NCERT 11.9.2 16Q

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Question

Between 1 and 31, m numbers have been inserted in such a way that the resulting sequence is an A.P. and the ratio of 7 th and (m - 1) th numbers is 5:9. Find the value of m.

Solution

Parameter	Value
First term of A.P $x(0)$	1
Common difference (d)	2
The value of m	14
General term x(n)	(2n-1)u(n)

TABLE 0

$$\text{First term } x(0) = 1 \tag{1}$$

$$\text{last term } x(n) = 31 \tag{2}$$

$$\text{number of terms}(n) = m + 2. \tag{3}$$

from equations **??**,**??** and **??**

$$x(n) = x(0) + nd \tag{4}$$

$$31 = 1 + (m+1)\,d \tag{5}$$

$$30 = (m+1)\,d \tag{6}$$

$$\frac{30}{m+1} = d \tag{7}$$

Now 7th and $(m-1)$th terms

$$\implies x_7 = x(0) + 7d \tag{8}$$

$$\implies x_{m-1} = x(0) + (m-1)\,d \tag{9}$$

Given

$$\frac{x_7}{x_{m-1}} = \frac{5}{9} \tag{10}$$

From equations **??** and **??** the augmented matrix is:

$$\begin{pmatrix} 1 & 7 & x_7 \\ 1 & m-1 & x_{m-1} \end{pmatrix} \xleftrightarrow{R_2 \leftarrow R_2 - R_1} \begin{pmatrix} 1 & 7 & x_7 \\ 0 & m-8 & x_{m-1} - x_7 \end{pmatrix} \tag{11}$$

$$\xleftrightarrow{R_2 \leftarrow \frac{1}{m-8} R_2} \begin{pmatrix} 1 & 7 & x_7 \\ 0 & 1 & \frac{x_{m-1} - x_7}{m-8} \end{pmatrix} \tag{12}$$

$$\xleftrightarrow{R_1 \leftarrow R_1 - 7R_2} \begin{pmatrix} 1 & 0 & x_7 - 7\left(\frac{x_{m-1} - x_7}{m-8}\right) \\ 0 & 1 & \frac{x_{m-1} - x_7}{m-8} \end{pmatrix} \tag{13}$$

$$\implies \begin{pmatrix} x(0) \\ d \end{pmatrix} = \begin{pmatrix} x_7 - 7\left(\frac{x_{m-1} - x_7}{m-8}\right) \\ \frac{x_{m-1} - x_7}{m-8} \end{pmatrix} \tag{14}$$

part 1

from equations **??** and **??**

$$x(0) = x_7 - \left(7\frac{x_{m-1} - x_7}{m-8}\right) \tag{15}$$

$$1 = x_7 - 7\left(\frac{x_{m-1} - x_7}{m-8}\right) \tag{16}$$

$$1 = x_7 - 7\left(\frac{x_7\left(\frac{9}{5}\right) - x_7}{m-8}\right) \tag{17}$$

$$1 = x_7\left((m-8) - \frac{28}{5}\right) \tag{18}$$

part 2

from equations **??** and **??**

$$d = \frac{x_{m-1} - x_7}{m-8} \tag{19}$$

$$\frac{30}{m+1} = \frac{x_7\left(\frac{4}{9}\right)}{m-8} \tag{20}$$

$$x_7 = \frac{75\,(m-8)}{2\,(m+1)} \tag{21}$$

$$\tag{22}$$

Substituting x_7 in part 1

$$m-8=\frac{75\left(m-8\right)\left(5m-68\right)}{10\left(m+1\right)} \tag{23}$$

$$2\left(m+1\right)=15\left(5m-68\right) \tag{24}$$

$$2m+2=75m-1020 \tag{25}$$

$$73m=1022 \tag{26}$$

$$m=14 \tag{27}$$

General term of AP as

$$x\left(n\right)=2n-1 \tag{28}$$

The Z-Transform Equation for x(n) is

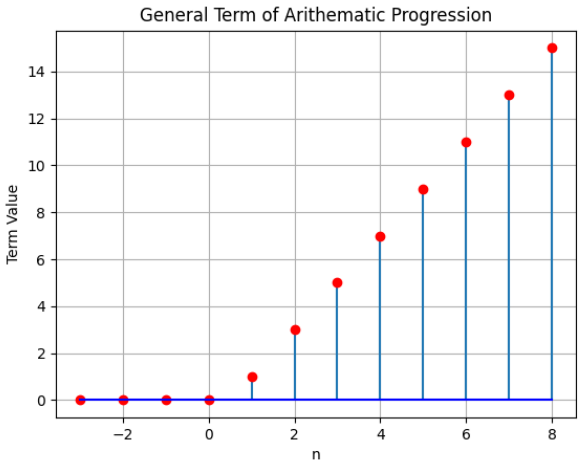

Fig. 0. Plot of x(n) vs n

$$X\left(z\right)=\sum_{n=-\infty}^{\infty}\left(2n-1\right)z^{-n}u\left(n\right) \tag{29}$$

$$\implies X\left(z\right)=\sum_{n=\infty}^{\infty}\left(2n\right)z^{-n}u\left(n\right)-\sum_{n=-\infty}^{n=\infty}z^{-n}u\left(n\right) \tag{30}$$

$$\implies X(z)=2\sum_{n=0}^{\infty}\frac{n}{z^{n}}-U\left(z\right) \tag{31}$$

The first part of summation is

$$\implies S_{\infty}=\frac{z^{2}}{\left(z-1\right)^{2}} \tag{32}$$

The second part of summation is

$$U\left(z\right)=\frac{1}{1-z^{-1}} \tag{33}$$

The result is,

$$X\left(z\right)=2S_{\infty}-U\left(z\right) \tag{34}$$

$$X\left(z\right)=\frac{z^{2}}{\left(z-1\right)^{2}}-\frac{1}{1-z^{-1}} \tag{35}$$

$$X\left(z\right)=\frac{z^{2}+z}{\left(z-1\right)^{2}} \tag{36}$$

(ROC) $|z|>1$.