Performance Bounds for Very Large Multiuser MIMO Systems

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Learn from yesterday, live for today, hope for tomorrow. The important thing is not to stop questioning.

-Albert Einstein

Abstract

The last ten years have seen significant advances of multiuser MIMO (MU-MIMO) in wireless communication. MU-MIMO is now being introduced in several new generation wireless standards (e.g., LTE-Advanced, 802.16m). The number of users is increasing with more and more applications. At the same time, high transmission data rates and communication reliability are required. Furthermore, there is a growing concern about green communication which relates to the effects of the radiation emitted from wireless devices on the human body. Therefore, future MU-MIMO systems have to satisfy three main requirements: i) serving many autonomous users in the same time-frequency resource, ii) having high data rate and communication reliability, and iii) less energy consumption/radiation. These are seemingly contradictory requirements since the more users are served, the more interference the systems will suffer from, and the higher the data rate is, the more power is required. MU-MIMO with very large antenna arrays seems to meet the above demands and hence, it can be considered as a promising technology for next generation wireless systems. With very large antenna arrays (we mean arrays comprising say a hundred of antennas), the channel vectors are nearly-orthogonal and hence, multiuser interference can be significantly reduced. As a result, many users can be simultaneously served with high data rate. In particular, with coherent processing, transmit power can be reduced dramatically owing to the array gain. In this thesis, we focus on performance bounds of MU-MIMO with very large antenna arrays. We study the fundamental limits on the system performance when using large antenna arrays under practical constraints such as low complexity processing, imperfect channel state information, intercell interference, and finite-dimensional channels.

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Part I

Introduction

Introduction

1 Motivation

In wireless communication, the transmitted signals are being attenuated by fading due to multipath propagation and by shadowing due to large obstacles in the signal path, yielding a fundamental challenge for reliable communication. Transmission with multiple-input multiple-output (MIMO) antennas is a well-known diversity technique to enhance the reliability of the communication. At the same time, with multiple antennas, multiple streams can be sent out and hence, we can obtain a multiplexing gain which systematically improves the capacity of the communication. As a result, MIMO systems have gained significant attention for the past decades, and are now being incorporated into several new generation wireless standards (e.g., LTE-Advanced, 802.16m).

In particular, multiuser MIMO (MU-MIMO) systems, where several users simultaneously communicate with a base station (BS) equipped with multiple antennas, have recently attracted substantial interest [1–5]. Such systems can achieve a spatial multiplexing gain even if each user has a single antenna [1]. Due to the small physical size and low cost requirement, user terminals can only support a single or very few antennas, while the BS can be equipped with a large number of antennas. The more antennas the BS is equipped with, the more degrees of freedom are offered and hence, more users can simultaneously communicate in the same time-frequency resource. The main question is whether we can obtain these gains with low complexity signal processing and low-cost hardware implementation? With large antenna

arrays, conventional signal processing techniques (e.g. maximum likelihood detection) become prohibitively complex due to high signal dimensions. Recently, in [6], Marzetta showed that simple linear processing is nearly-optimal when the number of BS antennas is large. More precisely, even with simple maximum-ratio combining (MRC) in the uplink or maximum-ratio transmission (MRT) in the downlink, the effects of fast fading, uncorrelated noise, and intracell interference tend to disappear as the number of BS station antennas increases. To illustrate with a quantitative result, [6] showed that for an unlimited number of BS antennas, in a multicell MU-MIMO with a frequency reuse factor of 7, and a bandwidth of 20 MHz, each user can achieve a downlink link average net throughput of 17 Mbits/sec. As a result, there has been a great deal of interest in MU-MIMO with very large antenna arrays [7–10].

By contrast to conventional MU-MIMO systems, *very large MU-MIMO systems* (a.k.a. *massive MU-MIMO*) use a very large number of antennas at the BS, i.e. a hundred or more antennas, to simultaneously serve tens of users in the same time-frequency resource. The main benefits of such very large systems are:

(i) *Improving the data rate and communication reliability*: The very large MU-MIMO systems inherit all gains from conventional MIMO, i.e., with M-antennas BS and K single-antenna users, we can achieve a diversity of order M and a multiplexing gain of $\min(M, K)$.

(ii) *Simple signal processing*: With an increasing number of BS antennas, channel hardening occurs (i.e., channel becomes more and more deterministic). As a consequence, the effect of thermal noise and small scale fading is averaged out. In particular, channel vectors are pairwisely orthogonal and hence, the effect of interuser interference can be eliminated with simple linear signal processing. As an example, multiuser detection in the uplink by simply projecting the received vectors onto each user's channel is nearly-optimal.

(iii) *Power efficiency*: For the uplink, coherent combining can achieve a very high array gain which allows for substantial reduction in the transmit power of each user. For the downlink, the BS can focus the energy into the spatial directions where the terminals are located. As a result, with a very large antenna array, the transmit power can be reduced by an order of magnitude, or more. For example, to obtain the same

quality-of-service as with a single-antenna BS, a 100-antenna array would need to radiate only 1% of the power.

The design and analysis of very large MU-MIMO systems is a fairly new subject that is attracting substantial interest [11–18].

Inspired by the above discussion, this thesis considers performance bounds for the uplink of very large MU-MIMO systems under practical constraints such as low complexity processing, imperfect channel state information (CSI), finite-dimensional channels, and intercell interference.

2 Background and Preliminaries

The thesis considers the uplink performance of MU-MIMO systems. Therefore, in this section, we will provide the basic background of MU-MIMO in terms of communication schemes, channel estimation, and signal detection, especially for the uplink.

2.1 Multiuser MIMO Systems

MIMO technology can provide a remarkable increase in data rate due to the spatial multiplexing gain, and in communication reliability through the diversity gain. It is now incorporated into practical cellular networks. Conventional cellular networks use orthogonal multiple-access techniques, i.e., each user is scheduled on a different time-frequency resource. However, when the BS is equipped with more antennas, more degrees of freedom are available and hence, more users can be scheduled on the same time-frequency resource. Such systems are referred as MU-MIMO systems (see Fig. 1).

Advantages of MU-MIMO

Recently, MU-MIMO has gained much attention because of following advantages:

- MU-MIMO allows for spatial multiplexing gain at the BS without the requirement of multiple-antennas at user terminals. This is important since users cannot support many antennas due to low-cost requirements and physical size limitations, whereas the BS can support many antennas.
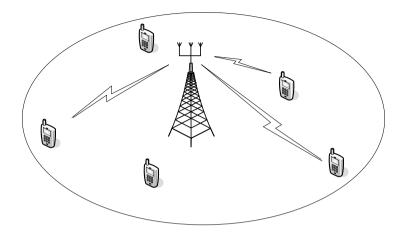
Figure 1: Multiuser MIMO Systems.

- MU-MIMO does not only reap all benefits of single-user MIMO (SU-MIMO) systems, but also overcomes most of propagation limitations in SU-MIMO such as ill-behavior channels. Specifically, by using scheduling schemes, we can reduce the limitations of ill-behavior channels. Furthermore, line-of-sight propagation, which causes significant reduction on the performance of SU-MIMO systems, is no longer a problem in MU-MIMO systems.

However, there is always a tradeoff between the system performance and the implementation complexity. The advantages of MU-MIMO come at a price.

Challenges

- Channel state information: in order to achieve high spatial multiplexing gain, the BS needs to process the received signals coherently. This requires accurate and timely acquisition of CSI. This can be challenging, especially in high mobility scenarios.

- There exists multiuser interference, hence complicated interference reduction or cancellation techniques should be used. For example, maximum likelihood multiuser detection [19] for uplink, dirty paper coding (DPC) techniques for downlink [20], and interference alignment [21].

- Since several users are served on the same time-frequency resource, scheduling schemes which optimally select the group of users depending on the precoding/detection schemes, CSI knowledge etc., should be considered. This increases the cost of the system implementation.

- Pilot contamination: in practical cellular networks, due to the limitation of the channel coherence interval, non-orthogonal pilot sequences have to be utilized in different cells. Therefore, the channel estimate obtained in a given cell is contaminated by pilots transmitted by users in other cells. This effect, called "pilot contamination", reduces the system performance [22].

2.2 Uplink Multiuser MIMO Systems

We consider the communication of a BS equipped with an array of M antennas and K single-antenna users.[1] In the uplink, the $M \times 1$ received vector at the BS is

$$\boldsymbol{y} = \sqrt{p_\mathrm{u}} \sum_{k=1}^{K} \boldsymbol{h}_k x_k + \boldsymbol{n} \tag{1}$$

$$= \sqrt{p_\mathrm{u}} \boldsymbol{H} \boldsymbol{x} + \boldsymbol{n} \tag{2}$$

where $\sqrt{p_\mathrm{u}} x_k$ is the transmitted signal from the kth user (the average power transmitted by each user is p_u), $\boldsymbol{h}_k \in \mathbb{C}^{M \times 1}$ is the channel vector between the kth user and the BS, $\boldsymbol{n} \in \mathbb{C}^{M \times 1}$ is the additive noise vector, $\boldsymbol{H} \triangleq [\boldsymbol{h}_1 \, ... \, \boldsymbol{h}_K]$, and $\boldsymbol{x} \triangleq [x_1 \, ... \, x_K]^T$. We assume that the elements of $\boldsymbol{h}_k$ and $\boldsymbol{n}$ are i.i.d. Gaussian distributed with zero mean and unit variance.

The BS will coherently detect the signals transmitted from K users by using the received signal vector $\boldsymbol{y}$ together with knowledge of the CSI. This CSI has to be estimated. The channel estimate can be obtained from uplink training.[2]

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[1]In general, each user can be equipped with multiple antennas. However, for simplicity of the analysis, we limit our systems to single-antenna users.

[2]In LTE, the channel is estimated at each user and then the user feedbacks this CSI to the BS. However, with very large antenna arrays at the BS, there is not enough time for CSI feedback. One possibility is to operate in TDD mode where the uplink and downlink use the same frequency spectrum. Assuming channel reciprocity, it is sufficient to obtain uplink channel estimates through uplink training.

We assume that the channel stays constant over T symbol durations. During each coherence interval, there are two phases (see Fig. 2). In the first phase, a part τ of the coherence interval is used for uplink training to estimate the channel of each user. In the second phase, all K users simultaneously transmit their data to the BS. The BS then detects the transmitted signals using the channel estimates acquired in the first phase.
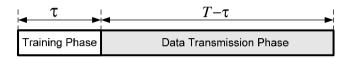

Figure 2: Uplink Transmission protocol.

Uplink Training Phase

A part of coherence interval is used for the uplink training. We assume that each user is assigned an orthogonal pilot sequence of τ symbols. This requires $\tau \geq K$. The pilot sequence used by the K users can be represented by a $\tau \times K$ matrix $\sqrt{p_\mathrm{p}}\boldsymbol{\Phi}$, which satisfies $\boldsymbol{\Phi}^H\boldsymbol{\Phi} = \boldsymbol{I}_K$, where $p_\mathrm{p} = \tau p_\mathrm{u}$, and $\sqrt{p_\mathrm{p}}\boldsymbol{\Phi}(t,k)$ is the signal transmitted by user k at time t. Here, we assume that the average transmit powers per pilot symbol and data symbol are the same. Then, the $M \times \tau$ received pilot matrix at the BS is given by

$$\boldsymbol{Y}_\mathrm{p} = \sqrt{p_\mathrm{p}}\boldsymbol{H}\boldsymbol{\Phi}^T + \boldsymbol{N}_\mathrm{p} \tag{3}$$

where $\boldsymbol{N}_\mathrm{p} \in \mathbb{C}^{M \times \tau}$ is the additive noise at the BS. Assume that the elements of $\boldsymbol{N}_\mathrm{p} \in C^{\tau \times K}$ are i.i.d. Gaussian distributed with zero mean and unit variance. The received pilot matrix $\boldsymbol{Y}_\mathrm{p}$ can be represented by $\boldsymbol{Y}_\mathrm{p}\boldsymbol{\Phi}^*$ and $\boldsymbol{Y}_\mathrm{p}\boldsymbol{\Phi}^*_\perp$, where $\boldsymbol{\Phi}^*_\perp$ is the orthogonal complement of $\boldsymbol{\Phi}^*$. Since $\boldsymbol{Y}_\mathrm{p}\boldsymbol{\Phi}^*_\perp$ only includes the noise part which is independent of $\boldsymbol{Y}_\mathrm{p}\boldsymbol{\Phi}^*$, $\boldsymbol{Y}_\mathrm{p}\boldsymbol{\Phi}^*$ is a sufficient statistic for the estimation of $\boldsymbol{H}$. Let $\tilde{\boldsymbol{Y}}_\mathrm{p} \triangleq \boldsymbol{Y}_\mathrm{p}\boldsymbol{\Phi}^*$. We have

$$\tilde{\boldsymbol{Y}}_\mathrm{p} = \sqrt{p_\mathrm{p}}\boldsymbol{H} + \boldsymbol{W} \tag{4}$$

where $\boldsymbol{W} \triangleq \boldsymbol{N}_\mathrm{p}\boldsymbol{\Phi}^*$ is an $M \times K$ complex Gaussian matrix whose elements are i.i.d. Gaussian distributed with zero mean and unit variance. Since $\boldsymbol{H}$ has independent columns, we can estimate each column of $\boldsymbol{H}$ independently. Let $\tilde{\boldsymbol{y}}_{\mathrm{p},k}$ and $\boldsymbol{w}_k$ be the kth columns of $\tilde{\boldsymbol{Y}}_\mathrm{p}$ and $\boldsymbol{W}$, respectively. Then

$$\tilde{\boldsymbol{y}}_{\mathrm{p},k} = \sqrt{p_\mathrm{p}}\boldsymbol{h}_k + \boldsymbol{w}_k. \tag{5}$$

i) Maximum a Posteriori Probability (MAP) Estimation:

With MAP estimation, we want to find the channel estimate of $\boldsymbol{h}_k$ which maximizes the posterior probability $p\left(\boldsymbol{h}_k | \tilde{\boldsymbol{y}}_{\mathrm{p},k}\right)$, i.e.,

$$\hat{\boldsymbol{h}}_k = \arg \max_{\boldsymbol{h}_k \in \mathbb{C}^M} p\left(\boldsymbol{h}_k | \tilde{\boldsymbol{y}}_{\mathrm{p},k}\right) = \arg \max_{\boldsymbol{h}_k \in \mathbb{C}^M} p\left(\tilde{\boldsymbol{y}}_{\mathrm{p},k} | \boldsymbol{h}_k\right) p\left(\boldsymbol{h}_k\right). \tag{6}$$

Since the elements of $\boldsymbol{h}_k$ and $\boldsymbol{w}_k$ are i.i.d. complex Gaussian distributed with zero mean and unit variance, we have

$$p\left(\boldsymbol{h}_k\right) = \frac{1}{\pi^M} \exp\left(-\|\boldsymbol{h}_k\|^2\right) \tag{7}$$

$$p\left(\tilde{\boldsymbol{y}}_{\mathrm{p},k} | \boldsymbol{h}_k\right) = \frac{1}{\pi^M} \exp\left(-\|\tilde{\boldsymbol{y}}_{\mathrm{p},k} - \sqrt{p_{\mathrm{p}}}\boldsymbol{h}_k\|^2\right). \tag{8}$$

Substituting (7) and (8) into (6), we obtain

$$\hat{\boldsymbol{h}}_k = \arg \min_{\boldsymbol{h}_k \in \mathbb{C}^M} \|\tilde{\boldsymbol{y}}_{\mathrm{p},k} - \sqrt{p_{\mathrm{p}}}\boldsymbol{h}_k\|^2 + \|\boldsymbol{h}_k\|^2 \tag{9}$$

$$= \frac{\sqrt{p_{\mathrm{p}}}}{p_{\mathrm{p}} + 1} \tilde{\boldsymbol{y}}_{\mathrm{p},k}. \tag{10}$$

ii) MMSE Channel Estimation:

With MMSE, the BS want to estimate the channel which minimizes the mean-square error. More precisely,

$$\hat{\boldsymbol{h}}_k = \arg \min_{\hat{\boldsymbol{h}}_k \in \mathbb{C}^M} \mathbb{E}_{\boldsymbol{h}_k, \tilde{\boldsymbol{y}}_{\mathrm{p},k}} \left\{ \|\hat{\boldsymbol{h}}_k - \boldsymbol{h}_k\|^2 \right\} \tag{11}$$

$$= \mathbb{E}\left\{\boldsymbol{h}_k | \tilde{\boldsymbol{y}}_{\mathrm{p},k}\right\} = \frac{\sqrt{p_{\mathrm{p}}}}{p_{\mathrm{p}} + 1} \tilde{\boldsymbol{y}}_{\mathrm{p},k}. \tag{12}$$

From (10) and (12), we can see that the MAP channel estimate coincides with the MMSE channel estimate. This is not surprising since it is well-known that when the conditional probability $p\left(\boldsymbol{h}_k | \tilde{\boldsymbol{y}}_{\mathrm{p},k}\right)$ is symmetric around the mean $\mathbb{E}\left\{\boldsymbol{h}_k | \tilde{\boldsymbol{y}}_{\mathrm{p},k}\right\}$, the MAP estimate coincides with the MMSE estimate [23].

2.3 Linear Receivers

In this section, we assume that the BS has perfect CSI knowledge. The BS wants to detect all signals transmitted from K users. To obtain optimal

performance, the maximum-likelihood (ML) multiuser detection can be used. More precisely,

$$\hat{\boldsymbol{x}} = \arg \min_{\boldsymbol{x} \in \mathcal{X}^K} \|\boldsymbol{y} - \sqrt{p_{\mathrm{u}}}\boldsymbol{H}\boldsymbol{x}\|^2 \tag{13}$$

where $\mathcal{X}$ is the finite alphabet of x_k, $k = 1, 2, ..., K$. The problem (13) is a finite-alphabet-constrained least-square (LS) problem. The BS has to search over all possible vectors $\boldsymbol{x}$. There are $|\mathcal{X}|^K$ such vectors. Therefore, ML has a complexity which is exponential in the number of users.

The BS can use linear detection schemes to reduce the decoding complexity. However, these schemes have lower detection reliability compared with ML detection. There is always a tradeoff between complexity and system performance. However, when the number of BS antennas is large, linear detectors are nearly-optimal [6, 7]. Therefore, in this thesis, we will consider linear detectors.

With linear detection schemes at the BS, the received signal $\boldsymbol{y}$ is separated in to K streams by multiplying it with a multiuser detection matrix. Each stream is then decoded independently. The complexity is of order of $K|\mathcal{X}|$. We now review some conventional linear multiuser detectors.

Maximum-Ratio Combining

With MRC, the BS wants to maximize the received signal-to-noise ratio (SNR) of each stream, ignoring the effect of multiuser interference. As a result, to detect the transmitted signal from the kth user, the received signal $\boldsymbol{y}$ is multiplied by the conjugate-transpose of the channel vector $\boldsymbol{h}_k$, i.e.,

$$\tilde{y}_k = \boldsymbol{h}_k^H \boldsymbol{y} = \sqrt{p_{\mathrm{u}}}\|\boldsymbol{h}_k\|^2 x_k + \sqrt{p_{\mathrm{u}}} \sum_{i \neq k}^{K} \boldsymbol{h}_k^H \boldsymbol{h}_i x_i + \boldsymbol{h}_k^H \boldsymbol{n}. \tag{14}$$

The received signal-to-interference-plus-noise ratio (SINR) of the kth stream for MRC is given by

$$\mathtt{SINR}_{\mathtt{mrc},k} = \frac{p_{\mathrm{u}}\|\boldsymbol{h}_k\|^4}{p_{\mathrm{u}} \sum_{i \neq k}^{K} |\boldsymbol{h}_k^H \boldsymbol{h}_i|^2 + \|\boldsymbol{h}_k\|^2} \tag{15}$$

$$\rightarrow \frac{\|\boldsymbol{h}_k\|^4}{\sum_{i \neq k}^{K} |\boldsymbol{h}_k^H \boldsymbol{h}_i|^2}, \text{ as } p_{\mathrm{u}} \rightarrow \infty. \tag{16}$$

- Advantage: the signal processing is very simple since the BS just multiplies the received vector with the conjugate-transpose of the channel matrix $\boldsymbol{H}$, and then detects each stream separately. More importantly, MRC can be implemented in a distributed manner. Furthermore, for small p_{u}, $\mathtt{SINR}_{\mathtt{mrc},k} \approx p_{\mathrm{u}}\|\boldsymbol{h}_k\|^2$. This implies that at low SNR, MRC can achieve the same array gain as in the case of a single-user system.

- Disadvantage: as discussed above, since MRC neglects the effect of multiuser interference, it performs poorly in interference-limited scenarios.

Zero-Forcing Receiver

By contrast to MRC, zero-forcing (ZF) receivers take the interuser interference into account, but neglect the effect of noise. With ZF, the multiuser interference is completely nulled out by projecting each stream onto the orthogonal space of the interuser interference. More precisely, the received vector is multiplied by the pseudo-inverse of the channel matrix $\boldsymbol{H}$ as

$$\tilde{\boldsymbol{y}} = \left(\boldsymbol{H}^H \boldsymbol{H}\right)^{-1} \boldsymbol{H}^H \boldsymbol{y} = \sqrt{p_{\mathrm{u}}}\boldsymbol{x} + \left(\boldsymbol{H}^H \boldsymbol{H}\right)^{-1} \boldsymbol{H}^H \boldsymbol{n}. \tag{17}$$

This scheme requires that $M \geq K$. We can see that each stream of $\tilde{\boldsymbol{y}}$ in (17) is free of multiuser interference. The received SINR of the kth stream is given by

$$\mathtt{SINR}_{\mathtt{zf},k} = \frac{p_{\mathrm{u}}}{\left[\left(\boldsymbol{H}^H \boldsymbol{H}\right)^{-1}\right]_{kk}} \tag{18}$$

- Advantage: the signal processing is simple and ZF works well in interference-limited scenarios.

- Disadvantage: since ZF neglects the effect of noise, it works poorly under noise-limited scenarios. Compared with MRC, ZF has a higher implementation complexity due to the computation of the pseudo-inverse of the channel gain matrix.

Minimum Mean-Square Error Receiver

Let $\boldsymbol{A}$ be an $M \times K$ linear detection matrix. The linear minimum mean-square error receiver aims to minimize the mean-square error between the estimate $\boldsymbol{A}^H \boldsymbol{y}$ and the transmitted signal $\boldsymbol{x}$. More precisely,

$$\boldsymbol{A}_{\text{mmse}} = \arg \min_{\boldsymbol{A} \in \mathbb{C}^{M \times K}} \mathbb{E} \left\{ \|\boldsymbol{A}^H \boldsymbol{y} - \boldsymbol{x}\|^2 \right\} \tag{19}$$

$$= \arg \min_{\boldsymbol{A} \in \mathbb{C}^{M \times K}} \sum_{k=1}^{K} \mathbb{E} \left\{ |\boldsymbol{a}_k^H \boldsymbol{y} - x_k|^2 \right\}. \tag{20}$$

where $\boldsymbol{a}_k$ is the kth column of $\boldsymbol{A}$. Therefore, the kth column of the MMSE detection matrix is [24]

$$\boldsymbol{a}_{\text{mmse},k} = \arg \min_{\boldsymbol{a}_k \in \mathbb{C}^{M \times 1}} \mathbb{E} \left\{ |\boldsymbol{a}_k^H \boldsymbol{y} - x_k|^2 \right\} \tag{21}$$

$$= \left(p_{\text{u}} \sum_{\substack{i=1 \\ i \neq k}}^{K} \boldsymbol{h}_i \boldsymbol{h}_i^H + \boldsymbol{I}_M \right)^{-1} \boldsymbol{h}_k \tag{22}$$

$$= c_k \left(\boldsymbol{H} \boldsymbol{H}^H + \frac{1}{p_{\text{u}}} \boldsymbol{I}_M \right)^{-1} \boldsymbol{h}_k \tag{23}$$

where

$$c_k \triangleq \frac{1}{p_{\text{u}} - p_{\text{u}} \boldsymbol{h}_k^H \left(\boldsymbol{H} \boldsymbol{H}^H + \frac{1}{p_{\text{u}}} \boldsymbol{I}_M \right)^{-1} \boldsymbol{h}_k}.$$

It is known that the MMSE receiver maximizes the received SINR. Therefore, among the MMSE, ZF, and MRC receivers, MMSE is the best. We can see from (23) that, at high SNR (high p_{u}), ZF approaches MMSE, while at low SNR, MRC performs as well as MMSE. Furthermore, the received SINR for the MMSE receiver is given by

$$\text{SINR}_{\text{mmse},k} = p_{\text{u}} \boldsymbol{h}_k^H \left(p_{\text{u}} \sum_{\substack{i=1 \\ i \neq k}}^{K} \boldsymbol{h}_i \boldsymbol{h}_i^H + \boldsymbol{I}_M \right)^{-1} \boldsymbol{h}_k. \tag{24}$$

Figure 3 shows the achievable sum-rates for MRC, ZF, and MMSE versus $\text{SNR} \triangleq p_{\text{u}}$, with $M = 5$ and $K = 5$. These curves are computed by using (15), (18), and (24), assuming that elements of $\boldsymbol{H}$ are i.i.d. Gaussian distributed with zero mean and unit variance. As expected, MMSE performs strictly better than ZF and MRC over the entire range of SNRs.
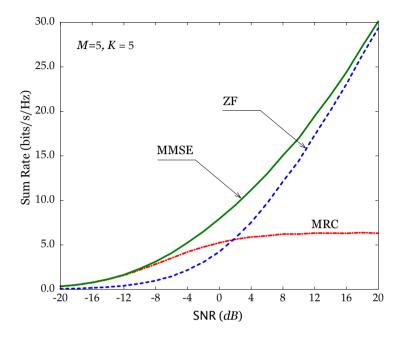
Figure 3: Performance of linear receivers.

2.4 Mathematical Preliminaries for Analysis of Very Large MIMO Systems

We now review some useful limit results about very long random vectors [25] which will be used for the analysis in the rest of the thesis.

- Let $\boldsymbol{p} \triangleq [p_1 \ ... \ p_n]^T$ and $\boldsymbol{q} \triangleq [q_1 \ ... \ q_n]^T$ be $n \times 1$ vectors whose elements are independent identically distributed (i.i.d.) random variables (RVs) with $\mathbb{E}\{p_i\} = \mathbb{E}\{q_i\} = 0$, $\mathbb{E}\left\{|p_i|^2\right\} = \sigma_p^2$, and $\mathbb{E}\left\{|q_i|^2\right\} = \sigma_q^2$, $i = 1, 2, ..., n$. Assume that $\boldsymbol{p}$ and $\boldsymbol{q}$ are independent.

  Applying the law of large numbers, we obtain

$$\frac{1}{n}\boldsymbol{p}^H\boldsymbol{p} \overset{a.s.}{\to} \sigma_p^2, \ \text{as } n \to \infty \tag{25}$$

$$\frac{1}{n}\boldsymbol{p}^H\boldsymbol{q} \overset{a.s.}{\to} 0, \ \text{as } n \to \infty. \tag{26}$$

  where $\overset{a.s.}{\to}$ denotes almost sure convergence.

Applying the Lindeberg-Lévy central limit theorem, we obtain

$$\frac{1}{\sqrt{n}} \boldsymbol{p}^H \boldsymbol{q} \overset{d}{\to} \mathcal{CN}\left(0, \sigma_p^2 \sigma_q^2\right), \text{ as } n \to \infty \tag{27}$$

where $\overset{d}{\to}$ denotes convergence in distribution.

- Let X_1, X_2, ... be a sequence of independent circular symmetric complex RVs, such that X_i has zero mean and variance σ_i^2. Further assume that the following conditions are satisfied: 1) $s_n^2 = \sum_{i=1}^n \sigma_i^2 \to \infty$, as $n \to \infty$; and 2) $\sigma_i/s_n \to 0$, as $n \to \infty$. Then by applying the Cramér's central limit theorem [25], we have

$$\frac{\sum_{i=1}^n X_i}{s_n} \to \mathcal{CN}\left(0, 1\right), \text{ as } n \to \infty. \tag{28}$$

3 Contributions of the Thesis

This thesis considers performance bounds of MU-MIMO with very large antenna arrays. The thesis consists two main contributions. The first one relates to energy and spectral efficiency when using very large antenna arrays, i.e., how much in power efficiency we can gain for perfect and imperfect CSI with different linear receivers in single and multi-cell scenarios. The second one relates to the pilot contamination effect for multicell systems.

Brief summaries of the papers included in this thesis are as follows:

Paper A: Energy and Spectral Efficiency of Very Large Multiuser MIMO Systems

Authored by Hien Quoc Ngo, Erik G. Larsson, and Thomas L. Marzetta.

To appear in the Transactions on Communications. This work is an extension of the conference paper [26].

A multiplicity of autonomous terminals simultaneously transmits data streams to a compact array of antennas. The array uses imperfect channel-state information derived from transmitted pilots to extract the individual data streams. The power radiated by the terminals can be made inversely proportional to the square-root of the number of base station antennas with

no reduction in performance. In contrast if perfect channel-state information were available the power could be made inversely proportional to the number of antennas. Lower capacity bounds for maximum-ratio combining (MRC), zero-forcing (ZF) and minimum mean-square error (MMSE) detection are derived. A MRC receiver normally performs worse than ZF and MMSE. However as power levels are reduced, the cross-talk introduced by the inferior maximum-ratio receiver eventually falls below the noise level and this simple receiver becomes a viable option. The tradeoff between the energy efficiency (as measured in bits/J) and spectral efficiency (as measured in bits/channel use/terminal) is quantified for a channel model that includes small-scale fading but not large-scale fading. It is shown that the use of moderately large antenna arrays can improve the spectral and energy efficiency with orders of magnitude compared to a single-antenna system.

Paper B: The Multicell Multiuser MIMO Uplink with Very Large Antenna Arrays and a Finite-Dimensional Channel

Authored by Hien Quoc Ngo, Erik G. Larsson, and Thomas L. Marzetta.

Submitted to the Transactions on Communications. This work is an extension of the conference paper [27].

We consider multicell multiuser MIMO systems with a very large number of antennas at the base station (BS). We assume that the channel is estimated by using uplink training. We further consider a physical channel model where the angular domain is separated into a finite number of distinct directions. We analyze the so-called pilot contamination effect discovered in previous work, and show that this effect persists under the finite-dimensional channel model that we consider. In particular, we consider a uniform array at the BS. For this scenario, we show that when the number of BS antennas goes to infinity, the system performance under a finite-dimensional channel model with P angular bins is the same as the performance under an uncorrelated channel model with P antennas. We further derive a lower bound on the achievable rate of uplink data transmission with a linear detector at the BS. We then specialize this lower bound to the cases of maximum-ratio combining (MRC) and zero-forcing (ZF) receivers, for a finite and an infinite number of BS antennas. Numerical results corroborate our analysis and show a comparison between the performances of MRC and ZF in terms of sum-rate.

Paper C: EVD-Based Channel Estimations for Multicell Multiuser MIMO with Very Large Antenna Arrays

Authored by Hien Quoc Ngo and Erik G. Larsson.

Published in the proceedings of the IEEE International Conference on Acoustics, Speed and Signal Processing (ICASSP), 2012 [28].

This paper consider a multicell multiuser MIMO with very large antenna arrays at the base station. For this system, with channel state information estimated from pilots, the system performance is limited by pilot contamination and noise limitation as well as the spectral inefficiency discovered in previous work. To reduce these effects, we propose the eigenvalue-decomposition-based approach to estimate the channel directly from the received data. This approach is based on the orthogonality of the channel vectors between the users and the base station when the number of base station antennas grows large. We show that the channel can be estimated from the eigenvalue of the received covariance matrix excepting the multiplicative factor ambiguity. A short training sequence is required to solved this ambiguity. Furthermore, to improve the performance of our approach, we investigate the join eigenvalue-decomposition-based approach and the Iterative Least-Square with Projection algorithm. The numerical results verify the effectiveness of our channel estimate approach.

3.1   Papers not Included in the Thesis

The following papers contain work done by the author but are not included in the thesis.

1. H. Q. Ngo and E. G. Larsson, "Linear Multihop Amplify-and-Forward Relay Channels: Error Exponent and Optimal Number of Hops", *IEEE Transactions on Wireless Communications*, vol. 10, no. 11, pp. 3834-3842, Nov 2011.

Abstract: We compute the random coding error exponent for linear multihop amplify-and-forward (AF) relay channels. Instead of considering only the achievable rate or the error probability as a performance measure separately, the error exponent results can give us insight into the fundamental tradeoff between the information rate and communication reliability in these channels. This measure enables us to determine what codeword length that is required to achieve a given level of communication reliability at a rate below the channel capacity. We first derive a general formula for the random

coding exponent of general multihop AF relay channels. Then we present a closed-form expression of a tight upper bound on the random coding error exponent for the case of Rayleigh fading. From the exponent expression, the capacity of these channels is also deduced. The effect of the number of hops on the performance of linear multihop AF relay channels from the error exponent point of view is studied. As an application of the random coding error exponent analysis, we then find the optimal number of hops which maximizes the communication reliability (i.e., the random coding error exponent) for a given data rate. Numerical results verify our analysis, and show the tightness of the proposed bound.

2. M. Matthaiou, G. C. Alexandropoulos, H. Q. Ngo, and E. G. Larsson, "Analytic framework for the effective rate of MISO fading channels," *IEEE Transactions on Communications*, vol. 60, no. 6, pp. 1741-1751, June 2012.

Abstract: The delay constraints imposed by future wireless applications require a suitable metric for assessing their impact on the overall system performance. Since the classical Shannon's ergodic capacity fails to do so, the so-called effective rate was recently established as a rigorous alternative. While prior relevant works have improved our knowledge on the effective rate characterization of communication systems, an analytical framework encompassing several fading models of interest is not yet available. In this paper, we pursue a detailed effective rate analysis of Nakagami-m, Rician and generalized-K multiple input single-output (MISO) fading channels by deriving new, analytical expressions for their exact effective rate. Moreover, we consider the asymptotically low and high signal-to-noise regimes, for which tractable, closed-form effective rate expressions are presented. These results enable us to draw useful conclusions about the impact of system parameters on the effective rate of different MISO fading channels. All the theoretical expressions are validated via Monte-Carlo simulations.

3. H. Q. Ngo, T. Q. Duong, and E. G. Larsson, "Uplink Performance Analysis of Multicell MU-MIMO with Zero-Forcing Receivers and Perfect CSI", *Proceedings of the IEEE Swedish Communication Technologies Workshop (Swe-CTW)*, 2011.

Abstract: We consider the uplink of a multicell multiuser MIMO system. The data detection is done by using the zero-forcing (ZF) technique, assuming the base station has perfect channel state information. We derive an exact closed-form expression for the uplink rate per user. We further study the asymptotic performance of the system. We show that, at high signal-to-noise ratio, the system is interference-limited and hence, we cannot improve the system performance by increasing the transmit power at each user. Instead, by increasing the number of base station antennas, the effects of interference and noise can be reduced, thereby improving the system performance. In particular, we show that, with very large antenna arrays at the base station, the transmit power of each user can be made inversely proportional to the number of base station antennas while maintaining a desired quality-of-service. Numerical results verify our analysis.

4. T. Q. Duong, H. Q. Ngo, H-J. Zepernick, and A. Nallanathan, "Distributed space-time coding in two-way fixed gain relay networks over Nakagami-m fading," *IEEE International Conference on Communicatons (ICC)*, Ottawa, Canada, June 2012.

Abstract: The distributed Alamouti space-time code in two-way fixed gain amplify-and-forward (AF) relay is proposed in this paper. In particular, closed-form expressions for approximated ergodic sum-rate and exact pairwise error probability (PWEP) are derived for Nakagami-m fading channels. To reveal further insights into array and diversity gains, an asymptotic PWEPis also obtained. Finally, numerical results are provided to corroborate the proposed theoretical analysis.

5. H. Q. Ngo, M. Matthaiou, and E. G. Larsson, "Performance Analysis of Large Scale MU-MIMO with Optimal Linear Receivers", *Proceedings of the IEEE Swedish Communication Technologies Workshop (SweCTW)*, 2012.

Abstract: We consider the uplink of multicell multiuser MIMO (MU-MIMO) systems with very large antenna arrays at the base station (BS). We assume that the BS estimates the channel through uplink training, and then uses this channel estimate to detect the signals transmitted from a multiplicity of autonomous users in its cell. By taking the correlation between the channel estimate and the interference from other cells into account,

we propose an optimal linear receiver (OLR) which maximizes the received signal-to-interference-plus-noise (SINR). Analytical approximations of the exact and lower bound on the achievable rate are then derived. The bound is very tight, especially at large number of BS antennas. We show that at low SINR, maximalratio combing (MRC) receiver performs as well as OLR, however at high SINR, OLR outperforms MRC. Compared with the typical minimum mean-square error receiver, our proposed OLR improves systematically the system performance, especially when the interference is large.

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