Multiple Output Selection-LAS Algorithm in Large MIMO Systems

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***Abstract*—We present a low-complexity algorithm for detection in large MIMO systems based on the likelihood ascent search (LAS) algorithm. The key idea in our work is to generate multiple possible solutions or outputs from which we select the best one. We propose two possible approaches to achieve this goal and both are investigated. Computer simulations demonstrate that the proposed algorithm, Multiple Output Selection-LAS, which has the same complexity order as that of conventional LAS algorithms, is superior in bit error rate (BER) performance to LAS conventional algorithms. For example, with 20 antennas at both the transmitter and receiver, the proposed MOS-LAS algorithm needs about 4 dB less SNR to achieve a target BER of 10^{-4} for 4-QAM.**

***Index Terms*—Large MIMO systems, maximum likelihood detection, likelihood ascent search (LAS), low-complexity detection.**

I. Introduction

It is well known that the capacity of multiple-input multiple-output (MIMO) channels grows linearly with the minimum of the number of antennas at the transmitter and the receiver sides [1]. Therefore one way to achieve very high spectral efficiency is to exploit large numbers of antennas at both the transmitter and receiver. The main bottleneck of such systems is the complexity of the receivers. A family of low-complexity detectors termed Likelihood Ascent Search (LAS) detectors have been proposed in [2] for large MIMO systems. The power of the LAS detector lies in the linear average per-bit complexity and the excellent BER performance in large MIMO systems (It has been proven [3] that the asymptotic BER performance of LAS detectors converges to that of maximum likelihood (ML) detector for example). The main disadvantage of LAS detectors, which is the motivation of our work, is that they need very large numbers of antennas to achieve the optimal BER performance, especially in high order modulation [3]. We consider a MIMO system with N_t transmit antennas and N_r receive antennas ($N_r \geq N_t$). The baseband system model is given by

$$\mathrm{y} = \mathrm{H}\mathrm{x} + v \tag{1}$$

where $\mathrm{y} = [\mathrm{y}_1, \mathrm{y}_2, \cdots \mathrm{y}_{N_r}]^{\mathrm{T}}$ is an $N_r \times 1$ receive signal vector, $\mathrm{x} = [\mathrm{x}_1, \mathrm{x}_2, \cdots \mathrm{x}_{N_r}]^{\mathrm{T}}$ is an $N_t \times 1$ transmit signal vector, v is the complex white Gaussian noise. The elements of the $N_r \times N_t$ channel matrix H are assumed i.i.d complex Gaussian random variables with zero mean and variance of unity. We assume ideal channel estimation and synchronization at the receiver end.

II. Conventional LAS Algorithm

A conventional LAS detector [2] starts from an initial solution vector x which can be the output from any known detector such as zero-forcing for example. It then searches through a sequence of solution vectors to refine the solution with monotonic likelihood ascent. At step n, the update algorithm for BPSK modulation using the LAS algorithm can be explained as follows. Given the initial vector $\mathrm{x}(0) \in \{+1, -1\}^{N_t}$ and the search candidate sets (SCS) $\mathrm{L}(n) \subseteq \{1, 2, \cdots N_t\}$, $\forall\ n \geq 0$, the jth bit of $\mathrm{x}(n+1)$ is given by

$$\mathrm{x}_j(n+1) = \begin{cases} +1 & \text{if } j \in L(n),\ \mathrm{x}_j(n) = -1, \\ & \text{and } \mathrm{M}(\mathrm{x}(n+1)) > \mathrm{M}(\mathrm{x}(n)) \\ -1 & \text{if } j \in L(n),\ \mathrm{s}_j(n) = +1, \\ & \text{and } \mathrm{M}(\mathrm{x}(n+1)) > \mathrm{M}(\mathrm{x}(n)) \\ \mathrm{x}_j(n) & \text{otherwise} \end{cases} \tag{2}$$

where $\mathrm{M}(\mathrm{x}) = ||\mathrm{y} - \mathrm{H}\mathrm{x}||^2$ is the likelihood metric for x. Note that the update rule (2) can be simplified and implemented more efficiently as in [2]. The LAS detector checks the candidate bits defined in the sequence of sets $\mathrm{L}(n)$ and updates $\mathrm{x}(n)$ according to (2). The LAS algorithm reaches a fixed point and terminates when there is no bit flipped in a certain period. Since the update algorithm (2) ensures monotonic likelihood ascent, it is guaranteed to converge to a local maximum likelihood (LML) point which will be the final output of the detector and will occur in a finite number of steps [4][5]. Obviously, the output LML point depends on the initial vector and the sequence of SCS. Specifying a sequence of $\mathrm{L}(n)$ for $n \geq 0$ and an initial vector x, one determines a particular LAS detector. $\mathrm{L}(n)$ should be designed such that all the bits are regularly checked in the sequence of SCS. One straightforward SCS is that each $\mathrm{L}(n)$ contains only one element, with element value modulo(n, N_t), so the detector checks all the N_t bits one by one in N_t steps. This sequence is then repeated until no bit is flipped in the last cycle. We should mention that checking (or updating) the bits in different orders may lead to different outputs. This approach to selecting the SCS has been shown to have good BER performance in the family of LAS algorithms [5]. It is also related to the "bit flipping" algorithms proposed in previous literature [6], [7], [8]. We only consider this straightforward SCS in this letter.

The update algorithm (2) can easily be adopted for M-PAM and M-QAM. For M-PAM similar likelihood ascent search can be performed over M-ary symbol vectors. Since M-QAM can be considered as quadrature PAM, we can use the same search algorithm as for PAM and treat the I and Q channels independently. A detailed description of LAS algorithm for M-QAM is given in [3].

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III. Multiple Output Selection-LAS Algorithm

An intuitive idea to improve the BER performance of conventional LAS algorithms is to generate different LML points, from which we select the best one as the final output. By doing so, we expect the probability that the output is a global maximum likelihood (GML) point is increased. We suggest the following two Multiple Output Selection-LAS (MOS-LAS) approaches to achieve this goal.

A. Multiple Initial Vectors (MIV)-LAS Algorithm

The procedure of this proposed algorithm is explained as follows:

1) Generate K initial vectors $\mathrm{p}_1,\ \mathrm{p}_2\ \cdots\ \mathrm{p}_K$.
2) Obtain K LML points $\mathrm{s}_1,\ \mathrm{s}_2\ \cdots\ \mathrm{s}_K$ by using LAS algorithm with K different initial vectors generated in 1).
3) Select the LML point with the minimum metric, i.e. $\hat{\mathrm{s}} = \arg \min_{i=1,2\cdots K} ||\mathrm{y}\text{-}\mathrm{H}\mathrm{s}_i||^2$.

We should note that the initial vectors are not necessarily generated from known detectors but could be random vectors. Random vectors have the advantage that they do not need channel inversion and that we can save computation. In fact we show later that by using multiple random initial vectors we can achieve better BER performance and less complexity at the same time compared to conventional LAS algorithms.

B. Multiple Search Candidate Sets (MSCS)-LAS Algorithm

Another approach to obtaining multiple solutions from LAS is to use multiple SCS (MSCS) with only one initial vector. In this approach we suggest that the initial vector be obtained using MMSE, which is generally the best approach to use when only one initial vector is available in LAS.

The procedure of MSCS-LAS algorithm can then be summarized as follows:

1) Obtain an initial vector p by using MMSE detector.
2) Obtain K LML points $\mathrm{s}_1,\ \mathrm{s}_2\ \cdots\ \mathrm{s}_K$ by using MMSE-LAS algorithm with K different sequences of SCS as $\mathrm{L}_1(n),\ \mathrm{L}_2(n),\ \cdots,\ \mathrm{L}_K(n)$. Here we update one bit each step for all the K different sequences of SCS, but with different orders.
3) Select the LML point with the minimum metric, i.e. $\hat{\mathrm{s}} = \arg \min_{i=1,2\cdots K} ||\mathrm{y}\text{-}\mathrm{H}\mathrm{s}_i||^2$.

Note that we do not guarantee the K LML points obtained in step 2) are all different.

The complexity of the LAS algorithm (excluding the initial vector) is mainly affected by the average number of steps required to reach to a fixed point. For MIV-LAS (except with multiple random vectors) algorithm the complexity is about K times the conventional LAS algorithms and is acceptable with small K. Because the average number of steps is very small for the LAS algorithm, using MMSE as the initial value, the complexity of the MSCS-LAS algorithm is comparable to the conventional LAS algorithms with small K (approx K<10 from simulation results).

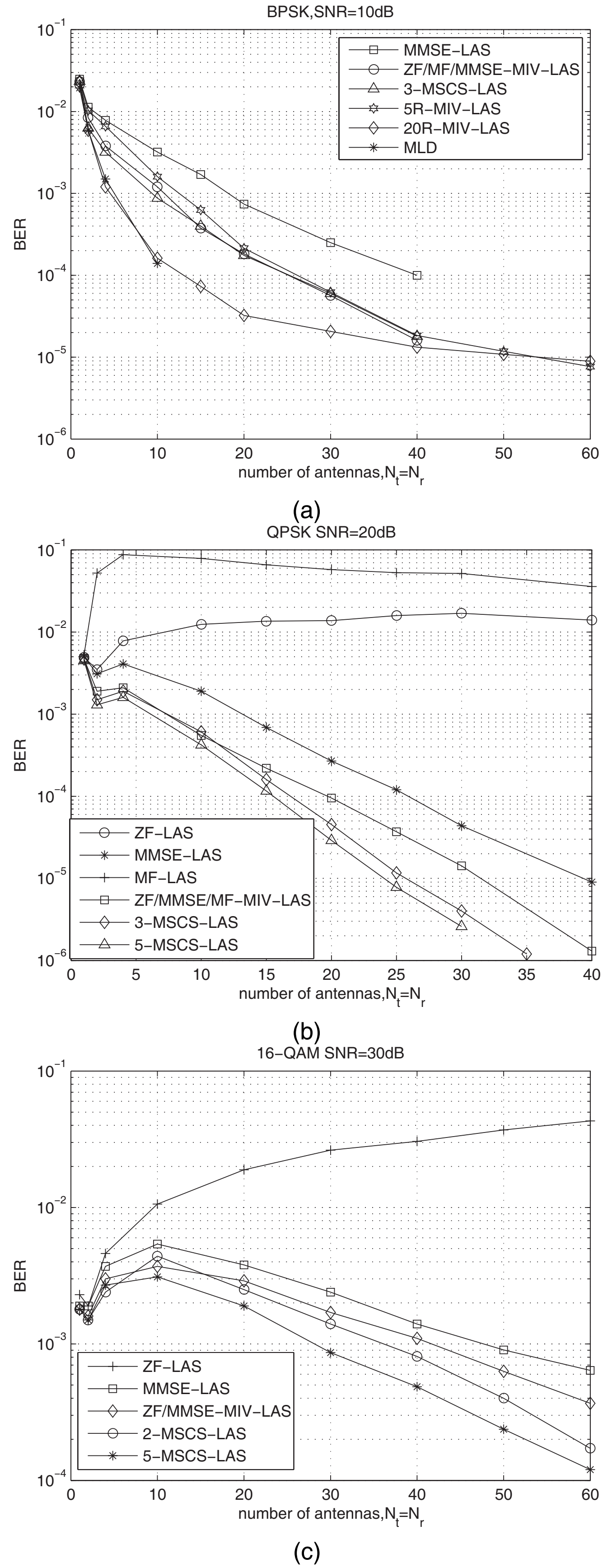

Fig. 1. The BER performance as a function of number of antennas ($N_r = N_t = N$) (a) BPSK modulation and SNR=10 dB. (b) 4-QAM modulation and SNR=20 dB. (c) 16-QAM modulation and SNR=30 dB.

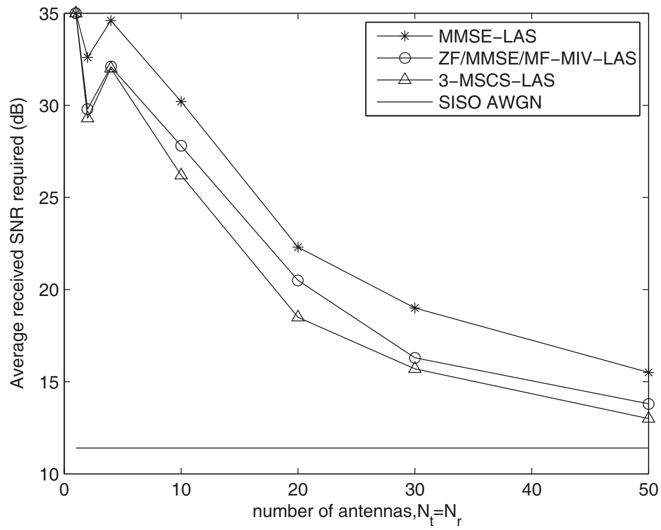

Fig. 2. Average received SNR required to achieve a target BER of 10^{-4} as a function of number of transmit/receive antennas ($N_t = N_r = N$) for 4-QAM.

IV. Numerical Results

In this section, we present the BER performance of the proposed MOS-LAS algorithms, and compare them with the conventional LAS algorithms. We investigate our two proposed approaches with different numbers of initial vectors and SCS.

Here we use ZF-LAS, MF-LAS, and MMSE-LAS to denote the conventional LAS algorithms with initial vectors generated by Zero Forcing, Matched Filter and Minimum Mean Square Error detectors respectively. We use KR-MIV-LAS to denote the MIV-LAS algorithms with K random vectors. We use ZF/MF/MMSE-MIV-LAS to denote the MIV-LAS algorithms with 3 initial vectors found using ZF, MF, and MMSE. We define ZF/MMSE-MIV-LAS detectors likewise for 2 initial vectors. For MSCS-LAS algorithms we use K-MSCS-LAS to denote the MSCS-LAS algorithm with K SCS.

Fig. 1(a)-(c) shows the BER performance as a function of the number of antennas for different modulations using MIV-LAS, MSCS-LAS, conventional LAS and/or ML algorithms. From Fig. 1(a)-(c) the following observations can be made:

1) The BER performance of the proposed MOS-LAS algorithms lie between that of the conventional MMSE-LAS algorithm and that of ML algorithm, and this shows clearly the advantage of our proposed MOS-LAS algorithm.
2) As shown in Fig. 1(a),the MIV-LAS algorithm performs similarly to MSCS-LAS algorithm in BPSK modulation. The low complexity of KR-MIV-LAS algorithm motivates us to use more random initial vectors to approach optimal performance. By using 20 random vectors, we can achieve ML performance for even less than 10 antennas.
3) Fig. 1(b) and (c) show BER performance for 4-QAM and 16-QAM, respectively. The ZF-LAS and MF-LAS algorithms perform poorly in high order modulations as does the KR-MIV-LAS algorithm (we did not plot the curve in the figures for clarity). But the performance gain achieved by using MSCS-LAS algorithm is still substantial. Even with only 2 or 3 SCS, MSCS-LAS algorithm outperforms MMSE-LAS and MIV-LAS algorithms with almost the same or even less complexity.

Fig. 2 shows average received SNR required to achieve a target BER of 10^{-4} for 4-QAM. We can see ZF/MMSE/MF-MIV-LAS algorithm has about 2dB SNR gain over the MMSE-LAS algorithm while 3-MSCS-LAS algorithm achieves as much as 4dB SNR gain.

V. Conclusions

A high-performance, low-complexity detector with two approaches is proposed in this letter. The first approach, MIV-LAS algorithm, with very low complexity with pure random initial vectors, is suggested to be used in BPSK modulation. The second approach, MSCS-LAS algorithm, performs well with low complexity and good BER performance and can be implemented as an efficient detector for the higher order modulations we considered. Simulation results verify that our proposed algorithm leads to an improved tradeoff between performance and complexity compared to the conventional LAS algorithms. A key advantage of our algorithms over conventional LAS is that they perform well even for low numbers of antennas.

References

[1] G. Foschini and M. Gans, "On limits of wireless communications in a fading environment when using multiple antennas," *Wireless Personal Commun.*, vol. 6, no. 3, pp. 311–335, 1998.
[2] K. Vardhan, S. Mohammed, A. Chockalingam, and B. Rajan, "A low-complexity detector for large mimo systems and multicarrier CDMA systems," *IEEE J. Sel. Areas Commun.*, vol. 26, no. 3, p. 473, 2008.
[3] S. Mohammed, A. Chockalingam, and B. Rajan, "Asymptotic analysis of the performance of LAS algorithm for large MIMO detection," online arXiv, vol. 806.
[4] Y. Sun, "A family of linear complexity likelihood ascent search detectors for CDMA multiuser detection," in *Proc. IEEE 6th Intl. Symp. on Spread Spectrum Tech. & App*, 2000.
[5] ——, "A family of likelihood ascent search multiuser detectors: an upper bound of bit error rate and a lower bound of asymptotic multiuser efficiency," *IEEE Trans. Commun.*, vol. 57, no. 6, pp. 1743–1752, June 2009.
[6] H. Fan, J. Murch, and R. Mow, "Near maximum likelihood detection schemes for wireless MIMO systems," *IEEE Trans. Wireless Commun.*, vol. 3, no. 5, pp. 1427–1430, 2004.
[7] H. Lim and B. Venkatesh, "An efficient local search heuristics for asynchronous multiuser detection," *IEEE Commun. Lett.*, vol. 7, no. 7, pp. 299–301, 2003.
[8] J. Hu and R. Blum, "A gradient guided search algorithm for multiuser detection," *IEEE Commun. Lett.*, vol. 4, no. 11, pp. 340–342, 2000.