GATE GE 81Q

EE23BTECH11021 - GANNE GOPI CHANDU*

Question

The value of the convolution of $f(x) = 3\cos(2x)$ and $g(x) = \frac{1}{3}\sin(2x)$ where $x \in [0, 2\pi)$, at $x = \frac{\pi}{3}$, is (Rounded off to 2 decimal places)

Solution

The f(x) = $3\cos(2x)$ the Fourier series is

$$c\,(n1) = \frac{1}{p}\int_0^{2\pi} f\,(x)\, e^{-j\frac{2\pi nx}{p}}\,dx \tag{1}$$

$$= \frac{3}{2\pi}\int_0^{2\pi} \cos\,(2x)\, e^{-j\frac{\pi nx}{\pi}}\,dx \tag{2}$$

$$= \frac{3}{2\pi}\int_0^{2\pi} \cos\,(2x)\,(\cos\,(nx) - j\sin\,(nx))\,dx \tag{3}$$

$$= \frac{3}{2\pi}\int_0^{2\pi} \cos\,(2x)\cos\,(nx) - j\cos\,(2x)\sin\,(nx)\,dx \tag{4}$$

$$= \frac{3}{2\pi}\int_0^{2\pi} \frac{1}{2}\,(\cos\,((2+n)\,x) + (\cos\,((2-n)\,x))) - j\,(\sin\,((2+n)\,x) - \sin\,(2-n)\,x)\,dx \tag{5}$$

$$= 0 \quad n1 \neq -2, 2 \tag{6}$$

Except c1(−2),c1(2) remaining all values are 0

$$c1\,(2) = \frac{3}{2\pi}\int_0^{2\pi} \cos\,(2x)\, e^{-j(\pi 2x)}\,dx \tag{7}$$

$$= \frac{3}{2\pi}\int_0^{2\pi} \cos\,(2x)\,(\cos\,(2x) - j\sin\,(2x))\,dx \tag{8}$$

$$= \frac{3}{2\pi}\int_0^{2\pi} \cos^2(2x) - j\sin\,(2x)\cos\,(2x)\,dx \tag{9}$$

$$= \frac{3}{2\pi}\left(\frac{2\pi}{2}\right) \tag{10}$$

$$c1\,(2) = \frac{3}{2} \tag{11}$$

and

$$\begin{aligned}
c1(-2) &= \frac{3}{2\pi}\int_0^{2\pi} \cos(2x)\, e^{j(\pi 2x)}\; dx && (12)\\
&= \frac{3}{2\pi}\int_0^{2\pi} \cos(2x)\left(\cos(2x) + j\sin(2x)\right)\; dx && (13)\\
&= \frac{3}{2\pi}\int_0^{2\pi} \cos^2(2x) + j\sin(2x)\cos(2x)\; dx && (14)\\
&= \frac{3}{2\pi}\left(\frac{2\pi - 0}{2}\right) && (15)\\
c1(-2) &= \frac{3}{2} && (16)
\end{aligned}$$

For $g(x) = \frac{1}{3}\sin(2x)$ the Fourier series is:

$$\begin{aligned}
c2(n2) &= \frac{1}{p}\int_0^{2\pi} g(x)\, e^{-j\frac{2\pi n x}{p}}\; dx && (17)\\
&= \frac{1}{6\pi}\int_0^{2\pi} \sin(2x)\, e^{-j\frac{\pi n x}{\pi}}\; dx && (18)\\
&= \frac{1}{6\pi}\int_0^{2\pi} \sin(2x)\left(\cos(nx) - j\sin(nx)\right)\; dx && (19)\\
&= \frac{1}{6\pi}\int_0^{2\pi} \sin(2x)\cos(nx) - j\sin(2x)\sin(nx)\; dx && (20)\\
&= \frac{1}{6\pi}\int_0^{2\pi} \frac{1}{2}\left(\sin((2+n)x) + (\sin((2-n)x))\right) - j\left(\cos((2-n)x) - \cos(2+n)x\right)\; dx && (21)\\
&= 0 \quad n2 \neq -2, 2 && (22)
\end{aligned}$$

Except c2(−2),c2(2) remaining all values are 0

$$\begin{aligned}
c2(2) &= \frac{1}{6\pi}\int_0^{2\pi} \sin(2x)\, e^{-j(\pi 2x)}\; dx && (23)\\
&= \frac{1}{6\pi}\int_0^{2\pi} \sin(2x)\left(\cos(2x) - j\sin(2x)\right)\; dx && (24)\\
&= \frac{1}{6\pi}\int_0^{2\pi} \sin(2x)\cos(2x) - j\sin^2(2x)\; dx && (25)\\
&= \frac{1}{6\pi}\left(\frac{2\pi - 0}{2}\right)(-j) && (26)\\
&= \frac{-j}{6} && (27)\\
& && (28)
\end{aligned}$$

and

$$c2\,(-2) = \frac{1}{6\pi}\int_0^{2\pi} \sin(2x)\, e^{j(\pi 2x)}\ dx \tag{29}$$
$$= \frac{1}{6\pi}\int_0^{2\pi} \sin(2x)\,(\cos(2x) + j\sin(2x))\ dx \tag{30}$$
$$= \frac{1}{6\pi}\int_0^{2\pi} \sin(2x)\cos(2x) + j\sin^2(2x)\ dx \tag{31}$$
$$= \frac{1}{6\pi}\left(\frac{2\pi}{2}\right)(j) \tag{32}$$
$$= \frac{j}{6} \tag{33}$$

By periodic convolution with Fourier series coefficients is (n)=c(n1)*c(n2)*p

$$c\,(2) = c1\,(2) * c2\,(2) * p \tag{34}$$
$$= \left(\frac{3}{2}\right)\left(\frac{-j}{6}\right)(2\pi) \tag{35}$$
$$= \frac{-j\pi}{2} \tag{36}$$

and

$$c\,(-2) = c1\,(-2) * c2\,(-2) * p \tag{37}$$
$$= \left(\frac{3}{2}\right)\left(\frac{j}{6}\right)(2\pi) \tag{38}$$
$$= \frac{j\pi}{2} \tag{39}$$

$$f * g\,(x) = \sum_{n=-N}^{N} c\,(n)\, e^{j\frac{\pi n x}{L}} \tag{40}$$
$$= c\,(-2)\, e^{-j2x} + c\,(2)\, e^{j2x} \tag{41}$$
$$= \frac{j\pi}{2} e^{-j2x} - \frac{j\pi}{2} e^{j2x} \tag{42}$$
$$= \frac{-j\pi}{2}\left(e^{j2x} - e^{-j2x}\right) \tag{43}$$
$$= \frac{-j\pi}{2}\,(sin\,(2x)\,2j) \tag{44}$$
$$= \pi\sin(2x) \tag{45}$$
$$\tag{46}$$

at $x = \frac{\pi}{3}$

$$= \frac{\sqrt{3}\pi}{2} \tag{47}$$
$$= 2.72 \tag{48}$$

Therefore the convolution of f(x) and g(x) is 2.72 For $n \neq 0$,

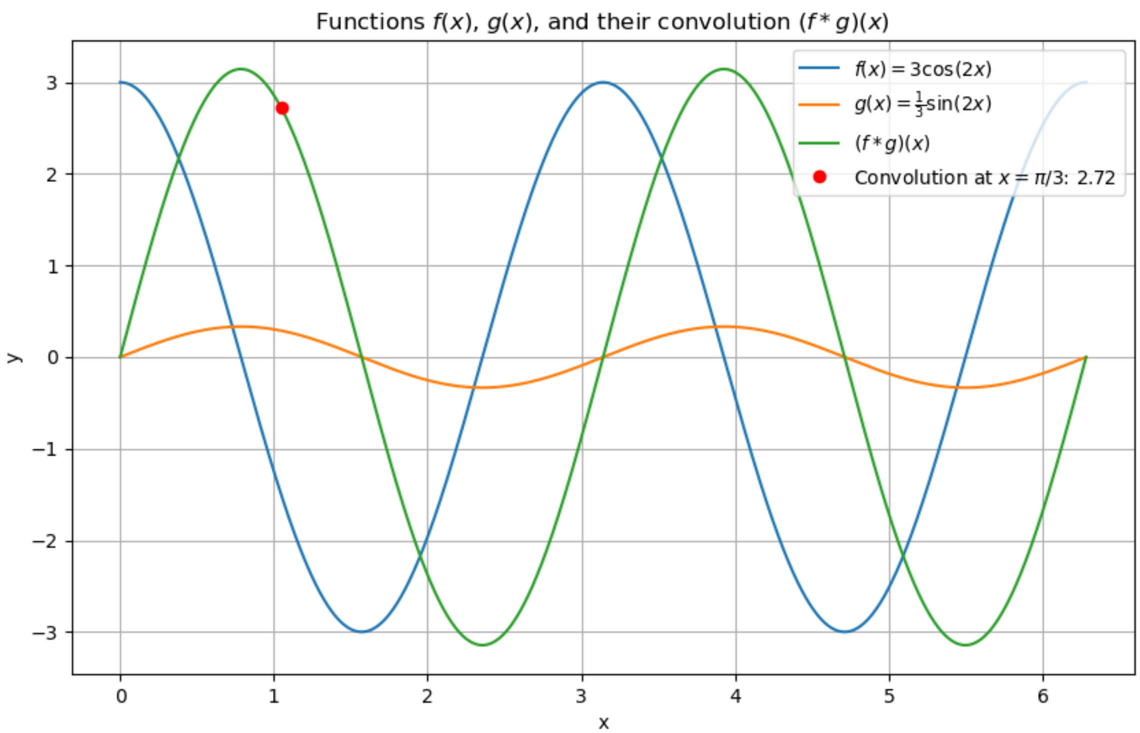

Fig. 0. Plot of y vs x