# On the Relationship Between MMSE-SIC and BI-GDFE Receivers for Large Multiple-Input Multiple-Output Channels

Ying-Chang Liang, Eng Yeow Cheu, Li Bai, and Guangming Pan

***Abstract*—A minimum mean-square error (MMSE)-based iterative soft interference cancellation (MMSE-SIC) receiver has been proposed to mitigate the interferences of the multiple-input multiple-output (MIMO) channels, with reduced complexity as compared to maximum-likelihood (ML) detection. On the other hand, the block-iterative generalized decision-feedback equalizer (BI-GDFE) attains close to the performance of the MMSE-SIC receivers with further reduced complexity. The BI-GDFE, however, requires an accurate estimate of the input-decision correlation (IDC), which is a statistical reliability metric of earlier-made decisions. To date, the BI-GDFE receiver is applicable only to phase-shift-keying (PSK) modulations due to the absence of a method to estimate the IDC for higher order quadrature amplitude modulations (QAMs). In this paper, we establish the relationship between the MMSE-SIC and BI-GDFE receivers and propose an algorithm to determine the IDC for BI-GDFE from the unconditional MMSE-SIC (U-MMSE-SIC). We further analyze and compare the asymptotic performances of the two receivers for large random MIMO channels and prove that for the limiting case, the output signal-to-interference-plus-noise ratios (SINRs) at each iteration for both receivers converge in probability to their respective deterministic limits. Our simulation results have shown that the bit error rate (BER) performance of the BI-GDFE receiver with the proposed IDC selection method achieves close to that of the U-MMSE-SIC receiver with similar convergence behavior and reaches the single-user matched filter bound (MFB) with several iterations for high enough signal-to-noise ratio (SNR).**

***Index Terms*—Asymptotic performance, iterative receivers, large systems, low complexity, multiple-input multiple-output (MIMO), random matrix theory.**

## I. Introduction

The input–output relationship for many communication systems can be modeled as a multiple-input multiple-output (MIMO) channel. These systems include, e.g., multiuser systems, multiple-transmit/multiple-receive antenna systems, single-carrier cyclic-prefix systems, and direct-sequence code-division multiple-access (CDMA) systems. When the inputs can be cooperatively preprocessed, and the outputs can be further processed in a cooperative manner, various joint transceiver designs can be applied, e.g., [16] and [17]. In this paper, we consider a multiuser system and refer to each of the inputs as an individual user's signal. As the data streams from the users interfere with each other, linear or nonlinear receivers have to be used to recover the transmitted symbols. While linear receivers are simple to implement, they usually produce a performance far off from the maximum-likelihood (ML) performance bound. The discouragingly high complexity of the ML detector, however, is infeasible for large system implementation as its complexity grows exponentially with the signal dimension. Closest lattice search schemes such as sphere decoding [6] have been designed to achieve ML performance; these detectors, however, become infeasible as well when the signal dimension becomes large.

Two receivers have been proposed recently to achieve near-ML detection with much reduced complexity; these are minimum mean-square-error (MMSE) receiver with soft interference cancellation (MMSE-SIC) [1], [2] and block-iterative generalized decision feedback equalizer (BI-GDFE) [3]–[5]. The conditional version of MMSE-SIC, called C-MMSE-SIC [1], requires the calculation of the filter weight for each user at each symbol interval and each iteration, thus its computational complexity is still very high. Choi *et al.* [7] proposed a block-wise soft interference cancellation to reduce the complexity of the MMSE-SIC as a compromise between the optimal maximum *a posteriori* (MAP) receiver and the symbolwise soft interference cancellation. In [8], a correlation factor for the MMSE-SIC receiver is introduced which further reduces the complexity of MMSE-SIC by employing the matrix inversion lemma and eigendecomposition to avoid the matrix inversion in each iteration. In [2], under the assumption of block fading channel, the unconditional MMSE-SIC (U-MMSE-SIC) receiver is proposed to approximate the instantaneous interference powers using their statistical means over the whole fading block consisting of many symbol intervals. Thus, it is not required to update the filter weights for every symbol interval; instead, the weights are the static for the whole fading block. However, the method in [2] deals with coded systems, and further it will introduce longer processing delay due to the averaging computation over the whole fading block.

The calculation of filter weights for any MMSE-SIC receiver relies on the soft estimates of the transmitted symbols at each iteration, thus it has to be implemented in an *online* manner. In

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BI-GDFE receiver [3]–[5], interference cancellation is carried out using the hard decisions derived in the previous iteration as well as the input-decision correlation (IDC), which is an indicator of the statistical reliability for the hard decisions. When the channel state information (CSI) is perfectly known, the theoretical IDC value can be pre-predicted based on the output signal-to-interference-plus-noise ratio (SINR) of the equalizer at each iteration. This alleviates the requirement for the exact values of the hard decisions in each iteration. In this perspective, the equalizer coefficients for BI-GDFE can be predetermined for each user at every iteration in an *offline* manner. For a block-fading channel, the equalizer coefficients for any particular user are the same for each symbol interval within the fading block. Therefore, the complexity of the BI-GDFE receiver is lower than that of the MMSE-SIC receiver.

For the BI-GDFE receiver, the selection of IDC is crucial as an inaccurate IDC may restrain the algorithm from convergence. When *the signal dimension goes to infinity* and if the signals are phase-shift-keying (PSK) modulated, the IDC is closely related to the symbol error rate/bit error rate (SER/BER) of the hard decisions made in each iteration [3]. Since the SER/BER depends on the SINR of the equalization output, the IDC can thus be determined by the output SINR of the equalizer at each iteration. However, the IDC formula derived for infinite signal dimension cannot be applied to the systems with finite signal dimension. In fact, for a finite signal dimension $K$, the IDC for one block of $K$ symbols is usually different from the IDC for another block of $K$ symbols, and thus, the IDC is a random variable. In [5], again for PSK modulations, an IDC determination algorithm has been proposed to take into consideration the randomness of the IDC as well as the equalization output SINR. The absence of accurate IDC values constitutes a major impediment to the reliable cancellation of interference by BI-GDFE receiver for higher order quadrature amplitude modulations (QAMs). It is therefore desirable to find a way to determine the IDC values such that the proposed BI-GDFE can apply to *higher order QAMs* with finite signal dimension.

In this paper, we establish the relationship between the MMSE-SIC and BI-GDFE receivers. To do so, we modify the C-MMSE-SIC receiver by replacing the instantaneous interference power matrix with a scaled identity matrix for each symbol interval. This receiver is referred to as the unconditional MMSE-SIC (U-MMSE-SIC) receiver in this paper, and this receiver will be used to compare with the BI-GDFE receiver in terms of relationship identification and computer simulations. Then, we show that the block model of BI-GDFE can be simplified to a form equivalent to that of the U-MMSE-SIC. Based on this relationship, we are able to formulate a simple algorithm to determine the IDC for BI-GDFE. Not only is the algorithm applicable to PSK modulations, the proposed IDC generation method can also be applied to higher order QAMs. We further analyze the asymptotic performance of the two receivers for large random MIMO channels and prove that for the limiting case, the output SINRs at each iteration for both receivers converge in probability to their respective deterministic limits. We show that for U-MMSE-SIC, the limiting SINR is determined by the statistics of the soft estimates, while for BI-GDFE, the limit is determined by the statistical reliability of the hard decisions.

The remainder of the paper is organized as follows. In Section II, we describe the generic channel model of interest. Section III reviews the MMSE-SIC and BI-GDFE receivers, and establishes the relationship between the U-MMSE-SIC receiver and BI-GDFE receiver. In Section IV, we present our algorithm for the selection of IDC values for BI-GDFE receiver. The asymptotic performances of the two receivers are analyzed in Section V. In Section VI, we compare the asymptotic convergence of U-MMSE-SIC and BI-GDFE numerically, and show the performance of BI-GDFE for MIMO channels with limited signal dimension based on the new IDC selection algorithm by computer simulations. This paper is concluded in Section VII.

The following notations are used throughout the paper. $\mathbb{E}\{\cdot\}$ stands for statistical expectation operation. The boldface is used to denote matrices and vectors. $\mathrm{Tr}(\mathbf{A})$, $\mathbf{A}^T$ and $\mathbf{A}^H$ denote the matrix trace, transpose operation, and conjugate transpose operation, respectively, for a matrix $\mathbf{A}$, and $\mathbf{A}(i,j)$ represents the element of $\mathbf{A}$ at the $i$th row and $j$th column; and $\mathrm{diag}(\mathbf{x})$ denotes a diagonal matrix with the diagonal elements to be vector $\mathbf{x}$. $\mathbf{I}_M$ denotes the $M \times M$ identity matrix.

## II. Channel Model

Consider a generic MIMO channel with signal dimension of $K$ and observation dimension of $N$ as follows:

$$\mathbf{x} = \mathbf{H}\mathbf{s} + \mathbf{n} \tag{1}$$

where

$$\mathbf{H} \in \mathbb{C}^{N \times K}$$

is the channel matrix with $\mathbf{H} = [\mathbf{h}_1, \mathbf{h}_2, \ldots, \mathbf{h}_K, \mathbf{s} = [s_1, s_2, \ldots, s_K]^T$ is the $K \times 1$ transmitted signal vector, $\mathbf{x} = [x_1, x_2, \ldots, x_N]^T$ is the $N \times 1$ received signal vector, and $\mathbf{n} = [n_1, n_2, \ldots, n_N]^T$ is the $N \times 1$ noise vector. To present the BI-GDFE and MMSE-SIC receivers, we make the following assumptions.

- AS1) The channel response $\mathbf{h}_k$ is represented as $\mathbf{h}_k = (1)/(\sqrt{N}[h_{1,k}, h_{2,k}, \ldots, h_{N,k}]^T$, where $h_{j,k}$ is with zero mean and unit variance.
- AS2) The transmitted symbol $s_k$ is independent and identically distributed (i.i.d.), zero mean, and with average power $\mathbb{E}\{|s_k|^2\} = \sigma_s^2$. Further, $s_k$ stems from a finite constellation of size $M$.
- AS3) The noise vector $\mathbf{n}$ is zero mean, circularly symmetric, complex Gaussian with covariance matrix $\mathbb{E}\{\mathbf{n}\mathbf{n}^H\} = \sigma_n^2 \mathbf{I}_N$.
- AS4) The transmitted symbols are statistically independent of the noises.

The signal-to-noise ratio (SNR) is defined as $\gamma = (\sigma_s^2)/(\sigma_n^2)$. Assumptions AS1) and AS2) will be revised in Section V in order to analyze the asymptotic performance of the receivers using random matrix theory. Assuming a block-fading channel, i.e., the channel is static over a frame of $T$ symbol blocks, our first objective is to detect the transmitted symbols for a given observation vector $\mathbf{x}$ assuming that the CSI $\mathbf{H}$ is perfectly known at the receiver.

## III. MMSE-SIC AND BI-GDFE RECEIVERS

In this section, we first review the principles of the MMSE-SIC and BI-GDFE receivers, and then establish the relationship between the two receivers.

### A. MMSE-SIC Receiver [1]

We first apply the MMSE-SIC receiver to detect the transmitted symbols for MIMO systems. Denote $\mathbf{w}_{k,\ell}$ as the MMSE weight vector of user $k$ at the $\ell$th iteration, and assume that the SNR $\gamma$ is perfectly known at the receiver. In the zeroth iteration, the conventional MMSE receiver is applied for user $k$, which is given by [11]

$$\mathbf{w}_{k,0} = \left[\mathbf{h}_k\mathbf{h}_k^H + \sum_{j\neq k}\mathbf{h}_j\mathbf{h}_j^H + \frac{1}{\gamma}\mathbf{I}_N\right]^{-1}\mathbf{h}_k. \quad (2)$$

The output of the MMSE receiver after *bias removing* is given by

$$\check{s}_{k,0} = \frac{\mathbf{w}_{k,0}^H\mathbf{x}}{\alpha_{k,0}} = s_k + e_{k,0} \quad (3)$$

where $\alpha_{k,0} = \mathbf{w}_{k,0}^H\mathbf{h}_k$ is the signal amplitude after MMSE receiver if no bias removing is carried out, and $e_{k,0} = (1)/(\alpha_{k,0})[\sum_{j\neq k}\mathbf{w}_{k,0}^H\mathbf{h}_j s_j + \mathbf{w}_{k,0}^H\mathbf{n}]$ contains the residual interference and noise after bias removing. The variance of $e_{k,0}$ is expressed as

$$\check{\sigma}_{k,0}^2 = \frac{\mathbf{w}_{k,0}^H\left[\mathbf{H}_k\mathbf{H}_k^H\sigma_s^2 + \sigma_n^2\mathbf{I}_N\right]\mathbf{w}_{k,0}}{|\alpha_{k,0}|^2} \quad (4)$$

where $\mathbf{H}_k$ denotes the matrix $\mathbf{H}$ with the $k$th column being deleted. Thus, the output SINR is given by

$$\beta_{k,0} = \frac{\sigma_s^2}{\check{\sigma}_{k,0}^2} = \mathbf{h}_k^H\left[\mathbf{H}_k\mathbf{H}_k^H + (1/\gamma)\mathbf{I}_N\right]^{-1}\mathbf{h}_k. \quad (5)$$

Having acquired the output of the MMSE receiver, the log-likelihood ratios (LLRs) of the modulation candidates are then derived. The soft estimate of user $k$'s symbol at the zeroth iteration can be generated as [7]

$$\begin{aligned}\tilde{s}_{k,0} = \mathbb{E}\{s_k|\check{s}_{k,0}\} &= \sum_{i=0}^{M-1} a_i p(s_k = a_i|\check{s}_{k,0}) \\ &= \frac{\sum_{i=0}^{M-1} a_i p(\check{s}_{k,0}|s_k = a_i)}{\sum_{i=0}^{M-1} p(\check{s}_{k,0}|s_k = a_i)}\end{aligned} \quad (6)$$

where $a_0, \ldots, a_{M-1}$ are the constellation points, and $p(x|y)$ denotes the probability of event $x$ occurs under condition $y$. When the signal dimension becomes large, the multiple access interference (MAI) after MMSE filtering and bias removing becomes asymptotic Gaussian [12], [15] with mean zero and variance $\check{\sigma}_{k,0}^2$, thus

$$p(\check{s}_{k,0}|s_k = a_i) = \frac{1}{\sqrt{2\pi}\check{\sigma}_{k,0}}\exp\left(-\frac{|\check{s}_{k,0} - a_i|^2}{2\check{\sigma}_{k,0}^2}\right). \quad (7)$$

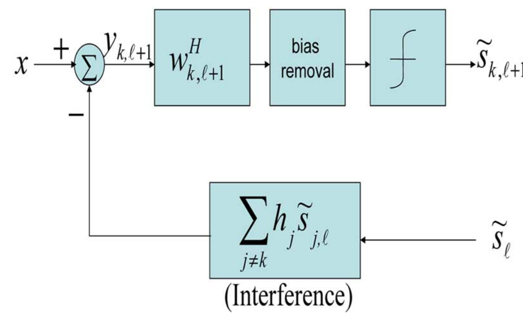


Fig. 1. Block diagram of MMSE-SIC receiver for detecting user $k$ at the $(\ell + 1)$th iteration.

At this point, we can assume that we have obtained the soft estimates $\tilde{s}_{k,\ell}$'s for all users at the $\ell$th iteration. The block diagram of the MMSE-SIC receiver is shown in Fig. 1 where soft cancellation of the MAI is carried out to produce the output for detecting user $k$ for subsequent iterations. This output is given by

$$\begin{aligned}\mathbf{y}_{k,\ell+1} &= \mathbf{x} - \sum_{j\neq k}\mathbf{h}_j\tilde{s}_{j,\ell} \\ &= \mathbf{h}_k s_k + \sum_{j\neq k}\mathbf{h}_j(s_j - \tilde{s}_{j,\ell}) + \mathbf{n}.\end{aligned} \quad (8)$$

Given the above output, subsequent iterations shall be carried out with a new optimal MMSE weight vector for every user $k$. The optimal MMSE weight vector $\mathbf{w}_{k,\ell+1}$ is obtained to minimize the mean-square-error (MSE) between $\mathbf{w}_{k,\ell+1}^H\mathbf{y}_{k,\ell+1}$ and $s_k$ and is given by

$$\mathbf{w}_{k,\ell+1} = \left[\mathbf{h}_k\mathbf{h}_k^H + \mathbf{H}_k\mathbf{D}_{k,\ell}\mathbf{H}_k^H + \frac{1}{\gamma}\mathbf{I}_N\right]^{-1}\mathbf{h}_k, \quad (9)$$

where

$$\mathbf{D}_{k,\ell} = \mathrm{diag}\left(d_{1,\ell}, \ldots, d_{k-1,\ell}, d_{k+1,\ell}, \ldots, d_{K,\ell}\right) \quad (10)$$

with

$$\begin{aligned}d_{j,\ell} &= \frac{1}{\sigma_s^2}\mathbb{E}\left\{|s_j - \tilde{s}_{j,\ell}|^2|\check{s}_{j,\ell}\right\} \\ &= \frac{1}{\sigma_s^2}\left[\mathbb{E}\{|s_j|^2|\check{s}_{j,\ell}\} - |\tilde{s}_{j,\ell}|^2\right]\end{aligned} \quad (11)$$

and $\mathbb{E}\{|s_j|^2|\check{s}_{j,\ell}\} = \sum_{i=0}^{M-1}|a_i|^2 p(s_j = a_i|\check{s}_{j,\ell})$. In (11), $d_{j,\ell}$ defines the normalized residual interference power for user $j$, which will vary for every symbol interval and every iteration, thus matrix $\mathbf{D}_{k,\ell}$ is termed instantaneous interference power matrix. Following (2) and (5), the MMSE output and output SINR for $(\ell + 1)$th iteration can be calculated accordingly. Thus, soft interference cancellation can be carried out iteratively. The above receiver is referred to as the conditional MMSE-SIC (C-MMSE-SIC) receiver [1].

Let us replace the instantaneous interference power matrix $\mathbf{D}_{k,\ell}$ in (10) with a scaled identity matrix

$$\bar{\mathbf{D}}_{k,\ell} = \left(1 - \delta_{k,\ell}^2\right)\mathbf{I}_{K-1} \quad (12)$$

where

$$\delta_{k,\ell}^2 = 1 - \frac{1}{(K-1)\sigma_s^2} \sum_{j \neq k} \left[ \mathbb{E}\{|s_j|^2 | \check{s}_{j,\ell}\} - |\tilde{s}_{j,\ell}|^2 \right]. \quad (13)$$

Here, $\bar{\mathbf{D}}_{k,\ell}$ is an approximation of $\mathbf{D}_{k,\ell}$ and $\delta_{k,\ell}^2$ can be considered as the average reliability for all users except user $k$, at iteration $\ell$. Thus, we can then design the weight vector as

$$\mathbf{w}_{k,\ell+1} = \left[ \mathbf{h}_k \mathbf{h}_k^H + \mathbf{H}_k \bar{\mathbf{D}}_{k,\ell} \mathbf{H}_k^H + \frac{1}{\gamma} \mathbf{I}_N \right]^{-1} \mathbf{h}_k. \quad (14)$$

Note from (11) and (13), we have $\mathrm{Tr}\,(\mathbf{D}_{k,\ell}) = \mathrm{Tr}(\bar{\mathbf{D}}_{k,\ell})$. Thus, in (12), we in fact use the average residual interference power to replace the instantaneous residual interference powers for all interferers. The above receiver is termed the unconditional MMSE-SIC (U-MMSE-SIC) receiver. Note this receiver is different from the one from [2] where the averaging is calculated not only over all users, but also over time. By doing so, the receiver may introduce longer processing delay. Finally, the definition of $\delta_{k,\ell}^2$ in (13) will enable us to link the U-MMSE-SIC receiver with the BI-GDFE receiver to be presented in the next subsection.

Using the above weight vector to obtain the output $\check{s}_{k,\ell+1} = \mathbf{w}_{k,\ell+1}^H \mathbf{y}_{k,\ell+1}$, the corresponding variance $\check{\sigma}_{k,\ell+1}^2$ of noise-plus-interference after bias removing can be generated as

$$\check{\sigma}_{k,\ell+1}^2 = \frac{\mathbf{w}_{k,\ell+1}^H \left[ \mathbf{H}_k \mathbf{D}_{k,\ell} \mathbf{H}_k^H \sigma_s^2 + \sigma_n^2 \mathbf{I}_N \right] \mathbf{w}_{k,\ell+1}}{\alpha_{k,\ell+1}^2}. \quad (15)$$

Using matrix inversion lemma, we derive output SINR as (16), shown at the bottom of the page. The LLRs of the modulation candidates are then computed and used in the determination of the soft symbols just like the way the soft symbols are calculated for C-MMSE-SIC. With the soft symbols, the same (8) is utilized to acquire the output decision of detecting user $k$ for the next iteration.

For every iteration, the soft symbols $\tilde{s}_{k,\ell}$ and the residual interference matrix $\mathbf{D}_{k,\ell}$ will vary. Thus, similar to C-MMSE-SIC, the U-MMSE-SIC receiver requires the calculation of the MMSE weights for each user, each symbol interval, and each detection iteration.

### *B. BI-GDFE Receiver*

The principle of BI-GDFE [5] is to estimate the transmitted symbols of each transmitted block of symbols in a parallel manner. The hard decisions made at the previous iteration are utilized to cancel out the interference on each of the symbols at the current iteration.

At $(\ell+1)$th iteration, the received signal $\mathbf{x}$ is passed to the feedforward equalizer (FFE) and at the same time, the decision-directed symbols $\hat{\mathbf{s}}_\ell$ from the $\ell$th iteration go through the feedback equalizer (FBE). The output from the FFE and that of the FBE in BI-FDGE are then combined and utilized to generate the filter output $\mathbf{z}_{\ell+1}$ for the $(\ell+1)$th iteration as

$$\mathbf{z}_{\ell+1} = \mathbf{F}_{\ell+1}^H \mathbf{x} - \mathbf{B}_{\ell+1} \hat{\mathbf{s}}_\ell \quad (17)$$

where $\mathbf{F}_{\ell+1}$ represents the FFE weight and $\mathbf{B}_{\ell+1}$ represents the FBE weight at the $(\ell+1)$th iteration. The underlying design of the BI-GDFE is to preserve and recover the desirable signal component $\mathbf{s}$ in the observation vector $\mathbf{x}$ by the FFE, and to cancel out the interference by the FBE.

Let $\mathbf{A}_{\ell+1}$ be the diagonal matrix whose diagonal elements are the same those of matrix $\mathbf{F}_{\ell+1}^H \mathbf{H}$. Denote $\rho_\ell$ as the IDC for the $\ell$th iteration, i.e., $\mathbb{E}\{\mathbf{s}\hat{\mathbf{s}}^H\} = \rho_\ell \sigma_s^2 \mathbf{I}_N$. It has been shown in [5] that the optimal FFE and FBE for $(\ell+1)$th iteration which maximizes the output SINR are given by

$$\mathbf{F}_{\ell+1} = \left[ (1-\rho_\ell^2) \mathbf{H}\mathbf{H}^H + (1/\gamma) \mathbf{I}_N \right]^{-1} \mathbf{H} \quad (18)$$

$$\mathbf{B}_{\ell+1} = \rho_\ell (\mathbf{F}_{\ell+1}^H \mathbf{H} - \mathbf{A}_{\ell+1}) \quad (19)$$

and the maximum SINR for symbol $s_k$ at $(\ell+1)$th iteration is shown to be

$$\gamma_{k,\ell+1} = \frac{|\mathbf{A}_{\ell+1}(k,k)|^2 \sigma_s^2}{\mathbf{G}_{\tilde{n}_{\ell+1}}(k,k)} \quad (20)$$

where $\mathbf{G}_{\tilde{n}_{\ell+1}}$ represents the noise-plus-interference covariance (NIC) matrix as

$$\mathbf{G}_{\tilde{n}_{\ell+1}} = \frac{\sigma_s^2 \left(1-\rho_\ell^2\right)}{\rho_\ell^2} \mathbf{B}_{\ell+1} \mathbf{B}_{\ell+1}^H + \sigma_n^2 \mathbf{F}_{\ell+1}^H \mathbf{F}_{\ell+1}. \quad (21)$$

Since the diagonal of the FBE $\mathbf{B}_{\ell+1}$ contains zeros, the previous decisions $\hat{\mathbf{s}}_\ell$ are only used in the cancellation of the interferences caused by the transmitted symbols. This will not affect the signal components contained at the slicer input $\mathbf{z}_{\ell+1}$.

The IDC $\rho_{-1}$ is set to 0 and thus the BI-GDFE functions as the conventional MMSE equalizer for the zeroth iteration. When all the symbols have been recovered correctly, the IDC coefficient becomes 1, and the FBE will cancel out all the interferences and FFE becomes single-user matched filter (SUMF).

### *C. Relationship Between U-MMSE-SIC and BI-GDFE*

In order to establish the relationship between the U-MMSE-SIC and BI-GDFE receivers, let us refer to the BI-GDFE combiner output $\mathbf{z}_{\ell+1}$ in (17) and the FBE $\mathbf{B}_{\ell+1}$ of BI-GDFE in (19). The block diagram of the BI-GDFE receiver

$$\beta_{k,\ell+1} = \frac{\left| \mathbf{h}_k^H \left[ \mathbf{H}_k \bar{\mathbf{D}}_{k,\ell} \mathbf{H}_k^H + (1/\gamma) \mathbf{I}_N \right]^{-1} \mathbf{h}_k \right|^2}{\mathbf{h}_k^H \left[ \mathbf{H}_k \bar{\mathbf{D}}_{k,\ell} \mathbf{H}_k^H + (1/\gamma) \mathbf{I}_N \right]^{-1} \left[ \mathbf{H}_k \mathbf{D}_{k,\ell} \mathbf{H}_k^H + (1/\gamma) \mathbf{I}_N \right] \left[ \mathbf{H}_k \bar{\mathbf{D}}_{k,\ell} \mathbf{H}_k^H + (1/\gamma) \mathbf{I}_N \right]^{-1} \mathbf{h}_k} \quad (16)$$

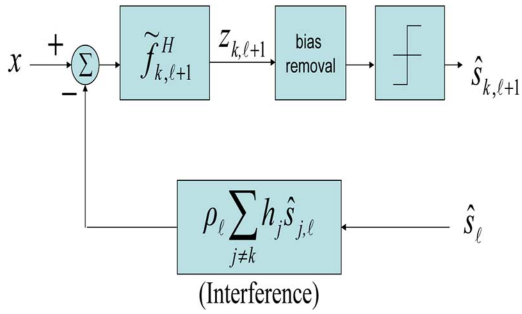


Fig. 2. Block diagram of BI-GDFE receiver for detecting user $k$ at the $(\ell+1)$th iteration.

is illustrated in Fig. 2. From these equations, we can re-express the $k$th component of the slicer input $\mathbf{z}_{\ell+1}$ for BI-GDFE as

$$\begin{aligned} z_{k,\ell+1} &= \mathbf{f}_{k,\ell+1}^H \mathbf{x} - \rho_\ell \sum_{j\neq k} \mathbf{f}_{k,\ell+1}^H \mathbf{h}_j \hat{s}_{j,\ell} \\ &= \mathbf{f}_{k,\ell+1}^H \left[ \mathbf{x} - \rho_\ell \sum_{j\neq k} \mathbf{h}_j \hat{s}_{j,\ell} \right] \end{aligned} \tag{22}$$

where $\mathbf{f}_{k,\ell+1}$ is the $k$th column of matrix $\mathbf{F}_{\ell+1}$. We further replace the weight vector of BI-GDFE for the $k$th user with

$$\tilde{\mathbf{f}}_{k,\ell+1} = \left[ \mathbf{h}_k \mathbf{h}_k^H + \left(1-\rho_\ell^2\right) \sum_{j\neq k} \mathbf{h}_j \mathbf{h}_j^H + \frac{1}{\gamma} \mathbf{I}_N \right]^{-1} \mathbf{h}_k. \tag{23}$$

According to matrix inversion lemma, $\tilde{\mathbf{f}}_{k,\ell+1}$ is a scalar version of $\mathbf{f}_{k,\ell+1}$, thus the above replacement will not change the performance of receiver, since in any case there is bias removal operation in the later stage.

Having acquired the simplified elemental component of the slicer input for BI-GDFE, we compare the output of detecting a particular user $k$ in (8) for U-MMSE-SIC (it shares this formula with C-MMSE-SIC) with the $k$ combiner output of BI-GDFE in (22). From Figs. 1 and 2, it can be seen that the operations of the MMSE-SIC and BI-GDFE receivers are analogous in design.

It can be further seen from the MMSE weight vector for user $k$ in (14) and the weight vector for user $k$ for BI-GDFE in (23) that both equations have the second term scaled by a factor. In (14), the second term is scaled by the normalized average power of the interfering soft symbols $\delta_{k,\ell}^2$, whereas in (23), the second term of the equation is scaled by the square of the IDC coefficient $\rho_\ell^2$. One interesting point that we can take note of from this observation is that both the scaling factor will approach 1 when the transmitted symbols are perfectly recovered. In (14), $\delta_{k,\ell}$ represents the reliability of the interfering soft symbols, and the IDC coefficient $\rho_\ell$ in (23) represents the statistical reliability factor of the hard decisions.

Despite the fact that the outputs of the BI-GDFE are hard decisions (symbols of discrete values in the constellation map of a particular modulation scheme), IDC scales down these hard decisions. With regard to this, hard decisions are scaled by $\rho_\ell$ at the FBE and these scaled decisions are then used in the cancellation process. The scaled decisions can be thought of as soft symbols. Hence, both $\delta_{k,\ell}$ and $\rho_\ell$ play the same role in measuring the correctness of recovered symbols.

## IV. IDC Generation Method for BI-GDFE

As can be seen from (18) and (19), BI-GDFE requires an accurate IDC coefficient $\rho_\ell$ to design the FFE and FBE. Thus, the selection of the IDC is critical to the guaranteed convergence of the BI-GDFE receiver. The IDC indicates the statistical reliability of the decision symbols. When signal dimension $K$ approaches infinity, IDC closely relates to the symbol/bit error rate (SER/BER) of the system. For quadrature phase-shift keying (QPSK) modulation and SINR of $\gamma_0$, we have [3]

$$\rho_\ell = 1 - 2\mathcal{Q}(\sqrt{\gamma_0}) \tag{24}$$

where $\mathcal{Q}(u) = (1/\sqrt{2\pi}) \int_u^\infty e^{-t^2/2} dt$. For finite signal dimension, the IDC value will fluctuate from one block of $K$ symbols to another block, thus (24) cannot be applied. In [5], a Monte Carlo-based method is proposed, which first generates the IDC values for $\mathcal{L}$ AWGN blocks with SINR $\gamma_0$($\mathcal{L}$ is chosen as a large number, say $\mathcal{L} = 20\,000$), then the mean and minimum values of the IDC are found from the IDC set, finally the average of these two values is considered as the IDC when the SINR value is $\gamma_0$. The simulations have shown that this IDC generation method can achieve guaranteed and fast convergence when the modulation is QPSK. However, for higher order QAM, this method does not work, and there are no effective methods available in the literature so far.

When the signal dimension $K$ is of finite size and for a given SINR, the IDC of BI-GDFE fluctuates from one block to another. Similarly, the reliability of the soft estimates from U-MMSE-SIC will also vary from one block to another. Notice that for BI-GDFE, we hope to pre-generate the IDC curve for a given SINR range and a given signal dimension. Based on the relationship between the scaling factor $\delta_{k,\ell}$ and IDC $\rho_\ell$ developed in the previous section, we propose an algorithm to estimate the IDC for BI-GDFE from the profile of the soft estimates of U-MMSE-SIC receiver. The IDC estimation is carried out over a MIMO channel (1) with signal dimension and observation dimension being equal to $K$, and for a particular modulation scheme. We first use Monte Carlo simulations to obtain, for U-MMSE-SIC, the profile defining the likelihood of a SINR-$\delta$ pair $(\gamma_\ell, \delta_{k,\ell})$ measured for each iteration. Then this profile is used to determine the SINR-IDC pair for BI-GDFE for the given signal dimension $K$.

The IDC generation algorithm is summarized as follows.

S1) Divide the SINR range into intervals of size $\Delta_\gamma$ where the $i$th interval is $[\gamma(i), \gamma(i+1)]$, and divide the $\delta$ range into intervals of size $\Delta_\delta$ where the $j$th interval is $[\delta(j), \delta(j+1)$. The SINR interval $i$ has nominal value of $\gamma_{\rm mid}\ (i)$ where $\gamma_{\rm mid}\ (i) = [\gamma(i+1) + \gamma(i)]/2$ and the $\delta$ interval $j$ has nominal value of $\delta_{\rm mid}(j)$ where $\delta_{\rm mid}\ (y) = [\delta(j+1) + \delta(j)]/2$. Initialize all the elements of a two-dimensional array $C_{i,j}$ to 0 for every SINR interval and $\delta$ interval. The two-dimensional array $C_{i,j}$ serves as a histogram which stores the frequency of occurrence for a SINR–$\delta$ pair for U-MMSE-SIC.

S2) For a $K \times K$ MIMO channel, generate a block of $K$ randomly modulated symbols. Using U-MMSE-SIC to acquire the SINR–$\delta$ pair $(\gamma_\ell, \delta_{k,\ell})$ where $\delta_{k,\ell}$ is calculated from (13) and $\gamma_\ell = (1)/(K) \sum_{k=1}^{K} \sigma_s^2 / \check{\sigma}_{k,\ell}^2$

with $\check{\sigma}^2_{k,\ell}$ being calculated from (15). Compute $\delta_\ell = (1)/(K)\sum_{k=1}^{K}\sqrt{\delta^2_{k,\ell}}$.

S3) Increment the frequency count $C_{i,j}$ at SINR interval $i$ and $\delta$ interval $j$ by 1 when the pair $(\gamma_\ell, \delta_\ell)$ satisfies the following conditions:

$$\gamma(i) \le \gamma_\ell < \gamma(i+1), \delta(j) \le \delta_\ell < \delta(j+1).$$

Repeat S2) and S3) for sufficiently large amount of realizations.

S4) Find the mean of the $\delta$ variable at every SINR interval $i$, denoted by $\bar{\delta}(i)$, such that

$$\bar{\delta}(i) = \frac{\sum_k C_{i,k}\delta_{\rm mid}(k)}{\sum_k C_{i,k}}.$$

S5) Find the lowest bound of the $\delta$ variable at every SINR interval $i$, denoted by $\delta_{\min}(i)$, such that

$$\text{ind} = \min_{C_{i,k}>0} k,$$
$$\delta_{\min}(i) = \delta_{\rm mid}\,(\text{ind}).$$

S6) Compute $\delta_{\rm ave}(i)$ by averaging $\bar{\delta}(i)$ and $\delta_{\min}(i)$ at every SINR interval $i$.

S7) Calculate IDC $\rho(i)$ at every SINR-interval $i$ by smoothing $\delta_{\rm ave}\,(i)$ as follows:

$$\rho(i) = \frac{1}{2\tau+1}\sum_{k=i-\tau}^{i+\tau}\delta_{\rm ave}\,(k)$$

where $\tau$ is a positive integer called smoothing factor.

In Step S6), we select the IDC as the average of the $\bar{\delta}(i)$ and $\delta^{\min}(i)$ to make tradeoff between convergence speed and guaranteed convergence, similar to the method proposed in [5]. Note that if the mean value is used, the BI-GDFE may not converge as we are having finite signal dimension. On the other hand, if the minimum value is used, the algorithm may converge very slowly. The jagged IDC is subsequently smoothed to remove any big fluctuation. The smoothed IDC values are then recorded against the SINR values and stored, for example, into a lookup table for the particular signal dimension and modulation scheme. Please note the generation of lookup table needs to be done once only.

Fig. 3 shows the SINR–IDC curves for $K = 32$ generated by the methods proposed in [3] and [5], and the IDC curve generated by our above algorithm for QPSK modulation scheme. It is seen that for a given SINR, the corresponding IDC value chosen from our proposed method is between those generated from [3] and [5]. The BER results of BI-GDFE using these three IDC generation methods will be compared in Section VI.

## V. Asymptotic Performance Analysis

In this section, we study the asymptotic performances of the U-MMSE-SIC receiver and the BI-GDFE receiver for *random* MIMO channels, assuming that $K \to \infty, N \to \infty$, and $K/N \to c$, where $c$ is a fixed constant. Note that here the constant $c$ can be greater than one.

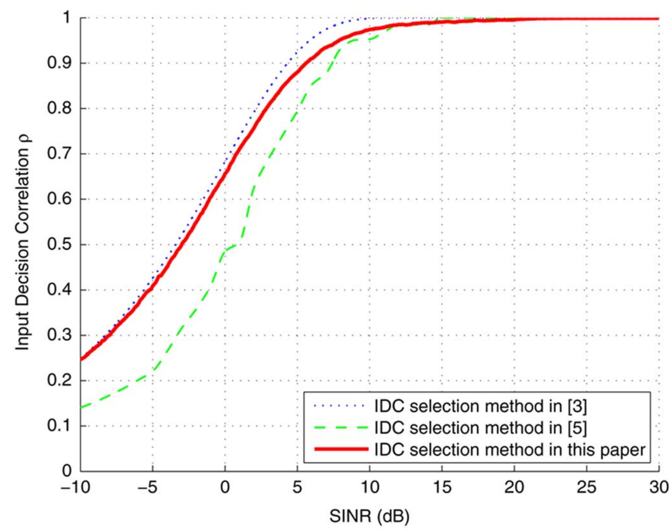


Fig. 3. IDC curves using different IDC determination methods for QPSK modulation with $K = 32$.

The following assumptions are made to derive the asymptotic performances of the receivers.

AS1') The channel responses $\mathbf{h}_k$ is represented as $\mathbf{h}_k = (1)/(\sqrt{N})[h_{1,k}, h_{2,k}, \ldots, h_{N,k}]^T$, where $h_{i,k}$'s are i.i.d., circularly symmetric, with zero mean, unit variance and finite fourth order moments, i.e., $\mathbb{E}\{|h_{1,1}|^2\} = 1$ and $\mathbb{E}\{|h_{1,1}|^4\} < \infty$.

AS2') The transmitted symbols $s_i$'s are i.i.d., zero mean and with unit amplitude $|s_i|^2 = 1$.

The analysis is based on random matrix theory [9], [10], which has been applied to wireless communications for limiting SINR analysis [11], multiple access interference analysis [12]. More details on the applications of random matrix theory in wireless communications can be found in [13]. In [14], the limiting SINR for mismatched MMSE receiver has been analyzed. In this paper, we apply that result to derive the limiting SINR for the iterative receivers studied in this paper.

### A. U-MMSE-SIC Receiver

For U-MMSE-SIC receiver, there are two important parameters: $\delta_\ell$ and output SINRs for each iteration. In this subsection, we will show that both of them converge to deterministic values for the limiting case.

Without loss of generality, we consider the detection performance of user 1. Let us define $\mathbf{R}_1 = \mathbf{H}_1\mathbf{H}_1^H + \mu^2\mathbf{I}_N$, and $\tilde{\mathbf{R}}_1 = \mathbf{H}_1\mathbf{D}_1\mathbf{H}_1^H + \sigma_n^2\mathbf{I}_N$ where $\mathbf{D}_1 = \text{diag}(p_2, \ldots, p_K)$ is a positive diagonal matrix with $(1)/(K-1)\,\text{Tr}(\mathbf{D}_1) = 1$. Here, $\mu^2$ defines the noise variance to be used in a simplified MMSE receiver, and $p_k$ defines the received power of user $k$. Since the nonzero norm of the linear receiver will not affect the output SINR, the weighting vector of U-MMSE-SIC receiver in (14) for user $k$ can take the generic form

$$\mathbf{w}_1 = \mathbf{R}_1^{-1}\mathbf{h}_1 \tag{25}$$

as long as the parameter $\mu^2$ can be properly chosen. With this receiver, the SINR in (16) can be represented in a *generic* formula

$$\hat{\beta}^{(a)} = \frac{\left|\mathbf{h}_1^H \mathbf{R}_1^{-1} \mathbf{h}_1\right|^2}{\mathbf{h}_1^H \mathbf{R}_1^{-1} \tilde{\mathbf{R}}_1 \mathbf{R}_1^{-1} \mathbf{h}_1}. \tag{26}$$

*Proposition 1 [11]:* Under assumption AS1'), when $(K)/(N) \to c > 0$ as $N \to \infty$, the following SINR:

$$\hat{\beta}^{(b)} = \mathbf{h}_1^H \mathbf{R}_1^{-1} \mathbf{h}_1 \tag{27}$$

converges in probability to a deterministic value, which is given by

$$\tilde{\beta}^{(b)}(\mu^2) = \int \frac{dF_c(x)}{x + \mu^2} \tag{28}$$

where $F_c(x)$ is the limit of the empirical distribution function for the eigenvalues of matrix $\mathbf{H}\mathbf{H}^H$.

The limiting theorem for the *generic* output SINR formula of (26) is stated as follows.

*Theorem 1:* Under Assumption AS1'), assume that the empirical distribution function of $\mathbf{D} = \mathrm{diag}\,(p_1, \ldots, p_K)$ converges to a probability distribution function $F_p(x)$ with $\int x dF_p(x) = 1$, and that $p_i$ is bounded by a constant. When $(K)/(N) \to c > 0$ as $N \to \infty, \hat{\beta}^{(a)}$ in (26) converges in probability to a constant $\tilde{\beta}^{(a)}(\mu^2, \sigma_n^2)$

$$\tilde{\beta}^{(a)}\left(\mu^2, \sigma_n^2\right) = \tilde{\beta}^{(b)}(\mu^2)\mathcal{K}\left(\mu^2, \sigma_n^2\right) \tag{29}$$

where

$$\mathcal{K}\left(\mu^2, \sigma_n^2\right) = \frac{c + \mu^2\left(1 + \tilde{\beta}^{(b)}(\mu^2)\right)^2}{c + \sigma_n^2(1 + \tilde{\beta}^{(b)}(\mu^2))^2}.$$

*Proof:* See [14].

According to [11], $\tilde{\beta}^{(b)}(\mu^2)$ is the positive solution of the following equation:

$$c - 1 - \frac{c}{1 + \tilde{\beta}^{(b)}(\mu^2)} + \mu^2 \tilde{\beta}^{(b)}(\mu^2) = 0 \tag{30}$$

which means

$$\tilde{\beta}^{(b)}(\mu^2) = \frac{(1-c)}{2\mu^2} - \frac{1}{2} + \sqrt{\frac{(1-c)^2}{4\mu^4} + \frac{(1+c)}{2\mu^2} + \frac{1}{4}}. \tag{31}$$

Calculating the derivation of $\tilde{\beta}^{(a)}(\mu^2, \sigma_n^2)$ over the variable $\mu^2$, and setting the result to be zero, we obtain that the maximum limiting SINR is achieved when we choose $\mu^2 = \sigma_n^2$.

For the limiting case, using Assumption AS2'), $\delta_{k,\ell} \to \bar{\delta}_\ell = \mathbb{E}\{|\tilde{s}_{j,\ell}|^2\}$. The limiting SINR for the U-MMSE-SIC receiver is stated as follows.

*Theorem 2:* Consider the MIMO channel model (1). Under the assumptions AS1'), AS2'), and AS3), the output SINR $\beta_{1,\ell+1}$ in (16) of user 1 using the U-MMSE-SIC receiver at the $(\ell+1)$th iteration converges to $\tilde{\beta}_{\ell+1}^{\mathrm{U-MMSE-SIC}}(\gamma)$ in probability as $K \to \infty, N \to \infty$, and $K/N \to c$, where

$$\tilde{\beta}_{\ell+1}^{\mathrm{U-MMSE-SIC}}(\gamma) = \frac{1}{\left(1 - \bar{\delta}_\ell^2\right)} \tilde{\beta}^{(b)}(\bar{\sigma}^2) \tag{32}$$

for $\bar{\delta}_\ell < 1, \bar{\sigma}^2 = 1/(\gamma(1 - \bar{\delta}_\ell^2))$, and $\tilde{\beta}_{\ell+1}^{\mathrm{U-MMSE-SIC}}(\gamma) = \gamma$ for $\bar{\delta}_\ell = 1$.

*Proof:* From (16), if $\bar{\delta}_\ell < 1$, we have

$$\lim_{K,N\to\infty} \beta_{1,\ell+1} = \frac{1}{1 - \bar{\delta}_\ell^2} \lim_{K,N\to\infty} \frac{\left(\mathbf{h}_1^H \mathbf{C}_1^{-1} \mathbf{h}_1\right)^2}{\mathbf{h}_1^H \mathbf{C}_1^{-1} \tilde{\mathbf{C}}_1 \mathbf{C}_1^{-1} \mathbf{h}_1} \tag{33}$$

where $\mathbf{C}_1 = \mathbf{H}_1 \mathbf{H}_1^H + \bar{\sigma}^2 \mathbf{I}_N, \tilde{\mathbf{C}}_1 = \mathbf{H}_1 \tilde{\mathbf{D}}_1 \mathbf{H}_1^H + \bar{\sigma}^2 \mathbf{I}_N$ with $(1)/(K-1)\ \mathrm{Tr}\ (\tilde{\mathbf{D}}_1) = 1$. Equation (32) is derived by applying Theorem 1 directly. On the other hand, if $\bar{\delta}_\ell = 1$, then the soft estimates are all correct estimates; thus, the interference has been completely cancelled out, thus SUMF achieves the matched filter bound (MFB), the SINR of which is equal to the input SNR $\gamma$ as $N \to \infty$.

### B. BI-GDFE Receiver

Similarly, we have the following theorem for the BI-GDFE receiver [5].

*Theorem 3:* Consider the MIMO channel model (1). Under the Assumptions AS1'), AS2'), and AS3), the output SINR $\theta_{1,\ell+1}$ of user 1 using the BI-GDFE receiver at the $(\ell+1)$th iteration converges to $\tilde{\beta}_{\ell+1}^{\mathrm{BI-GDFE}}(\gamma)$ in probability as $K \to \infty, N \to \infty$, and $K/N \to c$, where

$$\tilde{\beta}_{\ell+1}^{\mathrm{BI-GDFE}}(\gamma) = \frac{1}{\left(1 - \rho_\ell^2\right)} \tilde{\beta}^{(b)}(\bar{\sigma}^2) \tag{34}$$

for $\rho_\ell < 1, \bar{\sigma}^2 = 1/(\gamma(1 - \rho_\ell^2))$, and $\tilde{\beta}_{\ell+1}^{\mathrm{BI-GDFE}}(\gamma) = \gamma$ for $\rho_\ell = 1$.

## VI. Numerical Results and Comparisons

In this section, numerical results are presented to compare the asymptotic convergence of U-MMSE-SIC and BI-GDFE. We also compare the BER performance of BI-GDFE using the IDC generation algorithm proposed in this paper and those in [3] and [5].

### A. Asymptotic Performance

When the signal dimension $K$ goes to infinity, for the BI-GDFE receiver, the IDC parameters can be computed exactly, using (24) for QPSK modulation. However, there is no closed-form solution for $\bar{\delta}_\ell$. Fortunately, since the outputs of the U-MMSE-SIC at each iteration can be considered as the outputs of an AWGN channel $(z = s + n)$ with the SNR equal to the limiting SINR at $\ell$th iteration, using the soft estimate $\tilde{s}$ of $s$, the parameter $\bar{\delta}_\ell^2$ is determined as follows:

$$\bar{\delta}_\ell^2 = \mathbb{E}\{|\tilde{s}|^2\} \tag{35}$$

thus we can rely on computer simulations to derive the parameter $\bar{\delta}_\ell$, which is a function of SINR.

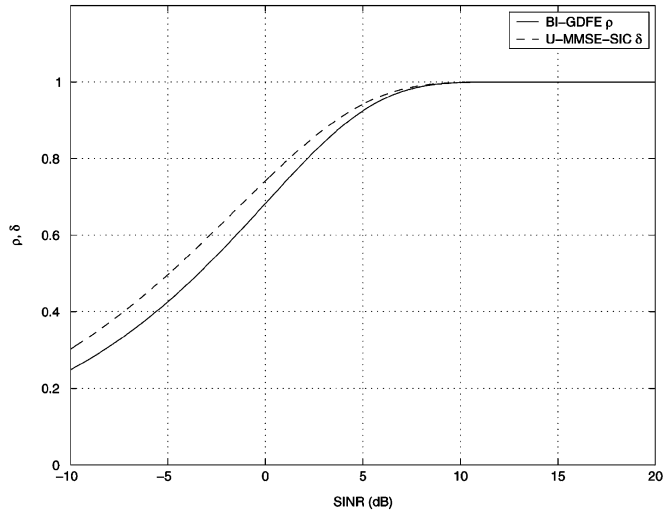


Fig. 4. Asymptotic IDC parameter $\rho$ for BI-GDFE and asymptotic parameter $\bar{\delta}$ for U-MMSE-SIC for QPSK modulation ($\delta$ should be read as $\bar{\delta}$ in the figure).

Once we have the asymptotic SINRs for each iteration, we are ready to study the asymptotic convergence of the U-MMSE-SIC and BI-GDFE receivers. The following steps will be involved.

1) Set $\ell = -1$ and $\rho_{-1} = 0$, and compute $\tilde{\beta}_0(\gamma) = \tilde{\beta}^{(b)}(\mu^2)$ using (31) with $\mu^2 = 1/\gamma$. Obtain $\bar{\delta}_0$ and $\rho_0$ based on lookup table.
2) Set $\ell + 1 \rightarrow \ell$.
3) Compute limiting SINR $\tilde{\beta}_\ell^{\mathrm{U-MMSE-SIC}}(\gamma)$ using (32) or $\tilde{\beta}_\ell^{\mathrm{BI-GDFE}}(\gamma)$ using (34).
4) Compute $\bar{\delta}_\ell$ using (35) or $\rho_\ell$ using (24) for QPSK modulation.
5) Go to (2), (3) and (4) until convergence.

Fig. 4 compares the IDC parameter $\rho$ for BI-GDFE and parameter $\bar{\delta}$ for U-MMSE-SIC with respect to SINR for QPSK modulation. It is seen that for a given SINR, $\bar{\delta} \geq \rho$, implying that the asymptotic output SINR for the next iteration for U-MMSE-SIC will be higher than that of the BI-GDFE receiver, if the output SINR for the current iteration is the same.

The asymptotic BER convergence behaviors of the two receivers are compared in Figs. 5 and 6 for $c = 1$ and $c = 2$, respectively. The BER results are derived using $Q$-function based on the asymptotic SINR at each iteration. Here we also plot the BER performance of an AWGN channel with SNR defined in Section II (this corresponds to the case when all interferences have been completely cancelled out). The BER curve for this AWGN channel serves as the lowest BER bound. The BER curve for MMSE receiver is also included for comparison. Note that the U-MMSE-SIC and BI-GDFE receivers start with the MMSE receiver and gradually achieve the AWGN bound with the increase of iterations. It is seen that for systems with reasonable load $c = 1$, the two receivers require almost the same number of iterations to converge to the AWGN bound for BER lower than $10^{-3}$. Further, the converged curves are almost the same for the two receivers. For heavily loaded systems $(c = 2)$, however, the U-MMSE-SIC receiver provides better BER performance than the BI-GDFE receiver for a given number of iterations. Also, the converged BER curve for U-MMSE-SIC seems

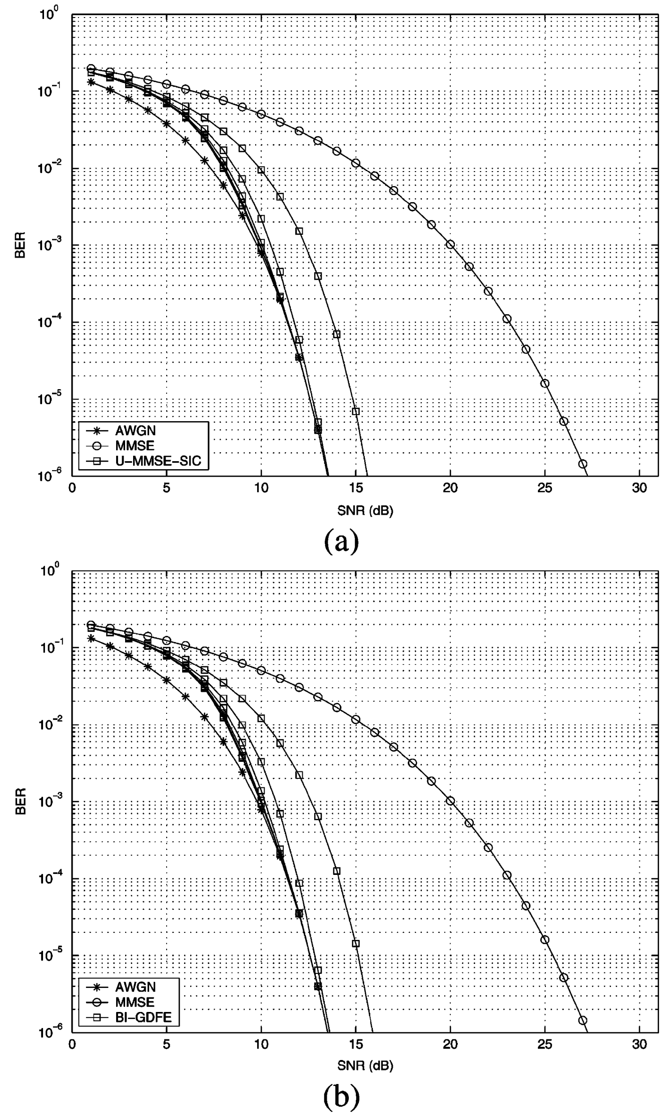


Fig. 5. Asymptotic convergence of BER for (a) U-MMSE-SIC receiver; (b) BI-GDFE receiver with $c = 1$ and QPSK modulation. Note the U-MMSE-SIC and BI-GDFE receivers start from MMSE and gradually shift to the AWGN bound with the increase of iterations of Step 1. While the two receivers have similar convergence speed, as seen in the second iteration, U-MMSE-SIC is slightly faster than BI-GDFE.

to be better than that for BI-GDFE. This is the return of using soft interference cancellation for the U-MMSE-SIC receiver.

Finally, Fig. 7 shows the convergence of multiuser efficiency defined as the output SINR normalized by the input SNR for U-MMSE-SIC and BI-GDFE receivers. Here $c = 2$ and QPSK modulation are used. The efficiency 1 is achieved when the input SNR is sufficiently high and enough iterations have been carried out.

### *B. BER Performance*

In the computer simulations, we assume that the receivers have perfect CSI, and a block fading channel model is used where the channel responses remain constant within each block of transmission.

We simulate the $32 \times 32$ random MIMO channels with QPSK modulation. The receivers compared include MMSE,

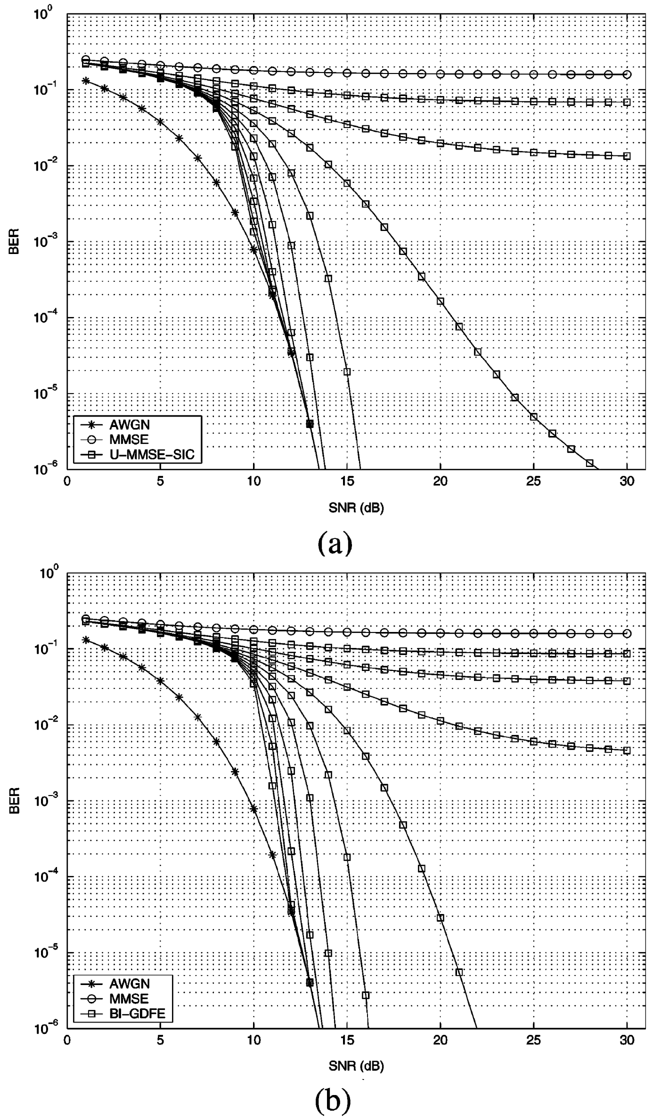


Fig. 6. Asymptotic convergence of BER for (a) U-MMSE-SIC receiver and (b) BI-GDFE receiver with $c = 2$ and QPSK modulation. Note the U-MMSE-SIC and BI-GDFE receivers start from MMSE and gradually shift to the AWGN bound with the increase of iterations of step 1. With SNR of 16 dB, at the fifth iteration, U-MMSE-SIC achieves BER of $10^{-6}$; however, BI-GDFE only achieves BER of $4 \times 10^{-3}$.

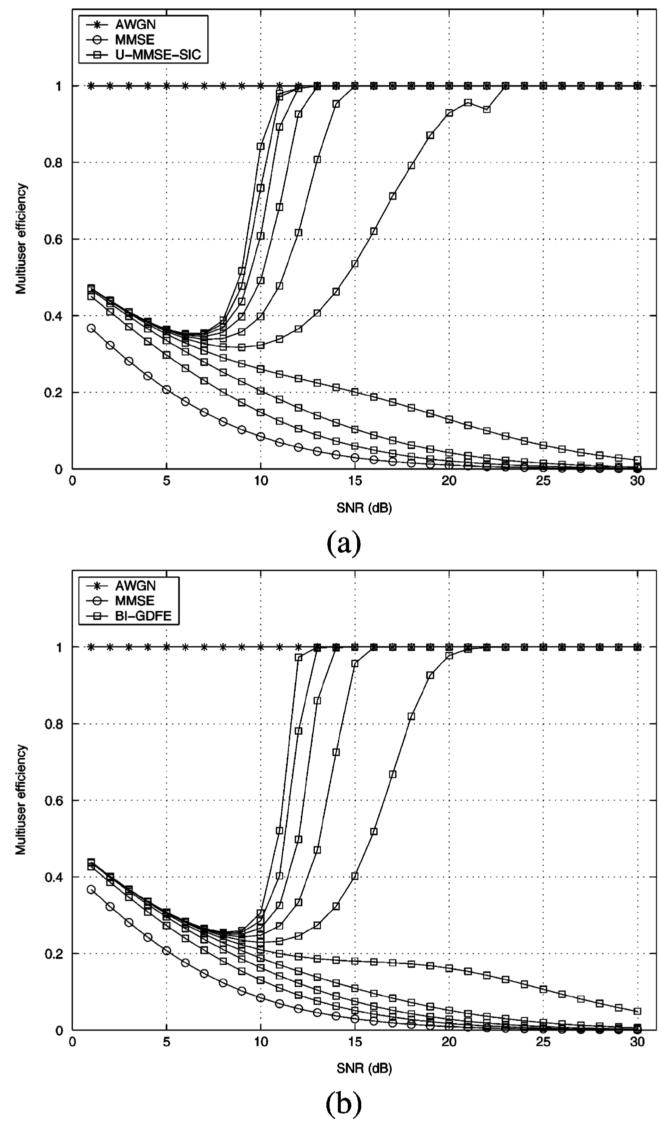


Fig. 7. Asymptotic convergence of multiuser efficiency for (a) U-MMSE-SIC receiver and (b) BI-GDFE receiver with $c = 2$ and QPSK modulation. Note the U-MMSE-SIC and BI-GDFE receivers start from MMSE and gradually shift to the AWGN bound with the increase of iterations of Step 1. U-MMSE-SIC requires five iterations to reach an efficiency of 1 at SNR of 23 dB; however, BI-GDFE requires six iterations to achieve an efficiency of 1 at the same SNR.

C-MMSE-SIC, U-MMSE-SIC and BI-GDFE with various IDC generation methods. Single user MFB is also used for comparison. It can be seen from Fig. 8 that U-MMSE-SIC always provides better performance than any BI-GDFE, with the cost of higher computational complexity. On the other hand, the BI-GDFE receiver using our proposed IDC selection algorithm achieves the best BER performance among the BI-GDFE receivers, and the achievable performance is closest to that of the U-MMSE-SIC receiver after five iterations. This is due to the fact that, for low SINR region, as seen from Fig. 3, the IDC value selected by the method proposed in this paper is larger than that chosen by method in [5]; thus, the BI-GDFE receiver with this paper's IDC method achieves faster convergence. Furthermore, for the high SINR region, the two IDC methods generate almost the same IDC value; thus, they both achieve guaranteed convergence.

We apply the same technique to generate the IDC values for 16 QAM. Fig. 9 shows the IDC values for 16 QAM with different signal dimension $K$. We then incorporate these IDC values into BI-GDFE and compare the BER performance of BI-GDFE with that of the MMSE-SIC receivers. The BER performances of both receivers based on the $64 \times 64$ and $128 \times 128$ single carrier cyclic prefix (SCCP) systems with 16 QAM are shown in Fig. 10(a) and (b), respectively. Note SCCP is one of the air interfaces for IEEE 802.16 wireless broadband access systems [5]. We discover that for $64 \times 64$ systems with a high SNR value (above around 22 dB), the BER curves of MMSE-SIC receivers tend to flatten. This is due to the fact that the denominator in (6) of the MMSE-SIC receiver approaches too close to zero at high SNR region. This small value is unreliable to provide a good measure of the correctness of the modulation candidates [7]. With larger signal dimension $(K = 128)$,

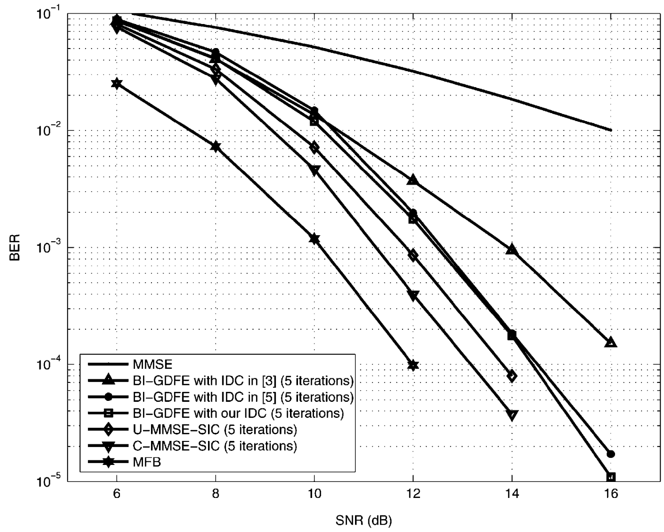


Fig. 8. Performance of $32 \times 32$ random MIMO system using different receivers for QPSK.

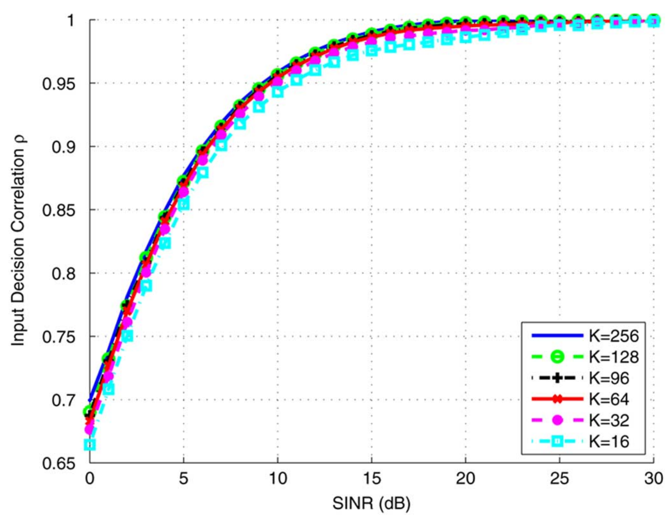


Fig. 9. IDC curves of different signal dimensions for 16 QAM.

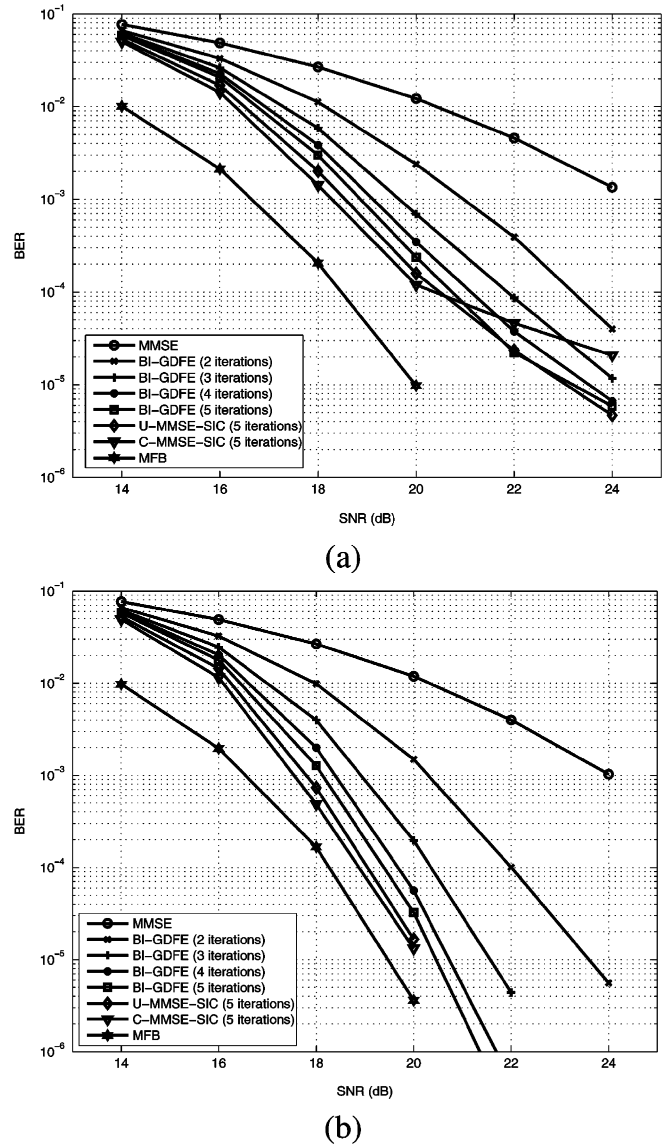


Fig. 10. Performance of SCCP systems with 16 QAM for (a) $N = K = 64$ and (b) $N = K = 128$.

we can see from Fig. 10(b) that there is no more flattening of the BER curves, and that the BI-GDFE receiver achieves close to the performance of MMSE-SIC receivers with similar behavior and outperforms the conventional MMSE.

In conclusion, our results have shown that for a system load of 1, the U-MMSE-SIC and BI-GDFE receivers have similar convergence behavior and both receivers require the similar number of iterations to reach the single user MFB. Considering the fact that BI-GDFE receiver has much lower complexity, and the weights for BI-GDFE can be precalculated, BI-GDFE is a promising candidate to achieve near-ML performance for MIMO channels.

## VII. Conclusion

In this paper, we have analyzed the MMSE-SIC and BI-GDFE receivers, and have identified a relationship between the two receivers. It is shown that the block model of BI-GDFE can be represented in a form equivalent to that of the MMSE-SIC receiver. This surprisingly simple remodeling of BI-GDFE provides an important insight in the derivation of a method to determine the IDC from the statistically normalized average power of the soft interference symbols acquired from MMSE-SIC. This IDC determination method can also be applied to higher order QAMs. We have also analyzed the limiting performances of the two receivers for random MIMO channels when $K \to \infty, N \to \infty$ and $K/N \to$ constant $c$. Numerical results have shown that the asymptotic convergence behaviors of both receivers for system load of 1 are similar and that both require the similar number of iterations to converge to AWGN bound. The computer simulations have also validated the effectiveness of BI-GDFE applied to MIMO systems with higher order QAMs using the proposed IDC determination method.

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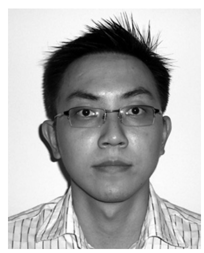

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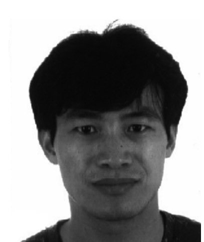

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