NCERT 11.9.2 16Q

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Question
Between 1 and 31, m numbers have been inserted in such a way that the resulting sequence is an A.P. and the ratio of 7 th and (m - 1) th numbers is 5:9. Find the value of m.
Solution
Let the m numbers between 1 and 31 be $A_1, A_2, \ldots, A_m$. Then, the resulting sequence $1, A_1, A_2, \ldots, A_m, 31$ is an arithmetic progression (A.P.). The first term of the A.P. is $a_1 = 1$, the last term is $b_{31} = 31$, and the number of terms is $n = m + 2$. Substitute the values of a, b, and n in the equation:

$$b = a + (n-1)d \tag{1}$$
$$31 = 1 + (m+2-1)d \tag{2}$$
$$30 = (m+1)d \tag{3}$$
$$\frac{30}{m+1} = d \tag{4}$$

Now, we know that $A_1 = a + d$, $A_2 = a + 2d$, $A_3 = a + 3d$, . Then 7th and $(m-1)$th terms are given by:

$$\implies A_7 = a + 7d \tag{5}$$
$$\implies A_{m-1} = a + (m-1)d \tag{6}$$

According to the conditions given in the question:

$$\frac{A_7}{A_{m-1}} = \frac{5}{9} \tag{7}$$

From equations 5 and 6:

$$\implies \frac{a+7d}{a+(m-1)d} = \frac{5}{9} \tag{8}$$

From equations 4 and 9:

$$\implies \frac{1 + 7\left(\frac{30}{m+1}\right)}{1 + (m-1)\left(\frac{30}{m+1}\right)} = \frac{5}{9} \tag{9}$$
$$\implies \frac{m+1+210}{m+1+30m-30} = \frac{5}{9} \tag{10}$$
$$\implies \frac{m+211}{31m-29} = \frac{5}{9} \tag{11}$$
$$\implies 9m + 1899 = 155m - 145 \tag{12}$$
$$\implies 155m - 9m = 1899 + 145 \tag{13}$$
$$\implies 146m = 2044 \tag{14}$$
$$\implies m = 14 \tag{15}$$

Therefore, $m = 14$ is the value of m.
General term of AP can also be written as

$$x(n) = 2n - 1 \tag{16}$$

The Z-Transform Equation for x(n) is

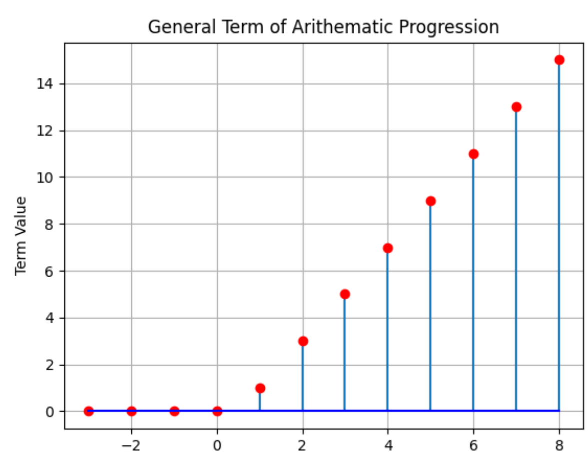

Fig. 0. Plot of general term of AP taken from Python

$$X(z) = \sum_{-\infty}^{\infty} (2n-1) z^{-n} u(n) \tag{17}$$

$$\implies X(z) = \sum_{n=0}^{\infty} (2n-1) z^{-n} \tag{18}$$

$$\implies X(z) = \sum_{n=0}^{\infty} (2n) z^{-n} - \sum_{n=0}^{n=\infty} z^{-n} \tag{19}$$

$$\implies X(z) = 2\sum_{n=0}^{\infty} \frac{n}{z^n} - \sum_{n=0}^{\infty} \frac{1}{z^n} \tag{20}$$

let us evaluate both the summations separately.let

$$S_\infty = \sum_{n=0}^{\infty} \frac{n}{z^n} \tag{21}$$

$$\implies \frac{S_\infty}{z} = \sum_{n=0}^{\infty} \frac{n}{z^{n+1}} \tag{22}$$

$$\tag{23}$$

on subtracting both the equations, we get

$$\implies S_\infty \left(1 - \frac{1}{z}\right) = \sum_{n=0}^{\infty} \frac{1}{z^n} \tag{24}$$

$$\implies S_\infty \left(1 - \frac{1}{z}\right) = \frac{z}{z-1} \tag{25}$$

$$\implies S_\infty = \frac{z^2}{(z-1)^2} \tag{26}$$

Now,

$$S^{|}_\infty = \sum_{n=0}^{\infty} \frac{1}{z^n} \tag{27}$$

$$\implies S^{|}_\infty = \frac{z}{z-1} \tag{28}$$

Now to get the desired result,

$$X(z) = 2S_\infty - S^{|}_\infty \tag{29}$$

$$X(z) = \frac{z^2 + z}{(z-1)^2} \tag{30}$$

The region of convergence (ROC) for this transfer function is $|z| > 1$.

Parameter	Value
First term of A.P (a_1)	1
Common difference (d)	2
The value of m	14

TABLE 0
Variables used