# Discrete Transforms – 활용

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# 학습 목표

- Cosine transform에 대해 설명할 수 있다.
- Fourier transform에 대해 설명할 수 있다.
- Spectrum의 특성에 대해 설명할 수 있다.
- 주파수 공간에서의 필터링 종류를 구분하여 설명할 수 있다.

# Cosine Transform

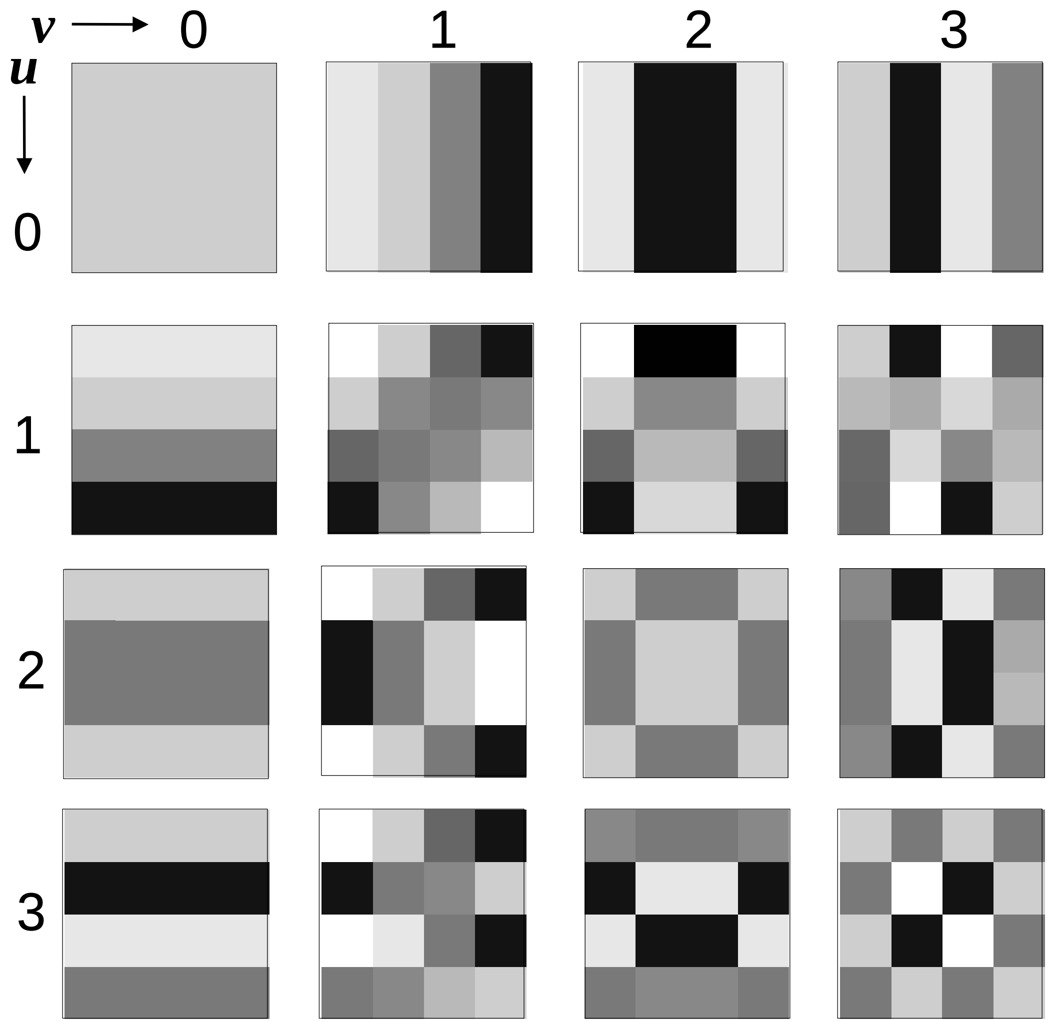
- Use only cosine function
- Use only real arithmetic
- Used in image compression (JPEG, MPEG)
- 2-D Discrete Cosine Transform (DCT)

$$C(u,v) = \alpha(u)\alpha(v)\sum_{r=0}^{N-1}\sum_{c=0}^{N-1} I(r,c)\left[\cos[\frac{(2r+1)u\pi}{2N}]\cos[\frac{(2c+1)v\pi}{2N}]\right]$$

$$\alpha(u),\alpha(v) = \begin{cases} \sqrt{\frac{1}{N}} & for\ u,v=0 \\ \sqrt{\frac{2}{N}} & for\ u,v=1,2,\ldots N-1 \end{cases}$$

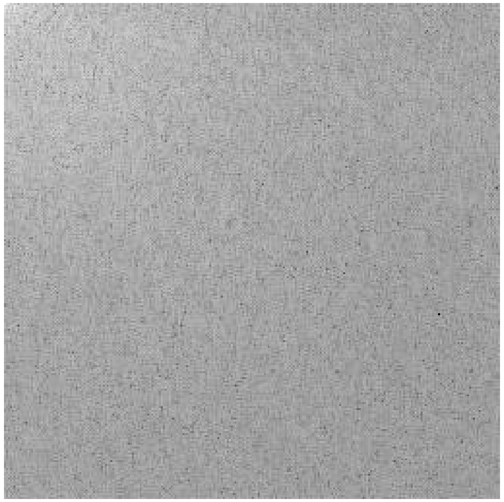
$$I(r,c) = \sum_{u=0}^{N-1}\sum_{v=0}^{N-1} \alpha(u)\alpha(v)C(u,v)\left[\cos[\frac{(2r+1)u\pi}{2N}]\cos[\frac{(2c+1)v\pi}{2N}]\right]$$

# Basis Images for DCT



Original Image

Cosine transform
linearly remapped image



Cosine transform
log remapped image

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# Fourier Transform (1)

- Most well-known and most widely used
- 2-D discrete Fourier transform
  - Decompose an image into a weighted sum of 2-D sinusoidal terms

$$F(u,v) = \frac{1}{N}\sum_{r=0}^{N-1}\sum_{c=0}^{N-1} I(r,c)e^{-j2\pi\frac{(ur+vc)}{N}}$$

- Euler's identity

$$e^{jx} = \cos x + j\sin x$$

$$F(u,v) = \frac{1}{N}\sum_{r=0}^{N-1}\sum_{c=0}^{N-1} I(r,c)\left[\cos\left(\frac{2\pi}{N}(ur+vc)\right) - j\sin\left(\frac{2\pi}{N}(ur+vc)\right)\right]$$

a complex spectral component

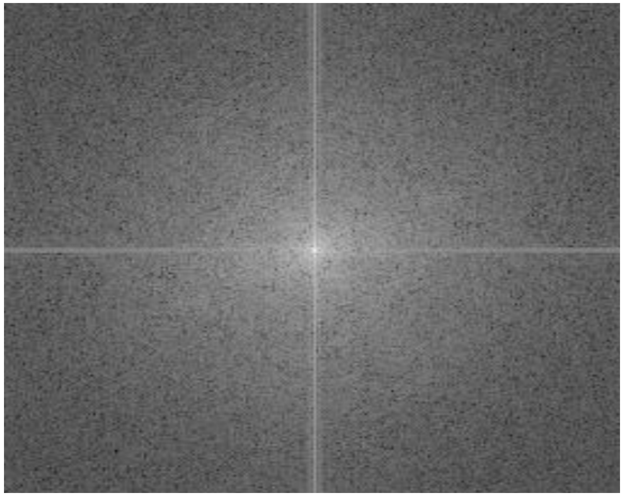
$$\mathrm{F}(u,v) = \mathrm{R}(u,v) + j\mathrm{I}(u,v)$$

Magnitude : related to contrast

$$|F(u,v)| = \sqrt{R^2(u,v) + I^2(u,v)}$$

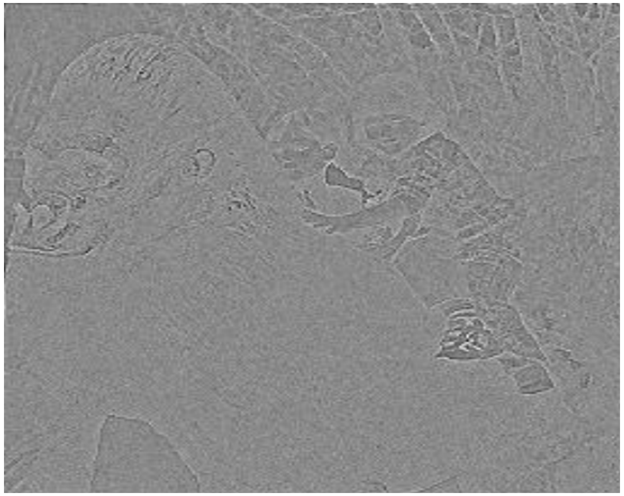
Phase : related to where objects are in an image

$$\phi(u,v) = \tan^{-1}\left[\frac{I(u,v)}{R(u,v)}\right]$$



Fourier Transform

Original Image



Phase-only Image

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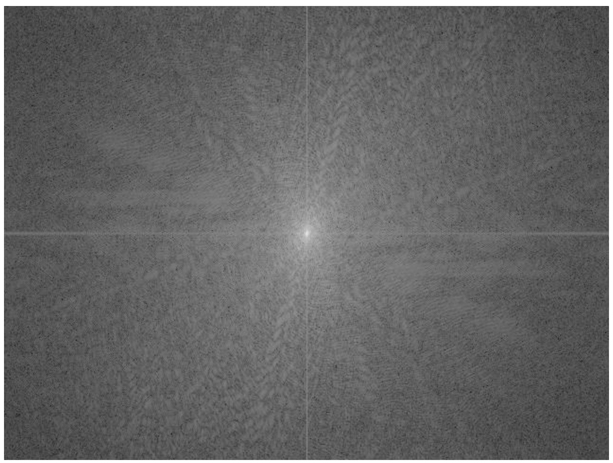
# Fourier Transform (2)

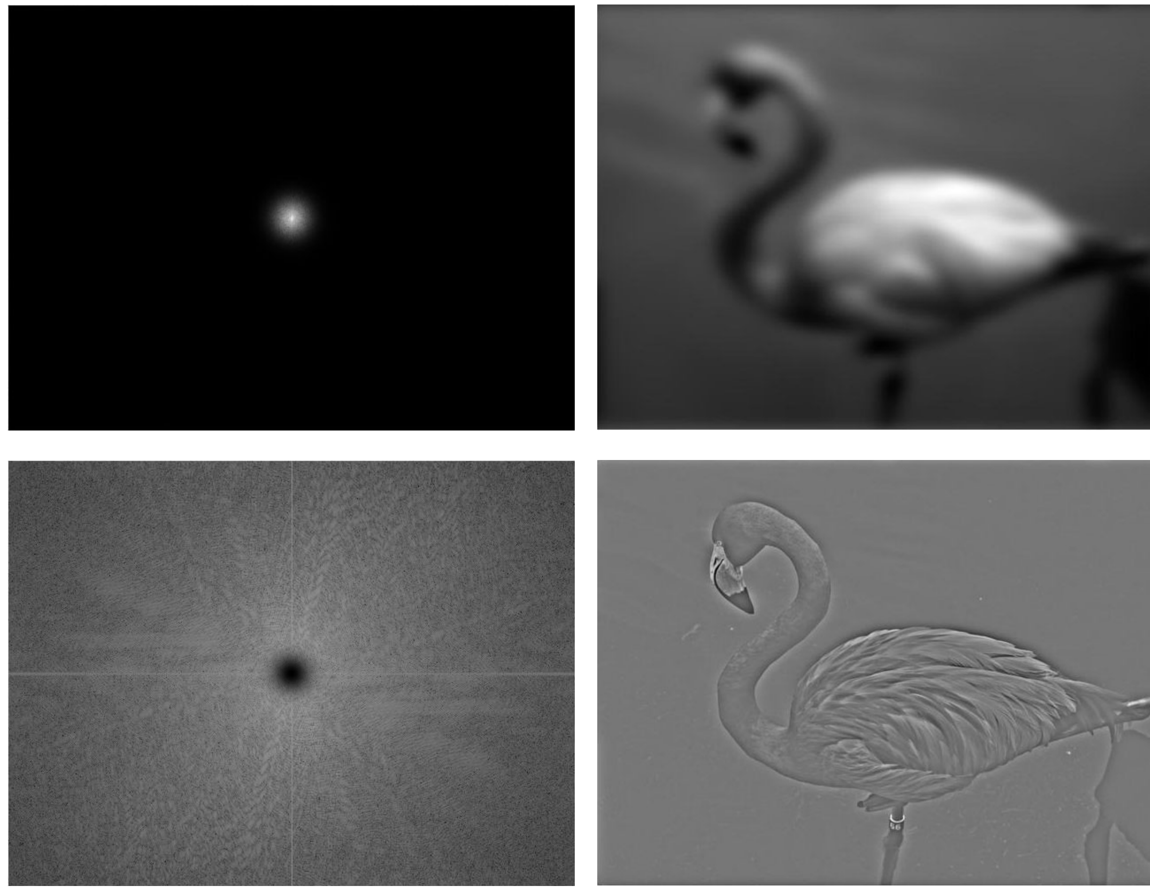
- 2-D discrete Inverse Fourier Transform
  - Get the original image back

$$I(r,c) = \frac{1}{N}\sum_{u=0}^{N-1}\sum_{v=0}^{N-1} F(u,v)e^{j2\pi\frac{(ur+vc)}{N}}$$

- Fourier transform & Inverse Fourier transform
  - basis functions' exponent is changed from -1 to +1
  - Same in the frequency and magnitude of the basis functions



Fourier Transform



# Fourier Transform의 특성

- Separability
  - If two dimensional transform is separable
    - By successive application of two one-dimensional transforms

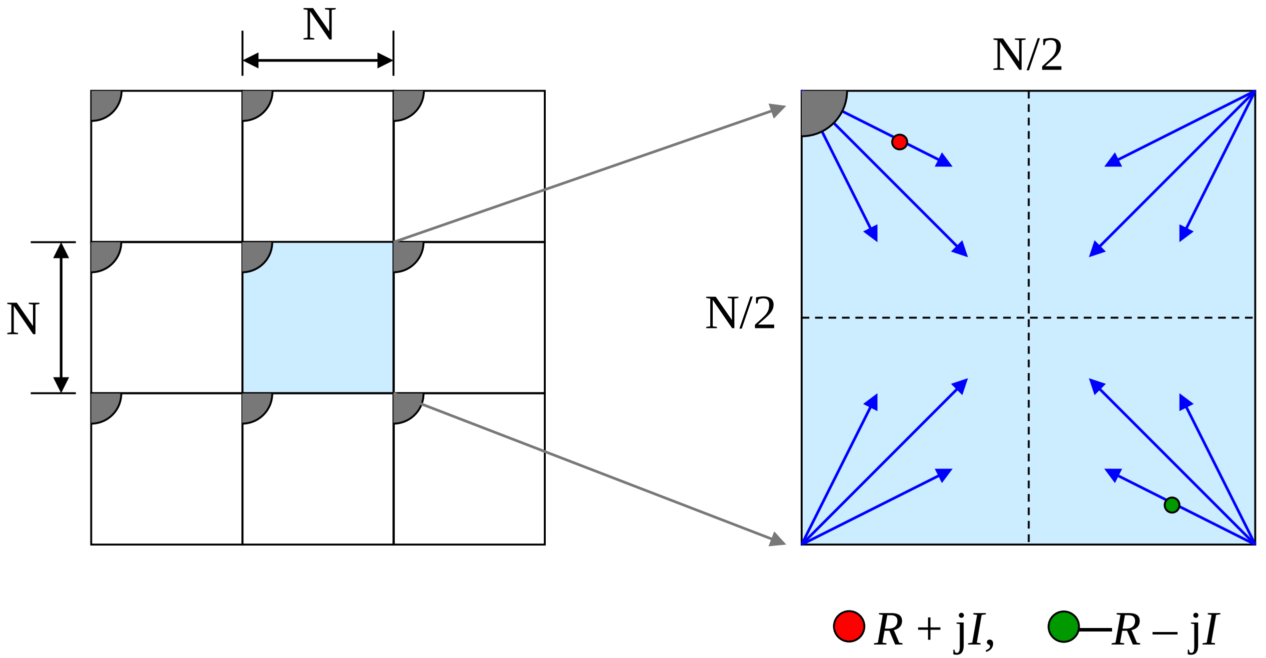
$$F(u,v) = \frac{1}{N}\sum_{r=0}^{N-1}\exp[-j2\pi ur/N]\left(\sum_{c=0}^{N-1} I(r,c)\exp[-j2\pi vc/N]\right)$$

$$F(u,v) = \frac{1}{N}\sum_{r=0}^{N-1} F(r,v)\exp[-j2\pi ur/N]$$

$$F(r,v) = N\left[\frac{1}{N}\sum_{c=0}^{N-1} I(r,c)\exp[-j2\pi vc/N]\right]$$

# Properties of Spectrum

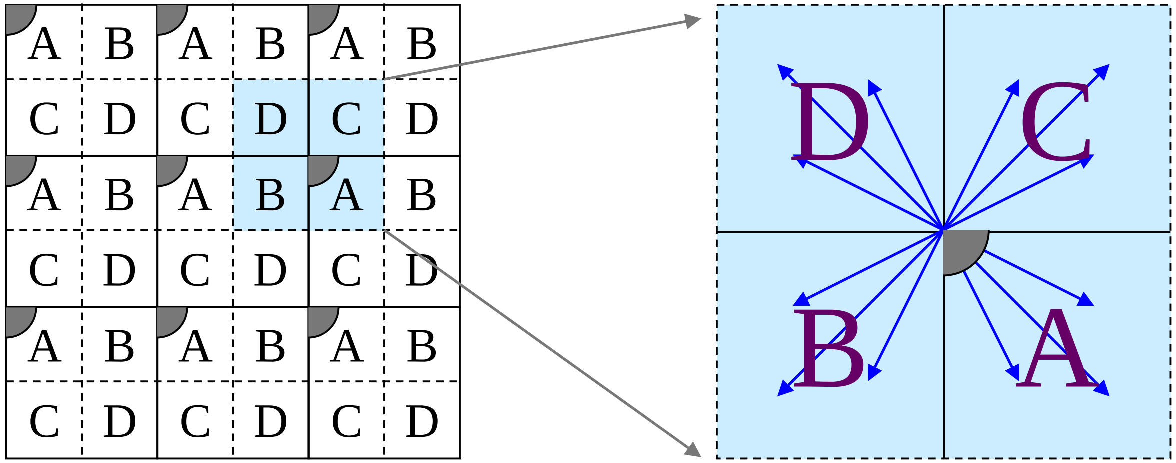
- Fourier Transform: Periodicity and Conjugate Symmetry
  - N×N spectrum is repeated in all directions to infinity
    - F(u,v) = F(u+N, v) = F(u,v+N) = F(u+N, v+N)
  - F(u,v) = F*(-u,-v)

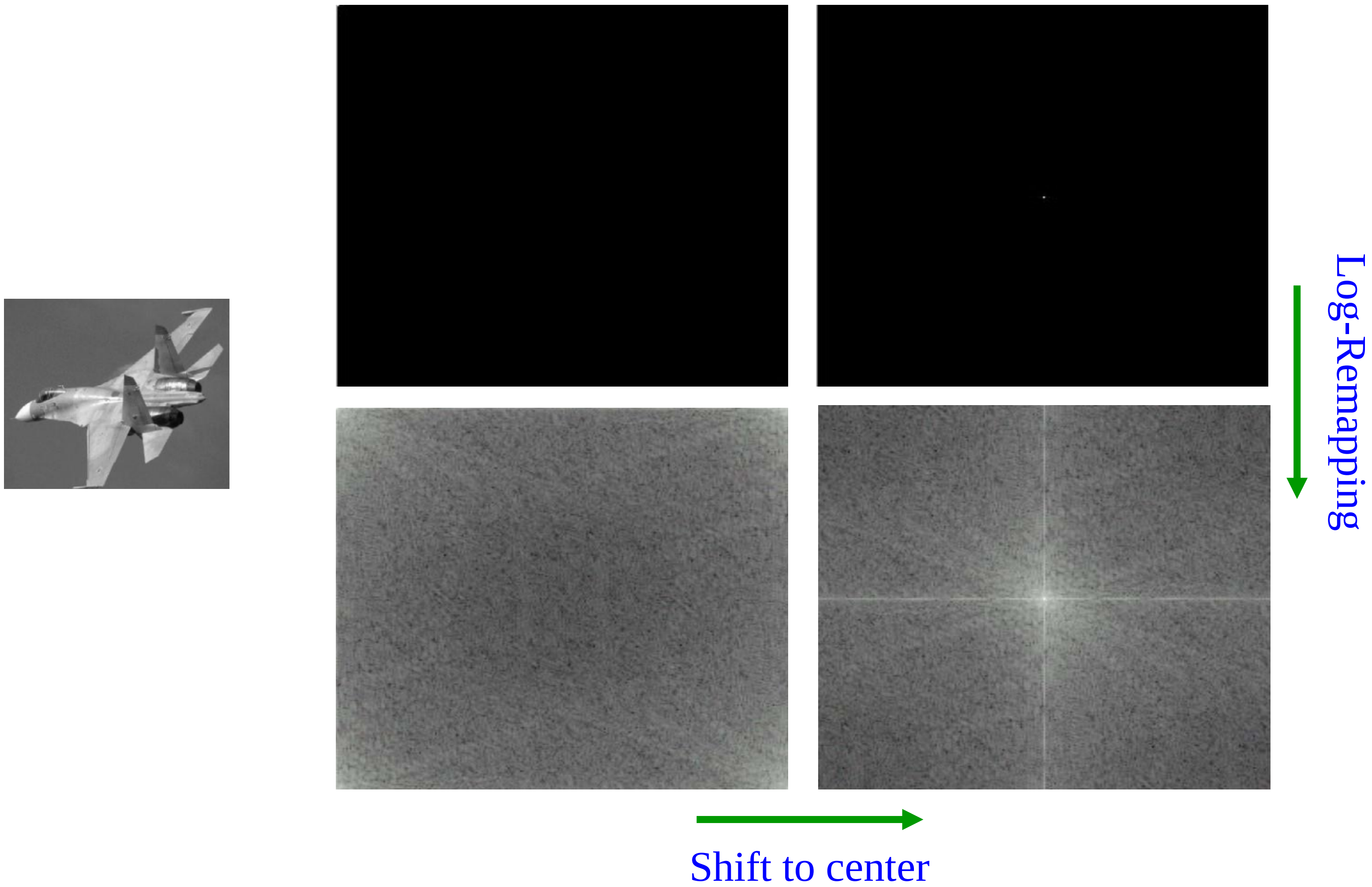


# Properties of Spectrum

- Frequency Translation
  - Shift the origin of the spectrum to the center for display and filtering purpose
- Log transform of spectrum
  - Greatly enhance the visual information in the spectrum

$$\log(u,v) = k \log\left[1 + \left|F(u,v)\right|\right]$$



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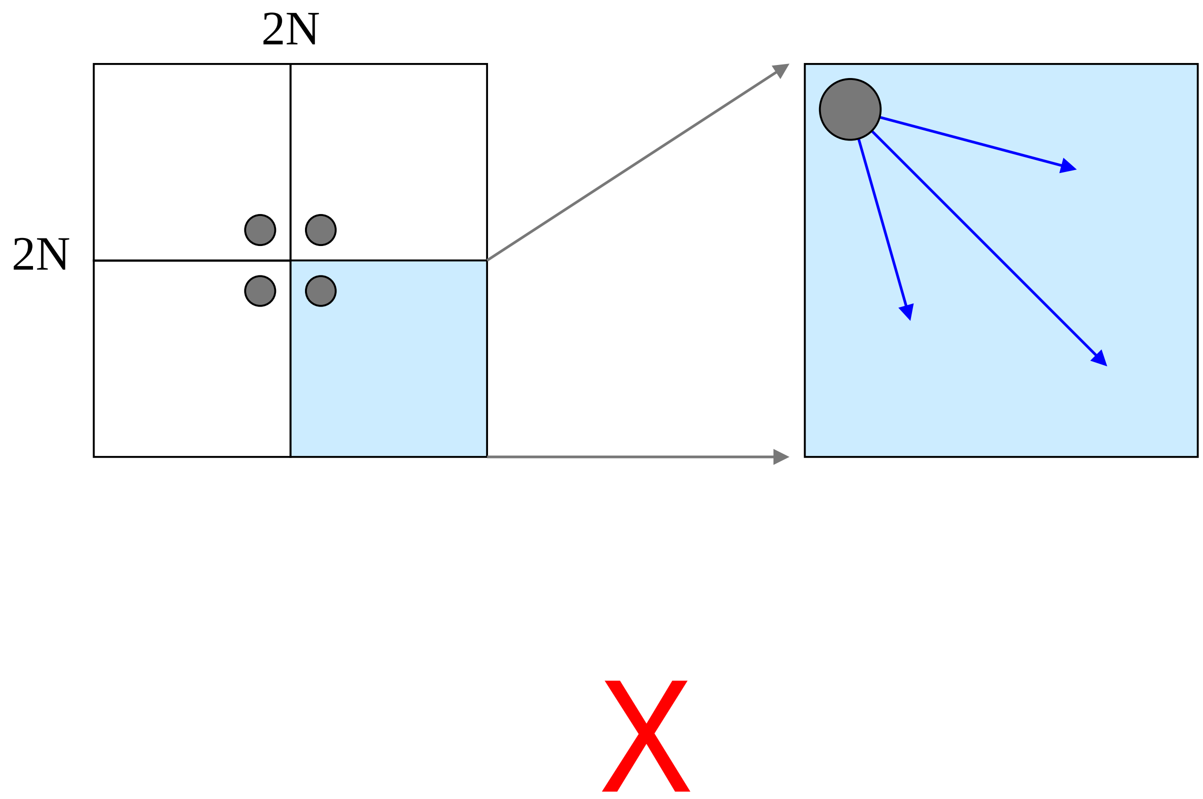
# Properties of Spectrum

- In Cosine Transform,



# Filtering

- Four Types of Filtering
  - Lowpass filtering
    - Remove high-frequency information, blurring an image
  - Highpass filtering
    - Remove low-frequency information, sharpen an image
  - Bandpass filtering
    - Extract frequency information in specific parts
  - Bandreject filtering
    - Eliminate frequency information in specific parts

# Filtering: Lowpass Filtering

- Pass low frequency and eliminate high frequency information ➡ Blur images
- Used for hiding effects caused by noise
- Performed by multiplying the spectrum by a low-pass filter and then applying the inverse transform to obtain the filtered image

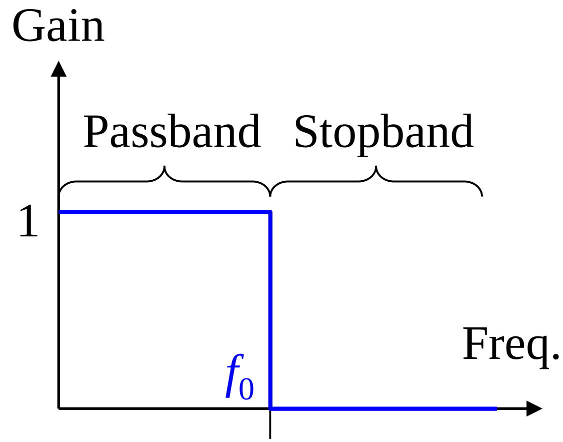
$$\mathbf{ILPF}(r,c) = \mathbf{T}^{-1}[\mathbf{T}(u,v)\mathbf{HLPF}(u,v)]$$

***Convolution Theorem***

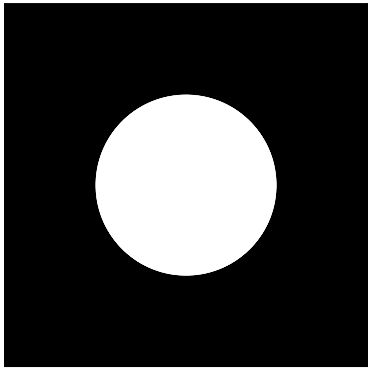
$$G(u,v) = H(u,v)F(u,v)$$
$$g(x,y) = h(x,y) * f(x,y)$$

# Filtering: Lowpass Filtering

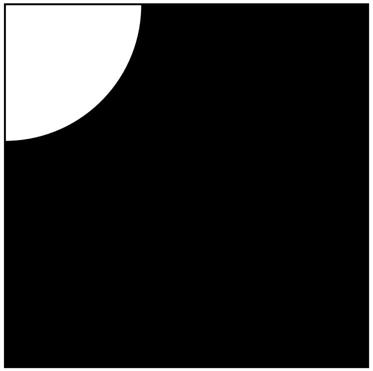
- Some Terminologies
  - Cutoff  frequency, f0
    - the frequency at which we start to eliminate information
  - Passband
    - not filtered out frequency
  - Stopband
    - filtered out frequency
  - Ideal filter
    - leave undesirable artifacts in image
  - Nonideal filter (Butterworth filter)
    - avoid the problem of ideal filter



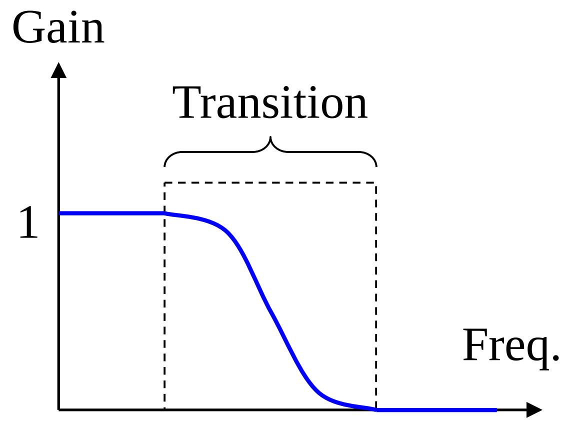

1D Lowpass Ideal Filter



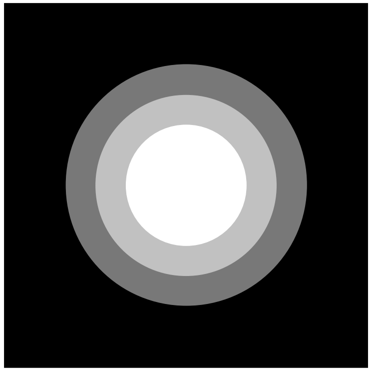
2D LPIF for Fourier spectrum



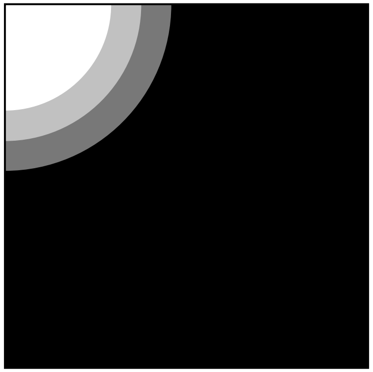
2D LPIF for Cosine



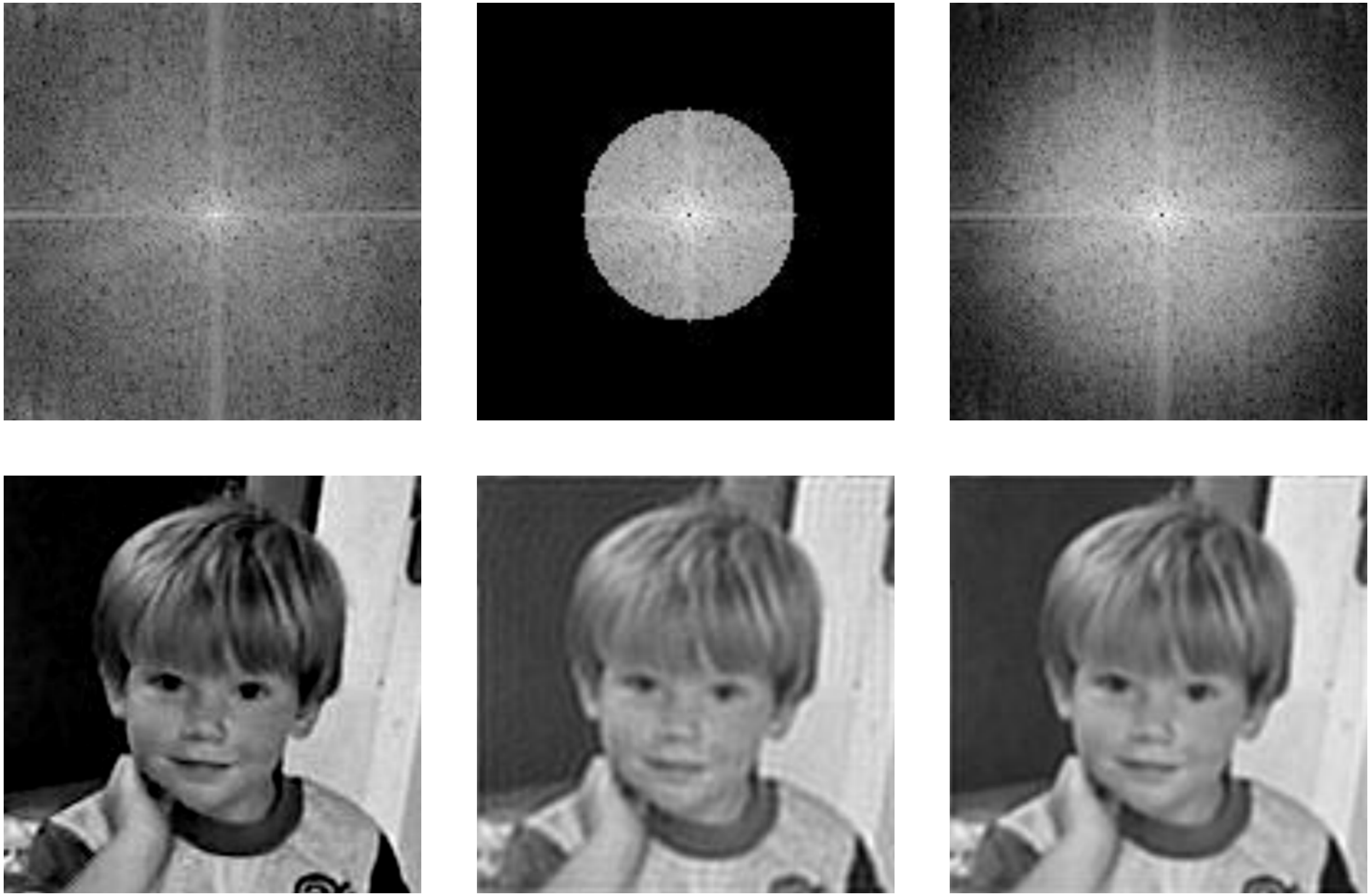

1D Lowpass Nonideal Filter



2D LPNIF for Fourier spectrum



2D LPNIF for Cosine



ripple artifact

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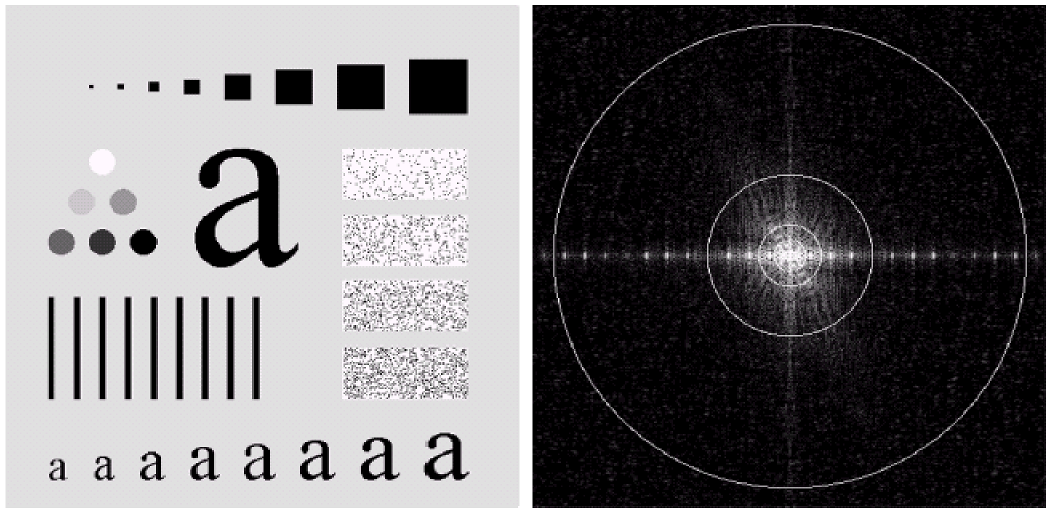


a b

**FIGURE 4.11** (a) An image of size 500 × 500 pixels and (b) its Fourier spectrum. The superimposed circles have radii values of 5, 15, 30, 80, and 230, which enclose 92.0, 94.6, 96.4, 98.0, and 99.5% of the image power, respectively.

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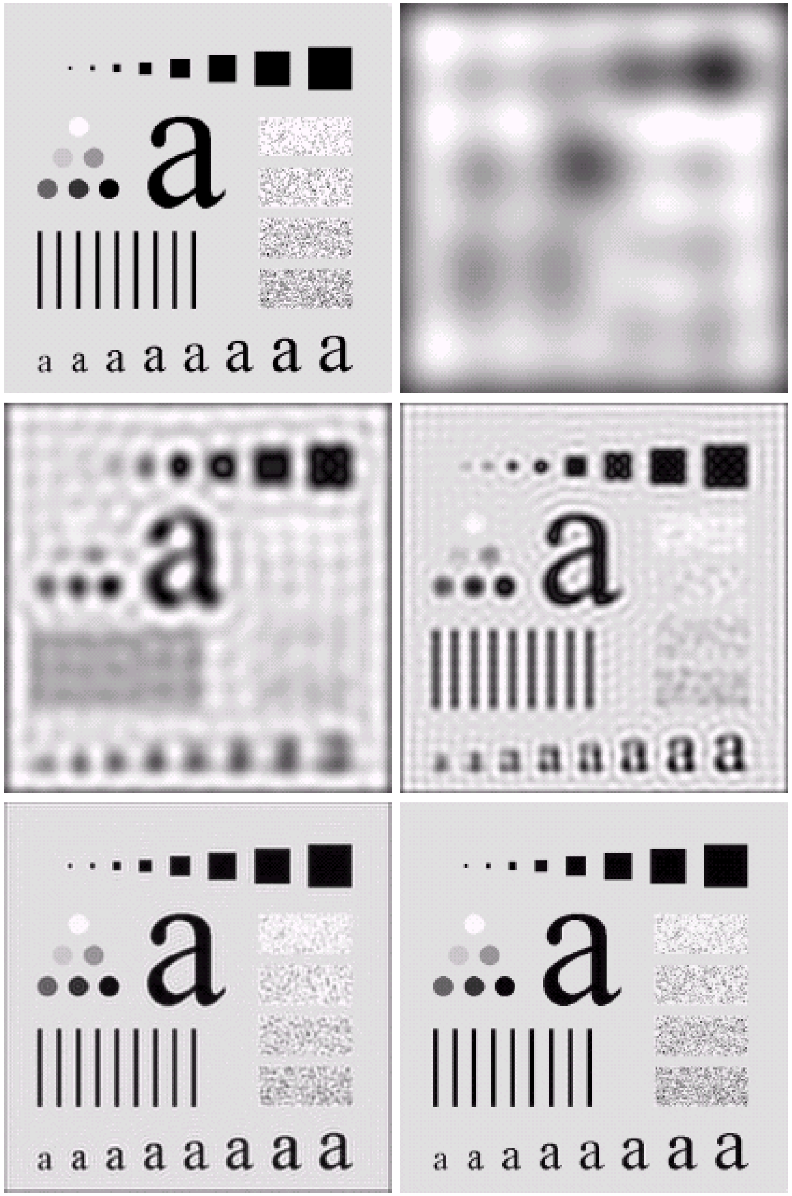

| | |
|---|---|
| a | b |
| c | d |
| e | f |

**FIGURE 4.12** (a) Original image. (b)–(f) Results of ideal lowpass filtering with cutoff frequencies set at radii values of 5, 15, 30, 80, and 230, as shown in Fig. 4.11(b). The power removed by these filters was 8, 5.4, 3.6, 2, and 0.5% of the total, respectively.

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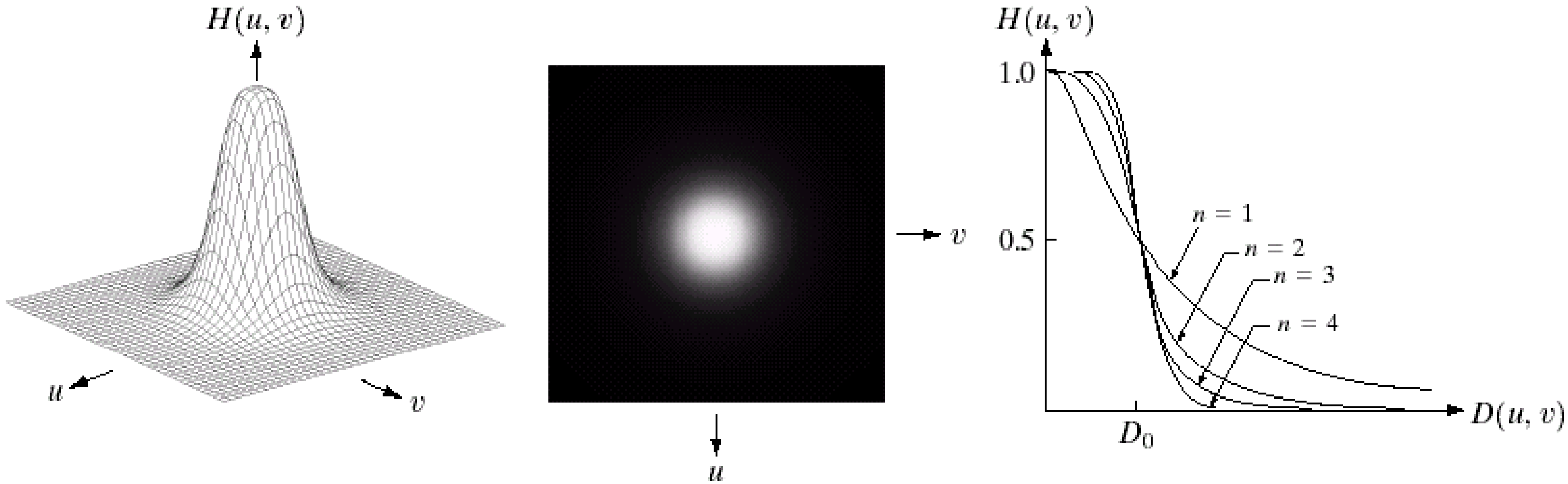


a b c

**FIGURE 4.14** (a) Perspective plot of a Butterworth lowpass filter transfer function. (b) Filter displayed as an image. (c) Filter radial cross sections of orders 1 through 4.

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Filter Order = 1

Filter Order = 3

Filter Order = 6

Filter Order = 8

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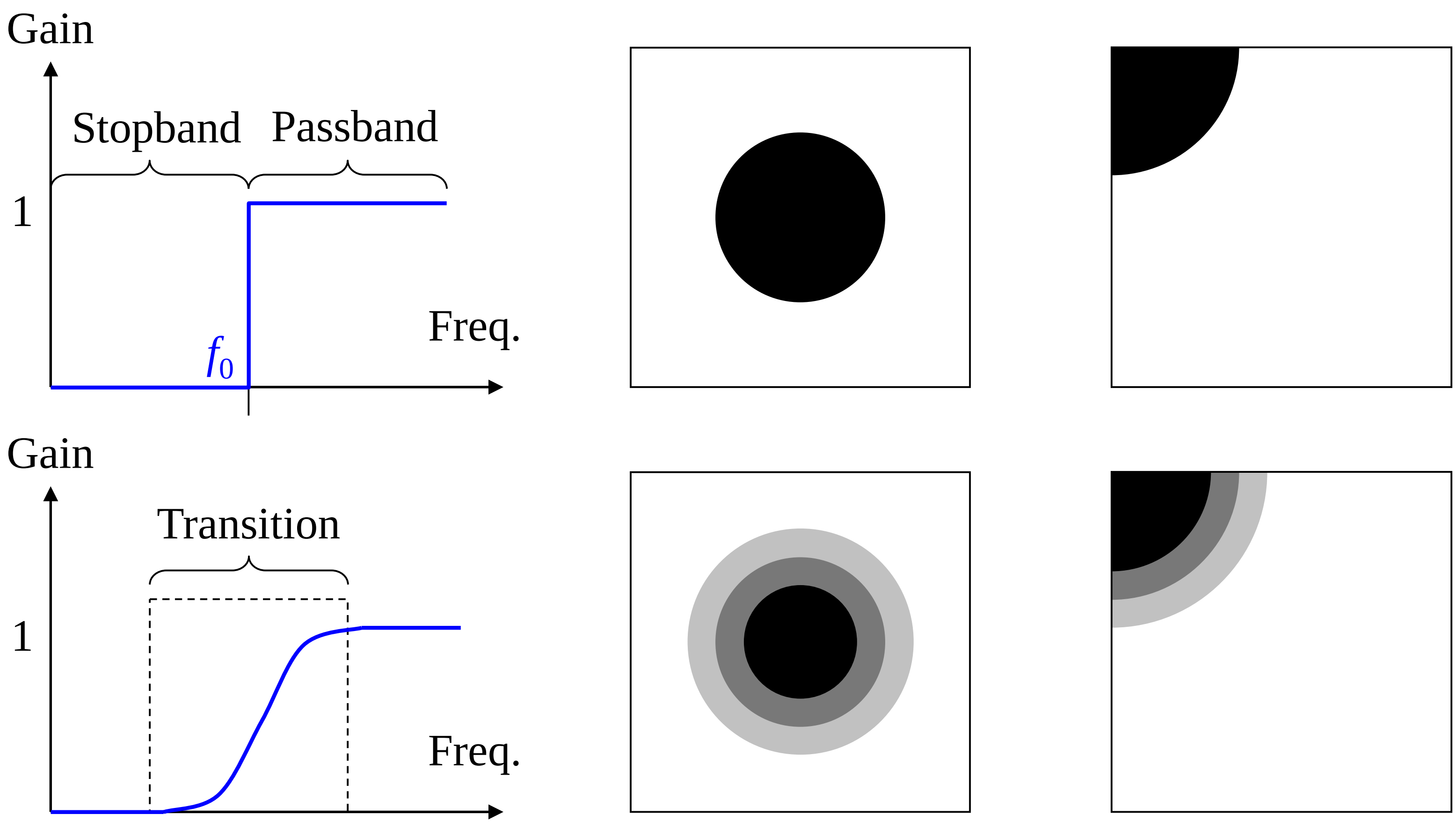
# Filtering: Highpass Filtering

- Pass high frequency information for edge enhancement



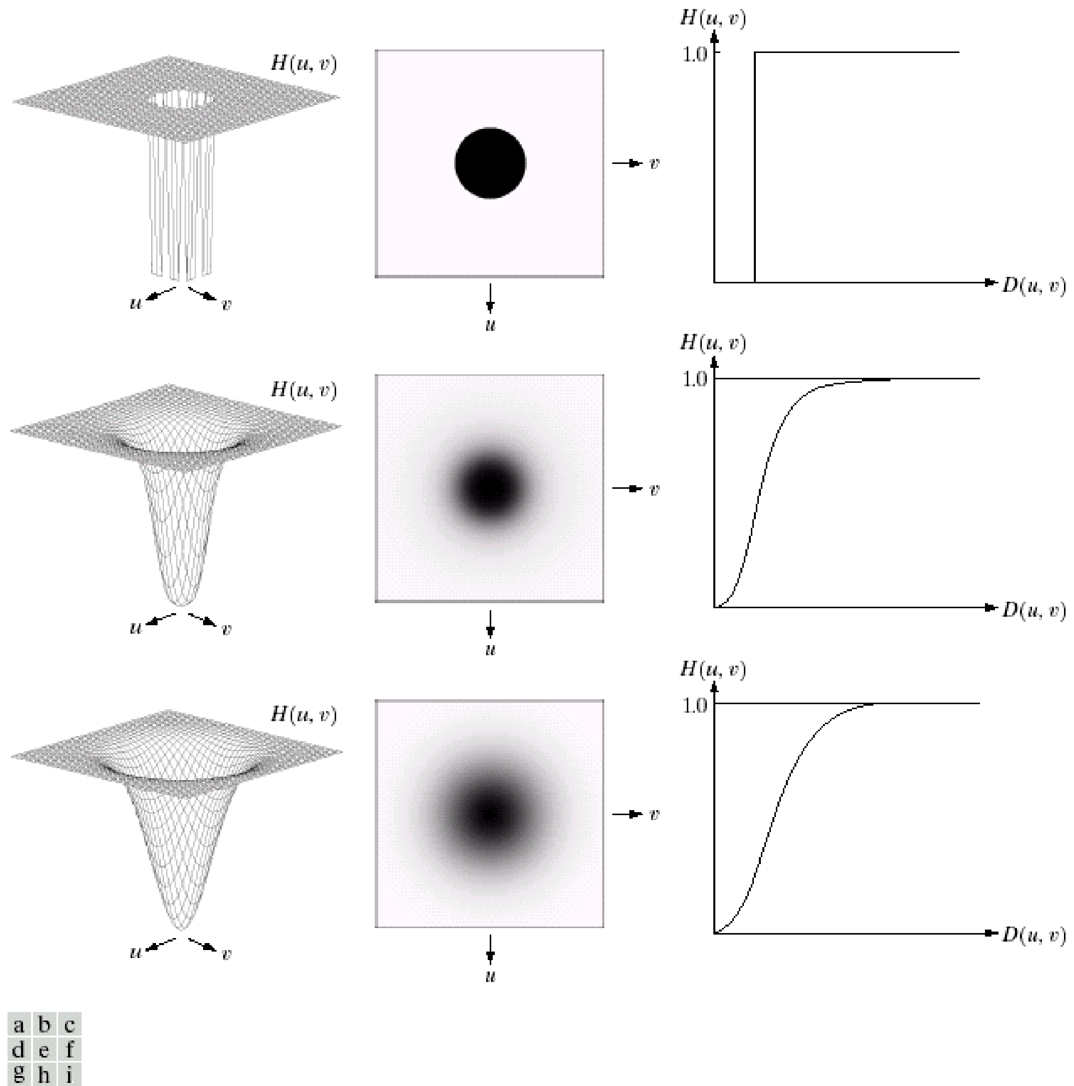



FIGURE 4.22 Top row: Perspective plot, image representation, and cross section of a typical ideal highpass filter. Middle and bottom rows: The same sequence for typical Butterworth and Gaussian highpass filters.

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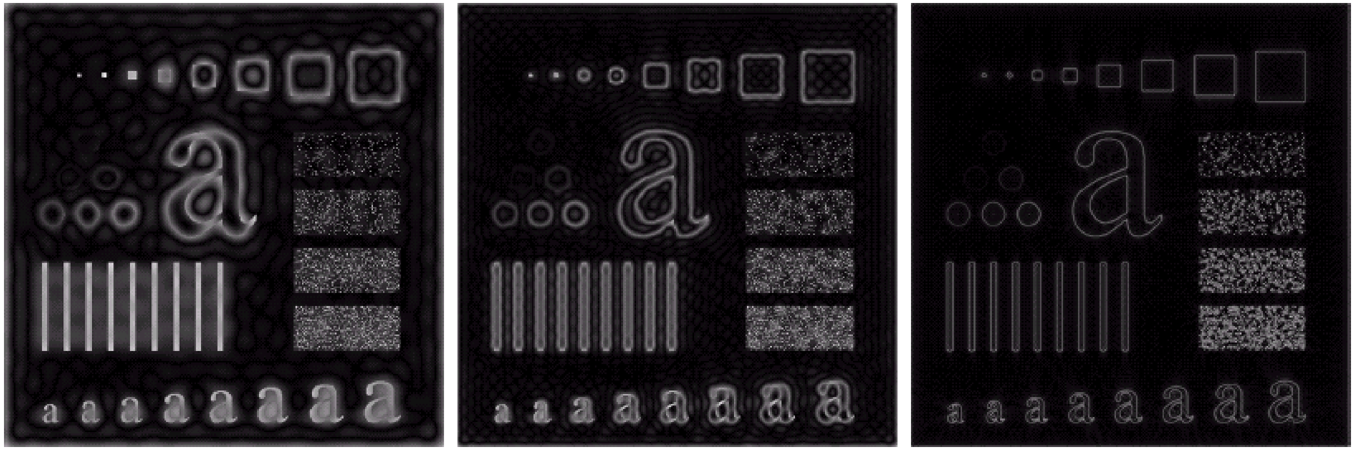
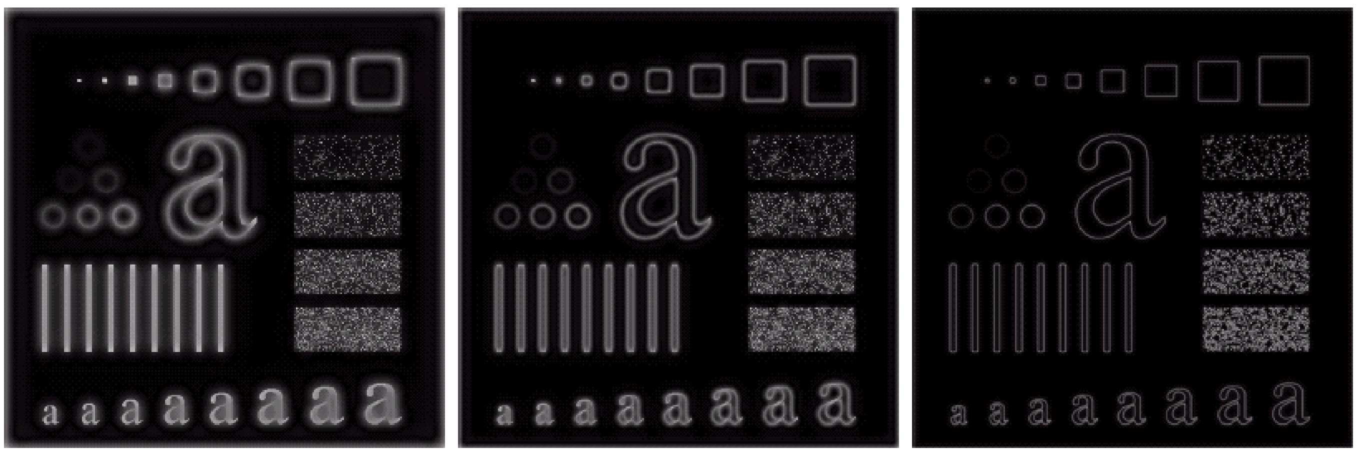


a b c

**FIGURE 4.24** Results of ideal highpass filtering the image in Fig. 4.11(a) with $D_0 = 15$, 30, and 80, respectively. Problems with ringing are quite evident in (a) and (b).

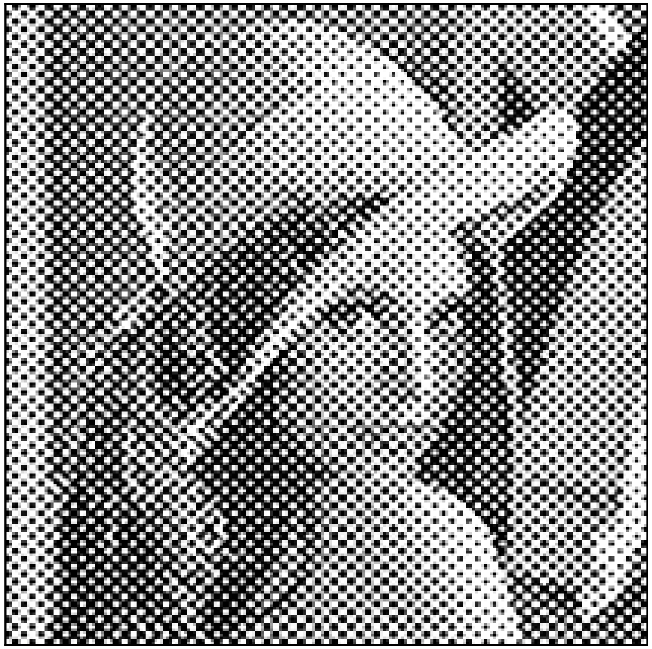


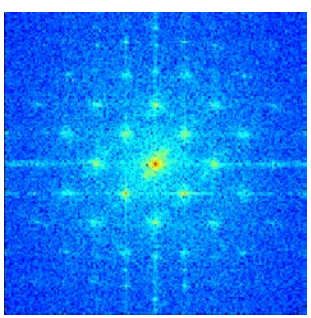
a b c

**FIGURE 4.25** Results of highpass filtering the image in Fig. 4.11(a) using a BHPF of order 2 with $D_0 = 15$, 30, and 80, respectively. These results are much smoother than those obtained with an ILPF.

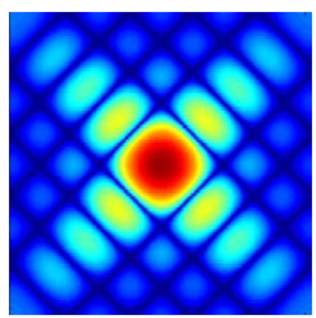
영상은 Digital Image Processing (Prentice Hall)에서 가져왔음

# Inverse Halftoning





X



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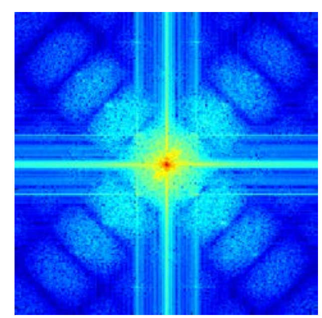

# 학습 정리

- Fourier transform
  - 사인 및 코사인 함수의 조합으로 된 기저 함수를 사용
  - 스펙트럼은 복소수 값으로 표현
  - 신호(영상) 분석에 주로 사용
- Consine transform
  - 코사인 함수를 기저 함수로 사용
  - 스펙트럼은 실수 값으로 표현
  - 영상 압축에 주로 사용
- 스펙트럼의 특성 (Properties of Spectrum)
  - Fourier Transform: Periodicity and Conjugate Symmetry
- Four Types of Filtering
  - Lowpass filtering, Highpass filtering, Bandpass filtering, Bandreject filtering

# Reference


- Scott E Umbaugh, **Computer Imaging**, CRC Press, 2005
- R. Gonzalez, R. Woods, **Digital Image Processing (2nd Edition),** Prentice Hall, 2002