A Modified Compressed Sampling Matching Pursuit Algorithm on Redundant Dictionary and Its Application to Sparse Channel Estimation on OFDM

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***Abstract*—This paper proposes a modified compressed sensing algorithm for sparse multipath channel estimation in wideband OFDM systems. By using a virtual channel representation, the sparse nature of a multipath channel is revealed and exploited. Unlike other proposed sparse channel estimation schemes, it is noted that a truly sparse assumption on the channel impulse response (CIR) must factor in the leakage of energy of each multipath component resulting from bandpass filtering and the resulting limited bandwidth. We propose to represent the CIR as a strongly correlated redundant frame so that the representation is truly sparse. However, the introduction of correlated frames complicates the canonical compressed sensing problem. A model-based modification to CoSaMP [1] algorithm is thus proposed for recovering the CIR. Simulation results are presented indicating a significant improvement over straightforward application of canonical compressed sensing techniques.**

I. Introduction

In a scattering environment, the radio signal from transmitter will be reflected, scattered and diffracted by surrounding objects and will undergo different delays, attenuations, Doppler shifts and ultimately arrive at the receiver from different directions. These effects result in a superposition of multiple independently delayed, attenuated and frequency shifted version of the transmitted signal at the receiver. On one hand, this superposition at the receiver could result in a deep fade in time, frequency or angular domains, which adversely affects the capacity and reliability of the communication system. On the other hand, multipath fading, if managed appropriately, can provide diversity and degree of freedom and, hence, increase the capacity, thereby improve the reliability of wireless communication system. To exploit multipath fading requires making the channel state information (CSI) available to the system (transmitter and/or receiver). Commonly, CSI is obtained by training-based channel estimation techniques using pilot tones. Traditional channel estimation schemes employ least-square (LS) or minimum mean square error (MMSE) methods, which inherently assume a richly scattered multipath channel.

However growing number of both theoretical and experimental studies [2], [3] indicate that practical wireless channels tend to exhibit sparseness in the high dimensional signal space inherent in a wideband communication system which enables high time resolution. Traditional algorithms thus perform poorly in sparse channel scenario. Recent theoretical development in compressed sensing (CS) [4]–[7] provides a framework to study and exploit the sparse structure of wireless communication channels. CS enables the efficient reconstruction of sparse signals from a small number of measurements. Thus, this provides the opportunity to either increase the accuracy of the channel estimate thereby improving the capacity and reliability or reduce the number of pilot symbols required to meet the desired system performance requirement.

Prior research formulated the pilot assisted channel estimation problem as a standard compressed sensing problem [3], [8]. Note that even if the physically underlying multipath channel is sparse, the band-limited nature of the wireless communication channel coupled with pulse-shaping causes the sampled impulse response of multipath channel to generally not be sparse in the classical sense. In [3] proved that this baseband channel is compressible or almost sparse. This motivates us to impose a model wherein that the impulse response is a superimposition of multiple time-delayed and amplitude-scaled replica of autocorrelation function of the impulse response of the pulse-shaping filter. This structure falls into the framework of model-based compressed sensing [9]. This may further increase channel estimation accuracy if exploited properly. Meanwhile a lot of works, such as [10]–[12], studied the performance of various sensing algorithms for sparse channel estimation. But they all assume the CIR's are strictly sparse which in general, as mentioned above, is not true.

The limited band and filtering effect in the wireless communication will result in non-strictly sparse discrete CIR.

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This phenomenon is analogous to the leakage phenomenon in spectral sensing problems. The latter is a result of finite length window in time domain. To combat this problem [13] proposed to solve the problem using a redundant frame which represents the signal of interest in a zero-padded DFT basis. We use the time-frequency dual version of this idea to represent the CIR in a time-domain redundant frame. But difficulty in recovering signal arises as the frame becomes redundant. We will try to alleviate this problem by proposing a modification to the CoSaMP [1] algorithm and applying the idea of model-based compressed sensing [9].

This paper is organized as follows. In Section II, we will review a widely used framework in the communications literature that discretize physical channel. In Section III, we formulate the channel estimation problem in OFDM system as a model-based CS problem; In Section IV, we propose a successive-canceling (SC) CoSaMP algorithm for recovering the CIR. In Section V, the simulation results are presented and some comparing will be done between the canonical CoSaMP algorithm and compressed sensing based algorithms. We then devote conclusion and some discussion to Section VI .

II. Multipath Wireless Channel

Lots of researches on the estimate of doubly-selective channels via pilot-based method have been done. Some of them are based on the assumption of underlying richly scattering environment. In contrast, in real world, a physical channel often exhibits sparseness in the channel impulse response. That is, the number of dominant resolvable paths is very small comparing to the dimension of the channel. This phenomenon is more prevalent in communication systems operating on wideband and ultra-wideband modes. In delay-Doppler domain, sparse channels are mostly zero or near-zero except in some small number of limited regions. In the sequel of this section, we will review a virtual channel representation presented in [2].

The virtual channel model for doubly-selective channels captures the interaction between the physical paths and the signal space. With arbitrarily high delay-Doppler resolution, each physical propagation path can be represented as a distinct pulse in the delay-Doppler domain. Virtual channel model provides a low-dimensional approximation of the underlying multipath environment through uniform sampling of the delay-Doppler domain at resolution commensurate with the signaling duration and bandwidth. This model provides us a framework to establish and analyze in theory the sparseness of effective channel and hence makes it possible to formulate the channel estimation problem as a compressive sensing problem.

A single-input single-output (SISO) communication channel can be characterized as a linear time-variant system with time-varying channel impulse response $h(t, \tau)$. Hence the correspond input-output relation of such system is given by [14],

$$\begin{aligned} y(t) &= \int_0^{\tau_{max}} h(t,\tau)x(t-\tau)d\tau \\ &= \int_0^{\tau_{max}} \int_{-\nu_{max}/2}^{\nu_{max}/2} C(\nu,\tau)x(t-\tau)e^{j2\pi\nu t} d\nu d\tau \qquad (1) \end{aligned}$$

where $x(t)$ and $y(t)$ denote the transmitted and received waveforms, respectively. And $C(\nu, \tau)$ is called delay-Doppler spreading function which is the Fourier transform of $h(t, f)$ with respect to t. And τ_{max} is the maximum excess delay induced by the channel; ν_{max} is two-sided Doppler spread which determines the maximal possible Doppler shift caused by the channel.

Furthermore, if we suppose there are N_{path} dominant resolvable propagation paths, the delay-Doppler spreading function of the channel is expressed as

$$C(\nu,\tau) = \sum_{i=1}^{N_{path}} \alpha_i \delta(\nu-\nu_i)\delta(\tau-\tau_i) \qquad (2)$$

and the transmitted and received waveforms are related by

$$r(t) = \sum_{i=1}^{N_{path}} \alpha_i e^{j2\pi\nu_i t} s(t-\tau_i) \qquad (3)$$

where $\alpha_i \in \mathbb{C}$, $\nu_i \in [-\nu_{max}/2, \nu_{max}/2]$ and $\tau_i \in [0, \tau_{max}]$ are the complex path gain, the delay and the Doppler shift associated with the i-th physical path, respectively. For a static channel, i.e. time-invariant channel, this formulation can be further simplified by letting $\nu_i = 0$ for $i = 1 \ldots N_{path}$ and we have

$$r(t) = \sum_{i=1}^{N_{path}} \alpha_i s(t-\tau_i) \qquad (4)$$

Let's now consider a cyclic-prefix (CP) OFDM system in the baseband. Let N_{cp} and N denote the length of cyclic prefix and number of subcarriers, respectively. Hence $N_{cp}+N$ is the symbol duration in terms of multiple of sample period. During one symbol duration the discrete-time OFDM transmit signal is given by

$$s[n] = \sum_{k=0}^{N-1} a_k e^{j2\pi\frac{kn}{N}}, \; with \; n = -N_{cp} \ldots N-1 \qquad (5)$$

where a_k is the symbol sent on the k-th subcarrier and the first N_{cp} time domain samples constitute the cyclic prefix. The discrete signal is subsequently converted into continuous-time transmit signal by some pulse-shaping filter, with impulse response $p_{tr}(t)$. For example, squared-root raised-cosine (SRRC) filter is frequently used. As a result, we have the continuous-time transmit signal

$$\begin{aligned} s(t) &= \sum_{m=-N_{cp}}^{N-1} s[m]p_{tr}(t-mT) \qquad (6) \\ &= \sum_{m=-N_{cp}}^{N-1} \sum_{k=0}^{N-1} a_k e^{j2\pi\frac{km}{N}} p_{tr}(t-mT) \qquad (7) \end{aligned}$$

Assuming the channel is static, we may combine eq.(4) and eq.(6). Then at the receiver's front end channel output is

$$r(t) = \sum_{i=1}^{N_{path}} \alpha_i \sum_{m=-N_{cp}}^{N-1} s[m] p_{tr}(t - \tau_i - mT) + w(t) \quad (8)$$

where $w(t)$ is a zero-mean, circularly-symmetric complex Gaussian process with power spectral density N_0.

The receiver will further match filter and sample the channel output to convert it into discrete-time signal

$$r[n] = \int_{-\infty}^{+\infty} r(t) p_{tr}^*(t - nT) dt \quad (9)$$

$$= \sum_{i=1}^{N_{path}} \alpha_i \sum_{-Ncp}^{N-1} s[m] p((n-m)T - \tau_i) + w[n] \quad (10)$$

where $p(s) = \int_{-\infty}^{\infty} p_{tr}(t) p_{tr}^*(t-s) dt$ is the autocorrelation of pulse-shaping function $p_{tr}(s)$. Notice that if we assume $p_{tr}(s)$ is a SRRC function, $p(s)$ is therefore a raised-cosine function. And the input-output relation can be written as a convolution between the discrete-time transmit signal and sampled impulse response of the effective channel,

$$r[n] = \sum_{-Ncp}^{N-1} s[m] h[n-m] + w[n] \quad (11)$$

where $h[n] = \sum_{i=1}^{N_{path}} \alpha_i p[T(n - \frac{\tau_i}{T})]$. It is reasonable to assume that $h[n] \approx 0$ for $n \geq N_{cp}$. Hence after removing the CP and taking FFT at the OFDM demodulator, the relationship between transmit symbols a_k's and the received symbols y_k's, in vector form, is given by

$$\boldsymbol{y} = \boldsymbol{W}^H \boldsymbol{X} \boldsymbol{W} \boldsymbol{h} \quad (12)$$

where $\boldsymbol{h} = [h[0]\ h[1]\ \ldots h[N_{cp}-1]]^T$, $\boldsymbol{W}$ consists of the first N_{cp} columns of an N by N unitary DFT matrix and $\boldsymbol{X} = diag\,[a_0\ a_1\ \ldots a_{N-1}]$.

III. Modeled Sparsity of Channel Impulse Response

We have established that, for a static channel, the sampled channel impulse response can be written as $h[n] = \sum_{i=1}^{N_{path}} \alpha_i p[T(n - \frac{\tau_i}{T})]$. In [3], it is proved that the autocorrelation function $p[nT - \tau_i]$ of commonly used pulse-shaping waveforms, such as SRRC and sinc function, has most energy located within the neighborhood of τ_i/T which consists of samples within $[\lfloor \tau_i/T \rfloor - \Delta + 1,\ \lfloor \tau_i/T \rfloor + \Delta]$. For $p[n] = sinc[n]$ and $\delta = 1$ by eq.(23) in [3] at least 0.93% of the energy of $p[n]$ is contained in the neighborhood of τ_i/T comprising two samples. If $p[n]$ is raised cosine this number can raise to 99%. Therefore, $h[n]$ can be considered a $2N_{path}$-sparse signal as $N_{path} \ll N_{cp}$.

By the last argument one may reconstruct $h[n]$ via compressed sensing technique. But it is still possible to improve this further. Indeed, the argument above does not consider the fact that the dominant taps around τ_i/T of $h[n]$ have strong dependency to each other through the autocorrelation function $p[n]$. By exploiting this higher level model, further improvement in the performance is expected. To exploit this property we propose consider in a finer, but still finite grid. Suppose τ_i take values in a large equi-spaced grid spans $[0, N_{cp}T]$. That is,

$$\tau_i \in \left\{0, \frac{T}{\beta}, \frac{2T}{\beta}, \ldots, (N_{cp} - \frac{1}{\beta})T\right\} \quad (13)$$

Combining this with eq.(11) we can express the channel impulse response as

$$h[n] = \sum_{i=1}^{N_{path}} \alpha_i p\left[T\left(n - \frac{\ell_i}{\beta}\right)\right],\ with\ \ell = 0, 1, ..., \beta N_{cp} \quad (14)$$

where $\ell_i = \tau_i \beta / T$. Stacking $h[n]$ into a vector gives

$$\boldsymbol{h} = \boldsymbol{P} \boldsymbol{\alpha} \quad (15)$$

where $\boldsymbol{\alpha}$ is a $\beta N_{cp} \times 1$ vector with the j-th entry $(\boldsymbol{\alpha})_j = \alpha_i$ if $j = \ell_i$ and $(\boldsymbol{\alpha})_j = 0$ otherwise. Hence $\boldsymbol{\alpha}$ by definition is a N_{path}-sparse vector while $\boldsymbol{h}$ is approximately $2N_{path}$-sparse. And P is a $N_{cp} \times \beta N_{cp}$ matrix defined as

$$\boldsymbol{P} = \begin{bmatrix} \boldsymbol{p}_1 & \boldsymbol{p}_2 & \ldots & \boldsymbol{p}_{\beta N_{cp}} \end{bmatrix} \quad (16)$$

where each column of $\boldsymbol{P}$ is $\boldsymbol{p}_j = [p(T(-\frac{j}{\beta})), p(T(1 - \frac{j}{\beta})), \ldots, p(T((N_{cp} - 1) - \frac{j}{\beta}))]^\top$.Combining eq.(12) and eq.(16) gives

$$\boldsymbol{y} = \boldsymbol{W}^H \boldsymbol{X} \boldsymbol{W} \boldsymbol{P} \boldsymbol{\alpha} \quad (17)$$

The pilot assisted channel sensing relation is established as

$$\boldsymbol{Y}_{pilot} = \boldsymbol{X}_{pilot} \boldsymbol{W}_{pilot} \boldsymbol{P} \boldsymbol{\alpha} \quad (18)$$

where $\boldsymbol{X}_{pilot}$ is a matrix with the pilot symbols on the diagonal, $\boldsymbol{W}_{pilot}$ is a partial unitary DFT matrix with only rows corresponding to pilot tone subcarriers and $\boldsymbol{Y}_{pilot}$ contains the received symbols on pilot subcarriers. Comparing this to canonical compressed sensing formulation $\boldsymbol{y} = \boldsymbol{\Phi} \boldsymbol{x}$, where x is an unknown sparse vector, we find that the sensing matrix is given by

$$\boldsymbol{\Phi} = \boldsymbol{X}_{pilot} \boldsymbol{W}_{pilot} \boldsymbol{P} \quad (19)$$

Unfortunately, this sensing matrix does not satisfy the so-called Restricted Isometry Property (RIP) [15] condition which quantifies the ability to preserve information under linear transformation of a sparse matrix. This is mostly due to the strong correlation between neighboring columns of matrix $\boldsymbol{P}$ in eq.(16). To alleviate this problem, we need to further restrict the vector $\boldsymbol{\alpha}$ to a subset of all N_{path}-sparse signals. To state this formally, let α_Ω represent the entries of $\boldsymbol{\alpha}$ to the set of index $\Omega \subseteq \{0, 1, \ldots \beta N_{cp}\}$, and let Ω^C denote the complement of the set Ω. We assume the Ω bears the following structure

Definition 1: The support set Ω of channel response vector $\boldsymbol{\alpha}$ is given by

$$\Omega = \{\omega_1, \omega_2, \ldots, \omega_{N_{path}}\}$$

Inputs: Basis matrix $\Phi^H\Phi$, signal residual estimate e, target sparsity K, signal dimension N
Output: support set estimate according to the signal model

$\Omega \leftarrow \emptyset$, $b = 0$, $I = \{0, 1, \ldots N-1\}$
while $|\Omega| \leq K$ **do**
  $\omega \leftarrow arg\,max_{i\in I}\{e_i\}$
  $\Omega \leftarrow \Omega \cup \{\omega\}$
  $I \leftarrow I \backslash \mathbb{E}()$ $\{\mathbb{E}(\Omega)$ selects the set of support index in I that are conflict with the ones already in Ω according to the model$\}$
  $b = (\Phi^H\Phi)|_\omega$
  $e \leftarrow e - \frac{e^\top b}{\|b\|_2^2} b$
end while
return Ω

Fig. 1. Structured Signal Approximate Algorithm via Successive Canceling Method

Inputs: CS matrix $\boldsymbol{\Phi}$, Measurement vector $\boldsymbol{Y}_{pilot}$, Signal Approximation Algorithm $\mathbb{M}$, Signal Sparsity K
Output: Structured Sparse approximation of the channel impulse response vector $\hat{\boldsymbol{\alpha}}$

$\hat{\alpha}_0 = 0, r = \boldsymbol{Y}_{pilot}, t = 0$
while $|\Omega| \leq K$ **do**
  $t = t + 1$
  $e \leftarrow \Phi^H r$
  $\Omega \leftarrow \mathbb{M}(e, \Phi^H\Phi, 2K)$
  $T \leftarrow \Omega \cup supp(\hat{\alpha}_{t-1})$
  $b|_T \leftarrow \Phi_T^\dagger Y_{pilot}, b|_{T^C} \leftarrow 0$
  $supp_i \leftarrow \mathbb{M}(e, \boldsymbol{I}, K)$ $\{\boldsymbol{I}$ is an identity matrix$\}$
  $\hat{\alpha}|_{supp} \leftarrow b|_{supp}, \hat{\alpha}|_{supp^C} \leftarrow 0$
  $r \leftarrow \boldsymbol{Y}_{pilot} - \Phi\hat{\alpha}_i$
end while
return $\hat{\alpha} \leftarrow \hat{\alpha}_i$

Fig. 2. Multipath Channel Estimation via Successive Canceling CoSaMP

where $\omega_i \in \{0, 1, \ldots \beta N_{cp}\}$, and they satisfies

$$(\forall i, j)\ \ |\omega_i - \omega_j| \geq \beta$$

and we collect all such allowed support sets into a collection

$$\left\{\Omega_1, \Omega_2, \ldots \Omega_{m_{N_{path}}}\right\} \tag{20}$$

where $m_{N_{path}} = \beta^{N_{path}}\binom{N_{cp}}{N_{path}}$ is the total number, which depends on the sparsity N_{path}, of allowed support configurations.
Hence the valid support of $\boldsymbol{\alpha}$ shall be one of these Ω's. Formally, $\boldsymbol{\alpha}$ shall satisfies $\boldsymbol{\alpha}_{\Omega^C} = 0$ and $\boldsymbol{\alpha}_\Omega \in \mathbb{C}^{N_{path}}$ with Ω satisfies Definition 1. And we claim that the sensing matrix Φ defined by eq.(19) satisfies the so-called $\mathcal{M}_{N_{path}}$ restricted isometry property ($\mathcal{M}_{N_{path}}$-RIP) in [9] with constant $\delta_{N_{path}}$ for all eligible $\boldsymbol{\alpha}$. The proof of the claim is out of the scope of this paper.

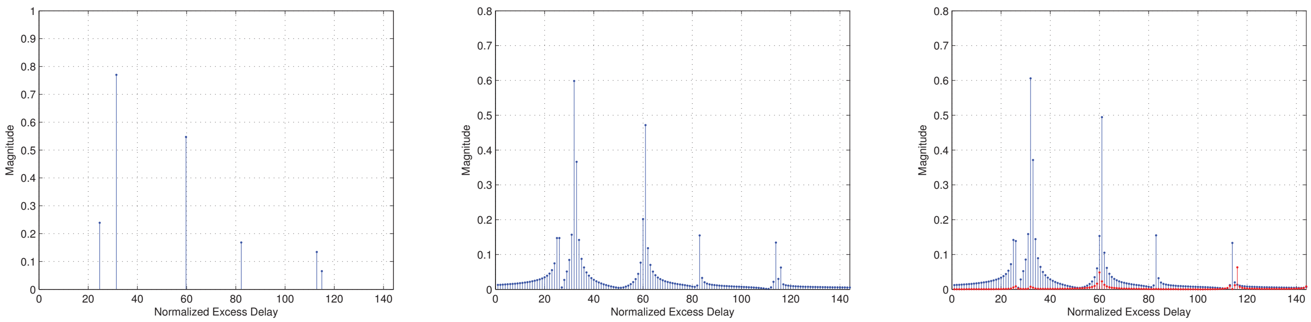
IV. Channel Estimation Algorithm based on Compressed Sensing

We choose to adopt the framework of Model-based CoSaMP algorithm proposed in [9] which is a modification of the CoSaMP algorithm [1]. Before we describe the Model-based CoSaMP algorithm in detail, we first propose a support set selecting algorithm, shown in Fig.1, which is inspired by successive canceling algorithm widely used in CDMA systems. We denote this algorithm by $\mathbb{M}(e, \Phi^H\Phi, K)$. The signal residual estimate $\boldsymbol{e}$ is obtained by matched filtering each column of the sensing matrix Φ with the compressed sensing measurement. In symbols $\boldsymbol{e} = \boldsymbol{\Phi}^H\boldsymbol{y} = \boldsymbol{\Phi}^H\boldsymbol{\Phi}\boldsymbol{x}$. Hence $\boldsymbol{e}$ can serve as a proxy for the signal because the matrix $\Phi^H\Phi$ has most of its large values in magnitude near the diagonal. So each peak of $\boldsymbol{e}$ points toward the approximate location of a large component of $\boldsymbol{x}$. To improve the approximation accuracy, inspired by the successive canceling method, each time we identify a large component in $\boldsymbol{x}$ we subtract its contribution to the proxy $\boldsymbol{e}$ hoping its interference to other component in the measurement will be mostly canceled. And the method $\mathbb{E}(\Omega)$ will select those index that are conflict with the index already exist in Ω in accordance with definition 1. Then the complement of $\mathbb{E}(\Omega)$, i.e. $I\backslash\mathbb{E}(\Omega)$, is the feasible index that can be added to Ω in the next iteration. We now in Fig.2 give our algorithm for the channel estimation problem formulated in last Section. Once we have the estimate of $\boldsymbol{\alpha}$, we may recover the discrete-time channel impulse response using eq.(15).

V. Simulation and Numerical Analysis

In the simulation an OFDM scheme with $N = 2048$ subcarriers, $N_{cp} = 144$ cyclic-prefix length and 4-QAM modulation is employed. This is specified in 3GPP LTE-A standard, specifically 3GPP TS 36.201. The multipath channel have $N_{path} = 12$ underlying physical multipath component. Each excess delay value τ_i is uniformly chosen from the set given in eq.(13) and the complex valued α_i are picked from i.i.d. zero-mean, unit-variance complex circularly symmetric Gaussian random variable. This channel model is widely accepted in modern wireless communication standards. For compressed sensing based channel estimation, 80 pilot tones are used in each OFDM symbol and their location in frequency domain is also randomly determined, while 144 pilot tones are arranged in comb pattern for MMSE based estimation. The CSI is estimated based on these pilot tones. Moreover, each OFDM symbol will be generated independently and will experience independent multipath channel. We first show a typical instance of the channel and its estimate via proposed SC-CoSaMP on a redundant frame in Fig.3. And then the performance of systems working with different channel estimation schemes are shown in Fig.4 where symbol error rate (SER) is used as the measure of performance.

Specifically, in Fig.4 the performance of channel estimation



(a) Physical Multipath Channel

(b) Sampled Multipath Channel

(c) Estimated Channel via SC-CoSaMP and its Estimation Error

Fig. 3. A realization of multipath channel and its recovery via proposed algorithm: a) truly sparse physical multipath channel; b) sampled channel impulse response; c) estimate of channel impulse response via proposed algorithm.

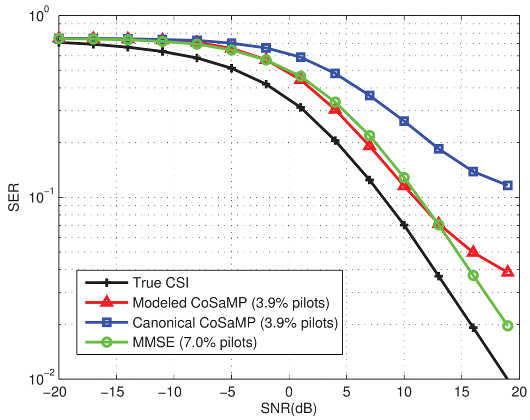


Fig. 4. SER performance of 1) True CSI is available to the receiver, 2) Channel Estimation via proposed scheme, 3) Channel Estimation via canonical CoSaMP algorithm

scheme based on the algorithm proposed in the paper is compared along with canonical CoSaMP and MMSE based algorithms. Moreover, the SER performance of an oracle receiver that knows the CSI exactly serves as a lower bound on the SER. The performance of such oracle system is only affected by the fading and additive white noise. The results shows that the performance of structured recovery scheme on a finer grid uniformly superior to the simple adoption of canonical compressed sensing algorithms. The reason for this gain is because the proposed scheme not only captures the largest components of the discrete-time CIR $h[n]$ but also be able to identify the smaller components. And a noteworthy observation is that by using only less than 4% pilots the proposed algorithm achieves performance that is comparable to what traditional MMSE algorithm using 7% pilots can do. This is especially true in low SNR region. It is also observed that in low-SNR region the system performance is limited by the AWGN channel while in high-SNR region the CSI mismatch contributes most to the SER.

VI. Conclusion

In this paper, we proposed a sparse multipath channel estimation scheme by applying model based compressed sensing technique to a finer sample grid. The simulation results indicate that with only 4% dedicated pilot sub-carriers the system performance is uniformly superior to previously proposed schemes based on canonical compressed sensing techniques.

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