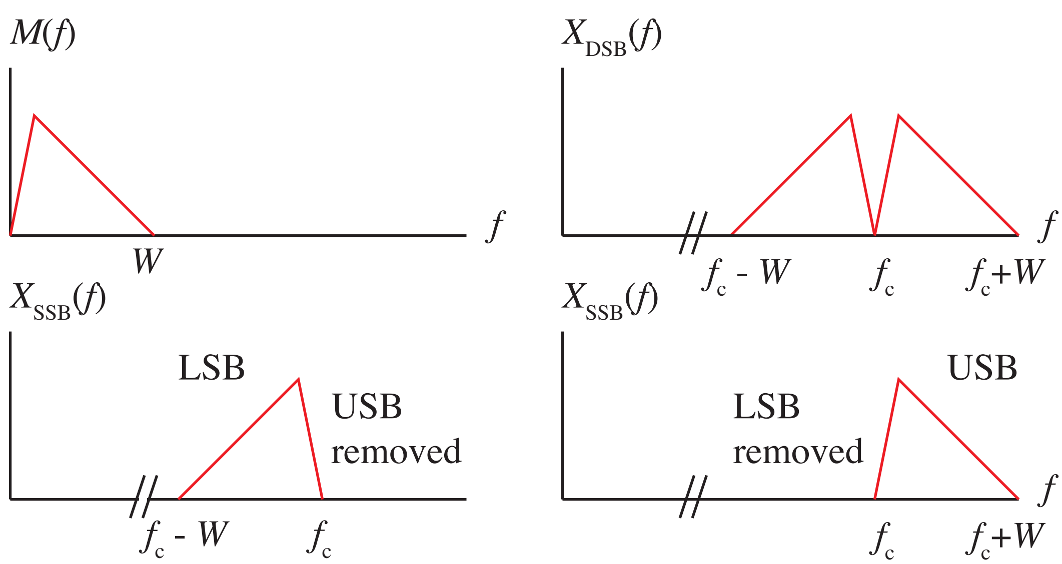
$$
\begin{aligned}
X_c(f) = {} & \frac{A_c}{2}\left[\delta(f - f_c) + \delta(f + f_c)\right] \\
& + \frac{aA_c}{4}\sum_{k=1}^{M} A_k\Big[e^{j\phi_k}\delta(f - (f_c + f_k)) \\
& \qquad + e^{-j\phi_k}\delta(f + (f_c + f_k))\Big] \text{ (USB terms)} \\
& + \frac{aA_c}{4}\sum_{k=1}^{M} A_k\Big[e^{j\phi_k}\delta(f - (f_c - f_k)) \\
& \qquad + e^{-j\phi_k}\delta(f + (f_c - f_k))\Big] \text{ (LSB terms)}
\end{aligned}
$$



Amplitude spectra

3.1.3 Single-Sideband Modulation

- In the study of DSB it was observed that the USB and LSB spectra are related, that is the magnitude spectra about f_c has even symmetry and phase spectra about f_c has odd symmetry
- The information is redundant, meaning that $m(t)$ can be reconstructed from one or the other sidebands
- Transmitting just the USB or LSB results in *single-sideband* (SSB)
- For $m(t)$ having lowpass bandwidth of W the bandwidth required for DSB, centered on f_c is $2W$
- Since SSB operates by transmitting just one sideband, the transmission bandwidth is reduced to just W



DSB to two forms of SSB: USSB and LSSB

- The filtering required to obtain an SSB signal is best explained with the aid of the *Hilbert transform*, so we divert from text

Chapter 3 back to Chapter 2 to briefly study the basic properties of this transform

Hilbert Transform

- The *Hilbert transform* is nothing more than a filter that shifts the phase of all frequency components by $-\pi/2$, i.e.,

$$H(f) = -j\,\mathrm{sgn}(f)$$

where

$$\mathrm{sgn}(f) = \begin{cases} 1, & f > 0 \\ 0, & f = 0 \\ -1, & f < 0 \end{cases}$$

- The Hilbert transform of signal $x(t)$ can be written in terms of the Fourier transform and inverse Fourier transform

$$\begin{aligned} \hat{x}(t) &= \mathcal{F}^{-1}\big[-j\,\mathrm{sgn}(f)X(f)\big] \\ &= h(t) * x(t) \end{aligned}$$

where $h(t) = \mathcal{F}^{-1}\{H(f)\}$

- We can find the impulse response $h(t)$ using the duality theorem and the differentiation theorem

$$\frac{d}{df}H(f) \overset{\mathcal{F}}{\longleftrightarrow} (-j2\pi t)h(-t)$$

where here $H(f) = -j\,\mathrm{sgn}(f)$, so

$$\frac{d}{df}H(f) = -2j\delta(f)$$

- Clearly,

$$\mathcal{F}^{-1}\{-2j\delta(f)\} = -2j$$

so

$$h(t) = \frac{-2j}{-j2\pi t} = \frac{1}{\pi t}$$

and

$$\frac{1}{\pi t} \stackrel{\mathcal{F}}{\longleftrightarrow} -j\,\mathrm{sgn}(f)$$

- In the time domain the Hilbert transform is the convolution integral

$$\hat{x}(t) = \int_{-\infty}^{\infty} \frac{x(\lambda)}{\pi(t-\lambda)}\,d\lambda = \int_{-\infty}^{\infty} \frac{x(t-\lambda)}{\pi\lambda}\,d\lambda$$

- Note that since the Hilbert transform of $x(t)$ is a $-\pi/2$ phase shift, the Hilbert transform of $\hat{x}(t)$ is

$$\hat{\hat{x}}(t) = -x(t)$$

why? observe that $(-j\,\mathrm{sgn}(f))^2 = -1$

Example 3.4: $x(t) = \cos\omega_0 t$

- By definition

$$\begin{aligned}\hat{X}(f) &= -j\,\mathrm{sgn}(f)\cdot\frac{1}{2}\big[\delta(f-f_0)+\delta(f+f_0)\big]\\ &= -j\frac{1}{2}\delta(f-f_c) + j\frac{1}{2}\delta(f+f_0)\end{aligned}$$

so from $e^{j\omega_0 t} \overset{\mathcal{F}}{\Rightarrow} \delta(f - f_0)$

$$\begin{aligned}\hat{x}(t) &= -j\frac{1}{2}e^{j\omega_0 t} + j\frac{1}{2}e^{-j\omega_0 t} \\ &= \frac{e^{j\omega_0 t} - e^{-j\omega_0 t}}{2j} = \sin\omega_0 t\end{aligned}$$

or

$$\widehat{\cos\omega_0 t} = \sin\omega_0 t$$

- It also follows that

$$\widehat{\sin\omega_0 t} = \widehat{\widehat{\cos\omega_0 t}} = -\cos\omega_0 t$$

since $\hat{\hat{x}}(t) = -x(t)$

Hilbert Transform Properties

1. The energy (power) in $x(t)$ and $\hat{x}(t)$ are equal

   The proof follows from the fact that $|Y(f)|^2 = |H(f)|^2|X(f)|^2$ and $|j\,\text{sgn}(f)|^2 = 1$

2. $x(t)$ and $\hat{x}(t)$ are orthogonal, that is

$$\begin{aligned}\int_{-\infty}^{\infty} x(t)\hat{x}(t)\,dt &= 0 \text{ (energy signal)} \\ \lim_{T\to\infty}\frac{1}{2T}\int_{-T}^{T} x(t)\hat{x}(t)\,dt &= 0 \text{ (power signal)}\end{aligned}$$

The proof follows for the case of energy signals by generalizing Parseval's theorem

$$\begin{aligned}\int_{-\infty}^{\infty} x(t)\hat{x}(t)\,dt &= \int_{-\infty}^{\infty} X(f)\hat{X}^*(f)\,df \\ &= \int_{-\infty}^{\infty} \underbrace{(j\,\text{sgn}(f))}_{\text{odd}}\underbrace{|X(f)|^2}_{\text{even}}\,df = 0\end{aligned}$$

3. Given signals $m(t)$ and $c(t)$ such that the corresponding spectra are

$$\begin{aligned}M(f) &= 0 \text{ for } |f| > W \text{ (a lowpass signal)} \\ C(f) &= 0 \text{ for } |f| < W \text{ ($c(t)$ a highpass signal)}\end{aligned}$$

then

$$\widehat{m(t)c(t)} = m(t)\hat{c}(t)$$

Example 3.5: $c(t) = \cos\omega_0 t$

- Suppose that $M(f) = 0$ for $|f| > W$ and $f_0 > W$ then

$$\begin{aligned}\widehat{m(t)\cos\omega_0 t} &= m(t)\widehat{\cos\omega_0 t} \\ &= m(t)\sin\omega_0 t\end{aligned}$$

Analytic Signals

- Define analytic signal $z(t)$ as

$$z(t) = x(t) + j\hat{x}(t)$$

where $x(t)$ is a real signal

- The *envelope* of $z(t)$ is $|z(t)|$ and is related to the envelope discussed with DSB and AM signals

- The spectrum of an analytic signal has single-sideband characteristics

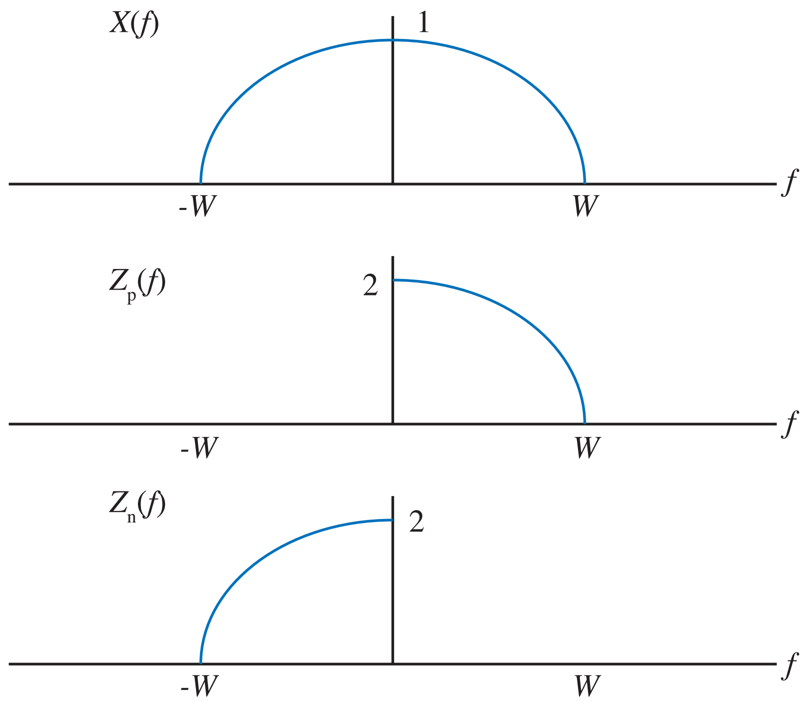
- In particular for $z_p(t) = x(t) + j\hat{x}(t)$

$$\begin{aligned} Z_p(f) &= X(f) + j\big\{-j\,\text{sgn}(f)X(f)\big\} \\ &= X(f)\big[1 + \text{sgn}(f)\big] \\ &= \begin{cases} 2X(f), & f > 0 \\ 0, & f < 0 \end{cases} \end{aligned}$$

<u>Note</u>: Only positive frequencies present

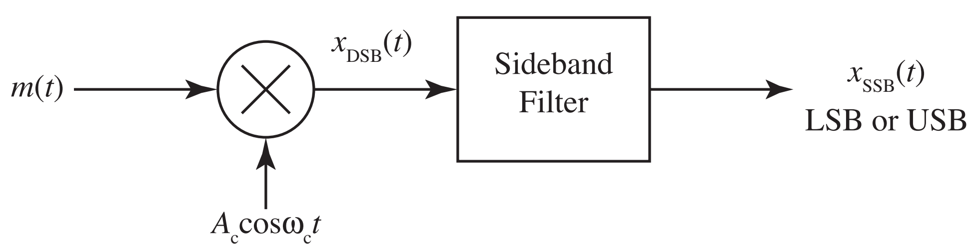
- Similarly for $z_n(t) = x(t) - j\hat{x}(t)$

$$\begin{aligned} Z_n(f) &= X(f)\big[1 - \text{sgn}(f)\big] \\ &= \begin{cases} 0, & f > 0 \\ 2X(f), & f < 0 \end{cases} \end{aligned}$$



The spectra of analytic signals can suppress positive or negative frequencies

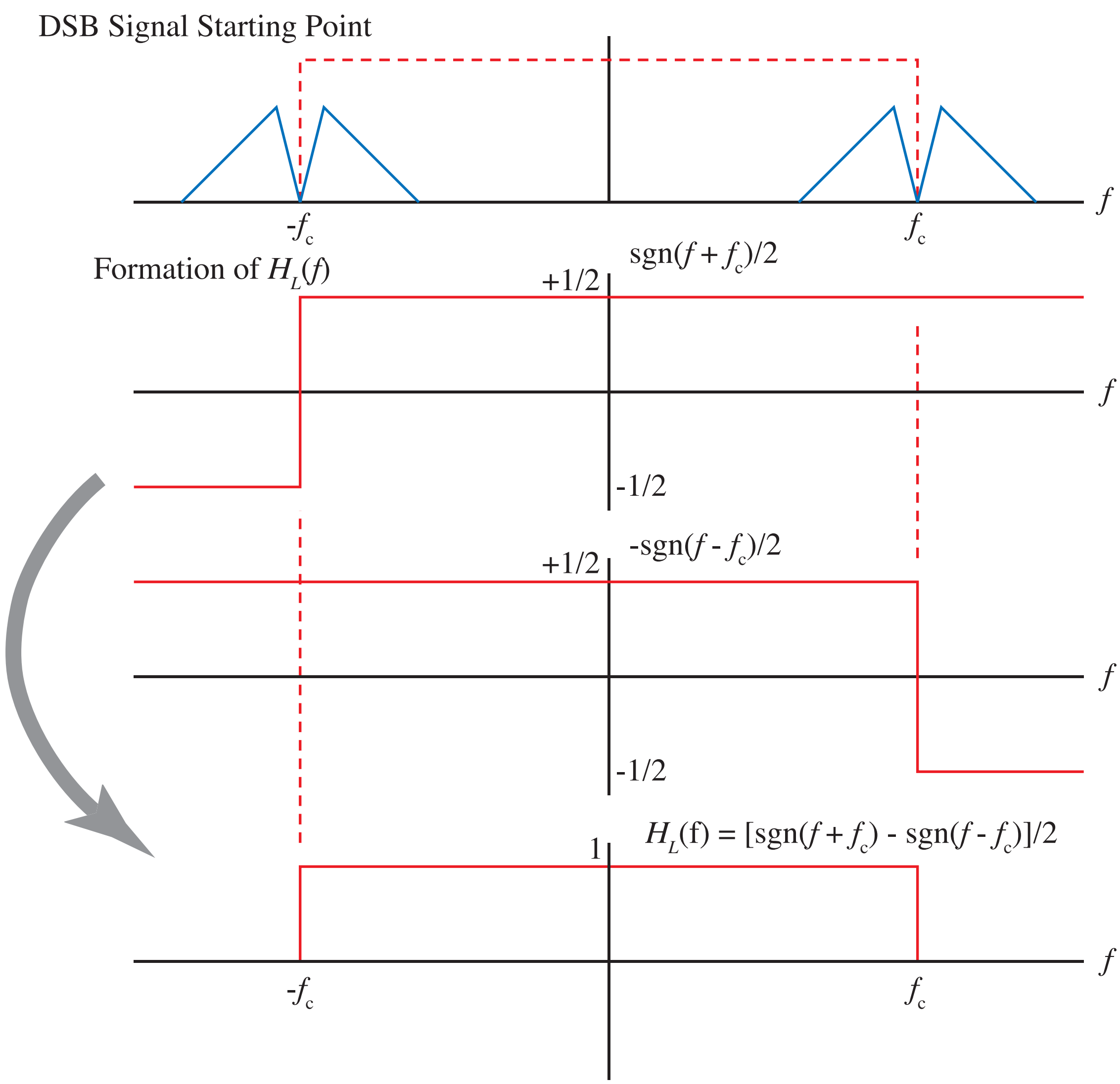
Return to SSB Development



Basic SSB signal generation

- In simple terms, we create an SSB signal from a DSB signal using a *sideband* filter

- The mathematical representation of LSSB and USSB signals makes use of Hilbert transform concepts and analytic signals



An ideal LSSB filter

- From the frequency domain expression for the LSSB, we can ultimately obtain an expression for the LSSB signal, $x_{c_{\text{LSSB}}}(t)$, in the time domain

- Start with $X_{\text{DSB}}(f)$ and the filter $H_L(f)$

$$
\begin{aligned}
X_{c_{\text{LSSB}}}(f) = \frac{1}{2}A_c\big[M(f + f_c) + M(f - f_c)\big] \\
\cdot \frac{1}{2}\big[\text{sgn}(f + f_c) - \text{sgn}(f - f_c)\big]
\end{aligned}
$$

$$\begin{aligned}
X_{c_{\text{LSSB}}}(f) = \frac{1}{4}A_c\big[&M(f+f_c)\text{sgn}(f+f_c) \\
&- M(f-f_c)\text{sgn}(f-f_c)\big] \\
&- \frac{1}{4}A_c\big[M(f+f_c)\text{sgn}(f-f_c) \\
&\quad - M(f-f_c)\text{sgn}(f+f_c)\big] \\
= \frac{1}{4}A_c\big[&M(f+f_c) + M(f-f_c)\big] \\
&+ \frac{1}{4}A_c\big[M(f+f_c)\text{sgn}(f+f_c) \\
&- M(f-f_c)\text{sgn}(f-f_c)\big]
\end{aligned}$$

- The inverse Fourier transform of the second term is DSB, i.e.,

$$\frac{1}{2}A_c m(t)\cos\omega_c t \stackrel{\mathcal{F}}{\longleftrightarrow} \frac{1}{4}A_c\big[M(f+f_c) + M(f-f_c)\big]$$

- The first term can be inverse transformed using

$$\hat{m}(t) \stackrel{\mathcal{F}}{\longleftrightarrow} -j\,\text{sgn}(f)\cdot M(f)$$

so

$$\mathcal{F}^{-1}\big\{M(f+f_c)\text{sgn}(f+f_c)\big\} = j\hat{m}(t)e^{-j\omega_c t}$$

since $m(t)e^{\pm j\omega_c t} \stackrel{\mathcal{F}}{\longleftrightarrow} M(f \pm f_c)$

- Thus

$$\begin{aligned}
&\frac{1}{4}A_c\mathcal{F}^{-1}\big\{M(f+f_c)\text{sgn}(f+f_c) - M(f-f_c)\text{sgn}(f-f_c)\big\} \\
&= \frac{1}{4}A_c\big[j\hat{m}(t)e^{-j\omega_c t} - j\hat{m}(t)e^{j\omega_c t}\big] = \frac{1}{2}\hat{m}(t)\sin\omega_c t
\end{aligned}$$

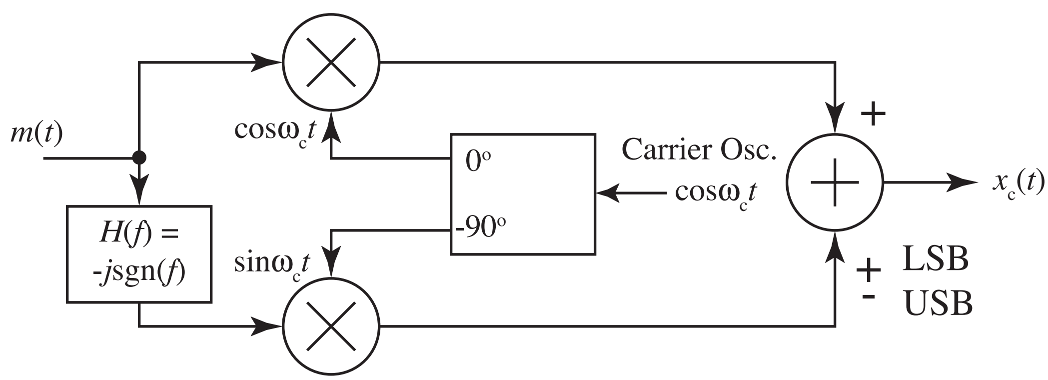
- Finally,

$$x_{c_{\text{LSSB}}}(t) = \frac{1}{2}A_c m(t)\cos\omega_c t + \frac{1}{2}A_c \hat{m}(t)\sin\omega_c t$$

- Similarly for USSB it can be shown that

$$x_{c_{\text{USSB}}}(t) = \frac{1}{2}A_c m(t)\cos\omega_c t - \frac{1}{2}A_c \hat{m}(t)\sin\omega_c t$$

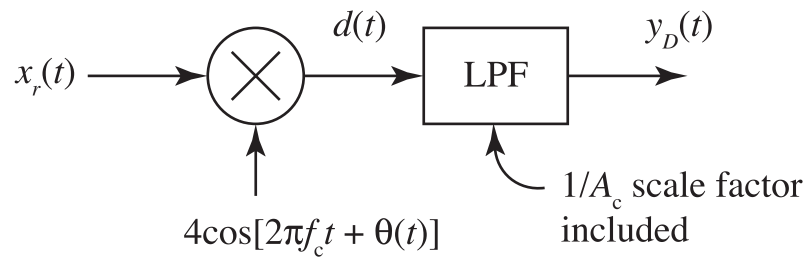
- The direct implementation of SSB is very difficult due to the requirements of the filter
- By moving the phase shift frequency from f_c down to DC (0 Hz) the implementation is much more reasonable (this applies to a DSP implementation as well)
- The phase shift is not perfect at low frequencies, so the modulation must not contain critical information at these frequencies



Phase shift modulator for SSB

Demodulation

- The coherent demodulator first discussed for DSB, also works for SSB



Coherent demod for SSB

- Carrying out the analysis to $d(t)$, first we have

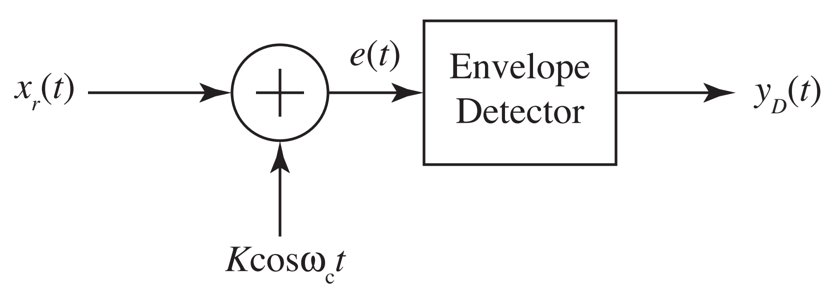
$$\begin{aligned} d(t) &= \frac{1}{2}A_c\big[m(t)\cos\omega_c t \pm \hat{m}(t)\sin\omega_c t\big]4\cos(\omega_c t + \theta(t)) \\ &= A_c m(t)\cos\theta(t) + A_c m(t)\cos[2\omega_c t + \theta(t)] \\ &\quad \mp A_c\hat{m}(t)\sin\theta(t) \pm A_c\hat{m}(t)\sin[2\omega_c t + \theta(t)] \end{aligned}$$

so

$$\begin{aligned} y_D(t) &= m(t)\cos\theta(t) \mp \hat{m}(t)\sin\theta(t) \\ &\overset{\theta(t)\text{ small}}{\simeq} m(t) \mp \hat{m}(t)\theta(t) \end{aligned}$$

  - The $\hat{m}(t)\sin\theta(t)$ term represents crosstalk

- Another approach to demodulation is to use *carrier reinsertion and envelope detection*



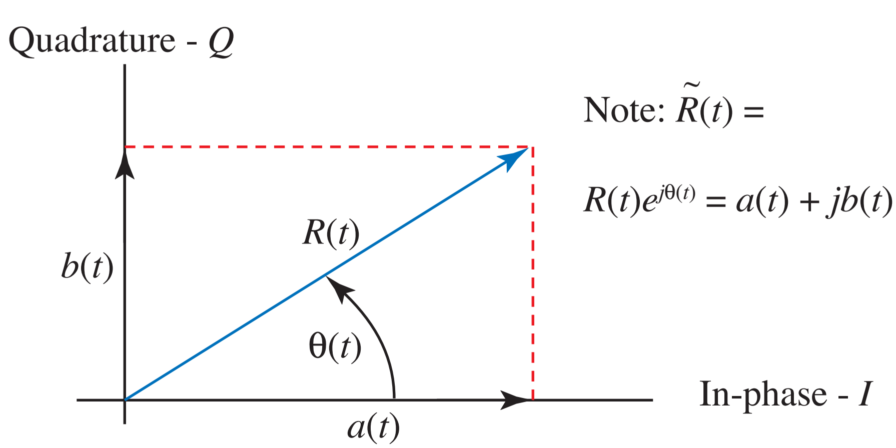
$$
\begin{aligned}
e(t) &= x_r(t) + K\cos\omega_c t \\
&= \left[\frac{1}{2}A_c m(t) + K\right]\cos\omega_c t \pm \frac{1}{2}A_c\hat{m}(t)\sin\omega_c t
\end{aligned}
$$

- To proceed with the analysis we must find the envelope of $e(t)$, which will be the final output $y_D(t)$

- Finding the envelope is a more general problem which will be useful in future problem solving, so first consider the envelope of

$$
\begin{aligned}
x(t) &= \underbrace{a(t)}_{\text{inphase}}\cos\omega_c t - \underbrace{b(t)}_{\text{quadrature}}\sin\omega_c t \\
&= \mathrm{Re}\left\{a(t)e^{j\omega_c t} + jb(t)e^{j\omega_c t}\right\} \\
&= \mathrm{Re}\left\{\underbrace{[a(t) + jb(t)]}_{\tilde{R}(t)=\text{complex envelope}} e^{j\omega_c t}\right\}
\end{aligned}
$$

- In a *phasor diagram* $x(t)$ consists of an *inphase* or direct component and a *quadrature* component



where the resultant $R(t)$ is such that

$$\begin{aligned} a(t) &= R(t)\cos\theta(t) \\ b(t) &= R(t)\sin\theta(t) \end{aligned}$$

which implies that

$$\begin{aligned} x(t) &= R(t)\big[\cos\theta(t)\cos\omega_c t - \sin\theta(t)\sin\omega_c t\big] \\ &= R(t)\cos\big[\omega_c t + \theta(t)\big] \end{aligned}$$

where $\theta(t) = \tan^{-1}[b(t)/a(t)]$

- The signal envelope is thus given by

$$R(t) = \sqrt{a^2(t) + b^2(t)}$$

- The output of an envelope detector will be $R(t)$ if $a(t)$ and $b(t)$ are slowly varying with respect to $\cos\omega_c t$

- In the SSB demodulator

$$y_D(t) = \sqrt{\left[\frac{1}{2}A_c m(t) + K\right]^2 + \left[\frac{1}{2}A_c \hat{m}(t)\right]^2}$$

- If we choose K such that $(A_c m(t)/2 + K)^2 \gg (A_c \hat{m}(t)/2)^2$, then

$$y_D(t) \simeq \frac{1}{2}A_c m(t) + K$$

- Note:
  - The above analysis assumed a phase coherent reference
  - In speech systems the frequency and phase can be adjusted to obtain intelligibility, but not so in data systems

– The approximation relies on the binomial expansion

$$(1+x)^{1/2} \simeq 1 + \frac{1}{2}x \text{ for } |x| \ll 1$$

Example 3.6: Noncoherent Carrier Reinsertion

- Let $m(t) = \cos\omega_m t$, $\omega_m \ll \omega_c$ and the reinserted carrier be $K\cos[(\omega_c + \Delta\omega)t]$

- Following carrier reinsertion we have

$$\begin{aligned} e(t) &= \frac{1}{2}A_c \cos\omega_m t \cos\omega_c t \\ &\quad \mp \frac{1}{2}A_c \sin\omega_c t \sin\omega_c t + K\cos\big[(\omega_c + \Delta\omega)t\big] \\ &= \frac{1}{2}A_c \cos\big[(\omega_c \pm \omega_m)t\big] + K\cos\big[(\omega_c + \Delta\omega)t\big] \end{aligned}$$

- We can write $e(t)$ as the real part of a complex envelope times a carrier at either ω_c or $\omega_c + \Delta\omega$

- In this case, since K will be large compared to $A_c/2$, we write

$$\begin{aligned} e(t) &= \frac{1}{2}A_c \mathrm{Re}\Big\{e^{\pm j\omega_m t}e^{j\omega_c t}\Big\} \\ &\quad + K\mathrm{Re}\Big\{1 \cdot e^{j(\omega_c + \Delta\omega)t}\Big\} \\ &= \mathrm{Re}\Big\{\underbrace{\Big(\frac{1}{2}A_c e^{j(\pm\omega_m - \Delta\omega)t} + K\Big)}_{\text{complex envelope } \tilde{R}(t)} e^{j(\omega_c + \Delta\omega)t}\Big\} \end{aligned}$$

- Finally expanding the complex envelope into the real and imaginary parts we can find the real envelope $R(t)$

$$\begin{aligned} y_D(t) &= \left[\left\{\frac{1}{2}A_c\cos[\pm\omega_m + \Delta\omega)t] + K\right\}^2 \right. \\ &\quad \left. + \left\{\frac{1}{2}A_c\sin[(\pm\omega_m + \Delta\omega)t]\right\}^2\right]^{1/2} \\ &\simeq \frac{1}{2}A_c\cos[(\omega_m \mp \Delta\omega)t] + K \end{aligned}$$

  where the last line follows for $K \gg A_c$

- Note that the frequency error $\Delta\omega$ causes the recovered message signal to shift up or down in frequency by $\Delta\omega$, but not both at the same time as in DSB, thus the recovered speech signal is more intelligible

---

3.1.4 Vestigial-Sideband Modulation

- Vestigial sideband (VSB) is derived by filtering DSB such that one sideband is passed completely while only a *vestige* remains of the other

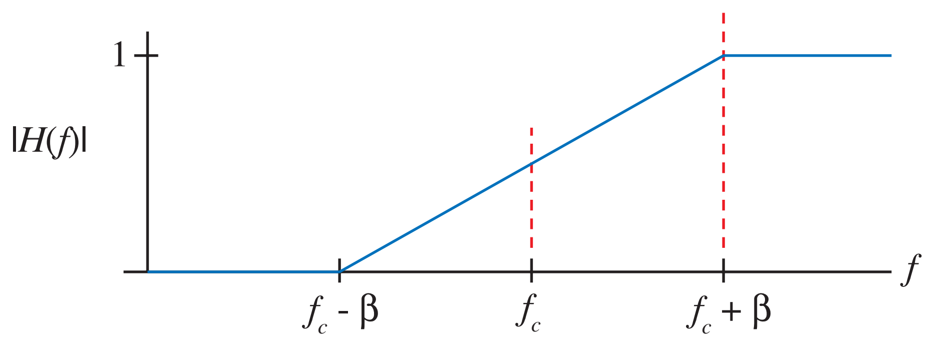
- Why VSB?

  1. Simplifies the filter design
  2. Improves the low-frequency response and allows DC to pass undistorted
  3. Has bandwidth efficiency advantages over DSB or AM, similar to that of SSB

- A primary application of VSB is the video portion of analog television (note HDTV replaces this in the US with 8VSB[1])

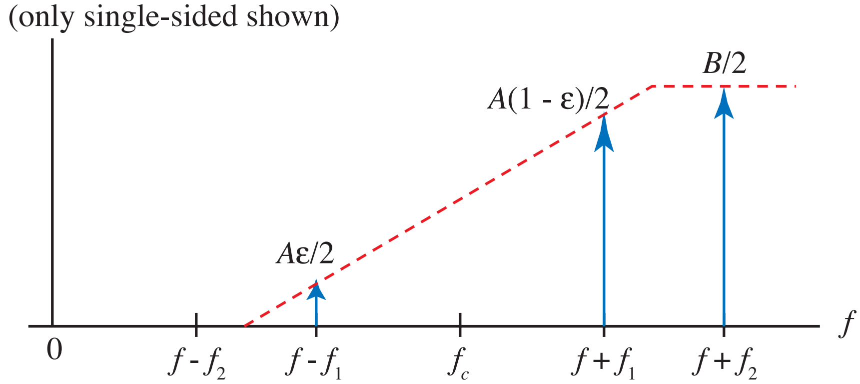
- The generation of VSB starts with DSB followed by a filter that has a 2β transition band, e.g.,

$$|H(f)| = \begin{cases} 0, & f < F_c - \beta \\ \frac{f-(f_c-\beta)}{2\beta}, & f_c - \beta \le f \le f_c + \beta \\ 1, & f > f_c + \beta \end{cases}$$



Ideal VSB transmitter filter amplitude response

- VSB can be demodulated using a coherent demod or using carrier reinsertion and envelope detection



Two-tone VSB signal

[1]http://www.tek.com/document/primer/fundamentals-8vsb

- Suppose the message signal consists of two tones

$$m(t) = A\cos\omega_1 t + B\cos\omega_2 t$$

- Following the DSB modulation and VSB shaping,

$$\begin{aligned} x_c(t) &= \frac{1}{2}A\epsilon\cos(\omega_c - \omega_1)t \\ &\quad + \frac{1}{2}A(1-\epsilon)\cos(\omega_c + \omega_1)t + \frac{1}{2}B\cos(\omega_c + \omega_2)t \end{aligned}$$

- A coherent demod multiplies the received signal by $4\cos\omega_c t$ to produce

$$\begin{aligned} e(t) &= A\epsilon\cos\omega_1 t + A(1-\epsilon)\cos\omega_1 t + B\cos\omega_2 t \\ &= A\cos\omega_1 t + B\cos\omega_2 t \end{aligned}$$

  which is the original message signal

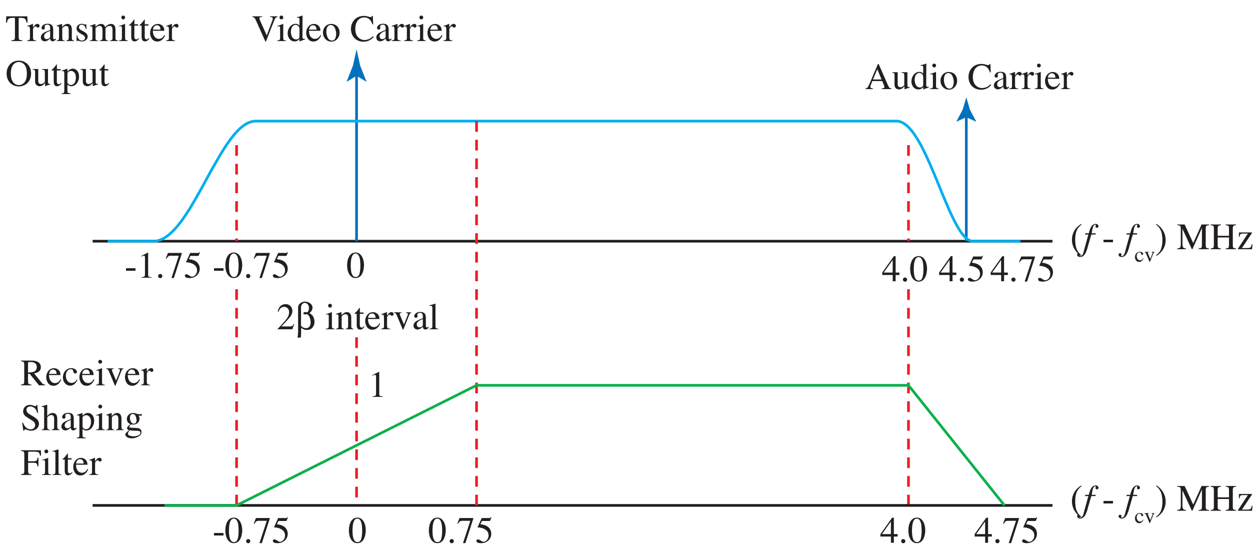
- The symmetry of the VSB shaping filter has made this possible

- In the case of broadcast TV the carrier in included at the transmitter to insure phase coherency and easy demodulation at the TV receiver (VSB + Carrier)

  – Very large video carrier power was required for typical TV station, i.e., greater than 100,000 W

  – To make matters easier still, the precise VSB filtering is not performed at the transmitter due to the high power requirements, instead the TV receiver did this

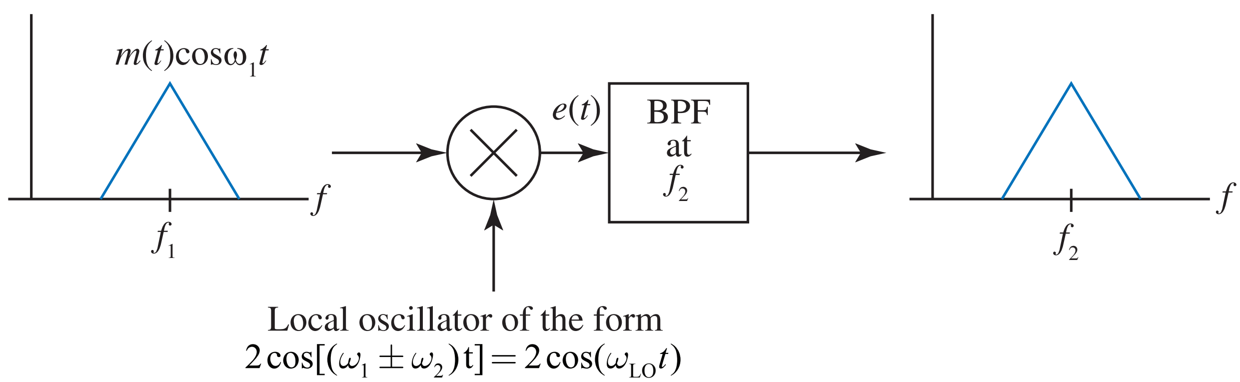
  – Further study is needed on taday's 8VSB



Broadcast TV transmitter spectrum and receiver shaping filter

3.1.5 Frequency Translation and Mixing

- Used to translate baseband or bandpass signals to some new center frequency



Frequency translation system

- Assuming the input signal is DSB of bandwidth $2W$ the mixer (multiplier) output is

$$
\begin{aligned}
e(t) &= m(t)\cos(\omega_1 t)\overbrace{2\cos(\omega_1 \pm \omega_2)t}^{\text{local osc (LO)}} \\
&= m(t)\cos(\omega_2 t) + m(t)\cos[(2\omega_1 \pm \omega_2)t]
\end{aligned}
$$

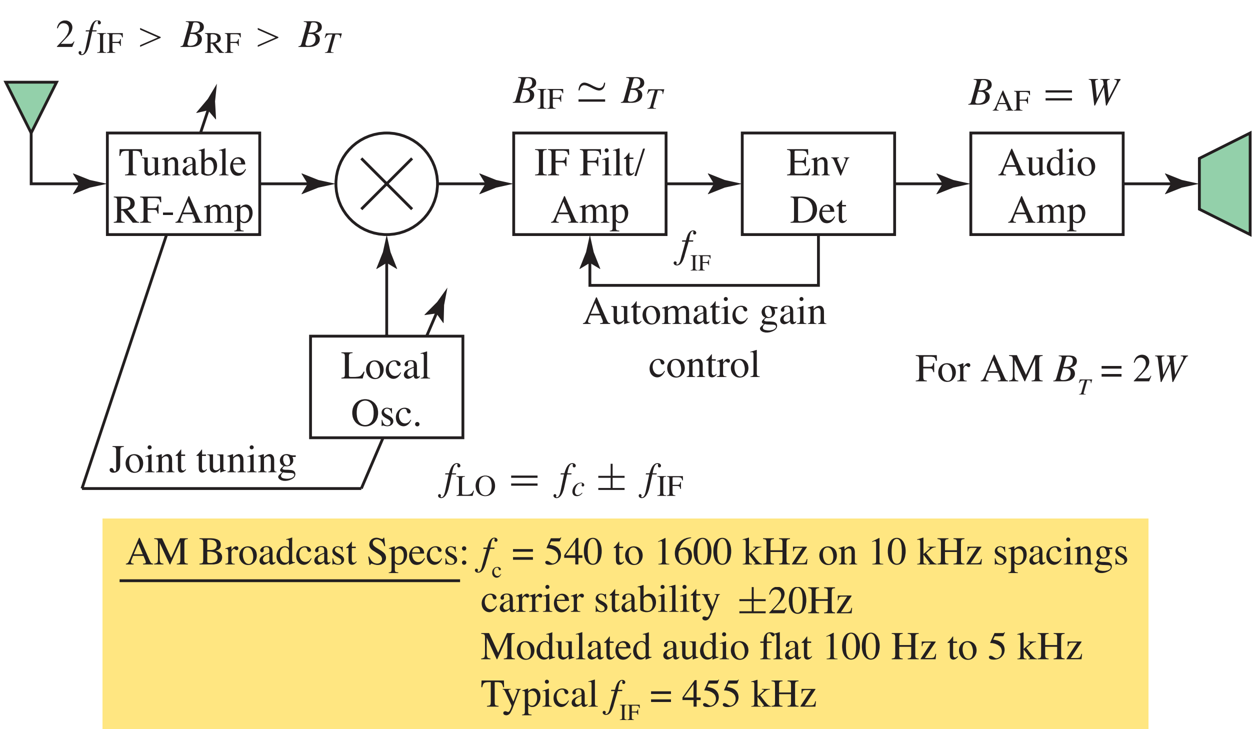
- The bandpass filter bandwidth needs to be at least $2W$ Hz wide
- Note that if an input of the form $k(t)\cos[(\omega_1 \pm 2\omega_2)t]$ is present it will be converted to ω_2 also, i.e.,

$$e(t) = k(t)\cos(\omega_2 t) + k(t)\cos[(2\omega_1 \pm 3\omega_2)t],$$

and the bandpass filter output is $k(t)\cos(\omega_2 t)$

- The frequencies $\omega_1 \pm 2\omega_2$ are the *image frequencies* of ω_1 with respect to $\omega_{\text{LO}} = \omega_1 \pm \omega_2$

Example 3.7: AM Broadcast Superheterodyne Receiver

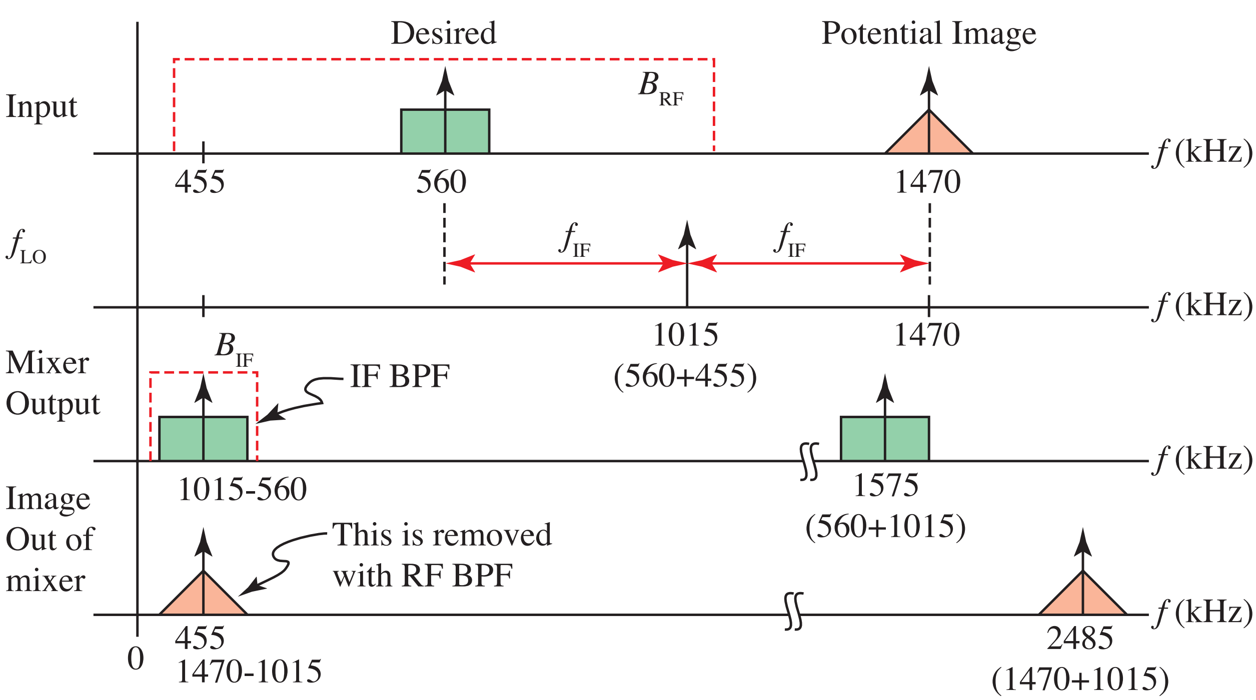


Classical AM superheterodyne receiver

- We have two choices for the local oscillator, *high-side* or *low-side* tuning

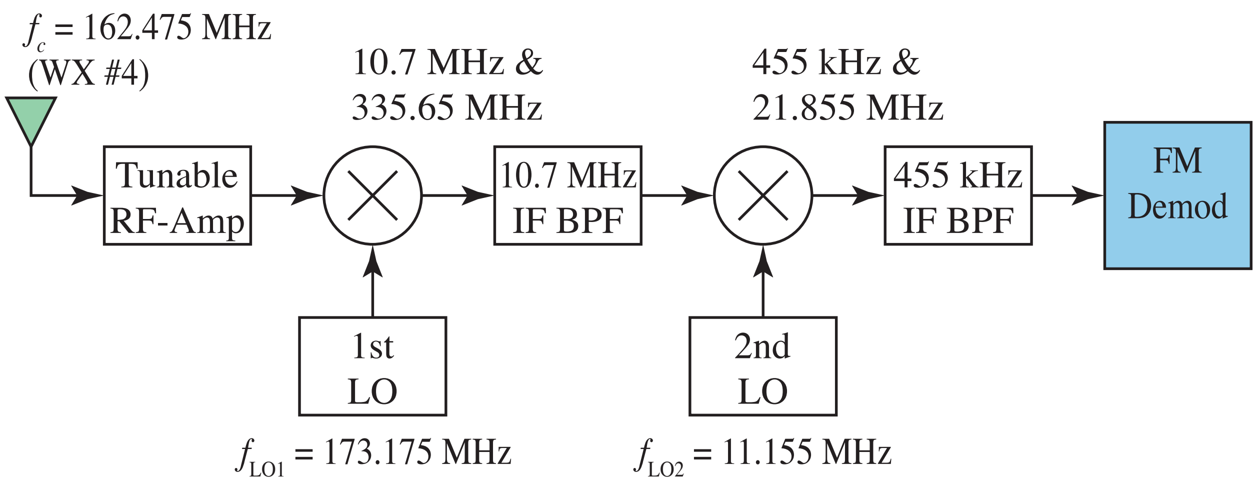
- Low-side: $540 - 455 \leq f_{\text{LO}} \leq 1600 - 455$ or $85 \leq f_{\text{LO}} \leq 1145$, all frequencies in kHz
- High-side: $540 + 455 \leq f_{\text{LO}} \leq 1600 + 455$ or $995 \leq f_{\text{LO}} \leq 2055$, all frequencies in kHz

- The high-side option is advantageous since the tunable oscillator or frequency synthesizer has the smallest frequency ratio $f_{\text{LO,max}}/f_{\text{LO,min}} = 2055/995 = 2.15$
- Suppose the desired station is at 560 kHz, then with high-side tuning we have $f_{\text{LO}} = 560 + 455 = 1015$ kHz
- The image frequency is at $f_{\text{image}} = f_c + 2f_{\text{IF}} = 560 + 2 \times 455 = 1470$ kHz (note this is another AM radio station center frequency



Receiver frequency plan including images

Example 3.8: A Double-Conversion Receiver



Double-conversion superheterodyne receiver (Lab 4)

- Consider a frequency modulation (FM) receiver that uses double-conversion to receive a signal on carrier frequency 162.475 MHz (weather channel #4 here in Colorado Springs)
  - Frequency modulation will be discussed in the next section
- The dual-conversion allows good image rejection by using a 10.7 MHz first IF and then can provide good selectivity by using a second IF at 455 kHz; why?
  - The ratio of bandwidth to center frequency can only be so small in a low loss RF filter
  - The second IF filter can thus have a much narrower bandwidth by virtue of the center frequency being much lower
- A higher first IF center frequency moves the image signal further away from the desired signal

- For high-side tuning we have $f_{\text{image}} = f_c + 2f_{\text{IF}} = f_c + 21.4$ MHz

- Double-conversion receivers are more complex to implement

Mixers

- The multiplier that is used to implement frequency translation is often referred to as a mixer

- In the world of RF circuit design the term mixer is more appropriate, as an ideal multiplier is rarely available

- Instead active and passive circuits that approximate signal multiplication are utilized

- The notion of *mixing* comes about from passing the sum of two signals through a nonlinearity, e.g.,

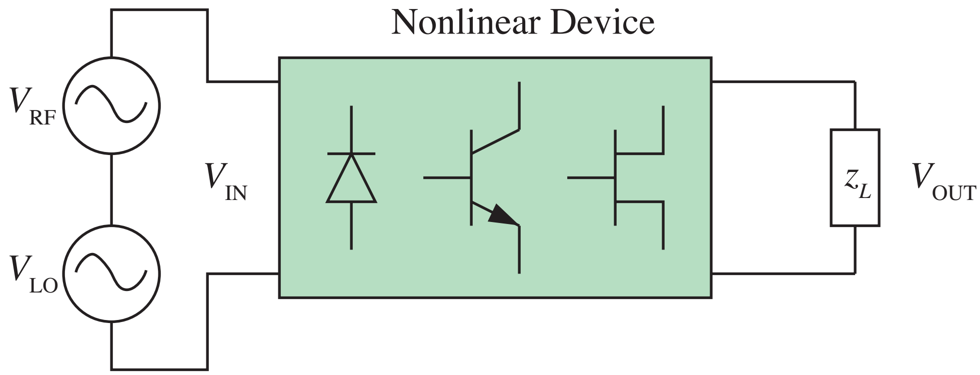
$$\begin{aligned} y(t) &= [a_1 x_1(t) + a_2 x_2(t)]^2 + \text{other terms} \\ &= a_1^2 x_1^2(t) + 2a_1 a_2 x_1(t) x_2(t) + a_2^2 x_2^2(t) \end{aligned}$$

- In this mixing application we are most interested in the center term

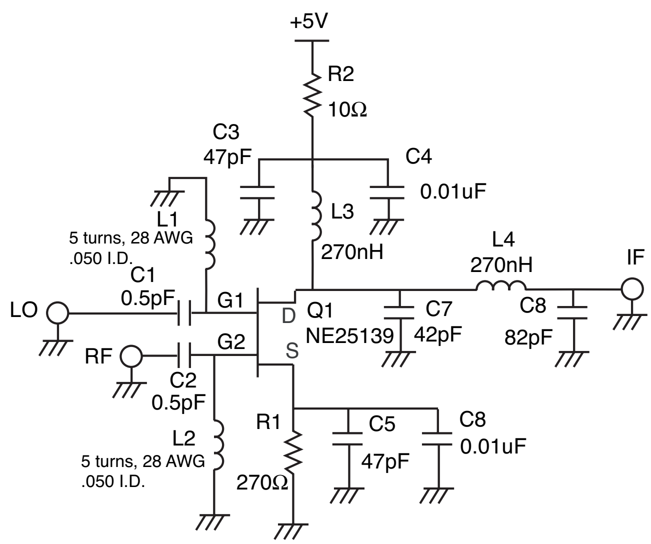
$$y_{\text{desired}}(t) = 2a_1 a_2 \big[x_1(t) \cdot x_2(t)\big]$$

- Clearly this mixer produces unwanted terms (first and third), and in general many other terms, since the nonlinearity will have more than just a square-law input/output characteristic

- A diode or active device can be used to form mixing products as described above, consider the dual-gate MEtal Semiconductor FET (MESFET) mixer shown below



Mixer concept

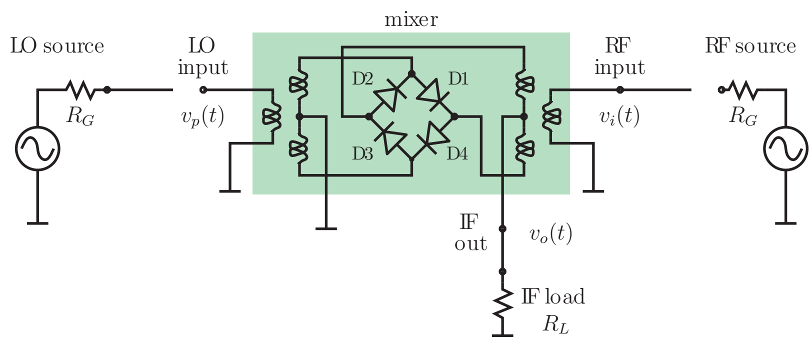


Dual-Gate MESFET Active Mixer

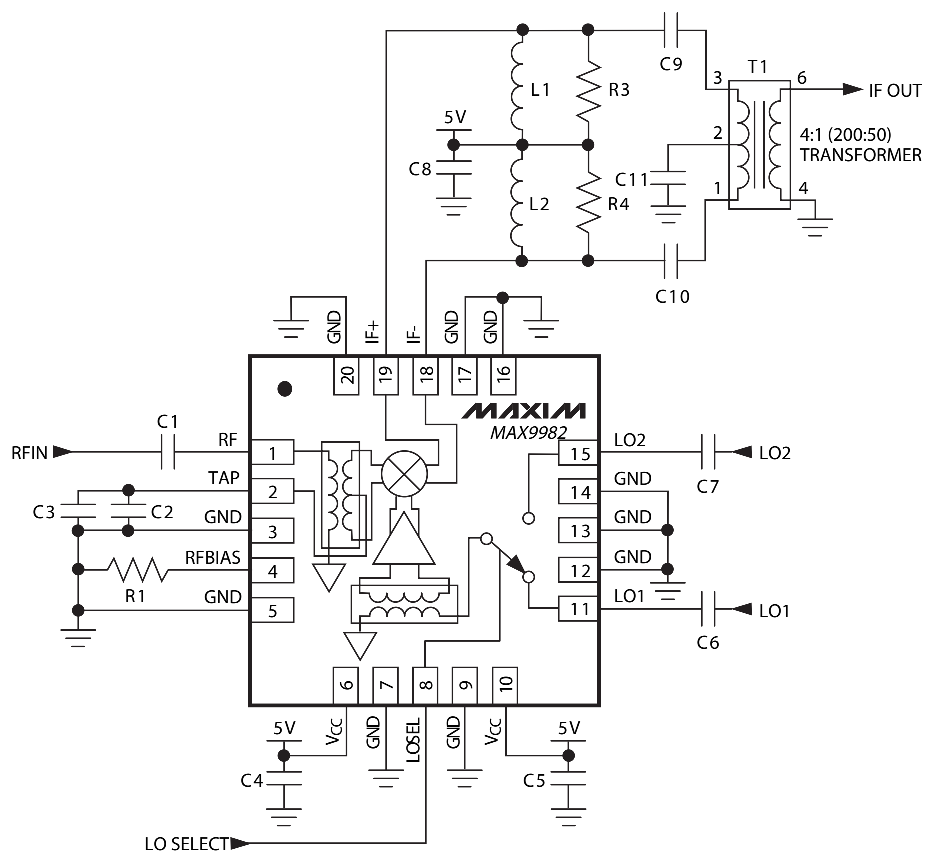
- The double-balanced mixer (DBM), which can be constructed using a diode ring, provides better isolation between the RF, LO, and IF ports

- When properly balanced the DBM also allows even harmonics to be suppressed in the mixing operation

- A basic transformer coupled DBM, employing a diode ring, is shown below, followed by an active version
- The DBM is suitable for use as a *phase detector* in phase-locked loop applications

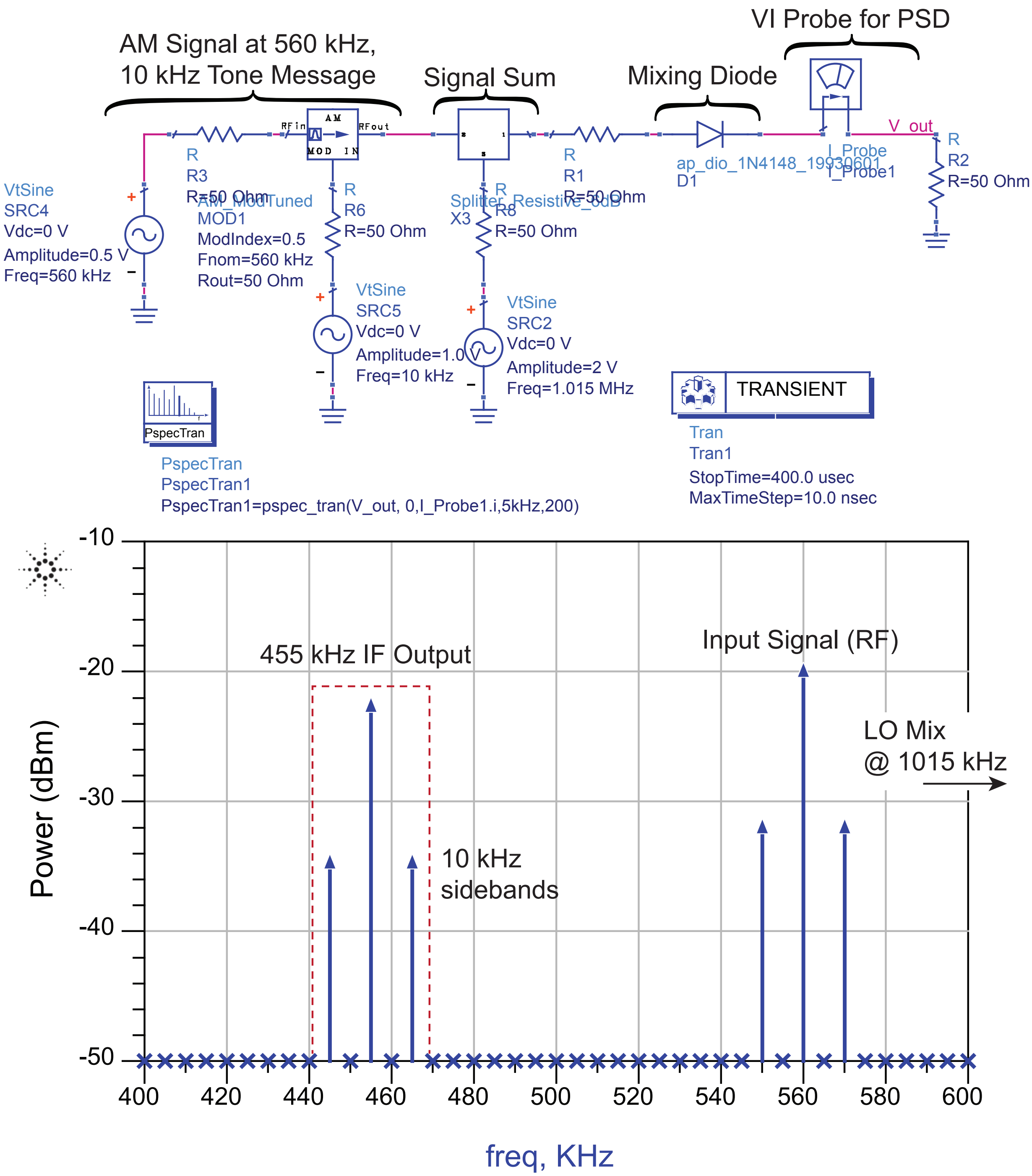


Passive Double-Balanced Mixer (DBM)



825 MHz to 915 MHz SiGe High-Linearity Active DBM

Example 3.9: Single Diode Mixer

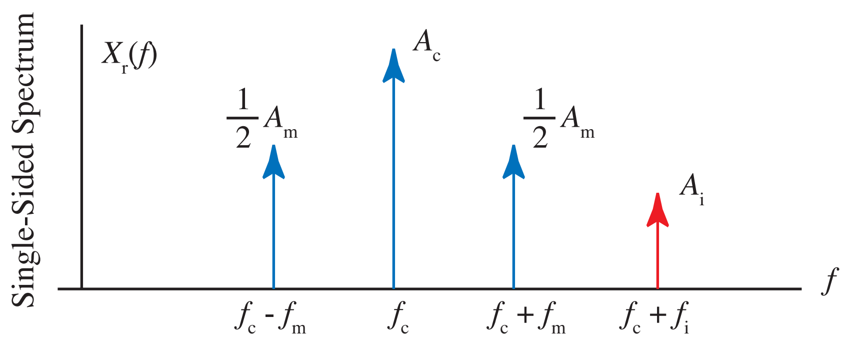


ADS single diode mixer simulation: 560 kHz → 455 kHz

3.2 Interference

Interference is a fact of life in communication systems. A through understanding of interference requires a background in random signals analysis (Chapter 6 of the text), but some basic concepts can be obtained by considering a single interference at $f_c + f_i$ that lies close to the carrier f_c

3.2.1 Interference in Linear Modulation



AM carrier with single tone interference

- If a single tone carrier falls within the IF passband of the receiver what problems does it cause?
- Coherent Demodulator

$$x_r(t) = \left[A_c \cos\omega_c t + A_m \cos\omega_m t \cos\omega_c t\right] + A_i \cos(\omega_c + \omega_i)t$$

  - We multiply $x_r(t)$ by $2\cos\omega_c t$ and lowpass filter

$$y_D(t) = A_m \cos\omega_m t + \underbrace{A_i \cos\omega_i t}_{\text{interference}}$$

- Envelope Detection: Here we need to find the received envelope relative to the strongest signal present

- Case $A_c \gg A_i$
- We will expand $x_r(t)$ in complex envelope form by first noting that

$$A_i \cos(\omega_c + \omega_i)t = A_i \cos\omega_i t \cos\omega_c t - A_i \sin\omega_i t \sin\omega_c t$$

now,

$$\begin{aligned} x_r(t) &= \text{Re}\Big\{\big(A_c + A_m \cos\omega_m t + A_i \cos\omega_i t \\ &\qquad - jA_i \sin\omega_i t\big)e^{j\omega_c t}\Big\} \\ &= \text{Re}\big\{\tilde{R}(t)e^{j\omega_c t}\big\} \end{aligned}$$

so

$$\begin{aligned} R(t) &= |\tilde{R}(t)| \\ &= \Big[(A_c + A_m \cos\omega_m t + A_i \cos\omega_i t)^2 \\ &\qquad + (A_i \sin\omega_i t)^2\Big]^{1/2} \\ &\simeq A_c + A_m \cos\omega_m t + A_i \cos\omega_i t \end{aligned}$$

assuming that $A_c \gg A_i$

- Finally,

$$y_D(t) \simeq A_m \cos\omega_m t + \underbrace{A_i \cos\omega_i t}_{\text{interference}}$$

- Case $A_i >> A_c$
- Now the interfering term looks like the carrier and the remaining terms look like sidebands, LSSB sidebands relative to $f_c + f_i$ to be specific

– From SSB envelope detector analysis we expect

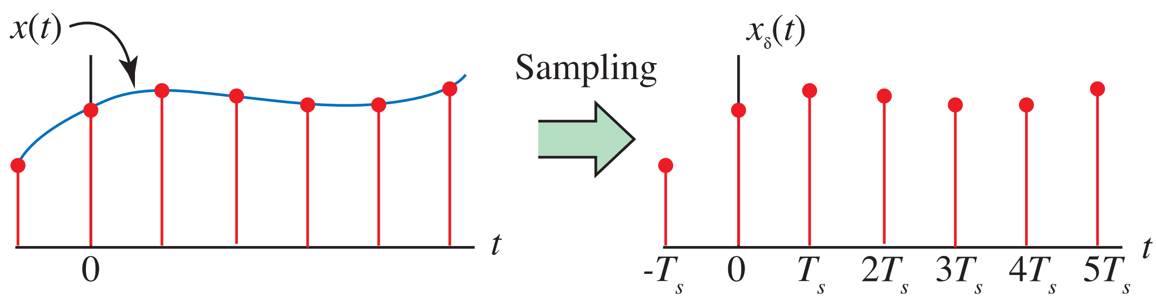
$$\begin{aligned} y_D(t) \simeq \frac{1}{2} A_m \cos(\omega_i + \omega_m)t + A_c \cos \omega_i t \\ + \frac{1}{2} A_m \cos(\omega_i - \omega_m)t \end{aligned}$$

and we conclude that the message signal is lost!

3.3 Sampling Theory

- We now return to text Chapter 2, Section 8, for an introduction/review of sampling theory
- Consider the representation of continuous-time signal $x(t)$ by the sampled waveform

$$x_\delta(t) = x(t)\left[\sum_{n=-\infty}^{\infty} \delta(t - nT_s)\right] = \sum_{n=-\infty}^{\infty} x(nT_s)\delta(t - nT_s)$$



- How is T_s selected so that $x(t)$ can be recovered from $x_\delta(t)$?
- **Uniform Sampling Theorem for Lowpass Signals**

  Given

$$\mathcal{F}\{x(t)\} = X(f) = 0, \quad \text{for } f > W$$

  then choose

$$T_s < \frac{1}{2W} \quad \text{or} \quad f_s > 2W \quad (f_s = 1/T_s)$$

  to reconstruct $x(t)$ from $x_\delta(t)$ and pass $x_\delta(t)$ through an ideal LPF with cutoff frequency $W < B < f_s - W$

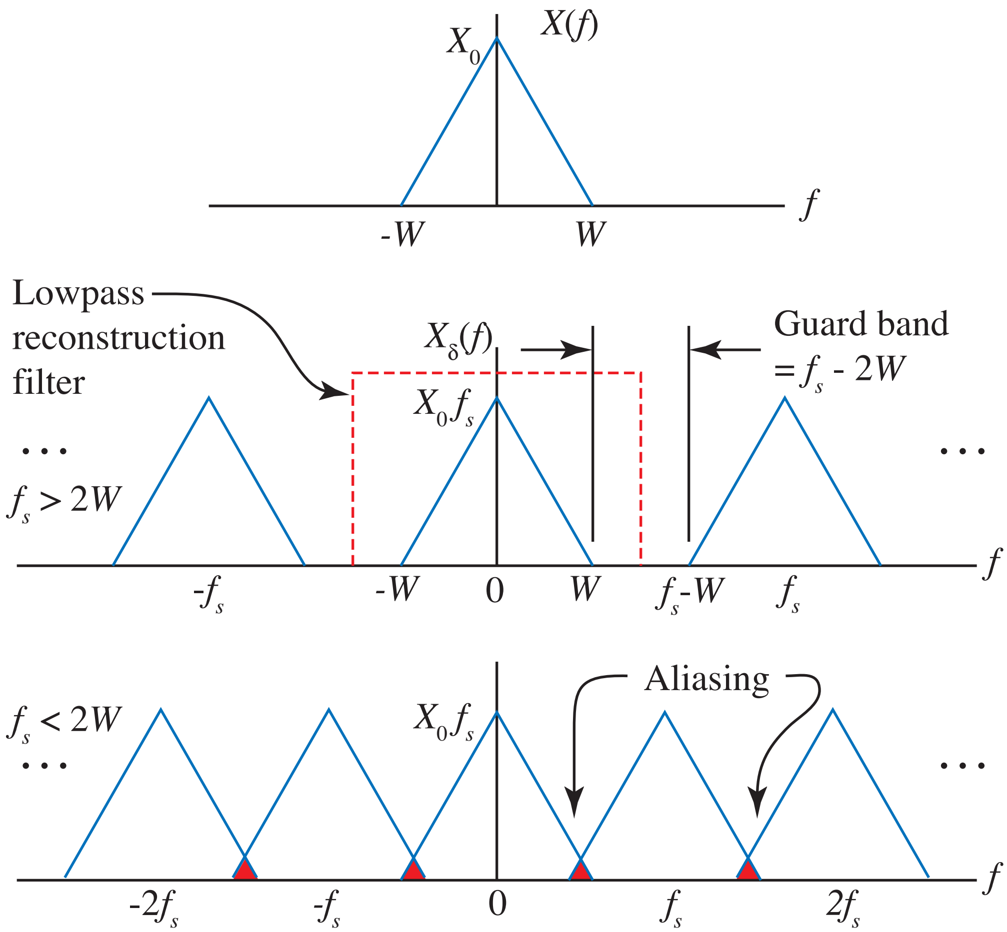
$$\begin{aligned} 2W &= \text{Nyquist frequency} \\ f_s/2 &= \text{folding frequency} \end{aligned}$$

proof:

$$X_\delta(f) = X(f) * \left[f_s \sum_{n=-\infty}^{\infty} \delta(f - nf_s) \right]$$

but $X(f) * \delta(f - nf_s) = X(f - nf_s)$, so

$$X_\delta(f) = f_s \sum_{n=-\infty}^{\infty} X(f - nf_s)$$



Spectra before and after sampling at rate f_s

- As long as $f_s - W > W$ or $f_s > 2W$ there is no *aliasing* (spectral overlap)

- To recover $x(t)$ from $x_\delta(t)$ all we need to do is lowpass filter the sampled signal with an ideal lowpass filter having cutoff frequency $W < f_{\text{cutoff}} < f_s - W$

- In simple terms we set the lowpass bandwidth to the folding frequency, $f_s/2$

- Suppose the *reconstruction* filter is of the form

$$H(f) = H_0 \Pi\left(\frac{f}{2B}\right) e^{-j2\pi f t_0}$$

  we then choose $W < B < f_s - W$

- For input $X_\delta(f)$, the output spectrum is

$$Y(f) = f_s H_0 X(f) e^{-j2\pi f t_0}$$

  and in the time domain

$$y(t) = f_s H_0 x(t - t_0)$$

- If the reconstruction filter is not ideal we then have to design the filter in such a way that minimal desired signal energy is removed, yet also minimizing the contributions from the spectral translates either side of the $n = 0$ translate

- The reconstruction operation can also be viewed as interpolating signal values between the available sample values

- Suppose that the reconstruction filter has impulse response $h(t)$, then

$$
\begin{aligned}
y(t) &= \sum_{n=-\infty}^{\infty} x(nT_s)h(t-nT_s) \\
&= 2BH_0 \sum_{n=-\infty}^{\infty} x(nT_s)\text{sinc}[2B(t-t_0-nT_s)]
\end{aligned}
$$

where in the last lines we invoked the ideal filter described earlier

- **Uniform Sampling Theorem for Bandpass Signals**

  If $x(t)$ has a single-sided bandwidth of W Hz and

  $$\mathcal{F}\{x(t)\} = 0 \quad \text{for} \quad f > f_u$$

  then we may choose

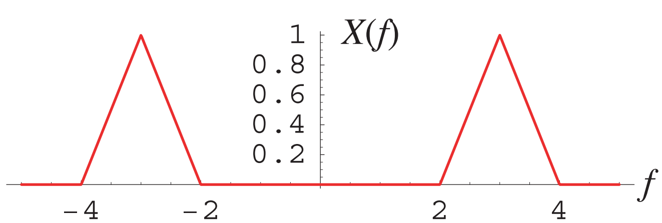
  $$f_s = \frac{2f_u}{m}$$

  where

  $$m = \left\lfloor \frac{f_u}{W} \right\rfloor,$$

  which is the greatest integer less than or equal to f_u/W

Example 3.10: **Bandpass signal sampling**



Input signal spectrum

- In the above signal spectrum we see that

$$W = 2, \quad f_u = 4 \quad f_u/W = 2 \Rightarrow m = 2$$

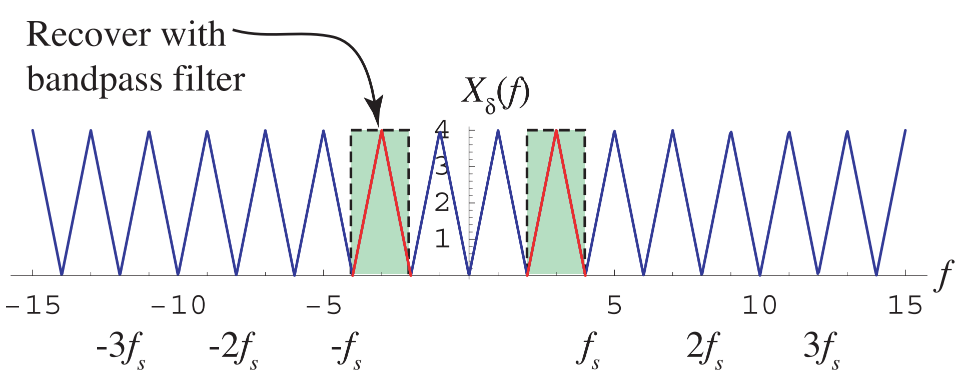
so

$$f_s = \frac{2(4)}{2} = 4$$

will work

- The sampled signal spectrum is

$$X_\delta(f) = 4 \sum_{n=-\infty}^{\infty} X(f - nf_s)$$



Spectrum after sampling

3.4 Analog Pulse Modulation

- The message signal $m(t)$ is sampled at rate $f_s = 1/T_s$
- A characteristic of the transmitted pulse is made to vary in a one-to-one correspondence with samples of the message signal
- A digital variation is to allow the pulse attribute to take on values from a finite set of allowable values

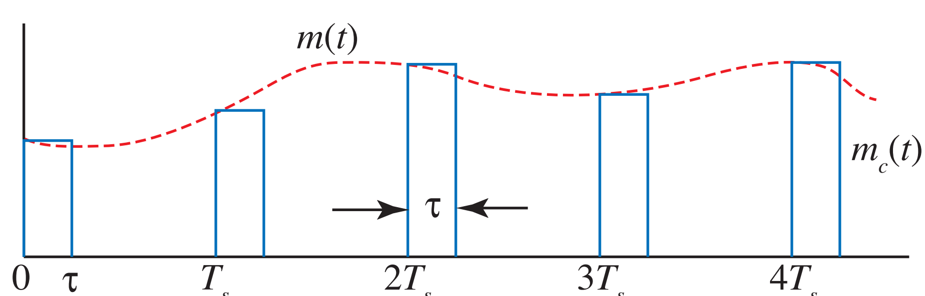
3.4.1 Pulse-Amplitude Modulation (PAM)

- PAM produces a sequence of flat-topped pulses whose amplitude varies in proportion to samples of the message signal
- Start with a message signal, $m(t)$, that has been uniformly sampled

$$m_\delta(t) = \sum_{n=-\infty}^{\infty} m(nT_s)\delta(t - nT_s)$$

- The PAM signal is

$$m_c(t) = \sum_{n=-\infty}^{\infty} m(nT_s)\Pi\left(\frac{t - (nT_s + \tau/2)}{\tau}\right)$$



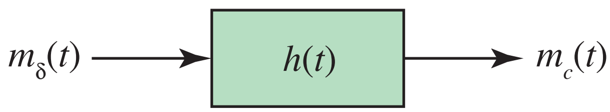
PAM waveform

- It is possible to create $m_c(t)$ directly from $m_\delta(t)$ using a *zero-order hold* filter, which has impulse response

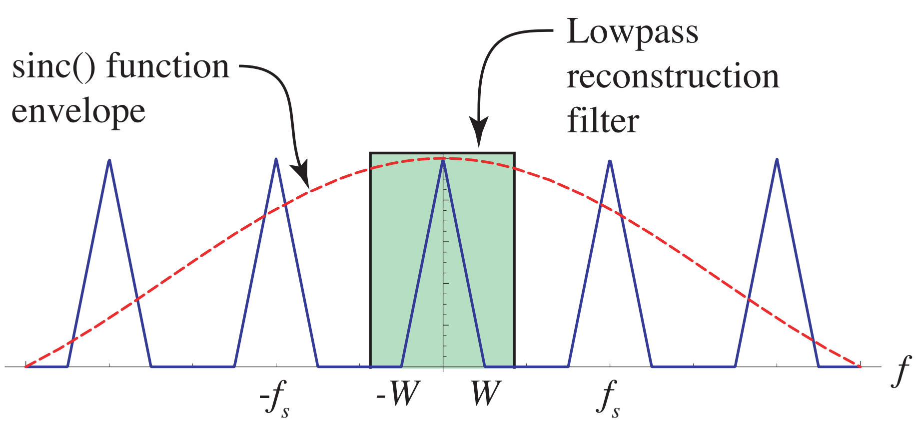
$$h(t) = \Pi\left(\frac{t - \tau/2}{\tau}\right)$$

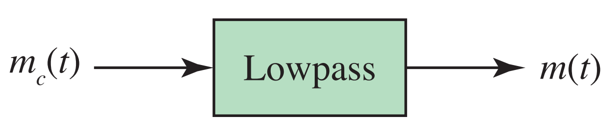
and frequency response

$$H(f) = \tau \operatorname{sinc}(f\tau)e^{-j\pi f\tau}$$



- How does $h(t)$ change the recovery operation from the case of ideal sampling?
  - If $\tau \ll T_s$ we can get by with just a lowpass reconstruction filter having cutoff frequency at $f_s/2 = 2/T_s$
  - In general, there may be a need for equalization if τ is on the order of $T_s/4$ to $T_s/2$

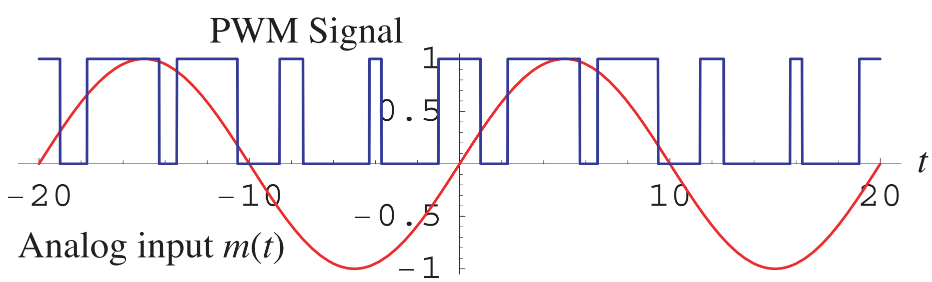




Recovery of $m(t)$ from $m_c(t)$

3.4.2 Pulse-Width Modulation (PWM)

- A PWM waveform consists of pulses with width proportional to the sampled analog waveform
- For bipolar $m(t)$ signals we may choose a pulse width of $T_s/2$ to correspond to $m(t) = 0$
- The biggest application for PWM is in motor control
- It is also used in *class D* audio power amplifiers
- A lowpass filter applied to a PWM waveform recovers the modulation $m(t)$

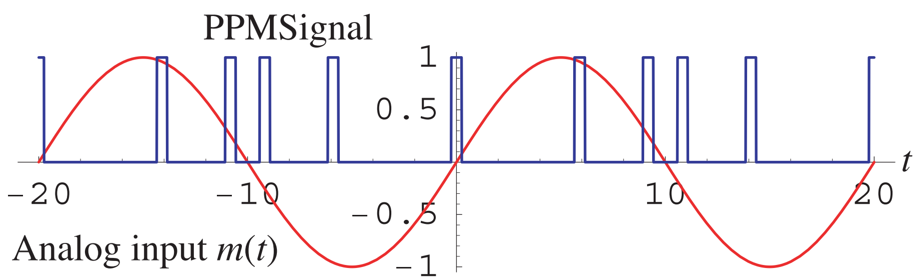


Example PWM signal

3.4.3 Pulse-Position Modulation

- With PPM the displacement in time of each pulse, with respect to a reference time, is proportional to the sampled analog waveform
- The time axis may be slotted into a discrete number of pulse positions, then $m(t)$ would be quantized
- Digital modulation that employs M slots, using nonoverlapping pulses, is a form of M-ary orthogonal communications

– PPM of this type is finding application in *ultra-wideband communications*



Example PPM signal

3.5 Delta Modulation and PCM

- This section considers two pure digital pulse modulation schemes

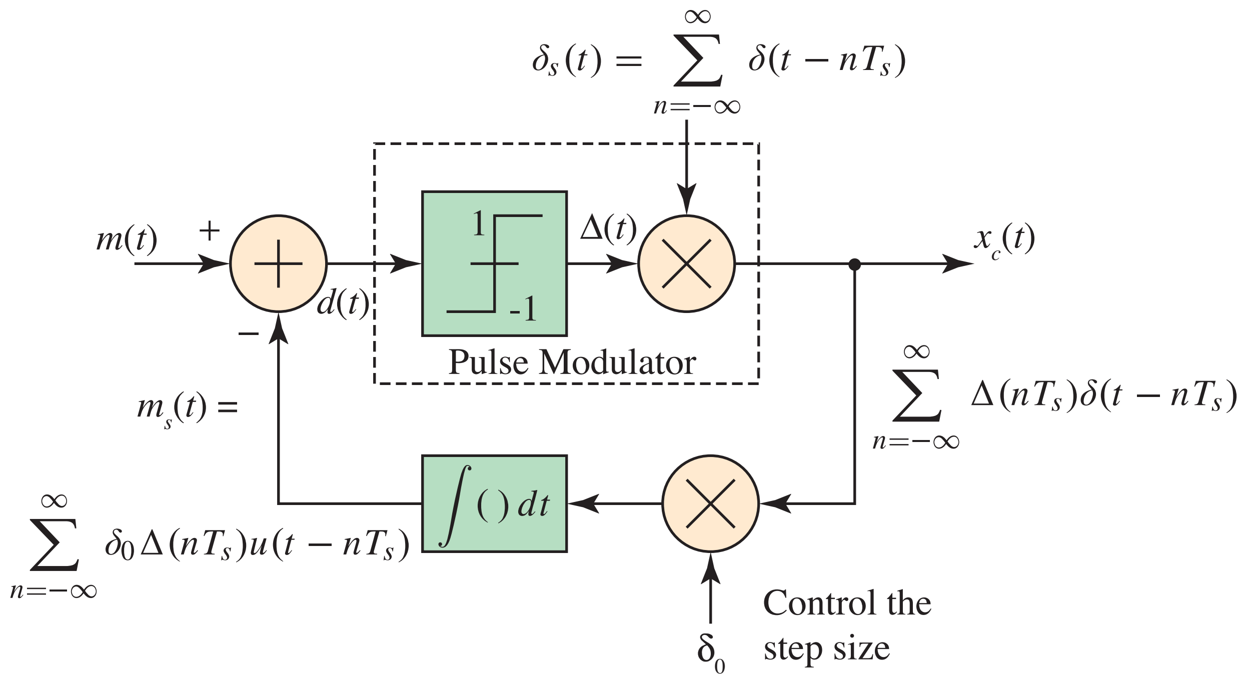
- Pure digital means that the output of the modulator is a binary waveform taking on only discrete values

3.5.1 Delta Modulation (DM)

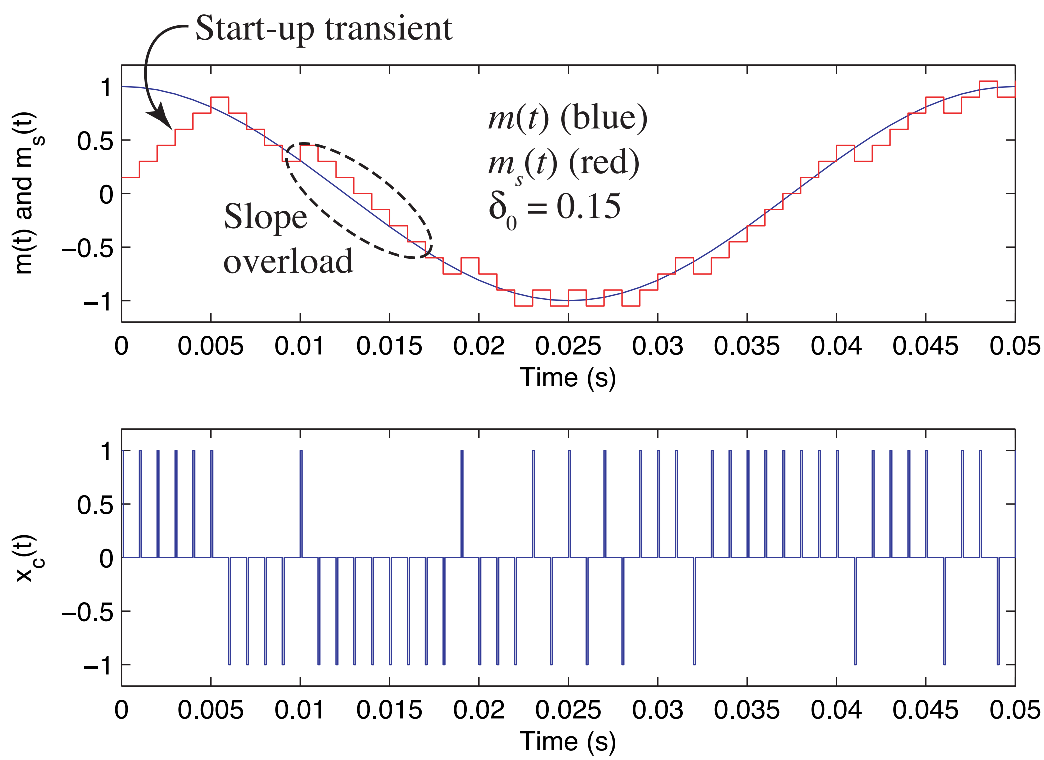
- The message signal $m(t)$ is encoded into a binary sequence which corresponds to changes in $m(t)$ relative to reference waveform $m_s(t)$

- DM gets its name from the fact that only the difference from sample-to-sample is encoded

- The sampling rate in combination with the step size are the two primary controlling modulator design parameters



Delta modulator with step size parameter δ_0



Delta modulator waveforms

- The maximum slope that can be followed is δ_0 / T_s

- A MATLAB DM simulation function is given below

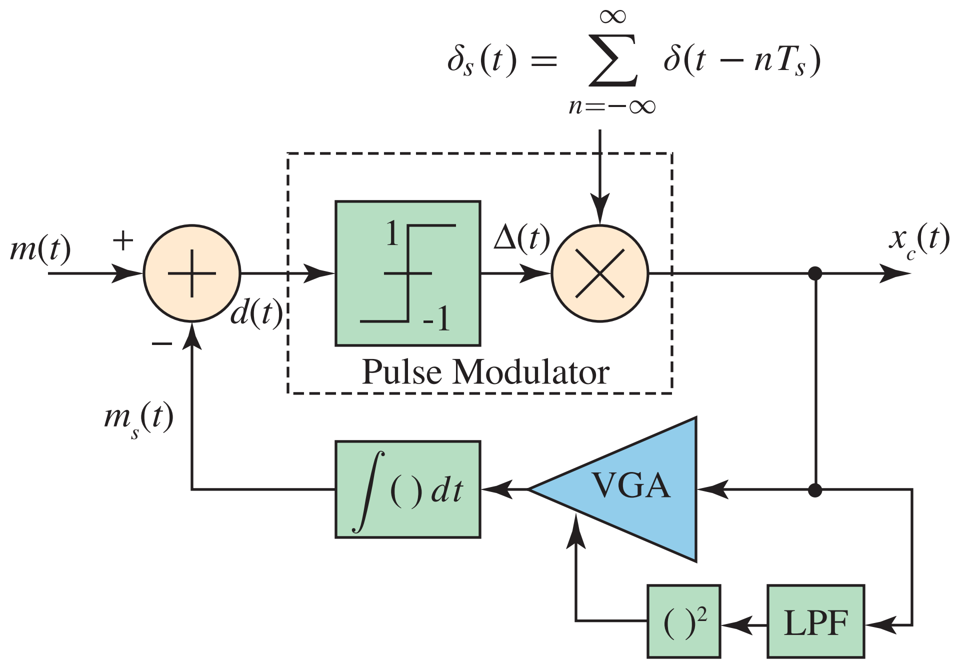
```
function [t_o,x,ms] = DeltaMod(m,fs,delta_0,L)
% [t,x,ms] = DeltaMod(m,fs,delta_0,L)
%
% Mark Wickert, April 2006

n = 0:(L*length(m))-1;
t_o = n/(L*fs);
ms = zeros(size(m));
x = zeros(size(m));
ms_old = 0; % zero initial condition
for k=1:length(m)
    x(k) = sign(m(k) - ms_old);
    ms(k) = ms_old + x(k)*delta_0;
    ms_old = ms(k);
end

x = [x; zeros(L-1,length(m))];
x = reshape(x,1,L*length(m));
```

- The message $m(t)$ can be recovered from $x_c(t)$ by integrating and then lowpass filtering to remove the stair step edges (lowpass filtering directly is a simplification)

- Slope overload can be dealt with through an adaptive scheme

  - If $m(t)$ is nearly constant keep the step size δ_0 small
  - If $m(t)$ has large variations, a larger step size is needed

- With adaptive DM the step size is controlled via a variable gain amplifier, where the gain is controlled by square-law detecting the output of a lowpass filter acting on $x_c(t)$

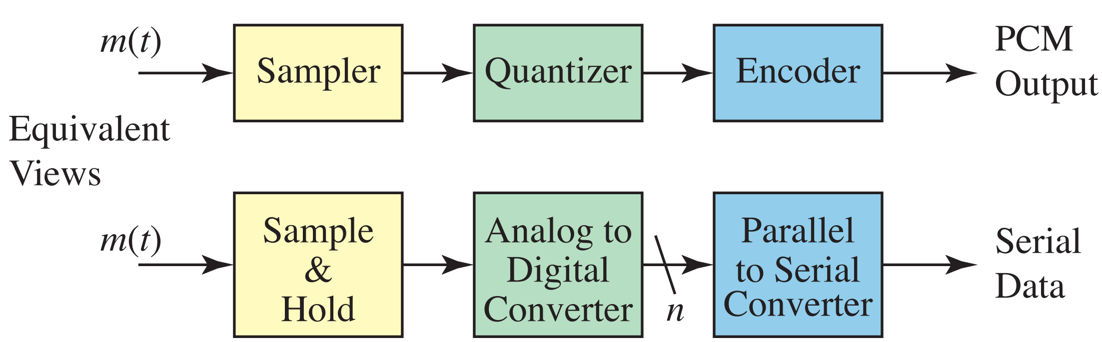


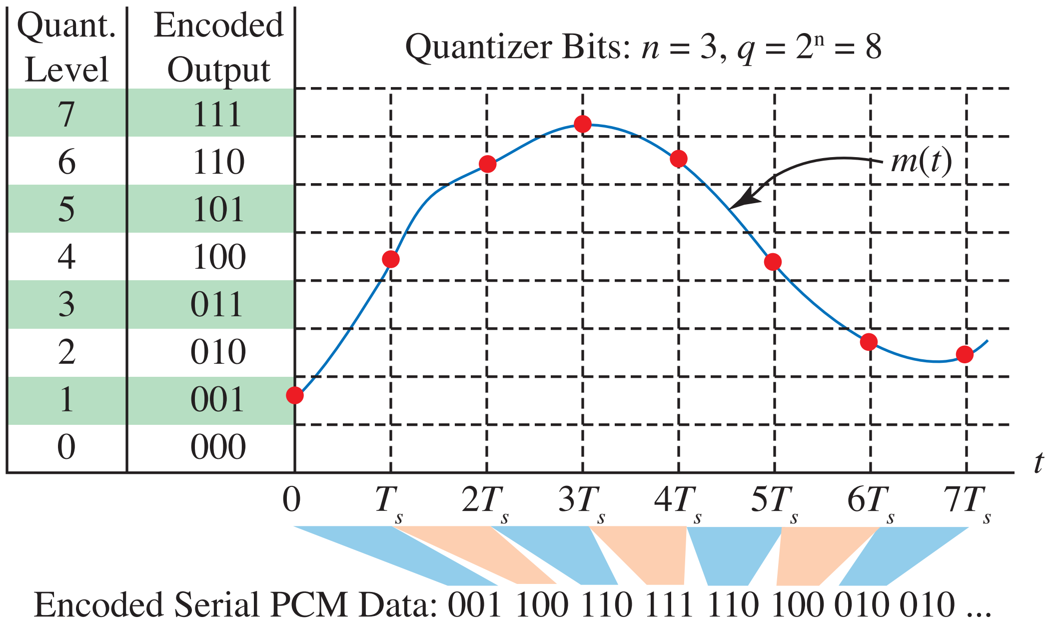
Means to obtain a variable step size DM

3.5.2 Pulse-Code Modulation (PCM)

- Each sample of $m(t)$ is mapped to a binary word by

  1. Sampling
  2. Quantizing
  3. Encoding





3-Bit PCM encoding

- Assume that $m(t)$ has bandwidth W Hz, then
  - Choose $f_s > 2W$
  - Choose n bits per sample ($q = 2^n$ quantization levels)
  - $\Rightarrow \geq 2nW$ binary digits per second must be transmitted
- Each pulse has width no more than

$$(\Delta\tau)_{\max} = \frac{1}{2nW},$$

so using the fact that the lowpass bandwidth of a single pulse is about $1/(2\Delta\tau)$ Hz, we have that the lowpass transmission bandwidth for PCM is approximately

$$B \simeq kWn,$$

where k is a proportionality constant

- When located on a carrier the required bandwidth is doubled

- Binary phase-shift keying (BPSK), mentioned earlier, is a popular scheme for transmitting PCM using an RF carrier

- Many other digital modulation schemes are possible

- The number of quantization levels, $q = \log_2 n$, controls the quantization error, assuming $m(t)$ lies within the full-scale range of the quantizer

- Increasing q reduces the quantization error, but also increases the transmission bandwidth

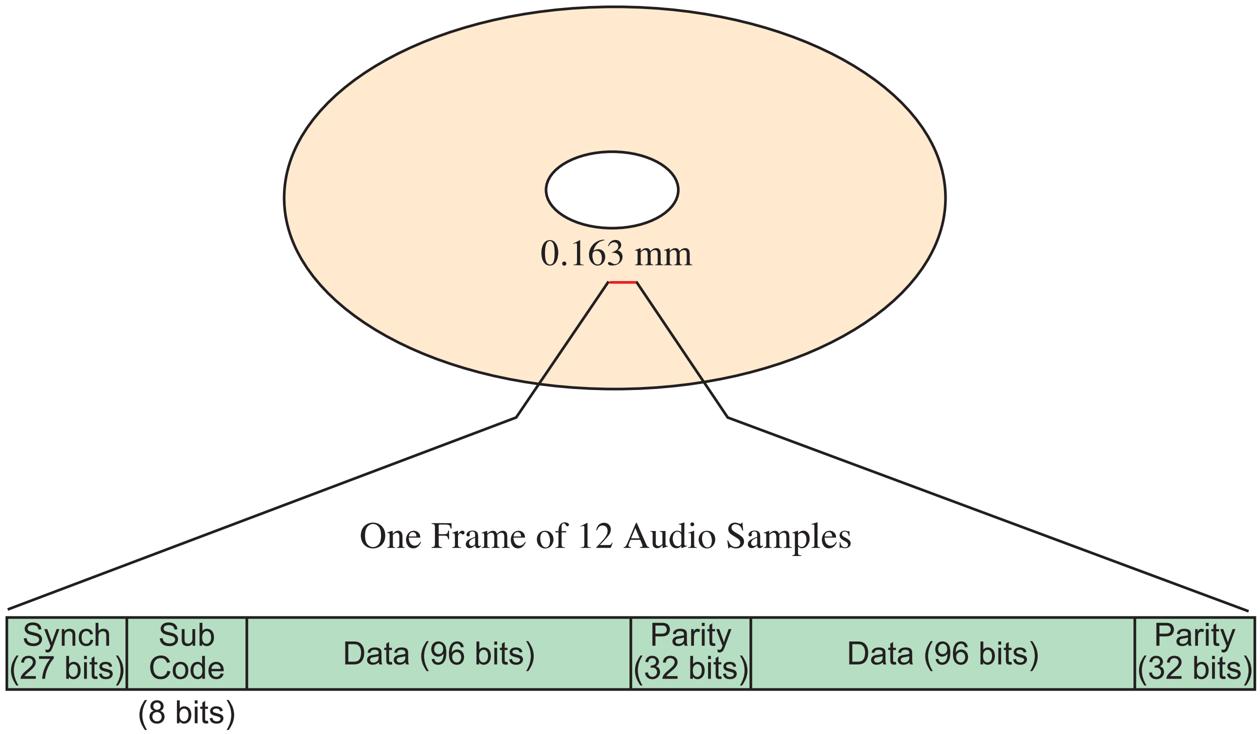
- The error between $m(kT_s)$ and the quantized value $Q[m(kT_s)]$, denoted $e(n)$, is the quantization error

- If $n = 16$, for example, the ratio of signal power in the samples of $m(t)$, to noise power in $e(n)$, is about 95 dB (assuming $m(t)$ stays within the quantizer dynamic range)

Example 3.11: **Compact Disk Digital Audio**

- CD audio quality audio is obtained by sampling a stereo source at 44.1 kHz

- PCM digitizing produces 16 bits per sample per L/R audio channel



CD recoding frame format

- The source bit rate is thus $2 \times 16 \times 44.1\text{ksps} = 1.4112$ Msps
- Data framing and error protection bits are added to bring the total bit count per frame to 588 bits and a serial bit rate of 4.3218 Mbps

3.6 Multiplexing

- It is quite common to have multiple information sources located at the same point within a communication system
- To simultaneously transmit these signals we need to use some form of multiplexing

- There is more than one form of multiplexing available to the communications engineer

- In this chapter we consider time-division multiplexing, while in Chapter 4 frequency division multiplexing is described

3.6.1 Time-Division Multiplexing (TDM)

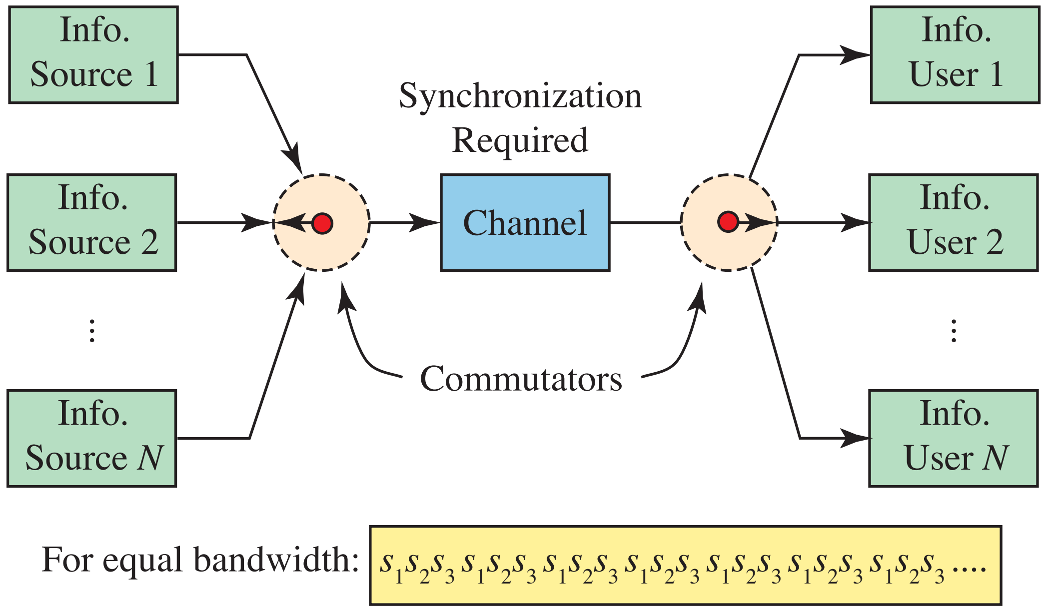
- Time division multiplexing can be applied to sampled analog signals directly or accomplished at the bit level

- We assume that all sources are sample at or above the Nyquist rate

- Both schemes are similar in that the bandwidth or data rate of the sources being combined needs to be taken into account to properly maintain real-time information flow from the source to user

- For message sources with harmonically related bandwidths we can interleave samples such that the wideband sources are sampled more often

- To begin with consider equal bandwidth sources



Analog TDM (equal bandwidth sources)

- Suppose that $m_1(t)$ has bandwidth $3W$ and sources $m_2(t)$, $m_3(t)$, and $m_4(t)$ each have bandwidth W, we could send the samples as
$$s_1 s_2 s_1 s_3 s_1 s_4 s_1 s_2 s_1 \ldots$$
with the commutator rate being $f_s > 2W$ Hz

- The equivalent transmission bandwidth for multiplexed signals can be obtained as follows
  - Each channel requires greater than $2W_i$ samples/s
  - The total number of samples, n_s, over N channels in Ts is thus
$$n_s = \sum_{i=1}^{N} 2W_i T$$
  - An equivalent signal channel of bandwidth B would produce $2BT = n_s$ samples in Ts, thus the equivalent base-

band signal bandwidth is

$$B = \sum_{i=1}^{N} W_i \text{ Hz}$$

which is the same minimum bandwidth required for FDM using SSB

- Pure digital multiplexing behaves similarly to analog multiplexing, except now the number of bits per sample, which takes into account the sample precision, must be included
- In the earlier PCM example for CD audio this was taken into account when we said that left and right audio channels each sampled at 44.1 ksps with 16-bit quantizers, multiplex up to

$$2 \times 16 \times 44,100 = 1.4112 \text{ Msps}$$

Example 3.12: Digital Telephone System

- The *North American* digital TDM hierarchy is based on a single voice signal sampled at 8000 samples per second using a 7-bit quantizer plus one signaling bit
- The serial bit-rate per voice channel is 64 kbps

North American Digital TDM Hierarchy)

Sys.	Digital Signal Number	Bit Rate R (Mb/s)	No. of 64 kbps PCM VF Channels	Transmission Media Used
	DS-0	0.064	1	Wire pairs
T1	DS-1	1.544	24	Wire pairs
T1C	DS-1C	3.152	48	Wire pairs
T2	DS-2	6.312	96	Wire pairs
T3	DS-3	44.736	672	Coax, radio, fiber
	DS-3C	90.254	1344	Radio, fiber
	DS-4E	139.264	2016	Radio, fiber, coax
T4	DS-4	274.176	4032	Coax, fiber
	DS-432	432.00	6048	Fiber
T5	DS-5	560.160	8064	Coax, fiber

- Consider the T1 channel which contains 24 voice signals
- Eight total bits are sent per voice channel at a sampling rate of 8000 Hz
- The 24 channels are multiplexed into a T1 frame with an extra bit for frame synchronization, thus there are $24 \times 8 + 1 = 193$ bits per frame

- Frame period is $1/8000 = 0.125$ ms, so the serial bit rate is $193 \times 8000 = 1.544$ Mbps
- Four T1 channels are multiplexed into a T2 channel (96 voice channels)
- Seven T2 channels are multiplexed into a T3 channel (672 voice channels)
- Six T3 channels are multiplexed into a T4 channel (4032 voice channels)