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# NCERT 11.9.2 16Q

# EE23BTECH11021 - GANNE GOPI CHANDU\*

### **Question**

Between 1 and 31, m numbers have been inserted in such a way that the resulting sequence is an A.P. and the ratio of 7 th and (m - 1) th numbers is 5:9. Find the value of m.

## Solution

Symbol	Value	description
x(0)	1	First term of A.P
d	2	Common difference
m	14	The value of m
x(n)	(2n-1)u(n)	General term

TABLE 0

First term x(0) = 1

last term x(n) = 31

number of terms(n) = m + 2.

from equations ??,?? and ??

$$x(n) = x(0) + nd \tag{4}$$

$$31 = 1 + (m+1)d \tag{5}$$

$$30 = (m+1)d (6)$$

$$\frac{30}{m+1} = d \tag{7}$$

Now 7th and (m-1)th terms

$$\implies x(7) = x(0) + 7d \tag{8}$$

$$\implies x(m-1) = x(0) + (m-1)d$$
 (9)

Given

$$\frac{x(7)}{x(m-1)} = \frac{5}{9} \tag{10}$$

From equations ?? and ?? the augmented matrix is:

$$\begin{pmatrix} 1 & 7 & x(7) \\ 1 & m-1 & x(m-1) \end{pmatrix} \xrightarrow{R_2 \leftarrow R_2 - R_1} \begin{pmatrix} 1 & 7 & x(7) \\ 0 & m-8 & x(m-1) - x_7 \end{pmatrix}$$
(11)

$$\stackrel{R_2 \leftarrow \frac{1}{m-8}R_2}{\longleftrightarrow} \begin{pmatrix} 1 & 7 & x(7) \\ 0 & 1 & \frac{x(m-1)-x_7}{m-8} \end{pmatrix}$$
(12)

$$\xrightarrow{R_1 \leftarrow R_1 - 7R_2} \begin{pmatrix} 1 & 0 & x_7 - 7\left(\frac{x(m-1) - x(7)}{m-8}\right) \\ 0 & 1 & \frac{x(m-1) - x(7)}{m-8} \end{pmatrix}$$
(13)

$$\implies {\binom{x(0)}{d}} = {\binom{x_7 - 7\left(\frac{x(m-1) - x(7)}{m-8}\right)}{\frac{x(m-1) - x(7)}{m-8}}}$$
(14)

(1)

from equations ?? and ??

$$x(0) = x(7) - \left(7\frac{x(m-1) - x(7)}{m-8}\right)$$
 (15)

$$1 = x(7) - 7\left(\frac{x(m-1) - x(7)}{m-8}\right)$$
 (16)

$$1 = x(7) - 7 \left( \frac{x(7)\left(\frac{9}{5}\right) - x(7)}{m - 8} \right) \tag{17}$$

$$1 = x(7)\left((m-8) - \frac{28}{5}\right) \tag{18}$$

part 2 from equations ?? and ??

$$d = \frac{x(m-1) - x(7)}{m-8} \tag{19}$$

$$\frac{30}{m+1} = \frac{x(7)\left(\frac{4}{9}\right)}{m-8} \tag{20}$$

$$x(7) = \frac{75(m-8)}{2(m+1)}$$
 (21)

(22)

(34)

Substituting x(7) in part 1

$$m - 8 = \frac{75(m - 8)(5m - 68)}{10(m + 1)}$$
 (23)

$$X(z) = \frac{z^2}{(z-1)^2} - \frac{1}{1-z^{-1}}$$
 (35)

$$2(m+1) = 15(5m-68) \tag{24}$$

$$(z-1)^{2} 1-z^{-1}$$

$$X(z) = \frac{z^{2}+z}{z^{2}} (36)$$

$$2m + 2 = 75m - 1020$$

$$X(z) = \frac{z^2 + z}{(z - 1)^2}$$

 $X(z) = 2S_{\infty} - U(z)$ 

$$73m = 1022$$

(26)(27)

(25)

$$m = 14$$

(ROC) |z| > 1.

General term of AP as

$$x(n) = 2n - 1 \tag{28}$$

The Z-Transform Equation for x(n) is

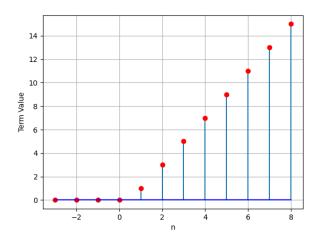


Fig. 0. Plot of x(n) vs n

$$X(z) = \sum_{n = -\infty}^{\infty} (2n - 1) z^{-n} u(n)$$
 (29)

$$\implies X(z) = \sum_{n=-\infty}^{\infty} (2n) z^{-n} u(n) - \sum_{n=-\infty}^{n=-\infty} z^{-n} u(n) \quad (30)$$

$$\implies X(z) = 2\sum_{n=0}^{\infty} \frac{n}{z^n} - U(z)$$
 (31)

The first part of summation is

$$\implies S(\infty) = \frac{z^2}{(z-1)^2}$$
 (32)

The second part of summation is

$$U(z) = \frac{1}{1 - z^{-1}} \tag{33}$$