Appendix A

Physical Noise Sources

Contents

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A.1 Physical Noise Sources

- In communication systems noise can come from both internal and external sources

- Internal noise sources include

    - Active electronic devices such as amplifiers and oscillators

    - Passive circuitry

- Internal noise is primarily due to the random motion of charge carriers within devices and circuits

- The focus of this chapter is modeling and analysis associated with internal noise sources

- External sources include

    - Atmospheric, solar, and cosmic noise

    - Man-made sources such as intentional or unintentional jamming

- To analyze system performance due to external noise location can be very important

- Understanding the impact on system performance will require on-site measurements

A.1.1 Thermal Noise

- Thermal noise is due to the random motion of charge carriers

- Nyquist's Theorem: States that the noise voltage across a resistor is

$$v_{\mathrm{rms}}^2 = \langle v_n^2(t) \rangle = 4kTRB \text{ v}^2$$
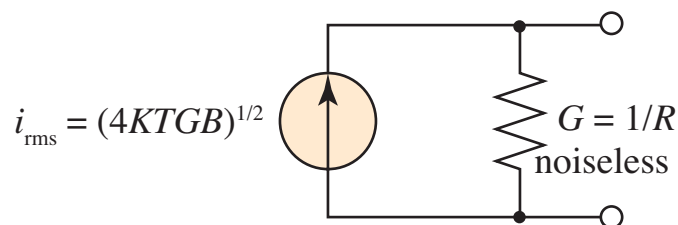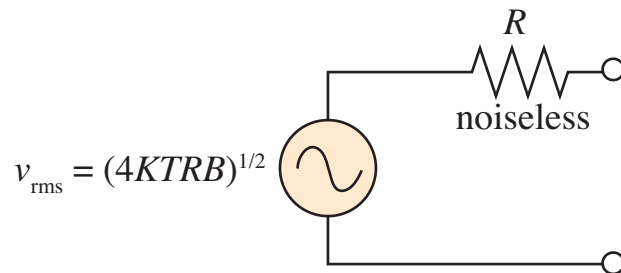
where

$$k = \text{Boltzmanns constant} = 1.38 \times 10^{-23} \text{ J/K}$$
$$T = \text{Temperature in Kelvin}$$
$$R = \text{resistance in ohms}$$
$$B = \text{measurement bandwidth}$$

Equivalent noise circuits: voltage and current

- Consider the following resistor network

Noise analysis for a resistor network

$$v_i^2 = 4KTR_iB$$
$$i = 1, 2, 3$$
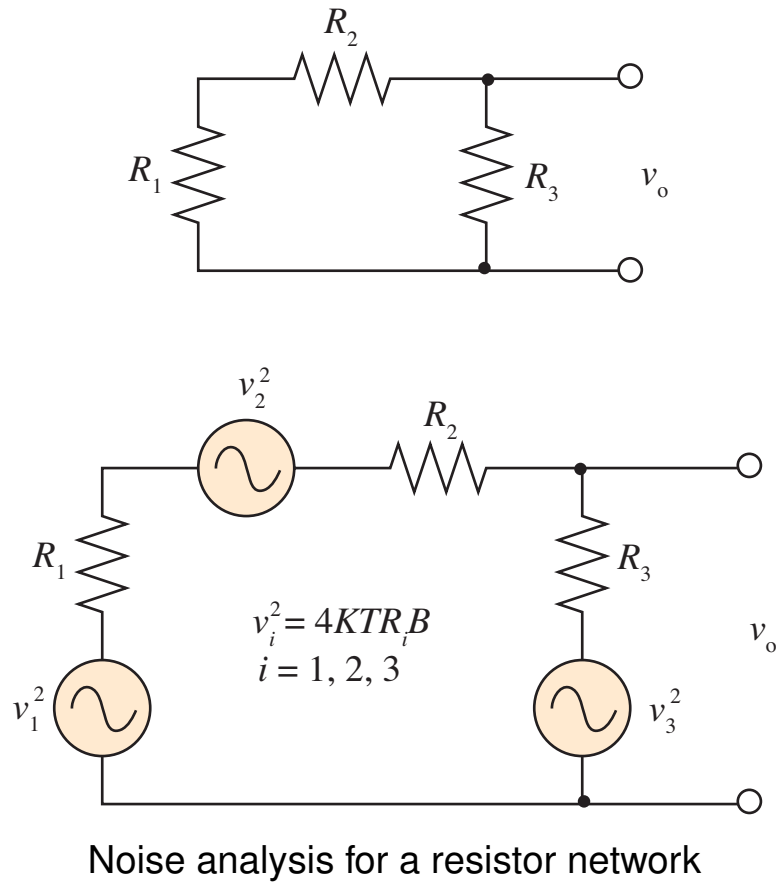
- Since the noise sources are independent, the total noise voltage, v_o can be found by summing the square of the voltage due to each noise source (powers due to independent sources add)

$$v_o^2 = v_{o1}^2 + v_{o2}^2 + v_{o3}^2$$

- The noise voltages, v_{o1}, v_{o2}, v_{o3}, can be found using superposition

A.1.2   Nyquist's Formula

Nyquist's formula for passive networks

- Consider a one-port R, L, C network with input impedance in the frequency domain given by $Z(f)$
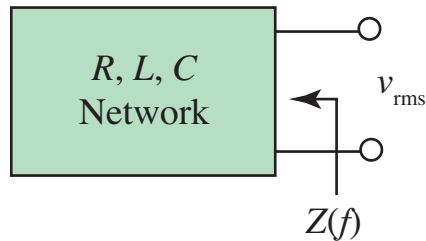
- Nyquist's theorem states that

$$v_{\text{rms}}^2 = \langle v_n^2 \rangle = 2kT \int_{-\infty}^{\infty} R(f)\, df$$

where

$$R(f) = \text{Re}\big[Z(f)\big]$$

- For a pure resistor network Nyquist's formula reduces to

$$\langle v_n^2 \rangle = 2kT \int_{-B}^{B} R_{\text{eq}}\, df = 4kTR_{\text{eq}}B$$

- In the previous example involving three resistors

$$R_{\text{eq}} = R_3 || (R_1 + R_2)$$

Example A.1: **Circuit simulation for noise characterization**

- Spice and Spice-like circuit simulators, e.g. Qucs, have the ability to perform noise analysis on circuit models

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- The analysis is included as part of an AC simulation (in Qucs for example it is turned off by default)

- When passive components are involved the analysis follows from Nyquist's formula

- The voltage that AC noise analysis returns is of the form

$$\frac{v_{\mathrm{rms}}}{\sqrt{\mathrm{Hz}}} = \sqrt{4kTR(f)}$$

where the B value has been moved to the left side, making the noise voltage a spectral density like quantity

- When active components are involved more modeling information is required

- Consider the following resistor circuit
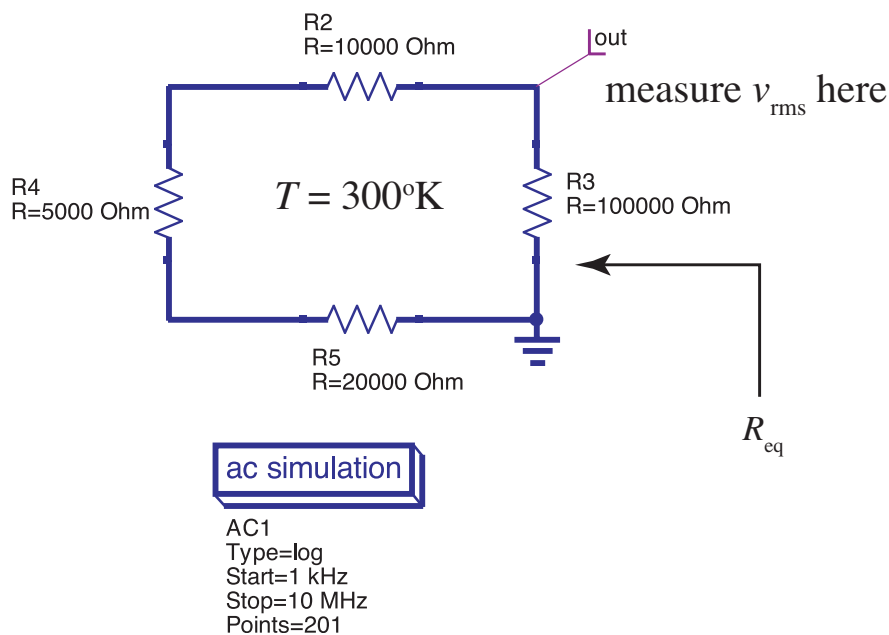
Pure resistor circuit

- To apply Nyquist's formula we need R_{eq}

$$R_{eq} = 100\text{K}||(10 + 5 + 20)\text{K}$$
$$= \frac{100 \cdot 35}{100 + 35}\text{K} = 25.93 \text{ K}$$

- In Nyquists formula the rms noise voltage normalized by B is

$$\frac{v_{rms}}{\sqrt{\text{Hz}}} = \sqrt{4kT \times 25.93 \times 10^3}$$
$$= 2.072 \times 10^{-8} \text{ v}/\sqrt{\text{Hz}}$$

assuming $T = 300°$ K

- Circuit simulation results are shown below
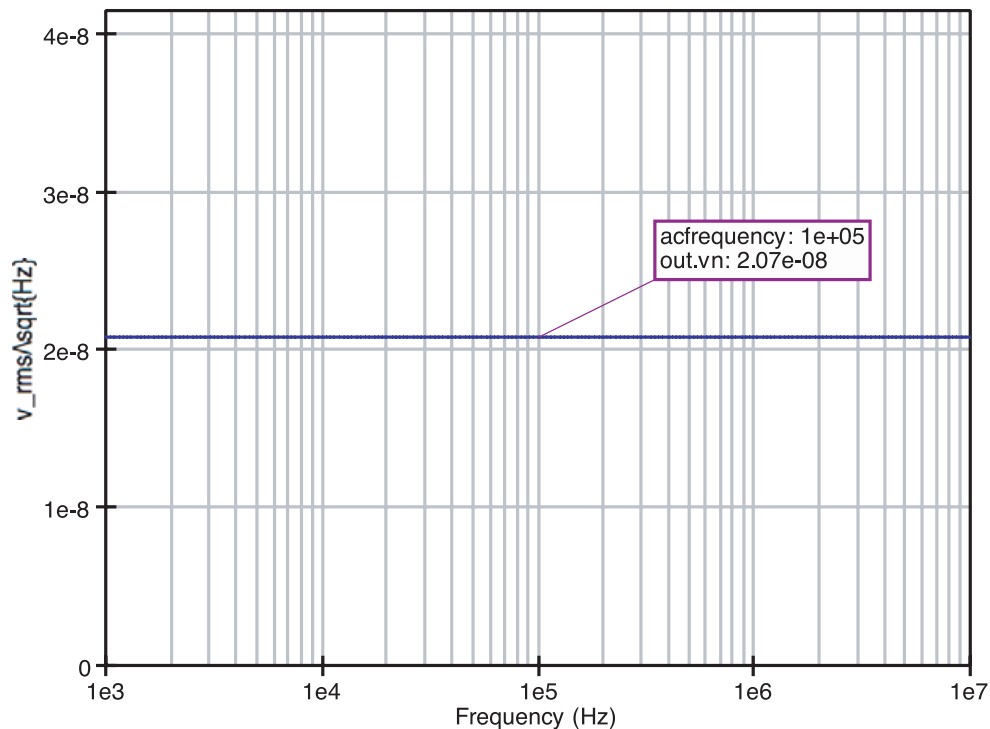
Resistor circuit RMS noise voltage $(v_{rms}/\sqrt{\text{Hz}})$

- Circuit simulation becomes particularly useful when reactive elements are included

- To demonstrate this we modify the resistor circuit by placing a 10 nf capacitor in shunt with the 5 K resistor

$v_{\text{rms}}/\sqrt{\text{Hz}}$ for a simple passive RC circuit

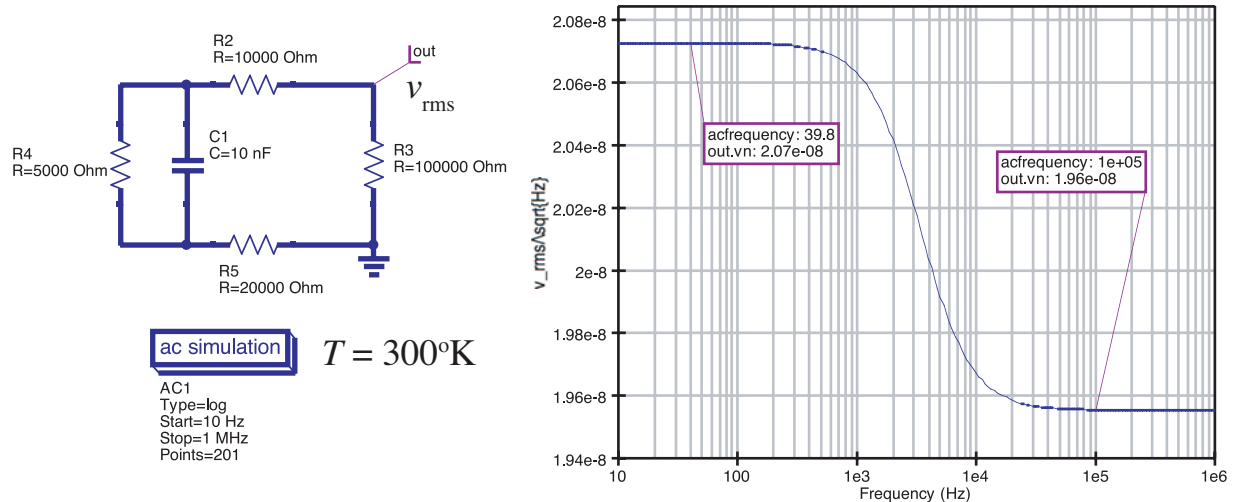- The input impedance of this circuit is

$$
Z(s) = \frac{\left(\dfrac{R_4 \cdot \frac{1}{C_1 s}}{R_4 + \frac{1}{C_1 s}} + R_2 + R_5 \right) R_3}{\dfrac{R_4 \cdot \frac{1}{C_1 s}}{R_4 + \frac{1}{C_1 s}} + R_2 + R_5 + R_3}
$$

- Here the noise voltage/$v_{\text{rms}}/\sqrt{\text{Hz}}$ takes on two limiting values depending upon whether the capacitor acts as an open circuit or a short circuit

- To get the actual rms noise voltage as measured by an AC voltmeter, we need to integrate the $v_{\text{rms}}/\sqrt{\text{Hz}}$ quantity, which can be accomplished with a true rms measuring instrument

$$
v_{\text{rms}}^2 = 4kT \int_0^\infty \text{Re}\{Z(f)\}\, df
$$

Example A.2: **Active circuit modeling**

- For Op-Amp based circuits noise model information is usually available from the data sheet[1]

- Circuit simulators include noise voltage and current sources just for this purpose

741 Noise Data
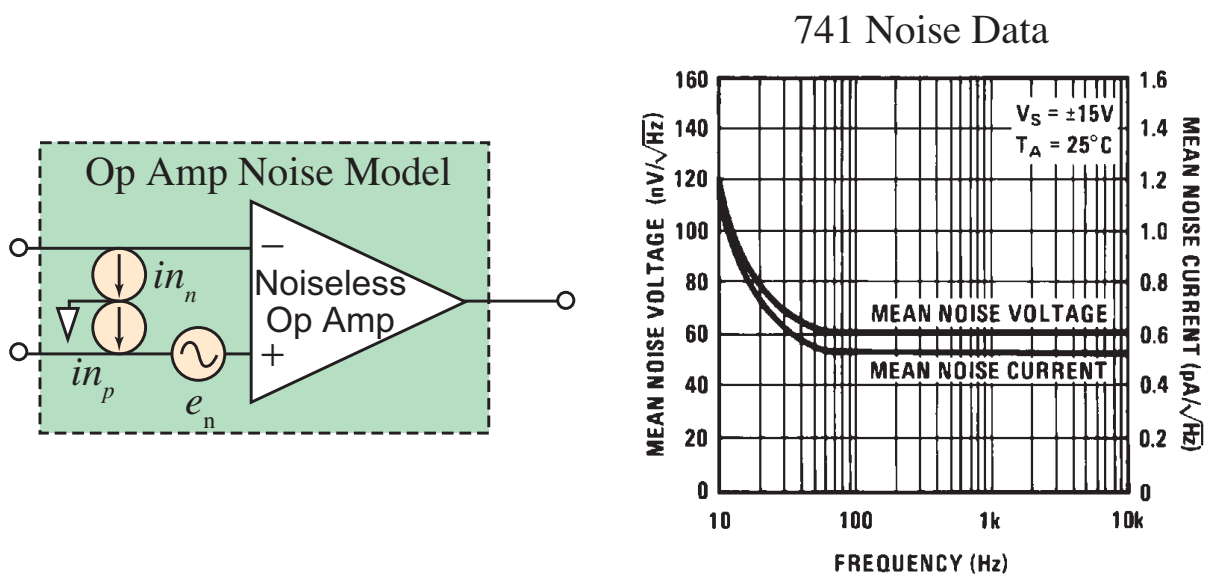
Op amp noise model with 741 data sheet noise information

- Consider an inverting amplifier with a gain of 10 using a 741 op-amp

- This *classic* op amp, has about a 1MHz gain-bandwidth product, so with a gain of 10, the 3 dB cutoff frequency of the amplifier is at about 100 kHz

- The noise roll-off is at the same frequency

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[1]Ron Mancini, editor, *Op Amps for Everyone: Design Reference*, Texas Instruments Advanced Analog Products, Literature number SLOD006, September 2000.

- The rms noise as $v/\sqrt{\text{Hz}}$ plotted below, is a function of the op amp noise model and the resistors used to configure the amplifier gain

- With relatively low impedance configured at the inputs to the op amp, the noise voltage e_n dominates, allowing the noise currents to be neglected

Noise voltage at the op amp output
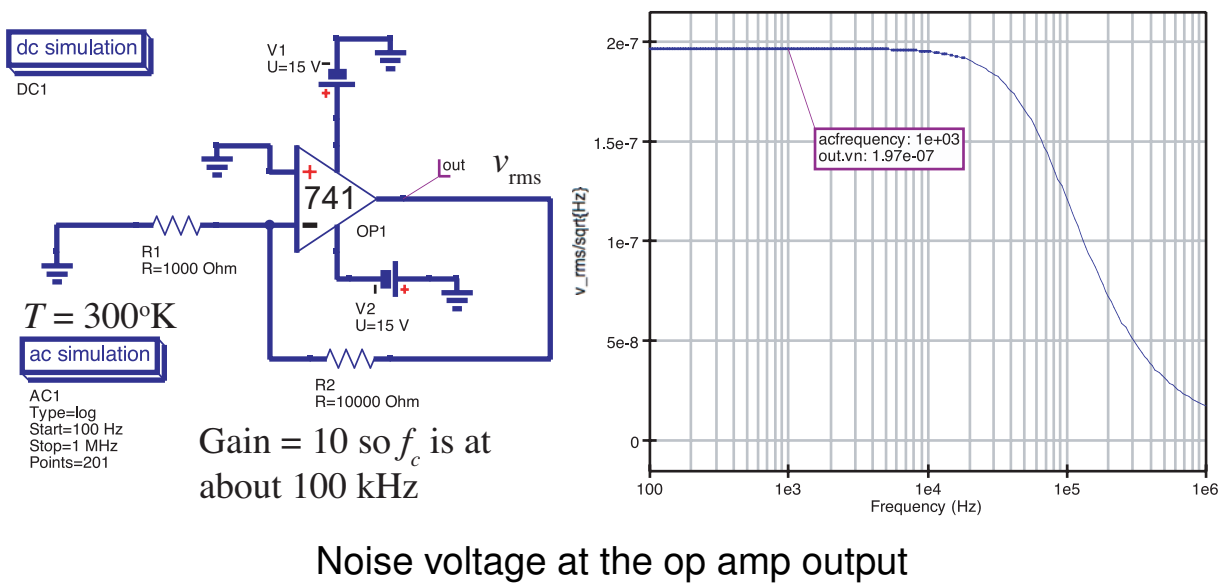
A.1.3 Shot Noise

- Due to the discrete nature of current flow in electronic devices

- Given an average current flow of I_d A,

$$i_{\text{rms}}^2 = \langle i_n^2(t) \rangle = 2e I_d B \text{ A}^2$$

where $e = 1.6 \times 10^{-19}$ is the charge on an electron

- Special Case: For a PN junction diode

$$I = I_s \left[\exp\left(\frac{eV}{kT} \right) - 1 \right] \text{ A}$$

where I_s is the reverse saturation current

- Assuming I_s and $I_s \exp(eV/kT)$ to be independent sources in terms of noise sources, then

$$
\begin{aligned}
i^2_{\text{rms,tot}} &= \left[2e I_s \exp\left(\frac{eV}{kT} \right) + 2e I_s \right] B \\
&= 2e \left[I + I_s \right] B \text{ A}^2
\end{aligned}
$$

- For $I \gg I_s$ the diode differential conductance is

$$g_o = \frac{dI}{dV} = \frac{eI}{kT},$$

thus

$$i_{\text{rms,tot}} \simeq 2eIB = 2kTg_oB$$

which is half the noise due to a pure resistance

A.1.4 Other Noise Sources

- Generation-Recombination Noise: Results from generated free carriers recombining in a semiconductor (like shot noise)

- Temperature-Fluctuation Noise: Results from fluctuating heat exchange between devices and the environment

- Flicker Noise: Has a spectral density of the form $1/f^\alpha \simeq 1/f$, also known as *pink noise*; the physics is not well understood

A.1.5 Available Power

(a)   (b)
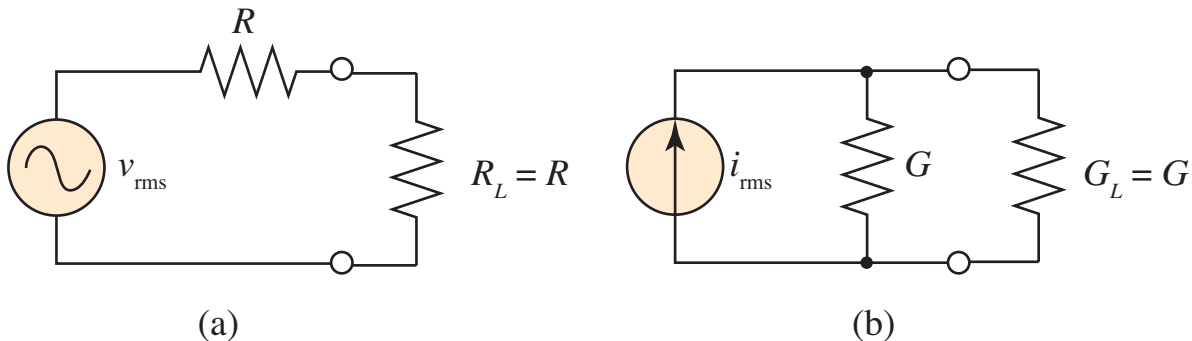
- Noise analysis is often focused around receiver circuitry where maximum power transfer is implemented, i.e., match the load and sources resistances

- Under these conditions the power delivered to the load is the *available power* P_a

$$\text{(a)} \quad P_a = \frac{\left(\frac{1}{2}i_{\text{rms}}\right)^2}{R} = \frac{i_{\text{rms}}^2}{4R} \stackrel{\text{also}}{=} \frac{\left(\frac{1}{2}v_{\text{rms}}\right)^2}{R} = \frac{v_{\text{rms}}^2}{4R}$$

$$\text{(b)} \quad P_a = \frac{\left(\frac{1}{2}i_{\text{rms}}\right)^2}{G} = \frac{i_{\text{rms}}^2}{4G}$$

- For a noisy resistor

$$v_{\text{rms}}^2 = 4kTRB,$$

so

$$P_{a,R} = \frac{4kTRB}{4R} = kTB \text{ W}$$

APPENDIX A. PHYSICAL NOISE SOURCES

Example A.3: **Fundamental Example**

- Consider *room temperature* to be $T_o = 290$ K, then the thermal noise power density is

$$\frac{P_{a,R}}{B} = 4.002 \times 10^{-21} \text{ W/Hz}$$

- For communication system analysis a popular measurement unit for both signal and noise power levels, is the *power ratio* in decibels (dB) referenced to

  (i) 1 W $\longleftrightarrow$ 0 dBW

$$= 10\log_{10}\left(\frac{P_{\text{Watts}}}{1 \text{ Watt}}\right); \quad P_{\text{Watt}} = 1$$

  (ii) 1 mW $\longleftrightarrow$ 0dBm

$$= 10\log_{10}\left(\frac{P_{\text{mW}}}{1 \text{ mW}}\right); \quad P_{\text{mW}} = 1 \text{ mW}$$

- In dB units thermal noise power spectral density under maximum power transfer is

$$\text{Pwr/Hz (dBW)} = 10\log_{10}\left(\frac{4.002 \times 10^{-21}}{1 \text{ W}}\right) \simeq -204 \text{ dBW/Hz}$$

$$\text{Pwr/Hz (dBm)} = 10\log_{10}\left(\frac{4.002 \times 10^{-21}}{1 \text{ mW}}\right) \simeq -174 \text{ dBm/Hz}$$
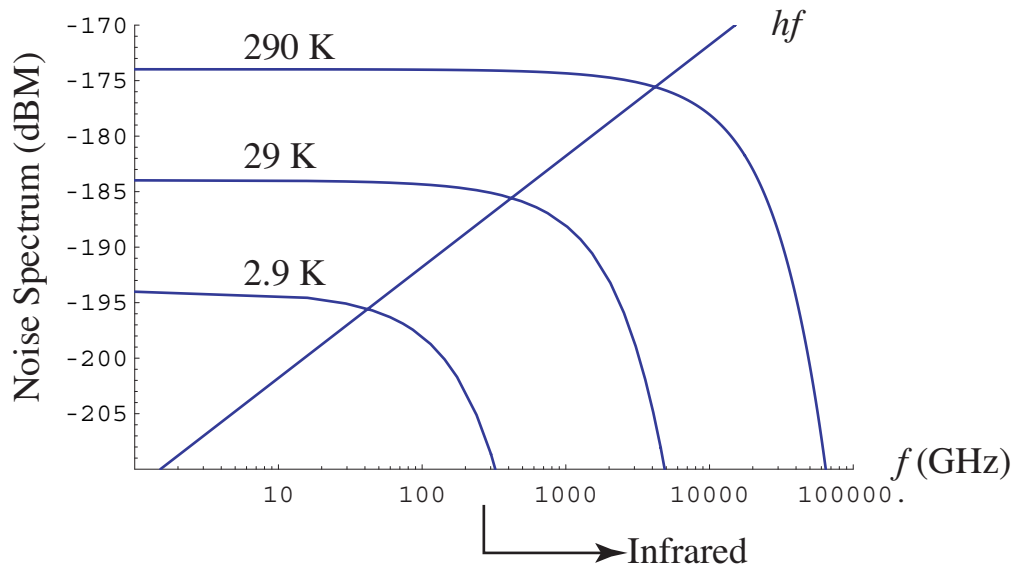
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A.1.6 Frequency Dependence

- If frequency dependence is included, then the available power spectral density is

$$S_a(f) \triangleq \frac{P_a}{B} = \frac{hf}{\exp\left(\frac{hf}{kT}\right) - 1} \text{ W/Hz}$$

where

$$h = \text{Planck's constant} = 6.6254 \times 10^{-34} \text{ J-sec}$$



Thermal noise spectral density, including quantum noise

A.1.7 Quantum Noise

- To account for quantum noise the term hf must be added

- Thermal noise dominates for most applications (i.e., $< 20\,\text{GHz}$), except in optical systems and some millimeter wave systems

A.2 Characterization of Noise in Systems

- In communication system modeling we wish to consider how the noise introduced by each subsystem enters into the overall noise level delivered to the demodulator

- In RF/microwave systems the concept of representing a system as a cascade of subsystems is particularly appropriate, since all connections between subsystems is done at a constant impedance level of say 50 ohms
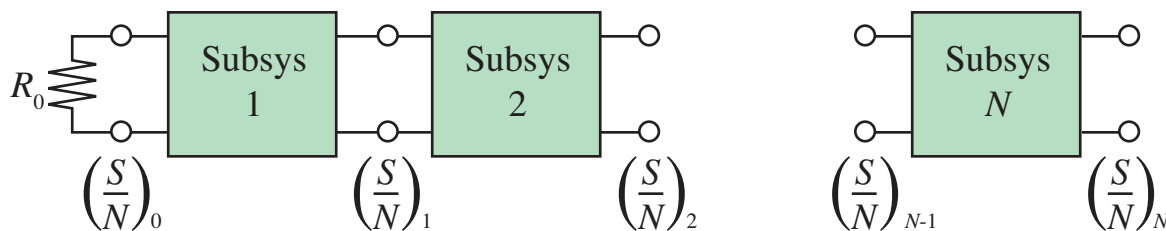
N-subsystem cascade analysis

A.2.1 Noise Figure of a System
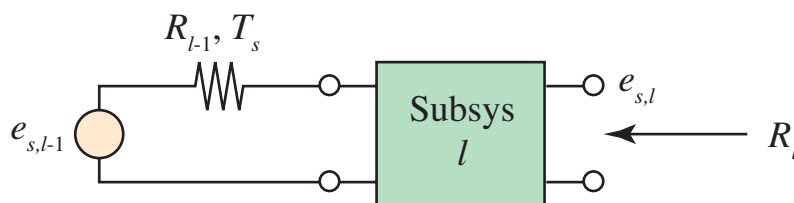
*l*th subsystem model

- For the *l*th subsystem define the *noise figure*, F_l, as

$$\left(\frac{S}{N}\right)_l = \frac{1}{F_l}\left(\frac{S}{N}\right)_{l-1}$$

- Ideally, $F_l = 1$, in practice $F_l > 1$, meaning that each subsystems generates some noise of its own

- In dB the noise figure (NF) is

$$F_{\text{dB}} = 10 \log_{10} F_l$$

- Assuming the subsystem input and put impedances (resistances) are matched, then our analysis may be done in terms of the available signal power and available noise power

- For the lth subsystem the available signal power at the input is

$$P_{sa,l-1} = \frac{e_{s,l-1}^2}{4R_{l-1}}$$

- Assuming thermal noise only, the available noise power is

$$P_{na,l-1} = kT_s B$$

where T_s denotes the *source temperature*

- Assuming that the lth subsystem (device) has power gain G_a, it follows that

$$P_{sa,l} = G_a P_{sa,l-1}$$

where we have also assumed the system is linear

- We can now write that

$$\left(\frac{S}{N}\right)_l = \frac{P_{sa,l}}{P_{na,l}} = \frac{1}{F_l}\frac{P_{sa,l-1}}{P_{na,l-1}} = \frac{1}{F_l}\left(\frac{S}{N}\right)_{l-1}$$

which implies that

$$F_l = \frac{P_{sa,l-1}}{P_{na,l-1}} \cdot \frac{P_{na,l}}{\underbrace{P_{sa,l}}_{G_a P_{sa,l-1}}} = \frac{P_{na,l}}{G_a \underbrace{P_{na,l-1}}_{kT_s B}}$$

- Now

$$P_{na,l} = G_a P_{na,l-1} + P_{\text{int},l}$$

  where $P_{\text{int},l}$ is internally generated noise

- Finally we can write that

$$F_l = 1 + \frac{P_{\text{int},l}}{G_a k T_s B}$$

  - Note that if $G_a \gg 1 \Rightarrow F_l \simeq 1$, assuming that G_a is independent of $P_{\text{int},l}$

- As a standard, NF is normally given with $T_s = T_0 = 290$ K, so

$$F_l = 1 + \frac{P_{\text{int},l}}{G_a k T_0 B}$$

A.2.2   Measurement of Noise Figure

- In practice NF is measured using one or two calibrated noise sources

Method #1

- A source can be constructed using a saturated diode which produces noise current

$$i_n^2 = 2 e I_d B \text{ A}^2$$

- The current passing through the diode is adjusted until the noise power at the output of the devide under test (DUT) is double the amount obtained without the diode, then we obtain

$$F = \frac{e I_d R_s}{2 k T_0}$$

where R_s is the diode series resistance and I_d is the diode current

Method #2

- The so-called Y-factor method requires 'hot'; and 'cold' calibrated noise sources and a precision variable attenuator
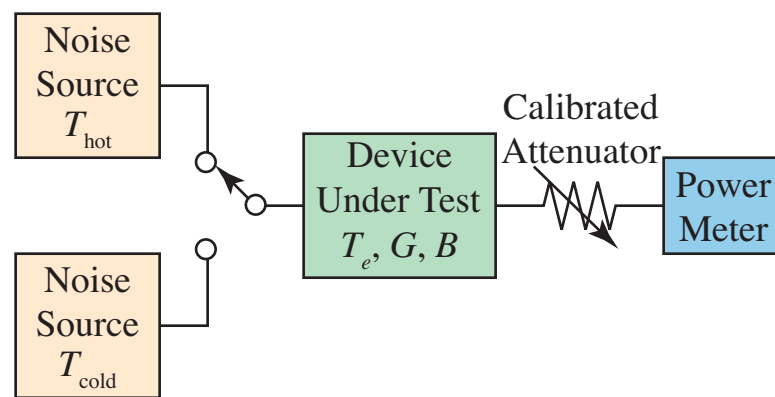
Y factor determination of NF

- From noise power measurements taken with the hot and cold sources we form the ratio

$$\frac{P_h}{P_c} = Y = \frac{k(T_{\text{hot}} + T_e)BG}{k(T_{\text{cold}} + T_e)BG} = \frac{T_{\text{hot}} + T_e}{T_{\text{cold}} + T_e}$$

- Solving for T_e

$$T_e = \frac{T_{\text{hot}} - Y T_{\text{cold}}}{Y - 1}$$

- The Y value is obtained by noting the attenuator setting change, ΔA dB, needed to maintain $P_c = P_h$ and calculating $Y = 10^{\Delta A/10}$
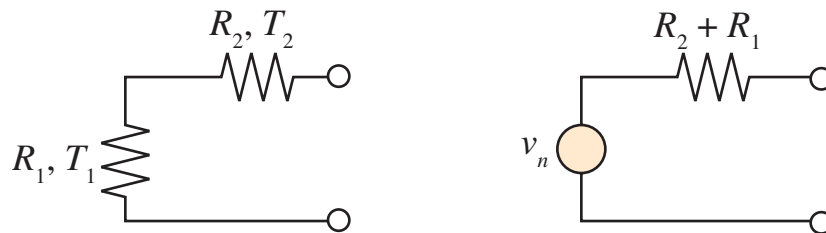

A.2.3   Noise Temperature

- The *equivalent noise temperature* of a subsystem/device, is defined as

$$T_n = \frac{P_{n,\mathrm{max}}}{kB}$$

with $P_{n,\mathrm{max}}$ being the maximum noise power of the source into bandwidth B

Example A.4: **Resistors in series and parallel**

- Find T_n for two resistors in series

R_2, T_2 $R_2 + R_1$

R_1, T_1 v_n

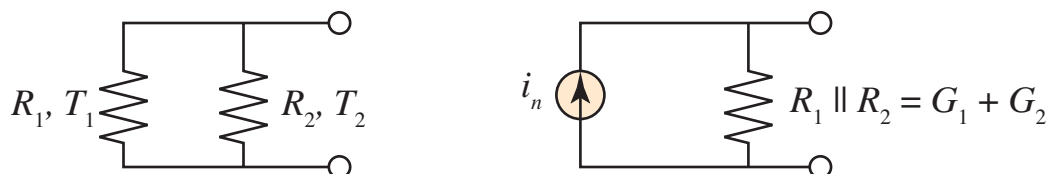$$\langle v_n^2 \rangle = 4kBR_1T_1 + 4kBR_2T_2$$

and

$$P_{na} = \frac{\langle v_n^2 \rangle}{4(R_1 + R_2)} = \frac{4k(T_1R_1 + T_2R_2)B}{4(R_1 + R_2)}$$

therefore

$$T_n = \frac{P_{na}}{kB} = \frac{T_1R_1 + T_2R_2}{R_1 + R_2}$$

- Find T_n for two resistors in parallel

R_1, T_1 R_2, T_2 i_n $R_1 \| R_2 = G_1 + G_2$

$$\langle i_n^2 \rangle = 4kBG_1T_1 + 4kBG_2T_2$$

and

$$P_{na} = \frac{\langle i_n^2 \rangle}{4(G_1 + G_2)} = \frac{4k(T_1G_1 + T_2G_2)B}{4(G_1 + G_2)}$$

therefore

$$T_n = \frac{T_1G_1 + T_2G_2}{G_1 + G_2} = \frac{T_1R_2 + T_2R_1}{R_1 + R_2}$$

A.2.4   Effective Noise Temperature

- Recall the expression for NF at stage l

$$F_l = 1 + \underbrace{\frac{P_{\text{int},l}}{G_a k T_0 B}}_{\text{internal noise}} = 1 + \frac{T_e}{T_0}$$

- Note: $P_{\text{int},l}/(G_a k B)$ has dimensions of temperature

- Define
$$T_e = \frac{P_{\text{int},l}}{G_a k B} = \text{effective noise temp.,}$$
which is a measure of the system *noisiness*
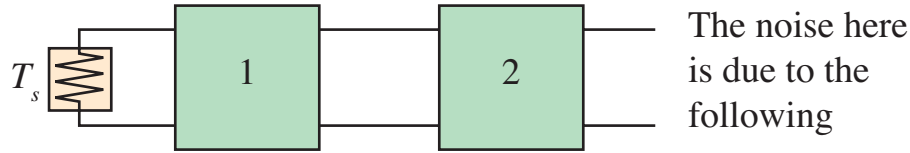
- Next we use T_e to determine the noise power at the output of the lth subsystem

- Recall that

$$\begin{aligned} P_{na,l} &= G_a P_{na,l-1} + P_{\text{int},l} \\ &= G_a k T_s B + G_a k T_e B \\ &= G_a k (T_s + T_e) B \end{aligned}$$

- This references all of the noise to the subsystem input by virtue of the G_a term

A.2.5   Cascade of Subsystems

- Consider two systems in cascade and the resulting output noise contributions

1. Amplified source noise $= G_{a_1} G_{a_2} k T_s B$
2. Internal noise from amplifier 1 $= G_{a_1} G_{a_2} k T_{e_1} B$
3. Internal noise from amplifier 2 $= G_{a_2} k T_{e_2} B$

- Thus

$$P_{na,2} = G_{a_1} G_{a_2} k \left(T_s + T_{e1} + \frac{T_{e_2}}{G_{a_1}} \right) B$$

which implies that

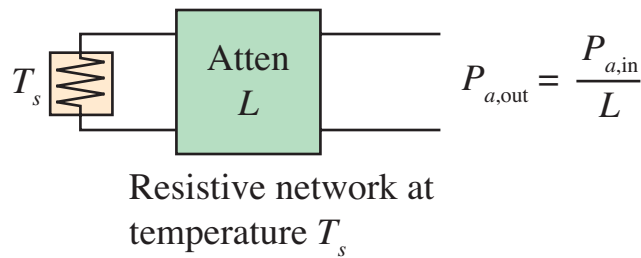$$T_e = T_{e_1} + \frac{T_{e_2}}{G_{a_1}}$$

and since $F = 1 + T_e / T_0$

$$F = 1 + \frac{T_{e_1}}{T_0} + \frac{T_{e_2}}{G_{a_1} T_0}$$

$$= F_1 + \frac{1 + \frac{T_{e_2}}{T_0} - 1}{G_{a_1}}$$

$$= F_1 + \frac{F_2 - 1}{G_{a_1}}$$

- In general for an arbitrary number of stages (*Frii's formula*)

$$F = F_1 + \frac{F_2 - 1}{G_{a_1}} + \frac{F_3 - 1}{G_{a_1} G_{a_2}} + \cdots$$

$$T_e = T_{e_1} + \frac{T_{e_2}}{G_{a_1}} + \frac{T_{e_3}}{G_{a_1} G_{a_2}} + \cdots$$

A.2.6 Attenuator Noise Temperature and Noise Figure

Resistive network at temperature T_s

$$P_{a,\text{out}} = \frac{P_{a,\text{in}}}{L}$$

Attenuator model

- Since the attenuator is resistive, we know that the impedances are matched and

$$P_{na,\text{out}} = kT_s B \text{ (independent of } R_s \text{ or } L)$$

- Let the *equivalent* temperature of the attenuator be T_e, then

$$P_{na,\text{out}} = G_a k (T_s + T_e) B$$
$$= \frac{1}{L} \underbrace{k(T_s + T_e)B}_{\text{looks like } P_{an,\text{in}}}$$

- Thus since $P_{an,\text{out}} \overset{\text{also}}{=} kT_s B$, it follows that

$$\frac{1}{L}(T_s + T_e) = T_s$$

or

$$T_e = (1 - L)T_s$$

- Now since

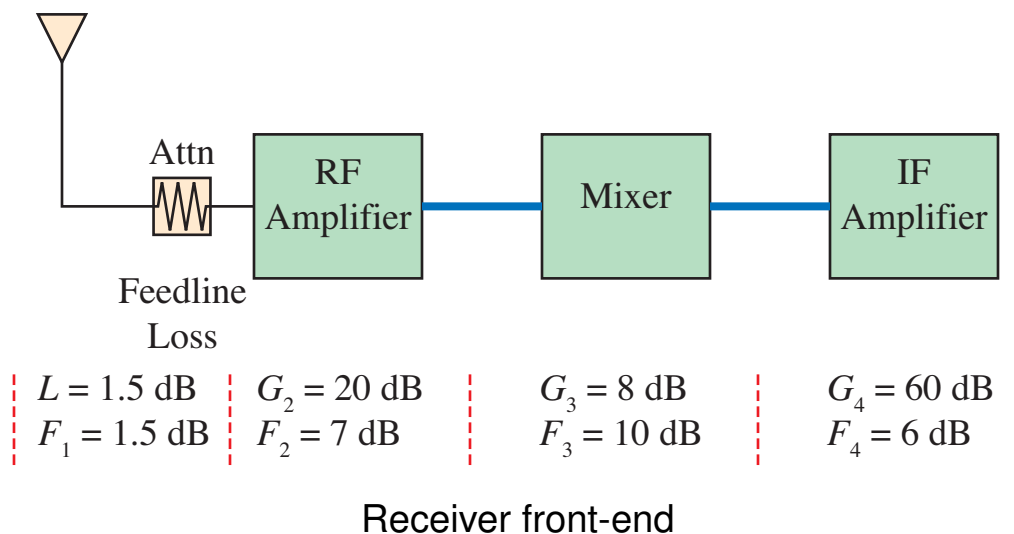$$F = 1 + \frac{T_e}{T_0} = 1 + \frac{(L-1)T_s}{T_0}$$

with $T_s = T_0$ (i.e., attenuator at room temperature)

$$F_{attn} = 1 + L - 1 = L$$

Example A.5: **6 dB attenuator**

- The attenuator analysis means that a 6 dB attenuator has a noise figure of 6 dB

Example A.6: **Receiver system**

Receiver front-end

- We need to convert from dB back to ratios to use Frii's formula

$$
\begin{array}{l|l}
G_1 = 10^{-1.5/10} = \frac{1}{1.41} & F_1 = 10^{1.5/10} = 1.41 \\
G_2 = 10^{20/10} = 100 & F_2 = 10^{7/10} = 5.01, \\
G_3 = 10^{8/10} = 6.3 & F_3 = 10 \\
G_4 = 10^{60/10} = 10^6 & F_4 = 3.98
\end{array}
$$

- The system NF is

$$
\begin{aligned}
F &= 1.41 + \frac{5.01 - 1}{1/1.41} + \frac{10 - 1}{100/1.41} + \frac{3.98 - 1}{(100)(6.3)/1.41} \\
&= 7.19 \text{ or } 8.57 \text{ dB}
\end{aligned}
$$

- The effective noise temperature is

$$
\begin{aligned}
T_e &= T_0(F - 1) = 290(7.19 - 1) \\
&= 1796.3 \text{ K}
\end{aligned}
$$

- To reduce the noise figure (i.e., to improve system performance) interchange the cable and RF preamp

- In practice this may mean locating an RF preamp on the back of the receive antenna, as in a satellite TV receiver

- With the system of this example,

$$
\begin{aligned}
F &= 5.01 + \frac{1.41 - 1}{100} + \frac{10 - 1}{100/1.41} + \frac{3.98 - 1}{(100)(6.3)/1.41} \\
&= 5.15 \text{ or } 7.12 \text{ dB} \\
T_e &= 1202.9 \text{ K}
\end{aligned}
$$

- <u>Note</u>: If the first component has a high gain then its noise figure dominates in the cascade connection

- Note: The antenna noise temperature has been omitted, but could be very important

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Example A.7: **Receiver system with antenna noise temperature**

$$F = 7.19 \text{ or } F_{dB} = 8.57 \text{ dB}, T_e = 1796.3 \text{ K}$$
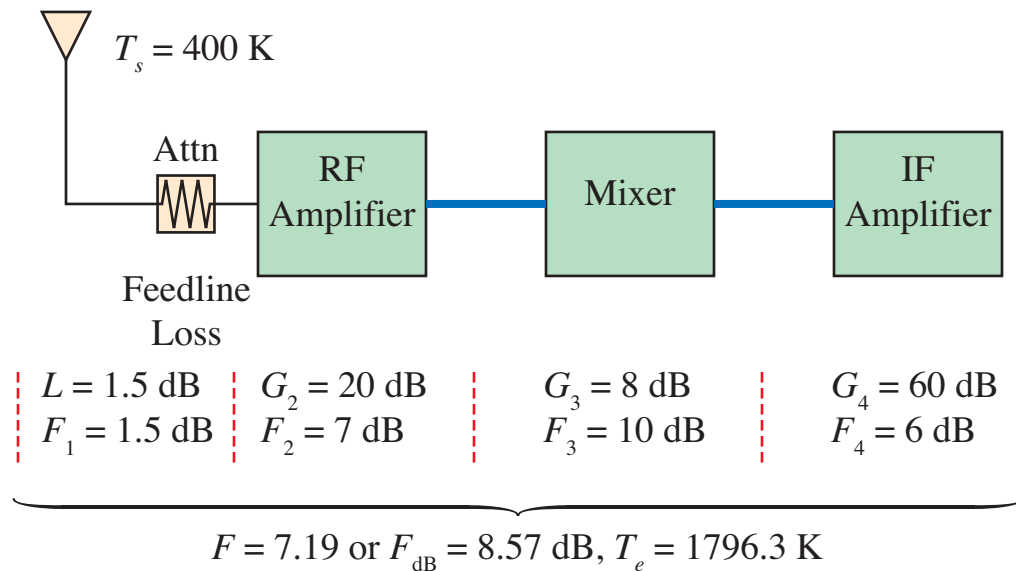
- Rework the previous example, except now we calculate available noise power and signal power with additional assumptions about the receiving antenna

- Suppose the antenna has an effective noise temperature of $T_s = 400$ K and the system bandwidth is $B = 100$ kHz

- What is the maximum available output noise power in dBm?

- Since

$$P_{na} = G_a k(T_s + T_e)B = (G_a)(kT_0)\left(\frac{T_s + T_e}{T_0}\right)(B)$$

where

$$G_{a,\text{dB}} = -1.5 + 20 + 8 + 60 = 86.5 \text{ dB}$$

$$kT_0 = -174 \text{ dBm/Hz}, \quad T_0 = 290 \text{ K}$$

we can write in dB that

$$P_{na,\text{dB}} = 86.5 - 174 + 10\log\left(\frac{400 + 1796.3}{290}\right) + 10\log_{10} 10^5$$

$$= -28.71 \text{ dBm}$$

- What must the received signal power at the antenna terminals be for a system output SNR of 20 dB?

- Let the received power be P_s or in dBm $P_{s,\text{dB}}$

$$10\log_{10}\left(\frac{G_a P_s}{P_{na}}\right) = 20$$

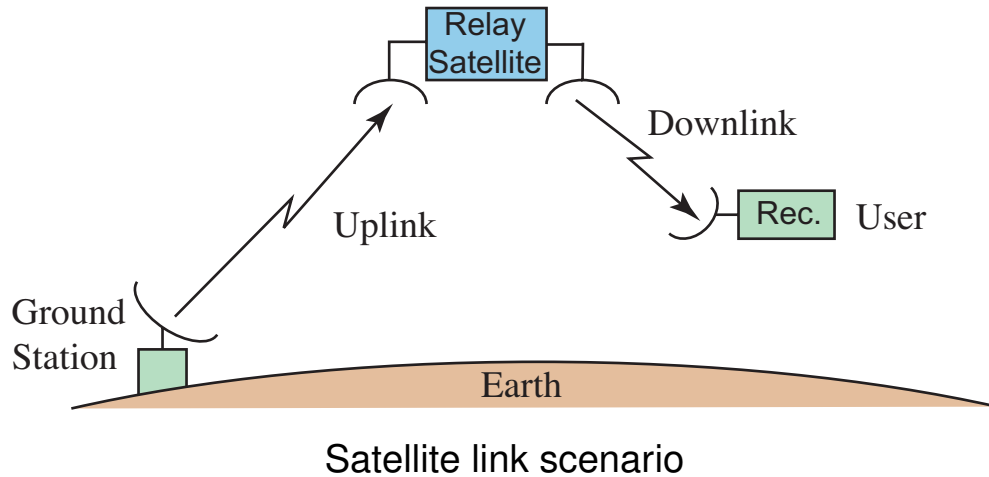- Solving for P_s in dbm

$$P_{s,\text{dB}} = 20 + P_{na,\text{dB}} - G_a, \text{dB}$$

$$= 20 + (-28.71) - 86.5 = -95.21 \text{ dBm}$$

A.3   Free-Space Propagation Channel

- A practical application of the noise analysis is in calculating the link budget for a free-space communications link

- This sort of analysis applies to satellite communications

Satellite link scenario

- Consider an *isotropic radiator* which is an ideal *omnidirectional* antenna

Power P_T is radiated uniformly in all directions (a point source)
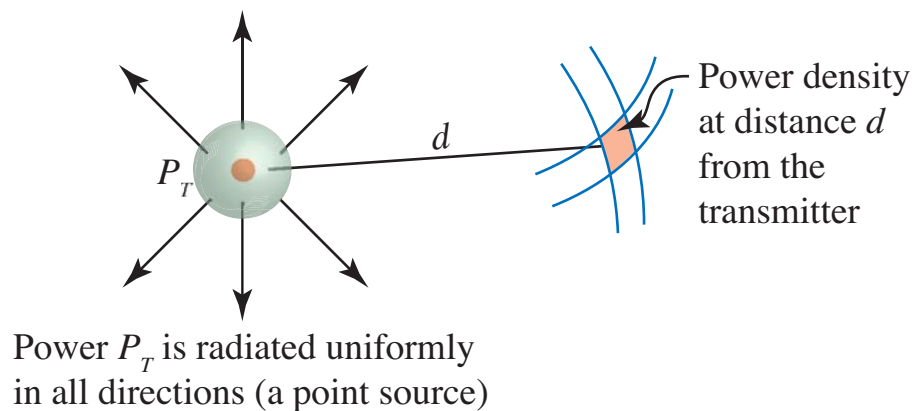
Omni antenna and received flux density

- The power density at distance d from the source (antenna) is

$$p_t = \frac{P_T}{4\pi d^2} \text{ W/m}^2$$

- An antenna with *directivity* (more power radiated in a particular direction), is described by a power gain, G_T, over an isotropic antenna

- For an *aperture-type* antenna, e.g., a parabolic dish antenna, with aperture area, A_T, such that

$$A_T \gg \lambda^2$$

  with λ the transmit wavelength, G_T is given by

$$G_T = \frac{4\pi A_T}{\lambda^2}$$

- Assuming a receiver antenna with aperture area, A_R, it follows that the received power is

$$\begin{aligned} P_R = p_t A_R &= \frac{P_T G_T}{4\pi d^2} \cdot A_R \\ &= \frac{P_T G_T G_R \lambda^2}{(4\pi d)^2} \end{aligned}$$

  since $A_R = G_R \lambda^2/(4\pi)$

- For system analysis purposes modify the P_R expression to include a *fudge factor* called the system loss factor, L_0, then we can write

$$P_R = \underbrace{\left(\frac{\lambda}{4\pi d}\right)^2}_{\text{Free space loss}} \frac{P_T G_T G_R}{L_0}$$

- In dB (actually dBW or dBm) we have

$$
\begin{aligned}
P_{R,\text{dB}} &= 10\log_{10} P_R \\
&= 20\log_{10}\left(\frac{\lambda}{4\pi d}\right) \\
&\quad + \underbrace{10\log_{10} P_T + 10\log_{10} G_T}_{\text{EIRP}} \\
&\quad + 10\log_{10} G_R - 10\log_{10} L_0
\end{aligned}
$$

where EIRP denotes the *effective isotropic radiated power*

Example A.8: **Free-Space Propagation**

- Consider a free-space link (satellite communications) where

$$
\text{Trans. EIRP} = (28 + 10) = 38 \text{ dBW}
$$
$$
\text{Trans. Freq} = 400 \text{ MHz}
$$

- The receiver parameters are:

$$
\text{Rec. noise temp.} = T_s + T_e = 1000 \text{ K}
$$
$$
\text{Rec. ant. gain} = 0 \text{ dB}
$$
$$
\text{Rec. system loss } L_0) = 3 \text{ dB}
$$
$$
\text{Rec. bandwidth} = 2 \text{ kHz}
$$
$$
\text{Path length } d = 41,000 \text{ Km}
$$

- Find the output SNR in the 2 kHz receiver bandwidth

- The received signal power is

$$P_{R,\text{dB}} = 20\log_{10}\left(\frac{3\times 10^8/4\times 10^8}{4\pi \times 41,000\times 10^3}\right) + 38 \text{ dBW} + 0 - 3$$

$$= -176.74 + 38 - 3 = -141.74 \text{ dBW}$$

$$= -111.74 \text{ dBm}$$

  - <u>Note</u>: $\lambda = c/f = 3\times 10^8/400\times 10^6$

- The receiver output noise power is

$$P_{na,\text{dB}} = 10\log_{10}(kT_0) + 10\log_{10}\left(\frac{T_s + T_e}{T_0}\right) + 10\log_{10} B$$

$$= -174 + 5.38 + 33$$

$$= -135.62 \text{ dBm}$$

- Hence

$$\text{SNR}_{\text{o, dB}} = 10\log_{10}\left(\frac{P_R}{P_{na}}\right)$$

$$= -111.74 - (-135.62)$$

$$= 23.88 \text{ dB}$$

.