NCERT 11.9.2 16Q

EE23BTECH11021 - GANNE GOPI CHANDU*

Question

Between 1 and 31, m numbers have been inserted in such a way that the resulting sequence is an A.P. and the ratio of 7 th and (m - 1) th numbers is 5:9. Find the value of m.

Solution

Symbol	Value	description
$x(0)$	1	First term of A.P
x(n)	31	last term
$\frac{x(7)}{x(m-1)}$	$\frac{5}{9}$	ratio of 7 th and (m-1) th numbers
n	m+2	number of terms
m	14	number of terms inserted

TABLE 0

The last term is

$$x(n) = x(0) + nd \tag{1}$$
$$31 = 1 + (m+1)\,d \tag{2}$$
$$30 = (m+1)\,d \tag{3}$$
$$\frac{30}{m+1} = d \tag{4}$$

Now 7th and $(m-1)$th terms

$$\implies x(7) = x(0) + 7d \tag{5}$$
$$\implies x(m-1) = x(0) + (m-1)\,d \tag{6}$$

From equations (5) and (6) the augmented matrix is:

$$\begin{pmatrix} 1 & 7 & x(7) \\ 1 & m-1 & x(m-1) \end{pmatrix} \xleftrightarrow{R_2 \leftarrow R_2 - R_1} \begin{pmatrix} 1 & 7 & x(7) \\ 0 & m-8 & x(m-1)-x(7) \end{pmatrix} \tag{7}$$

$$\xleftrightarrow{R_2 \leftarrow \frac{1}{m-8} R_2} \begin{pmatrix} 1 & 7 & x(7) \\ 0 & 1 & \frac{x(m-1)-x(7)}{m-8} \end{pmatrix} \tag{8}$$

$$\xleftrightarrow{R_1 \leftarrow R_1 - 7R_2} \begin{pmatrix} 1 & 0 & x(7) - 7\left(\frac{x(m-1)-x(7)}{m-8}\right) \\ 0 & 1 & \frac{x(m-1)-x(7)}{m-8} \end{pmatrix} \tag{9}$$

$$\implies \begin{pmatrix} x(0) \\ d \end{pmatrix} = \begin{pmatrix} x(7) - 7\left(\frac{x(m-1)-x(7)}{m-8}\right) \\ \frac{x(m-1)-x(7)}{m-8} \end{pmatrix} \tag{10}$$

part 1

From the table

$$x(0) = x(7) - 7\left(\frac{x(m-1)-x(7)}{m-8}\right) \tag{11}$$
$$1 = x(7) - 7\left(\frac{x(m-1)-x(7)}{m-8}\right) \tag{12}$$
$$1 = x(7) - 7\left(\frac{x(7)\left(\frac{9}{5}\right)-x(7)}{m-8}\right) \tag{13}$$
$$1 = x(7)\left((m-8) - \frac{28}{5}\right) \tag{14}$$

part 2

from equations (4) and from table

$$d = \frac{x(m-1)-x(7)}{m-8} \tag{15}$$
$$\frac{30}{m+1} = \frac{x(7)\left(\frac{4}{9}\right)}{m-8} \tag{16}$$
$$x(7) = \frac{75(m-8)}{2(m+1)} \tag{17}$$

Substituting (17) in (14)

$$m - 8 = \frac{75(m-8)(5m-68)}{10(m+1)} \tag{18}$$
$$2(m+1) = 15(5m-68) \tag{19}$$
$$2m + 2 = 75m - 1020 \tag{20}$$
$$73m = 1022 \tag{21}$$
$$m = 14 \tag{22}$$

General term of AP as

$$x(n) = (2n+1)\,u(n) \tag{23}$$

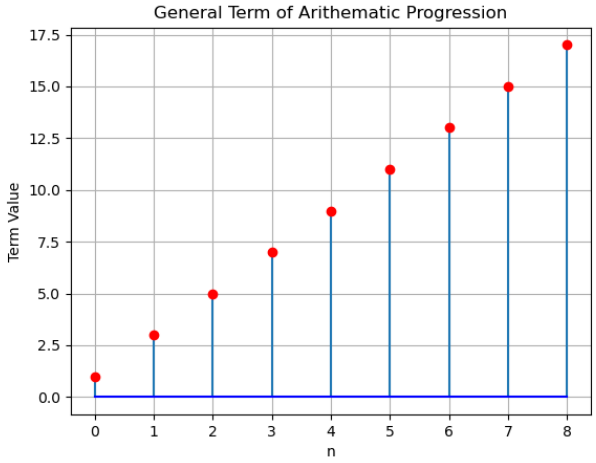

Fig. 0. Plot of x(n) vs n

The Z-Transform Equation for x(n) is

$$X(z) = \sum_{n=-\infty}^{\infty} (2n+1)\,z^{-n}u(n) \tag{24}$$

$$= \sum_{n=\infty}^{\infty} (2n)\,z^{-n}u(n) + \sum_{n=-\infty}^{n=\infty} z^{-n}u(n) \tag{25}$$

$$X(z) = 2\sum_{n=0}^{\infty} \frac{n}{z^n} + U(z) \tag{26}$$

The first part of summation is

$$S(\infty) = \sum_{n=0}^{\infty} \frac{n}{z^n} \tag{27}$$

$$\implies S(\infty) = \frac{z^2}{(z-1)^2} \tag{28}$$

The second part of summation is

$$U(z) = \frac{1}{1-z^{-1}} \tag{29}$$

The result is,

$$X(z) = 2S_{\infty} + U(z) \tag{30}$$

$$= \frac{2z^2}{(z-1)^2} + \frac{1}{1-z^{-1}} \tag{31}$$

$$X(z) = \frac{3z^2 - z}{(z-1)^2} \tag{32}$$

(ROC) $|z| > 1$.