

NCERT 11.9.2 16Q

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Question

Between 1 and 31, m numbers have been inserted in such a way that the resulting sequence is an A.P. and the ratio of 7th and $(m - 1)$ th numbers is 5:9. Find the value of m .

Solution

Symbol	Value	description
$x(0)$	1	First term of A.P
$x(n)$	31	$(n + 1)$ th term
$\frac{x(7)}{x(m-1)}$	$\frac{5}{9}$	ratio of 7 th and $(m - 1)$ th numbers
n	$m + 2$	number of terms

TABLE 0

The last term is

$$x(n) = x(0) + (n) d \quad (1)$$

$$\Rightarrow 31 = 1 + (m + 1) d \quad (2)$$

$$\Rightarrow 30 = (m + 1) d \quad (3)$$

$$\Rightarrow \frac{30}{m + 1} = d \quad (4)$$

Now 7th and $(m - 1)$ th terms

$$x(7) = x(0) + 7d \quad (5)$$

$$x(m - 1) = x(0) + (m - 1) d \quad (6)$$

From equations (5) and (6)

$$\frac{x(0) + 7d}{x(0) + (m - 1) d} = \frac{5}{9} \quad (7)$$

Substituting (4) in (7)

$$\Rightarrow \frac{1 + 7\left(\frac{30}{m+1}\right)}{1 + (m - 1)\left(\frac{30}{m+1}\right)} = \frac{5}{9} \quad (8)$$

$$\Rightarrow \frac{m + 1 + 210}{m + 1 + 30m - 30} = \frac{5}{9} \quad (9)$$

$$\Rightarrow \frac{m + 181}{31m - 29} = \frac{5}{9} \quad (10)$$

$$\Rightarrow 9m + 1899 = 155m - 145 \quad (11)$$

$$\Rightarrow 155m - 9m = 1899 + 145 \quad (12)$$

$$\Rightarrow 146m = 2044 \quad (13)$$

$$\Rightarrow m = 14 \quad (14)$$

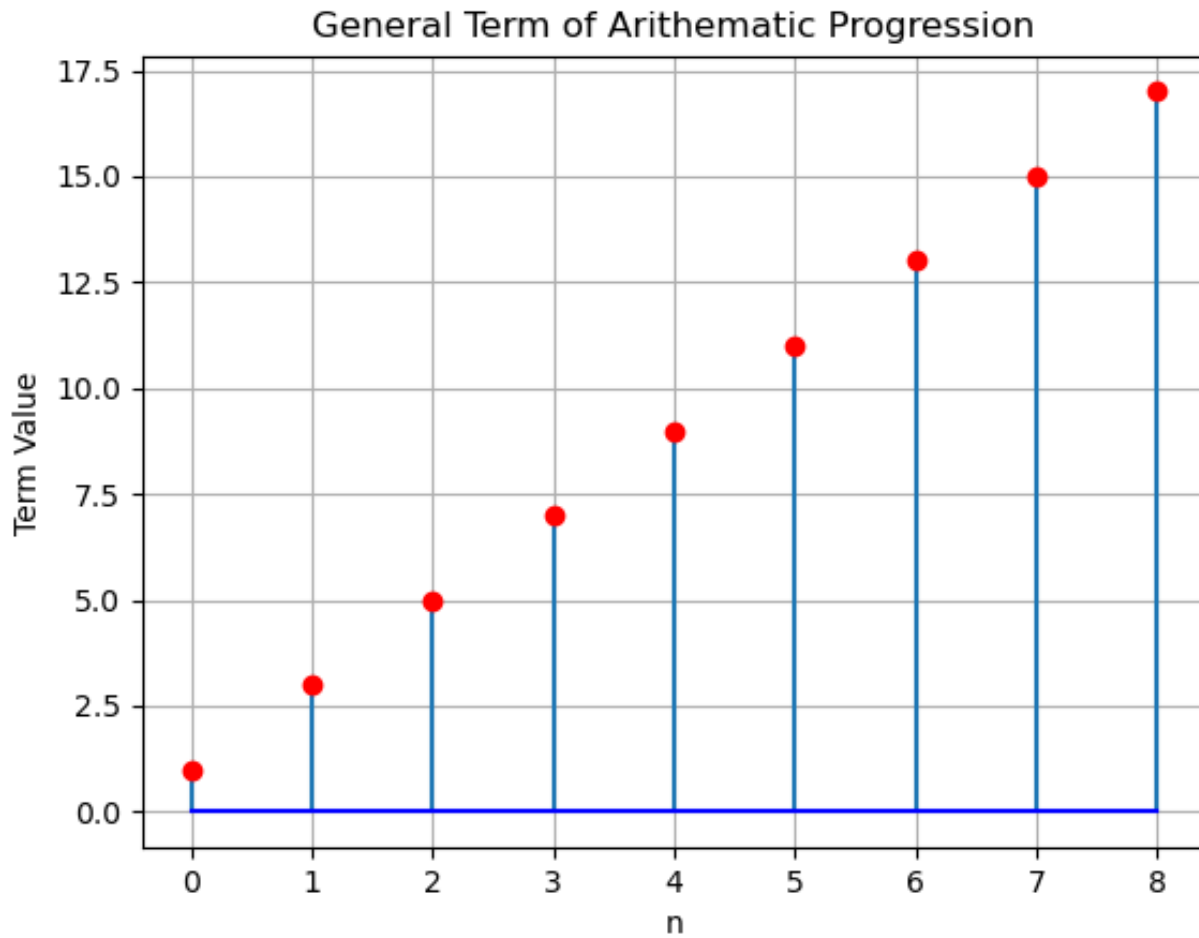


Fig. 0. Plot of $x(n)$ vs n

Therefore, $m = 14$.
General term of AP is

$$x(n) = (2n + 1) u(n) \quad (15)$$

$$x(n) = (2n) u(n) + u(n) \quad (16)$$

The Z-Transform is

$$X(z) = 2 \left(\frac{z}{(z-1)^2} \right) + U(z) \quad (17)$$

$$= \frac{2z}{(z-1)^2} + \frac{1}{1-z^{-1}} \quad (18)$$

$$X(z) = \frac{z^2 + z}{(z-1)^2} \quad |z| > 1 \quad (19)$$