

# NCERT 11.9.2 16Q

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Let the  $m$  numbers between 1 and 31 be  $A_1, A_2, \dots, A_m$ . Then, the resulting sequence  $1, A_1, A_2, \dots, A_m, 31$  is an arithmetic progression (A.P.). The first term of the A.P. is  $a_1 = 1$ , the last term is  $b_{31} = 31$ , and the number of terms is  $n = m + 2$ . Substitute the values of  $a$ ,  $b$ , and  $n$  in the equation:

$$b = a + (n - 1)d \quad (1)$$

$$31 = 1 + (m + 2 - 1)d \quad (2)$$

$$30 = (m + 1)d \quad (3)$$

$$\frac{30}{m + 1} = d \quad (4)$$

Now, we know that  $A_1 = a + d$ ,  $A_2 = a + 2d$ ,  $A_3 = a + 3d$ , . Then 7th and  $(m - 1)$ th terms are given by:

$$\Rightarrow A_7 = a + 7d \quad (5)$$

$$\Rightarrow A_{m-1} = a + (m - 1)d \quad (6)$$

According to the conditions given in the question:

$$\frac{A_7}{A_{m-1}} = \frac{5}{9} \quad (7)$$

From equations 5 and 6:

$$\Rightarrow \frac{a + 7d}{a + (m - 1)d} = \frac{5}{9} \quad (8)$$

From equations 4 and 9:

$$\Rightarrow \frac{1 + 7\left(\frac{30}{m+1}\right)}{1 + (m - 1)\left(\frac{30}{m+1}\right)} = \frac{5}{9} \quad (9)$$

$$\Rightarrow \frac{m + 1 + 210}{m + 1 + 30m - 30} = \frac{5}{9} \quad (10)$$

$$\Rightarrow \frac{m + 211}{31m - 29} = \frac{5}{9} \quad (11)$$

$$\Rightarrow 9m + 1899 = 155m - 145 \quad (12)$$

$$\Rightarrow 155m - 9m = 1899 + 145 \quad (13)$$

$$\Rightarrow 146m = 2044 \quad (14)$$

$$\Rightarrow m = 14 \quad (15)$$

Therefore,  $m = 14$  is the value of  $m$ .

General term of AP can also be written as

$$x(n) = 2n - 1 \quad (16)$$

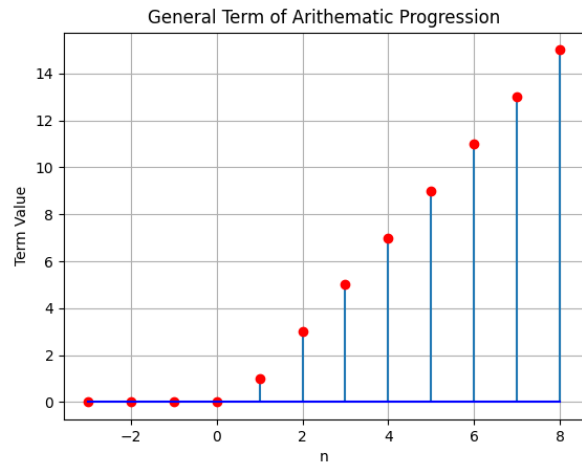


Fig. 0. Plot of general term of AP taken from Python

The Z-Transform Equation for  $x(n)$  is

$$X(z) = \sum_{n=-\infty}^{\infty} (2n - 1) z^{-n} u(n) \quad (17)$$

$$\Rightarrow X(z) = \sum_{n=0}^{\infty} (2n - 1) z^{-n} \quad (18)$$

$$\Rightarrow X(z) = \sum_{n=0}^{\infty} (2n) z^{-n} - \sum_{n=0}^{\infty} z^{-n} \quad (19)$$

$$\Rightarrow X(z) = 2 \sum_{n=0}^{\infty} \frac{n}{z^n} - \sum_{n=0}^{\infty} \frac{1}{z^n} \quad (20)$$

let us evaluate both the summations separately.let

$$S_{\infty} = \sum_{n=0}^{\infty} \frac{n}{z^n} \quad (21)$$

$$\Rightarrow \frac{S_{\infty}}{z} = \sum_{n=0}^{\infty} \frac{n}{z^{n+1}} \quad (22)$$

$$(23)$$

on subtracting both the equations, we get

$$\Rightarrow S_{\infty} \left(1 - \frac{1}{z}\right) = \sum_{n=0}^{\infty} \frac{1}{z^n} \quad (24)$$

$$\Rightarrow S_{\infty} \left(1 - \frac{1}{z}\right) = \frac{z}{z-1} \quad (25)$$

$$\Rightarrow S_{\infty} = \frac{z^2}{(z-1)^2} \quad (26)$$

Now,

$$S_{\infty}^{\downarrow} = \sum_{n=0}^{\infty} \frac{1}{z^n} \quad (27)$$

$$\Rightarrow S_{\infty}^{\downarrow} = \frac{z}{z-1} \quad (28)$$

Now to get the desired result,

$$X(z) = 2S_{\infty} - S_{\infty}^{\downarrow} \quad (29)$$

$$X(z) = \frac{z^2 + z}{(z-1)^2} \quad (30)$$

Parameter	Value
First term of A.P ( $a_1$ )	1
Common difference (d)	2
The value of m	14

TABLE 0  
VARIABLES USED