

NCERT 11.9.2 16Q

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Question

Between 1 and 31, m numbers have been inserted in such a way that the resulting sequence is an A.P. and the ratio of 7th and $(m - 1)$ th numbers is 5:9. Find the value of m .

Solution

Symbol	Value	description
$x(0)$	1	First term of A.P
$x(n)$	31	last term
$\frac{x(7)}{x(m-1)}$	$\frac{5}{9}$	ratio of 7th and $(m-1)$ th numbers
n	$m+2$	number of terms
m	14	number of terms inserted

TABLE 0

The last term is

$$x(n) = x(0) + nd \quad (1)$$

$$\Rightarrow 31 = 1 + (m + 1)d \quad (2)$$

$$\Rightarrow 30 = (m + 1)d \quad (3)$$

$$\Rightarrow \frac{30}{m + 1} = d \quad (4)$$

Now 7th and $(m - 1)$ th terms

$$x(7) = x(0) + 7d \quad (5)$$

$$x(m - 1) = x(0) + (m - 1)d \quad (6)$$

From equations (5) and (6) the augmented matrix is:

$$\left(\begin{array}{ccc} 1 & 7 & x(7) \\ 1 & m - 1 & x(m - 1) \end{array} \right) \xrightarrow{R_2 \leftarrow R_2 - R_1} \left(\begin{array}{ccc} 1 & 7 & x(7) \\ 0 & m - 8 & x(m - 1) - x(7) \end{array} \right) \quad (7)$$

$$\xrightarrow{R_2 \leftarrow \frac{1}{m-8} R_2} \left(\begin{array}{ccc} 1 & 7 & x(7) \\ 0 & 1 & \frac{x(m-1)-x(7)}{m-8} \end{array} \right) \quad (8)$$

$$\xrightarrow{R_1 \leftarrow R_1 - 7R_2} \left(\begin{array}{ccc} 1 & 0 & x(7) - 7\left(\frac{x(m-1)-x(7)}{m-8}\right) \\ 0 & 1 & \frac{x(m-1)-x(7)}{m-8} \end{array} \right) \quad (9)$$

$$\Rightarrow \left(\begin{array}{c} x(0) \\ d \end{array} \right) = \left(\begin{array}{c} x(7) - 7\left(\frac{x(m-1)-x(7)}{m-8}\right) \\ \frac{x(m-1)-x(7)}{m-8} \end{array} \right) \quad (10)$$

part 1

From the table

$$x(0) = x(7) - 7 \left(\frac{x(m-1) - x(7)}{m-8} \right) \quad (11)$$

$$\Rightarrow 1 = x(7) - 7 \left(\frac{x(m-1) - x(7)}{m-8} \right) \quad (12)$$

$$\Rightarrow 1 = x(7) - 7 \left(\frac{x(7) \left(\frac{9}{5} \right) - x(7)}{m-8} \right) \quad (13)$$

$$\Rightarrow 1 = x(7) \left((m-8) - \frac{28}{5} \right) \quad (14)$$

part 2

from equations (4) and from table

$$d = \frac{x(m-1) - x(7)}{m-8} \quad (15)$$

$$\Rightarrow \frac{30}{m+1} = \frac{x(7) \left(\frac{4}{9} \right)}{m-8} \quad (16)$$

$$\Rightarrow x(7) = \frac{75(m-8)}{2(m+1)} \quad (17)$$

Substituting (17) in (14)

$$m-8 = \frac{75(m-8)(5m-68)}{10(m+1)} \quad (18)$$

$$\Rightarrow 2(m+1) = 15(5m-68) \quad (19)$$

$$\Rightarrow 2m+2 = 75m-1020 \quad (20)$$

$$\Rightarrow 73m = 1022 \quad (21)$$

$$\Rightarrow m = 14 \quad (22)$$

General term of AP is

$$x(n) = (2n+1)u(n) \quad (23)$$

The Z-Transform Equation for $x(n)$ is

$$X(z) = \sum_{n=-\infty}^{\infty} (2n+1)z^{-n}u(n) \quad (24)$$

$$= \sum_{n=-\infty}^{\infty} (2n)z^{-n}u(n) + \sum_{n=-\infty}^{\infty} z^{-n}u(n) \quad (25)$$

$$X(z) = 2 \sum_{n=0}^{\infty} \frac{n}{z^n} + U(z) \quad (26)$$

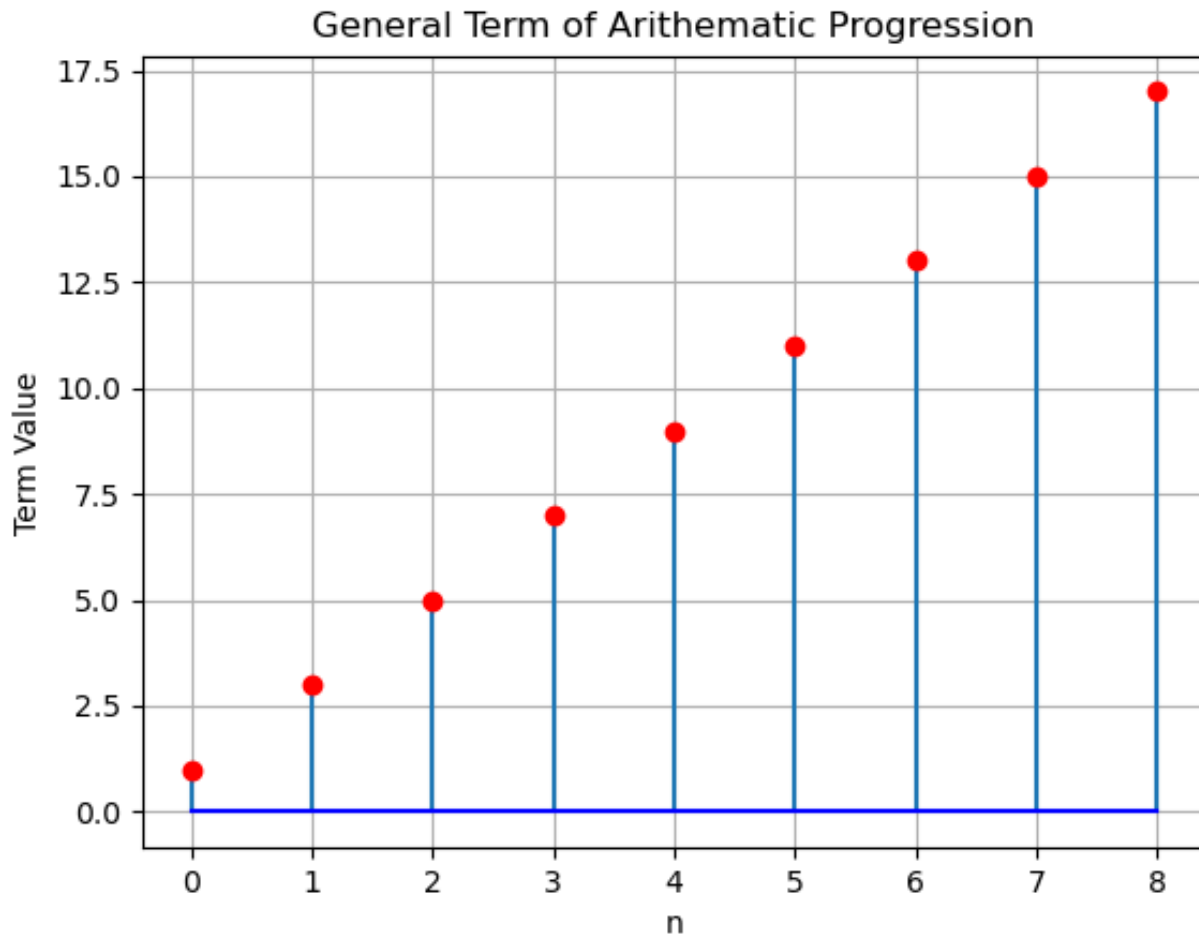


Fig. 0. Plot of $x(n)$ vs n

The first part of summation is

$$S(\infty) = \sum_{n=0}^{\infty} \frac{n}{z^n} \quad (27)$$

$$S(\infty) = \frac{z^2}{(z-1)^2} \quad (28)$$

The second part of summation is

$$U(z) = \frac{1}{1-z^{-1}} \quad (29)$$

The result is,

$$X(z) = 2S_{\infty} + U(z) \quad (30)$$

$$= \frac{2z^2}{(z-1)^2} + \frac{1}{1-z^{-1}} \quad (31)$$

$$X(z) = \frac{3z^2 - z}{(z-1)^2} \quad |z| > 1 \quad (32)$$