Channel Estimation for OFDM Transmission in Multipath Fading Channels Based on Parametric Channel Modeling

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***Abstract*—In this paper, we present an improved channel estimation algorithm for orthogonal frequency-division multiplexing mobile communication systems using pilot subcarriers. This algorithm is based on a parametric channel model where the channel frequency response is estimated using an L-path channel model. In the algorithm, we employ the ESPRIT (estimation of signal parameters by rotational invariance techniques) method to do the initial multipath time delays acquisition and propose an interpath interference cancellation delay locked loop to track the channel multipath time delays. With the multipath time delays information, a minimum mean square error estimator is derived to estimate the channel frequency response. It is demonstrated that the use of the parametric channel model can effectively reduce the signal subspace dimension of the channel correlation matrix for the sparse multipath fading channels and, consequently, improve the channel estimation performance.**

***Index Terms*—Channel estimation, fading radio channels, OFDM, wireless communication systems.**

I. Introduction

In RECENT years, there has been a lot of interest in applying orthogonal frequency-division multiplexing (OFDM) in wireless and mobile communication systems because of its various advantages in lessening the severe effects of frequency-selective fading [1]. However, the high-rate and spectrum efficient OFDM systems employing multilevel modulation schemes with nonconstant amplitude (e.g., 16QAM) generally require estimation and tracking of the fading channel parameters to perform coherent demodulation.

In OFDM systems, the channel estimation can be achieved by exploiting the correlation of the channel frequency response at different frequencies and times. In [2], [3], and [5], the channel estimators for OFDM systems have been proposed based on frequency-domain filtering and time-domain filtering. A singular-value decomposition-based channel estimator has also been presented in [4]. These methods do not make any assumptions about the channel model, and hence the dimension of the estimation problem can be quite large. However, the radio channel in a wireless communication system is often characterized by the multipath propagation. In large cells with high base station antenna platforms, the multipath propagation is aptly modeled by a few dominant specular paths, typically two to six [6]. Moreover, the high-speed data transmission in wireless communications potentially results in a sparse multipath fading channel. The sparsity of a multipath channel can be defined as the ratio of the time duration (in OFDM samples) spanned by the multipaths to the number of the multipaths [7]. A parametric channel model can then be used to represent this type of channel. When the channel correlation matrix is constructed based on the parametric channel model, the signal subspace dimension of the correlation matrix can be effectively reduced. Accordingly, the channel estimator performance can be improved. The parametric channel model approach has been applied to the Global System for Mobile Communications (GSM) system [6] and the high-speed digital video broadcast system [7] to improve the channel equalizer and estimator performance. It should be also noted that in mobile communications the multipath time delays are slowly time varying. In contrast, the amplitude and relative phase of each path are relatively fast time varying and subject to (Rayleigh) fading [8]. We can thus take this into account in designing the channel estimator.

In this paper, we propose an improved channel estimation method for OFDM transmission over the sparse multipath fading channels using pilot subcarriers. The channel estimator is based on a parametric channel model. That is, the channel frequency response of the multipath fading channel is modeled as the Fourier transform of a multipath finite impulse response (FIR). The channel estimator is derived to estimate the parameters which include the time delays, gains, and phases of the paths. Specifically, we first use the minimum description length (MDL) criterion to detect the number of paths in the channel. Then, we use the estimation of signal parameters by rotational invariance techniques (ESPRIT) to estimate the initial multipath time delays. Because of the slow time-varying nature of time delays, we propose an *interpath interference cancellation* (IPIC) delay locked loop (DLL) to track the channel multipath time delays. With the multipath time delays

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information, a minimum mean-square-error (MMSE) estimator is derived to estimate the channel frequency response.

This paper is organized as follows. Section II introduces the OFDM baseband model. Then, Section III derives the proposed channel estimation algorithm and analyzes its performance. Next, Section IV gives some simulation results that demonstrate the effectiveness of the proposed channel estimation method. We conclude the paper in Section V.

II. OFDM Baseband Model

Consider an OFDM system that consists of N subcarriers among which $N_u + 1$ subcarriers at the central spectrum are used for transmission and the other subcarriers at both edges form the guard bands. Each transmission subcarrier is modulated by a data symbol $X_{i,n}$, where i represents the OFDM symbol number and n represents the subcarrier number. The OFDM transmitters usually employ an inverse fast Fourier transform (IFFT) of size N for modulation.[1] In order to limit the transmit signal to a bandwidth smaller than $1/T$ where T is the sampling time interval of the OFDM signal (thus, allowing a simple "T-spaced" OFDM receiver), the subcarriers in the guard band are not used. The guard band also enables us to choose an appropriate analog transmission filter $G_T(w)$ to limit the periodic spectrum of the discrete time signal at the output of the IFFT. A guard interval is also added for every OFDM symbol to avoid intersymbol interference caused by multipath fading channels. As a result, the output baseband signal of the transmitter can be represented as

$$s(t) = \sum_{i=-\infty}^{\infty} \sum_{n=-N_u/2}^{N_u/2} X_{i,n} \Psi_{i,n}(t) \otimes g_T(t) \tag{1}$$

where $\otimes$ denotes the convolution, $g_T(t)$ is the impulse response of the transmission analog filter, and $\Psi_{i,n}(t)$ is the subcarrier pulse that can be described by

$$\Psi_{i,n}(t) = \begin{cases} e^{j2\pi \frac{n}{T_u}(t-\Delta-iT_s)}, & iT_s \le t < (i+1)T_s \\ 0, & \text{otherwise} \end{cases} \tag{2}$$

where $T_s = T_u + \Delta$ is the duration of a whole OFDM symbol including the guard interval, $1/T_u$ is the OFDM subcarrier spacing, and Δ is the guard interval length. We also have $T = T_u/N$.

It is assumed that the signal is transmitted over a multipath Rayleigh fading channel characterized by

$$h(\tau, t) = \sum_{\ell=1}^{L} h_\ell(t) \cdot \delta(\tau - \tau_\ell) \tag{3}$$

where $\{h_\ell(t)\}$ are the different path complex gains, $\{\tau_\ell\}$ are the different path time delays, and L is the number of paths. $\{h_\ell(t)\}$ are wide-sense stationary (WSS) narrow-band complex Gaussian processes with the so-called Jakes' power spectrum [9] and the different path gains are uncorrelated with respect to each other where the average energy of the total channel energy is normalized to one.

[1] N is usually a power of 2.

At the receiver side, with the assumptions that the guard interval duration is longer than the channel maximum excess delay, that the channel is quasi-stationary (i.e., the channel does not change within one OFDM symbol duration but varies from symbol to symbol), and that the synchronization is perfect, the nth subcarrier output during the ith OFDM symbol can be represented by

$$Y_{i,n} = X_{i,n} \cdot H_{i,n} \cdot G_T(n) G_R(n) + n_{i,n}, \quad -N_u/2 \le n \le N_u/2 \tag{4}$$

where $n_{i,n}$ is a white complex Gaussian noise with variance σ^2, $G_T(n)$ and $G_R(n)$ are the transmitter and receiver filter frequency response values at the nth subcarrier frequency $f_n = n/T_u$, and $H_{i,n}$ is the channel frequency response given by

$$H_{i,n} = \sum_{\ell=1}^{L} h_\ell(iT_s) \cdot e^{-j2\pi \frac{n\tau_\ell}{NT}} \tag{5}$$

where $h_\ell(iT_s)$ denotes the channel ℓth path gain during the ith OFDM symbol. If we assume that the $N_u + 1$ transmission subcarriers within the flat region of the transmitter and receiver filter frequency responses, (4) can be rewritten as

$$Y_{i,n} = X_{i,n} \cdot H_{i,n} + n_{i,n} \tag{6}$$

where $G_T(n)$ and $G_R(n)$ are assumed to be equal to one at the flat region. When the flat region assumption does not hold, we can also eliminate $G_T(n)$ and $G_R(n)$ from (4) with the *a priori* knowledge of the transmitter and receiver filters. In the following, we assume that (6) is correct.

III. Proposed Channel Estimation Method

The proposed channel estimator is shown in Fig. 1. The whole algorithm can be divided into two modes: acquisition mode and tracking mode, as shown in Fig. 1(a). The acquisition mode includes the detection of the number of paths based on the MDL principle and the acquisition of the initial multipath time delays through the ESPRIT method. The tracking mode includes tracking the multipath time delays via IPIC DLLs and estimating the channel frequency response via the MMSE estimator as shown in Fig. 1(b).

A. Pilot Pattern

M pilot subcarriers are evenly inserted into the $N_u + 1$ transmission subcarriers. If D_f is used to denote the interval in terms of the number of subcarriers between two adjacent pilots in the frequency domain, then

$$M = \left\lceil \frac{N_u + 1}{D_f} \right\rceil \tag{7}$$

where $\lceil x \rceil$ denotes the nearest integer that is larger than or equal to x. Let $\mathcal{P}$ denote the set that contains the position indexes of the M pilot subcarriers. Then

$$\mathcal{P} = \left\{ p(m) \,\middle|\, p(m) = \left(m - \frac{M-1}{2} \right) D_f, \quad m = 0, \ldots, M-1 \right\} \tag{8}$$

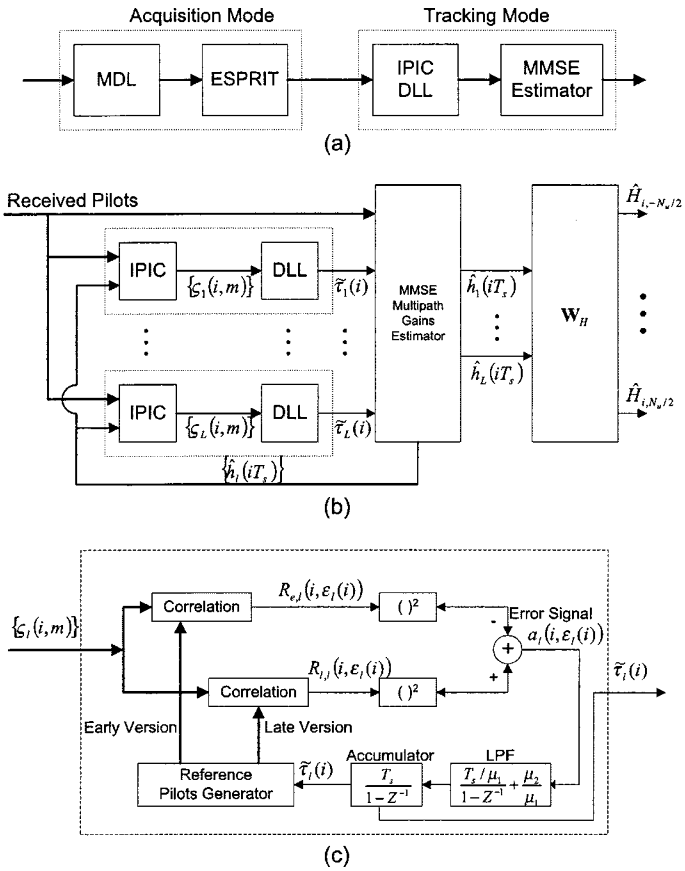

Fig. 1. The block diagrams of the proposed estimator: (a) the overall estimator block diagram; (b) the tracking mode block diagram; and (c) the DLL block diagram.

where D_f is assumed to be an even number. When D_f is an odd number, a similar set can be defined. At the pilot positions, we have that

$$X_{i,p(m)} = \Upsilon_m, \qquad m = 0, \ldots, M-1 \tag{9}$$

where $\{\Upsilon_m\}$ are the pilot subcarrier symbols that have the same amplitude.

According to the sampling theorem, the sampling rate in the frequency domain must fulfill the following requirement:

$$N/D_f > \tau_{\max}/T \tag{10}$$

where $\tau_{\max}/T$ denotes the normalized channel maximum excess delay. Let D_t also denote the spacing in terms of the number of OFDM symbols between two pilot subcarriers in the time domain. The sampling rate in the time domain should also fulfill the following requirement:

$$\frac{1}{D_t T_s} > 2 f_D \tag{11}$$

where f_D is the channel Doppler frequency. The pilot pattern design problem is a tradeoff between good channel estimation (closely spaced pilots) and high spectral and power efficiency (sparsely spaced pilots). In this paper, we set $D_t = 1$. However, the proposed method can be also extended for other $D_t > 1$ values in order to increase the system spectral and power efficiency. Fig. 2 shows a simple example of the pilot pattern with $N = 1024, N_u + 1 = 901, D_f = 32$, and $D_t = 1$.

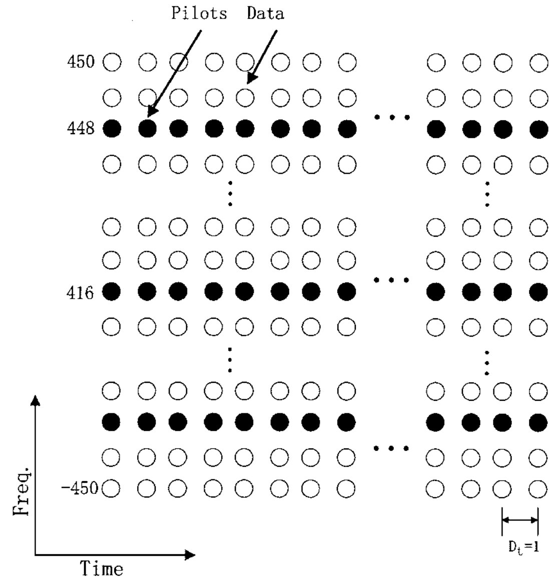

Fig. 2. Pilot pattern of OFDM signal.

B. Estimation of the Number of Paths

The pilot subcarriers can be employed to get the least squares (LS) estimates of the channel frequency response at the pilot subcarrier frequencies as

$$\begin{aligned} H'_{i,p(m)} &= \frac{Y_{i,p(m)}}{\Upsilon_m} \\ &= \sum_{\ell=1}^{L} h_\ell(iT_s) \cdot e^{-j2\pi \frac{p(m)\tau_\ell}{NT}} + n_{i,p(m)}/\Upsilon_m \end{aligned} \tag{12}$$

where $H'_{i,p(m)}$ is modeled as the summation of the complex-valued sinusoidal signals plus the complex-valued white noise. Let $\mathbf{H}'_{\mathrm{LS},P,i} = [H'_{i,p(0)}, \ldots, H'_{i,p(M-1)}]^T$ where $(\cdot)^T$ represents the transpose operation. The most prevalent technique for estimating the number of sinusoidal signals is based on the use of the MDL criterion [13], [14]. To use this criterion, $\mathbf{H}'_{\mathrm{LS},P,i}$ is organized into the following snap shot array:

$$\mathbf{Q}(i) = \begin{bmatrix} H'_{i,p(0)} & H'_{i,p(1)} & \cdots & H'_{i,p(K-1)} \\ H'_{i,p(1)} & H'_{i,p(2)} & \cdots & H'_{i,p(K)} \\ \vdots & \vdots & \cdots & \vdots \\ H'_{i,p(M-K)} & H'_{i,p(M-K+1)} & \cdots & H'_{i,p(M-1)} \end{bmatrix}. \tag{13}$$

Then, based on the forward–backward (FB) approach [12], the temporary sample correlation matrix is given by

$$\hat{\mathbf{R}}(i) = \frac{1}{2K}(\mathbf{Q}(i)\mathbf{Q}(i)^H + \mathbf{J}\overline{\mathbf{Q}(i)\mathbf{Q}(i)^H}\mathbf{J}) \tag{14}$$

where $\overline{(\cdot)}$ denotes complex conjugate only, $(\cdot)^H$ denotes Hermitian transpose, and $\mathbf{J}$ is a matrix with 1's on its anti-diagonal and 0's elsewhere. Note that the multipath time delays $\{\tau_\ell\}$ are quite stationary, while the amplitudes and the relative phases of the multipaths are subject to Rayleigh fading. Therefore, we can further average $\hat{\mathbf{R}}(i)$ over I OFDM symbols to get a better estimate of the channel correlation matrix as

$$\hat{\mathbf{R}} = \frac{1}{I}\sum_{i=0}^{i=I-1}\hat{\mathbf{R}}(i). \tag{15}$$

$\hat{\mathbf{R}}$ has the eigendecomposition form as

$$\hat{\mathbf{R}} = \sum_{k=1}^{k=M-K+1}\hat{\lambda}_k\hat{\mathbf{u}}_k\hat{\mathbf{u}}_k^H \tag{16}$$

where $\hat{\lambda}_1 \geq \cdots \geq \hat{\lambda}_{M-K+1}$ are the eigenvalues, and $\hat{\mathbf{u}}_1,\ldots,\hat{\mathbf{u}}_{M-K+1}$ are the corresponding eigenvectors. Then, for the FB approach, the MDL criterion can be described as [14]

$$\begin{aligned}\mathrm{MDL}(\rho) = &-I(M-K+1-\rho)\\ &\times\log\left\{\frac{\left(\prod_{k=\rho+1}^{M-K+1}\hat{\lambda}_k\right)^{1/(M-K+1-\rho)}}{\frac{1}{M-K+1-\rho}\sum_{k=\rho+1}^{M-K+1}\hat{\lambda}_k}\right\}\\ &+\frac{1}{4}\rho[2(M-K+1)-\rho+1]\log I.\end{aligned} \tag{17}$$

Finally, the number of paths is determined by

$$\hat{L} = \arg\min_{\rho\in\{0,\ldots,M-K\}}\mathrm{MDL}(\rho) \tag{18}$$

where $\arg\min_\rho \mathrm{MDL}(\rho)$ denotes the value ρ that minimizes $\mathrm{MDL}(\rho)$. In order to correctly detect the number of paths L, we must have that $M-K \geq L$.

C. Acquisition of Multipath Time Delays

The ESPRIT method [15] is used to acquire the initial multipath time delays $\{\tau_\ell\}$. First, the $\hat{L}$ eigenvectors associated with the largest eigenvalues of $\hat{\mathbf{R}}$ are organized into a matrix $\hat{\mathbf{U}} = [\hat{\mathbf{u}}_1,\ldots,\hat{\mathbf{u}}_{\hat{L}}]$. Then, let

$$\hat{\mathbf{U}}_1 = [\mathbf{I}_{M-K}\ 0]\hat{\mathbf{U}} \tag{19}$$

and

$$\hat{\mathbf{U}}_2 = [0\ \mathbf{I}_{M-K}]\hat{\mathbf{U}} \tag{20}$$

where $\mathbf{I}_{M-K}$ is the identity matrix of dimension $(M-K)\times(M-K)$, $[\mathbf{I}_{M-K}\ 0]$ and $[0\ \mathbf{I}_{M-K}]$ are of dimension $(M-K)\times(M-K+1)$, and $\hat{\mathbf{U}}_1$ and $\hat{\mathbf{U}}_2$ are of dimension $(M-K)\times\hat{L}$. After we calculate the $\hat{L}$ eigenvalues $\{\nu_\ell\}_{\ell=1}^{\hat{L}}$ of the matrix

$$\hat{\phi} = (\hat{\mathbf{U}}_1^H\hat{\mathbf{U}}_1)^{-1}\hat{\mathbf{U}}_1^H\hat{\mathbf{U}}_2 \tag{21}$$

the ℓth path time delay is given by

$$\hat{\tau}_\ell = \arg(\nu_\ell^*)N/(2\pi D_f)T, \qquad \ell = 1,\ldots,\hat{L} \tag{22}$$

where $\arg(\nu_\ell^*)$ denotes the phase angle of ν_ℓ^* in the interval $[0,2\pi)$ and ν_ℓ^* denotes the complex conjugate of ν_ℓ.

D. Tracking of Multipath Time Delays with IPIC DLL

Because of the complexity of the ESPRIT method, it can be only used as the initial acquisition method. On the other hand, the multipath time delays change slowly in mobile communications. Hence, we propose the low-complexity IPIC DLL to track the multipath time delays after the initial multipath time delays acquisition.

The DLL is a feedback technique that can automatically track the changes of slowly varying parameters. In the multipath fading channels, however, the interpath interference (IPI) dominates the DLL performance [10]. To increase the multipath time delays tracking accuracy, an IPIC method is proposed in this paper.

It follows from (12) that

$$Y_{i,p(m)} = \Upsilon_m\sum_{\ell=1}^{L}h_\ell(iT_s)\cdot e^{-j2\pi\frac{p(m)\tau_\ell}{NT}} + n_{i,p(m)} \tag{23}$$

where the received pilot subcarriers $Y_{i,p(m)}$ are affected by all the paths in the channel. In order to improve the DLL tracking performance, the estimated multipath gains and time delays are used to cancel the IPI. That is

$$\zeta_\ell(i,m) = Y_{i,p(m)} - \Upsilon_m\sum_{\substack{v=1\\v\neq\ell}}^{\hat{L}}\hat{h}_v(iT_s)\cdot e^{-j2\pi\frac{p(m)\tilde{\tau}_v(i)}{NT}},\quad m=0,\ldots,M-1 \tag{24}$$

where $\zeta_\ell(i,m)$ is the mth IPI canceled pilot subcarrier that is used to estimate the ℓth path delay for the $(i+1)$th OFDM symbol, $\tilde{\tau}_v(i)$ is the estimated multipath time delay for the vth path during the ith symbol based on the IPIC DLL, and $\hat{h}_v(iT_s)$ is the estimated multipath gain for the vth path during the ith symbol based on the MMSE estimator described in the next subsection.

After that, $\{\zeta_\ell(i,m)\}$ are input into the ℓth DLL whose structure is depicted in Fig. 1(c). The ℓth DLL operation can be described as follows.

First, $\{\zeta_\ell(i,m)\}$ are cross-correlated with the locally generated early and late reference pilot symbols. The early reference pilots based on the accumulator output during the ith OFDM symbol are

$$\Upsilon_{e,\ell}(i,m) = e^{-j2\pi\frac{p(m)}{N}(\tilde{\tau}_\ell(i)/T-\delta)}\Upsilon_m,\quad m=0,\ldots,M-1 \tag{25}$$

where $\delta(\delta<1)$ is the advanced (and retarded) interval normalized by the OFDM sample interval. Similarly, the local late reference pilots are

$$\Upsilon_{l,\ell}(i,m) = e^{-j2\pi\frac{p(m)}{N}(\tilde{\tau}_\ell(i)/T+\delta)}\Upsilon_m,\quad m=0,\ldots,M-1. \tag{26}$$

Then, the early branch cross-correlation output is

$$R_{e,\ell}(i,\epsilon_\ell(i)) = \sum_{m=0}^{M-1}\zeta_\ell(i,m)\Upsilon_{e,\ell}^*(i,m) \tag{27}$$

where $\epsilon_\ell(i) = (\tilde{\tau}_\ell(i)-\tau_\ell)/T$ is the normalized loop tracking error. Likewise, we get the late cross-correlation output

$R_{l,\ell}(i,\epsilon_\ell(i))$. Next, the cross-correlation output is squared to generate the tracking error signal as

$$\begin{aligned} a_\ell(i,\epsilon_\ell(i)) &= |R_{l,\ell}(i,\epsilon_\ell(i))|^2 - |R_{e,\ell}(i,\epsilon_\ell(i))|^2 \\ &= \mathrm{E}\left[a_\ell(i,\epsilon_\ell(i))\right] + n_{a_\ell(i,\epsilon_\ell(i))} \end{aligned} \tag{28}$$

where $\mathrm{E}[a_\ell(i,\epsilon_\ell(i))]$ is the useful component, $n_{a_\ell(i,\epsilon_\ell(i)}$ is a zero-mean disturbance called the loop noise, and $\mathrm{E}[\cdot]$ represents the statistical average operation. If the IPIC can cancel all the IPI, $\mathrm{E}[a_\ell(i,\epsilon_\ell(i))]$ is given by

$$\mathrm{E}[a_\ell(i,\epsilon_\ell(i))] = A^2\mathrm{E}[|h_\ell|^2]S_\ell(\epsilon_\ell(i)) \tag{29}$$

where

$$\begin{aligned} S_\ell(\epsilon_\ell(i)) = &\left|\sum_{m=0}^{M-1} e^{-j2\pi\frac{p(m)}{N}(\epsilon_\ell(i)+\delta)}\right|^2 \\ &- \left|\sum_{m=0}^{M-1} e^{-j2\pi\frac{p(m)}{N}(\epsilon_\ell(i)-\delta)}\right|^2 \end{aligned} \tag{30}$$

is the S-Curve or discriminator characteristic of the DLL, and $A = |\Upsilon_m|^2$. After that, $a_\ell(i,\epsilon_\ell(i))$ is smoothed by a low-pass loop filter (LPF) and further accumulated by an accumulator. The models of the accumulator and LPF are shown in Fig. 1(c). The accumulator output is the estimated ℓth path time delay, $\tilde{\tau}_\ell(i)$. The detailed DLL operation and analysis can be found in [16]. It is also shown in [16] that the DLL technique is a recursive solution to the maximum-likelihood (ML) estimation of the path time delay in the AWGN channel.

It should be noted that the design of the ℓth IPIC DLL needs the information of $\mathrm{E}[|h_\ell|^2]$. For the robust channel estimator design, we choose $\mathrm{E}[|h_\ell|^2] = 1/\hat{L}$.

E. MMSE Channel Estimator Based on Multipath Time Delays Information

When the MMSE estimator of $\{h_\ell\}$ is derived, we assume that the estimated $\hat{L}$ and $\{\tilde{\tau}_\ell\}$ are correct. First, the channel estimator based on the one-dimensional signal processing using the correlation in the frequency domain is derived. Then, the result is extended to the two-dimensional signal processing case. For the one-dimensional channel estimator derivation, the OFDM symbol index has been deleted.

1) Channel Estimation Model: Substituting $\hat{L}$ and $\{\tilde{\tau}_\ell\}$ into (12)

$$\mathbf{H}'_{\mathrm{LS},P} = \mathbf{W}_P\mathbf{h} + \mathbf{n}_P \tag{31}$$

where

$$\mathbf{W}_P = \begin{bmatrix} e^{-j2\pi\frac{p(0)}{N}\frac{\tilde{\tau}_1}{T}} & \cdots & e^{-j2\pi\frac{p(0)}{N}\frac{\tilde{\tau}_{\hat{L}}}{T}} \\ \vdots & \ddots & \vdots \\ e^{-j2\pi\frac{p(M-1)}{N}\frac{\tilde{\tau}_1}{T}} & \cdots & e^{-j2\pi\frac{p(M-1)}{N}\frac{\tilde{\tau}_{\hat{L}}}{T}} \end{bmatrix} \tag{32}$$

is the $M \times \hat{L}$ Fourier transform matrix, $\mathbf{h} = [h_1,\ldots,h_{\hat{L}}]^T$ is the $\hat{L}\times 1$ multipath gains vector to be estimated, and $\mathbf{n}_P = [n_{p(0)}/\Upsilon_0,\ldots,n_{p(M-1)}/\Upsilon_{M-1}]^T$ is the $M\times 1$ noise vector. We assume that $\mathbf{h}$ is a Gaussian random vector with zero mean. Likewise, $\mathbf{n}_P$ is a Gaussian random vector that is independent of $\mathbf{h}$. Then, the cross-correlation matrix of $\mathbf{h}$ and $\mathbf{H}'_{\mathrm{LS},P}$ is

$$\mathbf{R}_{hH'_{\mathrm{LS},P}} = \mathrm{E}\left[\mathbf{h}\mathbf{H}'^H_{\mathrm{LS},P}\right] = \mathbf{C}_h\mathbf{W}_P^H \tag{33}$$

where $\mathbf{C}_h = \mathrm{diag}([\sigma^2_{h_1},\ldots,\sigma^2_{h_{\hat{L}}}]^T)$ is the covariance matrix of $\mathbf{h}$. $\mathrm{diag}(\mathbf{x})$ denotes a diagonal matrix with the entries of the vector $\mathbf{x}$ along the main diagonal. Similarly, the autocorrelation matrix of $\mathbf{H}'_{\mathrm{LS},P}$ is

$$\mathbf{R}_{H'_{\mathrm{LS},P}H'_{\mathrm{LS},P}} = \mathrm{E}\left[\mathbf{H}'_{\mathrm{LS},P}\mathbf{H}'^H_{\mathrm{LS},P}\right] = \mathbf{W}_P\mathbf{C}_h\mathbf{W}_P^H + \frac{\sigma^2}{A}\mathbf{I}_M \tag{34}$$

where $\mathbf{I}_M$ is an identity matrix of size $M\times M$. Then, the MMSE estimator for $\mathbf{h}$ is [11]

$$\begin{aligned} \hat{\mathbf{h}} &= \mathbf{R}_{hH'_{\mathrm{LS},P}}\mathbf{R}^{-1}_{H'_{\mathrm{LS},P}H'_{\mathrm{LS},P}}\mathbf{H}'_{\mathrm{LS},P} \\ &= \left(\frac{\beta}{\mathrm{SNR}}\mathbf{C}_h^{-1} + \mathbf{W}_P^H\mathbf{W}_P\right)^{-1}\mathbf{W}_P^H\mathbf{H}'_{\mathrm{LS},P} \end{aligned} \tag{35}$$

where $\beta = \mathrm{E}[|X_{i,n}|^2]/A$ is the ratio of the average signal power to the pilot power, and $\mathrm{SNR} = \mathrm{E}[|X_{i,n}|^2]/\sigma^2$ is the average signal-to-noise ratio (SNR).

Hence, the MMSE estimator of the channel frequency response is

$$\begin{aligned} \hat{\mathbf{H}} &= \mathbf{W}_H\hat{\mathbf{h}} \\ &= \mathbf{W}_H\left(\frac{\beta}{\mathrm{SNR}}\mathbf{C}_h^{-1} + \mathbf{W}_P^H\mathbf{W}_P\right)^{-1}\mathbf{W}_P^H\mathbf{H}'_{\mathrm{LS},P} \end{aligned} \tag{36}$$

where

$$\mathbf{W}_{\mathbf{H}} = \begin{bmatrix} e^{-j2\pi\frac{-N_u/2}{N}\frac{\tilde{\tau}_1}{T}} & \cdots & e^{-j2\pi\frac{-N_u/2}{N}\frac{\tilde{\tau}_{\hat{L}}}{T}} \\ \vdots & \ddots & \vdots \\ e^{-j2\pi\frac{N_u/2}{N}\frac{\tilde{\tau}_1}{T}} & \cdots & e^{-j2\pi\frac{N_u/2}{N}\frac{\tilde{\tau}_{\hat{L}}}{T}} \end{bmatrix} \tag{37}$$

is the $(N_u+1)\times\hat{L}$ Fourier transform matrix and $\hat{\mathbf{H}} = [\hat{H}_{-N_2/2},\ldots,\hat{H}_{N_u/2}]^T$ is the estimated $(N_u+1)\times 1$ channel frequency response.

We note that the linear MMSE estimator based on the nonparametric channel model [2]–[5] can be also written as

$$\hat{\mathbf{H}}' = \mathbf{R}'_{HH'_{\mathrm{LS},P}}\mathbf{R}'^{-1}_{H'_{\mathrm{LS},P}H'_{\mathrm{LS},P}}\mathbf{H}'_{\mathrm{LS},P} \tag{38}$$

where $\mathbf{R}'_{HH'_{\mathrm{LS},P}}$ denotes the cross-correlation matrix between $\mathbf{H} = [H_{-N_u/2+1},\ldots,H_{N_u/2+1}]^T$ and $\mathbf{H}'_{\mathrm{LS},P}$ based on the nonparametric channel model, and $\mathbf{R}'_{H'_{\mathrm{LS},P}H'_{\mathrm{LS},P}}$ is the autocorrelation matrix of $\mathbf{H}'_{\mathrm{LS},P}$ based on the nonparametric model.

It follows from (36) and (38) that both the parametric and the nonparametric channel model-based estimator are linear transformations of $\mathbf{H}'_{\mathrm{LS},P}$. For the parametric model-based estimator, however, the multipath time delays are treated as "known parameters" (via estimation) so the channel correlation matrix of $\mathbf{H}'_{\mathrm{LS},P}$ has the form as (34). Because $\mathbf{W}_P$ only contains $\hat{L}$ linear independent column vectors, the eigenvalue decomposition (EVD) of (34) is

$$\mathbf{R}_{H'_{\mathrm{LS},P}H'_{\mathrm{LS},P}} = \mathbf{Q}\begin{bmatrix}\mathbf{\Lambda}_1 & \mathbf{0}\\ \mathbf{0} & \mathbf{0}\end{bmatrix}\mathbf{Q}^H + \frac{\sigma^2}{A}\mathbf{I}_M \tag{39}$$

where $\mathbf{\Lambda}_1$ is an $\hat{L}\times\hat{L}$ diagonal matrix of nonzero eigenvalues, and $\mathbf{Q}$ is a unitary matrix of dimension $(N_u+1)\times(N_u+1)$. Thus, we know that the signal subspace dimension of $\mathbf{R}_{H'_{\mathrm{LS},P}H'_{\mathrm{LS},P}}$ is $\hat{L}$ [12].

On the other hand, the multipath time delays are regarded as random variables for the nonparametric model-based method. Then, the EVD of the $\mathbf{R}'_{H'_{\mathrm{LS},P}H'_{\mathrm{LS},P}}$ is

$$\mathbf{R}'_{H'_{\mathrm{LS},P}H'_{\mathrm{LS},P}} = \mathbf{Q}'\begin{bmatrix}\mathbf{\Lambda}'_1 & \mathbf{0}\\ \mathbf{0} & \mathbf{\Lambda}'_2\end{bmatrix}\mathbf{Q}'^H + \frac{\sigma^2}{A}\mathbf{I}_M \tag{40}$$

where the $r\times r$ diag matrix $\mathbf{\Lambda}'_1$ contains the principal eigenvalues of $\mathbf{R}'_{H'_{\mathrm{LS},P}H'_{\mathrm{LS},P}}$. That is, the eigenvalues contained in $\mathbf{\Lambda}'_1$ are much larger than those in $\mathbf{\Lambda}'_2$. Hence, the signal subspace of its channel correlation matrix has the size of r that can be approximated by [4]

$$r \approx \Delta/T + 1 \tag{41}$$

which approximately equal to the length of the guard interval.

For the sparse multipath channel, we have that

$$\Delta/T + 1 > \hat{L}. \tag{42}$$

Hence, using the parametric channel model can reduce the signal subspace dimension. In Appendix A, it is shown that the reduction of the signal subspace dimension can improve the channel estimator performance.

2) Channel Estimation Mean Square Error: First, the covariance matrix of the estimated multipath gains $\hat{\mathbf{h}}$ is [11]

$$\mathbf{C}_{\hat{h}} = \left(\mathbf{C}_h^{-1} + \frac{\mathrm{SNR}}{\beta}\mathbf{W}_P^H\mathbf{W}_P\right)^{-1}. \tag{43}$$

Then, the channel estimation mean square error (MSE) is

$$\begin{aligned}\mathrm{MSE}_{\hat{H}} &= \frac{1}{N_u+1}\mathrm{E}[(\mathbf{H}-\hat{\mathbf{H}})^H(\mathbf{H}-\hat{\mathbf{H}})]\\ &= \frac{1}{N_u+1}\mathrm{tr}\left(\mathbf{C}_{\hat{h}}\mathbf{W}_H^H\mathbf{W}_H\right)\end{aligned} \tag{44}$$

where $\mathrm{tr}(\cdot)$ is the matrix trace.

3) Channel Estimation Under Mismatch: The channel estimator (36) requires knowledge of the channel multipath gains covariance matrix $\mathbf{C}_h$ and the system SNR. In practice, both $\mathbf{C}_h$ and SNR are not known. Hence, we can only design the channel estimator for fixed nominal values $\tilde{\mathbf{C}}_h$ and $\widetilde{\mathrm{SNR}}$. It has been shown in [4] and [5] that the channel estimator designed for the uniform channel power delay profile is robust to the $\mathbf{C}_h$ *mismatch*. Thus, we choose $\tilde{\mathbf{C}}_h = \mathrm{diag}([(1/\hat{L}),\ldots,(1/\hat{L})]^T)$.

As for the SNR mismatch, it has also been shown that the estimator designed for a high nominal $\widetilde{\mathrm{SNR}}$ value is preferable [4]. Therefore, substituting $\tilde{\mathbf{C}}_h$ and $\widetilde{\mathrm{SNR}}$ in (36), the robust generic channel estimator is given by

$$\hat{\mathbf{H}} = \mathbf{W}_H\left(\frac{\beta}{\widetilde{\mathrm{SNR}}}\tilde{\mathbf{C}}_h^{-1} + \mathbf{W}_P^H\mathbf{W}_P\right)^{-1}\mathbf{W}_P^{\mathbf{H}}\mathbf{H}'_{\mathrm{LS},P}. \tag{45}$$

4) Extension to Two-Dimensional Signal Processing: It should be noted that only the correlation in the frequency domain is used to estimate $\{h_\ell\}$ in the above derivation. However, the correlation in the time domain of $\{h_\ell(iT_s)\}$ can be also employed to further improve the channel estimation performance. For the nonparametric model-based estimators, with little performance degradation, the optimal two-dimensional filter can be simplified by two one-dimensional filters operating in the time and frequency domains, respectively [2], [3], [5]. In this paper, the parametric model-based two-dimensional estimator is also fulfilled by two separated filters operating in the frequency and time domains. Filtering in the frequency domain is followed by filtering in the time domain. That is, after we get $\hat{\mathbf{h}}$ using (35), $\hat{L}$ additional FIR filters operating in the time domain are applied to $\{\hat{\mathbf{h}}(iT_s)\}$ to further suppress the noise. For the ℓth path, the time-domain filtering operation is described as

$$\check{h}_\ell(i) = \sum_{k=0}^{L_t-1} c_\ell(k)\hat{h}_\ell(i-k) \tag{46}$$

where $\check{h}_\ell(i) = \check{h}_\ell(iT_s)$ is the resulted ℓth path gain after the time-domain filtering during the ith OFDM symbol, $\{c_\ell(k)\}$ are the L_t coefficients of the ℓth time-domain FIR filter, and $\hat{h}_\ell(i) = \hat{h}_\ell(iT_s)$ is the ℓth path gain sample obtained from (35) during the ith OFDM symbol. In order to reduce the channel estimation latency, only L_t-1 previously estimated $\hat{h}_\ell(i-k)$ are used in (46).

Applying the Wiener filter theory, the optimal filter coefficients in the MMSE sense can be found as

$$\mathbf{c}_\ell^T = \mathbf{q}_\ell\mathbf{R}_{\hat{h}_\ell\hat{h}_\ell}^{(t)-1} \tag{47}$$

where $\mathbf{c}_\ell = [c_\ell(0),\ldots,c_\ell(L_t-1)]^T$, $\mathbf{R}_{\hat{h}_\ell\hat{h}_\ell}^{(t)} = \mathrm{E}[\hat{\mathbf{h}}_\ell^{(t)}\hat{\mathbf{h}}_\ell^{(t)H}]$ is the autocorrelation matrix of $\hat{\mathbf{h}}_\ell^{(t)} = [\hat{h}_\ell(i),\ldots,\hat{h}_\ell(i-L_t+1)]^T$, and $\mathbf{q}_\ell = \mathrm{E}[h_\ell(i)\hat{\mathbf{h}}_\ell^{(t)H}] = [q_{\ell,0},\ldots,q_{\ell,L_t-1}]$.

For the robust channel estimator design, $\mathbf{c}_\ell$ should be calculated for the high nominal $\widetilde{\mathrm{SNR}}$ values. Then, substituting (31) into (35), the frequency-domain estimator (35) can be approximated as

$$\hat{\mathbf{h}}(i) \approx \mathbf{h}(i) + \left(\mathbf{W}_P^H\mathbf{W}_P\right)^{-1}\mathbf{W}_P^H\mathbf{n}_{i,P} \tag{48}$$

since $(\beta/\widetilde{\mathrm{SNR}})[\mathbf{C}_h^{-1}]_{\ell,\ell} \ll ([\mathbf{W}_P^H\mathbf{W}_P]_{\ell,\ell} = M)$ where $[\cdot]_{\ell,\ell}$ denotes the (ℓ,ℓ)th element of a matrix. Then

$$\hat{h}_\ell(i) \approx h_\ell(i) + \hat{n}_{i,\ell} \tag{49}$$

where $\hat{n}_{i,\ell}$ is a Gaussian noise term with variance $\sigma_{\hat{n}}^2 = ((\beta[(\mathbf{W}_P^H\mathbf{W}_P)^{-1}]_{\ell,\ell})/(\widetilde{\mathrm{SNR}}))$. To make $\sigma_{\hat{n}}^2$ be independent of $\mathbf{W}_P$, we can further let $(\mathbf{W}_P^H \mathbf{W}_P) \approx M\mathbf{I}_M$. Then, $\sigma_{\hat{n}}^2 = (\beta / M\widetilde{\mathrm{SNR}})$.

Next, we organize $\hat{h}_\ell(i)$ as

$$\hat{\mathbf{h}}_\ell^{(t)} = \mathbf{h}_\ell^{(t)} + \hat{\mathbf{n}}_\ell^{(t)} \tag{50}$$

where $\mathbf{h}_\ell^{(t)} = [h_\ell(i), \ldots, h_\ell(i - L_t + 1)]^T$, and $\hat{\mathbf{n}}_\ell^{(t)} = [\hat{n}_{i,\ell}, \ldots, \hat{n}_{i-L_t+1,\ell}]^T$ is a Gaussian noise vector with the covariance matrix $\mathbf{C}_{\hat{n}_\ell} = \sigma_{\hat{n}}^2 \mathbf{I}_{L_t}$. Then

$$\mathbf{R}_{\hat{h}_\ell \hat{h}_\ell}^{(t)} = \mathbf{R}_{h_\ell h_\ell}^{(t)} + \sigma_{\hat{n}}^2 \mathbf{I}_{L_t} \tag{51}$$

where $\mathbf{R}_{h_\ell h_\ell}^{(t)} = \mathrm{E}[\mathbf{h}_\ell^{(t)} \mathbf{h}_\ell^{(t)H}]$.

$\mathbf{R}_{h_\ell h_\ell}^{(t)}$ and $\mathbf{q}_\ell$ both depend on the channel Doppler power spectrum. It has been shown in [5] that the robust channel estimator design should use the uniform channel Doppler power spectrum. Then

$$q_{\ell,k} = \sigma_{h_\ell}^2 \frac{\sin(2\pi f_D T_s k)}{2\pi f_D T_s k} \tag{52}$$

and

$$\left[\mathbf{R}_{h_\ell h_\ell}^{(t)}\right]_{i,k} = q_{\ell,i-k}. \tag{53}$$

To derive a robust channel estimator, we also let $\sigma_{h_\ell}^2 = 1/\hat{L}$. Then, we can use the same coefficient set for the $\hat{L}$ FIR time-domain filters. Finally, similar to (36), the channel frequency response estimate output of the two-dimensional channel estimator is

$$\breve{\mathbf{H}} = \mathbf{W}_H \breve{\mathbf{h}}. \tag{54}$$

IV. Simulation Results

In this section, we will investigate the improved channel estimation algorithm performance in the multipath Rayleigh fading channel. We choose the "Vehicular A" channel environment [17] defined by ETSI for the evaluation of UMTS radio interface proposals (except when otherwise specified). The multipath time delays $\{\tau_\ell\}$ and the variance of the multipath gains $\{h_\ell(t)\}$ of the "Vehicular A" channel are shown in Table I. The channel maximum Doppler frequency is set to be $f_D = 100$ Hz (except when otherwise specified).

Throughout, a 16QAM-OFDM system is considered with $N = 1024$ subcarriers among which we use $N_u + 1 = 901$ subcarriers for transmission. The system also occupies a bandwidth of 5 MHz operating in the 2.4-GHz frequency band. The sample period is $T = 0.2\ \mu$s. Then, $T_u = 205\ \mu$s. The OFDM symbol has a guard interval with $\Delta/T = 16$ OFDM sample periods. We set $D_f = 32$, so there are $M = 29$ constant amplitude pilot subcarriers evenly inserted in the $N_u + 1$ subcarriers.

We choose the outermost four points from the 16QAM constellation as the symbols to be transmitted over the pilot subcarriers. Thus, $\beta = 1/1.8$ in (35). We also choose $K = 6$ in (13) and $I = 100$ in (15). For the DLLs, the normalized advance (retard) interval is $\delta = 1/2$, the loop damping ratio is $\vartheta = 1$, and the normalized one-sided loop bandwidth is $B_L = 0.005$. To evaluate the performance, the whole algorithm including the acquisition mode and the tracking mode is independently simulated 500 times for different channel data. During each time, the tracking mode lasts for 1000 OFDM symbols.

TABLE I
CHARACTERISTICS OF THE ETSI "VEHICULAR A" CHANNEL ENVIRONMENT

Tap	Time Delays (nsec)	Time Delays (T)	Average Power (dB)
1	0	0	0
2	310	1.55	-1
3	710	3.55	-9
4	1090	5.45	-10
5	1730	8.65	-15
6	2510	12.55	-20

Fig. 3 shows the probability of correct detection of the number of paths based on the MDL criterion. We can see that the MDL criterion can correctly detect the number of paths at high SNR values. At low SNR values, the paths with the smaller power are overwhelmed by the noise so they are undetectable. However, the MDL criterion did not overestimate the number of paths in the simulations.

The standard deviation of the initial time delay estimation errors normalized by T of the second path is shown in Fig. 4. As can be seen, the multipath time delays acquisition errors based on the ESPRIT method are small. Table II shows the probability of correct acquisition of different path time delays at $\mathrm{SNR} = 8$ dB where the MDL can only detect $\hat{L} < L$ paths. The ℓth path time delay is considered to be correctly acquired if $|\tau_\ell - \hat{\tau}_k|/T < 0.2,\ k = 1, \ldots, \hat{L}$. Because the $\hat{L}$ eigenvectors associated with the largest eigenvalues of $\hat{\mathbf{R}}$ are used to form (21), the ESPRIT method will estimate the time delays of the $\hat{L}$ dominant paths when $\hat{L} < L$. Hence, the MDL criterion and the ESPRIT method can adaptively estimate the channel parameters.

Figs. 3 and 4 also show the performances of MDL and ESPRIT when $\tau_2 = 0.5T$. For the reduced τ_2, the performance of the MDL criterion is affected very little and the ESPRIT method can still correctly acquire the initial path time delays. We then change the channel maximum Doppler frequency to $f_D = 20$ Hz. The corresponding performances are also shown in Figs. 3 and 4. The performance of the MDL criterion at $f_D = 20$ Hz is degraded compared with that of $f_D = 100$ Hz because the correlation matrix $\hat{\mathbf{R}}$ can be estimated more accurately when $f_D = 100$ Hz. We can average $\hat{\mathbf{R}}(i)$ in (15) over more OFDM symbols to improve the performance for small f_D values.

We then plot the standard deviation of the tracking errors normalized by T for the second IPIC DLL that tracks τ_2 in Fig. 5. For comparison, Fig. 5 also contains the curve of the tracking errors standard deviation of the normal DLL [10] that tracks the same path time delay. We can see that IPIC DLL can significantly reduce the tracking errors and, consequently, improve the channel estimation accuracy.

Next, Fig. 6 shows the SER curves of the one-dimensional channel estimator (36) designed for the correct channel statistics $\mathbf{C}_h$ and SNR, and the generic channel estimator (45) de-

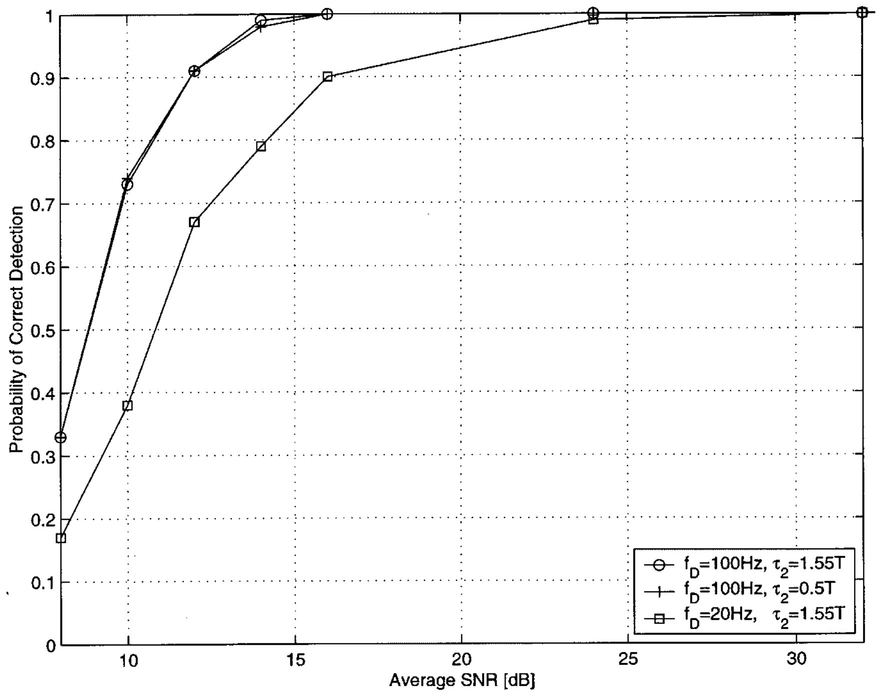

Fig. 3. The probability of correct detection of the number of paths based on the MDL criterion.

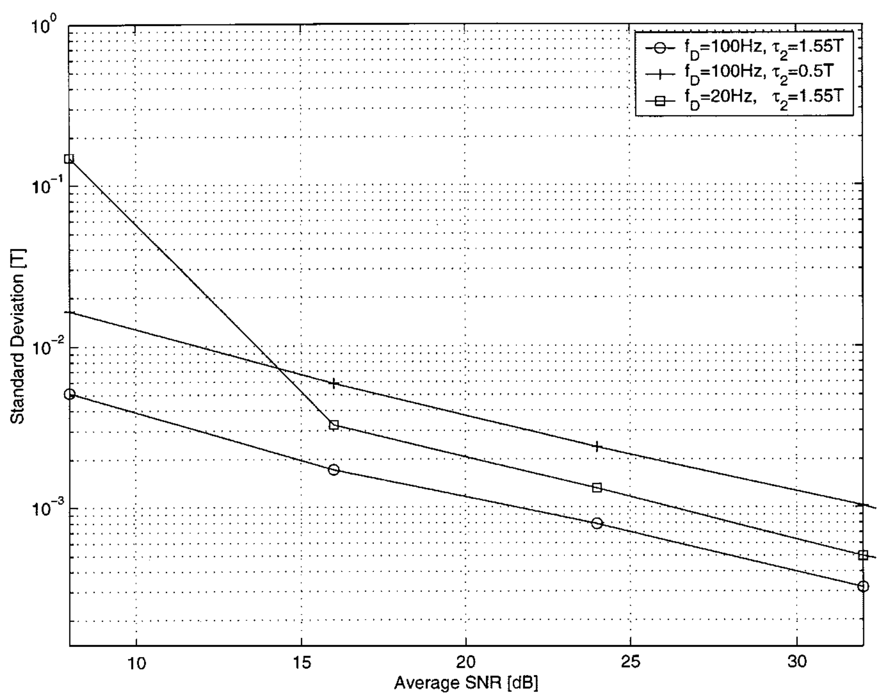

Fig. 4. The standard deviation of the initial path time delay estimation errors of the second path using the ESPRIT method.

TABLE II
PROBABILITY OF CORRECT ACQUISITION OF DIFFERENT PATH TIME DELAYS WITH ESPRIT METHOD AT $\mathrm{SNR} = 8$ dB

Tap	1	2	3	4	5	6
Probability	1	1	1	1	0.97	0.33

signed using the uniform power delay profile $\tilde{\mathbf{C}}_h$ and $\widetilde{\mathrm{SNR}} = 32$ dB. The MSE curves of these two estimators are also shown in Fig. 7. A close observation of the two figures indicates that the performance degradation of (45) compared with (36) is very small. It should be also noted that the performance is degraded very little when the DLLs only track the $\hat{L} < L$ dominant paths at low SNR values.

We also compare the SER and MSE performance of the proposed channel estimator with the one-dimensional non-parametric channel model-based channel estimator (38) in Figs. 6 and 7. In Fig. 6, the SER curve for the OFDM receiver assuming perfect channel knowledge, i.e., the 16QAM SER curve in Rayleigh fading channels, is also shown for reference.

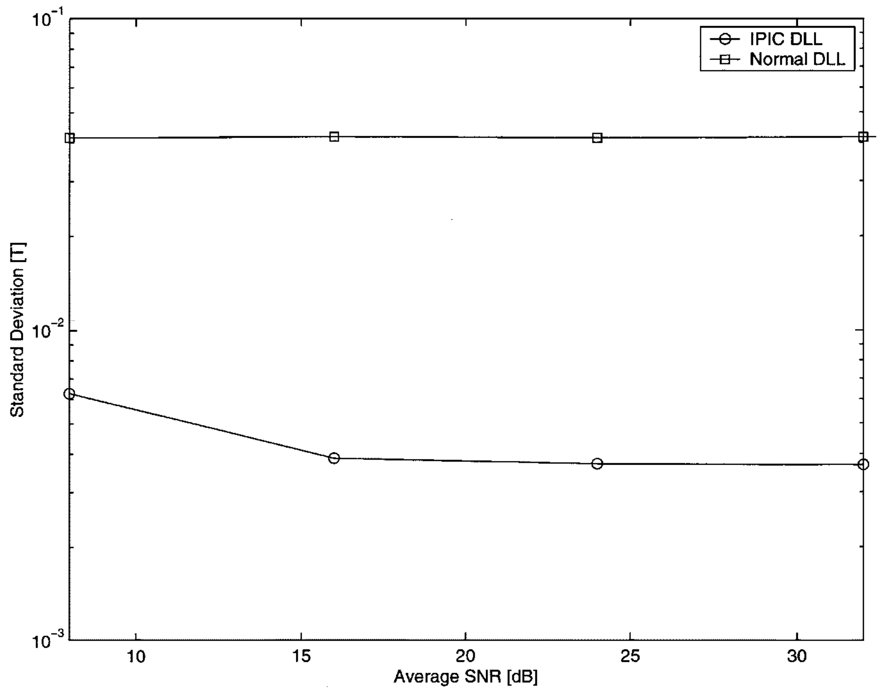

Fig. 5. Comparison of the standard deviations of the tracking errors normalized by T of the IPIC DLL and the normal DLL when they track τ_2.

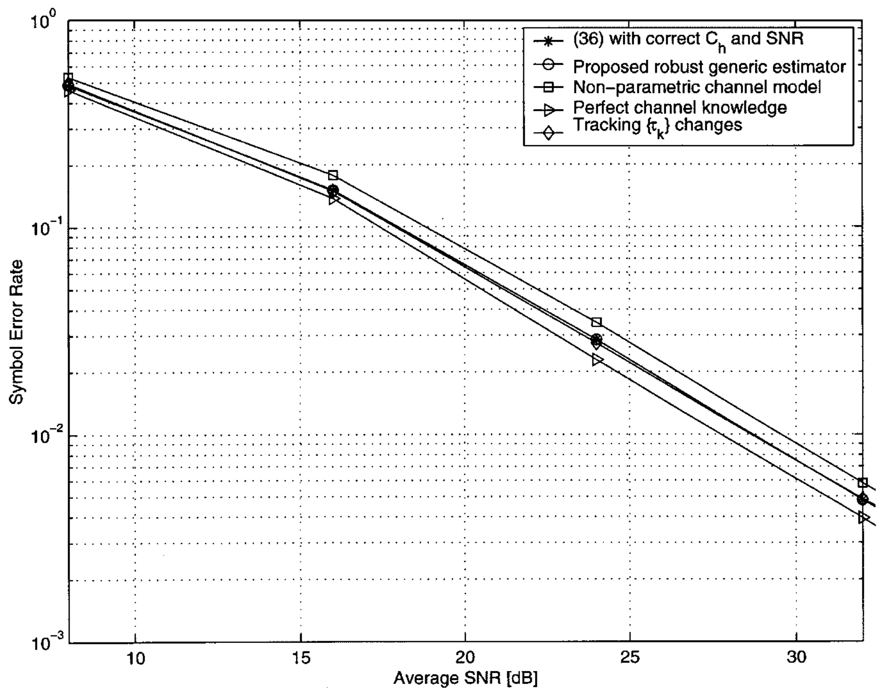

Fig. 6. The performances of the OFDM system using the proposed estimator (36) with the correct channel statistics $\mathbf{C}_h$ and SNR; using the proposed robust generic estimator (45) based on the parametric channel model; the MMSE estimator based on the nonparametric channel model; under perfect channel knowledge; and using (45) to track $\{\tau_k\}$ changes.

From the comparison, we can see that the proposed one-dimensional estimator has better performance. For instance, the SER performance improvement is about 1 dB and the MSE performance improvement is about 5 dB compared with the estimator based on the nonparametric model.

Fig. 7 also contains the MSE curves of the one-dimensional parametric estimator (45) and the nonparametric estimator (38) in a multipath fading channel with a relative small delay spread. The channel only consists of the first three paths of the UMTS "Vehicular A" channel model. Since the signal subspace dimension of $\mathbf{R}_{H'_{\mathrm{LS},P}H'_{\mathrm{LS},P}}$ is reduced, (45) has the better performance in the three paths channel than in the six-path channel. For the nonparametric model-based channel estimator, however, the performances are same in the two different channels.

In order to test the proposed two-dimensional channel estimator, we set $L_t = 10$ in (46). The MSE performance of

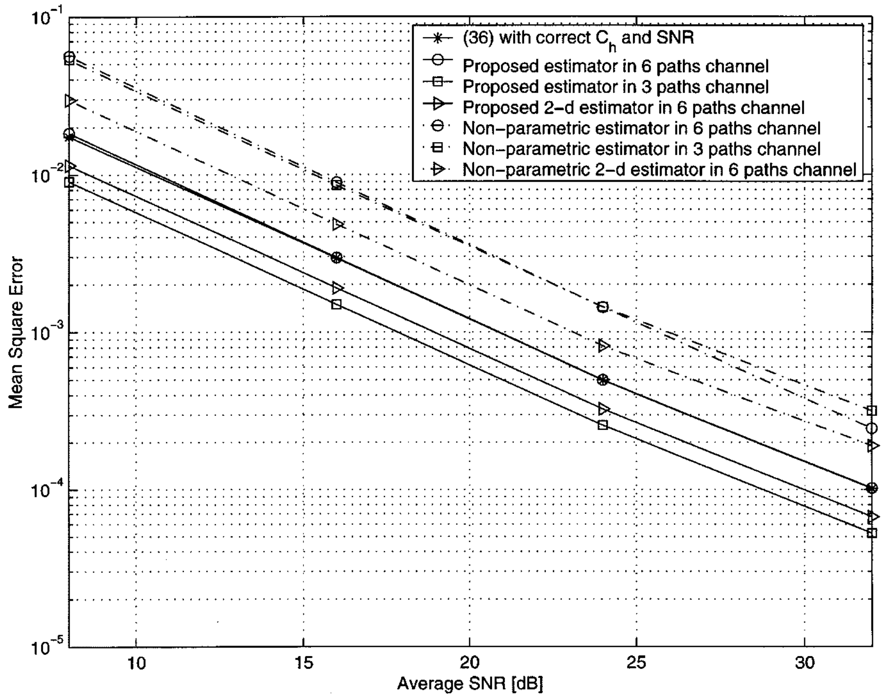

Fig. 7. The MSE performances of the proposed estimator (36) with the correct channel statistics $\mathbf{C}_h$ and SNR; the proposed robust generic estimator (45) based on the parametric channel model in the 6 paths channel and the 3 paths channel; the proposed two-dimensional parametric model-based estimator (54); the MMSE estimator (38) based on the nonparametric channel model in the 6 paths channel and the 3 paths channel; and the two-dimensional nonparametric model-based estimator.

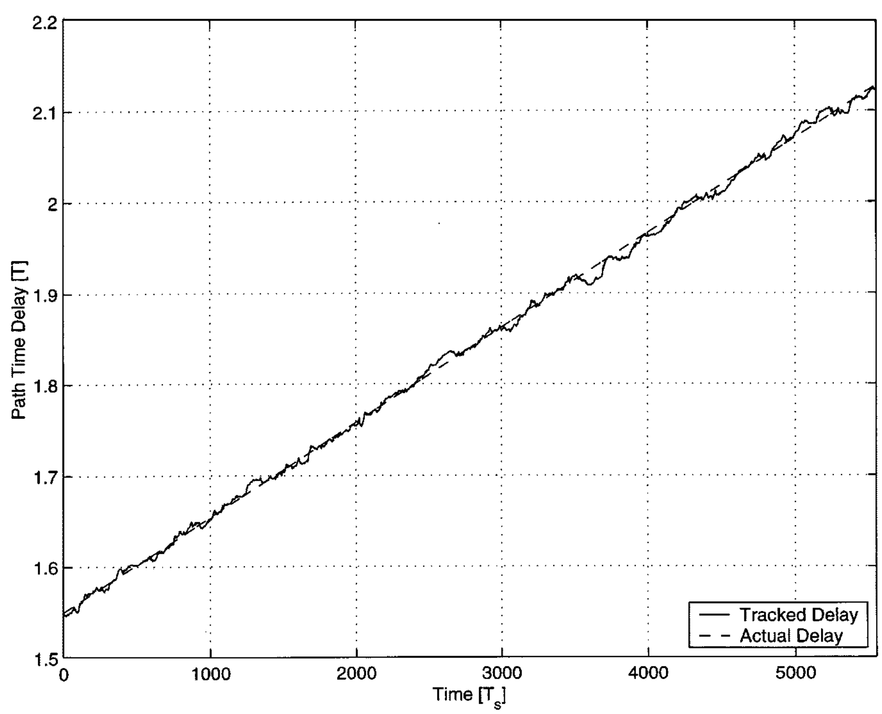

Fig. 8. Tracking the channel path time delay changes of τ_2 at $\mathrm{SNR} = 16$ dB.

the estimator (54) is shown in Fig. 7. For comparison, Fig. 7 also contains an MSE curve of a nonparametric two-dimensional channel estimator. For this estimator, we first apply a time-domain filter to every pilot subcarrier similar as (46) to reduce the noise effects. The time-domain filter also consists of $L_t = 10$ taps and only uses the previous pilot symbols to do the filtering operation. Then, a frequency-domain filter similar as (38) is used to fulfill the channel estimation. Both filters are designed using the Wiener filter theory. From the comparison, we can see that using an additional time-domain filter can improve the channel estimation performance for both the parametric and nonparametric channel estimators and that the performance improvements of the two kinds of estimator are similar. Thus, employing the parametric channel model for the two-dimensional channel estimator design can also improve the estimator performance.

Finally, we evaluate the proposed channel estimator ability of tracking the channel path time delay changes. We assume that the channel path time delays $\{\tau_k\}_{k=1}^{L}$ increase 100 ns/s, i.e., $0.5T$/s. For this simulation, the tracking mode lasts for 5550 OFDM symbols after the initial multipath time delays acquisition. The output of the second IPIC DLL that tracks τ_2 at $\mathrm{SNR} = 16$ dB is shown in Fig. 8. When the IPIC DLLs track the multipath time delays, the output of the MMSE estimator of (45) is used for 16QAM coherent detection. The corresponding SER curve is shown in Fig. 6. We can see that the proposed channel estimator can track the channel path time delays changes.

V. Conclusion

In this paper, we have presented an effective channel estimation algorithm using the pilot subcarriers for the sparse multipath fading channels. This algorithm is based on the use of a parametric channel model. The use of such model can effectively reduce the signal subspace dimension of the channel correlation matrix and, consequently, improve the channel estimation performance. The simulation results show that the MDL criterion and the ESPRIT method can adaptively estimate the initial channel parameters. Further, the IPIC DLL is shown to be an effective way to estimate and track the multipath time delays. Analysis and simulation results also demonstrate that the proposed channel estimation algorithm gives a substantial performance improvement in MSE over the nonparametric channel model-based methods.

Appendix A
Performance Improvement of the Parametric Channel Model-Based Channel Estimator

To illustrate the performance improvement of the channel estimator based on parametric channel modeling, we consider the MSE of the channel estimation at the pilot subcarrier frequencies. That is, for the parametric channel estimator, we consider

$$\mathrm{MSE}_{\hat{H}_P} = \frac{1}{M}\mathrm{E}[(\mathbf{H}_P - \hat{\mathbf{H}}_P)^H(\mathbf{H}_P - \hat{\mathbf{H}}_P)] \tag{55}$$

where $\mathbf{H}_P = [H_{p(0)}, \ldots, H_{p(M-1)}]^T$ and $\hat{\mathbf{H}}_P = [\hat{H}_{p(0)}, \ldots, \hat{H}_{p(M-1)}]^T$. It follows from (36) that

$$\hat{\mathbf{H}}_P = \mathbf{W}_P\left(\frac{\beta}{\mathrm{SNR}}\mathbf{C}_h^{-1} + \mathbf{W}_P^H\mathbf{W}_P\right)^{-1}\mathbf{W}_P^H\mathbf{H}'_{\mathrm{LS},P}. \tag{56}$$

The channel estimation MSE is given by (57), shown at the bottom of the page, where

$$\mathbf{R}_{H_PH_P} = \mathbf{W}_P\mathbf{C}_h\mathbf{W}_P^H. \tag{58}$$

Using the EVD of $\mathbf{R}_{H_PH_P}$, we can further simplify (57) as

$$\mathrm{MSE}_{\hat{H}_P} = \frac{1}{M}\sum_{m=1}^{M}\frac{\lambda_m\frac{\beta}{\mathrm{SNR}}}{\lambda_m + \frac{\beta}{\mathrm{SNR}}} \tag{59}$$

where $\{\lambda_m\}_{m=1}^{M}$ are the eigenvalues of $\mathbf{R}_{H_PH_P}$. From (39), we know that only the first $\hat{L}$ values of $\{\lambda_m\}_{m=1}^{M}$ are nonzero. Thus, we have

$$\mathrm{MSE}_{\hat{H}_P} = \frac{1}{M}\sum_{m=1}^{\hat{L}}\frac{1}{\frac{1}{\lambda_m} + \frac{\mathrm{SNR}}{\beta}}. \tag{60}$$

At high SNR values, $\mathrm{SNR}/\beta \gg 1/\lambda_\ell$. Therefore, we can further approximate $\mathrm{MSE}_{\hat{H}_P}$ as

$$\mathrm{MSE}_{\hat{H}_P} \approx \frac{\hat{L}\beta}{M\mathrm{SNR}}. \tag{61}$$

For the nonparametric model-based method, the linear MMSE estimator is given by [4]

$$\hat{\mathbf{H}}'_P = \mathbf{R}'_{H_PH_P}\left(\mathbf{R}'_{H_PH_P} + \frac{\sigma^2}{A}\mathbf{I}_M\right)^{-1}\mathbf{H}'_{\mathrm{LS},P} \tag{62}$$

and the channel estimation MSE is in (63), shown at the bottom of the page, where $\mathbf{R}'_{H_PH_P}$ is the channel correlation matrix based on the nonparametric channel model. If we assume that the multipath time delays are uniformly and independently distributed over the length of the guard interval and that all paths have the same average power, the correlation matrix $\mathbf{R}'_{H_PH_P} = [r'_{m,n}]$ can be written as [4]

$$r'_{m,n} = \begin{cases} 1, & p(m) = p(n) \\ \frac{1 - e^{-j2\pi\frac{\Delta(p(m)-p(n))}{TN}}}{j2\pi\frac{\Delta(p(m)-p(n))}{TN}}, & p(m) \neq p(n). \end{cases} \tag{64}$$

$$\mathrm{MSE}_{\hat{H}_P} = \frac{1}{M}\mathrm{tr}\left(\mathbf{R}_{H_PH_P}\left(\mathbf{I}_M - \left(\mathbf{R}_{H_PH_P} + \frac{\beta}{\mathrm{SNR}}\mathbf{I}_M\right)^{-1}\mathbf{R}_{H_PH_P}\right)\right) \tag{57}$$

$$\mathrm{MSE}_{\hat{H}'_P} = \frac{1}{M}\mathrm{tr}\left(\mathbf{R}'_{H_PH_P}\left(\mathbf{I}_M - \left(\mathbf{R}'_{H_PH_P} + \frac{\beta}{\mathrm{SNR}}\mathbf{I}_M\right)^{-1}\mathbf{R}'_{H_PH_P}\right)\right) \tag{63}$$

Similarly, using the EVD of $\mathbf{R}'_{H_P H_P}$, $\text{MSE}_{\hat{H}'_P}$ can be also written as

$$\text{MSE}_{\hat{H}'_P} = \frac{1}{M} \sum_{m=0}^{M-1} \frac{1}{\frac{1}{\lambda'_m} + \frac{\text{SNR}}{\beta}} \tag{65}$$

where $\{\lambda'_m\}_{m=0}^{M-1}$ are the eigenvalues of $\mathbf{R}'_{H_P H_P}$. From (41), we know $\mathbf{R}'_{H_P H_P}$ has $\Delta/T + 1$ principal eigenvalues. Using the same approximation as in (61), we have

$$\text{MSE}_{\hat{H}'_P} \approx \frac{(\Delta/T + 1)\beta}{M\text{SNR}}. \tag{66}$$

Hence, we know that the channel estimation MSE is related to the signal subspace dimension of the channel correlation matrix. The reduction of the signal subspace dimension can improve the MMSE channel estimator performance. The ratio of these two MSEs is

$$\frac{\text{MSE}_{\hat{H}_P}}{\text{MSE}_{\hat{H}'_P}} = \frac{\hat{L}}{\Delta/T + 1} \tag{67}$$

which depends on the multipath channel sparsity.

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