GATE GE 81Q

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Question

The value of the convolution of $f(x) = 3\cos(2x)$ and $g(x) = \frac{1}{3}\sin(2x)$ where $x \in [0, 2\pi)$, at $x = \frac{\pi}{3}$, is (Rounded off to 2 decimal places)

Solution

The convolution integral:

$$(f * g)(x) = \int_0^{2\pi} f(t)g(x-t)\,dt \tag{1}$$

Substitute the $f(x)$ and $g(x)$

$$(f * g)(x) = \int_0^{2\pi} 3\cos(2t) \cdot \frac{1}{3}\sin(2(x-t))\,dt \tag{2}$$

$$(f * g)\left(\frac{\pi}{3}\right) = \int_0^{2\pi} 3\cos(2t) \cdot \frac{1}{3}\sin\left(2\left(\frac{\pi}{3} - t\right)\right)\,dt \tag{3}$$

$$= \int_0^{2\pi} \left(\cos(2t) \cdot \sin\frac{\pi}{3}\cos t - \cos(2t)\cos\frac{\pi}{3} \cdot \sin(t)\right)\,dt \tag{4}$$

$$= \frac{1}{2}\int_0^{2\pi} \left(\sqrt{3}\cos(2t) \cdot \cos t - \cos(2t) \cdot \sin(t)\right)\,dt \tag{5}$$

$$= \int_0^{2\pi} \left(\frac{\sqrt{3}}{2}\cos(2t) \cdot \cos t\right)dt - \int_0^{2\pi} \left(\frac{1}{2}\cos(2t) \cdot \sin(t)\right)dt \tag{6}$$

$$= \int_0^{2\pi} \left(\frac{\sqrt{3}}{2} \cdot \frac{1}{2}\left[\cos(3t) + \cos(t)\right]\right)dt - \int_0^{2\pi} \left(\frac{1}{2} \cdot \frac{1}{2}\left[\sin(3t) - \sin(t)\right]\right)dt \tag{7}$$

$$= \frac{\sqrt{3}}{4}\int_0^{2\pi} \cos(3t)dt + \frac{\sqrt{3}}{4}\int_0^{2\pi} \cos(t)dt - \frac{1}{4}\int_0^{2\pi} \sin(3t)dt + \frac{1}{4}\int_0^{2\pi} \sin(t)dt \tag{8}$$

$$= \frac{\sqrt{3}}{4}\left[\frac{\sin(3t)}{3}\right]_0^{2\pi} + \frac{\sqrt{3}}{4}\left[\sin(t)\right]_0^{2\pi} + \frac{1}{4}\left[\frac{\cos(3t)}{3}\right]_0^{2\pi} - \frac{1}{4}\left[\cos(t)\right]_0^{2\pi} \tag{9}$$

$$= \frac{\sqrt{3}}{4}\left(\frac{\sin(6\pi)}{3} - \frac{\sin(0)}{3}\right) + \frac{\sqrt{3}}{4}\left(\sin(2\pi) - \sin(0)\right) + \frac{1}{4}\left(\frac{\cos(6\pi)}{3} - \frac{\cos(0)}{3}\right) - \frac{1}{4}\left(\cos(2\pi) - \cos(0)\right) \tag{10}$$

$$= 0 \tag{11}$$

Therefore, the value of the convolution of $f(x) = 3\cos(2x)$ and $g(x) = \frac{1}{3}\sin(2x)$ at $x = \frac{\pi}{3}$ is 0.

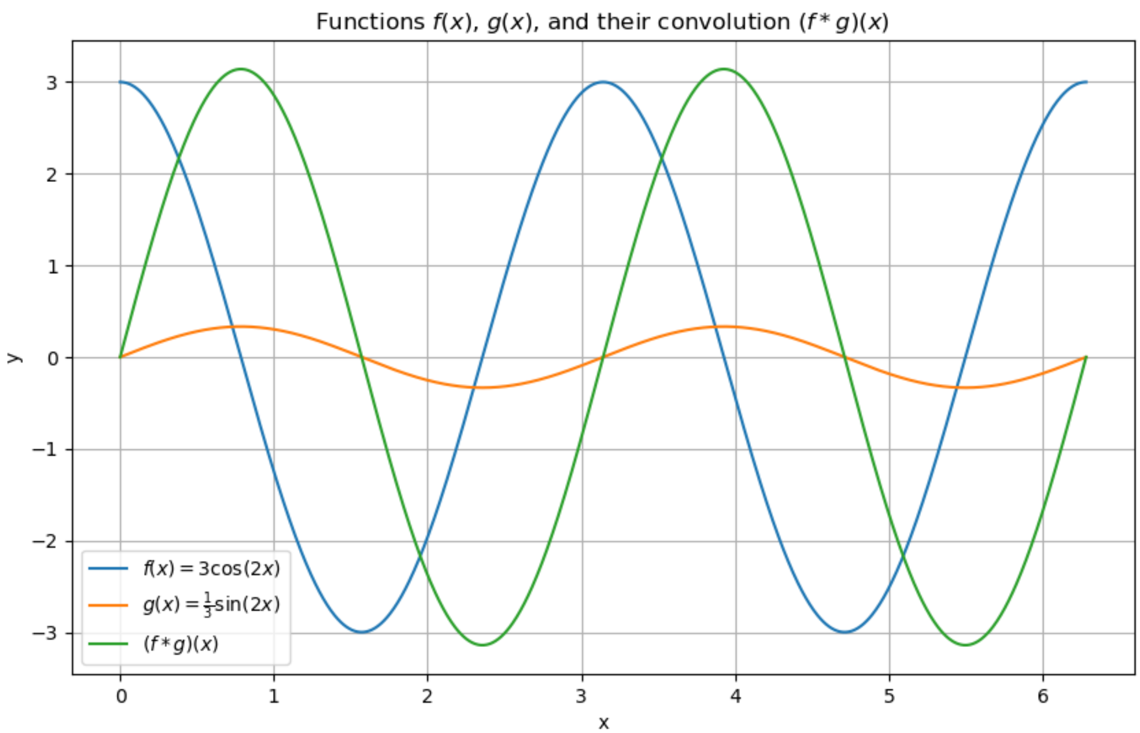

Fig. 0. Plot of y vs x