A GPS Signal Transmission Model for Improved Single Antenna Attitude Determination

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Summary: This paper describes the development of a GPS transmission model for improving the attitude accuracy by modelling the GPS transmission path. Various factors are considered, such as receiving antenna gain, distance dependent path loss, effects of the ionosphere, GPS satellite transmitting antenna gain and varying GPS satellite transmission power.

The subsequent sections present a new single antenna attitude system based on signal strength. By incorporating a precisely measured spherical antenna gain pattern and a GPS transmission model, attitude can be estimated using a least square search. Finally, the improved attitude system is analysed by comparing to previous results. This paper concludes that further analysis is required for modelling the transmission path, particularly for the ionosphere and the GPS transmitting antenna gain.

Keywords: attitude determination, GPS, SNR, single antenna, FedSat.

Introduction

The Federation Satellite (FedSat) is a low cost micro-satellite designed to conduct scientific experiments in a low Earth orbit (LEO) of approximately 800 km altitude. It was built and now operated by a group of research organisations, companies and universities comprising the Australian Cooperative Research Centre for Satellite Systems (CRCSS). Since the successful launch of the FedSat in December 2002, it has returned valuable data about the space environment for Australian scientists and engineers.

One of the main requirements of the FedSat project is the provision of navigation services through the use of a suitable space qualified GPS receiver. Developed by SpectrumAstro and JPL, "BlackJack" is a dual frequency GPS receiver designed to meet the requirements for precise orbit determination, real-time positioning, precise timing and Earth science research. In addition, the single antenna configuration of FedSat provides an excellent test platform for the development of single antenna GPS-based attitude system.

In recent years, substantial research effort has been directed towards single antenna GPS-based attitude systems [1-9]. The fundamental principles of these systems are based on the assumption that the GPS receiving antenna gain pattern varies as a function of the direction to the GPS satellite. The gain is at the highest along the antenna boresight vector with decreasing gain down to 90° off-boresight angle α as defined in Fig. 1. The azimuthal variations are very small and ignored. Thus, by utilizing the signal strength of the tracked signal and the geometry of the tracked satellite, orientation of the antenna boresight vector with respect to a reference coordinate system can be estimated.

The research has indicated great potential for use in space and other airborne applications, such as UAVs, due to its advantage of being low cost and simple implementation. Other research has demonstrated an attitude accuracy of 15° rms on several spacecraft missions [1, 3, 9].

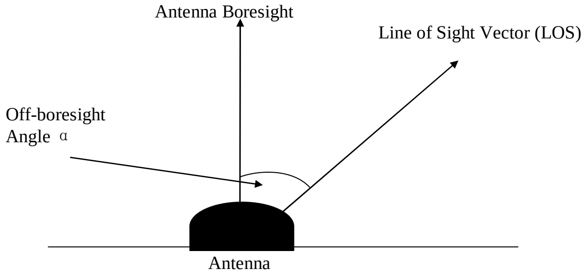

Fig. 1: Antenna boresight definition

The performances of single antenna attitude system, based on signal strength, have been shown to depend strongly on the quality of the calibration mapping function [3, 9]. Other influential factors include the number of GPS signals available, the geometry of tracked satellites and the receiver's antenna gain pattern.

Analysis from this research in a simulated environment shows that the accuracy of the system can be estimated within 5° rms with an improved calibration mapping function [9]. This is achieved by using the WelNavigate GS-720 GPS constellation simulator, a Novatel 3151R GPS receiver and a Novatel 501 antenna. In order to achieve this degree of accuracy in real applications, a model for GPS signal transmission is required.

The objective of this research is to investigate methods for improving the attitude accuracy by modelling the GPS transmission path. Factors such as receiving antenna gain, distance dependent path loss, effects of the ionosphere, GPS satellite transmitting antenna gain and varying GPS satellite transmission power are considered. In addition, a new attitude system is proposed based on a spherical antenna gain pattern and the transmission model developed. The performance of the system is analysed and compared with other single antenna attitude systems developed.

GPS Signal Transmission Model

A basic GPS signal transmission model is shown in Fig 2.

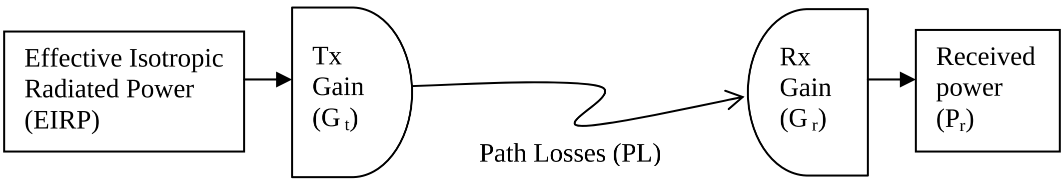

Fig 2: Block diagram for GPS signal transmission

The estimated received power (P_r) at the input of the low noise amplifier is equal to the sum of the transmitted power plus the GPS transmitting antenna gain (G_t), path losses (PL) and the receiving antenna gain (G_r).

$$P_r = EIRP + G_t + PL + G_r \tag{1}$$

The carrier to noise ratio (C/N_0) measured by GPS receiver is related to the received power (P_r), system noise temperature (T_{sys}), noise figure of receiver (L_{Nf}) and the losses in analog to digital (A/D) conversion (L_I) [12]. The equation can be written as:

$$C/N_0 = P_r - 10*\log_{10} T_{sys} + 228.6 + L_{Nf} + L_I \quad (2)$$

Substituting Equation 1 to Equation 2:

$$C/N_0 = EIRP + G_t + PL + G_r - 10*\log_{10} T_{sys} + 228.6 + L_{Nf} + L_I \quad (3)$$

Finally, with a reasonable assumption of EIRP = 30 dBW, T_{sys} = 190K, L_I = -1.5 dB [12], constant terms in Equation 3 can be grouped together to form a constant signal offset *c*. The carrier to noise measured by the GPS receiver can be expressed as:

$$C/N_0 = G_t + PL + G_r + c \quad (4)$$

Equation 4 shows that the C/N_0 measurements consist of three varying components, FedSat receiving gain (G_r), path losses (PL) and GPS transmitting gain (G_t). The model for each component will be individually discussed in the following subsections. After that, descriptions on the FedSat operating environment, where data were collected and used for model development, will be presented, followed by the GPS signal transmission model results.

FedSat Receiving Gain

Single antenna systems developed by previous researchers are based on two assumptions [3, 6-9]. Firstly, the gain is at the highest along the antenna boresight vector with decreasing gain down to 90° off-boresight. Secondly, the azimuthal variations are very small and ignored. However, due to the antenna mounting position, the receiving antenna gain pattern is often distorted by the signal reflected off the mounting surface. This distortion causes error in the calibration mapping function and in consequence decreases the accuracy of the attitude system.

In order to overcome the uncertainty of the receiving antenna gain, the FedSat antenna (S67-1575-14) was tested in an anechoic chamber. The impacts on the published antenna gain pattern due to the mounting object were precisely measured and recorded. The test setup and results were detailed in [10], but are described briefly here.

As shown in Fig 3, the antenna of FedSat is mounted on the negative velocity vector face of the spacecraft. This configuration allows FedSat to conduct upper atmosphere analysis to detect alterations in propagation characteristics in the Earth's ionosphere/plasmasphere. However, as discussed in later sections, this configuration causes an uneven distribution in the GPS data collected. In consequence, the performance of the system is slightly affected.

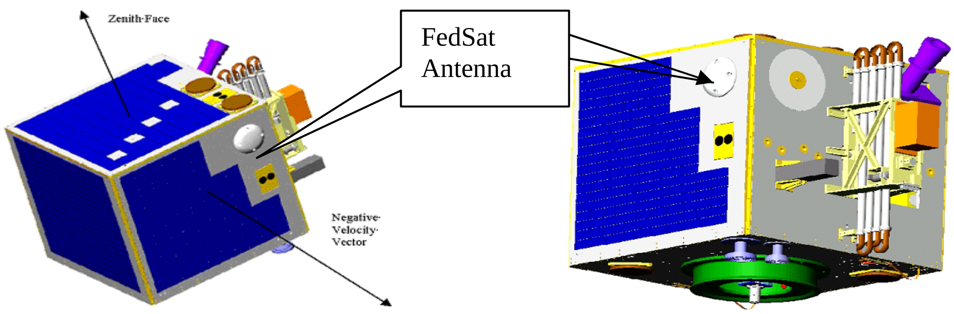

Fig. 3: FedSat GPS antenna mounting configuration

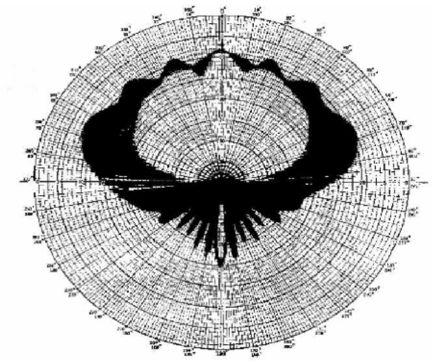

Fig. 4: Published antenna gain pattern for Sensor Systems S67-1575-14

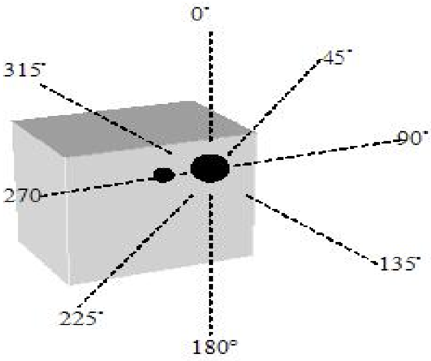

Fig 5: Definition of TinSat antenna plane slice

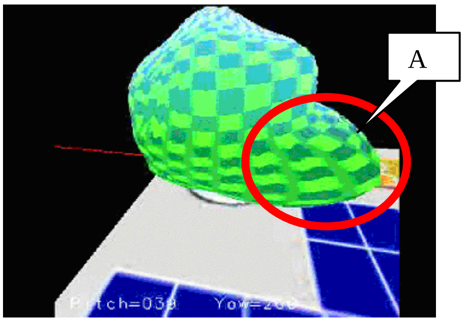

Fig. 6: FedSat spherical antenna gain pattern, view from pitch 38º, yaw 250º.

The antenna used in FedSat is a Sensor Systems S67-1575-14. It is a dual-band L1/L2 passive GPS antenna that provides coverage at 1227.6 MHz and 1575.42 MHz. The published antenna gain pattern is shown in Fig 4. Note that the theoretical gain pattern is in agreement with the assumption made by [3] discussed earlier this section.

During the anechoic chamber room test, the antenna was mounted on a mock-up model of FeSat-1 (called TinSat). The range control computer recorded the gain, phase and azimuth angle measurements to a series of files at 0.5° interval. The process was repeated through the four antenna plane slices defined in Fig. 5, i.e. 0°-180°, 45°-225°, 90°-270° and 135°-315°.

The measured FedSat spherical antenna gain pattern is created and shown in Fig 6 [10]. It can be clearly observed that the antenna gain has been heavily distorted in the circled Region A. The increase in gain measurement is the result of signals reflected off the mounting surface. A lookup table for gain as a function of azimuth and off-boresight angle is created using Matlab as shown in Fig. 15 in the transmission model result section.

Path Losses

The amount of loss in GPS signal strength is dependent on both the distance of travel and the type of transmitting medium. As the signal is travelling through space, the signal could encounter three different mediums; vacuum, ionosphere region and troposphere region. Each of these three mediums has different transmission losses associated with them.

The transmission in a vacuum is primary related to the distance travelled (distance dependent path loss). With the knowledge of distance between the GPS satellite and the receiving antenna, pseudo range (***R***), vacuum loss (***VL***) can be calculated as:

$$\boldsymbol{VL} \text{ (dB)} = 20 \log_{10} \boldsymbol{R} \quad (5)$$

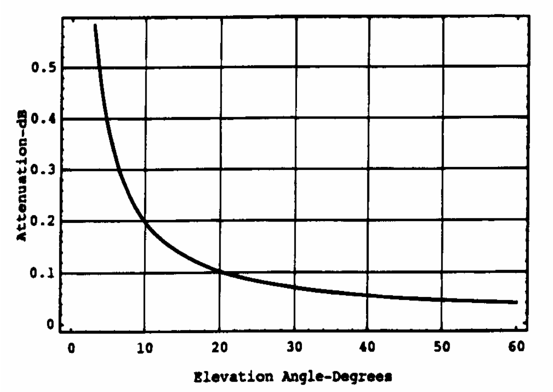

Fig. 7: Atmospheric attenuation

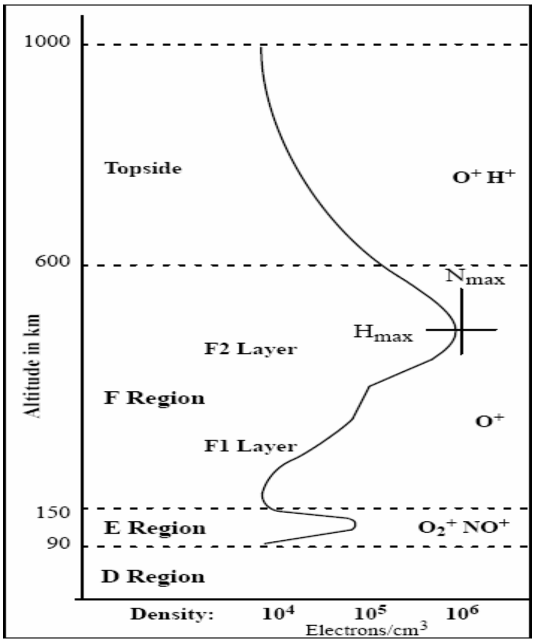

Fig 8: Various layers of the ionosphere and their electron content density

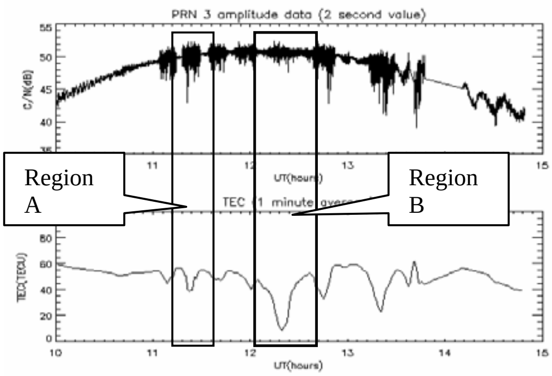

Fig 9: Carrier to Noise (C/No) Vs TEC Value

Atmospheric attenuation losses, including ionosphere and troposphere, in the GPS signal frequency range are primarily due to the presence of oxygen [16]. The typical signal attenuation is 0.035 dB at the zenith and increases by a factor of 10 at low elevation [16]. The model for atmospheric attenuation versus elevation in degrees is shown in Fig. 7. In addition, effects of water vapour, rain and nitrogen attenuation in this frequency are less than 0.5 dB at 5° elevation angle [16].

Since the orbit of FedSat is at an altitude of 800km, atmospheric attenuations are not considered here. However, if the attitude system is to be operating at low altitude, such as for aircrafts and UAVs, a model for atmospheric attenuation can be developed based on Fig. 7.

The ionosphere is classified into different layers in relation to its altitude and the electron content density as shown in Fig 8. One particular layer, F2, situated at approximately 500km Earth's surface is known to have most influence on the radio wave due to its highest concentration of electron [11].

Occasionally, the ionosphere can become highly turbulent and irregularities in the electron content can occur. These irregularities can cause amplitude and phase fluctuations in the GPS signal propagating through the ionospheric medium. The effects are called ionospheric scintillation and are mostly common near equator after sunset. The amplitude fading at GPS L1 frequency may exceed 20 dB and last for several hours [13]. Scintillation can also occur at high latitude during either day time or night.

The amplitude scintillation monitoring is accomplished with the S4 index, which is derived from detrended signal intensity [13]. In addition, the Total Electron Content (TEC) can be used to describe the amplitude scintillation, showing similar changes with the S4 index. However, as shown in Fig. 9, there is no clear relationship between TEC and the signal strength.

The upper plot of Region A in Fig. 9 shows a moderate variation in the signal strength. However, the variation of the TEC in the lower plot of Region A is minimal. In contrast, Region B shows a large variation in the TEC value with less variation in the signal strength.

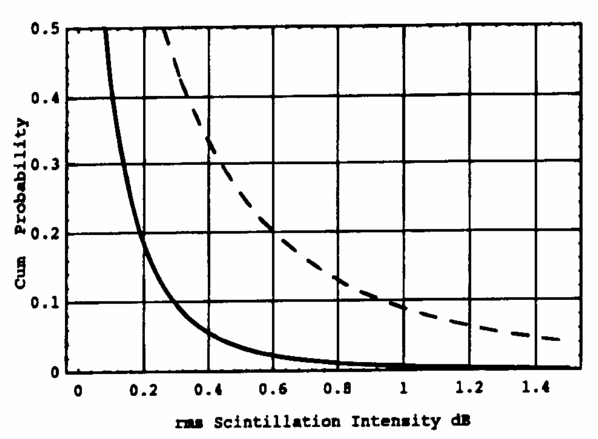

Fig 10: Cumulative probability for the scintillation.

Unfortunately, there is no way to predict the exact signal strength loss at a given time in the presence of scintillation. Fu [13] states that the variation in the signal strength caused by amplitude scintillation is not a linear function of the electron density due to the Fresnel filtering and saturation effect [13]. The variation in the signal strength can only be characterized by both the Nakagame-m distribution (solid curve in Fig. 10) for weak and moderate scintillation, and the Rayleigh distribution (dashed curve in Fig. 10) for strong scintillation [13].

The model for ionospheric scintillation can not be accomplished due to the nature of the error. However, several steps can be employed to reduce the impact of scintillation on the attitude system. Firstly, for a dual channel GPS receiver, TEC of ionosphere can be computed based on the pseudorange/phase measurements. By averaging fluctuations in the TEC value over a short time interval, a S4 value can be derived. For both S4 and TEC, if the value is above a pre-set threshold, it can be assumed that the ionospheric scintillation has occurred. Data collected with the presence of scintillations could either be averaged over a short time interval to reduce the signal strength fluctuation or otherwise discarded from the attitude algorithm. Secondly, a global scintillation model called WBMOD is available for single channel GPS receiver users. This model predicts regions and periods of increased scintillation, hence users are aware of data with presence of scintillation.

GPS Transmitting Gain

The GPS satellite transmission is designed to provide relatively uniform power on the surface of the Earth [15]. This is achieved by increasing the L-Band antenna gain from the nadir level to a maximum at an antenna theta angle of 10° (as shown in Fig 11) and then decreased slowly to about 16° [15]. The gain increased from nadir to edge-of earth (EOE) is approximately 2dB to compensate for the path loss due to the extra distance travelled.

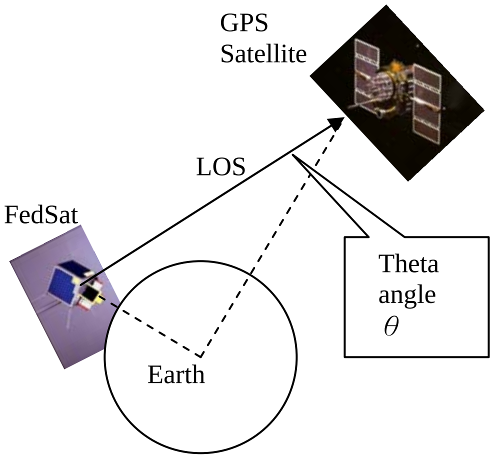

Fig. 11: Satellite geometry

The initial concept and design of the GPS antenna was performed by Rockwell International, Space Systems Division, for Block I, II and IIA satellites. The antenna array is comprised of 12 helical elements mounted on the Earth-facing satellite panel. Compared to the cylindrical group plane designed for Block I, the Block II/IIA changed to a conical design to provide better antenna efficiency. The Block IIR satellite was designed by Martin Marietta, Astro Space Division and changes in the ratio of inner and outer radii and the RF power feed ratio were made [17].

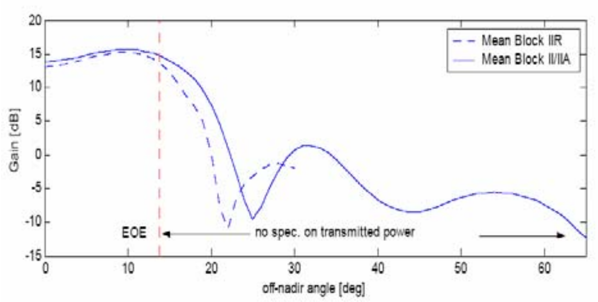

Fig. 12: Mean gain pattern for GPS Block II/IIA/IIR satellites.

The GPS antenna gain pattern has been previously modelled by [15, 17] and used in the single antenna attitude system developed by [3, 9]. Further experiment results in [12] shows that changes in Block IIR antenna design have an impact on the antenna gain pattern as shown in Fig. 12. The results were generated using data recorded by AMSAT OSCAR-40 satellite [12].

The results presented in [12] were created with an unknown receiving antenna gain pattern resulting in uncertainties in the model. Thus, the GPS transmitting gain model will be re-constructed using the FedSat data and the precisely measured receiving antenna gain pattern. As the GPS satellite antenna boresight is maintained within 0.5° of the nadir direction, the theta angle can be calculated based on the Line-of-Sight (LOS) vector between GPS satellite and FedSat, shown in Fig. 11.

Rearranging Equation 4, the GPS transmission gain and the GPS transmission power offset can be determined using the measured C/N_0, path losses and the receiving antenna gain.

$$G_t + c = C/N_0 - G_r + PL \qquad (6)$$

The procedures involved in determining the receiving antenna gain is as follows:

1. LOS vector is calculated based on the position of GPS satellite and FedSat in ECEF coordinates;
2. Transform the LOS vector to FedSat Local Level coordinate using FedSat position and velocity information;
3. LOS vector is transformed from Local Level coordinate to FedSat body coordinate using the attitude information available;
4. LOS vector in FedSat body coordinate is converted to azimuth (Rx_azimuth) and off-boresight angle (Rx_offBore) as defined in Fig 13; and
5. The receiving antenna gain is determined based on the azimuth angle (z, y plane) and off-boresight angle (x, z plane).

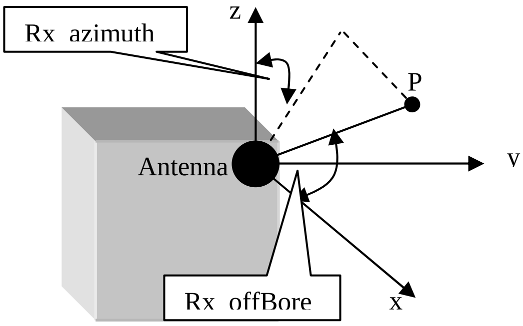

Fig 13: Definition of azimuth and off-boresight angle in FedSat body coordinates

In addition to the transmitting antenna gain, it is important to determine the transmitting satellite power differences. As mentioned in [15], the satellite power for the Block II satellites often exceeds the specified level because the satellite power is expected to degrade with time. Thus, newer satellites are designed to have a higher output power than their end-of-life specification, perhaps by as much as 6dB. Transmitting power differences for GPS satellites are determined as follows:

1. The receiving antenna gain value and path losses for each C/N_O measurements are calculated based on LOS vectors and satellite positions;
2. The transmitting antenna gain plus a constant (G_t + c) are calculated as shown in Equation 6;
3. Results from step 2 are separated with respect to the GPS PRN tracked;
4. Assume the mean transmitting gain pattern in Fig. 12 is correct and the constant value in Equation 6 is equal for data with same GPS PRN number. Results from one particular GPS PRN are subtracted by transmitting gain values determined based on the model in Fig 12;
5. Mean constant is determined by taking a mean value from step 4 results;
6. Steps 4 to 5 are repeated for all GPS PRN numbers; and
7. GPS satellite transmitting power differences are equal to the differences in the mean constants determined in step 5.

The transmitting antenna gain is then determined by subtracting results from step 2 by the mean constant determined. The resultant gain values are then grouped with respect to the type of GPS satellite, i.e. Block II/IIA and Block IIR satellites. Thus, with known GPS transmitting angle (Theta) and estimated transmitting antenna gain, models for GPS Block II/IIA and Block IIR antenna gain pattern can be derived with a best fit 4^{th} order polynomial function.

Environment for GPS Signal Transmission Model Development

Development of the transmission model is based on the FedSat data collected during 30/12/2002 to 28/02/03 period. Parameters used for model development are:

1. Carrier to noise ratio in dB.Hz.
2. FedSat attitude information.
3. Position and Velocity of the receiving antenna.
4. GPS satellite position calculated from ephemeris.

For 82.6% of the time period, numbers of GPS satellites tracked by FedSat GPS receiver were between 5 and 7 as shown in Table 1. For a small period of time (1.324%), GPS receiver was unable to compute the position of FedSat due to the number of satellites tracked being less than 4.

Table 1: Percentage of the number of satellites tracked

	Numbers	Percentage
Total Samples	62297	100%
> 4 Satellites Tracked	825	1.324%
4 Satellites Tracked	4563	7.345%
5 Satellites Tracked	13725	22.03%
6 Satellites Tracked	21730	34.88%
7 Satellites Tracked	15995	25.68%
8 Satellites Tracked	5459	8.763%

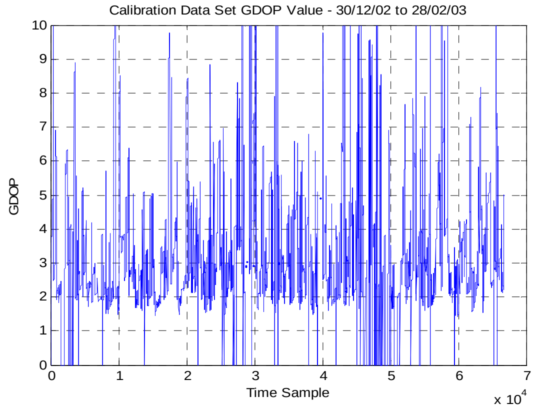

Fig. 14: GDOP for FedSat data – 30/12/02 to 28/02/03

Fig. 14 shows the computed geometry dilution of precision (GDOP) for the data collected. It shows that for approximately 90% of the data collected during this time period, GDOP values are less than 6. This is used to analyse the impact of the satellite geometry on the modelling performance.

GPS Signal Transmission Model Results and Discussion

The receiving antenna gain as a function of azimuth and off-boresight angle was created as a lookup table using Matlab as shown in Fig. 15. Since the antenna test performed only on four antenna slices, in between points were interpolated with Piecewise Cubic Hermite Interpolating Polynomial function (PCHIP). The lookup table was created for every 0.5° interval range from azimuth angle 0° to 360° and off-boresight angle0° to 130°.

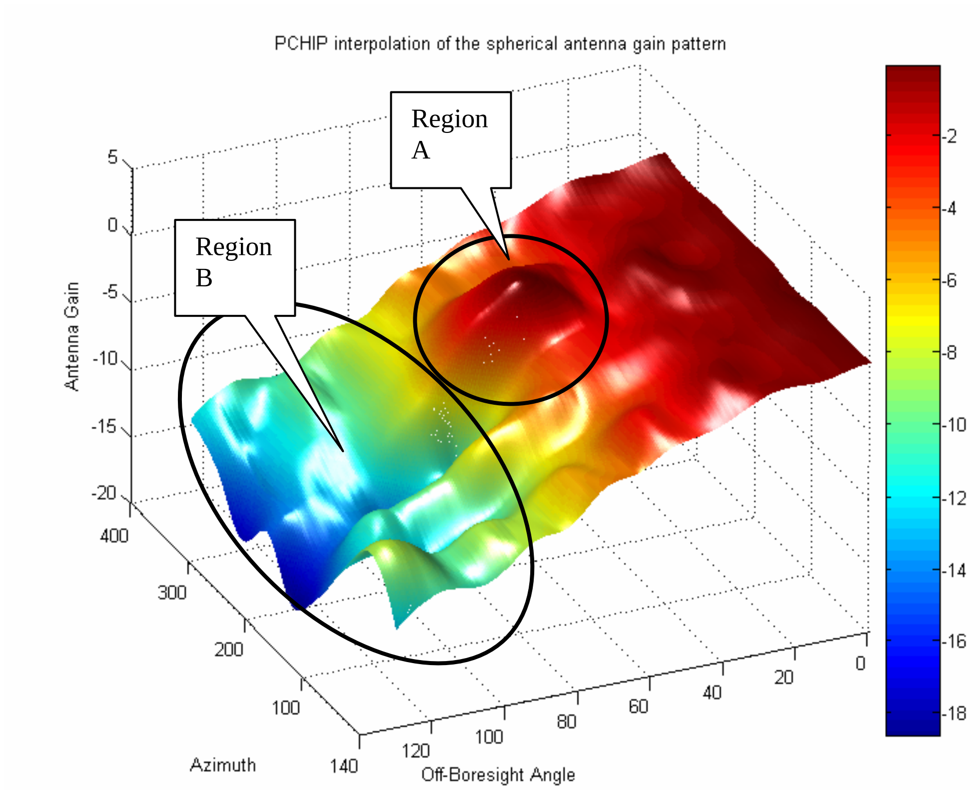

Fig. 15: PCHIP interpolation of the FedSat GPS sperical antenna gain pattern

As seen from Fig. 15, the antenna gain pattern is heavily distorted in Region A, located between an azimuth angle of 220° to 270° and off-boresight angle of 50° to 70°. This distortion corresponds to the distorted region highlighted in Fig. 6 and is caused by signal reflection. Region B in Fig 15 also shows a heavily distorted gain pattern. As this region is beyond 90° off-boresight angle, it means that some of the signals in this region were tracked by the GPS receiver by penetrating through the FedSat surface. As a result, large modelling errors are seen in this region.

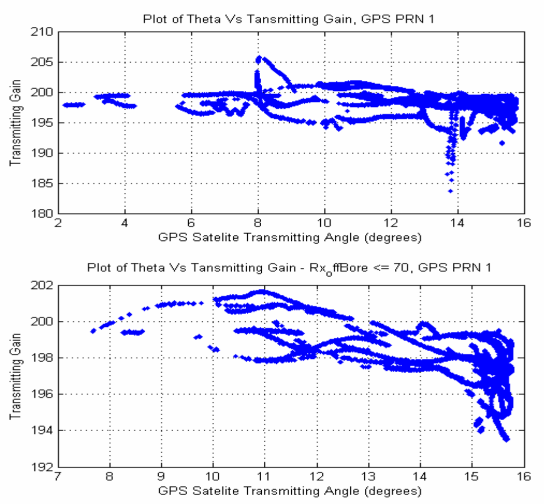

Fig. 16: Plot of Theta Vs transmitting gain.

Fig. 16 plots GPS transmitting angle (θ) verses transmitting gain plus constant (G_t + c) from GPS PRN 1 data measurements. The upper plot of Fig. 16 shows considerable differences between the published mean transmitting gain patterns in Fig. 12. Data seems randomly placed and no gain pattern could be extracted from the plot. Further investigation indicates that this was caused by the modelling error in the receiving antenna gain for receiving off-boresight angle larger than 70°. The lower plot of Fig. 16 shows a much improved result by eliminating data with receiving off-boresight angle larger than 70°.

Fig. 17 shows the distribution of the signal tracked by the GPS receiver. Data was divided into eight partitions, four quadrants with an off-boresight angle less than 90° and four quadrants with an off-boresight angle larger than 90°. The figure shows that approximately 16% of data was received with an off-boresight angle larger than 90°. Due to the modelling difficulties discussed earlier, these data were unusable. In addition, it was observed that approximately 78% of the data was tracked in the two upper quadrants. Due the negative velocity pointing antenna of FedSat, some parts of view in the two lower quadrants were blocked by the Earth and hence fewer signals could be tracked.

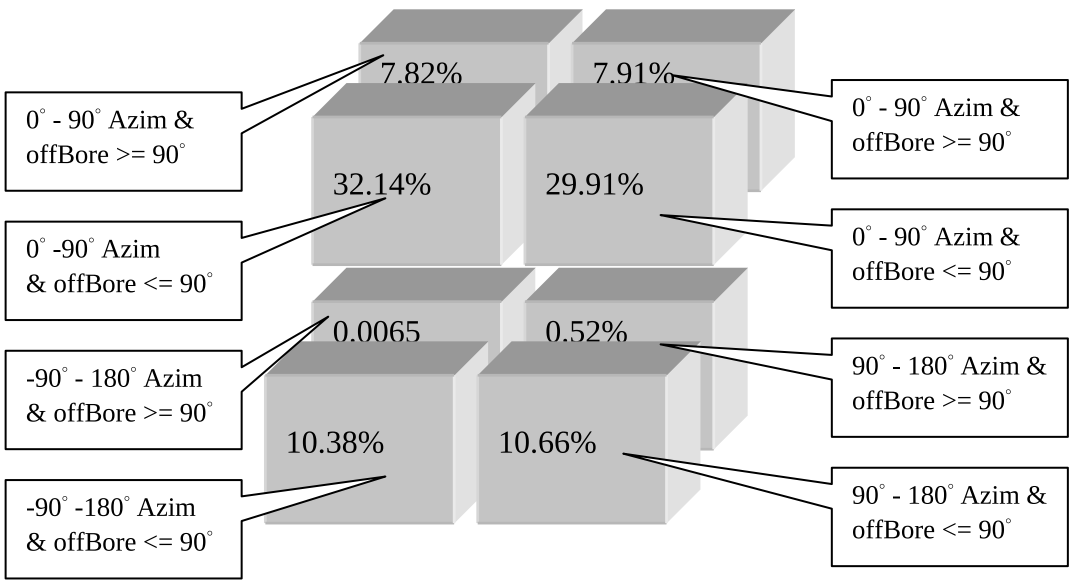

Fig 17: Distribution of signal tracked

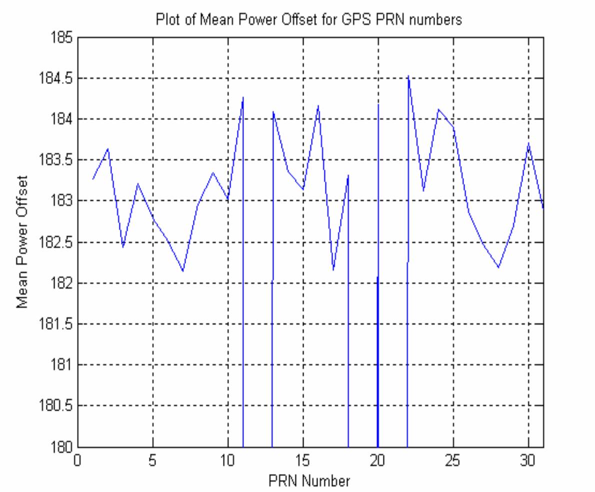

Fig. 18: Mean power offset for GPS PRN numbers

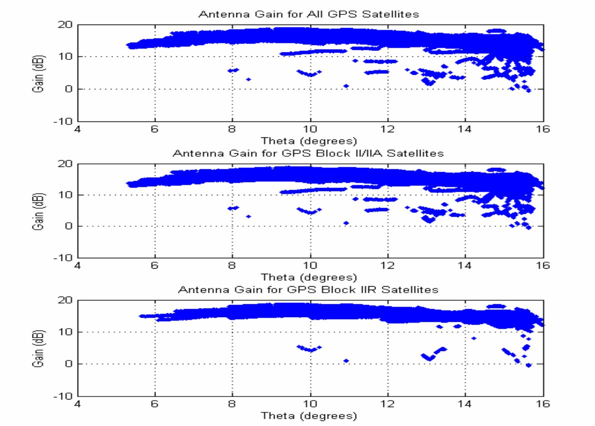

Fig. 19: The measured GPS antenna gain pattern for all, Block II/IIA and Block IIR satellites.

Fig. 18 is the mean power offset for all GPS PRN numbers. From the plot, PRN 22 has the highest power offset of 184.5 dB and is approximately 2.3 dB higher than the lower offset recorded by PRN 7.

During the period when data was taken, the GPS constellation consisted of 28 satellites, PRN 1 to 31 with exceptions of PRN 12, 19 and 21. PRN 2, 15 and 17 were the oldest Block II satellites, 16 of the Block IIA satellites (24, 25, 26, 27, 1, 29, 31, 7, 9, 5, 4, 6, 3, 10, 30 and 8) and 6 of the Block IIR satellite (13, 11, 20, 28, 14 and 18).

Models for GPS antenna gain pattern were constructed using signals received from receiving off-boresight less than 70°. By imposing this mask angle the receiving antenna removed the noisiest data as shown in figure 16. Models for all GPS satellites, Block II/IIA satellites and Block IIR satellites are shown in Fig 19. The shape and the amplitude of the measured GPS antenna gain pattern agree very well with the published patterns shown in Fig. 12.

As shown in Fig. 19, there are no signals being tracked by the GPS receiver at a transmitting angle (θ) less than 5° with a receiving off-boresight angle less than 70°. Due to the negative velocity pointing antenna configuration, any signal transmitted at nadir direction can only be tracked at a receiving off-boresight angle of 90°. As a consequence, these signals were excluded from the modelling process due to the 70° receiving off-boresight angle limit. The polynomial functions for GPS antenna models are shown in Equation 7 and 8.

$$\text{Block II/IIA Gain} = -0.0001947069x^4 + 0.0043173524x^3 - 0.044510881x^2 + 0.39060281244x + 13.720593 \qquad (7)$$

$$\text{Bock IIR Gain} = -0.0002417227x^4 + 0.0040297533x^3 - 0.03886506x^2 \; 0.466906505x + 13.0435705 \qquad (8)$$

Compared to models presented in [12], differences in the GPS antenna gain pattern for Block II/IIA satellites and Block IIR satellites are not clearly shown in Fig. 19. Evidence suggested differences in the result achieved were caused by ionosphere path losses. In previous models, two of the largest errors were the unknown receiving antenna gain and the spin-induced oscillations [12]. However, these errors were not of concern in this experiment. Thus, the only other cause to the modelling difference was due to the receiving antenna configuration. With a negative velocity pointing antenna, signals were more likely to travel through longer distances in the ionosphere region compared to signals tracked by an upward pointing antenna. The extra travelling distances in the ionosphere region increased the chance of presence of scintillation and caused uncertainties in the transmitting model created. Other modelling errors were due to factors such as GSP satellite attitude errors, mechanical antenna alignment errors and transmitter power output variations due to temperature variations, voltage variations and power amplifier variation.

Single Antenna GPS-Based Attitude Estimation

Single antenna attitude systems developed by previous researchers have been demonstrated successfully in several different satellite missions [1-8]. However, none of these systems has been tested in a platform similar to FedSat where the antenna boresight is in the negative velocity direction. In addition, none of these systems has a precisely measured receiving antenna gain pattern as the case of FedSat. Thus, a new single antenna attitude system is proposed for FedSat utilizing the advantage of spherical antenna gain and the transmitting model developed to maximize the performance.

With knowledge of positions of tracked GPS satellites and the GPS receiver, C/NO can be estimated based on Equation 4 with an estimated receiving antenna boresight. By rotating the estimated antenna boresight through the space; an optimal estimate of the attitude can be derived with least square search algorithm. Procedures are summarized as follows:

Least square error search with a spherical antenna gain pattern

1. At an instance, calculate the transmitting antenna gain, path losses and the transmitting power offsets for all tracked signals;
2. Form an estimation of the receiving antenna boresight vector;
3. The azimuth and the off-boresight angle of LOS vectors in local level coordinate are determined with respect to the estimated antenna boresight;
4. Receiving antenna gain is determined from lookup table based on the azimuth and the off-boresight angle;
5. Estimated signal strengths are then determined based on the transmitting antenna gain, path losses, the transmitting power offset and the receiving antenna gain;
6. Error associated with the estimated receiving antenna boresight is calculated as square root of the error between the measured and the estimated signal strengths;
7. Rotate the receiving antenna boresight estimation in space and repeat steps 2 to 6; and
8. Optimal estimation of the antenna boresight vector is determined as the least square error.

Two other single antenna attitude systems were evaluated and compared with results achieved by the least square error approach. Firstly, the approach was tested against the method developed by NASA Jet Propulsion Laboratory (JPL), in June 1998 [1]. The estimation of the antenna boresight direction was derived by a weighted average of the computed LOS vectors for the satellite antenna to each GPS satellite being tracked. The performance of the system was strongly dependent on the geometry of the tracked satellite [1]. The second approach involved utilization of the relationship between the strength of received signals and the direction-dependent antenna gain pattern [3]. With a calibrated mapping function, this SNR mapping approach estimated the antenna boresight vector using Newton-Raphson search. Methodology summaries for both systems are as follows:

JPL Approach [1] - (Sum of SNR Weighted Line-of-Sight Vectors)
1. Calculate the line-of-sight vector to each tracked satellite;
2. Normalize the line-of-sight vectors to a unit vector;
3. Multiply the unit vectors by the measured SNR for all observed satellites; and
4. Add SNR weighted vectors to obtain a vector sum.

SNR Mapping Approach [3]
1. Establish a SNR to elevation angle mapping function as the mean value of each SNR bin; the standard deviation within the bin provides the weight;
2. At any instance, look up estimated elevation for each observed SNR adjusted for the GPS antenna pattern and space losses;
3. Form the initial estimate of antenna boresight based upon elevation angles and LOS vectors in the local level frame; and
4. Use a Newton-Raphson search to find the optimal estimate of the boresight vector axis with respect to the reference frame.

Comparison of FedSat Attitude Results

The result of FedSat attitude estimation is based on the data collected between 30/12/2002 and 28/02/03. Based on the JPL approach [1], errors in the theta and phi estimation are shown in Fig 21. The theta and the phi angle in local level coordinate are defined as in Fig 20.

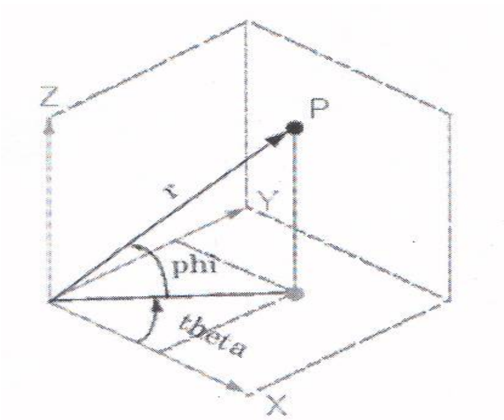

Fig. 20: Definition of theta and phi angle for attitude system estimation

The upper plot of Fig. 21 shows errors in the theta estimation are approximately 30° rms, with mean value close to 0°. Whereas the lower plot of Fig. 21 shows errors in the phi estimation are approximately 13° with mean value of 30°. These results are not too surprising; the large offset in the mean error of phi estimation was due to the distribution signal tracked as illustrated in Fig. 17. As approximately 78% of the data is situated in the upper two quadrants, results of LOS vectors summation will more likely be offset by a positive theta value.

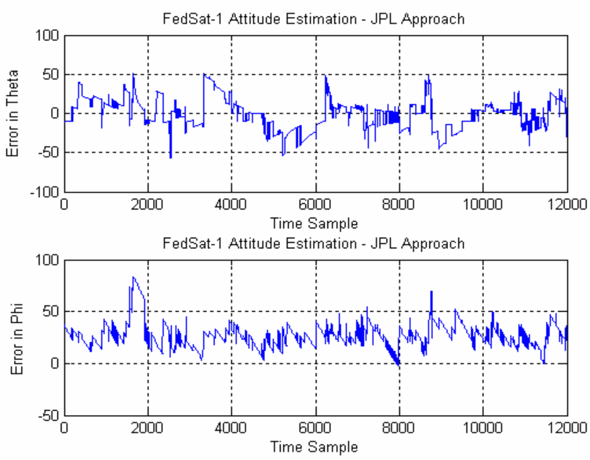

Fig. 21: FedSat attitude estimation using JPL approach

Table 2 shows the performance of the JPL approach with respect to the number of GPS satellites being tracked by the FedSat GPS receiver. Results have shown that there are no improvements on the accuracy of the estimated attitude as the number of tracked satellites increases. This indicated that the performance of the system was not dependent of the number of tracked satellites, but more dependent on evenness of the distribution of tracked signal in space. The unevenly distributed signal was due to the FedSat antenna configuration.

Table 2: Performance of the JPL approach with respect to the number of GPS satellites being tracked by the receiver.

	Data Samples	Mean Theta	STD Theta	Mean Phi	STD Phi
All time sample	12000	-0.2935	19.3197	26.8482	11.9593
satNumber = 4	813	-1.5948	22.0034	30.0239	12.8247
satNumber = 5	2022	-4.2233	25.7547	35.8275	16.8606
satNumber = 6	3623	-0.7847	20.2314	27.6489	8.3288
satNumber = 7	3745	0.7930	15.1603	23.4960	8.6308

Attitude estimation results using the SNR mapping approach are shown in Fig. 22, 23 and Table 3. The upper plot of Fig. 22 shows the mapping function created with data collected by FedSat from 30/12/02 to 28/02/03. The x-axis represents the receiving antenna off-boresight angle and y-axis is the received C/N_O measurement.

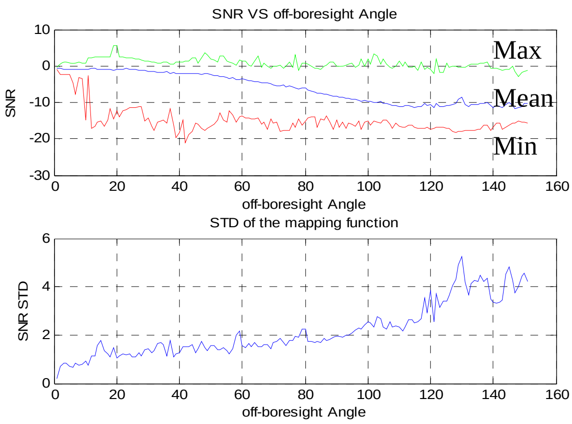

Fig. 22: SNR to elevation angle calibration results.

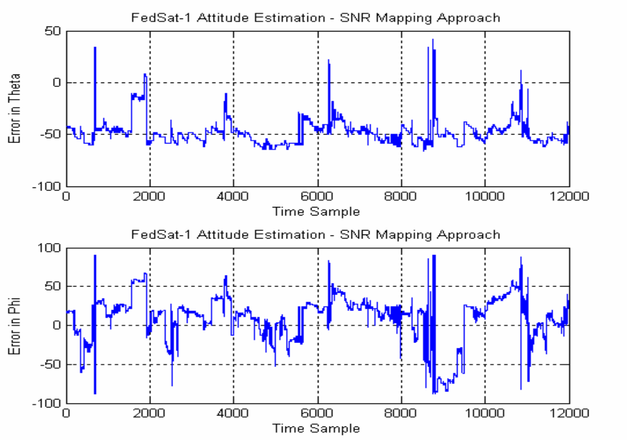

Fig. 23: FedSat attitude estimation using SNR mapping approach

Fig. 22 clearly shows the variation in observed C/N_O for receiving antenna off-boresight angles with respect to reference frame for all measurements made, at all azimuths. The green, blue and red line in the plot represent the maximum, mean and minimum observed C/N_O respectively. The lower plot of Fig. 22 shows the RMS error of the mean observed C/NO. Errors in the mapping function were gradually increased as the receiving antenna off-boresight angle increased. However, as the receiving off-boresight angles went beyond 90°, errors in the modelling raised sharply, reflecting the large azimuthal error shown in Region B, Fig. 15.

Fig. 23 shows the error in the theta angle (upper plot) and the phi angle (lower plot) estimation using SNR mapping approach. This figure shows error in the theta angle is fluctuating around -50° mean with several large outliers. On the other hand, the mean error in the phi angles is around 10°.

Table 3 shows the performance of the SNR mapping approach with respect to the number of GPS satellites being tracked by the GPS receiver. Mean error in the theta and the phi angle shows no improvements as the number of tracked satellites increases. However, the standard deviation (STD) errors in the theta and the phi angle are improved. The STD error in the theta angle improves from 14° to 8.3° when the numbers of GPS satellites being track by the GPS receiver are 4 and 7 respectively. On the other hand, the STD error in phi and improves from 38.6° to 22.7° when 4 and 7 GPS satellites being tracked respectively.

Table 3: Performance of the SNR mapping approach with respect to the number of GPS satellites being tracked by the receiver.

	Data Samples	Mean Theta	STD Theta	Mean Phi	STD Phi
All time sample	12000	-48.2397	11.6168	9.2277	29.7058
satNumber = 4	813	-45.1169	14.0675	-4.2706	38.6596
satNumber = 5	2022	-41.1711	15.8284	8.3393	40.3494
satNumber = 6	3623	-49.6511	9.9408	5.3265	27.4542
satNumber = 7	3745	-50.2211	8.2887	13.0195	22.7311

The new proposed attitude system for FedSat is based on the spherical antenna gain pattern and the GPS signal transmitting model developed in earlier sections. Attitude estimation results are shown in Fig. 24. In addition, the performance of the attitude system is analysed with respect to the number of GPS satellites being tracked and the GDOP value as shown in Table 4 and Table 5 respectively.

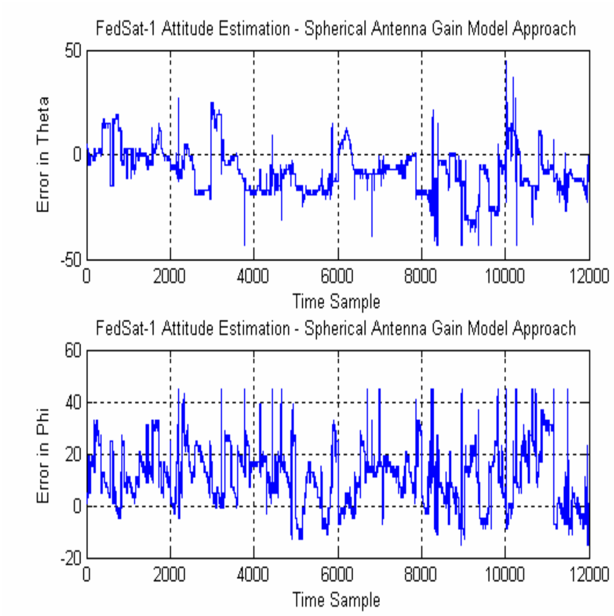

Fig. 24: *FedSat attitude estimation using spherical antenna gain pattern approach*

Compared to results achieved by the JPL [1] and the SNR mapping [3] approach, the spherical antenna gain pattern approach is capable of producing a better overall performance. As shown in Table 4, the mean error in theta and phi angle are relatively low, without the large theta angle offset seen in the SNR mapping approach and the large phi angle offset seen in the JPL approach. On the other hand, the STD error in theta and phi angle shows a similar pattern. Compared to the JPL approach, the STD error in phi angle degrades slightly, however, a dramatic improvement is seen in the STD error in theta angle. In the other case, this pattern is reversed when comparing to the SNR mapping approach.

Table 4: Performance of the spherical antenna gain pattern approach with respect to the number of GPS satellites being tracked by the receiver.

	Data Samples	Mean Theta	STD Theta	Mean Phi	STD Phi
All time sample	12000	-3.8308	13.4588	5.6587	14.1473
satNumber = 4	813	0.7274	16.8853	1.5032	18.7572
satNumber = 5	2022	-2.8919	14.1623	5.9246	14.5599
satNumber = 6	3623	-7.2803	11.1584	7.2330	10.5257
satNumber = 7	3745	-7.3402	10.6339	7.2382	9.7965

In addition, the performance of the spherical antenna gain pattern approach is analyzed with respect to the GDOP value. As shown in Table 5, the performance of the new attitude system, in both mean and STD error of the theta and the phi angle, improves as the GDOP value decreases.

Table 5: Performance of the spherical antenna gain pattern approach with respect to the geometry dilution of precision.

	Mean Theta	STD Theta	Mean Phi	STD Phi	Data Sample
GDOP < 3	8.2531	10.4336	9.2260	11.5558	1971
GDOP < 4	-7.3729	11.2051	8.3716	12.0645	3778
5 > GDOP > 3	-6.3493	11.4006	7.7774	12.5237	3113
10 > GDOP > 5	-5.2619	12.9227	8.7168	15.4627	2343
GDOP > 10	-1.4284	15.3572	10.5304	17.6205	795

The performance of the JPL and the SNR mapping approach achieved in this experiment has been degraded when compared to results achieved previously [1, 3, 9]. This is caused by the FedSat antenna configuration in the negative velocity direction. On the other hand, the new attitude system based on spherical antenna gain pattern is less subject to effects of the negative velocity pointing antenna configuration. Analysed results, as shown in Table 4 and Table 5, indicates that the performance of the system is dependent on the number of GPS satellites being tracked by the receiver. In addition, the performance is also affected by the geometry of the GPS satellites tracked.

Recommendations

For satellite missions intended to employ a GPS-based single antenna attitude system, it is recommended to use an upward pointing antenna to improve the number of GPS in view. This will avoid unbalanced distribution of the data as shown in Fig 17. In addition, the distance that GPS signals travel in the ionosphere region will be decreased. This will reduce the presence of ionospheric scintillation in the data set and hence increase the performance of the GPS antenna gain model. As a result, the performance of the attitude system will be improved.

There are still some uncertainties in the model of GPS transmitting gain pattern. As discussed in [13], the transmitting antenna gain is affected by mechanical antenna alignment errors, the noon-turn manoeuvre and the effect of the position of the solar array. At various GPS satellite transmitting theta angle (angle between nadir vector and LOS vector), the transmitting gain value can vary as much as 2 dB due to these factors. Thus, it is recommended to obtain an accurate spherical gain pattern of the GPS Block II/IIA/IIR satellite. In addition, as the modernized GPS will be launched in mid-2005, an anechoic chamber test on the modernized GPS satellite antenna will be beneficial.

Since the number of tracked satellites is one of the factors that affect the performance of the attitude system, it is also recommended to investigate an attitude system that incorporates both the GNSS satellites and the future Galileo satellite. This can be analysed in a simulation environment.

Conclusions

Based on the data collected by FedSat, a GPS signal transmission model was developed. The model consisted of parameters such as the FedSat receiving antenna gain pattern, path losses, the GPS transmitting antenna gain and the GPS transmitting power differences. The FedSat receiving antenna gain used was a close representation of the performance in a real application and hence eliminated uncertainties existing in earlier models developed. Path losses in vacuum, ionosphere and troposphere were investigated. However signal strength losses caused by amplitude scintillation were excluded from the model due to the nature of the error. The transmitting antenna gain pattern for GPS Block II/IIA and Block IIR satellites were developed. However, slight differences in the GPS antenna gain pattern were not clearly shown in this experiment, evidence suggested that this was caused by the FedSat antenna configuration and ionosphere path modelling. Finally, the GPS satellite transmission power differences were shown to vary by approximately 2 dB.

Initial analysis indicate that the negative velocity pointing antenna of FedSat had a severe impact on the performance of single antenna attitude systems based on the JPL and the SNR mapping approach [1, 3]. Both the mean and the STD error of these two attitude systems were degraded when compared to results achieved in an upward pointing antenna configuration. On the other hand, the new attitude system based on a spherical antenna gain pattern achieved better attitude estimation results compared to the JPL and the SNR mapping approach. Based on a precisely measured receiving antenna gain pattern and the GPS signal transmission model developed, the new attitude system can achieved an accuracy of 14° rms. Further analysis showed that the performance of the system was dependent of the number and the geometry of the GPS satellite being tracked by the receiver.

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