GATE GE 81Q

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Question

The value of the convolution of $f(x) = 3\cos(2x)$ and $g(x) = \frac{1}{3}\sin(2x)$ where $x \in [0, 2\pi)$, at $x = \frac{\pi}{3}$, is (Rounded off to 2 decimal places)

Solution

The Fourier transform of given functions is

$$F(\omega) = \mathcal{F}[f(x)] = 3\pi[\delta(\omega - 2\pi) + \delta(\omega + 2\pi)] \tag{1}$$

$$G(\omega) = \mathcal{F}[g(x)] = \frac{\pi}{3}[\delta(\omega - 2\pi) - \delta(\omega + 2\pi)] \tag{2}$$

Multiply the $F(\omega)$ and $G(\omega)$

$$H(\omega) = F(\omega) \cdot G(\omega) \tag{3}$$

$$= 3\pi \cdot \frac{\pi}{3}[\delta(\omega - 2\pi) + \delta(\omega + 2\pi)] \cdot [\delta(\omega - 2\pi) - \delta(\omega + 2\pi)] \tag{4}$$

$$= \pi^2[\delta(\omega - 2\pi) \cdot \delta(\omega - 2\pi) - \delta(\omega - 2\pi) \cdot \delta(\omega + 2\pi) \tag{5}$$

$$+ \delta(\omega + 2\pi) \cdot \delta(\omega - 2\pi) - \delta(\omega + 2\pi) \cdot \delta(\omega + 2\pi)] \tag{6}$$

$$= \pi^2[(\delta(\omega - 2\pi))^2 - (\delta(\omega + 2\pi))^2] \tag{7}$$

$$= 0 \tag{8}$$

Take the inverse Fourier Transform

$$h(x) = \mathcal{F}^{-1}[H(\omega)] = 0 \tag{9}$$

So, the convolution of $f(x)$ and $g(x)$ at $x = \frac{\pi}{3}$ is 0.

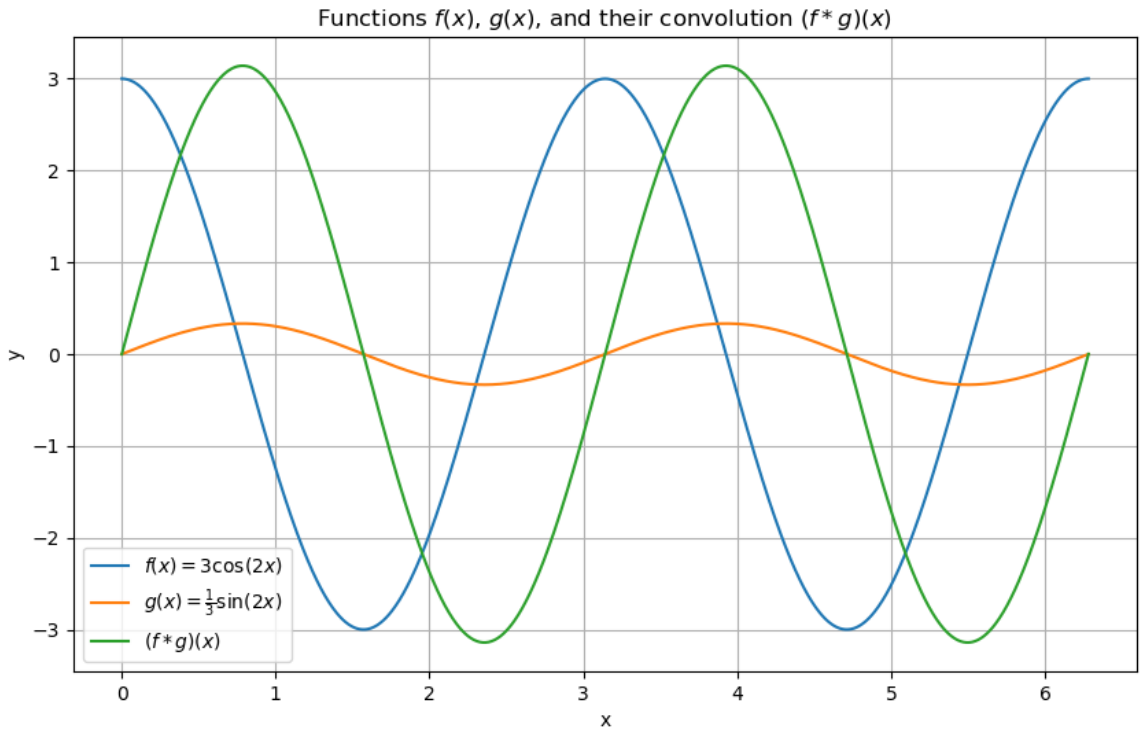

Fig. 0. Plot of y vs x