GATE GE 81Q

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Question

The value of the convolution of $f(x) = 3\cos(2x)$ and $g(x) = \frac{1}{3}\sin(2x)$ where $x \in [0, 2\pi)$, at $x = \frac{\pi}{3}$, is (Rounded off to 2 decimal places)

Solution

The f(x) = $3\cos(2x)$ the Fourier series is

$$f(x) = a_0 + \sum_{n=1}^{\infty} a_n \cos(2nx) \tag{1}$$

where $b_n = 0$ since f(x) is even function

$$a_0 = \frac{1}{\pi}\int_0^{2\pi} 3\cos(2x)dx = 0 \tag{2}$$

$$a_n = \frac{1}{\pi}\int_0^{2\pi} 3\cos(2x)\cos(2nx)dx \tag{3}$$

For $g(x) = \frac{1}{3}\sin(2x)$ the Fourier series is:

$$g(x) = \sum_{n=1}^{\infty} b_n \sin(2nx) \tag{4}$$

The $a_0 = a_n = 0$ since g(x) is a odd function

$$b_n = \frac{1}{\pi}\int_0^{2\pi} \frac{1}{3}\sin(2x)\sin(2nx)dx \tag{5}$$

Let's calculate the b_n coefficients for $g(x)$:

$$b_n = \frac{1}{\pi}\int_0^{2\pi} \frac{1}{3}\sin(2x)\sin(2nx)dx \tag{6}$$

$$b_n = \frac{1}{3\pi}\left[\frac{1}{2}\int_0^{2\pi} (\cos((2n-1)x) - \cos((2n+1)x))\, dx\right] \tag{7}$$

$$b_n = \frac{1}{3\pi}\left[\frac{1}{2}\left(\left[\frac{\sin((n-1)2x)}{2(n-1)} - \frac{\sin((n+1)2x)}{2(n+1)}\right]_0^{2\pi}\right)\right] \tag{8}$$

$$b_n = \frac{1}{3\pi}\left[\frac{1}{2}(0 - 0 - 0 + 0)\right] = 0 \tag{9}$$

Since all b_n coefficients are zero

$$(f * g)(x) = 0 \tag{10}$$

So, the convolution of $f(x)$ and $g(x)$ at $x = \frac{\pi}{3}$ is 0.

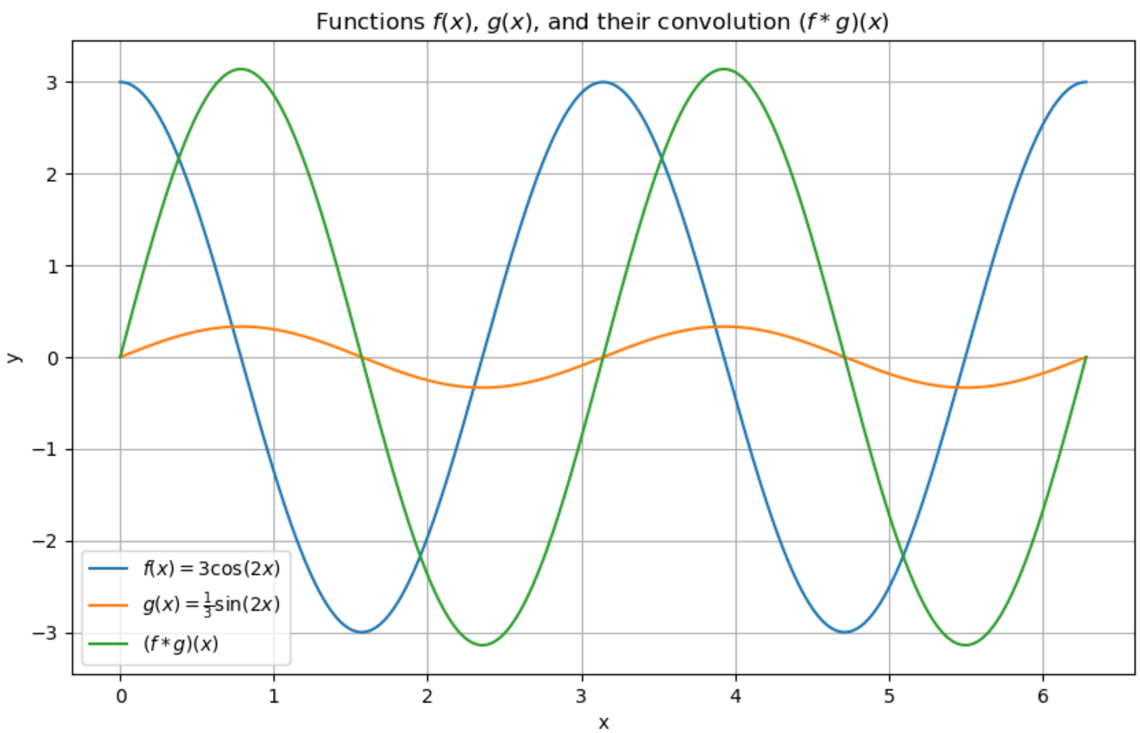

Fig. 0. Plot of y vs x