GATE GE 81Q

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Question

The value of the convolution of $f(x) = 3\cos(2x)$ and $g(x) = \frac{1}{3}\sin(2x)$ where $x \in [0, 2\pi)$, at $x = \frac{\pi}{3}$, is (Rounded off to 2 decimal places)

Solution

$$f(w) = 3\cos(2w) \tag{1}$$
$$= \frac{3}{2}e^{j2w} + \frac{3}{2}e^{-j2w} \tag{2}$$
$$f(w) = \sum_{k=-\infty}^{\infty} c_k e^{-jwk} \tag{3}$$

by comparing (2) and (3)

$$c_2 = c_{-2} = \frac{3}{2} \tag{4}$$
$$c_k = 0 \quad o.w \tag{5}$$

$$g(w) = \frac{1}{3}\sin(2w) \tag{6}$$
$$= \frac{1}{6}e^{j2w} - \frac{1}{6}e^{-j2w} \tag{7}$$
$$g(w) = \sum_{k=-\infty}^{\infty} d_k e^{-jwk} \tag{8}$$

by comparing (7) and (8)

$$d_2 = \frac{-j}{6} \tag{9}$$
$$d_{-2} = \frac{j}{6} \tag{10}$$
$$d_k = 0 \quad o.w \tag{11}$$

The periodic convolution is multiply Fourier series coefficients is c(n)=c(k)*d(k)*p

$$c_2 = c_2 * d_2 * p \tag{12}$$
$$= \left(\frac{3}{2}\right)\left(\frac{-j}{6}\right)(2\pi) \tag{13}$$
$$= \frac{-j\pi}{2} \tag{14}$$

and

$$c_{-2} = c_{-2} * d_{-2} * p \tag{15}$$
$$= \left(\frac{3}{2}\right)\left(\frac{j}{6}\right)(2\pi) \tag{16}$$
$$= \frac{j\pi}{2} \tag{17}$$

$$f * g(x) = \sum_{n=-N}^{N} c_n e^{j\frac{2\pi nx}{p}} \tag{18}$$
$$= c_{-2}e^{-j2x} + c_2 e^{j2x} \tag{19}$$
$$= \frac{j\pi}{2}e^{-j2x} - \frac{j\pi}{2}e^{j2x} \tag{20}$$
$$= \frac{-j\pi}{2}\left(e^{j2x} - e^{-j2x}\right) \tag{21}$$
$$= \frac{-j\pi}{2}(sin(2x)\,2j) \tag{22}$$
$$= \pi\sin(2x) \tag{23}$$
$$\tag{24}$$

at $x = \frac{\pi}{3}$

$$= \frac{\sqrt{3}\pi}{2} \tag{25}$$
$$\approx 3 \tag{26}$$

Therefore the convolution of f(x) and g(x) is 3

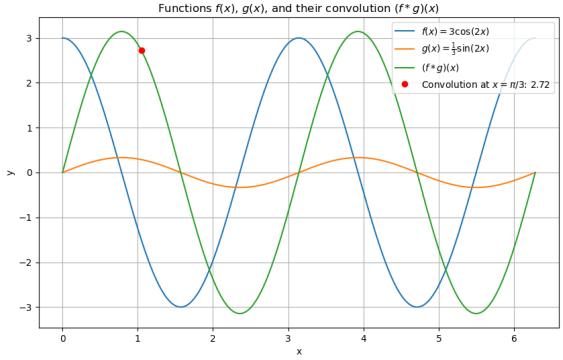

Fig. 0. Plot of y vs x