NCERT 11.9.2 16Q

EE23BTECH11021 - GANNE GOPI CHANDU*

Question

Between 1 and 31, m numbers have been inserted in such a way that the resulting sequence is an A.P. and the ratio of 7 th and (m - 1) th numbers is 5:9. Find the value of m.

Solution

Parameter	Value
First term of A.P $x(0)$	1
Common difference (d)	2
The value of m	14
General term x(n)	(2n-1)u(n)

TABLE 0

$$\text{First term } x(0) = 1 \tag{1}$$

$$\text{last term } x(n) = 31 \tag{2}$$

$$\text{number of terms}(n) = m + 2. \tag{3}$$

From 1,2,3

$$x(n) = x(0) + nd \tag{4}$$

$$31 = 1 + (m+1)\,d \tag{5}$$

$$30 = (m+1)\,d \tag{6}$$

$$\frac{30}{m+1} = d \tag{7}$$

Now 7th and $(m-1)$th terms

$$\implies x_7 = x(0) + 7d \tag{8}$$

$$\implies x_{m-1} = x(0) + (m-1)\,d \tag{9}$$

Given that

$$\frac{x_7}{x_{m-1}} = \frac{5}{9} \tag{10}$$

From 5 and 6:

$$\implies \frac{x(0) + 7d}{x(0) + (m-1)\,d} = \frac{5}{9} \tag{11}$$

From 4 and 9:

$$\implies \frac{1 + 7\left(\frac{30}{m+1}\right)}{1 + (m-1)\left(\frac{30}{m+1}\right)} = \frac{5}{9} \tag{12}$$

$$\implies \frac{m + 1 + 210}{m + 1 + 30m - 30} = \frac{5}{9} \tag{13}$$

$$\implies \frac{m + 181}{31m - 29} = \frac{5}{9} \tag{14}$$

$$\implies 9m + 1899 = 155m - 145 \tag{15}$$

$$\implies 155m - 9m = 1899 + 145 \tag{16}$$

$$\implies 146m = 2044 \tag{17}$$

$$\implies m = 14 \tag{18}$$

Therefore, $m = 14$.

General term of AP as

$$x(n) = 2n - 1 \tag{19}$$

The Z-Transform Equation for x(n) is

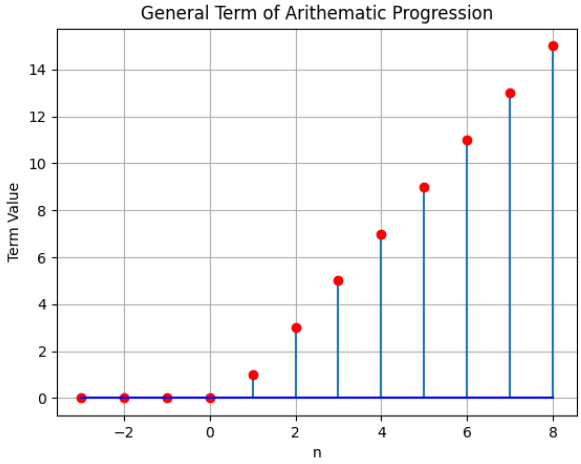

Fig. 0. Plot of general term of AP taken from Python

$$X(z) = \sum_{n=-\infty}^{\infty} (2n-1)\,z^{-n}u(n) \tag{20}$$

$$\implies X(z) = \sum_{n=\infty}^{\infty} (2n)\,z^{-n}u(n) - \sum_{n=-\infty}^{n=\infty} z^{-n}u(n) \tag{21}$$

$$\implies X(z) = 2\sum_{n=0}^{\infty} \frac{n}{z^n} - U(z) \tag{22}$$

The first part of summation is

$$\implies S_\infty = \frac{z^2}{(z-1)^2} \tag{23}$$

The second part of summation is

$$U(z) = \frac{1}{1-z^{-1}} \tag{24}$$

The result is,

$$X(z) = 2S_\infty - U(z) \tag{25}$$

$$X(z) = \frac{z^2}{(z-1)^2} - \frac{1}{1-z^{-1}} \tag{26}$$

$$X(z) = \frac{z^2+z}{(z-1)^2} \tag{27}$$

(ROC) $|z| > 1$.