

NCERT 11.9.2 16Q

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Question

Between 1 and 31, m numbers have been inserted in such a way that the resulting sequence is an A.P. and the ratio of 7th and $(m - 1)$ th numbers is 5:9. Find the value of m .

Solution

Symbol	Value	description
$x(0)$	1	First term of A.P
$x(n)$	31	last term
d	2	Common difference
n	$m+2$	number of terms
m	14	number of terms inserted

TABLE 0

$$\begin{pmatrix} 1 & 7 & x(7) \\ 1 & m-1 & x(m-1) \end{pmatrix} \xrightarrow{R_2 \leftarrow R_2 - R_1} \begin{pmatrix} 1 & 7 & x(7) \\ 0 & m-8 & x(m-1) - x_7 \end{pmatrix} \quad (8)$$

$$\xrightarrow{R_2 \leftarrow \frac{1}{m-8} R_2} \begin{pmatrix} 1 & 7 & x(7) \\ 0 & 1 & \frac{x(m-1) - x_7}{m-8} \end{pmatrix} \quad (9)$$

$$\xrightarrow{R_1 \leftarrow R_1 - 7R_2} \begin{pmatrix} 1 & 0 & x_7 - 7 \left(\frac{x(m-1) - x(7)}{m-8} \right) \\ 0 & 1 & \frac{x(m-1) - x(7)}{m-8} \end{pmatrix} \quad (10)$$

$$\Rightarrow \begin{pmatrix} x(0) \\ d \end{pmatrix} = \begin{pmatrix} x_7 - 7 \left(\frac{x(m-1) - x(7)}{m-8} \right) \\ \frac{x(m-1) - x(7)}{m-8} \end{pmatrix} \quad (11)$$

The last term is

part 1
from equation (7)

$$x(n) = x(0) + nd \quad (1)$$

$$31 = 1 + (m+1)d \quad (2)$$

$$30 = (m+1)d \quad (3)$$

$$\frac{30}{m+1} = d \quad (4)$$

$$x(0) = x(7) - 7 \left(\frac{x(m-1) - x(7)}{m-8} \right) \quad (12)$$

$$1 = x(7) - 7 \left(\frac{x(m-1) - x(7)}{m-8} \right) \quad (13)$$

$$1 = x(7) - 7 \left(\frac{x(7) \left(\frac{9}{5} \right) - x(7)}{m-8} \right) \quad (14)$$

$$1 = x(7) \left((m-8) - \frac{28}{5} \right) \quad (15)$$

Now 7th and $(m - 1)$ th terms

$$\Rightarrow x(7) = x(0) + 7d \quad (5)$$

$$\Rightarrow x(m-1) = x(0) + (m-1)d \quad (6)$$

part 2
from equations (4) and (7)

Given

$$\frac{x(7)}{x(m-1)} = \frac{5}{9} \quad (7)$$

$$d = \frac{x(m-1) - x(7)}{m-8} \quad (16)$$

$$\frac{30}{m+1} = \frac{x(7) \left(\frac{4}{9} \right)}{m-8} \quad (17)$$

$$x(7) = \frac{75(m-8)}{2(m+1)} \quad (18)$$

From equations (5) and (6) the augmented matrix is:

$$(19)$$

Substituting $x(7)$ in part 1

$$m - 8 = \frac{75(m - 8)(5m - 68)}{10(m + 1)} \quad (20)$$

$$2(m + 1) = 15(5m - 68) \quad (21)$$

$$2m + 2 = 75m - 1020 \quad (22)$$

$$73m = 1022 \quad (23)$$

$$m = 14 \quad (24)$$

General term of AP as

$$x(n) = 2n - 1 \quad (25)$$

The result is,

$$X(z) = 2S_{\infty} - U(z) \quad (31)$$

$$X(z) = \frac{z^2}{(z - 1)^2} - \frac{1}{1 - z^{-1}} \quad (32)$$

$$X(z) = \frac{z^2 + z}{(z - 1)^2} \quad (33)$$

(ROC) $|z| > 1$.

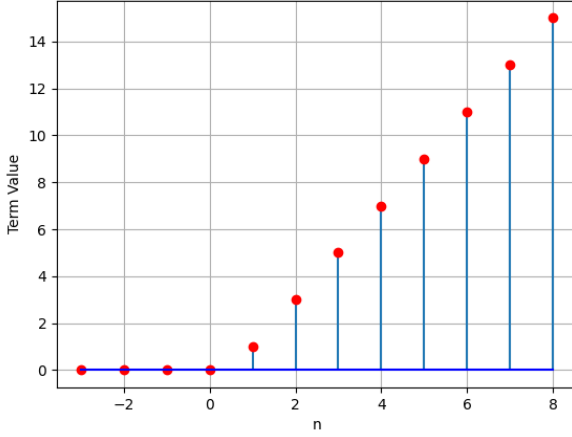


Fig. 0. Plot of $x(n)$ vs n

The Z-Transform Equation for $x(n)$ is

$$X(z) = \sum_{n=-\infty}^{\infty} (2n - 1) z^{-n} u(n) \quad (26)$$

$$\Rightarrow X(z) = \sum_{n=-\infty}^{\infty} (2n) z^{-n} u(n) - \sum_{n=-\infty}^{\infty} z^{-n} u(n) \quad (27)$$

$$\Rightarrow X(z) = 2 \sum_{n=0}^{\infty} \frac{n}{z^n} - U(z) \quad (28)$$

The first part of summation is

$$\Rightarrow S(\infty) = \frac{z^2}{(z - 1)^2} \quad (29)$$

The second part of summation is

$$U(z) = \frac{1}{1 - z^{-1}} \quad (30)$$