Inference of Heartbeats in BCG and ECG signals using Variable Rate Particle Filtering

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1 Model

Here we start with a nice simple model. A sequence of changepoints $\{\tau_k\}$ represents the start of each beat. Associated parameter sequences $\{a_k\}$ and $\{p_k\}$ represent the amplitude and pulse (mean beat period) for the corresponding beat. Finally, $\{\mathbf{w}_k\}$ is a vector representing the waveform of the pulse. This will consist of about 30 discrete samples (which, at 30Hz sampling rate, is about a second long).

The continuous time signal can be written as,

$$s(t) = a_{K(t)} b(t, \tau_{K_t})^T \mathbf{w}_{K(t)}, \tag{1}$$

where $b(t, \tau_{K_t})^T$ is an interpolation matrix which reconstructs a continuous time waveform from the discrete samples. For example, if we used sinc-interpolation,

$$b(t, \tau_{K_t})_n = \mathrm{sinc}\left(\frac{t - nT_s - \tau_{K(t)}}{T_s}\right), \tag{2}$$

where $T_s = \frac{1}{f_s}$ is the sampling period.

The waveform $\mathbf{w}_k$ is expected to change only slowly over time. We use a Gaussian transition density with a tight covariance matrix,

$$p(\mathbf{w}_k|\mathbf{w}_{k-1}) = \mathcal{N}(\mathbf{w}_k|\mathbf{w}_{k-1}, Q_w). \tag{3}$$

In addition, we assume that:

- τ_k has some distribution (probably gamma or inverse-gamma) with mean $\tau_{k-1} + p_{k-1}$.
- p_k has some distribution (maybe gamma or inverse gamma) with mean p_{k-1}.
- A_k has some distribution (maybe Rayleigh) with mean A_{k-1}.

Finally, we assume a Gaussian observation model,

$$y_n = \mathcal{N}(y_n|s(t_n), \sigma_y^2). \tag{4}$$

2 Inference Algorithm

Within this model we can do inference using a variable rate particle filter. In addition, because of the linear-Gaussian model assumptions, we can Rao-Blackwellise the waveform variable, $\mathbf{w}_k$, which will give us a huge dimensionality reduction. Hooray! The resulting algorithm will be a modification of that of [Morelande and Gordon, 2009].

We want our particle filter to estimate the changepoint times and also the parameters,

$$\mathbf{u}_k = \begin{bmatrix} p_k \\ a_k \end{bmatrix}. \tag{5}$$

As usual, we bundle all these into a single variable,

$$\theta_n^- = \{\tau_j, u_j \forall j : 0 \leq \tau_j < t_n\}. \tag{6}$$

So the target distribution is,

$$p(\theta_n^- | y_{1:n}). \tag{7}$$

A standard variable rate particle filter can now be applied. The likelihood is a little bit tricky,

$$\begin{aligned} p(y_n|\theta_n^-, y_{1:n-1}) &= \int p(y_n|\theta_n^-, \mathbf{w}_{K_n}) p(\mathbf{w}_{K_n}|\theta_n^-, y_{1:n-1}) d\mathbf{w}_{K_n} \\ &= \int p(y_n|s(t_n)) p(\mathbf{w}_{K_n}|\theta_n^-, y_{1:n-1}) d\mathbf{w}_{K_n} \\ &= \int \mathcal{N}(y_n|a_{K_n} b(t, \tau_{K_n})^T \mathbf{w}_{K_n}, \sigma_y^2) \mathcal{N}(\mathbf{w}_{K_n}|\mathbf{m}_{n-1}, \mathbf{P}_{n-1}). \end{aligned} \tag{8}$$

A Kalman filter is maintained for each particle to estimate the density over $\mathbf{w}_{K_n}$. If no changepoint occurs between t_{n-1} and t_n, then,

$$\begin{aligned} p(\mathbf{w}_{K_n}|\theta_n^-, y_{1:n}) &\propto p(y_n|\mathbf{w}_{K_n}, \theta_n^-) p(\mathbf{w}_{K_n}|\theta_n^-, y_{1:n-1}) \\ &= \mathcal{N}(y_n|a_{K_n} b(t, \tau_{K_n})^T \mathbf{w}_{K_n}, \sigma_y^2) \mathcal{N}(\mathbf{w}_{K_n}|\mathbf{m}_{n-1}, \mathbf{P}_{n-1}). \end{aligned} \tag{9}$$

Alternatively, if a changepoint does occur between t_{n-1} and t_n, then,

$$\begin{aligned} p(\mathbf{w}_{K_n}|\theta_n^-, y_{1:n}) &= p(y_n|\mathbf{w}_{K_n}, \theta_n^-) \int p(\mathbf{w}_{K_n}|\mathbf{w}_{K_n-1}) p(\mathbf{w}_{K_n-1}|\theta_n^-, y_{1:n-1}) d\mathbf{w}_{K_n-1} \\ &= \mathcal{N}(y_n|a_{K_n} b(t, \tau_{K_n})^T \mathbf{w}_{K_n}, \sigma_y^2) \int \mathcal{N}(\mathbf{w}_{K_n}|\mathbf{w}_{K_n-1}, Q_w) \mathcal{N}(\mathbf{w}_{K_n-1}|\mathbf{m}_{n-1}, \mathbf{P}_{n-1}) \end{aligned} \tag{10}$$

So everything's Gaussian, and can be calculated in closed form. Jolly good.