B06211129 余沛熙

1. (a) $f_Z(z) = \frac{1}{\sqrt{2\pi}} \cdot e^{-\frac{z^2}{2}}$

(b) $P(-1 \le z \le 1) = 1 - 2(1 - P(z \le 1))$

$= 1 - 0.3174$

$= 0.6826$

(c) $2(1-Y) = 0.05 \Rightarrow Y = 0.975$

$P(z \le X) = 0.975$

$X = 1.96$

(d) $f_\theta(\theta) = \frac{1}{\sqrt{2}\,\Gamma(\frac{1}{2})}\, \theta^{\frac{1}{2}} e^{\frac{\theta^2}{2}}$

$= \frac{1}{\sqrt{2\pi}}\, \theta^{\frac{-1}{2}} e^{\frac{-\theta^2}{2}}$

(e) $E[Q] = V = 1$

(f) $\mathrm{std}[Q] = \sqrt{\sigma^2} = \sqrt{2V} = \sqrt{2}$

(g) $P(Q < 1) = 1 - P(Q \ge 1)$

$= 1 - 0.242 = 0.758$

2. (a) $f_T(t) = \begin{cases} e^{-t}, & t > 0 \\ 0, & t < 0 \end{cases}$

(b) $E[T] = \beta = 1$

(c) $\mathrm{std}[T] = \sqrt{\sigma^2} = \sqrt{1^2} = 1$

(d) $P(T > 1) = e^{-1} = 0.3679$

(e) $f_{T_3}(t) = \frac{t^2 e^{-t}}{\Gamma(3)} = \frac{t^2 e^{-t}}{2}, \quad t > 0$

(f) $E[T_3] = 3 \cdot 1 = 3$

(g) $\mathrm{std}[T_3] = \sqrt{\sigma^2} = \sqrt{3 \cdot 1^2} = \sqrt{3}$

(h) $P(T_3 > 3) = 1 - \sum_{t=1}^{3} P(T_3 = t)$

$= 1 - 0.6787$

$= 0.3213$

(i) $P(T_3 > 7) = 1 - \sum_{t=1}^{7} P(T_3 = t)$

$= 1 - 0.9764$

$= 0.0236$