Performance of Massive MIMO V-BLAST with Channel Correlation and Imperfect CSI

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***Abstract*—In an uplink massive multiple-input-multiple-output (MIMO) deployment with distributed single-antenna users and a large base-station array, we consider the performance of Vertical Bell Laboratories Layered Space Time (V-BLAST) with maximum ratio combining (MRC) and V-BLAST with zero forcing (ZF). In the performance evaluation, we include the effects of imperfect channel state information (CSI) and channel correlation since these system imperfections are of particular importance in massive MIMO. The main contribution is that the performance of MRC V-BLAST is shown to approach that of ZF V-BLAST under a range of imperfect CSI levels, different channel powers and different types of array, as long as the channel correlations are not too high.**

***Index Terms*– MIMO, massive MIMO, ZF, MRC, V-BLAST, correlation, channel estimation.**

I. Introduction

It has been shown that large multi-input multi-output systems (MIMO) or massive MIMO (MM) increase the degrees of freedom (DOF), throughput and reliability of wireless systems [1], [2]. MM is one of the key technologies proposed for 5^{th} generation wireless systems (5G). The system performance of MM strongly depends on the accuracy of the channel state information (CSI). With imperfect channel estimation, the system performance can degrade severely due to pilot contamination [2] and latency effects [3]. Also, if the large arrays are located within a small area, then the channel correlations may become quite high impacting system performance [2]. Finally, simple processing is very important for MM. Hence, our focus is to study a low complexity, MRC-based MM receiver with imperfect CSI and channel correlation.

We consider an uplink MM deployment with a co-located base station (BS) array and distributed single antenna users. Several studies which consider channel estimation and correlated channels have appeared in [4], [5] for traditional MIMO and in [6], [7] for MM. One of the most popular schemes in traditional MIMO is the Vertical Bell Laboratories Layered Space Time (V-BLAST) approach, which is a multi-layer symbol detection scheme. It uses nonlinear (serial cancellation) and linear combining (interference suppression) and has been studied with imperfect channel estimation [8] and with correlated channels [9]. Most of the work on traditional V-BLAST discusses zero forcing (ZF) or minimum-mean-square-error (MMSE) combining and rarely considers maximum ratio combining (MRC) [10]. On the other hand, very little work has appeared on V-BLAST with MM probably because of the complexity of V-BLAST with large numbers of antennas. However, there is a low complexity approach to V-BLAST with MM [11] and successive interference cancellation methods in MM are described briefly in [2]. Hence, in this paper, we contribute as follows. We integrate MRC with V-BLAST for MM and compare its performance with ZF with V-BLAST. This work extends the promising low complexity MRC V-BLAST approach in [11], which assumed independent channels and perfect CSI, to the more realistic MM scenario of correlated channels and imperfect CSI. Furthermore, our results show that MRC with V-BLAST can approach the performance of ZF with V-BLAST in a MM context. In addition, we provide analytical results for antenna spacing in linear and square arrays which are useful for array dimensioning.

The rest of the paper is organized as follows. Section II introduces the system model, Section III describes the MM deployment and the channel model, and Section IV gives an overview of V-BLAST. We provide analysis in Section V, give simulation results in Section VI, and conclude in Section VII.

II. System model

We consider an uplink system with N_t single antenna user equipments (UEs) and a base station (BS) with N_r receive antennas. The transmit vector, $\mathbf{x}$, of size $N_t \times 1$ is $\mathbf{x} = [x_1, x_2, \ldots x_{N_t}]^T$ and the channel matrix of size $N_r \times N_t$ is $\mathbf{H} = [\mathbf{h}_1 \cdots \mathbf{h}_{N_t}]$. The noise vector, $\mathbf{n}$, of size $N_r \times 1$ has independent and identically distributed (i.i.d.) complex Gaussian elements, i.e., $n_i \sim \mathcal{CN}(0, \sigma^2)$. The received signal, $\mathbf{y} \in \mathbb{C}^{N_r}$, is given by

$$\mathbf{y} = \mathbf{H}\mathbf{x} + \mathbf{n} = \sum_{j=1}^{N_t} \mathbf{h}_j x_j + \mathbf{n}. \tag{1}$$

The channel coefficient, h_{ij}, from UE j to receive antenna i has link gain $\mathrm{E}[|h_{ij}|^2] = P_j$. The transmitted signal power is $\mathrm{E}[|x_j|^2] = E_s = 1$ and the noise power is $\mathrm{E}[|n_i|^2] = \sigma^2$. The ratio of the transmit signal power to the noise power (transmit SNR) is $1/\sigma^2$. In this paper, we consider linear combiners at the receiver employed in a V-BLAST scheme. For ease of exposition, we give the details of the linear combiner for the first stage of V-BLAST, i.e., when the received signal is given by (1) and the V-BLAST ordering procedure has selected the m^{th} signal, x_m, to be detected. Here, the estimated symbol is $\hat{x}_m = Q(\mathbf{w}_m^H \mathbf{y})$, where Q is a quantizer which maps its argument to the closest signal point using Euclidean distance and $\mathbf{w}_m^H$ represents the complex conjugate transpose of the linear combiner, $\mathbf{w}_m$. The SINR for the m^{th} user is

$$\mathrm{SINR_m} = \frac{\mathrm{E}[|\mathbf{w}_m^H \mathbf{h}_m x_m|^2]}{\mathrm{E}[|\mathbf{w}_m^H(\sum_{j \neq m} \mathbf{h}_j x_j + \mathbf{n})|^2]}, \tag{2}$$

where $\mathbf{w}_m$ is the m^{th} column of the linear combining matrix, $\mathbf{W}$. When perfect CSI is available, the MRC combiner is defined as $\mathbf{W} = \mathbf{H}$ [12] and the ZF combiner is defined as $\mathbf{W} = \mathbf{H}(\mathbf{H}^H\mathbf{H})^{-1}$ [12]. At the receiver, only an estimate, $\hat{\mathbf{H}}$, of the true channel, $\mathbf{H}$, is available. We consider the imperfect channel estimation model in [13], $\hat{\mathbf{H}} = r_0\mathbf{H} + \sqrt{1 - r_0^2}\mathbf{E}$, where r_0 is the correlation coefficient between the true channel, $\mathbf{H}$, and the estimated channel, $\hat{\mathbf{H}}$. The error matrix, $\mathbf{E} = [\mathbf{e}_1 \cdots \mathbf{e}_{N_t}]$, has the same statistics as $\mathbf{H}$ and is independent of $\mathbf{H}$. The combiners use the estimated channel so that the MRC combiner is defined as $\mathbf{W} = \hat{\mathbf{H}}$ and the ZF combiner is defined as $\mathbf{W} = \hat{\mathbf{H}}(\hat{\mathbf{H}}^H\hat{\mathbf{H}})^{-1}$.

III. Deployment and Channel Model

Assume a large co-located array at the BS serving N_t users in the coverage region. The channel matrix is defined as

$$\mathbf{H} = \mathbf{R}_r^{\frac{1}{2}}\mathbf{U}\mathbf{P}^{\frac{1}{2}}, \tag{3}$$

where the elements of $\mathbf{U}$ are i.i.d. $\mathcal{CN}(0, 1)$, $\mathbf{P} = \mathrm{diag}(P_1, P_2, \ldots, P_{N_t})$ and P_j is the link gain of user j. Traditionally, P_j is modeled by path loss, shadowing, etc. Here, we employ a simple model [14] so that P_j is defined as $P_j = \mathcal{A}\beta^{j-1}$, $j = \{1, 2, \ldots, N_t\}$ where $\mathcal{A}$ is the link gain of the strongest UE and β is a parameter which controls the rate of decay of the link gains and $0 < \beta \leq 1$. This model gives $P_1 > P_2 > \cdots > P_{N_t}$ so that the link gains of the users are ordered. This has no effect on the generality of the results since the user order is arbitrary. As $\beta \to 1$ the link gains become equal and as $\beta \to 0$ the link gains are dominated by one strong UE. We use this simple model in order to control the P_j's with a single parameter, β, which has a physical interpretation. Moreover, this model is useful because of the importance of the decay rate in V-BLAST, where it is well-known that performance is heavily dependent on the differences between the link gains. $\mathbf{R}_r$ is the channel correlation matrix at the receiver. In the following, we provide two simple models for the correlation matrix, depending on the type of array used at the receiver.

A. Case 1: Uniform Linear Array (ULA)

Let $\mathbf{R}_r = (R_{ij})$, then the element R_{ij} is defined by $R_{ij} = \alpha_u^{|i-j|}$, where α_u is the correlation between channels at adjacent antennas and $0 < \alpha_u < 1$. This is the simple exponential correlation model given in [15].

B. Case 2 : Square Array (SQ)

Here, we assume a regularly spaced square array and the element R_{ij} is defined by $R_{ij} = \alpha_s^{d_{ij}}$, where α_s is a correlation parameter and d_{ij} is the distance between the i^{th} and j^{th} antenna. To make the models comparable and ensure the correlation at a fixed separation is identical, we select $\alpha_s = \alpha_u^{1/\Delta}$, where Δ is the distance between adjacent antennas.

IV. V-BLAST

V-BLAST is an iterative procedure that utilizes linear detection at each iteration. In V-BLAST, the traditional approach to ordering the streams is to detect the stream at each stage with the highest output SINR first [16]. Suppose the ordered set $S = \{k_1, k_2, \ldots, k_{N_t}\}$ is a permutation of the integers 1, 2, …, N_t specifying the order in which the transmitted symbols in $\mathbf{x}$ are extracted. The index, k_i, is chosen such that the SINR/SNR of the detected symbol at the i^{th} stage is maximized. Under the assumption of perfect channel estimation, let $\mathbf{H}_i$ represent $\mathbf{H}$ with columns $k_1, k_2, \ldots, k_{i-1}$ replaced by zeros and let $\mathbf{W}_i$, the weight matrix for the i^{th} stage, be given by $\mathbf{H}_i$ for MRC and $\mathbf{H}_i(\mathbf{H}_i^H\mathbf{H}_i)^{-1}$ for ZF. The optimal ordering of V-BLAST with MRC, which is based on maximizing the SINR at stage i, is defined by

$$k_i^{(\mathrm{MRC})} = \underset{m \notin \{k_1, k_2, \ldots, k_{i-1}\}}{\arg\max} \mathrm{SINR}_m^{(\mathrm{MRC})}. \tag{4}$$

Assuming perfect cancellation at each stage, the SINR, based on (2), for user m at the i^{th} stage is

$$\mathrm{SINR}_m^{(\mathrm{MRC})} = \frac{|\mathbf{h}_m^H\mathbf{h}_m|^2}{\sum_{j \neq m} \mathbf{h}_m^H\mathbf{h}_j\mathbf{h}_j^H\mathbf{h}_m + \sigma^2\mathbf{h}_m^H\mathbf{h}_m}, \tag{5}$$

where $j \notin \{k_1, k_2, \ldots, k_{i-1}\}$ in the summation in (5).

The optimal V-BLAST ordering for ZF, which is based on maximizing SNR, is defined by [16]

$$k_i^{(\mathrm{ZF})} = \underset{j \notin \{k_1, k_2, \ldots, k_{i-1}\}}{\arg\min} ||(\mathbf{W}_i)_j^{(\mathrm{ZF})}||^2, \tag{6}$$

where $(\mathbf{W}_i^{(\mathrm{ZF})})_j$ represents column j of $\mathbf{W}_i^{(\mathrm{ZF})}$, i.e., the ZF combiner at stage i. Again this assumes perfect cancellation at each stage. With imperfect channel estimation, a similar strategy can be used except that the estimated channel replaces the perfect channel. Hence, $\hat{\mathbf{H}}_i$ is used instead of $\mathbf{H}_i$ where $\hat{\mathbf{H}}_i$ represents $\hat{\mathbf{H}}$ with columns $k_1, k_2, \ldots, k_{i-1}$ replaced by zeros and $\mathbf{W}_i$ is calculated from the estimated channel. For ZF, this means that detection is performed using $\mathbf{W}_i^{(\mathrm{ZF})} = \hat{\mathbf{H}}_i(\hat{\mathbf{H}}_i^H\hat{\mathbf{H}}_i)^{-1}$ and this estimated combining matrix is used in (6) for ordering [17]. For MRC, detection is performed using $\mathbf{W}_i^{(\mathrm{MRC})} = \hat{\mathbf{H}}_i$ and ordering uses (5) with the true channels replaced by their estimates. A simple way to assess the computational complexity of these two techniques is by counting the number of complex multiplications. In ZF, computing $\mathbf{H}^H\mathbf{H}$ requires $O(N_rN_t^2)$ multiplies and the inverse computation requires $O(N_t^3)$. This is repeated in each of the N_t layers and since $N_r \gg N_t$, the total complexity becomes $O(N_rN_t^3)$. In MRC, the dominant computation is ordering which requires $O(N_rN_t)$ multiplies. Repetition of the ordering in the N_t layers gives a total complexity $O(N_rN_t^2)$. As expected, the MRC complexity is less than ZF by a factor equal to the number of transmit antennas. For example, if $N_t = 100$, the complexity reduction is 2 orders of magnitude.

Recent results on low-complexity V-BLAST for MM [11] also show the same order of complexity reduction.

V. Analysis

A. Analysis of Imperfect CSI

In [11], it was shown that MRC V-BLAST and ZF V-BLAST can have similar performance with perfect CSI. In this section, we analyze the effects of imperfect CSI to see if this conclusion remains true in the presence of channel estimation errors. Consider stage $m+1$ of V-BLAST detection where the symbols $x_{k_1}, x_{k_2}, \ldots, x_{k_m}$ have already been detected and assume that all m symbols in the previous stages are detected correctly. This assumption is used to simplify the analysis and to obtain insights into the effects of imperfect CSI. In the simulations, the full V-BLAST procedure is used and error propagation may occur. The remaining users have link gains $P_{k_{m+1}}, P_{k_{m+2}}, \ldots, P_{k_{N_t}}$ which are denoted $P_{(m+1)}, P_{(m+2)}, \ldots, P_{(N_t)}$. Similarly, $P_{(1)}, P_{(2)}, \ldots, P_{(m)}$ are the link gains of users $k_1, k_2, \ldots, k_m$. Consider the noise inflation which occurs in V-BLAST with imperfect CSI in both the detection and cancellation stages. The noise inflation is defined as the difference in noise power between the imperfect CSI case and the perfect CSI case.

1) Detection Stage: During detection, adapting the result in [13], the noise inflation for a ZF combiner is $(1-r_0^2)\sum_{j=1}^{N_t-m} P_{(j)}$. For an MRC combiner, adapting the result in [18], the noise inflation during detection is $(1-r_0^2)P_{(m+1)}$.

2) Cancellation Stage: The signal to be detected at stage $m+1$ is x_{m+1} and the corresponding received signal is

$$\mathbf{y}^{(m+1)} = \sum_{j=m+1}^{N_t} \mathbf{h}_{k_j} x_{k_j} + \mathbf{n}^{(m)}, \tag{7}$$

where $\mathbf{n}^{(m)} = \mathbf{n} + (\mathbf{h}_{k_1} - \hat{\mathbf{h}}_{k_1})x_{k_1} + \cdots + (\mathbf{h}_{k_m} - \hat{\mathbf{h}}_{k_m})x_{k_m}$ and $\mathbf{h}_j = r_0\hat{\mathbf{h}}_j + \sqrt{1-r_0^2}\mathbf{e}_j$, where $\mathbf{e}_j$ is the j^{th} column of $\mathbf{E}$. The definition of $\mathbf{n}^{(m)}$ uses the assumption that $\hat{x}_{k_j} = x_{k_j}$ for $j = 1, 2, \ldots, m$. Substituting for $\mathbf{h}_j$ in $\mathbf{n}^{(m)}$ gives

$$\begin{aligned}
\mathbf{n}^{(m)} &= \mathbf{n} + \sum_{j=1}^{m}((r_0-1)\hat{\mathbf{h}}_{k_j} + \sqrt{1-r_0^2}\mathbf{e}_{k_j})x_{k_j} \\
&= (r_0-1)\sum_{j=1}^{m}\hat{\mathbf{h}}_{k_j}x_{k_j} + \mathbf{n} + \sqrt{1-r_0^2}\sum_{j=1}^{m}\mathbf{e}_{k_j}x_{k_j}.
\end{aligned}$$

The covariance matrix of $\mathbf{n}^{(m)}$, denoted $\Sigma^{(\mathrm{m})}$, is given by

$$\begin{aligned}
\Sigma^{(\mathrm{m})} &= (r_0-1)^2\mathrm{E}[\sum_{j=1}^{m}\hat{\mathbf{h}}_{k_j}x_{k_j}\sum_{j=1}^{m}x_{k_j}^H\hat{\mathbf{h}}_{k_j}^H] \\
&+ \mathrm{E}[\mathbf{n}\mathbf{n}^H] + (1-r_0^2)\mathrm{E}[\sum_{j=1}^{m}\mathbf{e}_{k_j}x_{k_j}\sum_{j=1}^{m}x_{k_j}^H\mathbf{e}_{k_j}^H] \\
&= (r_0-1)^2\sum_{j=1}^{m}\hat{\mathbf{h}}_{k_j}\hat{\mathbf{h}}_{k_j}^H + \sigma^2\mathbf{I} \\
&+ (1-r_0^2)\sum_{j=1}^{m}P_{(j)}\mathbf{R}_r.
\end{aligned} \tag{8}$$

In (8), we have used $\mathrm{E}[\mathbf{e}_j\mathbf{e}_j^H] = P_{(j)}\mathbf{R}_r$ since $\mathbf{e}_j$ has the same statistics as $\mathbf{h}_j$. Taking the $(1,1)^{\text{th}}$ element of (8) gives the equivalent noise at stage $m+1$ as

$$(r_0-1)^2\sum_{j=1}^{m}|\hat{h}_{k_j1}|^2 + \sigma^2 + (1-r_0^2)\sum_{j=1}^{m}P_{(j)}, \tag{9}$$

where $\hat{h}_{k_j1}$ is the first element of $\hat{h}_{k_j}$. Hence, the noise inflation due to cancellation is

$$(r_0-1)^2\sum_{j=1}^{m}|\hat{h}_{k_j1}|^2 + (1-r_0^2)\sum_{j=1}^{m}P_{(j)}. \tag{10}$$

Combining the noise inflation due to cancellation in (10) with the noise inflation that during detection ($(1-r_0^2)P_{(m+1)}$ for MRC and $(1-r_0^2)\sum_{j=1}^{N_t-m}P_{(j)}$ for ZF) gives the overall noise inflation results:

$$\mathrm{MRC}: (r_0-1)^2\sum_{j=1}^{m}|\hat{h}_{k_j1}|^2 + (1-r_0^2)\sum_{j=1}^{m+1}P_{(j)},$$

$$\mathrm{ZF}: (r_0-1)^2\sum_{j=1}^{m}|\hat{h}_{k_j1}|^2 + (1-r_0^2)\sum_{j=1}^{N_t}P_{(j)}.$$

Hence, imperfect CSI has a greater effect on ZF which suffers from a noise inflation at stage $m+1$ which is greater than MRC by $(1-r_0^2)\sum_{j=m+2}^{N_t}P_{(j)}$. A similar analysis was given in [19], but the simple form of (10) was not given.

The analysis shows that ZF is affected more by imperfect CSI and that the difference is weighted by the total remaining link gain, i.e., $\sum_{j=m+2}^{N_t}P_{(j)}$. When the $P_{(j)}$ terms are small, then the difference is small and the effect on ZF and MRC will be similar. When the $P_{(j)}$ terms are large then there is a larger difference in the effect of imperfect CSI. However, when the $P_{(j)}$ terms are large the SERs are likely to be small and will not contribute greatly to the overall V-BLAST SER. For example, in Figure 2 with $\beta = 0.5$, the SERs were also calculated for each layer. In the first few layers, where strong users are present, the $P_{(j)}$ terms are large but the SERs are low (below 10^{-3} for SNR ≤ 0 dB). Hence, the overall SER (approximately 2×10^{-1} at SNR = 0 dB) is not affected by these results. In contrast, in the later stages, the SERs are dominant and here the $P_{(j)}$ terms are much smaller. Overall, the analysis suggests that the effects of imperfect CSI are likely to be similar for ZF and MRC and this is supported by the simulations in Section VI. This is also supported in related work [20] where the asymptotic performance of minimum mean squared error (MMSE) and MRC receivers is shown to be equivalent (and ZF behaves similarly to MMSE [1]).

B. Analysis of Correlation

When comparing a ULA with a SQ array, the SQ array will suffer if the antenna spacing is held constant as there are many more near-neighbors in a square layout. Hence, it is of interest in a MM context to investigate SQ and ULA dimensions which lead to similar levels of correlations. To avoid relying on any particular correlation model, we define a ULA and a SQ to be "equivalent" if the median inter-element spacing is identical. Furthermore, since closed form results for median spacing in a SQ appear to be intractable, we employ a continuous approximation to the problem where the antenna locations are uniformly located on a line segment of length b_{ULA}, uniform on $[0, b_{\text{ULA}}]$ for ULA, and on a square of width b_{SQ} uniform on $[0, b_{\text{SQ}}] \times [0, b_{\text{SQ}}]$ for SQ. The continuous approximation is particularly is appealing for MM where a large number of antennas are considered.

For the ULA a random inter-element spacing can be given as $Z_{\text{ULA}} = |X_1 - X_2|$, where X_1 and X_2 are i.i.d uniform variables on $[0, b_{\text{ULA}}]$. The CDF of Z_{ULA} is well-known as $X_1 - X_2$ is triangular. Using standard transformation theory [21, page 136 and 137] the CDF of Z_{ULA} is given by

$$F_{Z_{\text{ULA}}}(z) = 2b_{\text{ULA}}^{-2}(b_{\text{ULA}}z - z^2/2), \quad 0 \le z \le b_{\text{ULA}}. \quad (11)$$

For the SQ case, the inter-element spacing is defined by $Z_{\text{SQ}} = \sqrt{(X_1 - X_2)^2 + (Y_1 - Y_2)^2}$, where X_1, X_2, Y_1 and Y_2 are i.i.d uniform variables on $[0, b_{\text{SQ}}]$. Using standard transformation theory [21, page 86-142 and 135-142], we obtain the CDF of Z_{SQ} as

$$F_{Z_{\text{SQ}}}(z) = b_{\text{SQ}}^{-4}(\pi b_{\text{SQ}}^2 z^2 - \tfrac{8}{3} b_{SQ} z^3 + \tfrac{z^4}{2}), \quad 0 \le z \le b_{\text{SQ}}. \quad (12)$$

To find the medians $(m_{\text{ULA}}, m_{\text{SQ}})$, solve $F_{Z_{\text{ULA}}}(m_{\text{ULA}}) = \frac{1}{2}$ using (11) and solve $F_{Z_{\text{SQ}}}(m_{\text{ULA}}) = \frac{1}{2}$ using (12), giving

$$m_{\text{ULA}} = (1 - \frac{\sqrt{2}}{2})b_{\text{ULA}}, \quad (13)$$

$$m_{\text{SQ}} = 0.512\, b_{\text{SQ}}. \quad (14)$$

To match the median spacing, $m_{\text{ULA}} = m_{\text{SQ}}$ and we obtain

$$b_{\text{SQ}} = \frac{1 - \frac{\sqrt{2}}{2}}{0.512} b_{\text{ULA}} = 0.572 b_{\text{ULA}}. \quad (15)$$

This suggests that in order for a SQ to have a similar level of correlation as a ULA, the width of the square should be around 57% of the length of the linear array. A SQ array with the same antenna spacing as the ULA has a width of approximately 9% of the ULA length. In contrast, using (15), the "Adjusted SQ" array has a width which is 57% of the ULA length. This preliminary observation has some impact on MM design as the SQ is not as compact as might be hoped in comparison to a ULA. The usefulness of this result is also demonstrated in Section VI.

VI. Results

In this section, we consider the performance of MM receivers with imperfect CSI and correlated channels using numerical simulations. Performance is measured by the symbol error rate (SER) assuming quadrature phase shift keying modulation. The results were averaged over the users and up to 10^5 independent channel realizations. The system size is $N_r = 100$, $N_t = 10$ and the baseline parameters are: $\beta = 0.5$, $\alpha_u = 0.1$, array type = ULA. Where other parameters are used, they are given in the figure captions. In the figures, the SNR is the SNR of the strongest UE, given by $\frac{\mathcal{A}}{\sigma^2}$, where $\mathcal{A} = 1$ without loss of generality.

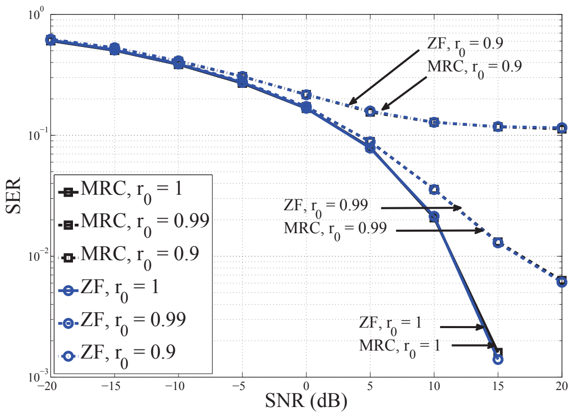

Fig. 1: SER vs. SNR with MRC V-BLAST and ZF V-BLAST for different CSI accuracy (r_0).

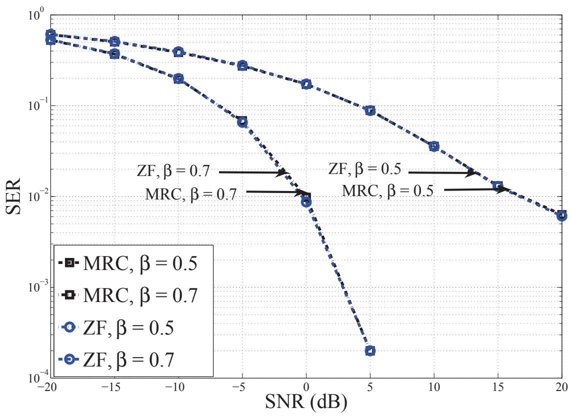

Fig. 2: SER vs. SNR with MRC V-BLAST and ZF V-BLAST for different user power distributions (β), $r_0 = 0.99$.

Figure 1 shows SER vs. SNR with MRC V-BLAST and ZF V-BLAST for different values of r_0, $r_0 \in \{1, 0.99, 0.9\}$. The effect of imperfect CSI on MRC V-BLAST and ZF V-BLAST is almost identical here when $\alpha_u = 0.1$ is used. Hence, with small correlation, the performance of MM using MRC V-BLAST and ZF V-BLAST is virtually the same and it is better to use MRC V-BLAST, which has a lower complexity. However, the MM performance is strongly affected by the level of CSI. When $r_0 = 0.9$, the performance of the system is extremely poor.

Figure 2 shows SER vs. SNR with MRC V-BLAST and ZF V-BLAST for different values of β, $\beta \in \{0.5, 0.7\}$ and $r_0 = 0.99$. The effect of changing the channel powers on MRC V-BLAST and ZF V-BLAST is very similar with small correlation ($\alpha_u = 0.1$). However, the system performance is strongly affected by β, with a better performance achieved when channel powers are less distributed ($\beta = 0.7$).

Figure 3 shows SER vs. SNR with MRC V-BLAST for SQ and ULA and $\alpha_u \in \{0.1, 0.9\}$. Inter-antenna spacing is the same for both configuration. The effect of changing the type of array on MRC V-BLAST and ZF V-BLAST is similar. From

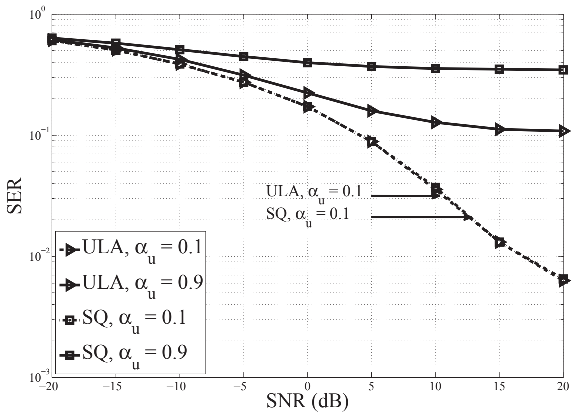

Fig. 3: SER vs. SNR with MRC V-BLAST with different values of antenna correlation (α_u) and array types (SQ, ULA), $r_0 = 0.99$.

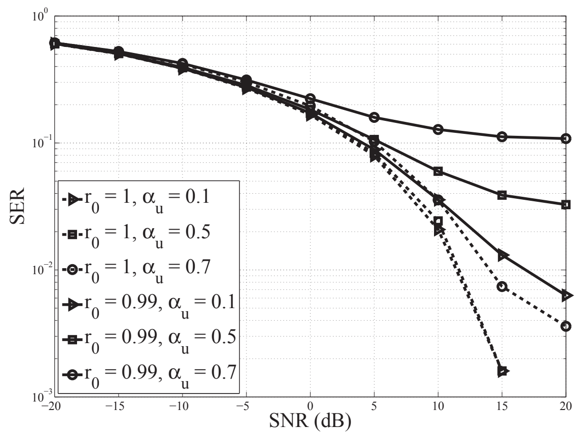

Fig. 5: SER vs. SNR with MRC V-BLAST for different values of antenna correlation (α_u) and CSI accuracy (r_0).

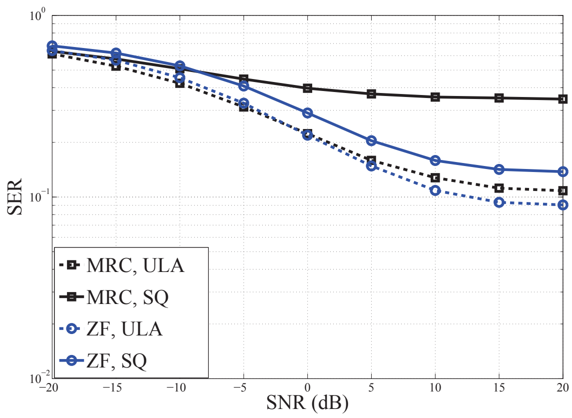

Fig. 4: SER vs. SNR with MRC V-BLAST and ZF V-BLAST for different array types (SQ, ULA), $r_0 = 0.99$ and $\alpha_u = 0.9$.

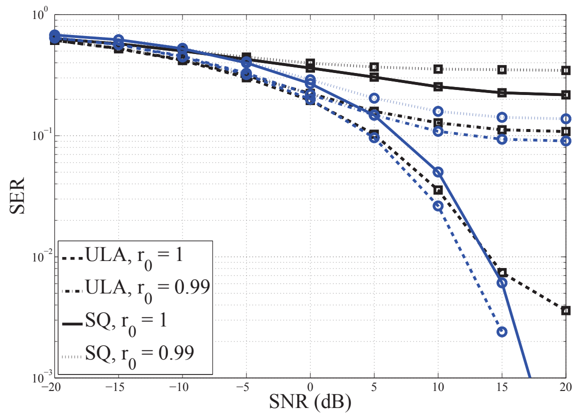

Fig. 6: SER vs. SNR with MRC V-BLAST and ZF V-BLAST for different values of CSI accuracy (r_0) and array types (SQ, ULA). $\alpha_u = 0.9$, MRC V-BLAST is denoted by squares and black lines and ZF V-BLAST by circles and blue lines.

Figure 3, we observe that performance deteriorates as antenna correlation increases and that the ULA has lower SER for high α_u. For low α_u values, SQ and ULA are similar.

Figure 4 shows SER vs. SNR with MRC V-BLAST and ZF V-BLAST for different array types (SQ and ULA), $r_0 = 0.99$ and $\alpha_u = 0.9$. Here, the correlation is high ($\alpha_u = 0.9$) and the increased levels of correlation found in a SQ configuration mean that the SERs for SQ are worse than for the corresponding ULA for both MRC V-BLAST and MRC V-BLAST. The difference between ULA and SQ is increased as α_u increases, i.e., for $\alpha_u = 0.1$, the average correlation between two antennas is 0.016 (SQ) and 0.012 (ULA). However, for $\alpha_u = 0.9$, the average correlation is 0.60 (SQ) and 0.17 (ULA).

Figure 5 shows SER vs. SNR for $r_0 \in \{1, 0.99\}$ and $\alpha_u \in \{0.1, 0.5, 0.7\}$. By increasing the correlation level, the performance of the receiver deteriorates, especially in the imperfect CSI case. The correlation level has little effect in the case of perfect channel estimation and $\alpha_u = 0.1$ and 0.5.

Figure 6 shows SER vs. SNR with MRC V-BLAST and ZF V-BLAST for different values of r_0, $r_0 \in \{1, 0.99\}$ and array types (SQ and ULA), $\alpha_u = 0.9$. Squares and black lines denote MRC V-BLAST and circles and blue lines denote ZF V-BLAST. Figure 6 demonstrates that MRC V-BLAST does not always perform in a similar way to ZF V-BLAST. For example, the ZF V-BLAST, SQ results with $r_0 = 1$ are far better than the MRC V-BLAST, SQ, $r_0 = 1$ results. Also, when $\alpha_u = 0.9$ and $r_0 = 1$ the performance of ZF V-BLAST with ULA is much better than it is with SQ. Both effects are due to correlation. Firstly, with equal antenna spacing the SQ has higher correlations and so the ZF results are worse for SQ compared to ULA. Secondly, correlation has a much greater impact on MRC for $r_0 = 1$ as ZF removes the interference while interference increases with correlation for MRC. To see this effect, the interference term $\sum_{j \neq m} \mathbf{h}_m^H \mathbf{h}_j \mathbf{h}_j^H \mathbf{h}_m$ in (5) is zero for ZF where as for MRC, each $\mathbf{h}_m^H \mathbf{h}_j \mathbf{h}_j^H \mathbf{h}_m$ is non-zero and is increased when $\mathbf{h}_m$ and $\mathbf{h}_j$ are correlated. This can be quantified numerically by considering the power of the interference term, $\mathrm{E}[\sum_{j \neq m} \mathbf{h}_m^H \mathbf{h}_j \mathbf{h}_j^H \mathbf{h}_m]$. From (3), we see

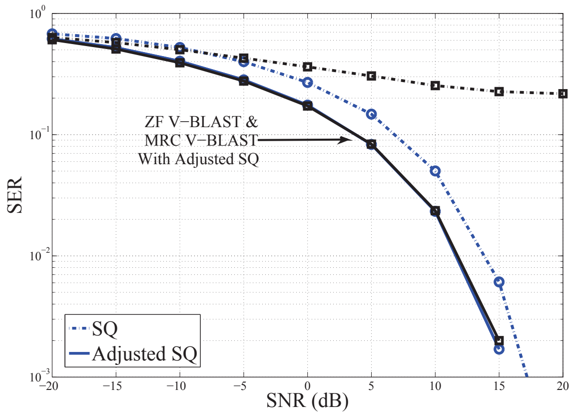

Fig. 7: SER vs. SNR with MRC V-BLAST and ZF V-BLAST for different SQ array sizes (SQ, Adjusted SQ). $\alpha_u = 0.9$, $r_0 = 1$, MRC V-BLAST is denoted by squares and black lines and ZF V-BLAST by circles and blue lines.

that $\mathrm{E}[\sum_{j\neq m} \mathbf{h}_m^H \mathbf{h}_j \mathbf{h}_j^H \mathbf{h}_m] = \mathrm{tr}(R_r^2) P_m \sum_{j\neq m} P_j$. For independent channels this power is $N_r P_m \sum_{j\neq m} P_j$ whereas for perfectly correlated channels the power is $N_r^2 P_m \sum_{j\neq m} P_j$. This shows the potential interference increase due to correlation and is particularly noticeable in MM where N_r is large. For the ULA, an exact calculation is possible giving $\mathrm{E}[\sum_{j\neq m} \mathbf{h}_m^H \mathbf{h}_j \mathbf{h}_j^H \mathbf{h}_m] = \frac{N_r - \alpha_u^2 (N_r - 2\alpha_u^{2N_r} + 2)}{(1-\alpha_u^2)^2} P_m \sum_{j\neq m} P_j$.

Figure 7 shows SER vs. SNR with MRC V-BLAST and ZF V-BLAST for different SQ array sizes (SQ, Adjusted SQ), $\alpha_u = 0.9$, $r_0 = 1$. MRC V-BLAST is denoted by squares and black lines and ZF V-BLAST by circles and blue lines. By adjusting the width of the SQ array, MRC V-BLAST and ZF V-BLAST can behave similarly and the Adjusted SQ SER results are similar for both ULA and SQ (See Figure 6). This shows that the simple closed form analysis in Section V-B may be beneficial in terms of dimensioning arrays. Note that the MRC V-BLAST receiver may require a larger array dimension than the ZF V-BLAST receiver. Nevertheless, given enough spacing, the performance of MRC V-BLAST can approach ZF V-BLAST performance. We showed that by equating median antenna spacings (15) that a SQ array needed to be increased in width from 9% of the ULA length (for equal inter-element spacing) to 57% in order to achieve similar performance.

VII. Conclusion

We have investigated V-BLAST with MRC and ZF in a massive MIMO system with imperfect CSI and channel correlation. We found that MRC V-BLAST performance can approach ZF V-BLAST, despite its lower complexity, for a range of imperfect CSI levels, different channel powers and different types of array, as long as the correlation is not too high. When the correlation level is increased, the performance of V-BLAST using either MRC or ZF is degraded and the gap between the two can become substantial. When the correlation is high, the uniform linear array also offers large gains over the square array if the same spacing is used. However, if the square array is dimensioned using the closed form result derived, then MRC V-BLAST and ZF V-BLAST with SQ or ULA are shown to behave similarly.

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