Limiting SIR Performance of MMSE Receivers for Large MIMO Systems with Spatial Multiplexing and Non-Constant Modulus Modulations

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***Abstract*— This paper is concerned with the limiting signal-to-interference-plus-noise (SIR) performance of MMSE receivers for large MIMO systems with spatial multiplexing and higher order non-constant modulus modulations. Using random matrix theory, we derive the limiting SIR performance of a suboptimal MMSE receiver when applied to large MIMO systems with unequally-powered users. It has been proven that the limiting SIR converges to a deterministic value when K and N go to infinity with $K/N = y$ being a constant, where K is the number of transmit antennas and N is the number of receive antennas. The limiting SIR result is then used to quantify the SIR loss for different MMSE receivers applied to MIMO systems with non-constant modulus modulations. Numerical results have shown that loss in limiting SIR is small for the suboptimal receiver as compared to optimal receiver.**

***Keywords*—MMSE, large systems, code division multiple access (CDMA), random spreading, MIMO, random matrix theory, limiting spectral theory.**

I. Introduction

Tse and Hanly [1], and Verdú and Shamai [2] published two seminarial papers in 1999 on the applications of random matrix theory (RMT) in wireless communications. Using limiting spectral results of large random matrices [6] - [10], it was shown that for frequency-flat, synchronous CDMA uplink with random spreading codes, the output signal-to-interference-plus-noise ratios (SIRs) using well-known linear receivers, such as matched filter, decorrelator and MMSE receiver, converge to deterministic values for large systems, i.e., when both spreading gain and number of users go to infinity, and their ratio goes to a deterministic constant. The limiting results provide us a fundamental guidance in predicting the system performance and designing the system parameters, without requiring the knowledge of the specific spreading sequences [3]. Other applications of RMT in wireless applications include asymptotic distribution of interference-plus-noise [4] and output SIR [5] of MMSE receivers.

In this paper, we are interested in quantifying the limiting SIRs of MMSE receivers applied to large MIMO systems with spatial multiplexing and non-constant modulus modulations. The typical non-constant modulus modulations include high order quadrature amplitude modulation (QAM), such as 16QAM and 64QAM, for which the the instantaneous powers of each symbol are not constant, and they depend on the constellation points the symbols are modulated. We apply the results of [1] and [2] to quantify the performance of optimal MMSE receiver, which assumes that the instantaneous power profile is available to the receiver. The performance of the optimal MMSE receiver provides an upper bound among all linear receivers. The limiting performance, according to [1], depends on the *statistical interference power profile* of the transmitted symbols, rather than the *instantaneous power profile. We first derive the statistical power profile of 16QAM and 64QAM for large systems, and quantify the limiting performance of the optimal MMSE receiver applied to those systems.*

We then consider a *suboptimal and practical MMSE receiver* which treats the instantaneous powers of all symbols as equally-valued, even though they are actually not. We study the limiting SIR performance of this MMSE receiver applied to large systems with unequally-powered users. From the limiting SIR results, we also derive the optimal noise variance with which the suboptimal MMSE receiver provides the highest limiting SIR within the category of the suboptimal MMSE receivers. The results obtained in this paper can also be used to quantify the mismatch effect when the noise power is overestimated or underestimated in designing the suboptimal MMSE receiver. We quantify the SIR loss of the suboptimal MMSE receiver for large MIMO systems by comparing the limiting SIR of the optimal MMSE receiver with that for suboptimal MMSE receiver derived in this paper.

This paper is organized as follows. In Section II, we present the channel model and review the optimal MMSE receiver. The limiting performance of SIR is reviewed in Section III and we quantify the limits for large MIMO systems with high order modulations.

The following notations are used in this paper: $(\cdot)^T$ for transpose; $(\cdot)^*$ for conjugate or conjugate and transpose; $\mathrm{E}[\cdot]$ for expectation; $tr(\cdot)$ for trace; $\mathrm{diag}(x)$ for diagonal matrix with diagonal elements being x.

II. Channel Model and Optimal MMSE Receiver

A. Channel Model

Let us consider the following MIMO channel:

$$x(n) = \sum_{i=1}^{K} h_i s_i(n) + \epsilon(n), \quad (1)$$

where K is the number of transmit antennas, and N is the number of receive antennas, $s_i(n)$ is the transmitted symbol through transmit antenna i, $h_i \in \mathcal{C}^{N\times 1}$ is the channel vector of for symbol $s_i(n)$, $\epsilon(n) \in \mathcal{C}^{N\times 1}$ is the additive white Gaussian noise vector, and $x(n) \in \mathcal{C}^{N\times 1}$ is the received signal vector. The following assumptions are made throughout this paper.

(AS1) The channel responses h_i's can be represented as

$$h_i = \frac{1}{\sqrt{N}}[X_{1i}, X_{2i}, \cdots, X_{Ni}]^T, \quad (2)$$

where X_{ki}'s are zero mean, iid and with unit variance, i.e., $\mathrm{E}[|X_{ki}|^2] = 1$ for all k's and i's.

(AS2) The transmitted symbols $s_i(n)$'s are M-ary modulated, and they are iid, zero mean and with average power P_i.

(AS3) The noise vector $\epsilon(n)$ is iid, zero mean, circularly symmetric complex Gaussian with covariance matrix $\mathrm{E}[\epsilon(n)\epsilon^*(n)] = \tilde{\sigma}^2 I$.

The use of normalization factor $1/\sqrt{N}$ in (1) implies that the signal-to-noise ratio (SNR), $\gamma_i = \frac{P_i}{\tilde{\sigma}^2}$, defines the average received SNR for user i. Besides the MIMO antenna systems, channel model (1) can also be used to describe the CDMA systems, and the assumption (AS1) is valid for CDMA uplink with random spreading codes, which is the case for 3G WCDMA systems.

B. Optimal MMSE Receiver

Rewrite (1) as

$$x(n) = Hs(n) + \epsilon(n), \quad (3)$$

where $H = [h_1, \cdots, h_K]$ and $s(n) = [s_1(n), \cdots, s_K(n)]^T$. We are then interested in recovering the transmitted symbols, $s_1(n), \cdots, s_K(n)$, from the received signal vector $x(n)$. For notation simplicity, the time index "(n)" will be dropped off when there is no confusion.

For linear receivers, the equalization output of user k is given by

$$\tilde{s}_k = w_k^* x, \quad (4)$$

where $w_k \in \mathcal{C}^{N\times 1}$ is the weighting vector for user k. Using (1), $\tilde{s}_k$ can be represented as

$$\tilde{s}_k = w_k h_k s_k + \sum_{j\neq k} w_k^* h_j s_j + w_k^* \epsilon. \quad (5)$$

Thus the equalization output consists of two components: (i) the desired signal component, $w_k^* h_k s_k$; and (ii) the MAI-plus-noise component, $\sum_{i\neq k} w_k^* h_i s_i + w_k^* \epsilon$.

Denote $\alpha_k = w_k^* h_k$, and σ_k^2 as the variance of the MAI-plus-noise component, respectively. One measure for quantifying the receiver performance is the output SIR of the equalizer, which is given by

$$SIR_k = \frac{|\alpha_k|^2 P_k}{\sigma_k^2}. \quad (6)$$

Notice that

$$\sigma_k^2 = w_k^*(H_k \tilde{D}_k H_k^* + \tilde{\sigma}^2 I) w_k, \quad (7)$$

where $\tilde{D}_k = \mathrm{diag}(P_1, \cdots, P_{k-1}, P_{k+1}, \cdots, P_K)$, and $H_k = [h_1, \cdots, h_{k-1}, h_{k+1}, \cdots, h_K]$, we derive the following

$$SIR_k = \frac{|w_k^* h_k|^2 P_k}{w_k^*(H_k \tilde{D}_k H_k^* + \tilde{\sigma}^2 I) w_k}. \quad (8)$$

Define the mean-square-error (MSE) function for user k as: $J(w_k) = \mathrm{E}[|s_k - w_k^* x|^2]$. The MMSE receiver, which minimizes the MSE function, $J(w_k)$, is represented as

$$\begin{aligned} w_k &= (\mathrm{E}[xx^*])^{-1}\mathrm{E}[xs_k^*] \\ &= (H\tilde{D}H^* + \sigma^2 I)^{-1} h_k P_k, \end{aligned} \quad (9)$$

where $\tilde{D} = \mathrm{diag}(P_1, \cdots, P_K)$. Let $A = H\tilde{D}H^* + \tilde{\sigma}^2 I$ and $C_k = H_k \tilde{D}_k H_k^* + \tilde{\sigma}^2 I$. Using matrix inversion lemma and notice that $A = C_k + P_k h_k h_k^*$, we have

$$A^{-1} = C_k^{-1} - \frac{C_k^{-1} h_k (C_k^{-1} h_k)^*}{(1/P_k) + h_k^* C_k^{-1} h_k}, \quad (10)$$

$$C_k^{-1} = A^{-1} + \frac{A^{-1} h_k (A^{-1} h_k)^*}{(1/P_k) - h_k^* A^{-1} h_k}. \quad (11)$$

Thus the MMSE receiver for user k can be written as

$$w_k = A^{-1} h_k P_k = \frac{C_k^{-1} h_k}{1/P_k + h_k^* C_k^{-1} h_k}. \quad (12)$$

The MMSE receiver also maximizes the output SIR, which is given by

$$SIR_k^{(o)} = h_k^* C_k^{-1} h_k P_k \quad (13)$$

$$= \frac{h_k^* A^{-1} h_k P_k}{1 - h_k^* A^{-1} h_k P_k}. \quad (14)$$

From (6), (9) and (13), it is easy to verify that

$$\alpha_k = \frac{\beta_k}{1+\beta_k}, \quad \text{and} \quad \sigma_k^2 = \frac{\beta_k P_k}{(1+\beta_k)^2}. \quad (15)$$

Only one matrix inversion A^{-1} needs to be calculated if the MMSE weights and output SIRs for all K users are required. The matrix inversion can further be calculated through successive use of matrix inversion lemma. This is very promising in terms of complexity saving since in MIMO environments, we need to decode all data symbols concurrently, the complexity of which could be otherwise very high.

III. Limiting SIRs for Optimal MMSE Receiver

In this section, we first introduce the limiting SIR results for optimal MMSE receiver [1], [2] for unequally-powered systems.

Proposition 3.1 [1]: Consider the random MIMO channel (1). Let $\beta^{(N)}$ be the (random) SIR of the MMSE receiver for user 1 with receive power P_1. Under assumptions (AS1)-(AS3), $\beta^{(N)}$ converges to β^* in probability for the limiting case, where β^* is the unique positive solution to the equation

$$\beta^* = \frac{P_1}{\sigma^2 + y\mathrm{E}_P[I(P, P_1, \beta^*)]}, \tag{16}$$

with $I(P, P_1, \beta^*) = \frac{PP_1}{P_1 + P\beta^*}$. Here, $\mathrm{E}_P[\cdot]$ denotes taking expectation with respect to the limiting empirical distribution of the received powers of the interference, and σ^2 is the variance of the AWGN noise.

For non-constant modulus modulations, consider the channel model (1), and denote $P_i = |s_i(n)|^2$ as the received power of symbol i at time n. For M-ary QAM modulations, even though some points may have large instantaneous power, all points will have almost equal chance of being disturbed (except the edge points), thus the effective signal powers for all constellation points are all equal to $\bar{P} = \frac{1}{M}\sum_{i=1}^{M} P_i$. Therefore, the effective SIR for detecting all points can be represented as

$$SIR_k = \frac{|w_k^* h_k|^2}{w_k^*(H_k D_k H_k^* + \mu^2 I)^{-1} w_k}, \tag{17}$$

where $D_k = \mathrm{diag}(P_1/\bar{P}, \cdots, P_{k-1}/\bar{P}, P_{k+1}/\bar{P}, \cdots, P_K/\bar{P})$, and $\mu^2 = \sigma^2/\bar{P}$. The optimal MMSE receiver requires the knowledge of the instantaneous powers, $P_1, \cdots, P_K$, of all transmitted symbols, and it can be represented as

$$w_k = (H_k D_k H_k^* + \mu^2 I)^{-1} h_k. \tag{18}$$

Note $\frac{1}{K-1}\mathrm{tr}(D_k) = 1$. The output SIR for optimal MMSE is given by

$$\beta_k^{(o)} = h_k^*(H_k D_k H_k^* + \mu^2 I)^{-1} h_k. \tag{19}$$

To make use of *Proposition 3.1*, the limiting distribution of the interference powers needs to be derived. Dividing the M constellation points into multiple groups, each with a different amplitude. Suppose there are L groups, and denote M_i as the number of points in the ith group with power Q_i. When the transmitted symbols are iid, and uniformly chosen from the alpha-bet set, when $K \to \infty$, the empirical distribution of the instantaneous powers tends to a deterministic distribution: $p(P = Q_i) = M_i/M$ for $i = 1, \cdots, L$.

Using *Proposition 3.1*, the limiting SIR $\beta_k^{(o)}$ of the optimal MMSE receiver is determined by

$$\beta^{(o)} = \frac{1}{\mu^2 + y\sum_{i=1}^{L} \frac{M_i}{M} \frac{Q_i/\bar{P}}{1 + (Q_i/\bar{P})\beta^{(o)}}}. \tag{20}$$

We then specifically consider 16QAM and 64QAM modulations, which are two most popular modulation schemes used for high data rate transmissions.

A. 16QAM

There exist three groups of constellation points for 16QAM:

- Group 1 of constellation points $(\pm 1 \pm j)$ with $M_1 = 4$ and $Q_1 = 2$;
- Group 2 of constellation points $(\pm 1 \pm 3j)$ and $(\pm 3 \pm j)$ with $M_2 = 8$ and $Q_2 = 10$;
- Group 3 of constellation points $(\pm 3 \pm 3j)$ with $M_3 = 4$ and $Q_3 = 18$.

Thus $\bar{P} = 10$. Using (20), the limiting effective SIR β of the MMSE output can be determined by the positive solution of the following equation

$$\beta = \frac{1}{\mu^2 + \frac{y}{4}\frac{(1/5)}{1+(1/5)\beta} + \frac{y}{2}\frac{1}{1+\beta} + \frac{y}{4}\frac{(9/5)}{1+(9/5)\beta}}. \tag{21}$$

B. 64QAM

There exist nine groups of constellation points for 64QAM:

- Group 1 of constellation points $(\pm 1 \pm j)$ with $M_1 = 4$ and $Q_1 = 2$;
- Group 2 of constellation points $(\pm 1 \pm 3j)$ and $(\pm 3 \pm j)$ with $M_2 = 8$ and $Q_2 = 10$;
- Group 3 of constellation points $(\pm 3 \pm 3j)$ with $M_3 = 4$ and $Q_3 = 18$;
- Group 4 of constellation points $(\pm 1 \pm 5j)$ and $(\pm 5 \pm j)$ with $M_4 = 8$ and $Q_4 = 26$;
- Group 5 of constellation points $(\pm 5 \pm 3j)$ and $(\pm 3 \pm 5j)$ with $M_5 = 8$ and $Q_5 = 34$;
- Group 6 of constellation points $(\pm 1 \pm 7j)$, $(\pm 7 \pm j)$ and $(\pm 5 \pm 5j)$ with $M_6 = 12$ and $Q_6 = 50$;
- Group 7 of constellation points $(\pm 3 \pm 7j)$ and $(\pm 7 \pm 3j)$ with $M_7 = 8$ and $Q_7 = 58$;
- Group 8 of constellation points $(\pm 5 \pm 7j)$ and $(\pm 7 \pm 5j)$ with $M_8 = 8$ and $Q_8 = 74$;
- Group 9 of constellation points $(\pm 7 \pm 7j)$ with $M_9 = 4$ and $Q_9 = 98$.

Thus $\bar{P} = 42$, and the limiting effective SIR can be calculated with (20) accordingly.

IV. A Suboptimal MMSE Receiver

A. A Suboptimal MMSE Receiver

The optimal MMSE receiver shown in (9) yields the highest output SIR among the linear receivers. This receiver, however, requires the instantaneous power profile of the transmitted symbols, thus it is difficult to implement as the power profile is not available in practice.

A practical solution is to treat all transmitted symbols or interferers as they are equally powered when designing the receiver. This solution is called suboptimal MMSE receiver. The weighting vector of suboptimal MMSE receiver for user k is then given by

$$w_k = B_k^{-1} h_k. \tag{22}$$

where $B_k = H_k H_k^* + \mu^2 I$. Note if $P_1 = P_2 = \cdots = P_N = P$ and $\mu^2 = \tilde{\sigma}_2/P$, then (9) is equivalent to (22). For unequally-powered systems, the determination of parameter μ^2 will be discussed later. With the suboptimal receiver, from (6), the output SIR for user k can be represented as

$$SIR_k^{(s)} = \frac{\delta_k |h_k^* B_k^{-1} h_k|^2}{h_k^* B_k^{-1}(H_k D_k H_k^* + \sigma^2 I) B_k^{-1} h_k}, \quad (23)$$

where $\delta_k = P_k/\bar{P}_k$, $D_k = \mathrm{diag}(P_1/\bar{P}_k, ..., P_{k-1}/\bar{P}_k, P_{k+1}/\bar{P}_k, \cdots, P_K/\bar{P}_k)$ with $\bar{P}_k = \frac{1}{K-1}\sum_{i\neq k} P_i$ and $\sigma^2 = \tilde{\sigma}^2/\bar{P}_k$. Note $(1/(K-1))\mathrm{tr}D_k = 1$, and δ_k denotes the ratio between the desired user's power to the average of the interference powers. In the sequel, we will derive the limiting SIR and asymptotic SIR distribution for the suboptimal MMSE receiver. Without loss of generality, we only consider the demodulation of user 1, and assume that $\delta_1 = 1$. If $\delta_1 \neq 1$, the limiting SIR will be scaled by a factor of δ_1, while the variance of the asymptotic distribution of the SIR will be scaled by a factor of δ_1^2.

B. Limiting SIR Analysis

In this subsection, we derive the limiting SIR for the suboptimal MMSE receiver when $N \to \infty$ and $K \to \infty$ with $\frac{K}{N} \to$ constant $y > 0$. Before presenting the main theorem for the suboptimal MMSE receiver, we briefly state the limiting SIR results for equally powered systems, i.e., when $D_k = I$.

Proposition 4.1: Suppose that:

(1) for all i and j, X_{ij}'s are iid with $\mathrm{E}[X_{11}] = 0$, $\mathrm{E}[|X_{11}|^2] = 1$ and $\mathrm{E}[|X_{11}|^4] < \infty$; and

(2) $\frac{K}{N} \to y > 0$ (y is a constant) as $N \to \infty$.

Then the following SIR,

$$SIR_1 = h_1^* B_1^{-1} h_1, \quad (24)$$

converges in probability to a deterministic value, which is given by

$$\beta(\mu^2) = \int \frac{dF_y(x)}{x+\mu^2}, \quad (25)$$

where $F_y(x)$ is the limit of the empirical distribution function of random matrix HH^*.

According to [1] and [2], $\beta(\mu^2)$ is the positive solution of following equation

$$y - 1 - \frac{y}{1+\beta(\mu^2)} + \mu^2\beta(\mu^2) = 0. \quad (26)$$

which is given by

$$\beta(\mu^2) = \frac{(1-y)}{2\mu^2} - \frac{1}{2} + \sqrt{\frac{(1-y)^2}{4\mu^4} + \frac{(1+y)}{2\mu^2} + \frac{1}{4}}. \quad (27)$$

Eq.(26) is also equivalent to the following equation, which will be used frequently in the sequel.

$$\frac{1}{\beta(\mu^2)} = y - 1 + \mu^2(1+\beta(\mu^2)) = \frac{y}{1+\beta(\mu^2)} + \mu^2. \quad (28)$$

In [5], we have proven that the limiting theorem for the output SIR of the suboptimal MMSE receiver.

Theorem 4.1: Suppose that:

(1) for all i and j, X_{ij}'s are iid with $\mathrm{E}[X_{11}] = 0$, $\mathrm{E}[|X_{11}|^2] = 1$ and $\mathrm{E}[|X_{11}|^4] < \infty$;

(2) $\frac{K}{N} \to y > 0$ as $N \to \infty$; and

(3) the empirical distribution function of $D = \mathrm{diag}(P_1/\bar{P}, ..., P_K/\bar{P})$ where $\bar{P} = (1/K)\sum_{i=1}^K P_i$ converges to a probability distribution function $F_p(x)$ with $\int x dF_p(x) = 1$, and the powers of all users are bounded by a constant.

Then the $SIR_1^{(s)}$ in (23), denoted as $\beta_1^{(s)}$, converges in probability to a deterministic constant $\beta^{(s)}(\mu^2, \sigma^2)$ which satisfies

$$\beta^{(s)}(\mu^2, \sigma^2) = \beta(\mu^2)\mathcal{K}(\mu^2, \sigma^2) \quad (29)$$

where

$$\mathcal{K}(\mu^2, \sigma^2) = \frac{y + \mu^2(1+\beta(\mu^2))^2}{y + \sigma^2(1+\beta(\mu^2))^2}.$$

For notation simplicity, we use β to denote $\beta(\mu^2)$, $\beta^{(s)}$ to $\beta^{(s)}(\mu^2, \sigma^2)$, and $\mathcal{K}$ to $\mathcal{K}(\mu^2, \sigma^2)$.

Remark 4.1: The convergence in mode in Theorem 4.1 can be strengthened to convergence with probability one via Borel-Cantelli lemma under mild moment condition. Moreover, one can prove the convergence in probability of $\beta_1^{(s)}$ under the finite second moment of spreading codes and convergence with probability one under finite fourth moment condition of spreading codes after suitable truncation and centralization; please refer to the results of [11].

Theorem 4.2: Under the conditions of Theorem 4.1, the limiting SIR $\beta^{(s)}$ is maximized when $\mu^2 = \sigma^2$.

Proof: Calculating the differentiation of $\beta^{(s)}$ over the variable μ^2, and setting $\frac{d\beta^{(s)}}{d\mu^2} = 0$ yields

$$\begin{aligned} & \left[\frac{y}{(1+\beta)^2} + \mu^2\right]\left[\frac{y}{(1+\beta)^2} + \sigma^2\right] \\ + \;& \frac{2y\beta}{(1+\beta)^3}(\mu^2 - \sigma^2) + \frac{\beta}{\beta'}\left[\frac{y}{(1+\beta)^2} + \sigma^2\right] = 0 \end{aligned} \quad (30)$$

From (26), the derivative of β over μ^2 can be calculated as follows:

$$-\frac{\beta}{\beta'} = \frac{y}{(1+\beta)^2} + \mu^2, \quad (31)$$

Thus the solution becomes $\mu^2 = \sigma^2$. Thus the maximum limiting SIR is achieved when μ^2 is chosen as σ^2.

V. Numerical Results

Numerical results for limiting SIRs are shown in Fig. 1 - Fig.4 for optimal and suboptimal MMSE receivers when applied to large MIMO systems with 16QAM and various loading factor y. For a given SNR, the limiting SIRs decreases with the increase of loading factor y. On the other hand, for a given loading factor y, the limiting SIR increases with the increase of SNR, but approaches to certain limits when y is large. It is also seen that when $y = 1$ and SNR = 20dB, there is about 1 dB SIR loss for the suboptimal receiver as compared to the optimal MMSE receiver. This loss becomes less when the SNR decreases. Further, when $y = 0.5$, the suboptimal MMSE receiver performs almost equally well as the optimal receiver.

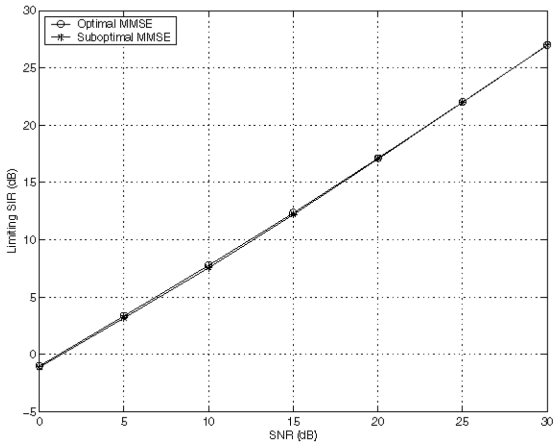

Fig. 1. Comparison of limiting SIRs of optimal and suboptimal MMSE receivers applied to 16QAM with $y = 0.5$.

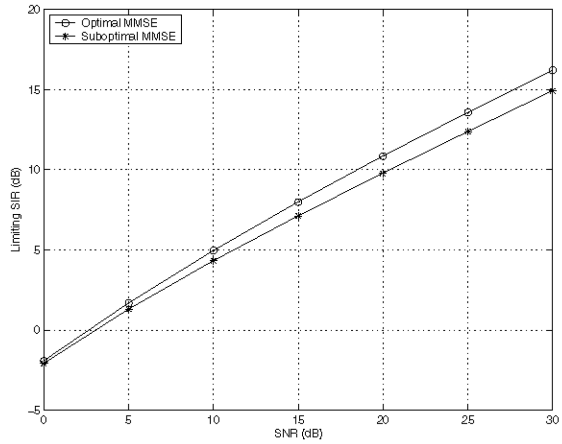

Fig. 2. Comparison of limiting SIRs of optimal and suboptimal MMSE receivers applied to 16QAM with $y = 1$.

VI. Conclusions

In this paper, the limiting SIRs for optimal MMSE receiver and one suboptimal MMSE receiver are compared for large MIMO systems with *higher order non-constant modulus modulations*. We have derived the theoretical formula for the limiting SIR expressions. Numerical comparison results have shown that for 16QAM modulation, the loss in limiting SIR is small for the suboptimal receiver as compared to optimal receiver if the loading factor is small, however, this loss becomes large for high SNR region.

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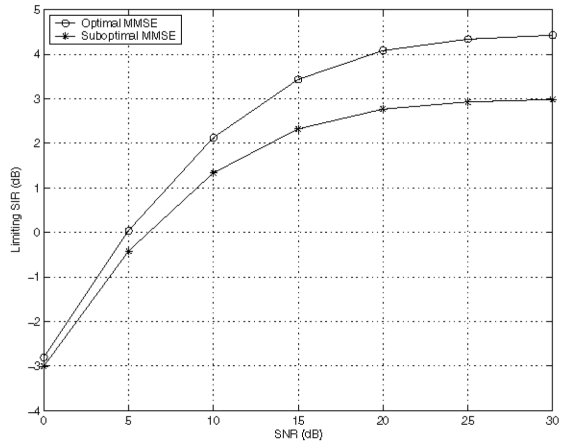

Fig. 3. Comparison of limiting SIRs of optimal and suboptimal MMSE receivers applied to 16QAM with $y = 1.5$.

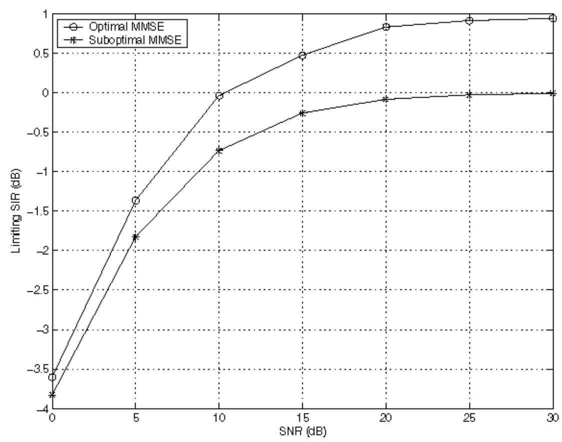

Fig. 4. Comparison of limiting SIRs of optimal and suboptimal MMSE receivers applied to 16QAM with $y = 2$.