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A Method Based on Rotatory Array Configuration to Solve DOA Estimation Ambiguities in Sparse Linear Array

Multiple Joint MUSIC DOA Estimation

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Content

1. A Review of Last Presentation;
2. Encountered Problems;
3. An Introduction to MJ-MUSIC;
4. Numerical Simulation Results and Discussion;

1. A Review of Last Presentation

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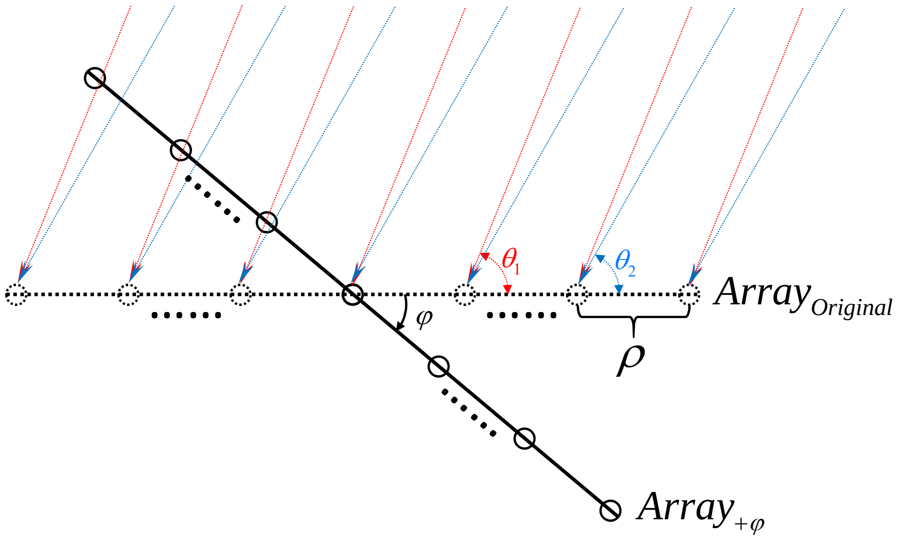

Fig. 1-1 Illustration of DOA Estimation of Rotatory Array

As shown in the left illustration, assume that two signals arriving in the original array, annotated as $Array_{Original}$, at incident angle θ_1 and θ_2 respectively. Additionally, because of disobeying the "*Spatial Sampling Theorem*", there should exist a set of false peaks, $\{\tilde{\theta}_i\}$, in the DOA estimation. When this array rotates clockwise around the center with respect to the original one by angle φ and incident signal maintains the same, the incident angle to this new array $Array_{+\varphi}$ apparently should be $\theta_1 + \varphi$ and $\theta_2 + \varphi$. What if $Array_{+\varphi}$ had a set of false peaks $\{\tilde{\theta}_i + \tilde{\varphi}_i\}$ and $\tilde{\varphi}_i \neq \varphi$, we may could cancel the false peaks by multiplying the DOA estimations of each array.

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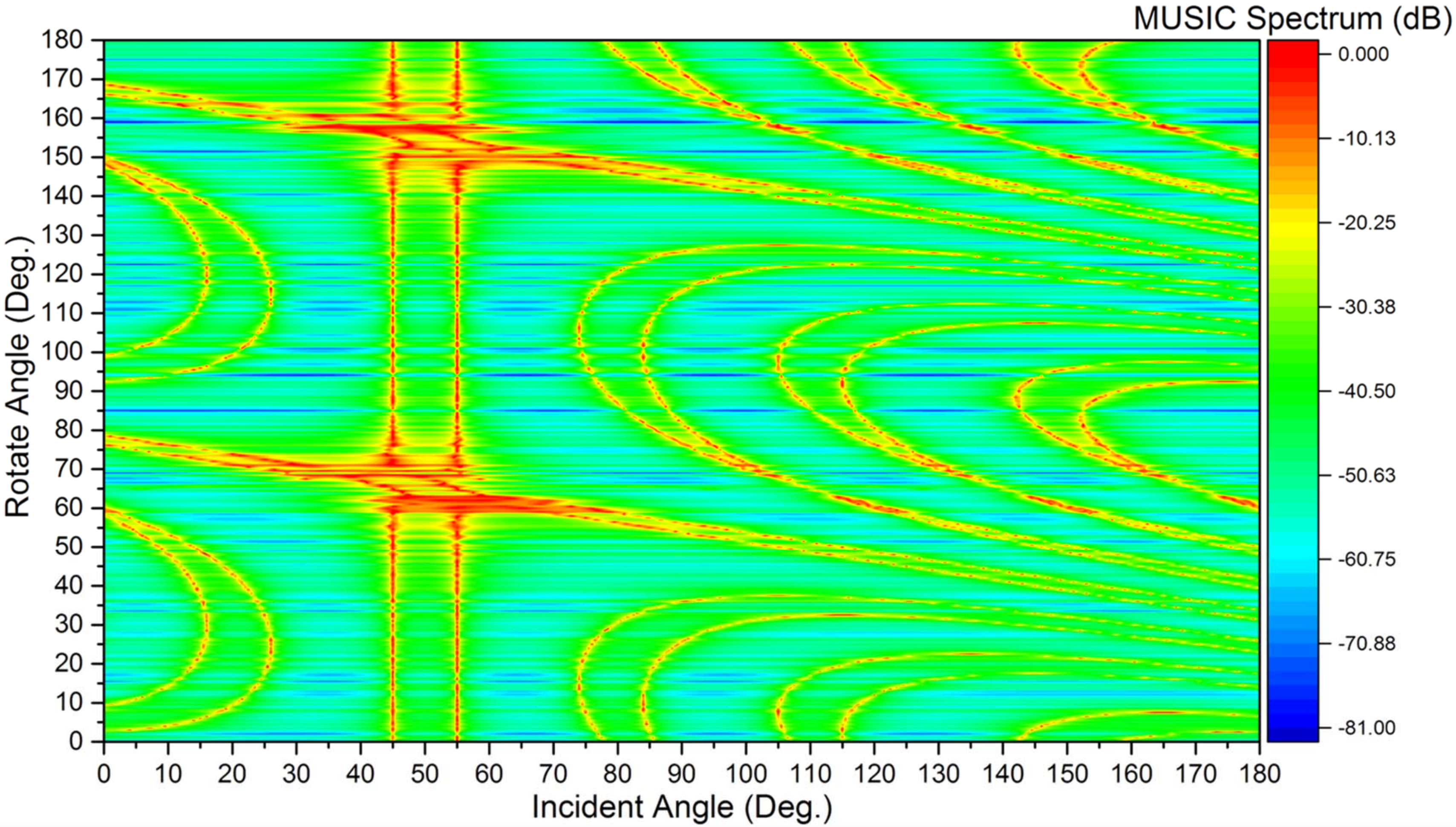

Fig. 1-2 Incident Angle = [45, 55], SNR=10, Element Number=3

1. A Review of Last Presentation

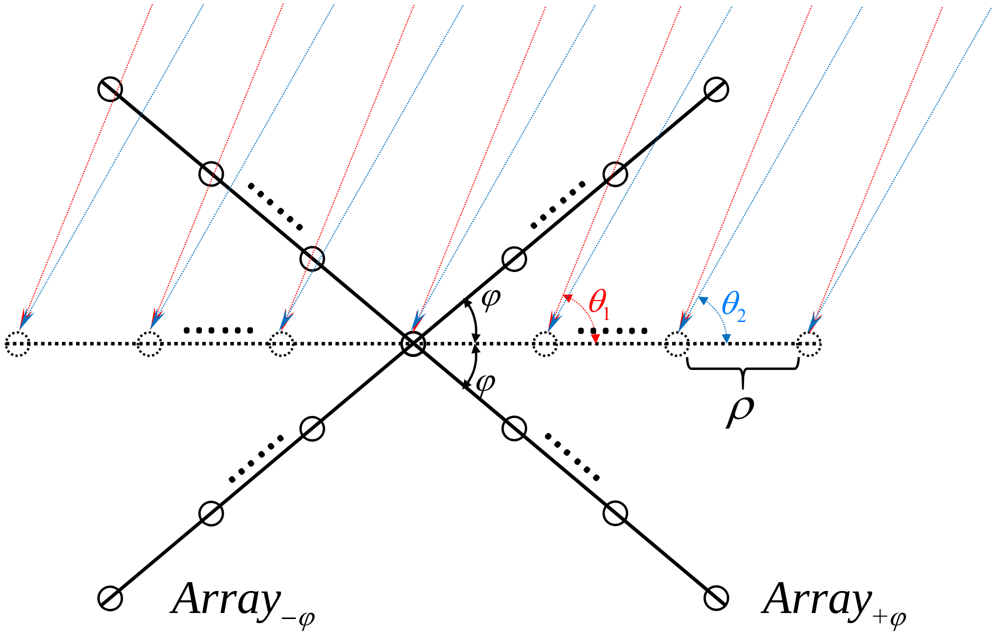

Fig. 1-3 Array Configuration

As shown in the left, there are two symmetrical array, $Array_{+\varphi}$ and $Array_{-\varphi}$, clockwise and anti-clockwise rotated with angle φ respect to the horizontal axis.

Known that $\cos(\theta+\varphi)+\cos(\theta-\varphi)=2\cos\theta\sin\varphi$

Also $\boldsymbol{a}(\theta)=[1 \quad e^{-j2\pi\rho\cos\theta} \quad \dots \quad e^{-j2\pi(N-1)\rho\cos\theta}]^T$,

And $\boldsymbol{b}(\theta,\varphi)=[1 \quad e^{-j2\pi\rho\, 2\cos\varphi\cos\theta} \quad \dots \quad e^{-j2\pi(N-1)\rho 2\cos\varphi\cos\theta}]^T$,

Thus we have: $\boldsymbol{a}(\theta+\varphi)=diag[\boldsymbol{a}^*(\theta-\varphi)]\boldsymbol{b}(\theta,\varphi)$

When the interval between each element to wavelength ratio ρ is larger than 0.5, assuming false peak exist at angle $\tilde{\theta}+\varphi$ to $Array_{+\varphi}$, which means the steering vector $\boldsymbol{a}(\tilde{\theta}+\varphi)$ is a linear combination of $\boldsymbol{a}(\theta_1+\varphi)$, $\boldsymbol{a}(\theta_2+\varphi)$, …, $\boldsymbol{a}(\theta_M+\varphi)$ where $M<N$, $\tilde{\theta}$, θ_1,…, $\theta_M \in [0,180]$.

SEEK PROVEMENT:

$\boldsymbol{a}(\tilde{\theta}-\varphi)$ is not a linear combination of $\boldsymbol{a}(\theta_1-\varphi)$, $\boldsymbol{a}(\theta_2-\varphi)$, …, $\boldsymbol{a}(\theta_M-\varphi)$

1. A Review of Last Presentation

证明：

反证法：

假设 $\mathbf{a}(\tilde{\theta}-\varphi)$ 是 $\mathbf{a}(\theta_1-\varphi),\mathbf{a}(\theta_2-\varphi),\ldots,\mathbf{a}(\theta_M-\varphi)$ 的线性组合。

则

$$\mathbf{a}(\tilde{\theta}+\varphi)=\sum_{i=1}^{M}k_i\mathbf{a}(\theta_i+\varphi)=\sum_{i=1}^{M}k_i diag\left[\mathbf{a}^*(\theta_i-\varphi)\right]\mathbf{b}(\theta_i,\varphi)=diag\left[\mathbf{a}^*(\tilde{\theta}-\varphi)\right]\mathbf{b}(\tilde{\theta},\varphi)$$

又因为 $\mathbf{a}(\tilde{\theta}-\varphi)=\sum_{r=1}^{M}c_r\mathbf{a}(\theta_r-\varphi)$，可得 $diag\left[\mathbf{a}^*(\tilde{\theta}-\varphi)\right]=\sum_{r=1}^{M}c_r^*diag\left[\mathbf{a}^*(\theta_r-\varphi)\right]$，

因此有 $\sum_{i=1}^{M}k_i diag\left[\mathbf{a}^*(\theta_i-\varphi)\right]\mathbf{b}(\theta_i,\varphi)=\sum_{r=1}^{M}c_r^*diag\left[\mathbf{a}^*(\theta_r-\varphi)\right]\mathbf{b}(\tilde{\theta},\varphi)$

展开有：

$$\begin{bmatrix} \sum_{i=1}^{M}k_i \\ \sum_{i=1}^{M}k_i e^{j2\pi\rho\cos(\theta_i-\varphi)}e^{-j2\pi\rho 2\cos\theta_i\cos\varphi} \\ \vdots \\ \sum_{i=1}^{M}k_i e^{j2\pi(N-1)\rho\cos(\theta_i-\varphi)}e^{-j2\pi(N-1)\rho 2\cos\theta_i\cos\varphi} \end{bmatrix}=\begin{bmatrix} \sum_{r=1}^{M}c_r^* \\ e^{-j2\pi\rho 2\cos\tilde{\theta}\cos\varphi}\sum_{r=1}^{M}c_r^*e^{j2\pi\rho\cos(\theta_i-\varphi)} \\ \vdots \\ e^{-j2\pi(N-1)\rho 2\cos\tilde{\theta}\cos\varphi}\sum_{r=1}^{M}c_r^*e^{j2\pi(N-1)\rho\cos(\theta_i-\varphi)} \end{bmatrix}$$

进而得方程组：

$$\begin{cases} \sum_{i=1}^{M}\left((k_ie^{-j2\pi 0\rho 2\cos\theta_i\cos\varphi}-c_i^*e^{-j2\pi 0\rho 2\cos\tilde{\theta}\cos\varphi})e^{j2\pi 0\rho\cos(\theta_i-\varphi)}\right)=0 \\ \sum_{i=1}^{M}\left((k_ie^{-j2\pi\rho 2\cos\theta_i\cos\varphi}-c_i^*e^{-j2\pi\rho 2\cos\tilde{\theta}\cos\varphi})e^{j2\pi\rho\cos(\theta_i-\varphi)}\right)=0 \\ \cdots \\ \sum_{i=1}^{M}\left((k_ie^{-j2\pi(N-1)\rho 2\cos\theta_i\cos\varphi}-c_i^*e^{-j2\pi(N-1)\rho 2\cos\tilde{\theta}\cos\varphi})e^{j2\pi(N-1)\rho\cos(\theta_i-\varphi)}\right)=0 \end{cases}$$

由于 $\{\mathbf{a}(\theta_1-\varphi)\quad \mathbf{a}(\theta_2-\varphi)\quad \ldots\quad \mathbf{a}(\theta_M-\varphi)\}$ 是信号空间的一组基，互相列线性无关，因此要满足上面方程，必然有 $(k_ie^{-j2\pi\rho n\cos\theta_i\cos\varphi}-c_i^*e^{-j2\pi\rho n\cos\tilde{\theta}\cos\varphi})=0$，其中 $i=1\sim M;n=0\sim N-1$，可推出：

1. A Review of Last Presentation

$$\mathbf{b}(\tilde{\theta},\varphi)=\begin{bmatrix} e^{-j2\pi 0\rho 2\cos\tilde{\theta}\cos\varphi} \\ e^{-j2\pi\rho 2\cos\tilde{\theta}\cos\varphi} \\ \vdots \\ e^{-j2\pi(N-1)\rho 2\cos\tilde{\theta}\cos\varphi} \end{bmatrix}=\begin{bmatrix} \dfrac{\sum_{r=1}^{M} k_r e^{-j2\pi 0\rho 2\cos\theta_r\cos\varphi}}{\sum_{i=1}^{M} c_i^*} \\ \dfrac{\sum_{r=1}^{M} k_r e^{-j2\pi\rho 2\cos\theta_r\cos\varphi}}{\sum_{i=1}^{M} c_i^*} \\ \vdots \\ \dfrac{\sum_{r=1}^{M} k_r e^{-j2\pi(N-1)\rho 2\cos\theta_r\cos\varphi}}{\sum_{i=1}^{M} c_i^*} \end{bmatrix}=\sum_{r=1}^{M}\frac{k_r}{C}\mathbf{b}(\theta_r,\varphi)$$

由上式可以看出 $\mathbf{b}(\theta,\varphi)$ 是 $\{\mathbf{b}(\theta_1,\varphi),\mathbf{b}(\theta_2,\varphi),\ldots,\mathbf{b}(\theta_M,\varphi)\}$ 的线性组合。

当 $0<2\rho\cos\varphi<0.5$ 时，向量组 $\{\mathbf{b}(\theta,\varphi)\}$ 在 $\theta\in[0,180]$ 一定列线性无关，无法得到上述结论，因此与题设矛盾。

因此 $0<2\rho\cos\varphi<0.5$ 时，原假设不成立，$\mathbf{a}(\tilde{\theta}-\varphi)$ 不是 $\mathbf{a}(\theta_1-\varphi),\mathbf{a}(\theta_2-\varphi),\ldots,\mathbf{a}(\theta_M-\varphi)$ 的线性组合。

2. Encountered Problems

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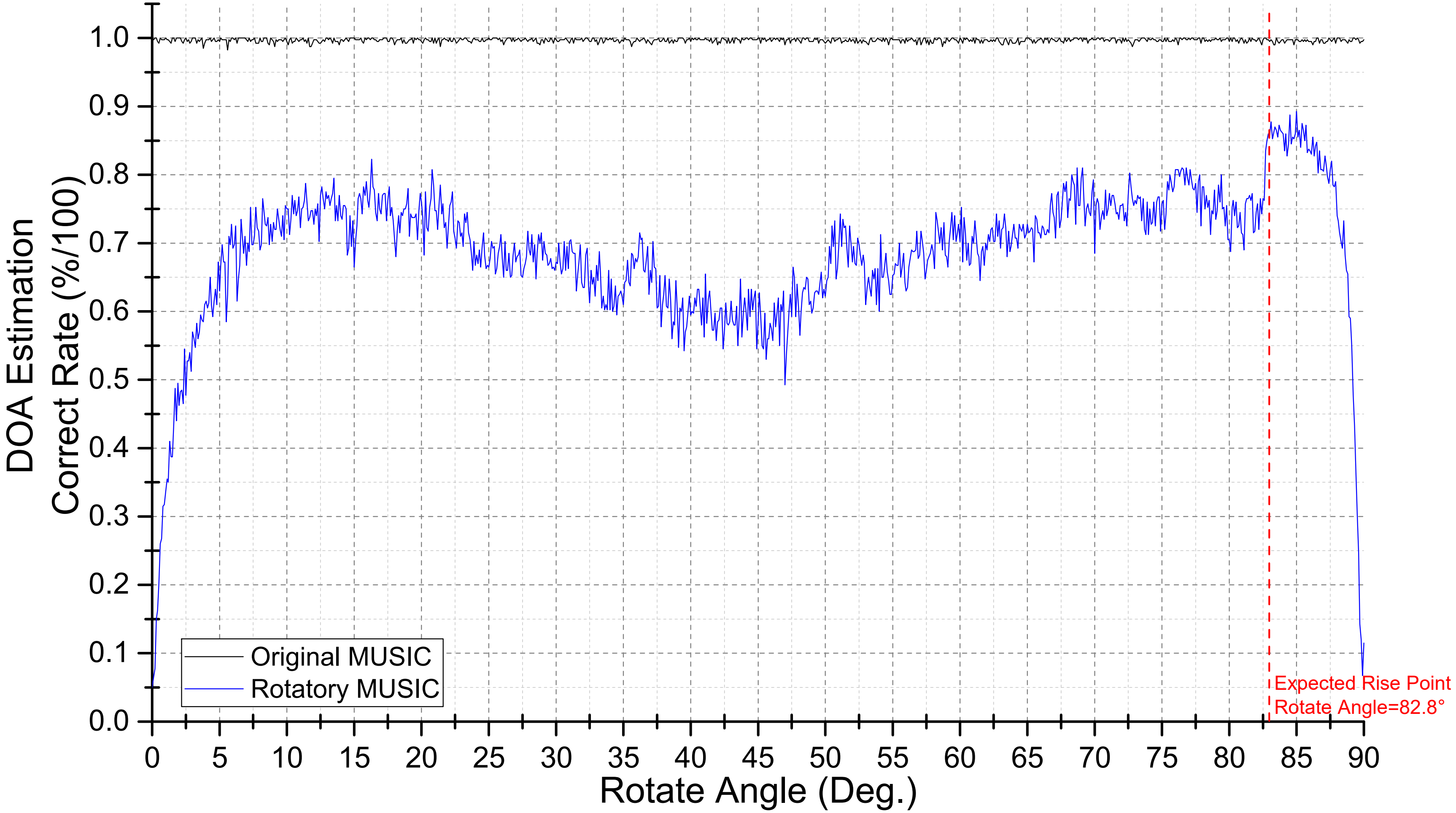

Fig. 2-1 Statistical Correct Rate of Joint DOA Estimation from Rotate Angle = 0 to 90 with Step Length = 0.1
200 Loops for Each Rotate Angle, Time Snapshots = 200 for Each Rotate Angle, Element Interval = 2×λ, SNR = 0dB

2. Encountered Problems

The Rotatory MUSIC mentioned before can be expressed as:

$$P_{RotatoryMUSIC}(\theta,\varphi) = P_{+}(\theta,\varphi)P_{-}(\theta,\varphi)$$
$$= \frac{1}{\mathbf{a}_{+}^{H}(\theta,\varphi)\mathbf{U}_{+}\mathbf{U}_{+}^{H}\mathbf{a}_{+}(\theta,\varphi)} \frac{1}{\mathbf{a}_{-}^{H}(\theta,\varphi)\mathbf{U}_{-}\mathbf{U}_{-}^{H}\mathbf{a}_{-}(\theta,\varphi)}$$

In order to improve the estimation performance of rotatory MUSIC algorithm, I was trying to optimize the DOA estimation based on using the redundant spatial information. Noticed that the group delay of each steering vector in space classification seems to have the potential.

$$\mathbf{\Phi}_{n}(\theta) = \sum_{n=1}^{N-M} \arg\left[\mathbf{u}_{n}^{H}\mathbf{a}(\theta)\right]$$

$$P_{MGD}(\theta) = \frac{\partial \mathbf{\Phi}_{n}}{\partial \theta} P_{MUSIC}(\theta)$$

2. Encountered Problems

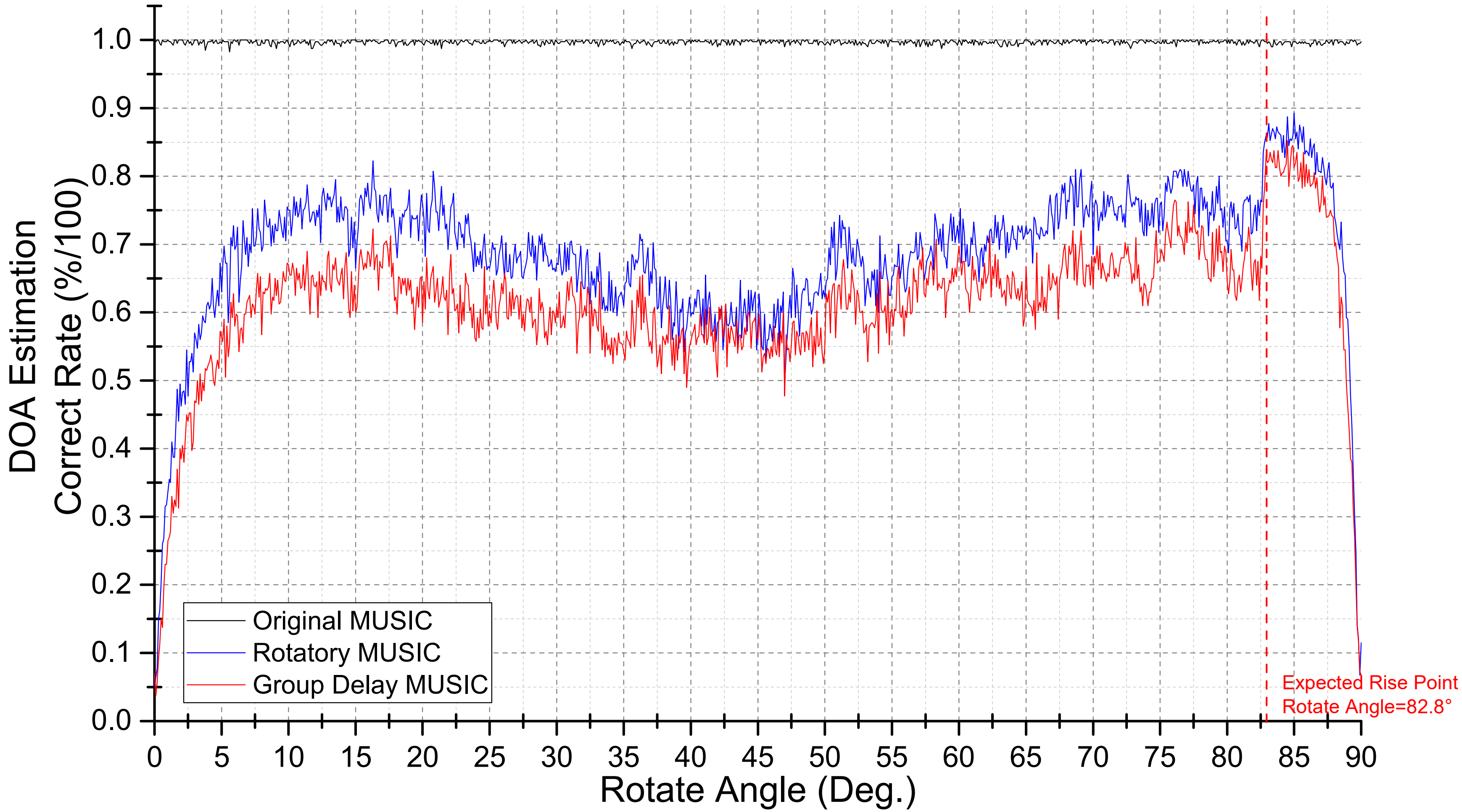

Fig. 2-2 Statistical Correct Rate of Joint DOA Estimation from Rotate Angle = 0 to 90 with Step Length = 0.1
200 Loops for Each Rotate Angle, Time Snapshots = 200 for Each Rotate Angle, Element Interval = 2×λ, SNR = 0dB

3. An Introduction to MJ-MUSIC

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The methods and results discussed before were either not good enough or can not reflect a correct rate rise at expected rotate angle. However those still inspired us that importing redundant information within these two arrays to resolving DOA estimation ambiguity may have great potential.

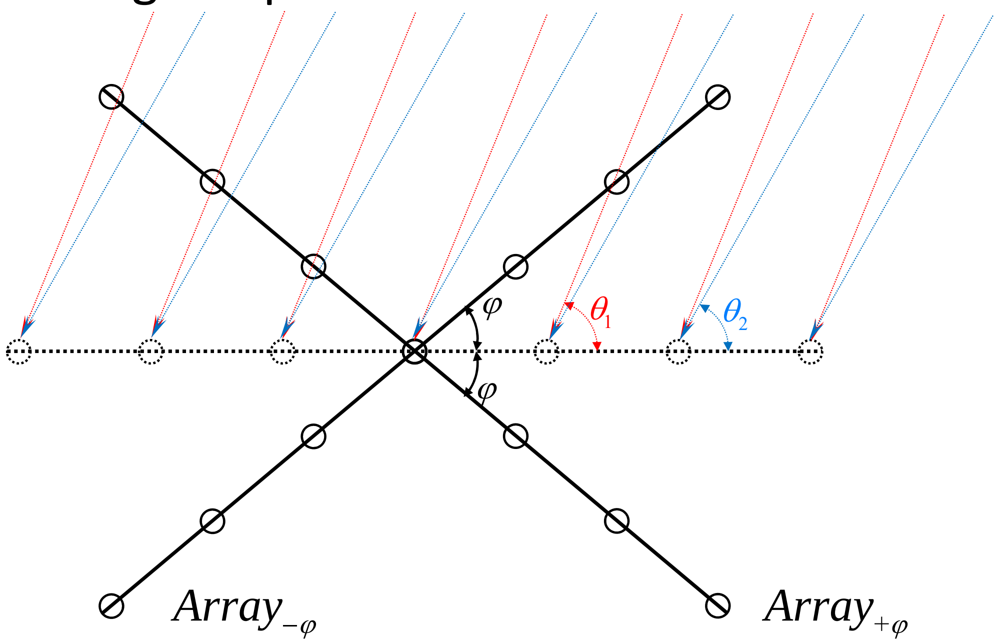

Fig. 3-1 Array Configuration

Same array configuration as before, the received signal vectors are shown below as:

$$\mathbf{X}_{+\varphi} = \mathbf{A}_{+\varphi}\mathbf{S} + \mathbf{N}_{+\varphi}$$

$$\mathbf{X}_{-\varphi} = \mathbf{A}_{-\varphi}\mathbf{S} + \mathbf{N}_{-\varphi}$$

$$\mathbf{X}_{c} = \mathbf{X}_{+\varphi} + \mathbf{X}_{-\varphi} = \left(\mathbf{A}_{-\varphi} + \mathbf{A}_{+\varphi}\right)\mathbf{S} + \mathbf{N}_{+\varphi} + \mathbf{N}_{-\varphi}$$

$$\mathbf{R}_{c} = E\left[\mathbf{X}_{c}\mathbf{X}_{c}^{H}\right]$$

$$= \left(\mathbf{A}_{-\varphi} + \mathbf{A}_{+\varphi}\right)E\left[\mathbf{S}\mathbf{S}^{H}\right]\left(\mathbf{A}_{-\varphi} + \mathbf{A}_{+\varphi}\right)^{H} + 2\sigma^{2}\mathbf{I}$$

3. An Introduction to MJ-MUSIC

As to the array manifold matrix:

$$\mathbf{A}_{+\varphi}=\left[\mathbf{a}(\theta_1+\varphi)\quad \mathbf{a}(\theta_2+\varphi)\quad \dots\quad \mathbf{a}(\theta_M+\varphi)\right]\qquad \mathbf{A}_{-\varphi}=\left[\mathbf{a}(\theta_1-\varphi)\quad \mathbf{a}(\theta_2-\varphi)\quad \dots\quad \mathbf{a}(\theta_M-\varphi)\right]$$

$$\mathbf{A}_c=\left[\mathbf{a}(\theta_1+\varphi)+\mathbf{a}(\theta_1-\varphi)\quad \mathbf{a}(\theta_2+\varphi)+\mathbf{a}(\theta_2-\varphi)\quad \dots\quad \mathbf{a}(\theta_M+\varphi)+\mathbf{a}(\theta_M-\varphi)\right]$$

Where:

$$\mathbf{a}_c(\theta,\varphi)=\mathbf{a}(\theta+\varphi)+\mathbf{a}(\theta-\varphi)=\begin{bmatrix} 1 \\ e^{-j2\pi\rho\cos(\theta+\varphi)} \\ \vdots \\ e^{-j2\pi(N-1)\rho\cos(\theta+\varphi)} \end{bmatrix}+\begin{bmatrix} 1 \\ e^{-j2\pi\rho\cos(\theta-\varphi)} \\ \vdots \\ e^{-j2\pi(N-1)\rho\cos(\theta-\varphi)} \end{bmatrix}$$

$$=2\begin{bmatrix} 1 \\ \cos\left[2\pi\rho\sin\varphi\sin\theta\right]e^{-j2\pi\rho\cos\varphi\cos\theta} \\ \vdots \\ \cos\left[2\pi(N-1)\rho\sin\varphi\sin\theta\right]e^{-j2\pi(N-1)\rho\cos\varphi\cos\theta} \end{bmatrix}$$

3. An Introduction to MJ-MUSIC

As to the combined correlation matrix:

$$\mathbf{R}_c = E\left[\mathbf{X}_c\mathbf{X}_c^H\right] = \left(\mathbf{A}_{-\varphi} + \mathbf{A}_{+\varphi}\right)E\left[\mathbf{S}\mathbf{S}^H\right]\left(\mathbf{A}_{-\varphi} + \mathbf{A}_{+\varphi}\right)^H + 2\sigma^2\mathbf{I}$$

Decompose this combined correlation matrix and scan the noise sub-space by using combined steering vectors from 0~180°, we arrived at a MUSIC-liked spectrum:

$$P_c(\theta,\varphi) = \frac{1}{\mathbf{a}_c^H(\theta,\varphi)\mathbf{U}_c\mathbf{U}_c^H\mathbf{a}_c(\theta,\varphi)}$$

Multiply this MUSIC-liked spectrum by the MUSIC spectrums from clockwise and anti-clockwise rotated arrays, we have Multiple Joint MUSIC (*MJ-MUSIC*) spectrum:

$$P_{MJ-MUSIC}(\theta,\varphi) = P_c(\theta,\varphi)P_+(\theta,\varphi)P_-(\theta,\varphi)$$

However, the linear independent property of combined steering vector still not be proven. Numerical simulation results indicate that this property meets the expectation before.

3. An Introduction to MJ-MUSIC

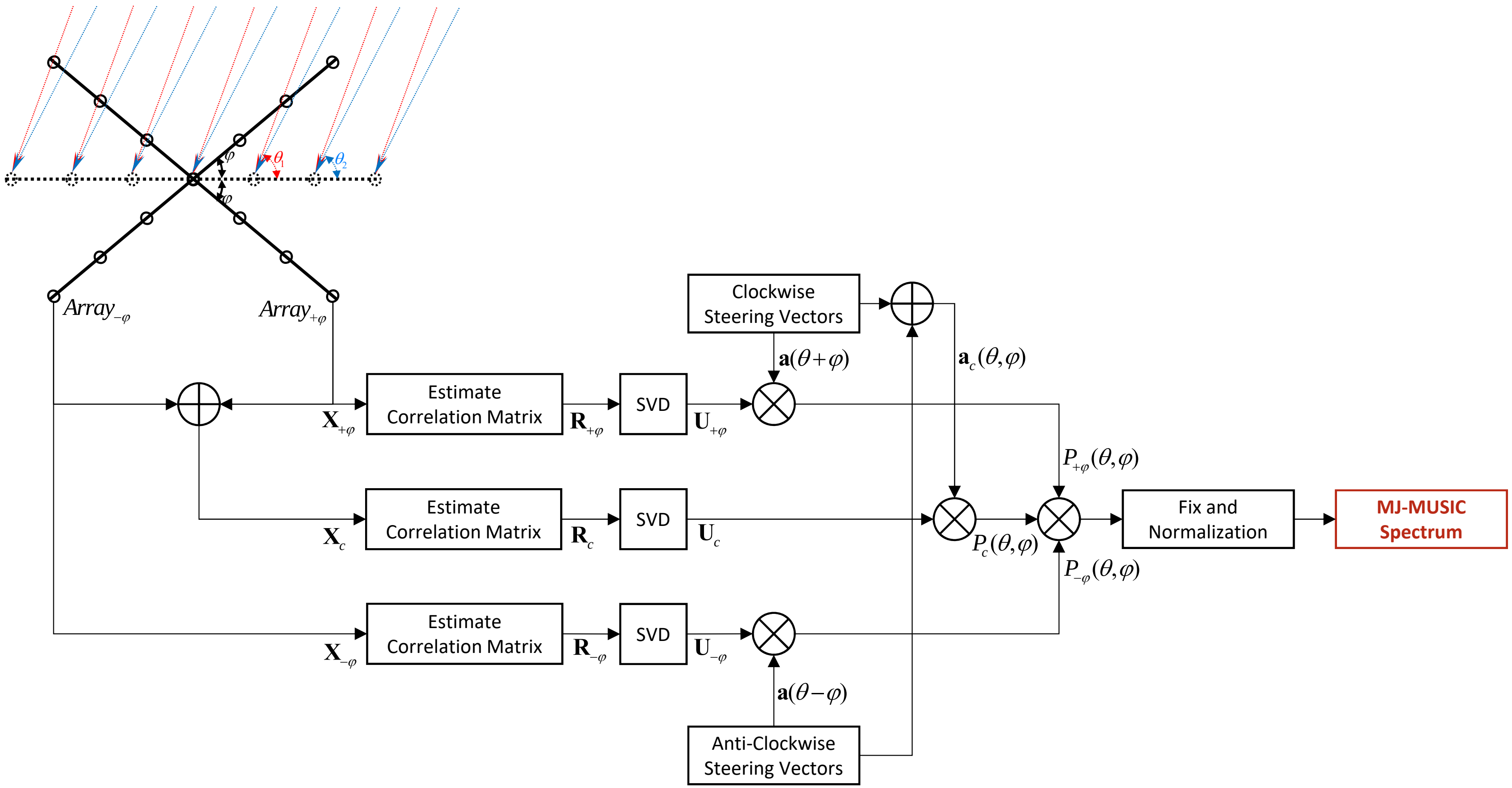

Fig. 3-2 Processing Block Chart of MJ-MUSIC DOA Estimation

4. Results and Discussion

4. Numerical Simulation Results and Discussion

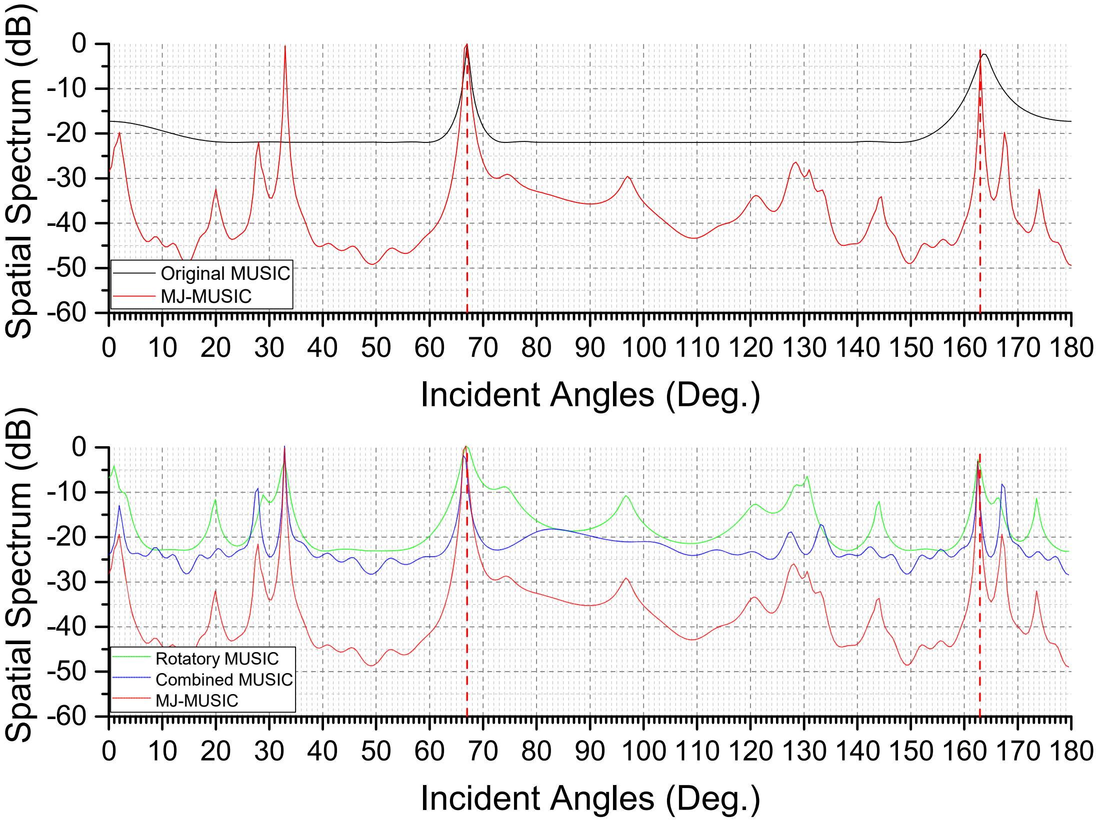

Fig. 4-1 Simulation Results, Incident Angle = [67,163],
Element Interval = 2×λ , Rotate Angle = 82.1 deg. , SNR=0dB

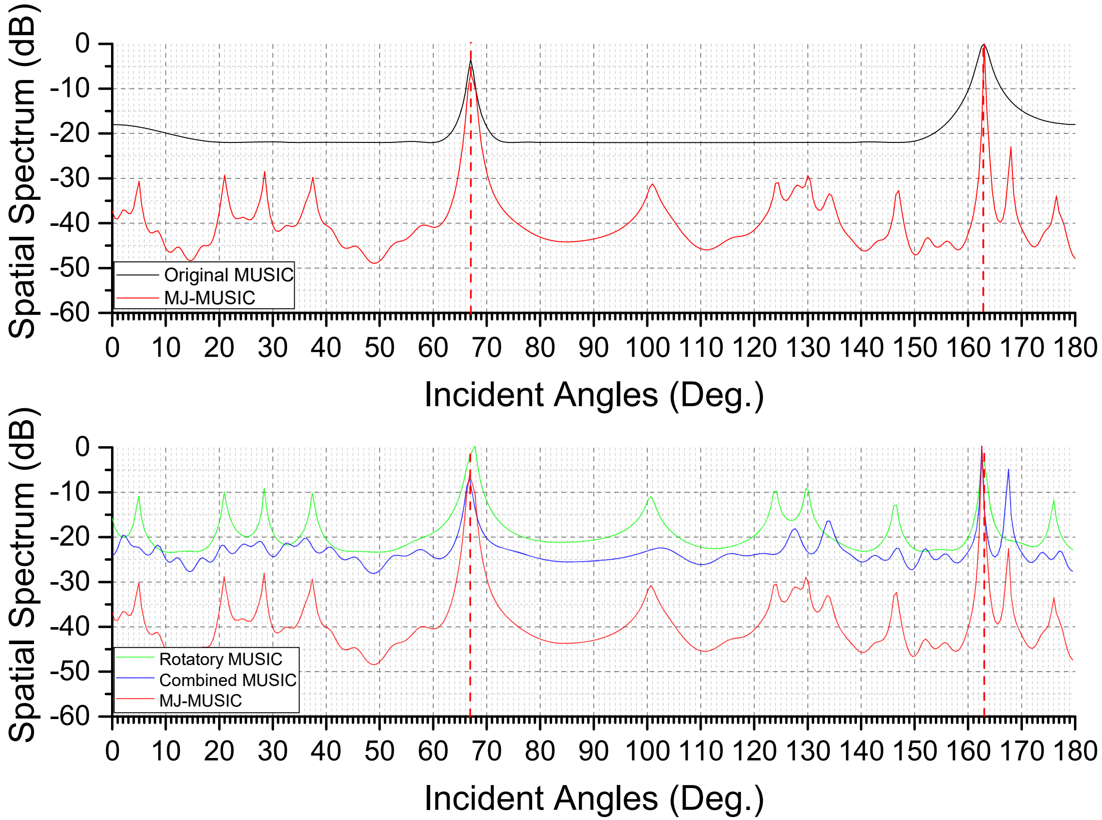

Fig. 4-2 Simulation Results, Incident Angle = [67,163],
Element Interval = 2×λ , Rotate Angle = 83.9 deg. , SNR=0dB

4. Numerical Simulation Results and Discussion

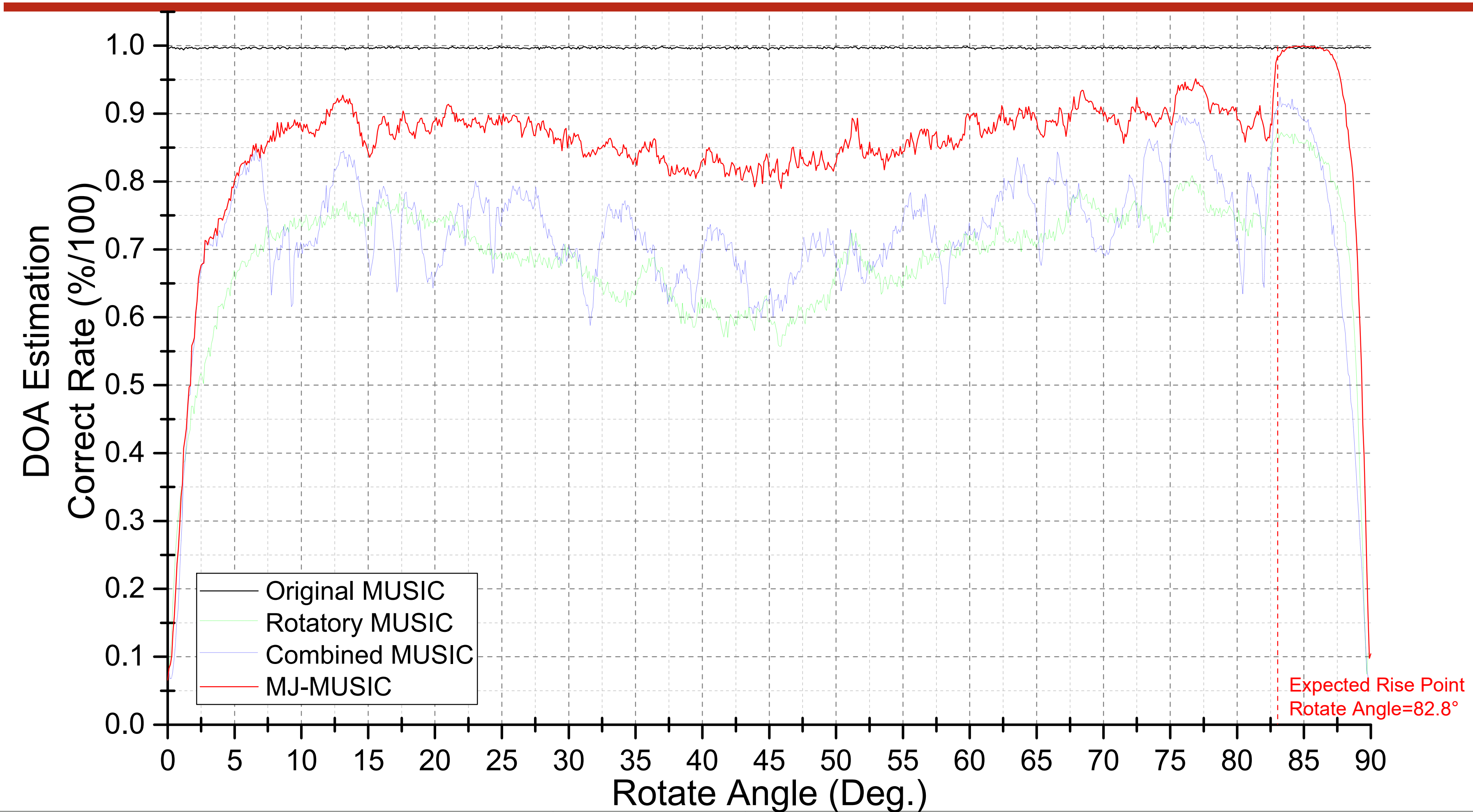

Fig. 4-3 Statistical Correct Rate of DOA Estimation from Rotate Angle = 0 to 90 with Step Length = 0.1
200 Loops for Each Rotate Angle, Time Snapshots = 200 for Each Rotate Angle, Element Interval = 2×λ, SNR = 0dB

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Thank You!

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