NCERT 11.9.2 16Q

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Question

Between 1 and 31, m numbers have been inserted in such a way that the resulting sequence is an A.P. and the ratio of 7 th and (m - 1) th numbers is 5:9. Find the value of m.

Solution

Symbol	Value	description
$x(0)$	1	First term of A.P
x(n)	31	last term
d	2	Common difference
n	m+2	number of terms
m	14	number of terms inserted

TABLE 0

The last term is

$$x(n) = x(0) + nd \tag{1}$$
$$31 = 1 + (m+1)\,d \tag{2}$$
$$30 = (m+1)\,d \tag{3}$$
$$\frac{30}{m+1} = d \tag{4}$$

Now 7th and $(m-1)$th terms

$$\implies x(7) = x(0) + 7d \tag{5}$$
$$\implies x(m-1) = x(0) + (m-1)\,d \tag{6}$$

Given

$$\frac{x(7)}{x(m-1)} = \frac{5}{9} \tag{7}$$

From equations (5) and (6) the augmented matrix is:

$$\begin{pmatrix} 1 & 7 & x(7) \\ 1 & m-1 & x(m-1) \end{pmatrix} \xleftrightarrow{R_2 \leftarrow R_2 - R_1} \begin{pmatrix} 1 & 7 & x(7) \\ 0 & m-8 & x(m-1) - x_7 \end{pmatrix} \tag{8}$$

$$\xleftrightarrow{R_2 \leftarrow \frac{1}{m-8} R_2} \begin{pmatrix} 1 & 7 & x(7) \\ 0 & 1 & \frac{x(m-1) - x_7}{m-8} \end{pmatrix} \tag{9}$$

$$\xleftrightarrow{R_1 \leftarrow R_1 - 7R_2} \begin{pmatrix} 1 & 0 & x_7 - 7\left(\frac{x(m-1)-x(7)}{m-8}\right) \\ 0 & 1 & \frac{x(m-1)-x(7)}{m-8} \end{pmatrix} \tag{10}$$

$$\implies \begin{pmatrix} x(0) \\ d \end{pmatrix} = \begin{pmatrix} x_7 - 7\left(\frac{x(m-1)-x(7)}{m-8}\right) \\ \frac{x(m-1)-x(7)}{m-8} \end{pmatrix} \tag{11}$$

part 1
from equation (7)

$$x(0) = x(7) - 7\left(\frac{x(m-1) - x(7)}{m-8}\right) \tag{12}$$
$$1 = x(7) - 7\left(\frac{x(m-1) - x(7)}{m-8}\right) \tag{13}$$
$$1 = x(7) - 7\left(\frac{x(7)\left(\frac{9}{5}\right) - x(7)}{m-8}\right) \tag{14}$$
$$1 = x(7)\left((m-8) - \frac{28}{5}\right) \tag{15}$$

part 2
from equations (4) and (7)

$$d = \frac{x(m-1) - x(7)}{m-8} \tag{16}$$
$$\frac{30}{m+1} = \frac{x(7)\left(\frac{4}{9}\right)}{m-8} \tag{17}$$
$$x(7) = \frac{75\,(m-8)}{2\,(m+1)} \tag{18}$$

Substituting (18) in (15)

$$m-8=\frac{75\,(m-8)\,(5m-68)}{10\,(m+1)} \tag{19}$$

$$2\,(m+1)=15\,(5m-68) \tag{20}$$

$$2m+2=75m-1020 \tag{21}$$

$$73m=1022 \tag{22}$$

$$m=14 \tag{23}$$

General term of AP as

$$x\,(n)=(2n-1)\,u(n) \tag{24}$$

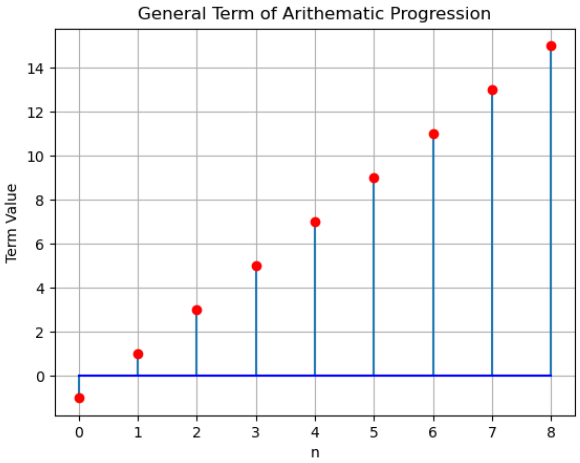

Fig. 0. Plot of x(n) vs n

The Z-Transform Equation for x(n) is

$$X\,(z)=\sum_{n=-\infty}^{\infty}(2n-1)\,z^{-n}u\,(n) \tag{25}$$

$$=\sum_{n=\infty}^{\infty}(2n)\,z^{-n}u\,(n)-\sum_{n=-\infty}^{n=\infty}z^{-n}u\,(n) \tag{26}$$

$$X(z)=2\sum_{n=0}^{\infty}\frac{n}{z^n}-U\,(z) \tag{27}$$

The first part of summation is

$$S\,(\infty)=\sum_{n=0}^{\infty}\frac{n}{z^n} \tag{28}$$

$$\implies S\,(\infty)=\frac{z^2}{(z-1)^2} \tag{29}$$

The second part of summation is

$$U\,(z)=\frac{1}{1-z^{-1}} \tag{30}$$

The result is,

$$X\,(z)=2S_{\infty}-U\,(z) \tag{31}$$

$$=\frac{2z^2}{(z-1)^2}-\frac{1}{1-z^{-1}} \tag{32}$$

$$X\,(z)=\frac{z^2+z}{(z-1)^2} \tag{33}$$

(ROC) $|z|>1$.