Massive MIMO Systems Tutorial

Part I:

Theory and Analysis

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Part II:

Propagation aspects

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Slides are available at:
http://www.ieee-icc.org/tutorials/private.html

Part II:
Propagation aspects of Massive MIMO Systems

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Propagation aspects

- What do we mean by favorable propagation conditions?
- Conventional MIMO vs. massive MIMO from a propagation perspective, what are the differences?
- Spatial resolution, influence of antenna configuration
- Channel richness; is that a problem for massive MIMO?
- What are the specific propagation phenomena that have to be taken into account in large array channel modeling
  - Received power levels
  - Singular values
  - Antenna correlation
  - Near field effects

Favorable propagation conditions?

- In MU-MIMO ($\boldsymbol{H} \Rightarrow \boldsymbol{G}$), "favorable propagation" if

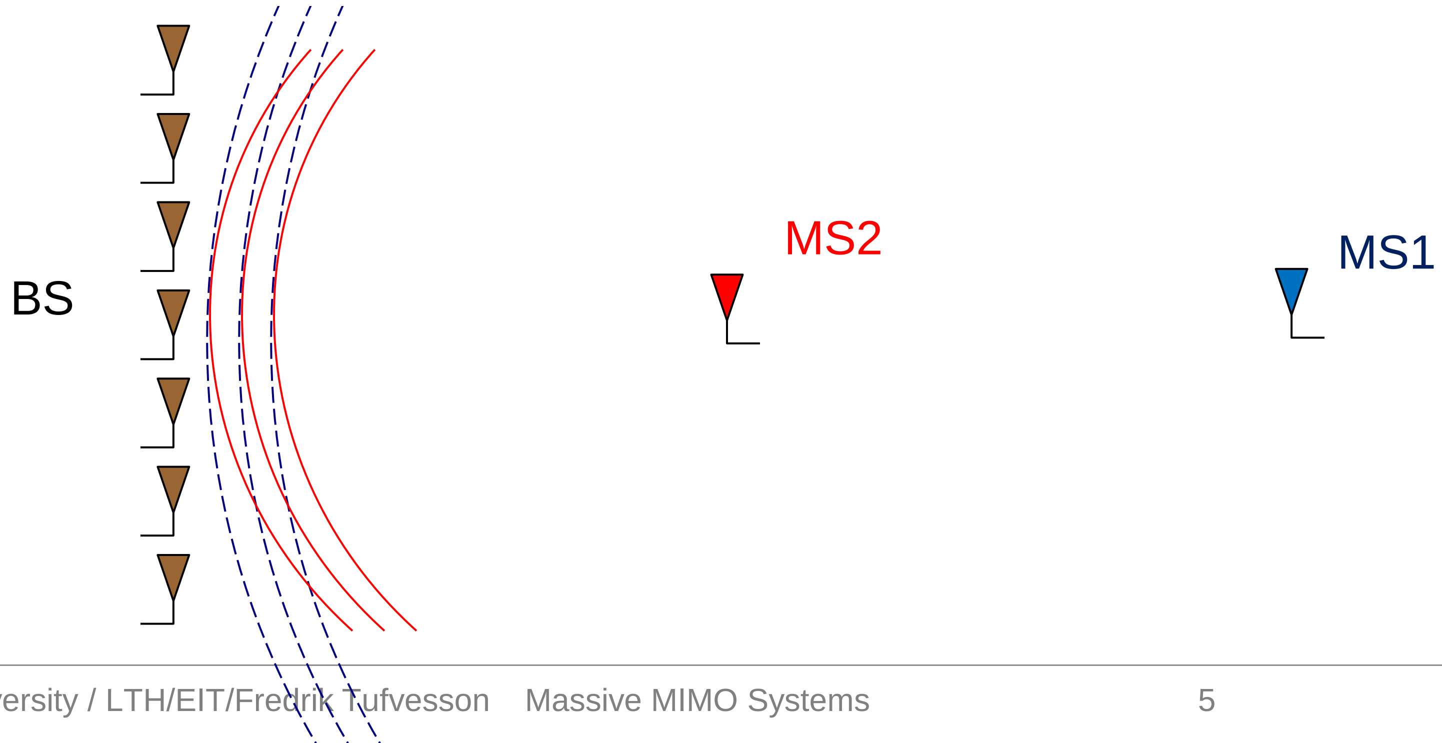
$$\frac{\boldsymbol{G}^H\boldsymbol{G}}{M} \approx \begin{bmatrix} \beta_1 & 0 & \cdots & 0 \\ 0 & \beta_2 & \ddots & \vdots \\ \vdots & \ddots & \ddots & 0 \\ 0 & \cdots & 0 & \beta_K \end{bmatrix} \triangleq \boldsymbol{D}, \quad M \gg K$$

- $\boldsymbol{H}$ i.i.d. $\Rightarrow$ favorable propagation.

- but...
  - the channel is not i.i.d. for a single user
  - different users have different received power levels
  - variations of statistics occur over a large array (i.e. non WSSUS)
  - large-scale fading can occur over a large array
  - different users can have correlated small-scale *and* large-scale fading even if they are far away from each other

Physically large arrays open up a new dimension

- In conventional MIMO we use the spatial domain:
  - Angle of Arrival and Angle of Departure
- With physically large arrays we can also use the *range* to the latest scattering point
  - spherical wave fronts instead of plane wave fronts



A small measurement example, compact array

- 128 port antenna array indoors
- outdoor single antenna users
- 2.6 GHz center frequency

Users outdoors at street level

Receive antenna array

4 circles of 16 dual polarized antennas

Receive antenna indoor

System Model

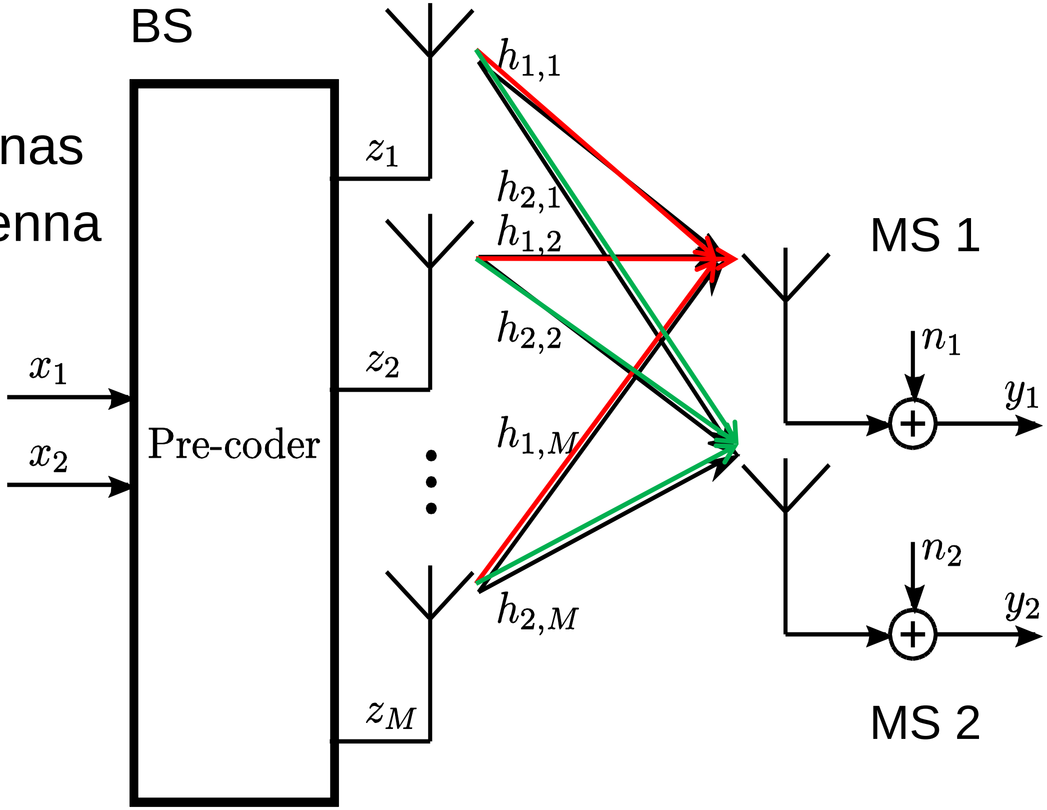
- Base station with *M* antennas
- *K*=2 users with single antenna
- Signal model

$$\mathbf{y} = \sqrt{\rho}\mathbf{H}\mathbf{z} + \mathbf{n}$$

$$\mathbf{z} = \mathbf{U}\mathbf{x}$$

- Gram matrix

$$\mathbf{H}\mathbf{H}^H = \begin{bmatrix} 1+g & \delta \\ \delta^* & 1-g \end{bmatrix}$$



g measures channel power imbalance

δ measures correlation between the two channels

Precoding Schemes

- DPC

$$C_{\text{DPC}} = \begin{cases} \log_2\left[1+\rho+\dfrac{\rho^2\left(1-g^2-|\delta|^2\right)^2+4g^2}{4\left(1-g^2-|\delta|^2\right)}\right], & |\delta|^2 \le 1-g^2-\dfrac{2g}{\rho} \\ \log_2[1+\rho(1+g)], & |\delta|^2 > 1-g^2-\dfrac{2g}{\rho} \end{cases}$$

- ZF

$$\mathbf{W}_{\text{ZF}} = \mathbf{H}^H\left(\mathbf{H}\mathbf{H}^H\right)^{-1}$$

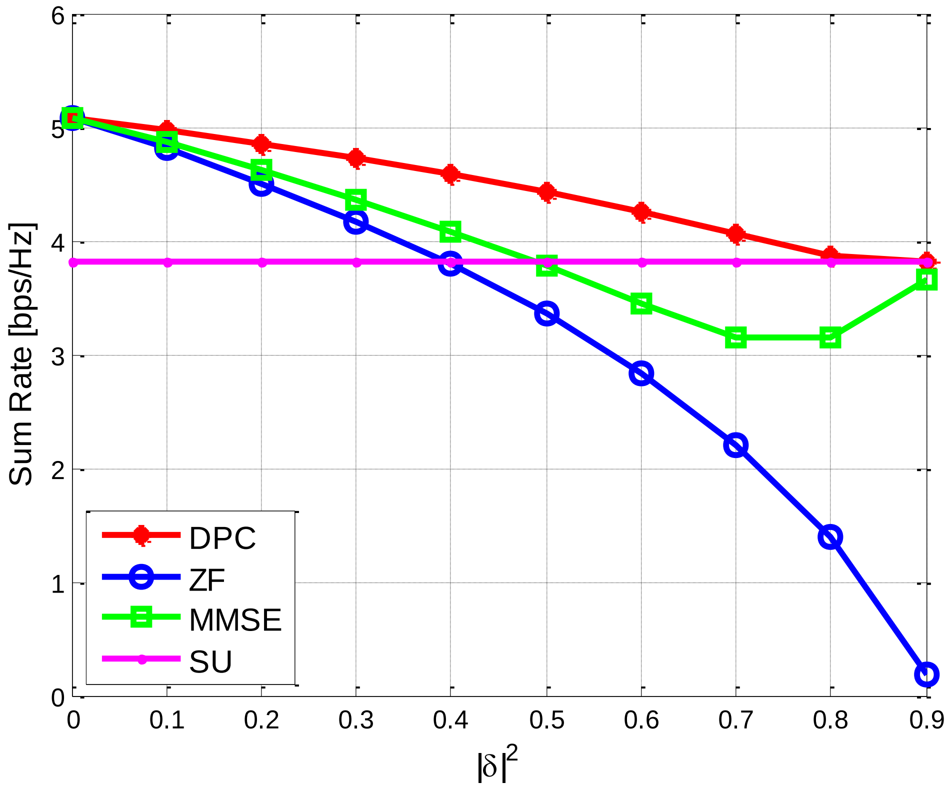
$$C_{\text{ZF}} = \begin{cases} \log_2\left[\dfrac{\left(2+\rho\left(1-g^2-|\delta|^2\right)\right)^2}{4\left(1-g^2\right)}\right], & |\delta|^2 \le 1-g^2-\dfrac{2g}{\rho} \\ \log_2\left[1+\dfrac{\rho\left(1-g^2-|\delta|^2\right)}{1-g}\right], & |\delta|^2 > 1-g^2-\dfrac{2g}{\rho} \end{cases}$$

- MMSE

$$\mathbf{W}_{\text{MMSE}} = \mathbf{H}^H\left(\mathbf{H}\mathbf{H}^H+\alpha\mathbf{I}\right)^{-1}$$

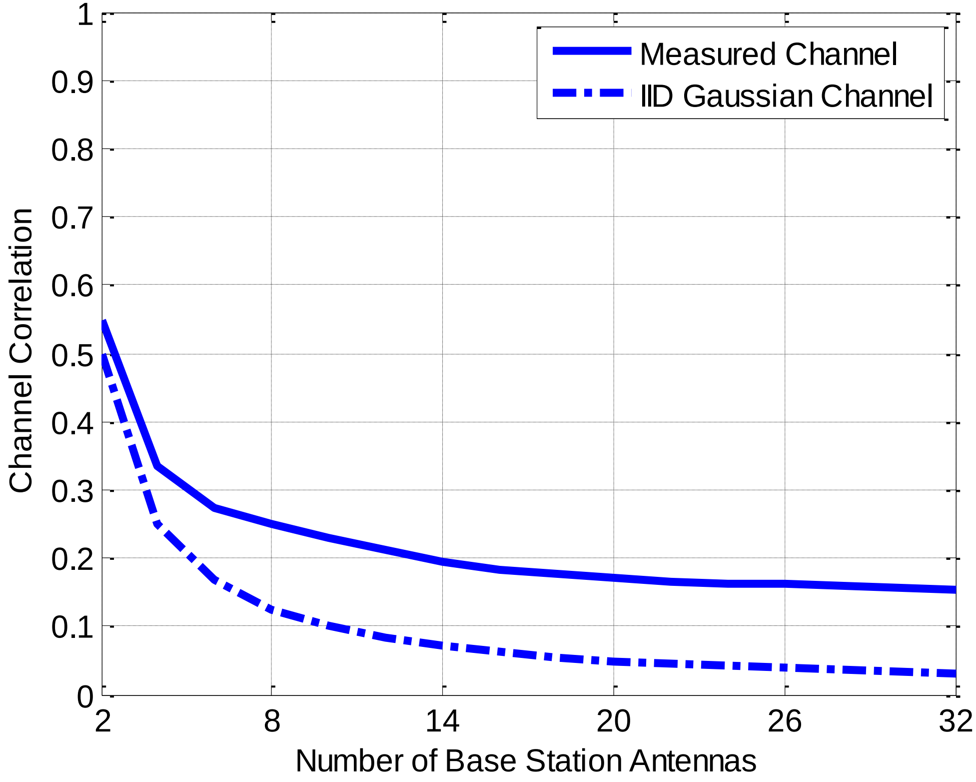
Performance Comparison

- Numerical evaluation



Performance Comparison

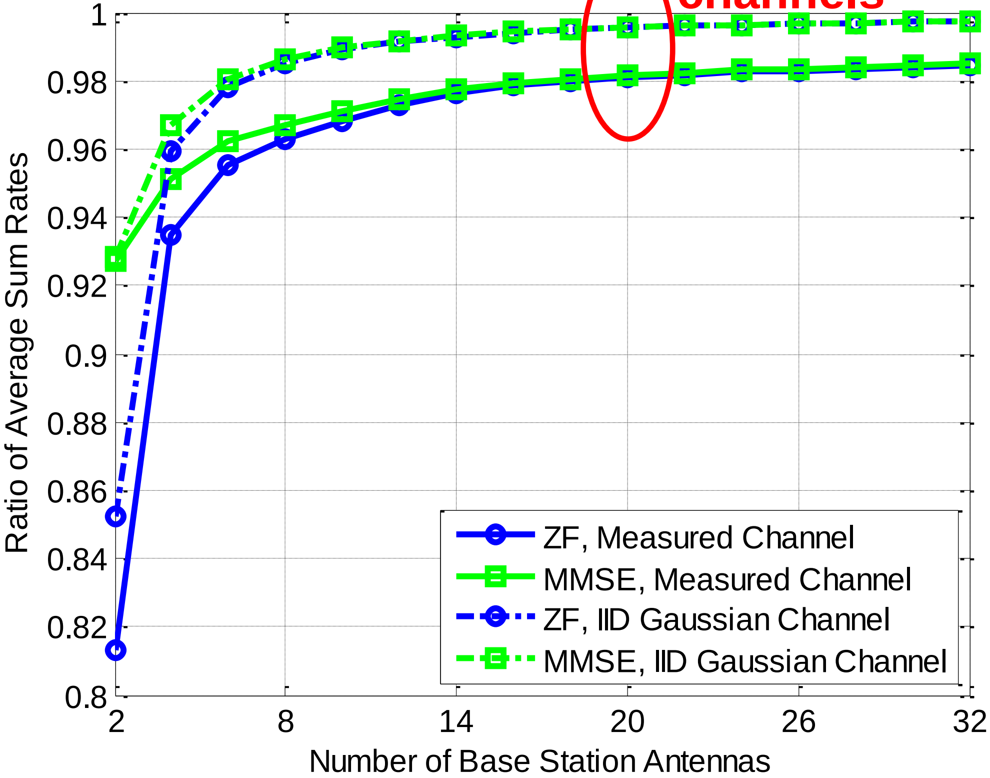
- Measured channels



Performance Comparison

- Measured channels

Linear precoding sum rates as high as 98% of DPC capacity in measured channels



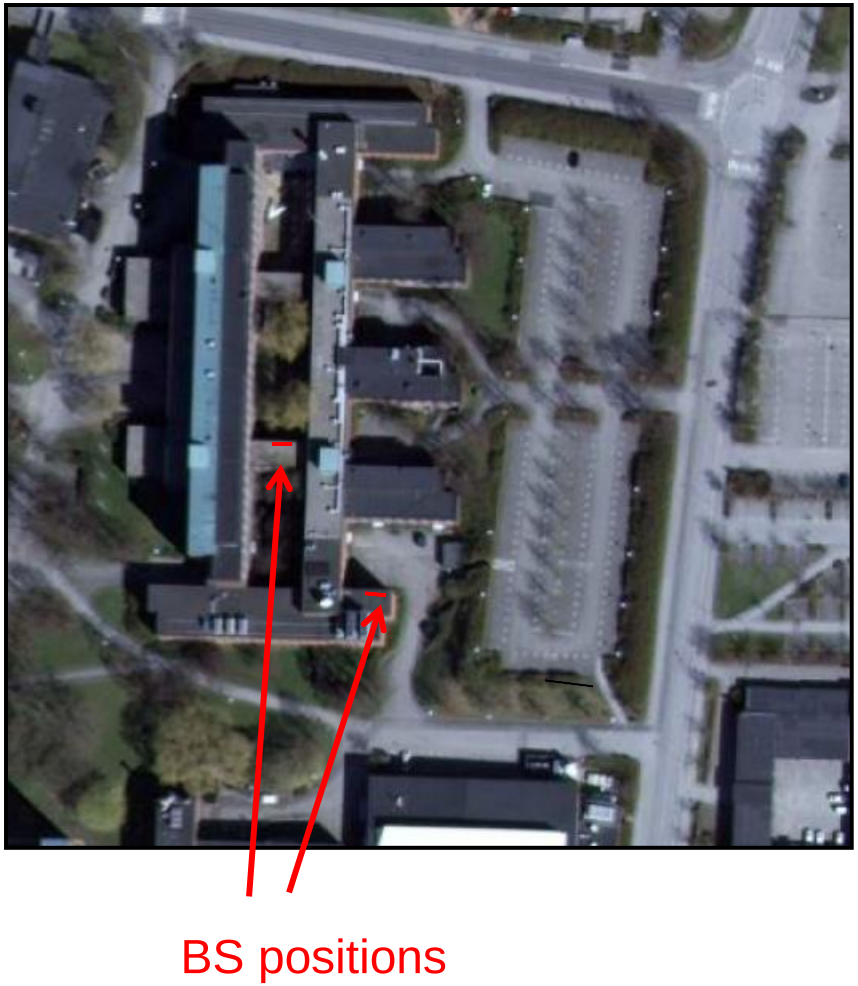
Summary

- User channels can be decorrelated by using reasonably large antenna arrays at base station
- Linear precoding can achieve almost the same sum rate as optimal but complex DPC technique
- Clear benefits can be seen with a relatively limited number of antennas in realistic propagation environments, given that M>>K

Non-stationarities and other channel properties for physically large arrays

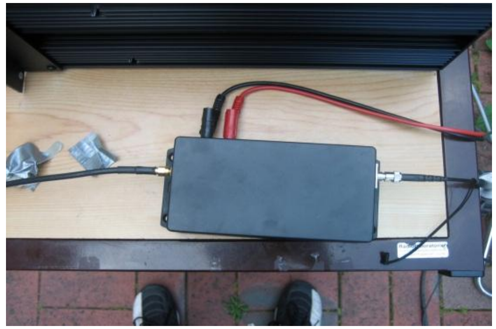
Measurement setup

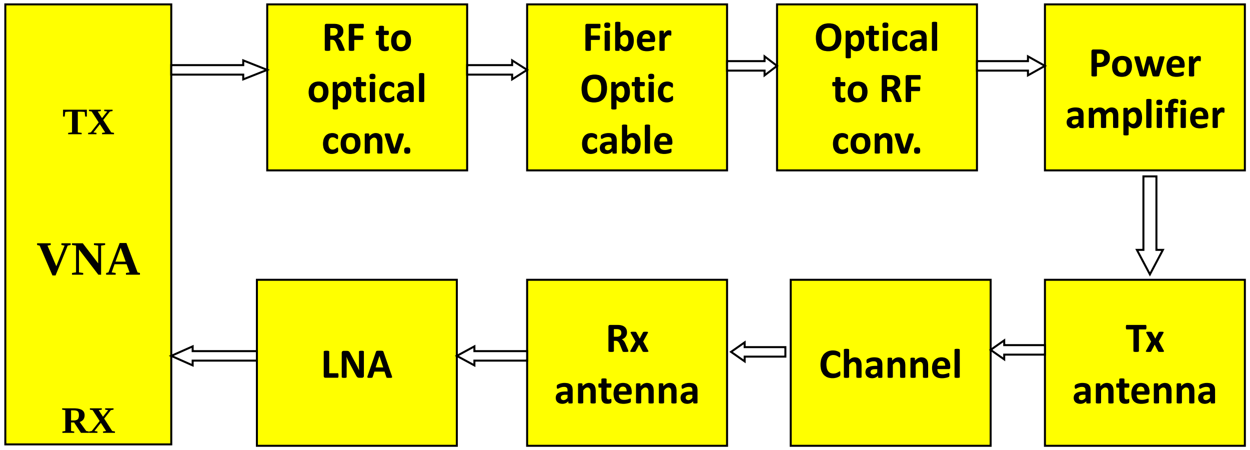
- 128 element virtual linear array as base station, 7.3 m long
- 36 different single antenna user positions in a court yard, 30 positions around building
- 2.6 GHz center frequency, 50 MHz bandwidth
- LOS and NLOS in an outdoor, but controlled environment.
- Extreme but not unrealistic scenarios

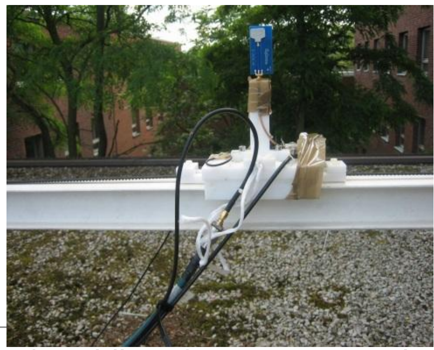


Measurement principle: virtual array

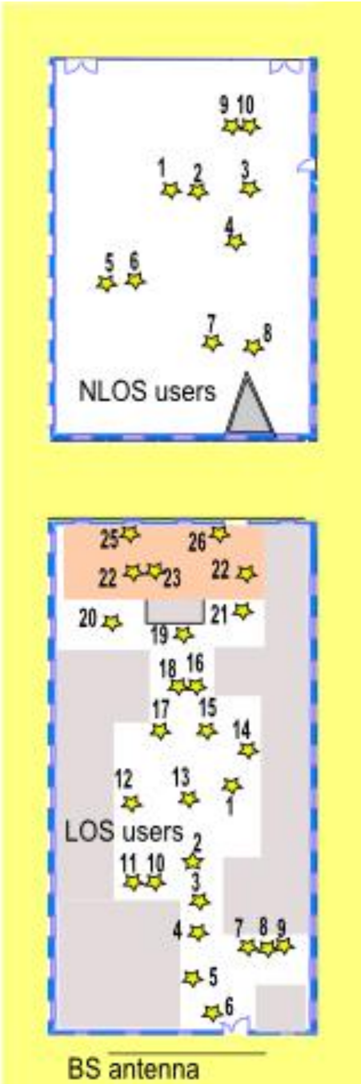






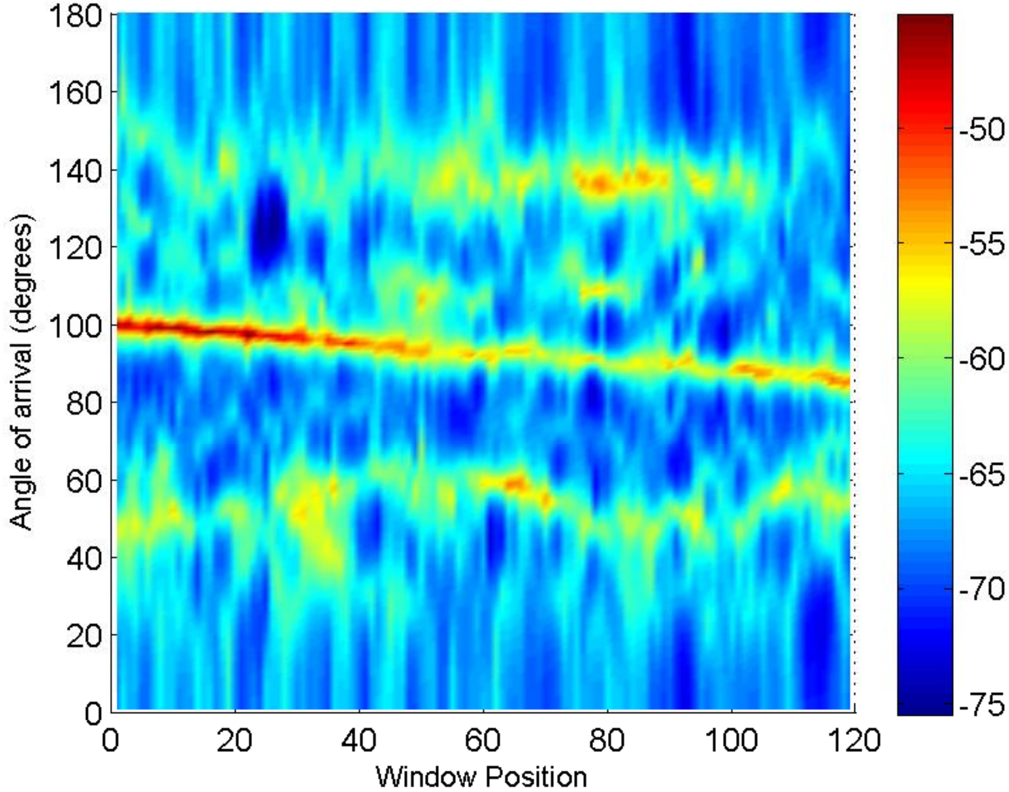
Scenario I: court yards



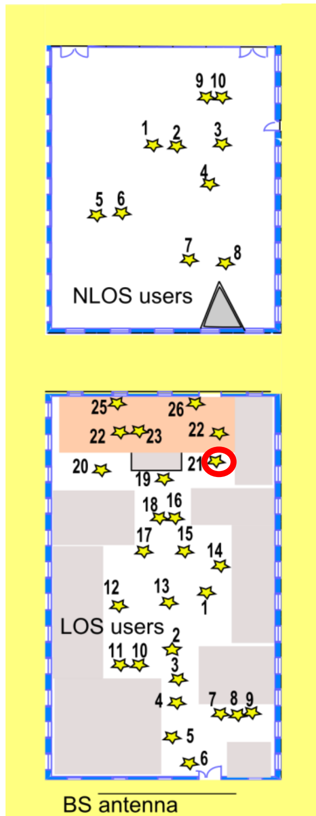
We observe large differences over the array

- A detailed study if received signals revealed that we had large variations over the array:
  - received power level
  - small scale fading distribution
  - angular power spectra
- This implies that the channel can not be seen as wide sense stationary over the large array, which has implications for
  - modeling
  - simulation
  - theoretical analysis
- For the following analysis we use a 10 element window, in which we can assume that a plane wave assumption and WSSUS holds

Angular power spectrum over the array

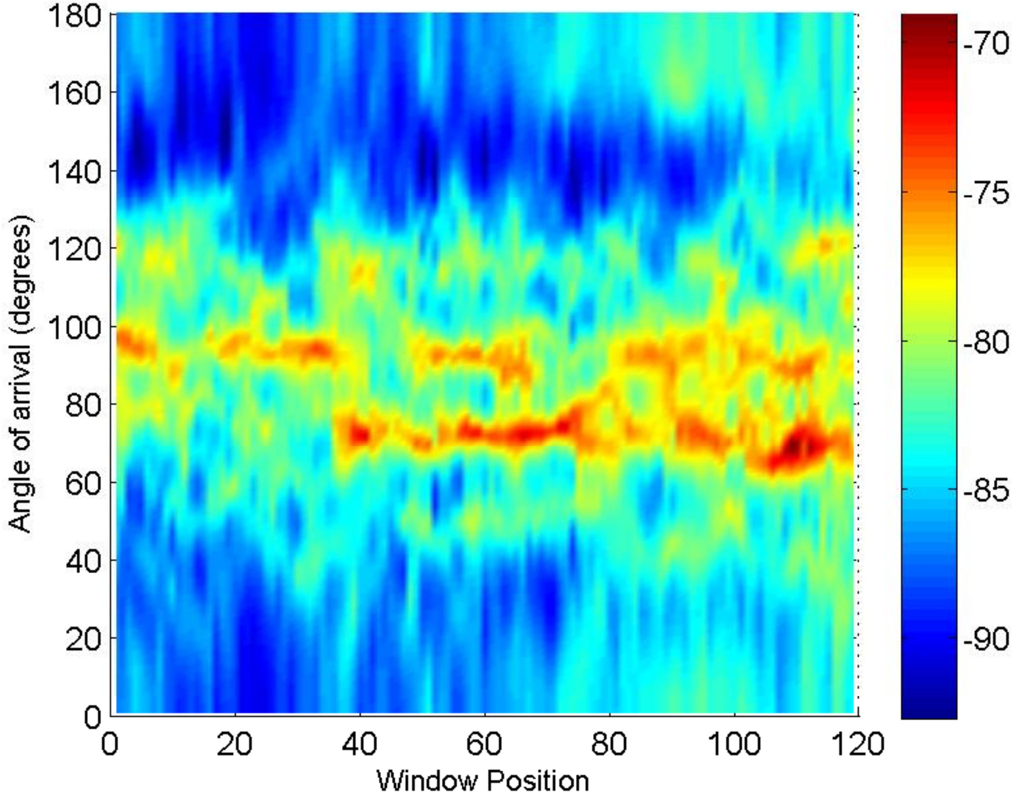


LOS court,
user 21

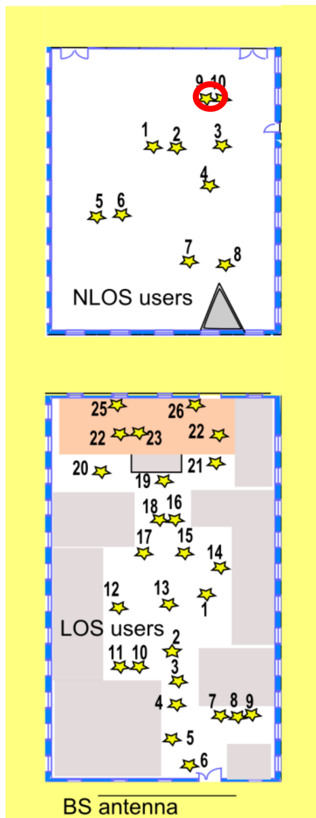


Scatterers come and go
LOS component varies in strength
Angle of arrival changes over the array

Angular power spectrum, 10 ant. sub-array



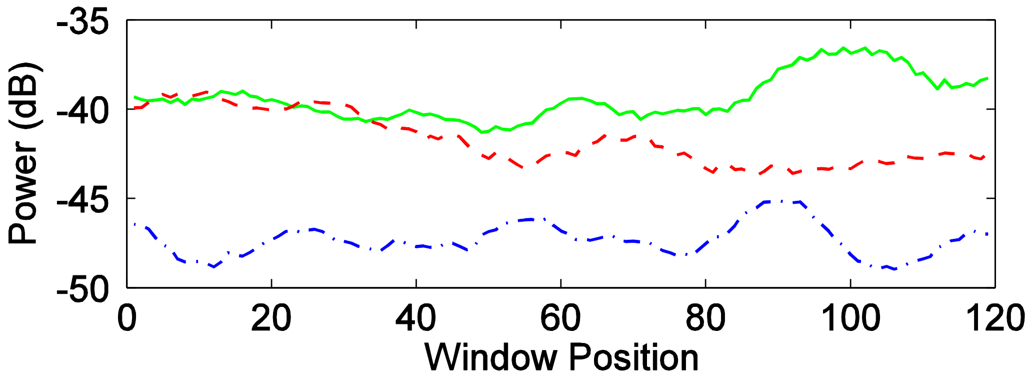
NLOS court, user 9



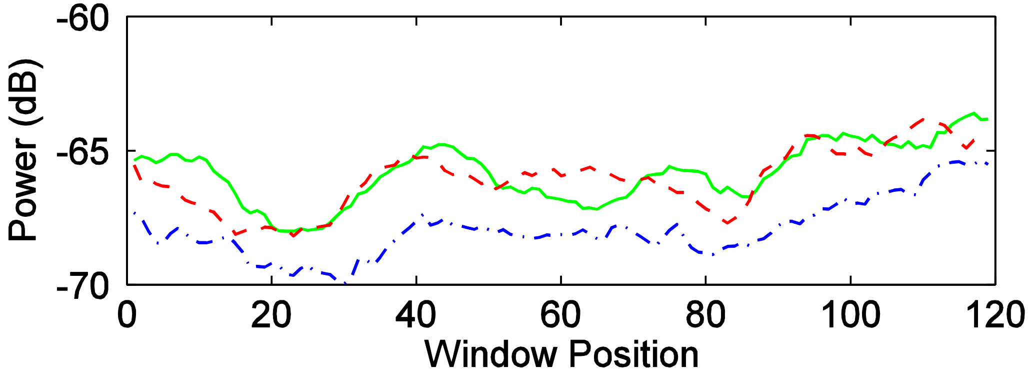
- Angular spread is limited
- Scatterers come and go
- Large variations over the array

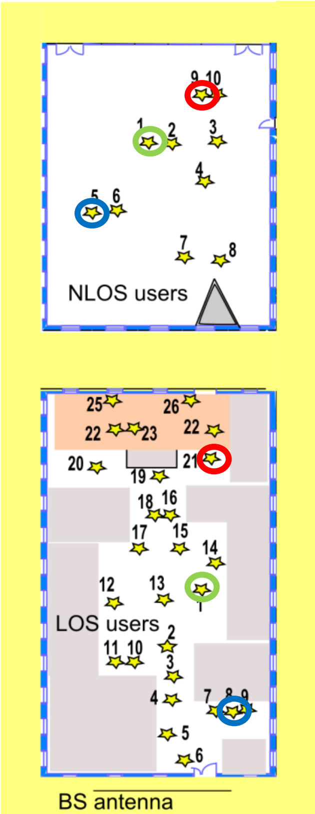
Received power level, 10 antenna sub-array

LOS



NLOS

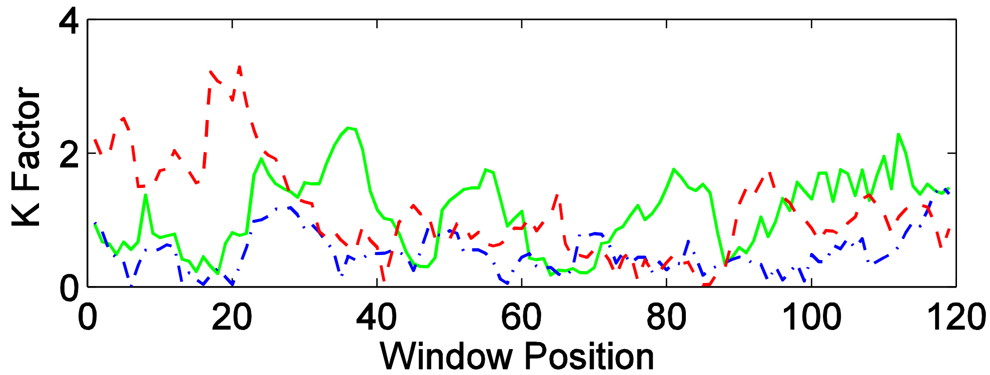




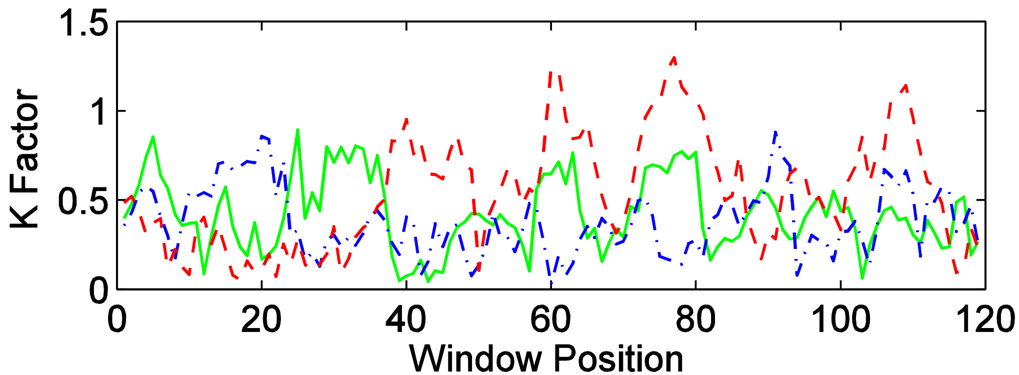
Power variations of 5 dB over the array
Interaction with nearby objects

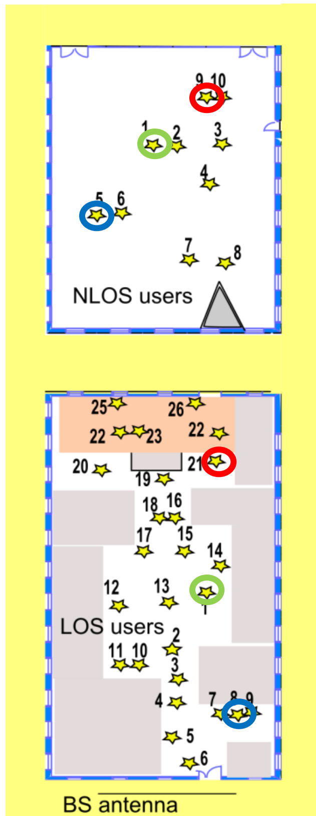
Ricean K-factor, 10 antenna sub-array

LOS



NLOS





The received amplitudes can be described as Ricean/Rayleigh with varying K-factor
NLOS court generally have low K-factors

Near field effects improve de-correlation

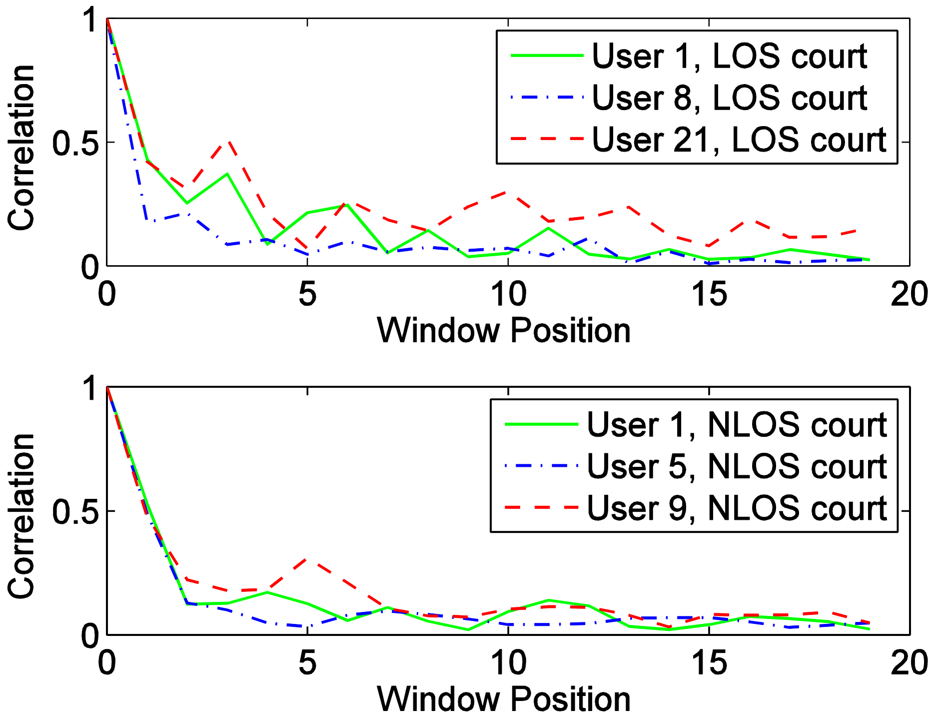
- Objects (scatterers) within the Rayleigh distance contribute to make the large antenna even larger
- Users within the Rayleigh distance will create spherical wavefronts at the array
- Scatterers are not visible over the whole array
- There is large scale fading over the array
- Rayleigh distance of the antenna $d_R=2(L_a)^2/\lambda$=945 m

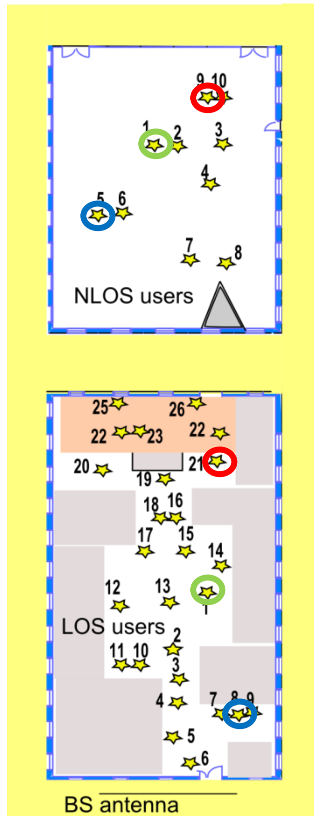
Antenna correlation

Antenna correlation coefficients, assuming WSSUS over the array (which is not strictly correct)

LOS

NLOS

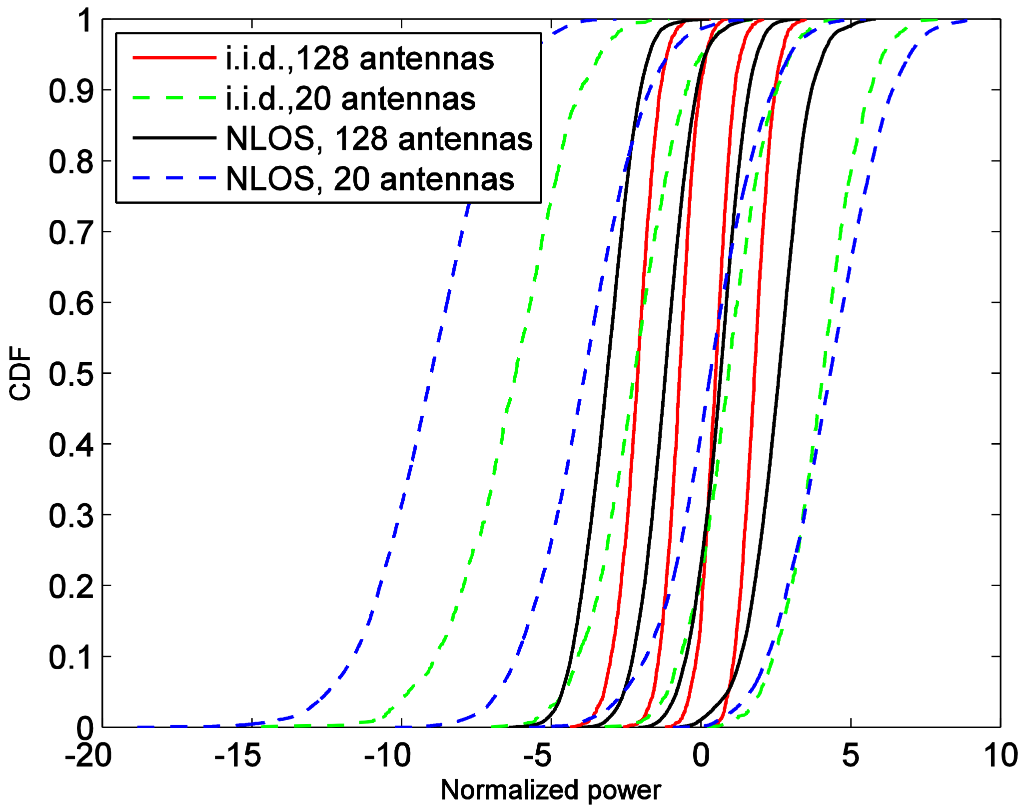




The antenna correlation is reasonably low, and somewhat larger in the NLOS court due to the limited angular spread

Eigenvalue distribution, NLOS and i.i.d.

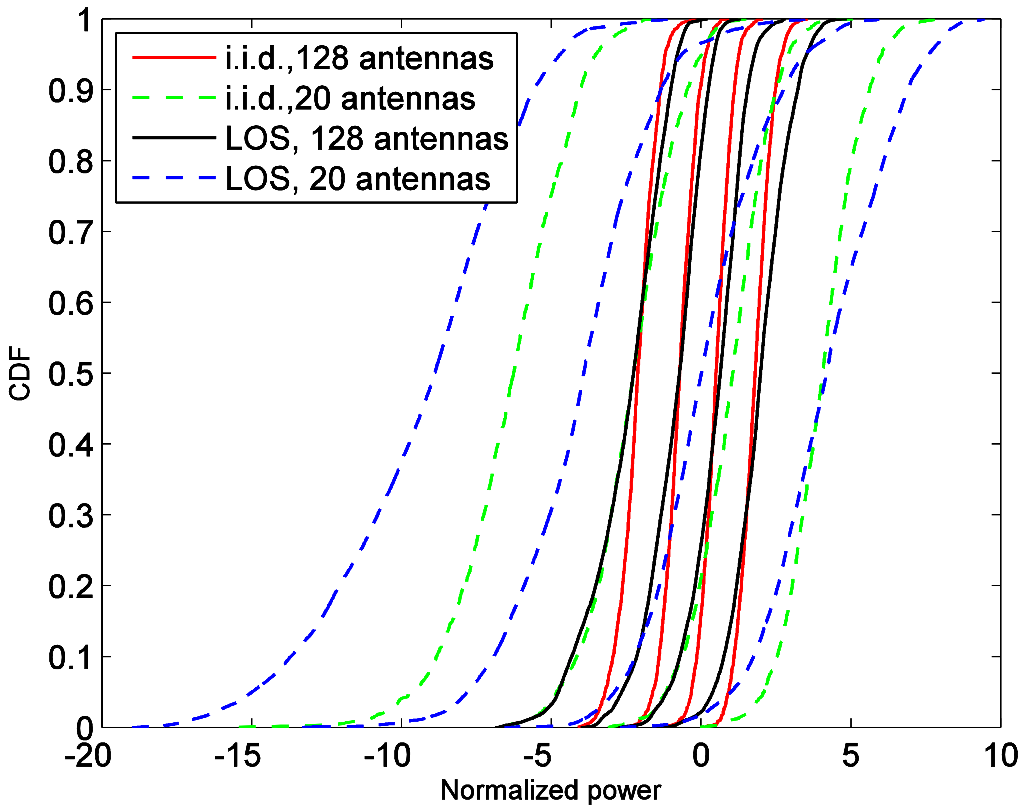
1000 permutations of 4 randomly selected users out of the 26 available
128 and 20 base station antennas, respectively



Stable eigenvalues in the 128 antenna case, not too different from the i.i.d. case

Eigenvalue distribution, LOS and i.i.d.

1000 permutations of 4 randomly selected users out of the 26 available
128 and 20 base station antennas, respectively



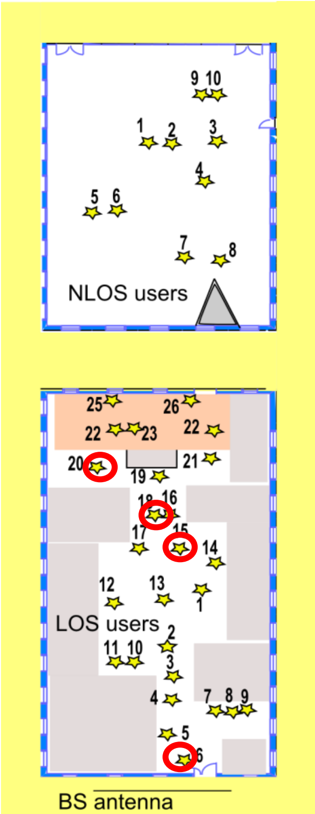
Very similar performance
The excess antennas makes the eigenmodes stable

User correlation, orthogonality between users

Example of user correlation coefficients, 128 and 20 antennas, resp.
LOS court, users 6, 20, 15, 18

$$R_{128}^{LOS} = \begin{pmatrix} 1.0000 & 0.0207 & 0.0716 & 0.0308 \\ 0.0207 & 1.0000 & 0.0119 & 0.0125 \\ 0.0716 & 0.0119 & 1.0000 & 0.0166 \\ 0.0308 & 0.0125 & 0.0166 & 1.0000 \end{pmatrix}$$

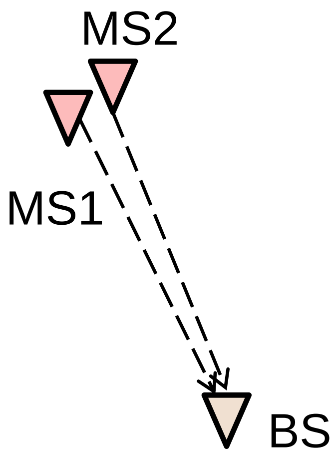
$$R_{20}^{LOS} = \begin{pmatrix} 1.0000 & 0.0634 & 0.0933 & 0.0208 \\ 0.0634 & 1.0000 & 0.0320 & 0.1199 \\ 0.0933 & 0.0320 & 1.0000 & 0.0264 \\ 0.0208 & 0.1199 & 0.0264 & 1.0000 \end{pmatrix}$$



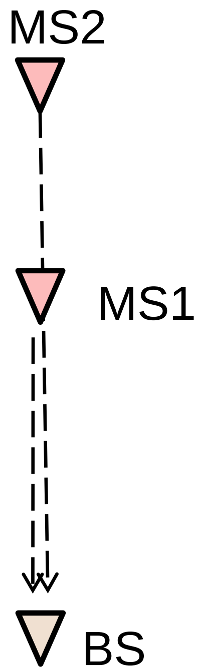
On the average the correlation goes down with the number of antennas, the correlation is reasonably low.

In conventional multi -link MIMO high correlation can occur when…

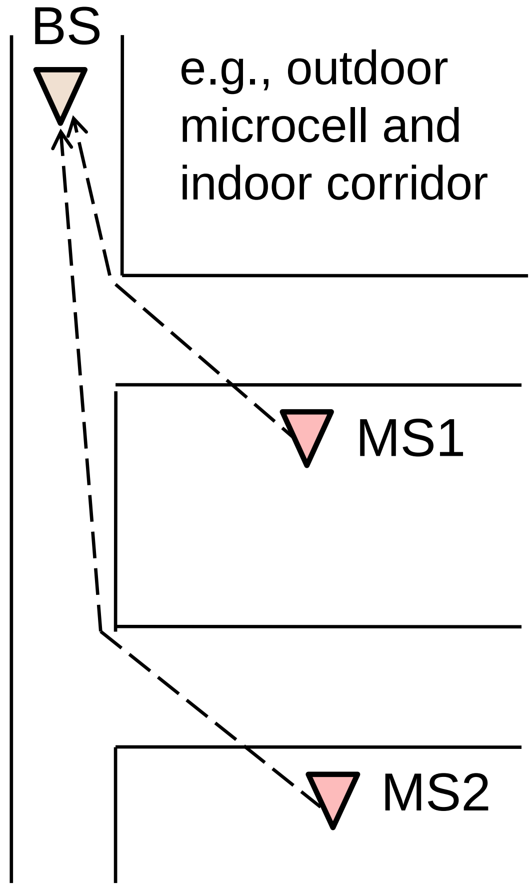
Two MS are close to each other



Two MS and BS are on the same line



Wave-guiding effects occur

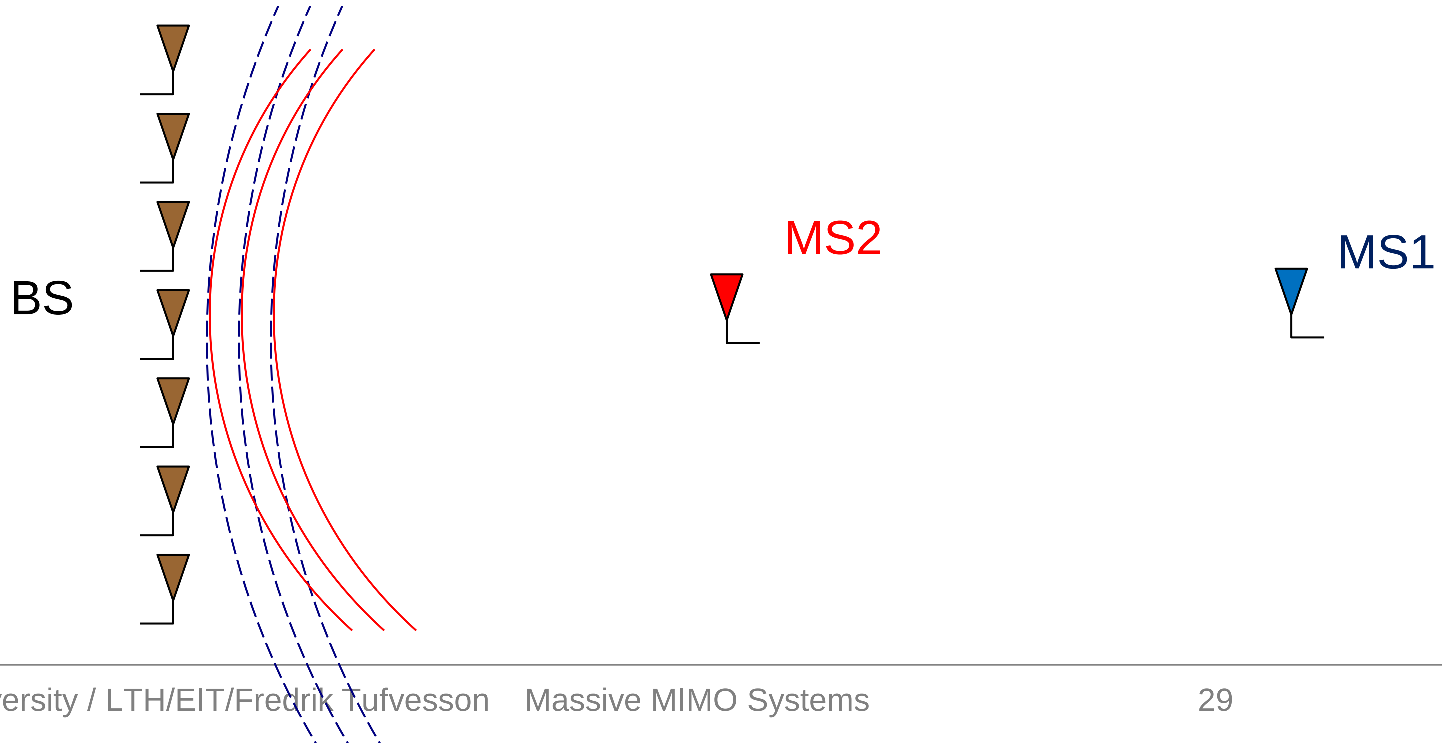


When can we suffer from high correlation between users in massive MIMO ?

- If the angles of the dominating multipath components does not differ enough to give a relative phase shift of more than π over the array
  - Less than 0.5 degree difference is enough for the previous measurement setup

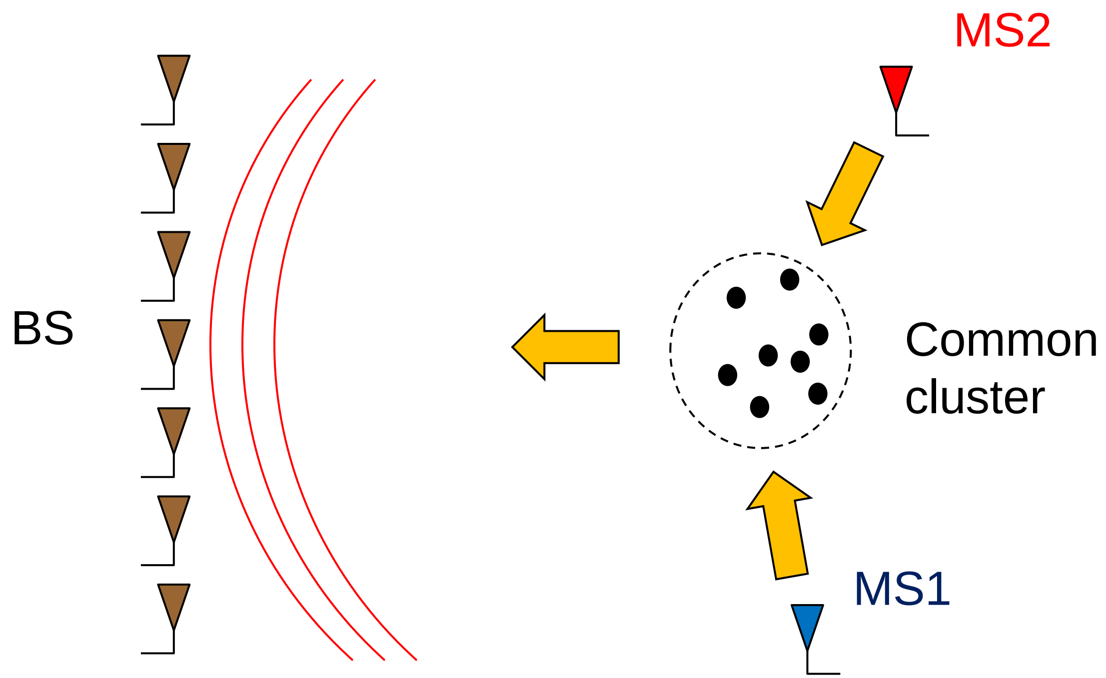
When can we suffer from high correlation between users in massive MIMO ?

- If the angles of the dominating multipath are the same, but the difference in distance is not enough to give spherical de-correlation



When can we suffer from high correlation between users in massive MIMO ?

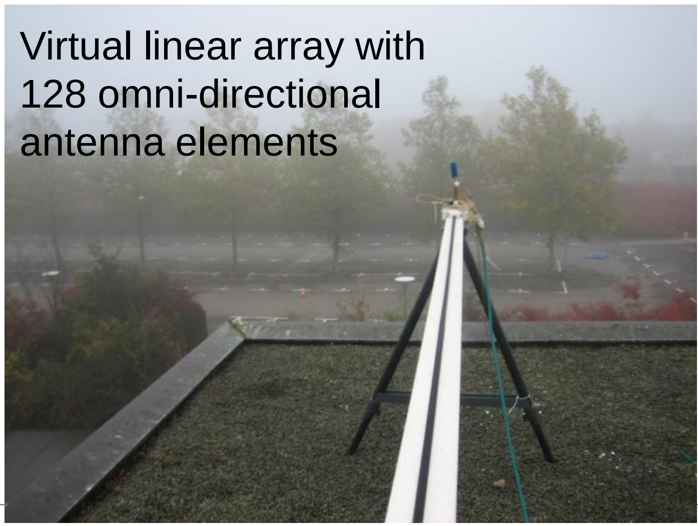
- When there are strong common clusters that we are not able to resolve (small cluster angular spread)

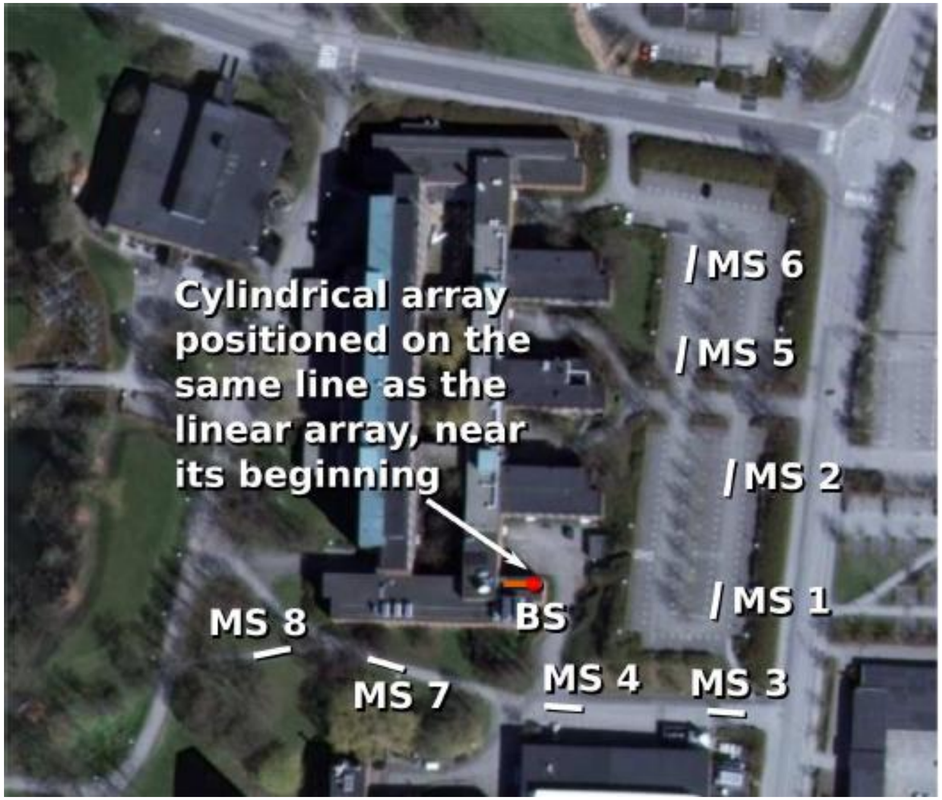


Consequences for modeling

- The environment is not stationary over a physically large array
  - scatterers come and go
- We need to include a LOS component with varying power to account for Rayleigh/Ricean variations
- We should introduce large scale fading over the array
- A modified COST 2100 geometric model could work as a base line for modeling, or use detailed ray-tracing
- The propagation situation is actually better than expected due to the spatial non-stationarities

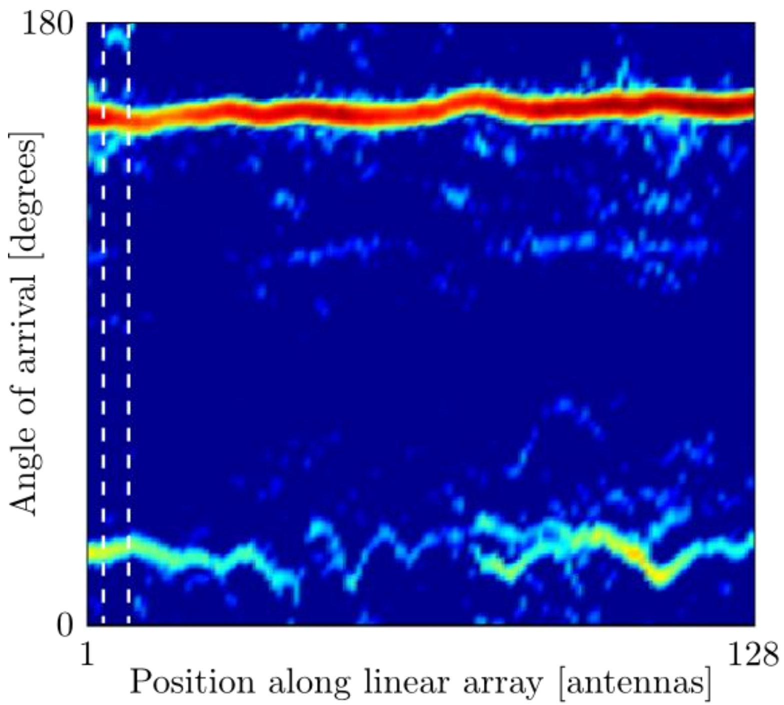
Scenario II: Campus



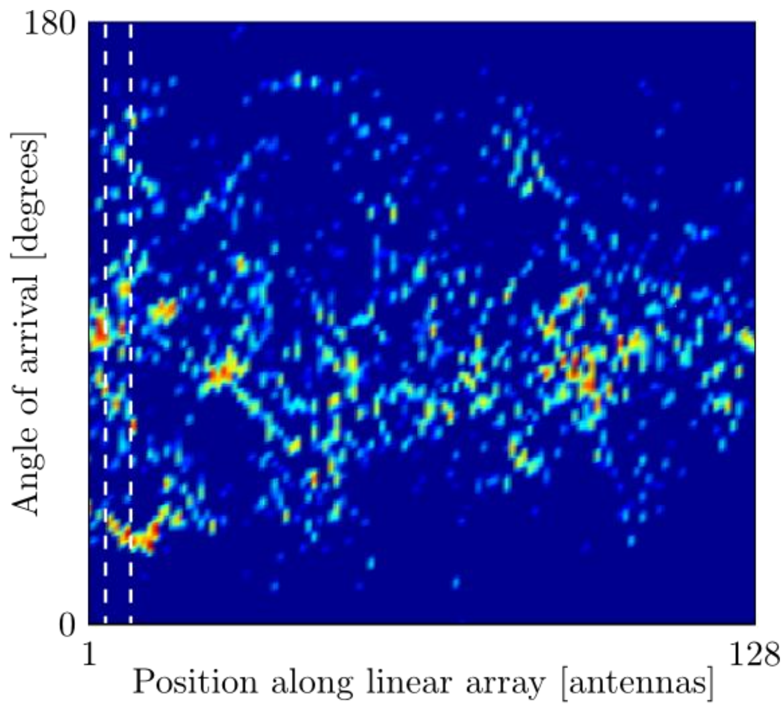


- Single omni-directional antenna at user side
- Center frequency at 2.6 GHz
- Bandwidth of 50 MHz

Angular power spectrum in LOS and NLOS



a) LOS scenario (MS 2)

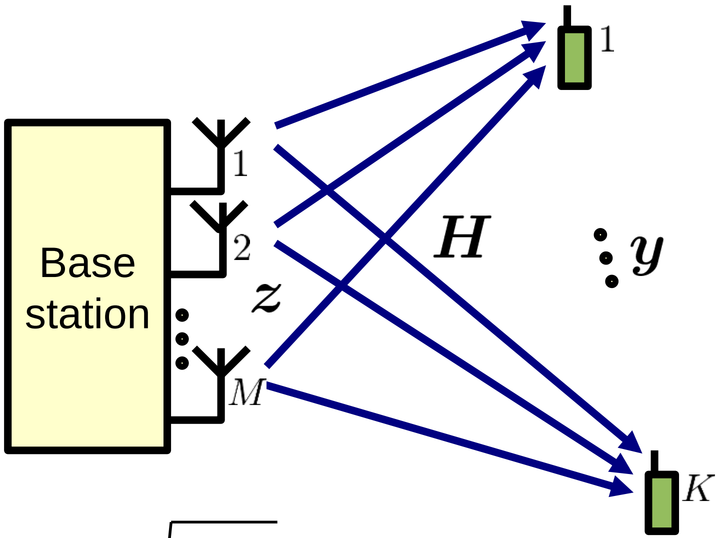


b) NLOS scenario (MS 7)

A "difficult" scenario, potentially with **low rank and high correlation between users**

A "better" scenario with rich scattering and likely low correlation between users

Capacity/sum-rate in the downlink



$$\boldsymbol{y} = \sqrt{\frac{\rho K}{M}}\boldsymbol{H}\boldsymbol{z} + \boldsymbol{n}$$

Increased array gain when *M* increases is harvested as reduced transmit power

- Dirty-paper coding (DPC) capacity

$$C_{\mathrm{DPC}} = \max_{\boldsymbol{P}} \log_2 \det\left(\boldsymbol{I} + \frac{\rho K}{M}\boldsymbol{H}^H \boldsymbol{P}\boldsymbol{H}\right)$$

Total power constraint: $\sum_{i=1}^{K} P_i = 1$

- Zero-forcing (ZF) sum-rate

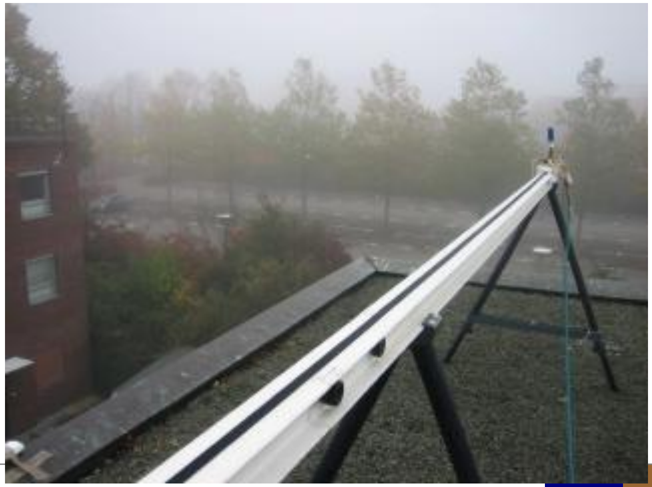
$$C_{\mathrm{ZF}} = \max_{\boldsymbol{P}} \sum_{i=1}^{K} \log_2\left(1 + \frac{\rho K}{M} P_i\right)$$

Total power constraint: $\sum_{i=1}^{K} P_i \left[\left(HH^H\right)^{-1}\right]_{ii} = 1$

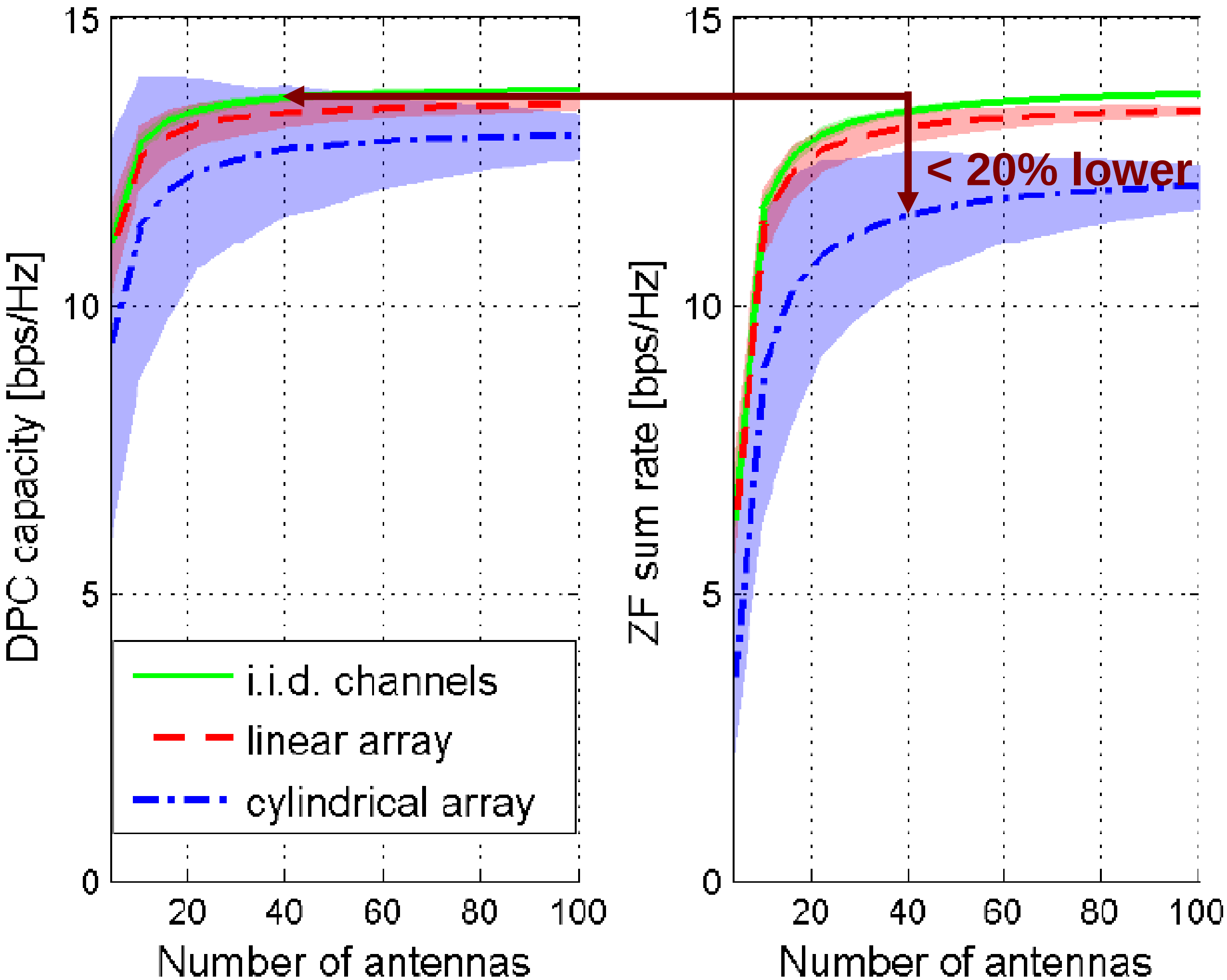
- ρ = 10 dB (interference-free per-user SNR)

Compact vs. physically large arrays

- Cylindrical antenna array
  - Compact: 30 cm wide x 30 cm high
  - Can resolve elevation
  - Directional dual polarized patch antenna elements $\lambda/2$ spacing,
- Linear antenna array
  - Physically large: 8 m long
  - Can not resolve elevation
  - Omni-directional antenna elements $\lambda/2$ spacing



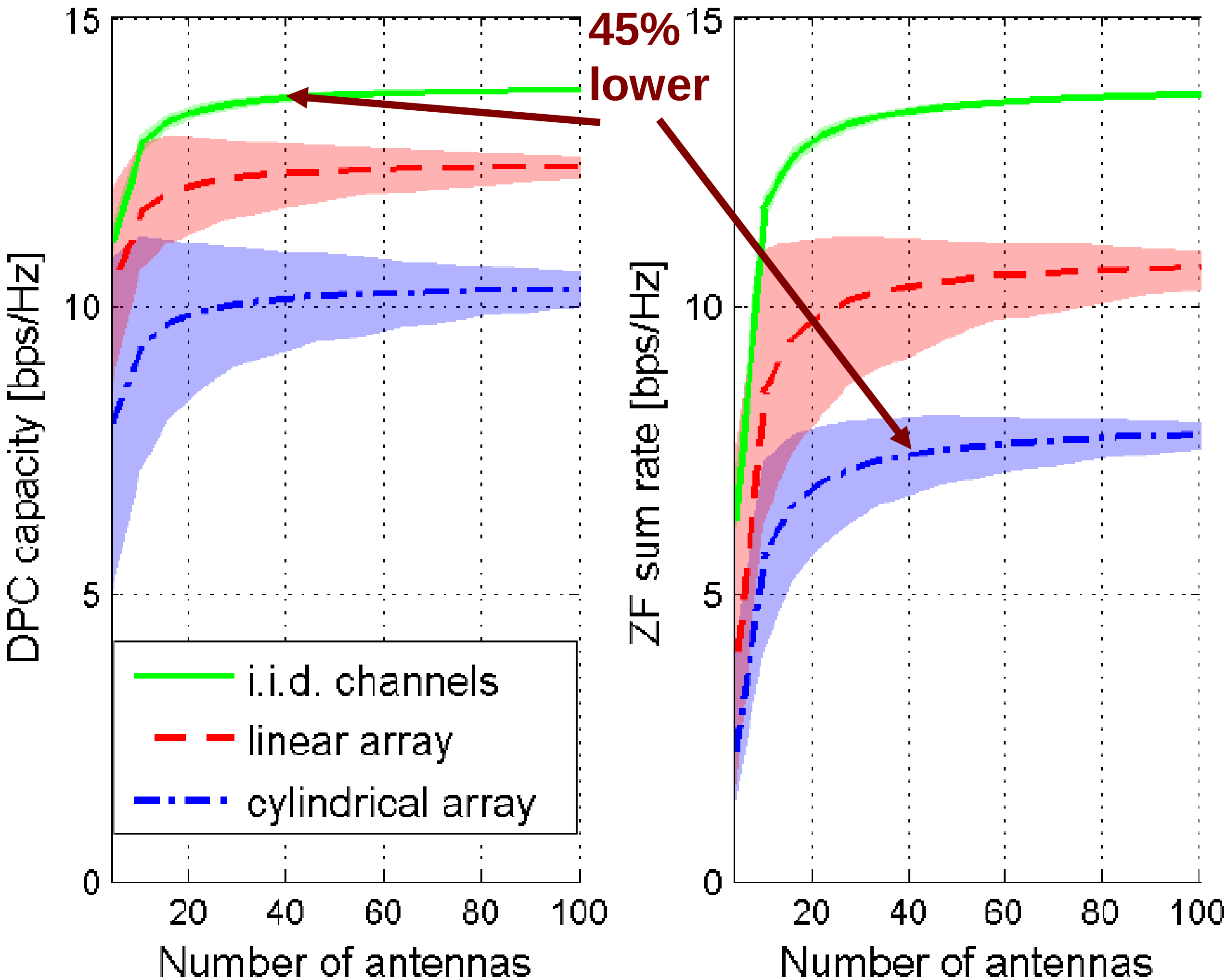
DPC and ZF in NLOS (MS 7), Distance between 4 users 1.5-2 m



Lines: average capacity/sum-rate over 50 MHz and 2000 random selections of *M* antennas

Shaded regions: 5%-95% outage

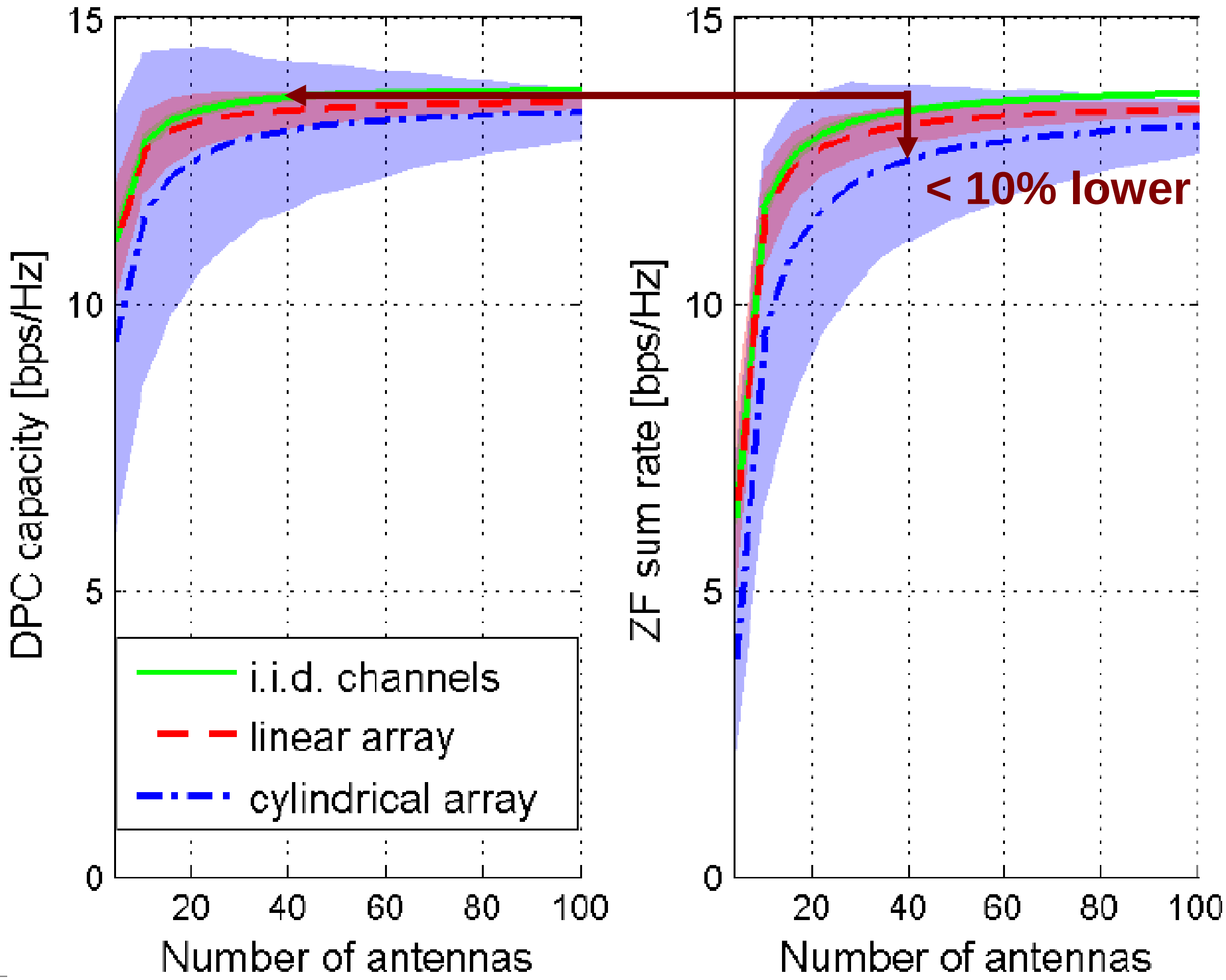
DPC and ZF in LOS (MS 2), tricky case
Distance between 4 users 1.5-2 m



Lines: average capacity/sum-rate over 50 MHz and 2000 random selections of *M* BS antennas

Shaded regions: 5%-95% outage

DPC and ZF in LOS (MS 1-4) Distance between 4 users >10 m



Lines: average capacity/sum-rate over 50 MHz and 2000 random selections of *M* antennas

Shaded regions: 5%-95% outage

Some observations

- In the studied realistic propagation environment, we have characteristics that allow for efficient use of very-large MIMO, even with less-complex linear precoding scheme
  - In the most “difficult” case studied, closely spaced users with LOS, the worst combination of cylindrical array with ZF reaches 55% of ideal performance
  - In other cases, both linear and cylindrical arrays with ZF reach 80-90% of ideal performance
- The limit for “large” MIMO, in terms of number of BS antennas, is in a reasonable range of about 10 times the number of users

Channel Models for massive MIMO

Important channel properties

Property	Single-link MIMO	Multi-link MIMO	massive MIMO
Fading correlation	Small-scale fading	Large-scale-fading	Small- and **large-scale fading**
Spatial / temporal correlation	Intra-link	Inter-link	Intra- and **inter-link**

Channel models for massive MIMO

Massive MIMO extension of existing channel models?

- Non-physical models
  - Analytical model (e.g., Kronecker, i.i.d.)
    - hard to model large-scale fading and non-stationarities
  - Stochastic models (e.g., Tapped-delay line)
    - hard to model non-stationarities
- Physical models (e.g., Ray-tracing, Geometry based stochastic channel models)
  - Related to distribution of scatterers
  - Good for narrowband power and finding dominant components, inaccurate for weak components, diffraction, diffuse scattering; these effects strongly influence performance
  - spherical waves inherent in the models

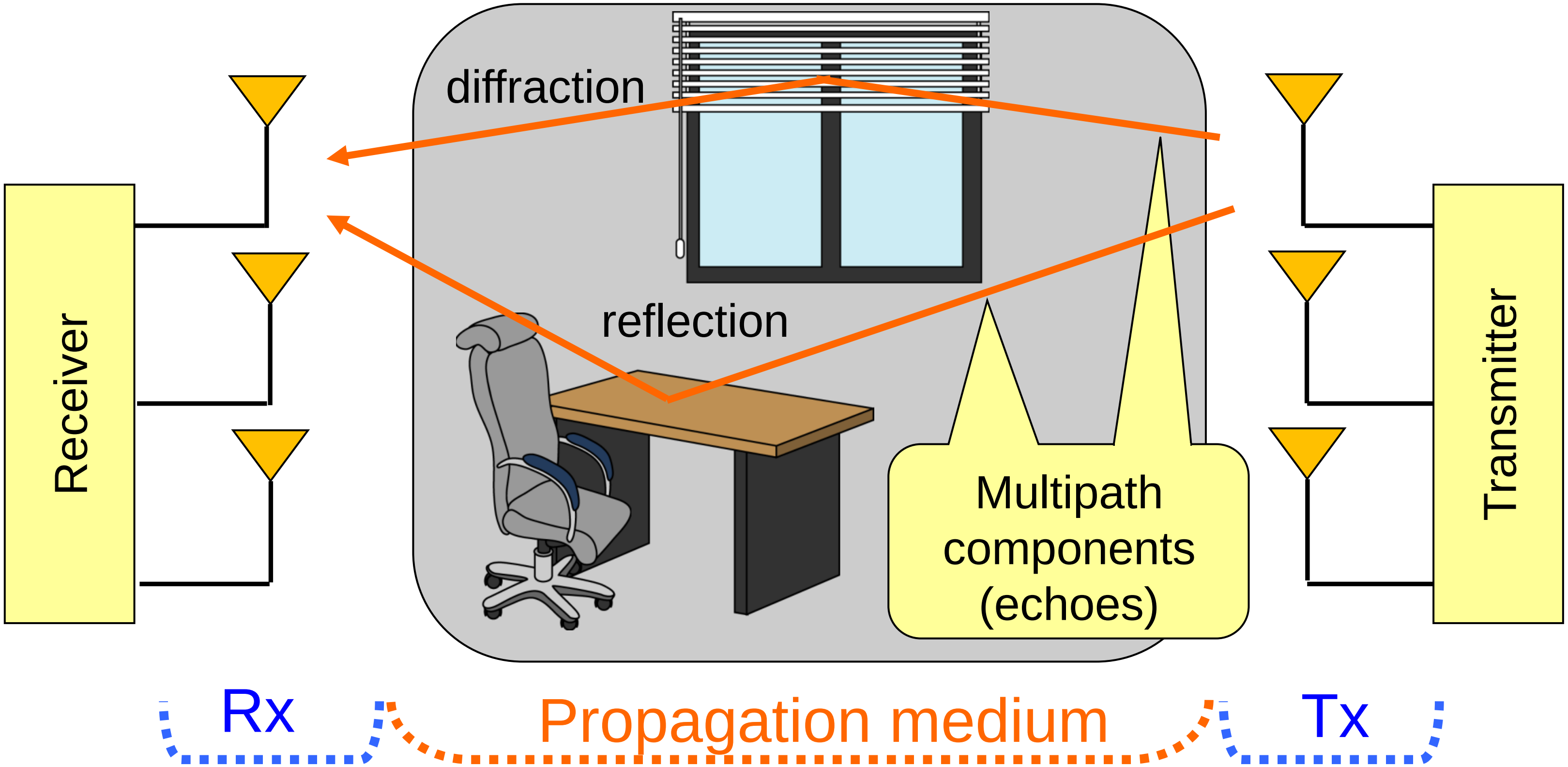
Comparison of approaches

- Modeling of transfer matrix
  - Depends on antenna configuration
  - Is what channel sounder measures, and wireless system “sees”
- Double-directional model
  - Models multipath components with DOA and DOD
  - Independent of antenna configuration
  - Basic setup requires more parameters, but realistic models easier to implement
  - Note that DOA and DOD changes over the array, as opposed to the classical approach

- Conversion between models
  - Nonphysical model can easily be obtained from physical model
    - sum the contributions from the multipath components
  - Getting physical from nonphysical model is difficult (high-resolution algorithms)

Double-Directional Propagation Modeling

- Separation of antennas and propagation (Steinbauer et al., 2001)



$$\mathbf{H}(ss) = \sum_{l=1}^{L(ss)} \mathbf{a}_l^{\mathrm{RX}} \left(\mathbf{g}_l^{\mathrm{RX}}\right)^T \bullet e^{\mathrm{j}(2\pi f \tau_l + \xi)} \bullet \mathbf{\Gamma}_l \bullet \mathbf{g}_l^{\mathrm{TX}} \left(\mathbf{a}_l^{\mathrm{TX}}\right)^T$$

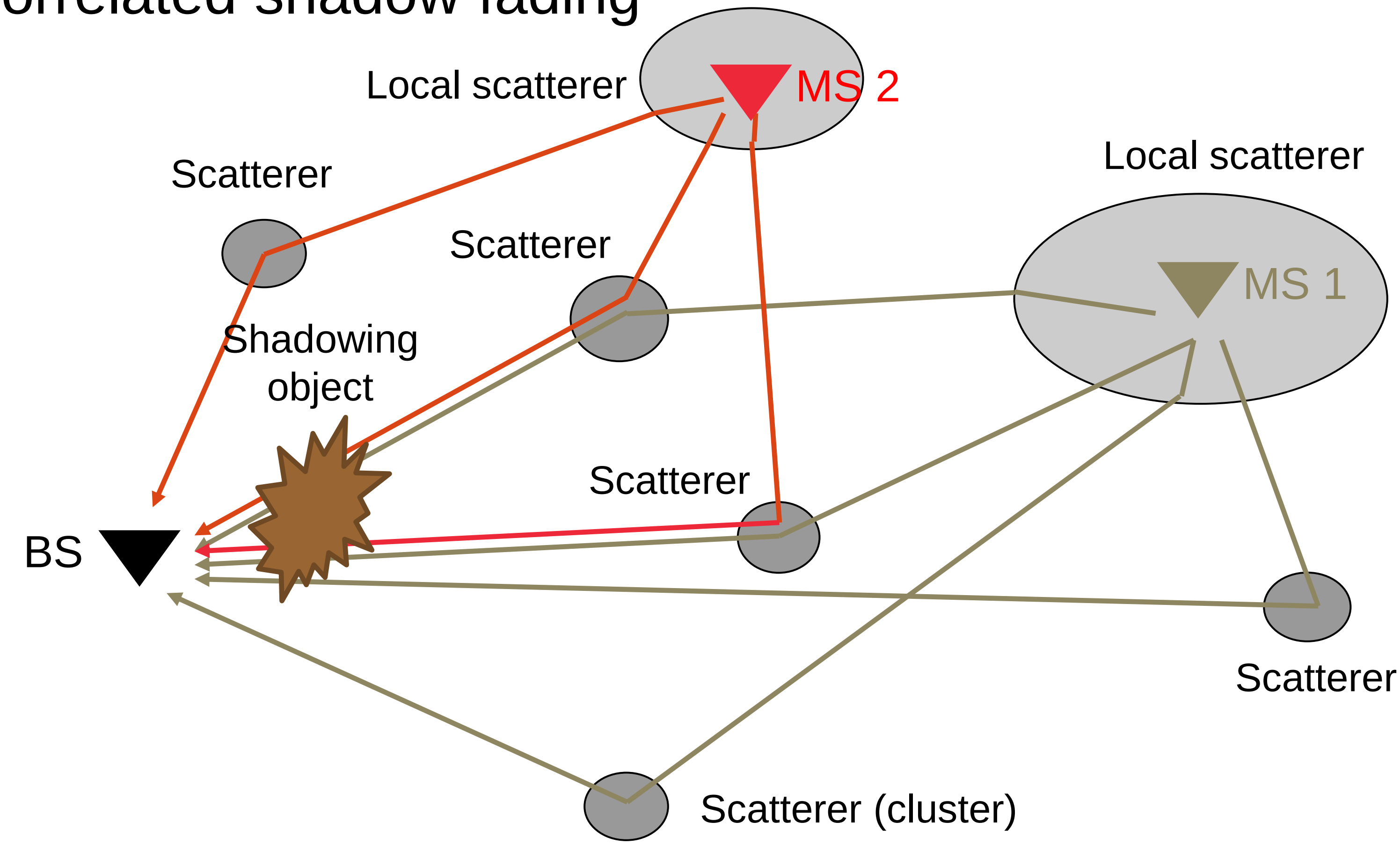
MIMO channel | Multi paths | Rx array | Antenna gain | Propagation time | Polarimetric weight | Antenna gain | Tx array

GSCM Philosophy

- Parametric approach, WSSUS not required
  - create a virtual map of scatterers and trace each contribution
- Based on clustering approach
  - scatterers with similar parameters are grouped together
- Multi-layer approach:
  - Radio environments
  - Large-scale effects
  - Small-scale effects

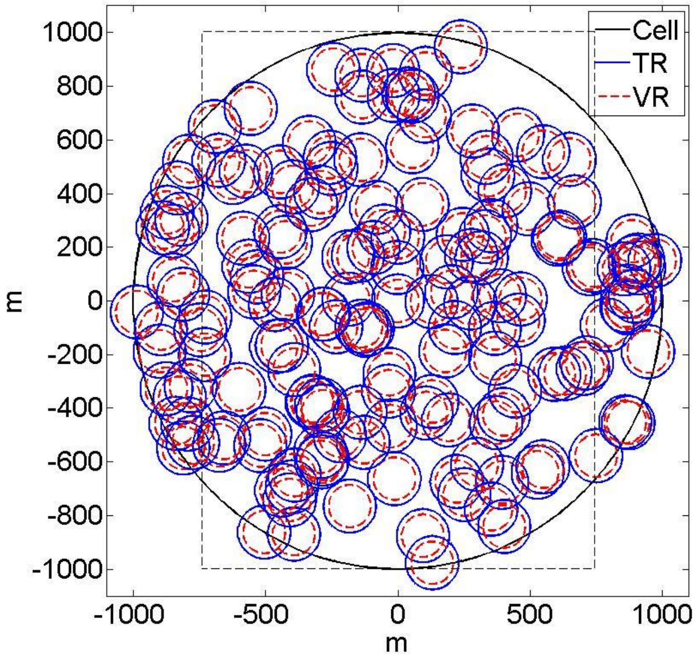
Geometry-Based Stochastic Channel Model

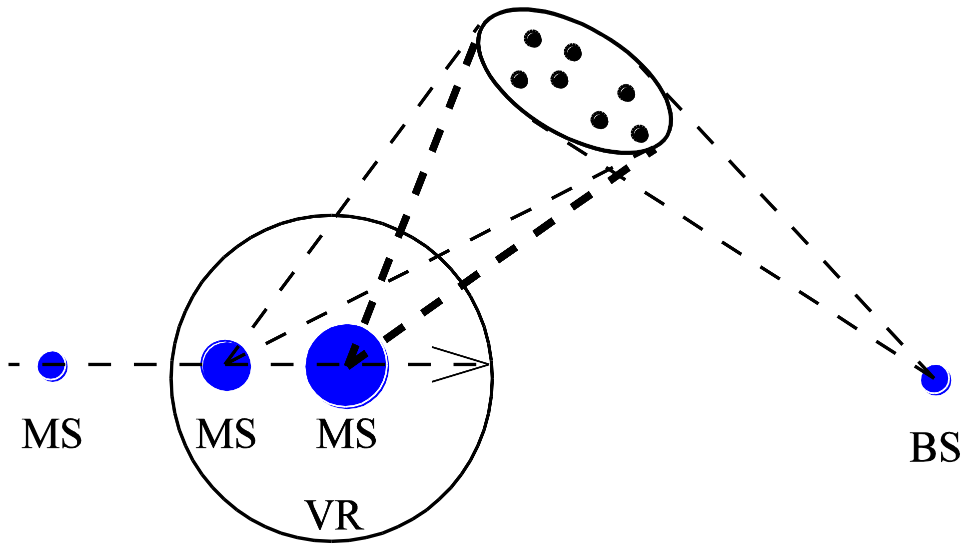
- Signals propagate through the same scatterers
  - → Inter-link spatial correlation
  - → Correlated shadow fading



Visibility Regions (VRs)

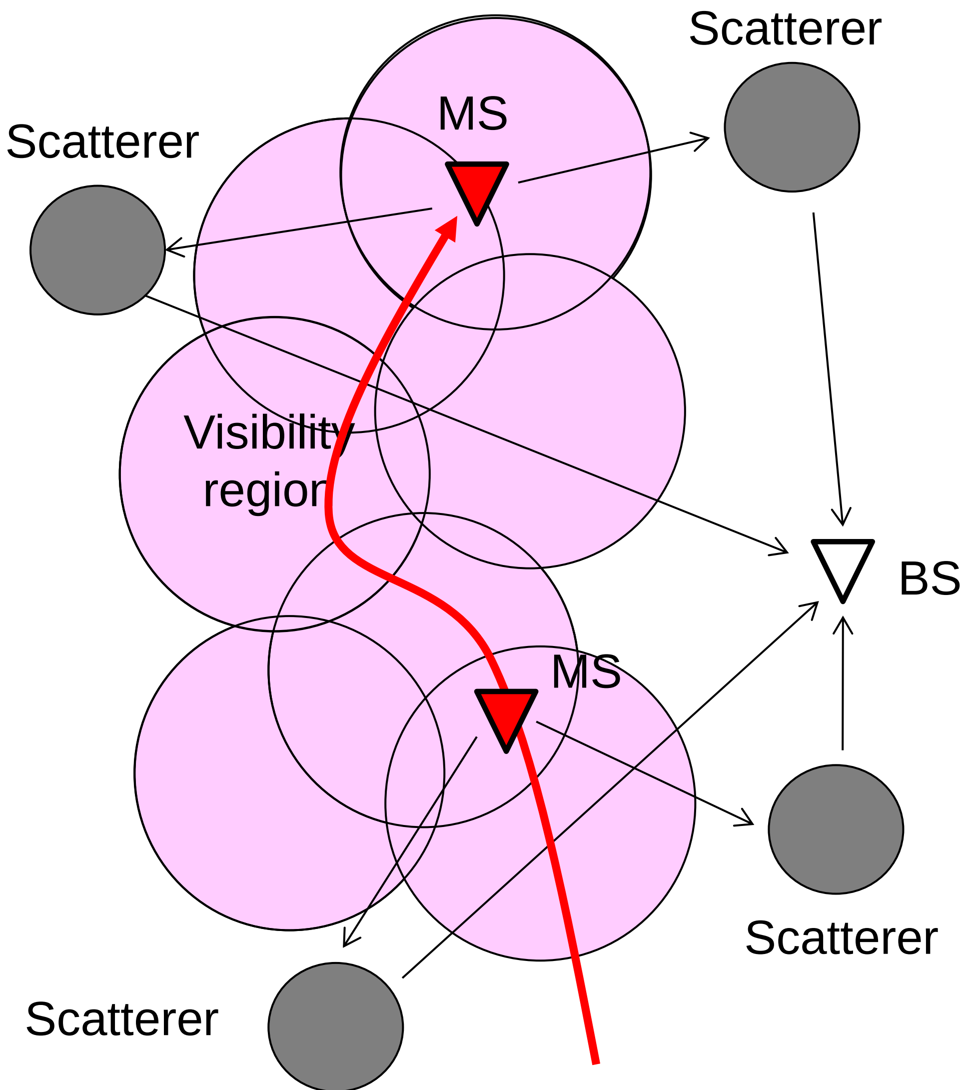
- Each cluster is associated with one visibility region: when the MS moves into the VR of a cluster, the cluster becomes visible
- There is a transition region when entering a visibility region





The COST 2100 Channel Model

- Geometry-based stochastic channel model
- relies on COST 259/273 approach, basis for WINNER and 3GPP/3GPP2 SCM models
- Pros
  - Allows dynamic channel simulation
  - Scalable for massive MIMO extension
- Cons
  - Lacking parameters for some interesting scenarios
- A Matlab implementation is available at Google code



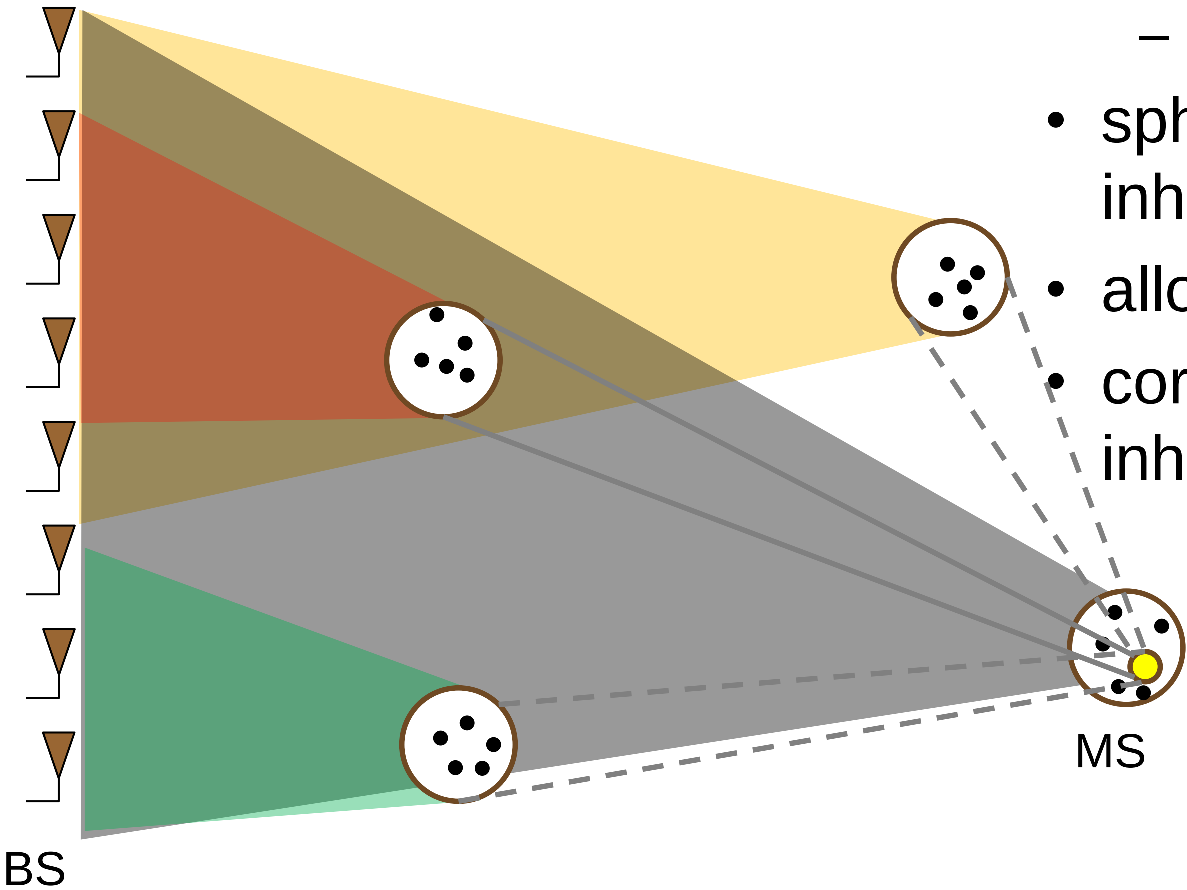
MIMO capability

- Multipath components are described also in the angular domain
- Because the COST 2100 channel model is double directional, it can be used to simulate MIMO channels for any array configuration
- The COST 2100 model is independent of antennas: the channel is combined with the antenna array description to provide the MIMO channel matrix

$$\mathbf{H}(t,\tau) = \sum_{n\in\mathscr{C}} \sum_{p} \alpha_{np}\delta(\tau - \tau_{np})\, \mathbf{s}_{MS}(\boldsymbol{\Omega}_{np}^{MS})\, \mathbf{s}_{BS}^{T}(\boldsymbol{\Omega}_{np}^{BS})$$

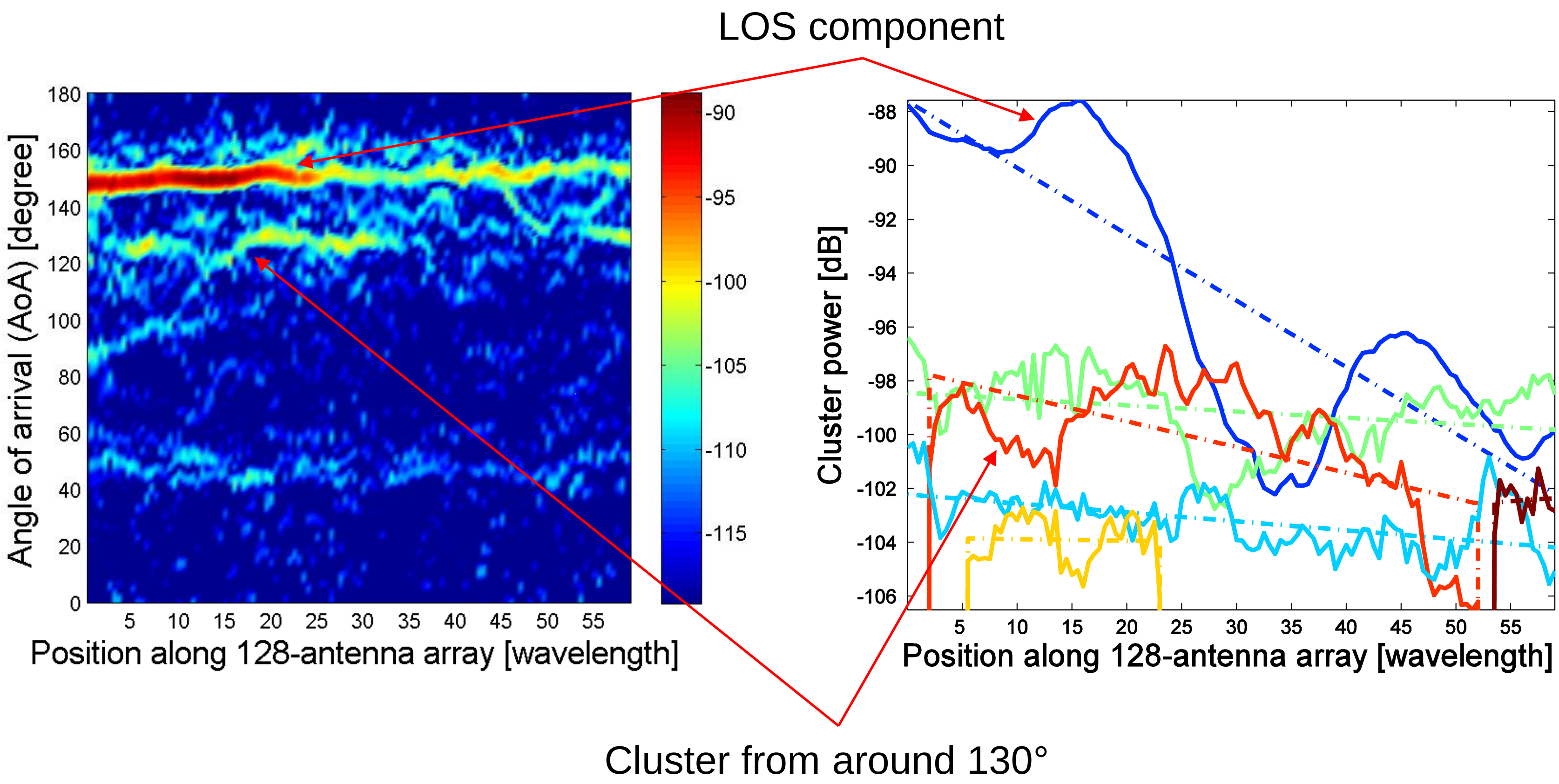
$$\mathbf{H}(t,f) = \sum_{n\in\mathscr{C}} \sum_{p} \alpha_{np}e^{-j2\pi f\tau_{np}}\, \mathbf{s}_{MS}(\boldsymbol{\Omega}_{np}^{MS})\, \mathbf{s}_{BS}^{T}(\boldsymbol{\Omega}_{np}^{BS})$$

Extension of the COST 2100 model

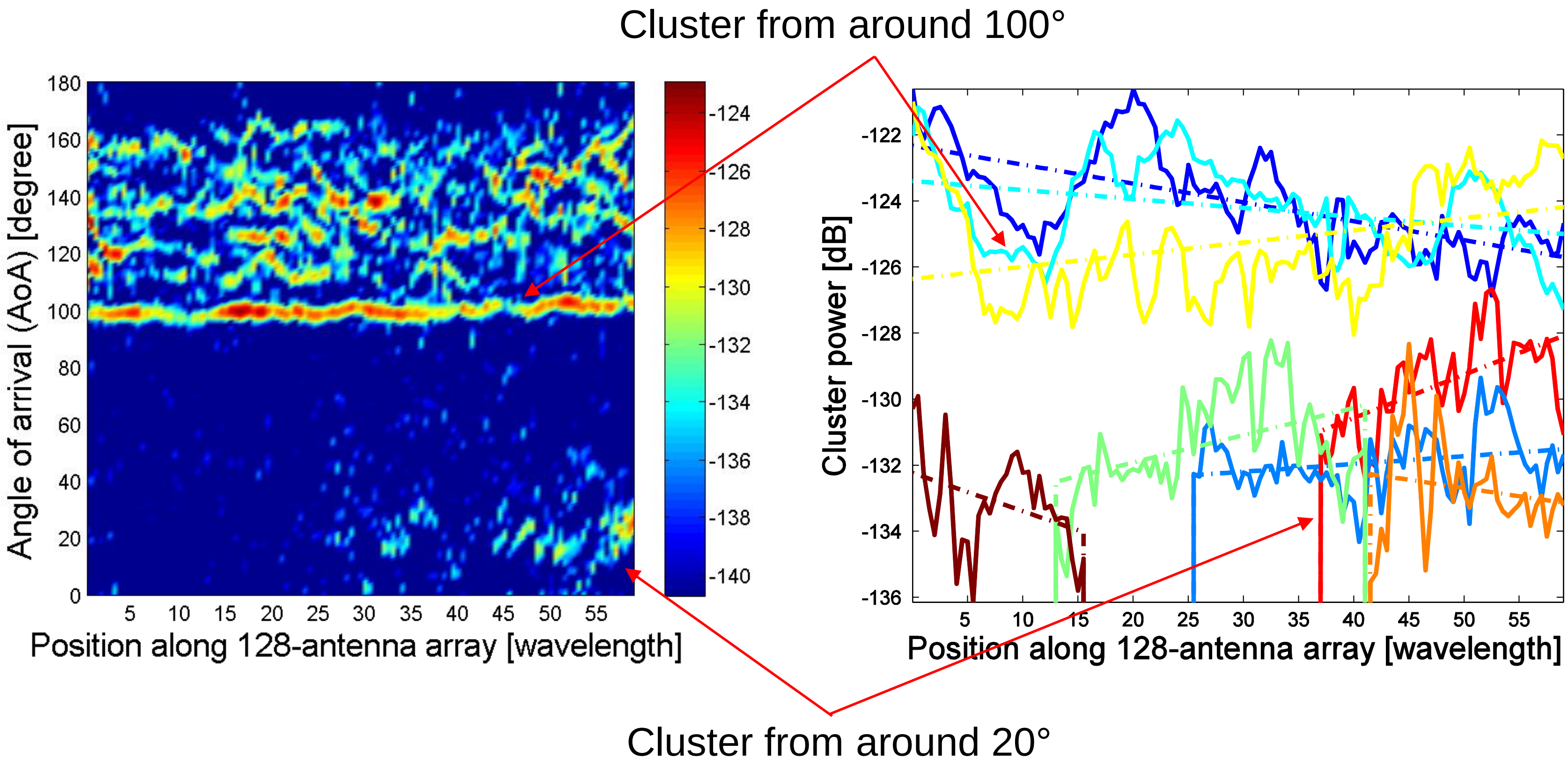


- Deploy visibility regions also at the base station
  - varying power levels
- spherical wavefronts inherent in the model
- allows for dynamic users
- correlation properties inherent

Angular power spectrum and cluster power variations (LOS scenario)

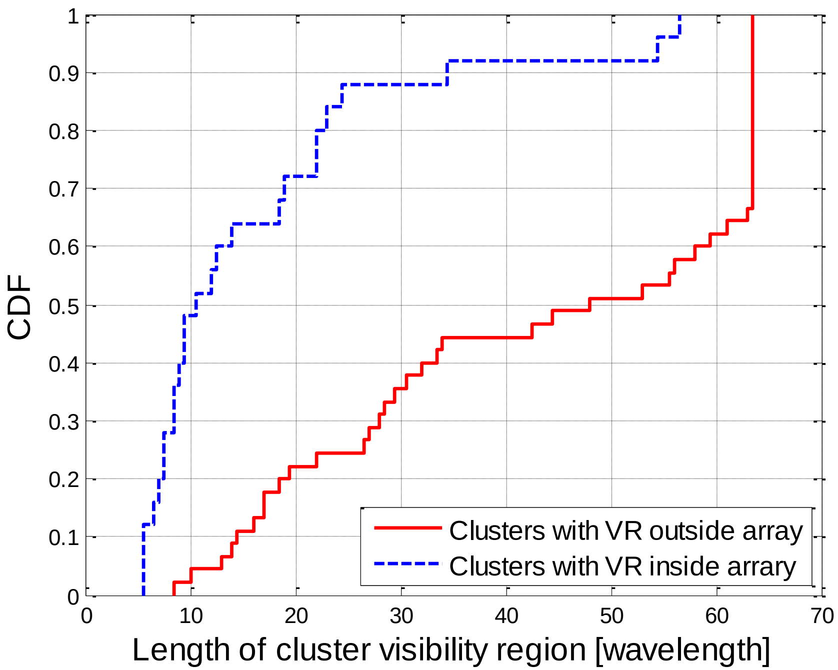


Angular power spectrum and cluster power variations (NLOS scenario)

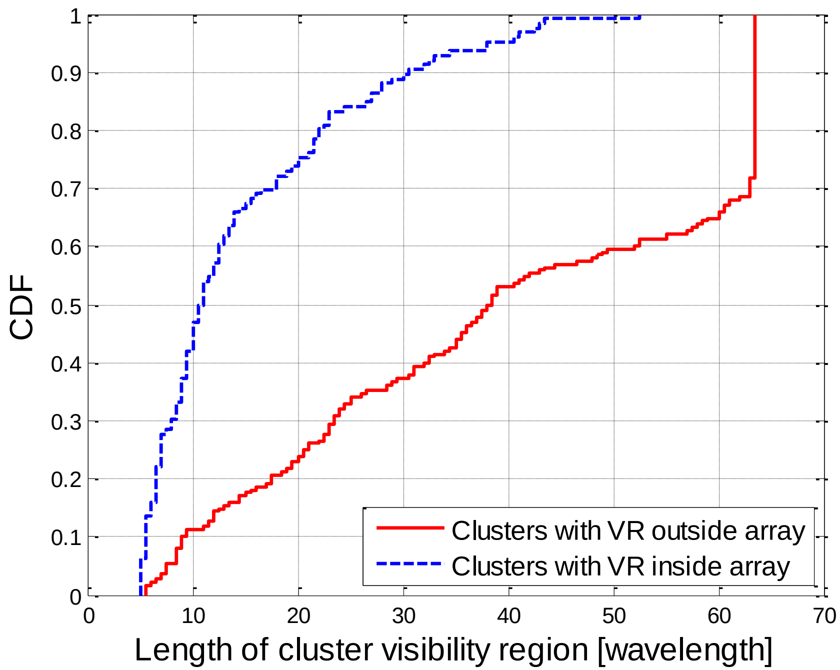


Distribution of cluster visibility regions

LOS group



NLOS group



Summary and Conclusions I

- Massive MIMO channels are different from conventional MIMO
  - variations over the array
  - non-WSSUS
  - spherical wavefronts
- Standard MIMO models can not be used if one want to capture a realistic channel behavior
- Power imbalances between users have to be taken into account

Summary and Conclusions II

- A modified COST 2100 channel model can model the channel behavior well
  - introduce visibility regions over the array
- Correlated large scale fading might be important for performance prediction
- The non-stationarities actually make the situation better than expected
- It seems that the channel is rich enough to provide enough degrees of freedom

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- Katsuyuki Haneda (Aalto University, Finland)
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Selected references I

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Selected references II

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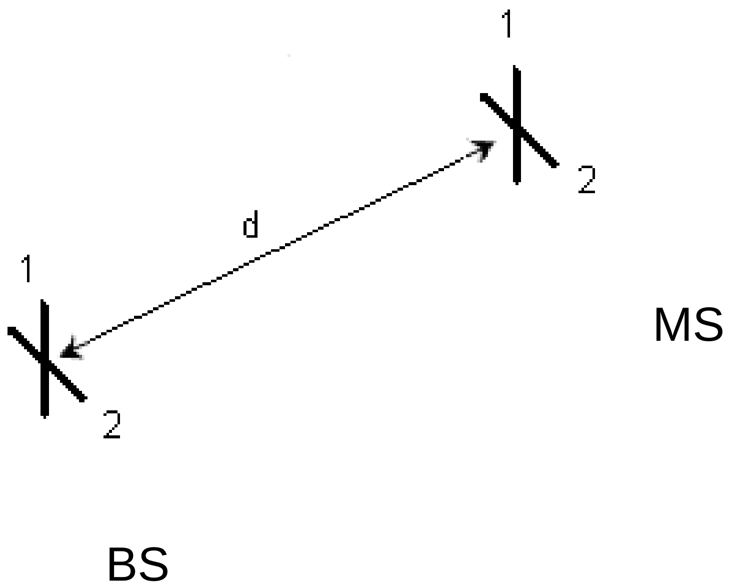
Questions?

Modeling polarization

- For systems relaying on polarization diversity, the channel model should take the depolarization induced by scattering and by antennas into account
- Each MPC must be described by a matrix (instead of a scalar)



Modeling polarization

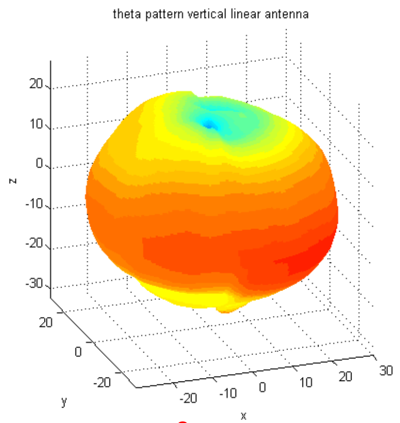
- Polarization of a single MPC is decomposed
- Antenna array polarization and orientation □ array pattern matrix
- Channel depolarization due to interactions □ polarization matrix

$$\mathbf{H}(t,\tau)=\underbrace{\exp\left(j2\pi\nu_k t\right)}_{\text{Doppler shift}}\underbrace{\mathbf{C}_\mathbf{r}\left(\theta_{r,k},\phi_{r,k}\right)}_{\text{project. on Rx array}}\begin{bmatrix}\gamma_{\theta\theta,k} & \gamma_{\theta\phi,k}\\ \gamma_{\phi\theta,k} & \gamma_{\phi\phi,k}\end{bmatrix}\underbrace{\mathbf{C}_\mathbf{t}\left(\theta_{t,k},\phi_{t,k}\right)^T}_{\text{project. on Tx array}}\delta\left(\tau-\tau_k\right)$$

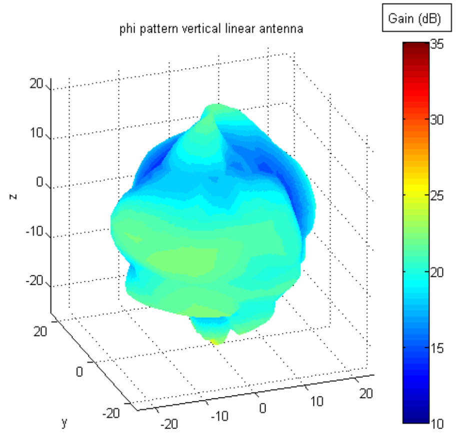
$$\mathbf{C}_\mathbf{r}\left(\theta_{r,k},\phi_{r,k}\right)=\begin{bmatrix}c_{\theta 1}\left(\theta_{r,k},\phi_{r,k}\right) & c_{\phi 1}\left(\theta_{r,k},\phi_{r,k}\right)\\ c_{\theta 2}\left(\theta_{r,k},\phi_{r,k}\right) & c_{\phi 2}\left(\theta_{r,k},\phi_{r,k}\right)\\ \vdots & \vdots\\ c_{\theta n_r}\left(\theta_{r,k},\phi_{r,k}\right) & c_{\phi n_r}\left(\theta_{r,k},\phi_{r,k}\right)\end{bmatrix}$$

θ-pattern of array

φ-pattern of array



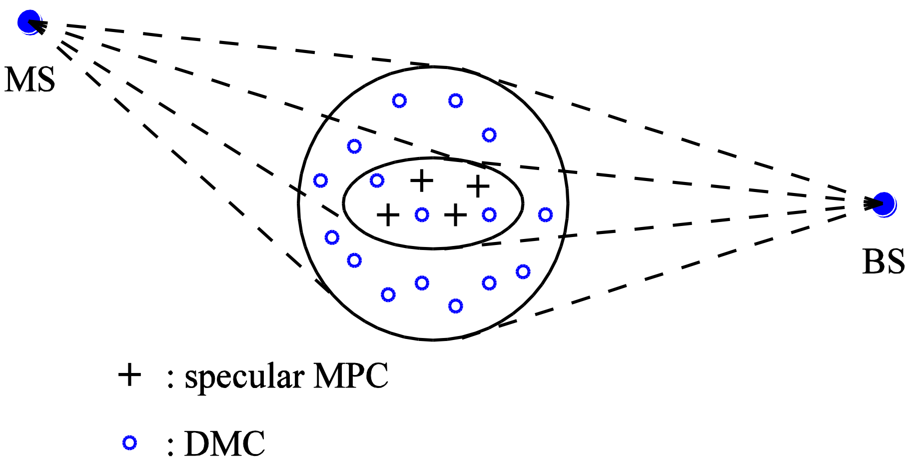
θ-pattern of a vertical linear antenna

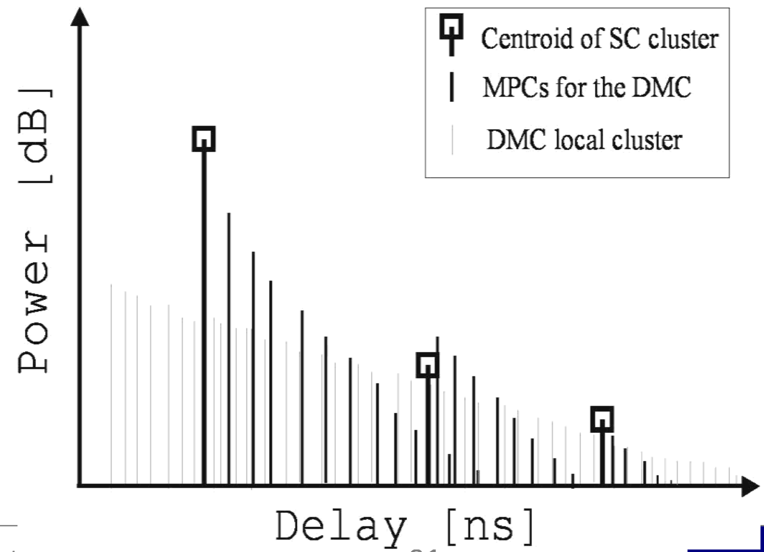


φ-pattern of a vertical linear antenna

Dense multipath components

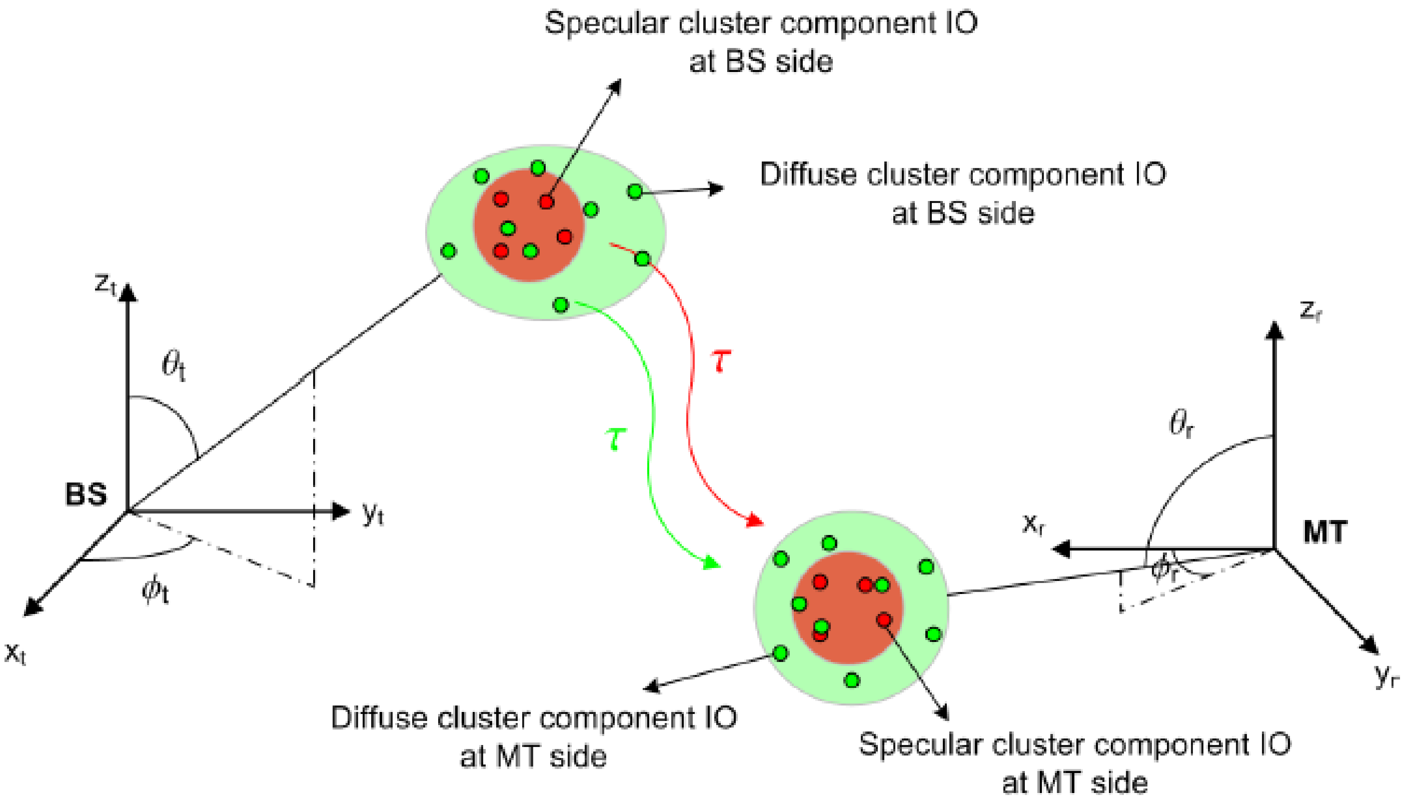
- DMC = dense or diffuse multipath components, caused by diffuse scattering or by non resolved specular paths
- Several studies found that the DMCs are clustered around the specular MPCs
- Clustering of DMC extends the cluster spectrum with extra power decaying in angular and time domains





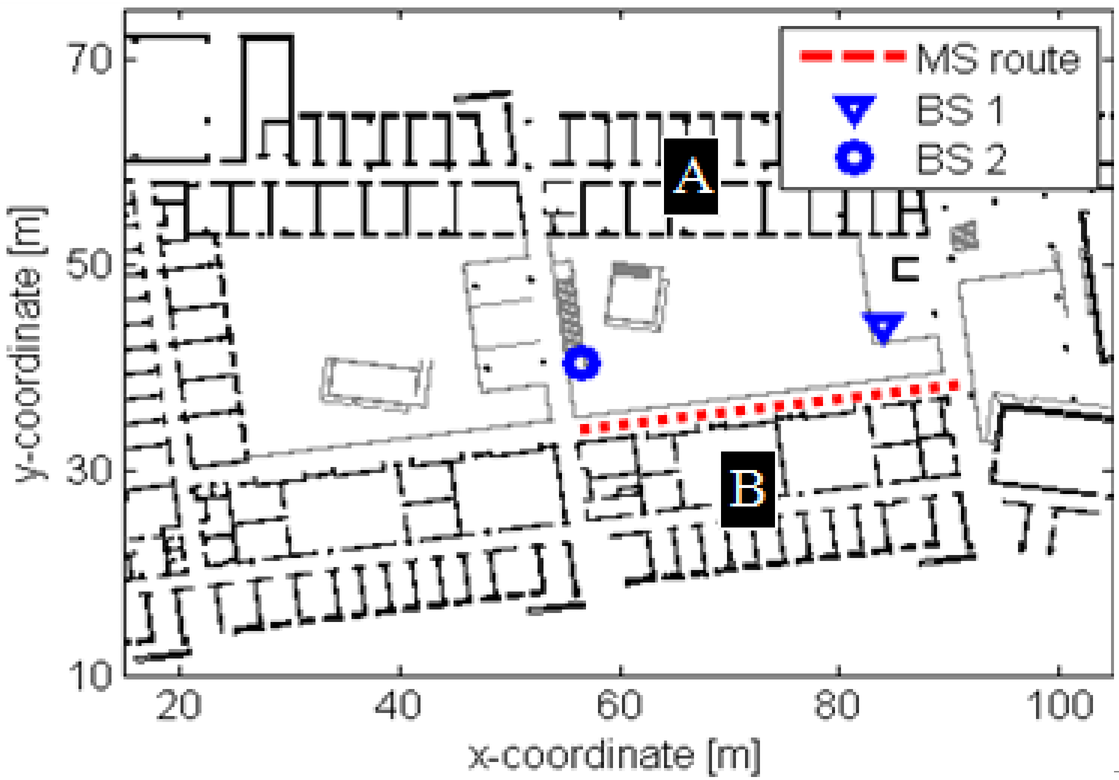
Putting DMC and polarization together

- The COST 2100 model combines the polarization and DMC behavior
- The DMC is also polarization selective !

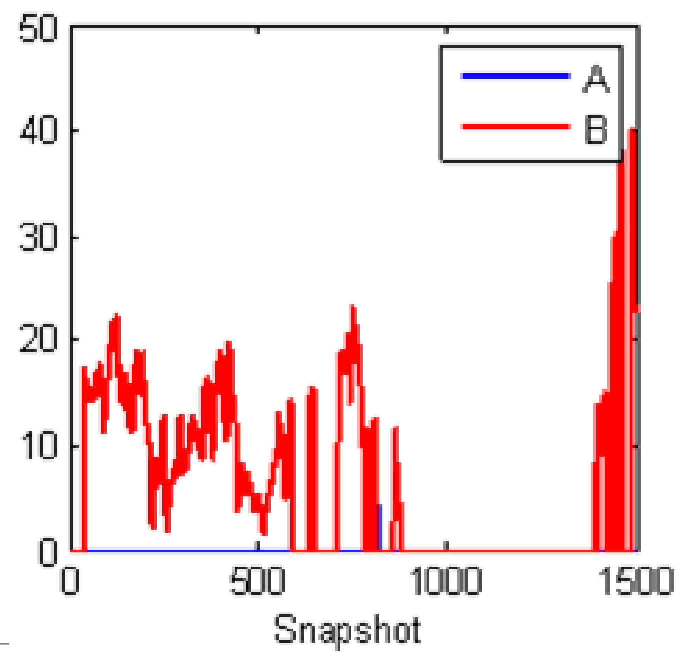


Multi-link aspects

- Multi-link channel sounding by Aalto and Lund University
- Existence of common scatterers or clusters of scatterers: some clusters are seen by different pairs of Tx-Rx
- The presence of common clusters causes inter-link correlations



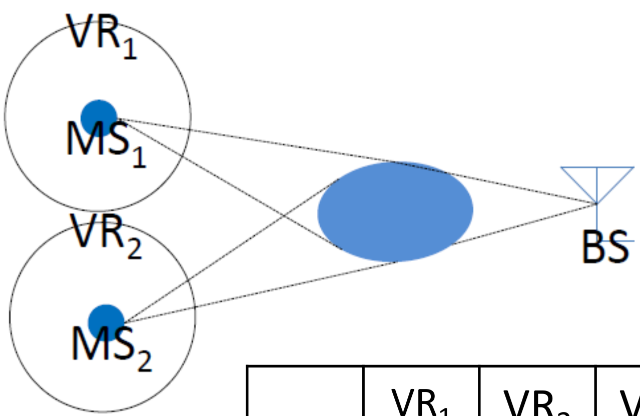
Power propagated through the same scatterer [%]



Multi-link aspects

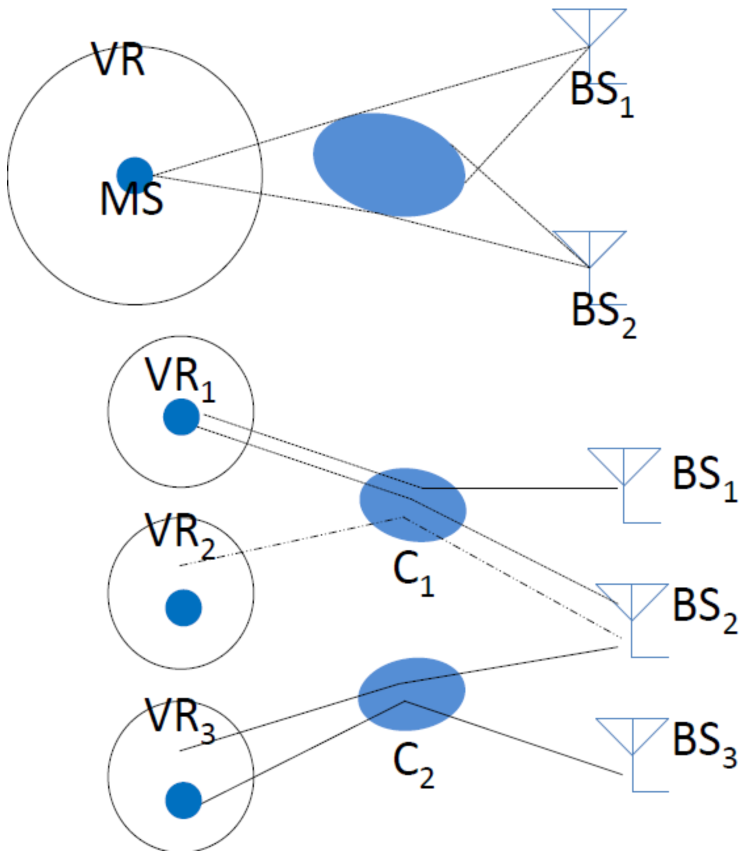
- Link-common clusters are connected to multiple VRs, each VR determining cluster connections to BSs

MS-common clusters:



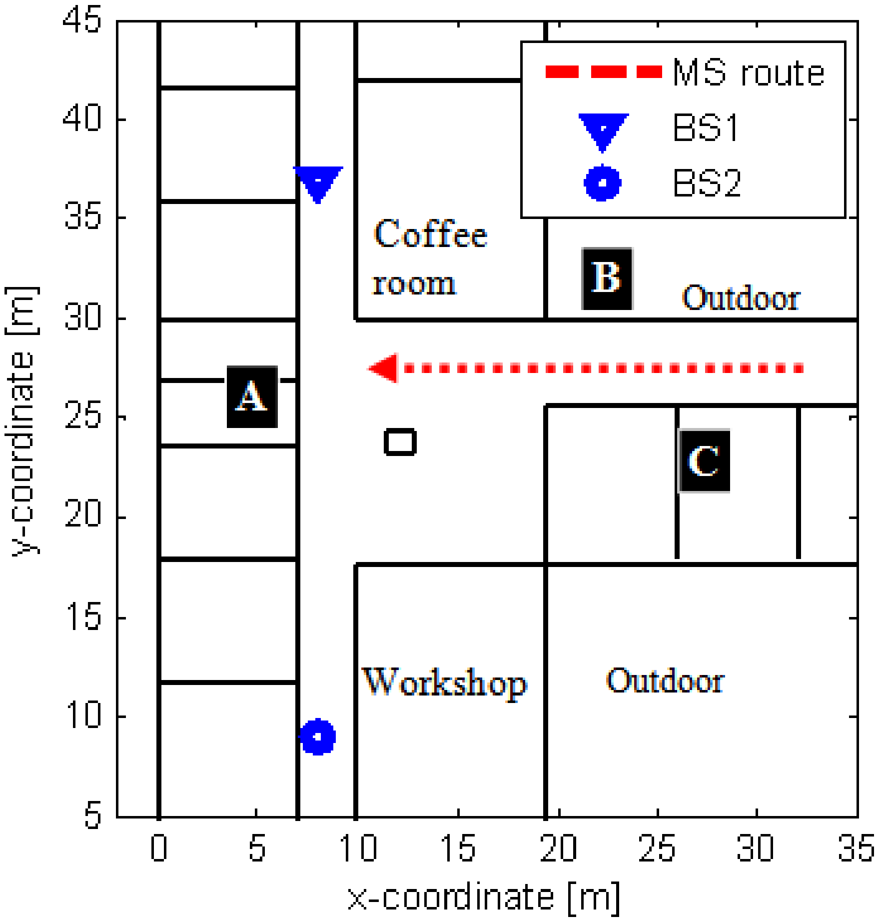
	VR_1	VR_2	VR_3	…
BS_1	C_1	-	-	…
BS_2	C_1	C_1	C_2	…
BS_3	-	-	C_2	…
…	…	…	…	…

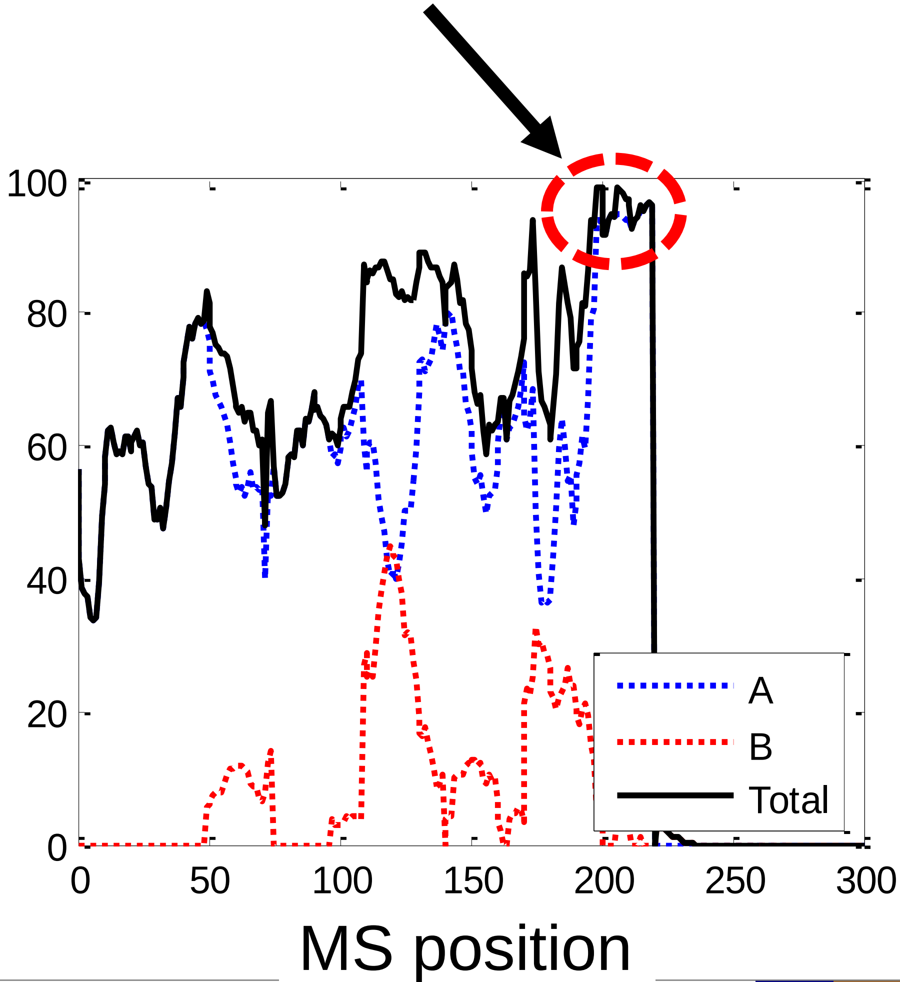
BS-common clusters:



Observations in Measurements

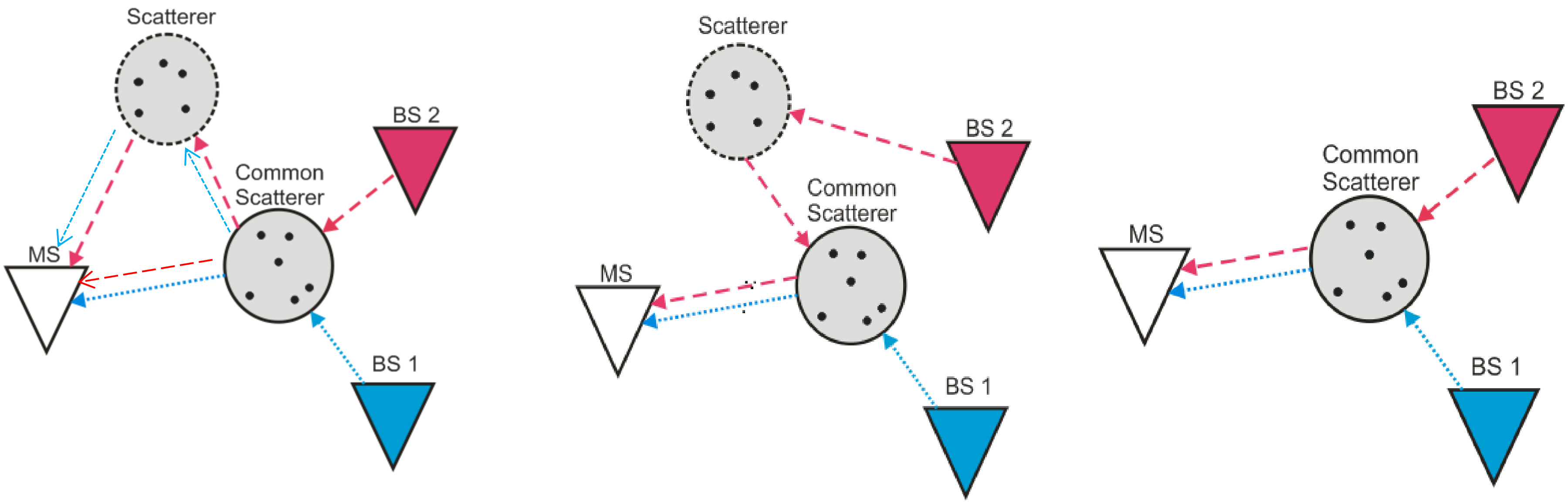
- Waveguiding in a corridor led to complete overlap of scatterers (all power propagates through the same scatterers!)





Common Scatterers (Clusters)

- Classification



(a)
BS-common

(b)
MS-common

(c)
BS-MS-common

Review: Multi-Link MIMO modeling