Discrete Transforms – 개요

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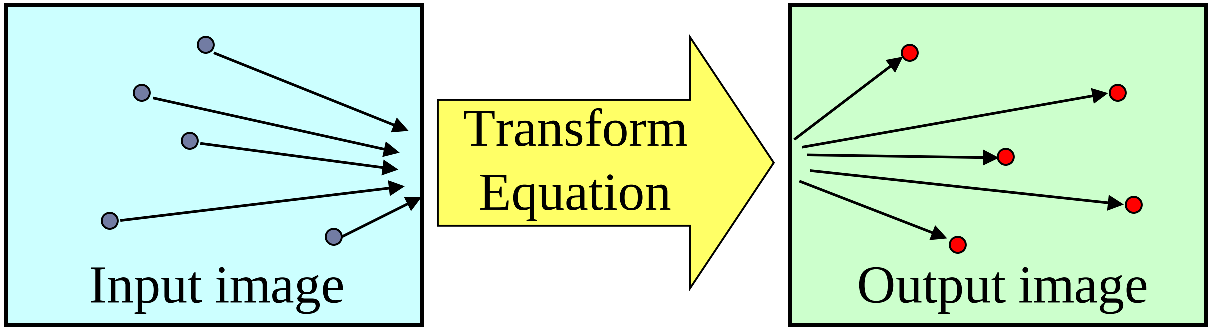
학습 목표

- 변환의 의미와 절차를 설명할 수 있다.
- 공간 주파수의 의미를 설명할 수 있다.

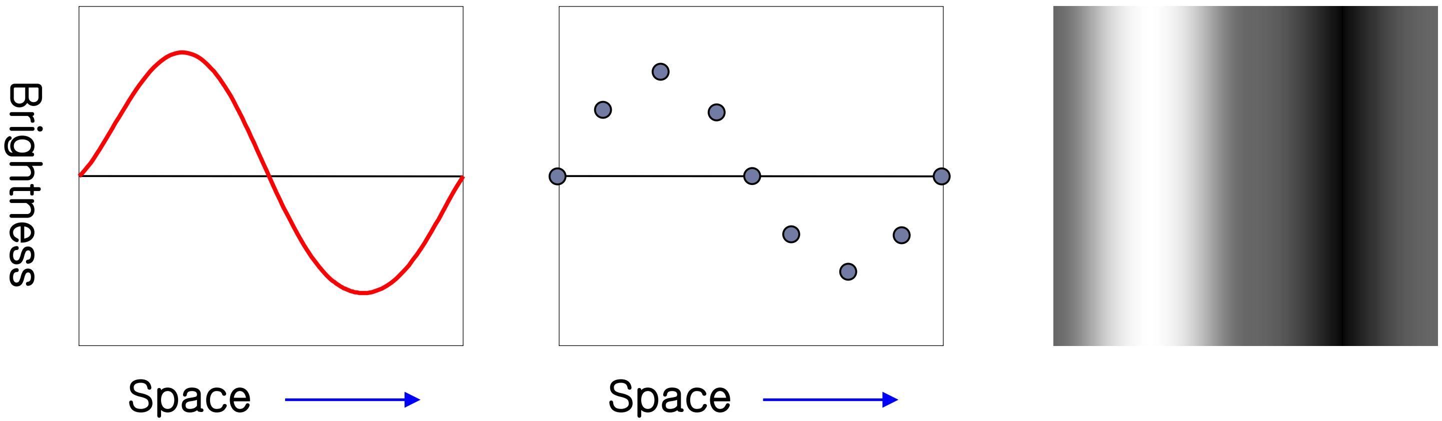
변환 (transform)

- 변환 수식에 의해 주어진 데이터(영상)을 다른 공간으로 매핑하는 과정 ➡ discrete transform의 형태를 가짐
- 주파수 변환 (frequency transform)
  - 공간(spatial) 도메인의 영상 데이터를 주파수 도메인으로 매핑
  - 입력 영상의 모든 픽셀들은 출력 데이터의 각 값에 기여



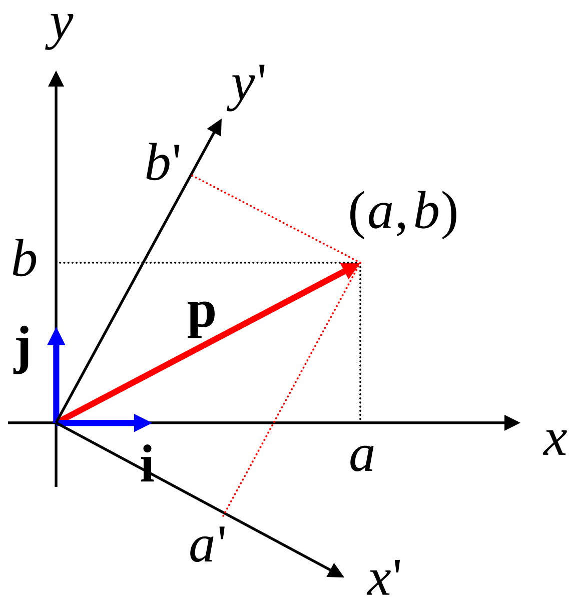
Basis function

- Transforms are based on basis functions
- Typically sinusoidal or rectangular form
- basis vector : 1-D sampling of basis function
- basis image or basis matrix : 2-D sampling of basis function



Process of Transform

- Projecting the image into the basis images
- Projecting process is an inner product



$$\text{basis vector set} = \{\mathbf{i}, \mathbf{j}\}$$

$$\|\mathbf{i}\| = \|\mathbf{j}\| = 1 \;\&\; \mathbf{i}\cdot\mathbf{j} = 0$$

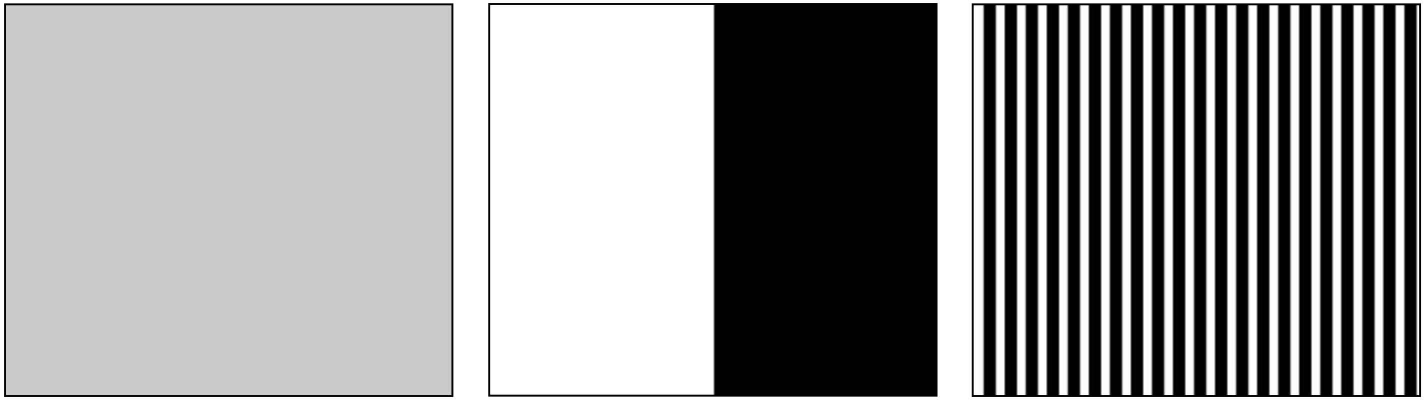
$$\mathbf{p}\cdot\mathbf{i} = \begin{bmatrix} a \\ b \end{bmatrix} \cdot \begin{bmatrix} 1 \\ 0 \end{bmatrix} = a$$

$$\mathbf{p}\cdot\mathbf{j} = \begin{bmatrix} a \\ b \end{bmatrix} \cdot \begin{bmatrix} 0 \\ 1 \end{bmatrix} = b$$

$$(a,b) \Rightarrow (a',b')$$

공간 주파수 (Spatial Frequency)

- The ways in which image brightness levels change in space
- High spatial frequency: rapidly changing brightness level
- Low spatial frequency: slowly changing brightness level
- Zero frequency: image with a constant value



f = 0, g = 202   f = 1, square wave   f = 20, square wave

General Form of Transformation

- Forward Transformation

$$\mathbf{T}(u,v) = \sum_{r=0}^{M-1} \sum_{c=0}^{N-1} \mathbf{I}(r,c)\mathbf{B}(r,c;u,v)$$

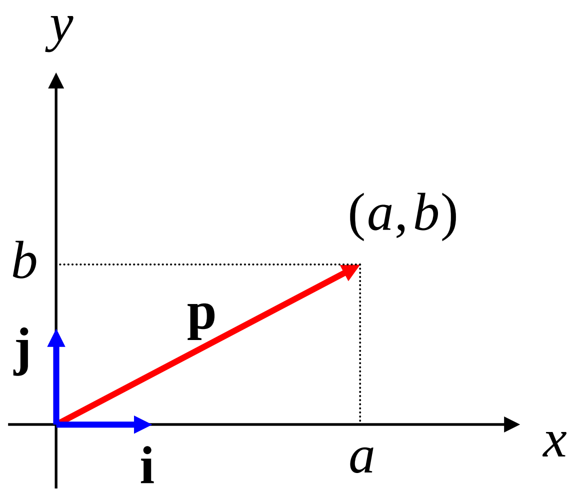
  - u, v : frequency variables
  - $\mathbf{T}(u,v)$: transform coefficients
  - $\mathbf{B}(r,c;u,v)$: basis images

- Backward (Inverse) Transformation

$$\mathbf{I}(r,c) = \sum_{u=0}^{M-1} \sum_{v=0}^{N-1} \mathbf{T}(u,v)\mathbf{B}^{-1}(r,c;u,v)$$

General Form of Transformation

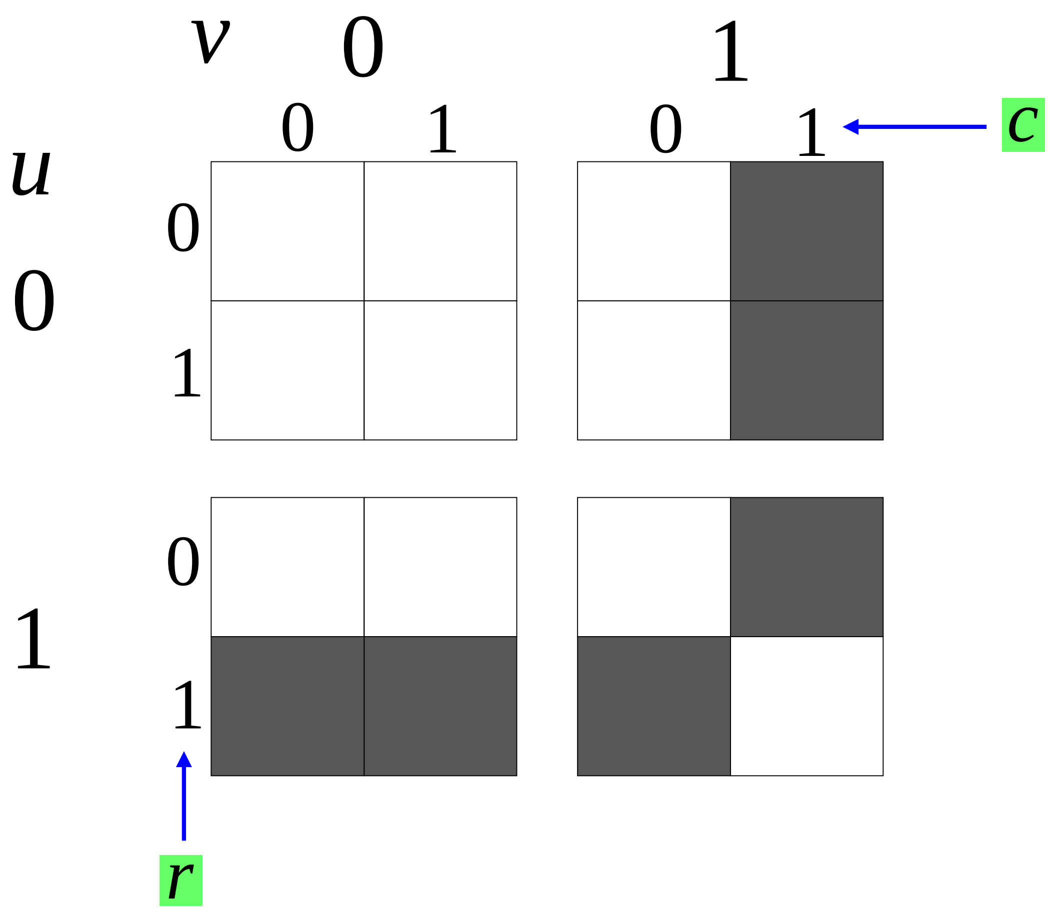
- $\mathbf{T}(u,v)$ is the projections of $\mathbf{I}(r,c)$ onto each $\mathbf{B}(u,v)$
  - Represent similarity of the image to the basis image
    - The more alike they are, the bigger the coefficient
  - $\mathbf{B}(u,v)$'s are orthogonal to each other
  - Image is decomposed into a weighted sum of the basis images, where $\mathbf{T}(u,v)$'s are the weights



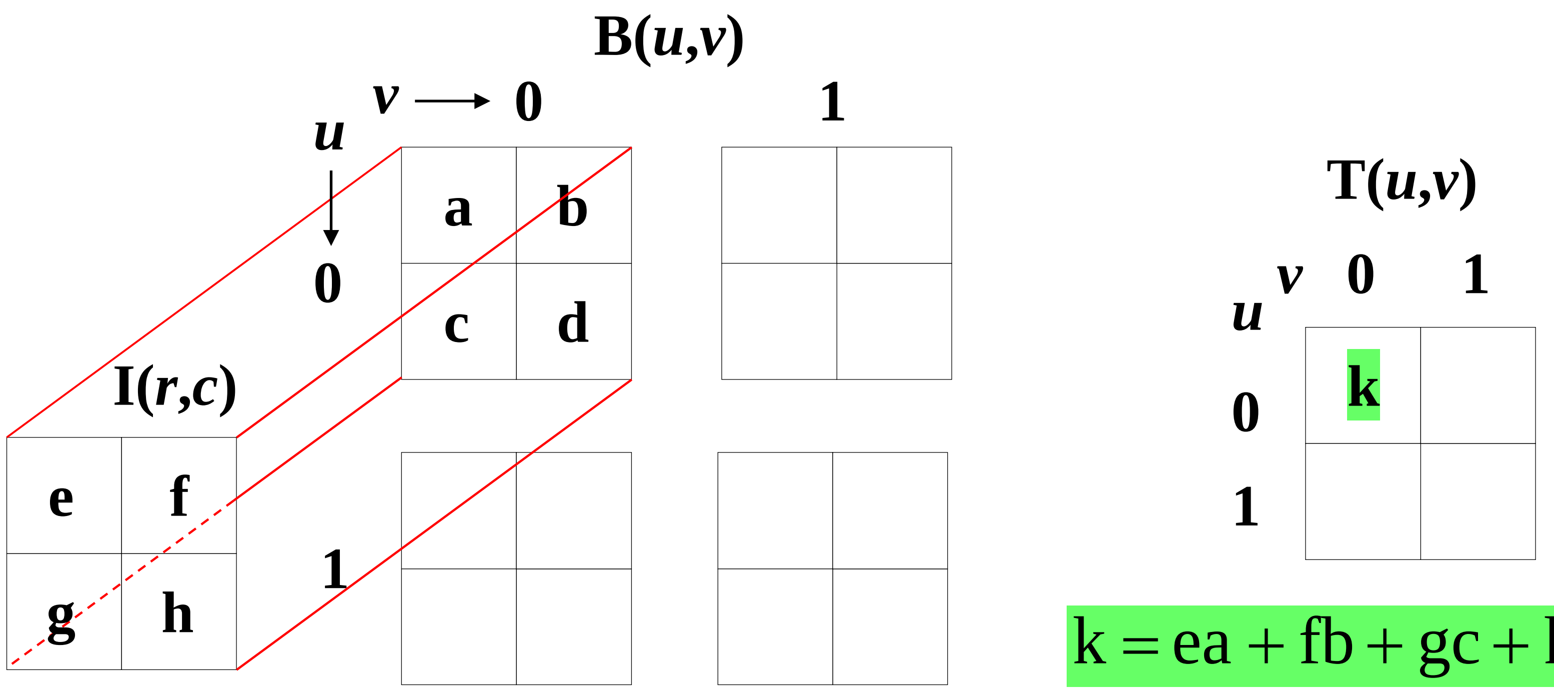
$$\mathbf{p}\cdot\mathbf{i}=\begin{bmatrix}a\\b\end{bmatrix}\cdot\begin{bmatrix}1\\0\end{bmatrix}=a \quad \mathbf{p}\cdot\mathbf{j}=\begin{bmatrix}a\\b\end{bmatrix}\cdot\begin{bmatrix}0\\1\end{bmatrix}=b$$

$$\mathbf{p}=a\,\mathbf{i}+b\,\mathbf{j}=(\mathbf{p}\cdot\mathbf{i})\,\mathbf{i}+(\mathbf{p}\cdot\mathbf{j})\,\mathbf{j}$$

A set of basis vector: $\mathbf{B}(r,c;u,v)$

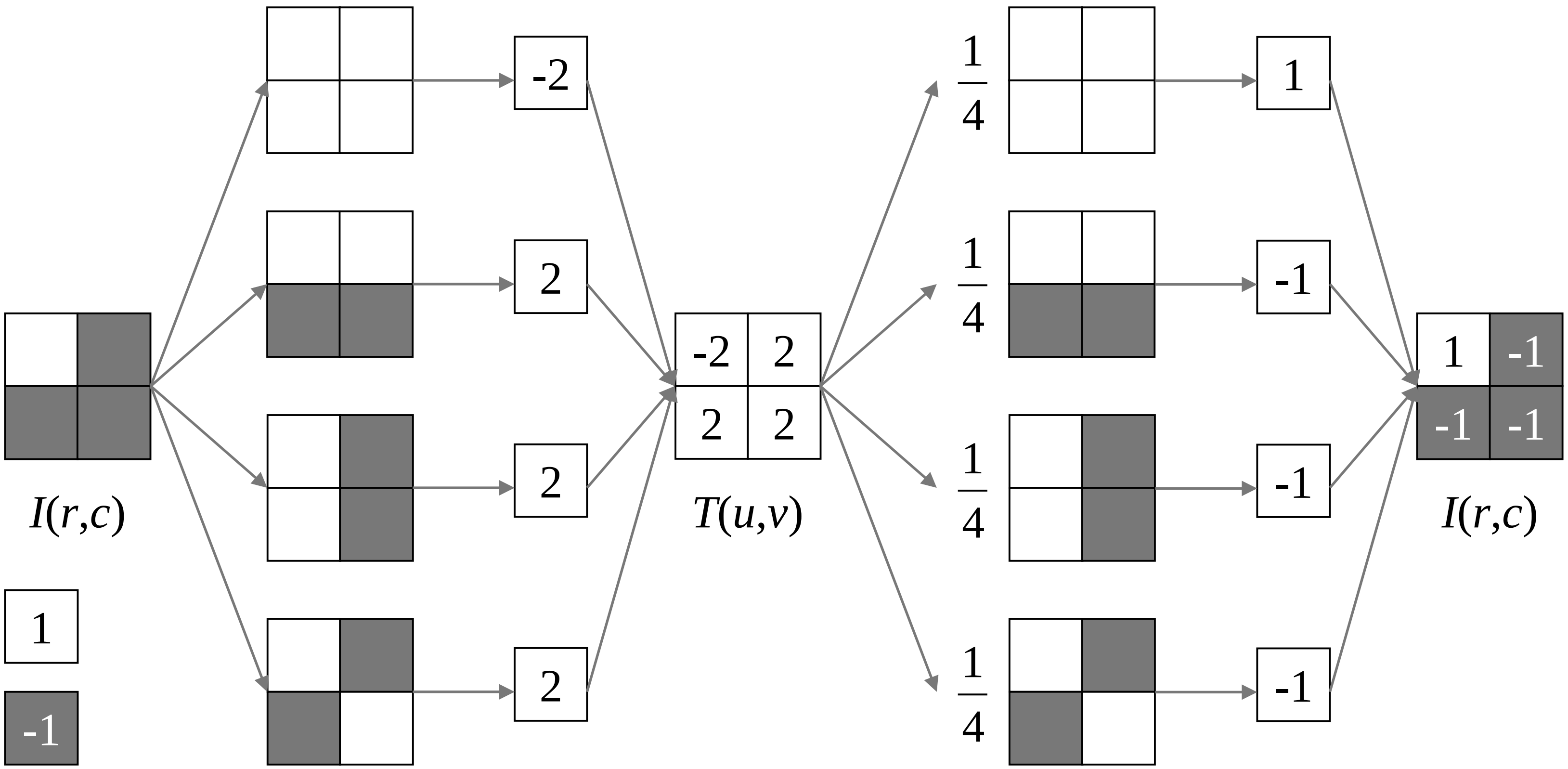


Transform Coefficients



$$k = ea + fb + gc + hd$$

Example



학습 정리

- 변환 (transform)
  - 변환 수식에 의해 주어진 데이터를 다른 공간으로 매핑하는 과정
- 기저 함수 (Basis function)
  - 변환에 사용되는 기반 함수
  - 주로 주파수의 변화 정도를 표현
- 변환 절차 (Process of Transform)
  - 기저 영상에 영상을 투영하여 처리
- 공간주파수
  - 공간에서 영상의 밝기가 변화는 정도를 나타냄
  - zero frequency, low frequency, high frequency

Reference

- Scott E Umbaugh, **Computer Imaging**, CRC Press, 2005
- R. Gonzalez, R. Woods, **Digital Image Processing (2nd Edition),** Prentice Hall, 2002