GATE EC 41Q

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Question

Consider the signals $x[n] = 2^{n-1}u[-n+2]$ and $y[n] = 2^{-n+2}u[n+1]$, where $u[n]$ is the unit step sequence. Let $X(e^{j\omega})$ and $Y(e^{j\omega})$ be the discrete-time Fourier transform of $x[n]$ and $y[n]$, respectively. The value of the integral

$$\frac{1}{2\pi}\int_0^{2\pi} X(e^{j\omega})Y(e^{-j\omega})d\omega$$

(rounded off to one decimal place) is.

Solution

$x[n] * y[n] \xleftrightarrow[transform]{Fourier} X(e^{j\omega})Y(e^{j\omega})$

$x[n] \xleftrightarrow[transform]{Fourier} X(e^{j\omega})$

$y[n] \xleftrightarrow[transform]{Fourier} Y(e^{j\omega})$

The $y(n) = y(e^{j\omega})$

By using the time reversal property:

$y[-n] = y(e^{-j\omega})$

Let

$$z\left(e^{j\omega}\right) = X(e^{j\omega})Y(e^{-j\omega}) \tag{1}$$

$$z[n] = x[n] * y[-n] \tag{2}$$

$$z[t] = \frac{1}{2\pi}\int_0^{2\pi} X(e^{j\omega})Y(e^{-j\omega})e^{j\omega t}d\omega. \tag{3}$$

By using the Central Ordinate Theorem:

$$z[0] = \frac{1}{2\pi}\int_0^{2\pi} Z(e^{j\omega})d\omega = X(e^{j\omega})Y(e^{-j\omega})d\omega \tag{4}$$

$$x[n] = 2^{n-1}u[-n+2] \tag{5}$$

$$y[n] = 2^{-n+2}u[n+1] \tag{6}$$

$$z[n] = x[n] * y[-n] \tag{7}$$

$$z[n] = (2^{n-1}u[-n+2]) * (2^{n+2}u[-n+1]) \tag{8}$$

$$= \sum_{k=-\infty}^{2} 2^{k-1}u[-k+2] \cdot 2^{n-k+2}u[-n+k+1 \tag{9}$$

$$= \sum_{k=-\infty}^{2} 2^{k-1} \cdot 2^{n-k+2}u[-n+k+1] \tag{10}$$

$$= \sum_{k=-\infty}^{2} 2^{k-1+n-k+2}u[-n+k+1] \tag{11}$$

$$= \sum_{k=-\infty}^{2} 2^{n+1}u[-n+k+1] \tag{12}$$

$$\frac{1}{2\pi}\int_0^{2\pi} X(e^{j\omega})Y(e^{-j\omega})d\omega = z[0] \tag{13}$$

Putting $n = 0$, we get:

$$z[0] = \sum_{k=-\infty}^{2} 2 \cdot u[k+1] \tag{14}$$

$$= 2(1) = 2 \times 4 \tag{15}$$

$$= 8 \tag{16}$$