

GATE GE 81Q

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Question

The value of the convolution of $f(x) = 3 \cos(2x)$ and $g(x) = \frac{1}{3} \sin(2x)$ where $x \in [0, 2\pi)$, at $x = \frac{\pi}{3}$, is (Rounded off to 2 decimal places)

Solution

The convolution integral:

$$(f * g)(x) = \int_0^{2\pi} f(t)g(x-t) dt \quad (1)$$

Substitute the $f(x)$ and $g(x)$

$$(f * g)(x) = \int_0^{2\pi} 3 \cos(2t) \cdot \frac{1}{3} \sin(2(x-t)) dt \quad (2)$$

Substitute $x = \frac{\pi}{3}$:

$$(f * g)\left(\frac{\pi}{3}\right) = \int_0^{2\pi} 3 \cos(2t) \cdot \frac{1}{3} \sin\left(2\left(\frac{\pi}{3} - t\right)\right) dt \quad (3)$$

$$\text{by using identity } \sin(\alpha - \beta) = \sin \alpha \cos \beta - \cos \alpha \sin \beta \quad (4)$$

$$(5)$$

then we have

$$(6)$$

$$= \int_0^{2\pi} \left(\cos(2t) \cdot \sin \frac{\pi}{3} \cos t - \cos(2t) \cos \frac{\pi}{3} \cdot \sin(t) \right) dt \quad (7)$$

$$= \frac{1}{2} \int_0^{2\pi} \left(\sqrt{3} \cos(2t) \cdot \cos t - \cos(2t) \cdot \sin(t) \right) dt \quad (8)$$

$$= \int_0^{2\pi} \left(\frac{\sqrt{3}}{2} \cos(2t) \cdot \cos t \right) dt - \int_0^{2\pi} \left(\frac{1}{2} \cos(2t) \cdot \sin(t) \right) dt \quad (9)$$

$$= \int_0^{2\pi} \left(\frac{\sqrt{3}}{2} \cdot \frac{1}{2} [\cos(3t) + \cos(t)] \right) dt - \int_0^{2\pi} \left(\frac{1}{2} \cdot \frac{1}{2} [\sin(3t) - \sin(t)] \right) dt \quad (10)$$

$$= \frac{\sqrt{3}}{4} \int_0^{2\pi} \cos(3t) dt + \frac{\sqrt{3}}{4} \int_0^{2\pi} \cos(t) dt - \frac{1}{4} \int_0^{2\pi} \sin(3t) dt + \frac{1}{4} \int_0^{2\pi} \sin(t) dt \quad (11)$$

$$= \frac{\sqrt{3}}{4} \left[\frac{\sin(3t)}{3} \right]_0^{2\pi} + \frac{\sqrt{3}}{4} [\sin(t)]_0^{2\pi} + \frac{1}{4} \left[\frac{\cos(3t)}{3} \right]_0^{2\pi} - \frac{1}{4} [\cos(t)]_0^{2\pi} \quad (12)$$

$$= \frac{\sqrt{3}}{4} \left(\frac{\sin(6\pi)}{3} - \frac{\sin(0)}{3} \right) + \frac{\sqrt{3}}{4} (\sin(2\pi) - \sin(0)) + \frac{1}{4} \left(\frac{\cos(6\pi)}{3} - \frac{\cos(0)}{3} \right) - \frac{1}{4} (\cos(2\pi) - \cos(0)) \quad (13)$$

$$= 0 \quad (14)$$

Therefore, the value of the convolution of $f(x) = 3 \cos(2x)$ and $g(x) = \frac{1}{3} \sin(2x)$ at $x = \frac{\pi}{3}$ is 0.

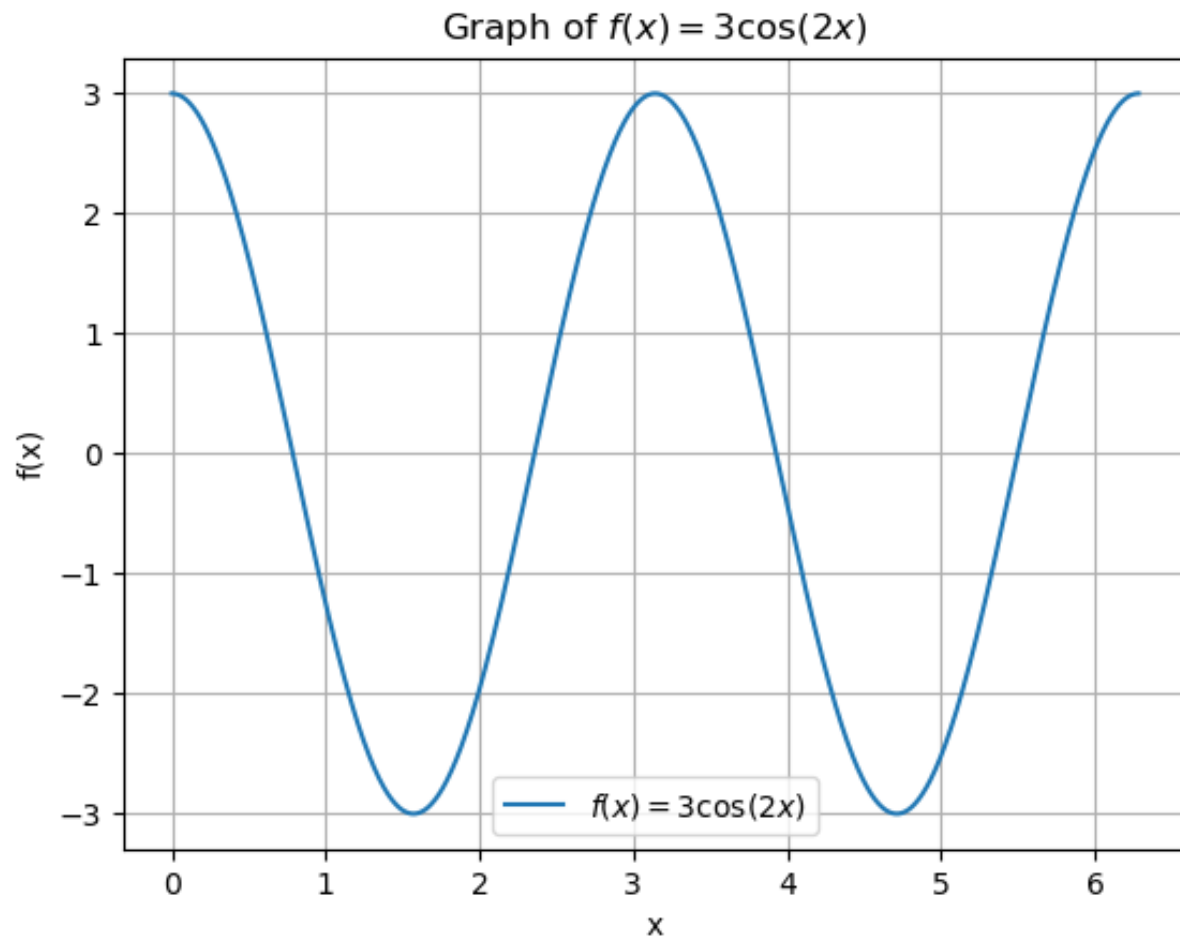


Fig. 0. Plot of $f(x)$ vs x

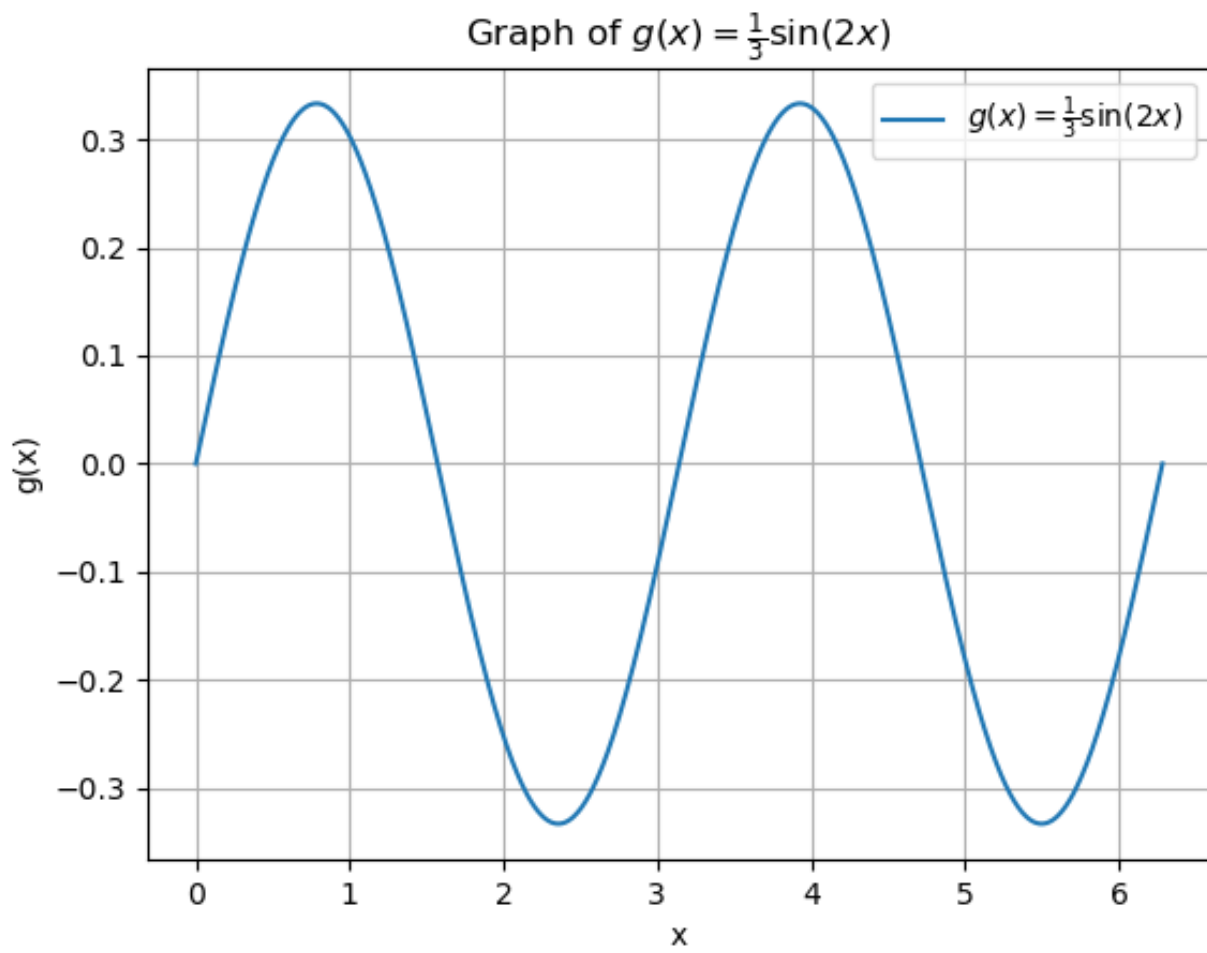


Fig. 0. Plot of $g(x)$ vs x

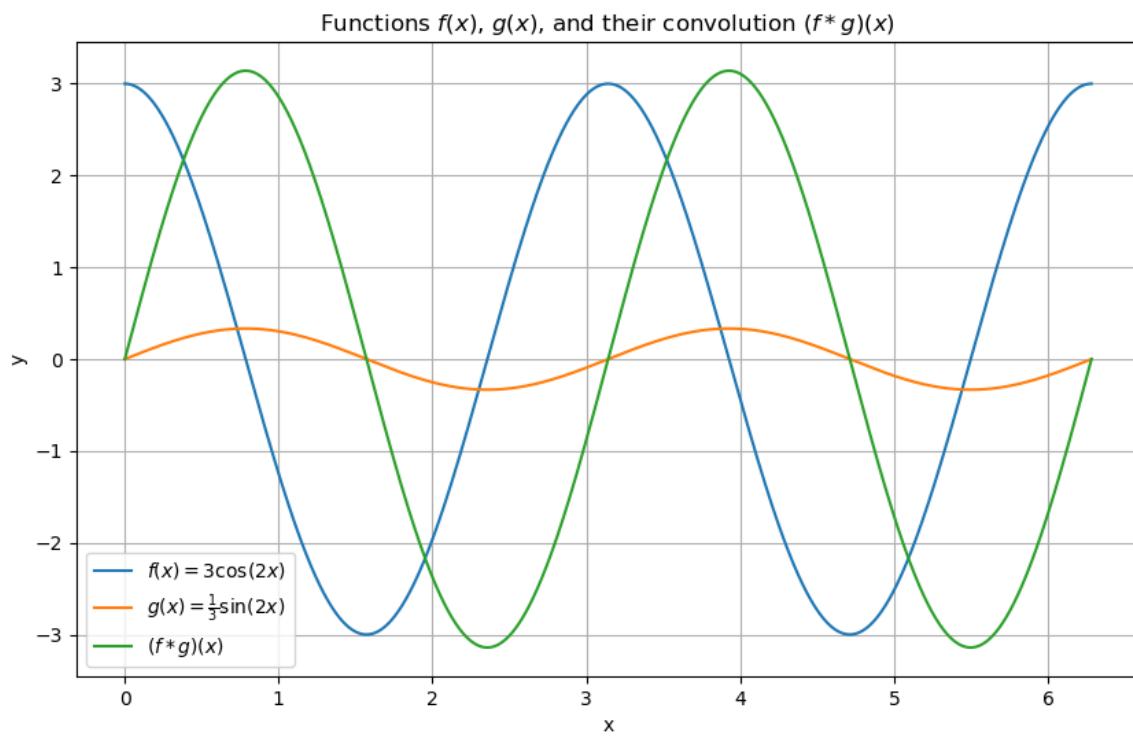


Fig. 0. Plot of y vs x