EE 779 Advanced Topics in Signal Processing
Assignment 4
Assigned: 03/10/16, Due: 10/10/16
Indian Institute of Technology Bombay

Note

- Most of these problems are from Chapter 6 in [1]. A scan of this uploaded in moodle.

Problems

1. [$*$]Consider a ULA comprising M sensors, with inter-element spacing equal to d. Let λ denote the wavelength of the signals impinging on the array. The spatial-frequency resolution of the beamforming used with this ULA is given by

$$\Delta\omega_s = \frac{2\pi}{M} \quad \Longleftrightarrow \quad \Delta f_s = \frac{1}{M}.$$

Hint: Use Taylor series expansion for $\sin(\theta + \Delta\theta)$.

(a) Use this to show that the direction-of-arrival (DOA) resolution of beamforming for signals coming from *broadside* ($\theta = 0$) is

$$\Delta\theta \simeq \sin^{-1}(1/L)$$

where L is the array's length measured in wavelengths:

$$L = \frac{(M-1)d}{\lambda}.$$

Explain how the DOA resolution approximately reduces to

$$\Delta\theta \simeq \frac{\lambda}{L}.$$

(b) Next, show that, for signals impinging from an *arbitrary direction* θ, the DOA resolution of beamforming is approximately

$$\Delta\theta \simeq \frac{1}{L|\cos\theta|}.$$

Hence, for signals coming from nearly end-fire ($\theta = \pm\pi/2$), the DOA resolution is much worse.

2. [$*$]Consider an $M-$element array, with M odd, shaped as an "L" with element spacing d. Thus, the array elements are located at points $(0,0),(0,d),\ldots,(0,d(M-1)/2)$ and $(d,0),(2d,0),\ldots,(d(M-1)/2,0)$. Determine the array's beampattern ($W(\theta) = a^H(\theta_0)a(\theta)$) at angle θ_0, by assuming the sensor at $(0,0)$ as the reference. Provide plots of $f(\theta) = |W(\theta)|^2$ at $\theta_0 = 90°$ and $\theta_0 = 45°$.

3. In words, MUSIC (for both frequency and DOA estimation) says that the direction vectors $\{a(\theta_k)\}$ belong to the subspace spanned by the columns of $\mathbf{V}_s$ (signal space eigenvectors). Therefore, we can think of estimating the DOAs by choosing θ (a generic DOA variable) so that the distance between $a(\theta)$ and the closest vector in the span of $\hat{\mathbf{V}}_s$ is minimized, i.e.,

$$\min_{\beta,\theta} \|a(\theta) - \widehat{\mathbf{V}}_s\beta\|^2,$$

where $\|.\|$ denotes the Euclidean vector norm. Note that the dummy vector variable β is defined in such a way that $\hat{\mathbf{V}}_s\beta$ is closest to $a(\theta)$ in Euclidean norm. Show that the DOA estimation method derived from the subspace-fitting criterion is the same as MUSIC.

Hint: Show that

$$\|a(\theta) - \widehat{\mathbf{V}}_s\beta\|^2 = \|\beta - \widehat{\mathbf{V}}_s a(\theta)\|^2 + a^H(\theta)\widehat{\mathbf{V}}_n\widehat{\mathbf{V}}_n^H a(\theta).$$

Simulations

1. Problem C6.15 in S & M [1].
2. Problem C6.17 in S & M [1]. Data file (`submarine.mat`) is uploaded in moodle.

Reference

1. Petre Stoica and Randolph Moses, "Spectral analysis of signals", Prentice Hall, 2005. (Indian edition available)
2. Monson H. Hayes, "Statistical signal processing and modeling", Wiley India Pvt. Ltd., 2002. (Indian edition available)