GATE EC 41Q

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Question
Consider the signals $x[n] = 2^{n-1}u[-n+2]$ and $y[n] = 2^{-n+2}u[n+1]$, where $u[n]$ is the unit step sequence. Let $X(e^{j\omega})$ and $Y(e^{j\omega})$ be the discrete-time Fourier transform of $x[n]$ and $y[n]$, respectively. The value of the integral

$$\frac{1}{2\pi}\int_0^{2\pi} X(e^{j\omega})Y(e^{-j\omega})d\omega$$

(rounded off to one decimal place) is.

Solution
$x[n] * y[n] \xleftrightarrow[transform]{Fourier} X(e^{j\omega})Y(e^{j\omega})$
$x[n] \xleftrightarrow[transform]{Fourier} X(e^{j\omega})$
$y[n] \xleftrightarrow[transform]{Fourier} Y(e^{j\omega})$
The $y(n) = y(e^{j\omega})$
By using the time reversal property:
$y[-n] = y(e^{-j\omega})$
Let assume

$$z[n] = x[n] * y[-n] \tag{1}$$
$$Z\left(e^{j\omega}\right) = X(e^{j\omega})Y(e^{-j\omega}) \tag{2}$$

$$z[n] = \frac{1}{2\pi}\int_0^{2\pi} Z(e^{j\omega})e^{j\omega n}d\omega \tag{3}$$
$$= \frac{1}{2\pi}\int_0^{2\pi} X(e^{j\omega})Y(e^{-j\omega})e^{j\omega n}d\omega. \tag{4}$$

By using the Central Ordinate Theorem:

$$z[0] = \frac{1}{2\pi}\int_0^{2\pi} X(e^{j\omega})Y(e^{-j\omega})d\omega \tag{5}$$

Given

$$x[n] = 2^{n-1}u[-n+2] \tag{6}$$
$$y[n] = 2^{-n+2}u[n+1] \tag{7}$$

$$z[n] = x[n] * y[-n] \tag{8}$$
$$z[n] = (2^{n-1}u[-n+2]) * (2^{n+2}u[-n+1]) \tag{9}$$
$$= \sum_{k=-\infty}^{2} 2^{k-1}u[-k+2] \cdot 2^{n-k+2}u[-n+k+1 \tag{10}$$
$$= \sum_{k=-\infty}^{2} 2^{k-1} \cdot 2^{n-k+2}u[-n+k+1] \tag{11}$$
$$= \sum_{k=-\infty}^{2} 2^{k-1+n-k+2}u[-n+k+1] \tag{12}$$
$$= \sum_{k=-\infty}^{2} 2^{n+1}u[-n+k+1] \tag{13}$$

Putting $n = 0$, we get:

$$\frac{1}{2\pi}\int_0^{2\pi} X(e^{j\omega})Y(e^{-j\omega})d\omega = z[0] \tag{14}$$
$$= \sum_{k=-\infty}^{2} 2 \cdot u[k+1] \tag{15}$$
$$= \sum_{k=-1}^{2} 2(1) = 2 \times 4 \tag{16}$$
$$= 8 \tag{17}$$