

GATE EC 41Q

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Question

Consider the signals $x[n] = 2^{n-1}u[-n+2]$ and $y[n] = 2^{-n+2}u[n+1]$, where $u[n]$ is the unit step sequence. Let $X(e^{j\omega})$ and $Y(e^{j\omega})$ be the discrete-time Fourier transform of $x[n]$ and $y[n]$, respectively. The value of the integral

$$\frac{1}{2\pi} \int_0^{2\pi} X(e^{j\omega})Y(e^{-j\omega})d\omega$$

(rounded off to one decimal place) is.

Solution

Symbol	Value	description
$x[n]$	$2^{n-1}u[-n+2]$	Discrete time signal
$y[n]$	$2^{-n+2}u[n+1]$	Discrete time signal

TABLE 0

$$x[n] * y[n] \xleftrightarrow[\text{transform}]{\text{Fourier}} X(e^{j\omega})Y(e^{j\omega}) \quad (1)$$

$$x[n] \xleftrightarrow[\text{transform}]{\text{Fourier}} X(e^{j\omega}) \quad (2)$$

$$y[n] \xleftrightarrow[\text{transform}]{\text{Fourier}} Y(e^{j\omega}) \quad (3)$$

$$\text{They}(n) = y(e^{j\omega}) \quad (4)$$

By using the time reversal property:

$$y[-n] = y(e^{-j\omega}) \quad (5)$$

Let assume

$$z[n] = x[n] * y[-n] \quad (6)$$

$$Z(e^{j\omega}) = X(e^{j\omega})Y(e^{-j\omega}) \quad (7)$$

$$z[n] = \frac{1}{2\pi} \int_0^{2\pi} Z(e^{j\omega})e^{j\omega n}d\omega \quad (8)$$

$$= \frac{1}{2\pi} \int_0^{2\pi} X(e^{j\omega})Y(e^{-j\omega})e^{j\omega n}d\omega. \quad (9)$$

putting n=0, we get

$$z[0] = \frac{1}{2\pi} \int_0^{2\pi} X(e^{j\omega})Y(e^{-j\omega})d\omega \quad (10)$$

$$z[n] = x[n] * y[-n] \quad (11)$$

$$= \sum_{k=-\infty}^2 2^{k-1}u[-k+2] \cdot 2^{n-k+2}u[-n+k+1] \quad (12)$$

$$= \sum_{k=-\infty}^2 2^{k-1} \cdot 2^{n-k+2}u[-n+k+1] \quad (13)$$

$$= \sum_{k=-\infty}^2 2^{k-1+n-k+2}u[-n+k+1] \quad (14)$$

$$= \sum_{k=-\infty}^2 2^{n+1}u[-n+k+1] \quad (15)$$

Putting $n = 0$, we get:

$$\frac{1}{2\pi} \int_0^{2\pi} X(e^{j\omega})Y(e^{-j\omega})d\omega = z[0] \quad (16)$$

$$= \sum_{k=-\infty}^2 2 \cdot u[k+1] \quad (17)$$

$$= \sum_{k=-1}^2 2(1) = 2 \times 4 \quad (18)$$

$$= 8 \quad (19)$$