4DK4 Lab 1

Caleb Gannon 400137271

Experiment Results and Analysis

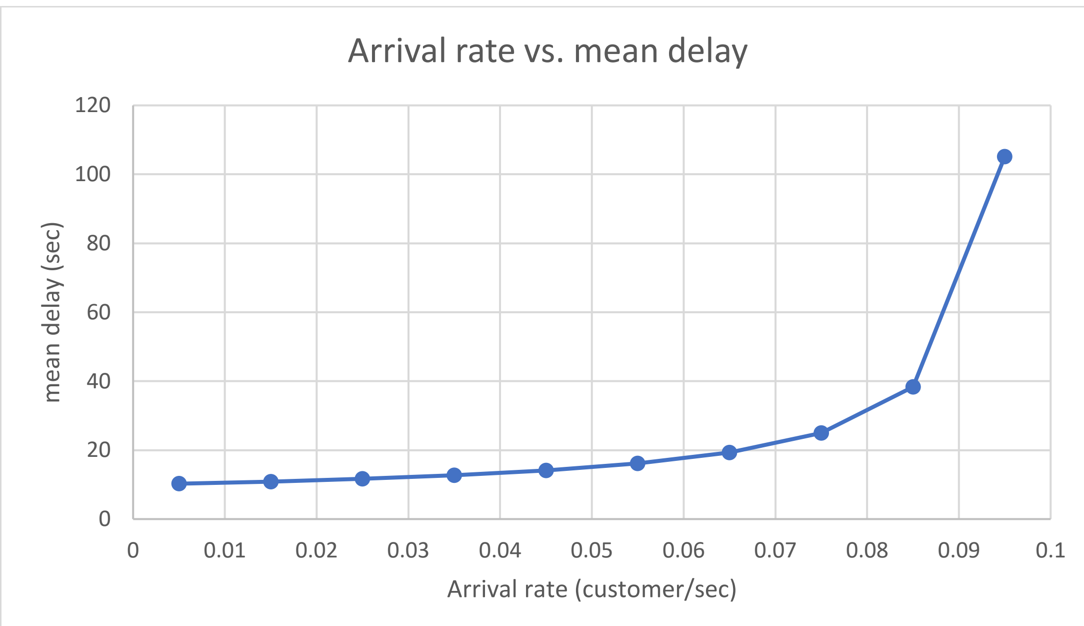
Section 2

Plot mean delay vs. ARRIVAL RATE

I used an array of arrival rates to evenly space the arrival rate intervals for 10 trials while maintaining the condition:

$$0 < \mathtt{ARRIVAL\_RATE} \times \mathtt{SERVICE\_TIME} < 1$$

The range was [0.05, 0.95].



Data is in the appendix at mean delay vs. ARRIVAL RATE Data.

My student number 400137271 was used as one of the seeds. The mean service time was averaged across all the seeds and plotted above.

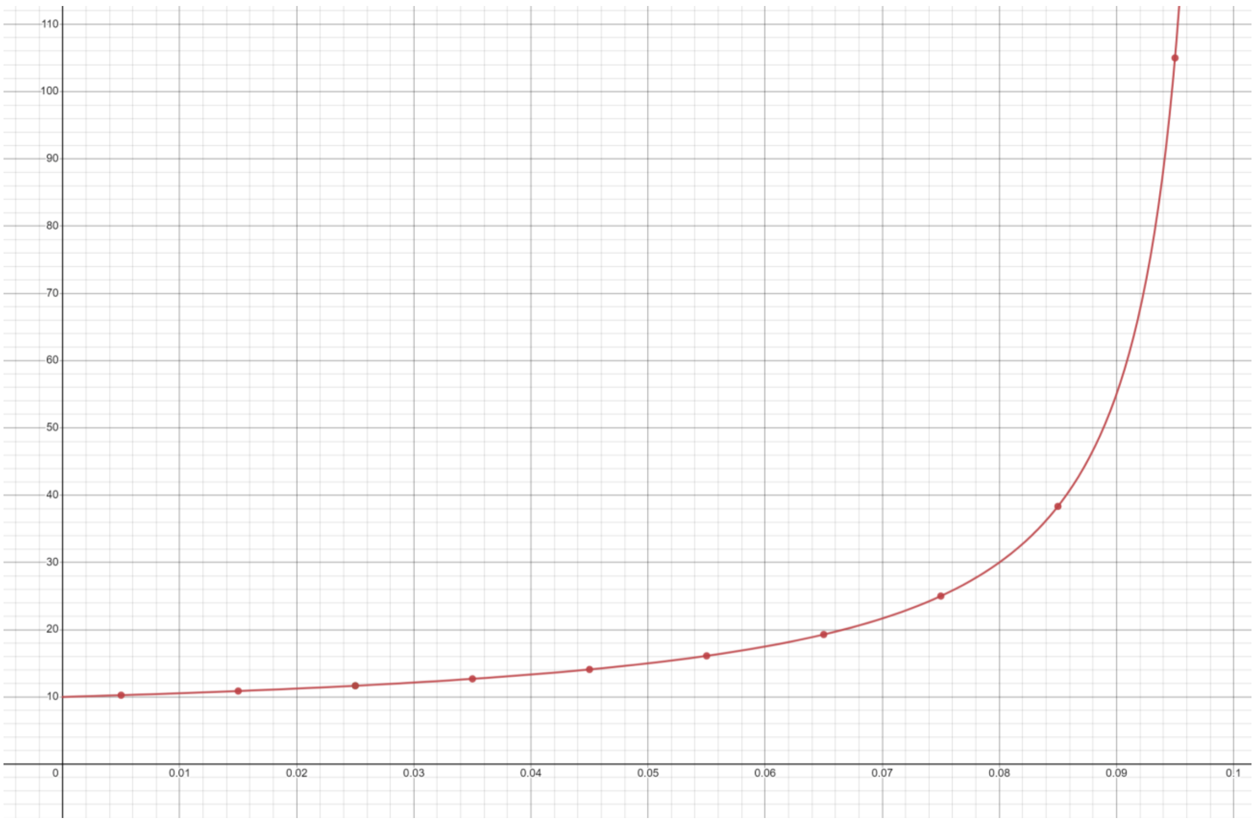
The averages are in the appendix at AVG: mean delay vs. ARRIVAL RATE Data.

At **low arrival rates**, we can see that the mean delay is approaching the value of the SERVICE_RATE, which is 10. This is because at low arrival rates, it is more likely that the system will be empty when a new customer arrives. This means the waiting time $\overline{W}$ trends to zero, so $\overline{T} = \overline{W} + \overline{X} = \overline{X}$.

At **high arrival rates**, we can see that the mean delay is approaching infinity. There will be a **vertical asymptote** at arrival_rate = 0.1. This is because the server utilization is approaching 100%, so customers cannot be cleared as fast as new ones come in. The buffer will continue to fill indefinitely.

Using $\bar{T} = \bar{W} + \bar{X}$ and $\bar{W} = \frac{\lambda \overline{X^2}}{2(1-\lambda \bar{X})}$ we can derive the equation for the graph above. We know that $\bar{X}$ is equal to the mean service time: a constant distribution of mean 10. In a constant distribution $\overline{X^2} = \bar{X}^2$. Therefore, $\overline{X^2}$ = 100. We know that for SERVICE_TIME = 10, the equation for mean delay is:

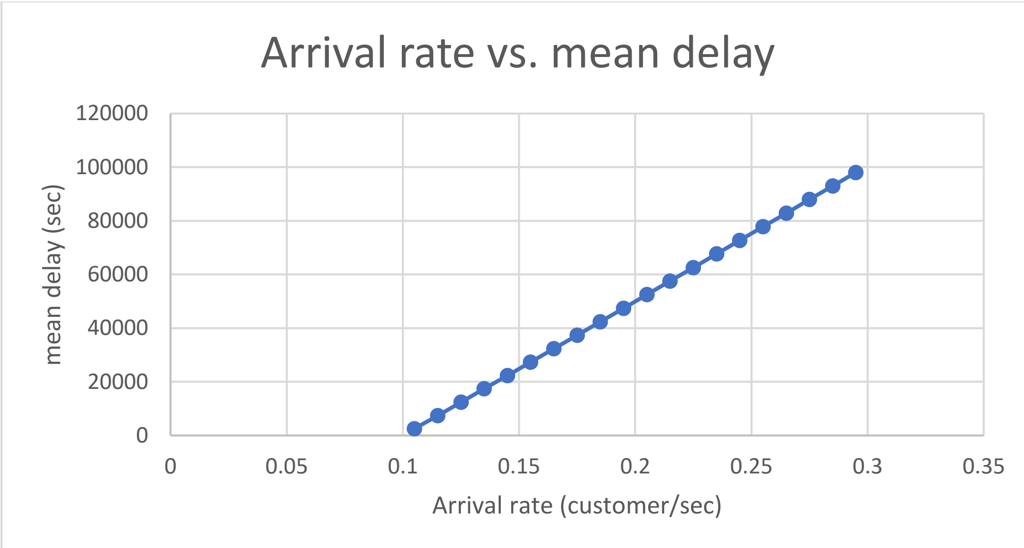
$$\bar{T}(\lambda) = \frac{\lambda \cdot 100}{2(1 - \lambda \cdot 10)} + 10 \qquad (1)$$



The observed values and the model values for mean delay have a correlation of 999999.9ppm.

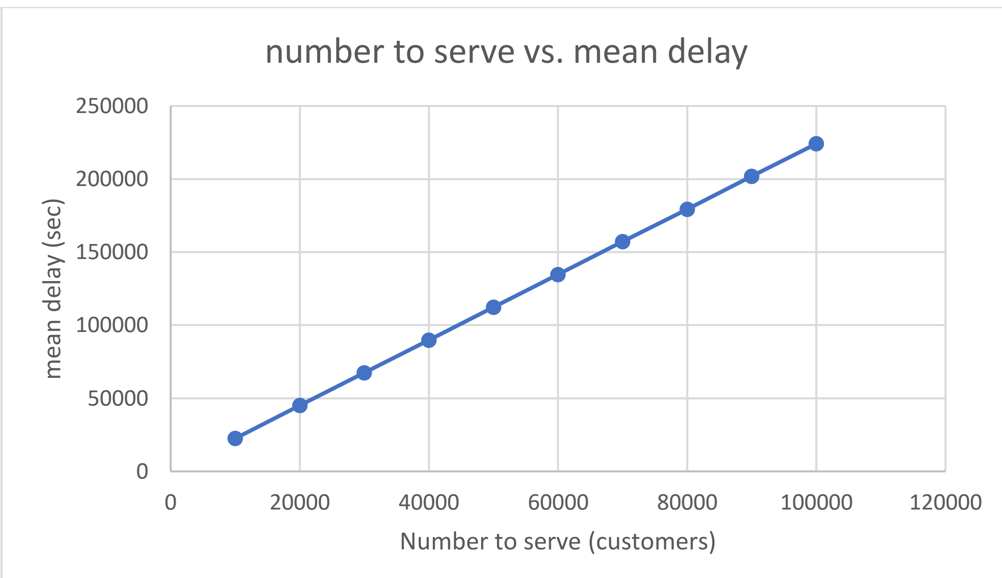
Section 3

When the arrival rate is greater than 1/SERVICE_TIME, the mean delay increases to infinity.



This plot has a `NUMBER_TO_SERVE` of 10,000.

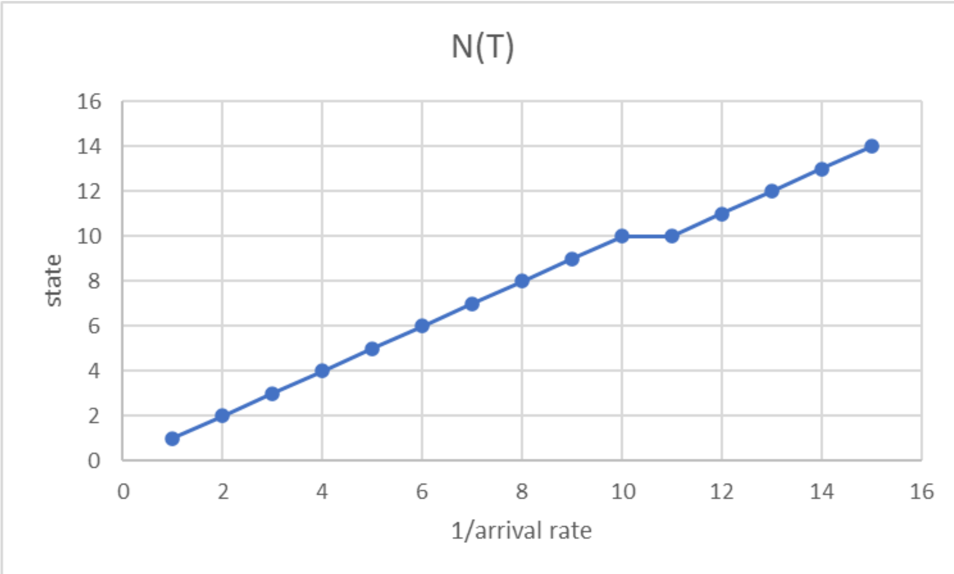
As we **increase the run length** at a fixed interval, we also see the mean delay increase to infinity.



We see these graphs because the server utilization is at 100%. This means that the queue will fill very quickly.

We also know that $\overline{T} = \frac{I}{N_s}$ where I is the total delay, or the area under N(t).

Because server utilization is at 100%, we know that N(t) will increase by 1 every $\frac{1}{\lambda}$. We know that it will decrease every mean SERVICE_TIME interval.



This means we can get the area under the graph roughly equal to:

$$I \cong \frac{N_A \cdot (1 + N_A)}{2} \cdot \frac{1}{\lambda} - \frac{N_S \cdot (1 + N_S)}{2} \cdot \bar{X}$$

Some derivation work follows, to obtain a general model of T.

$$N_A = \lambda \cdot Clock$$

$$N_A = \lambda \cdot \bar{X} \cdot N_S$$

$$\bar{T} \cong \frac{\lambda \cdot \bar{X} \cdot N_S \cdot (1 + \lambda \cdot \bar{X} \cdot N_S)}{2 \cdot N_S} \cdot \frac{1}{\lambda} - \frac{N_S \cdot (1 + N_S)}{2 \cdot N_S} \cdot \bar{X}$$

$$\bar{T} \cong \frac{\bar{X} + \lambda \cdot \bar{X}^2 \cdot N_S}{2} - \frac{(1 + N_S)}{2} \cdot \bar{X}$$

$$\bar{T} \cong \frac{\bar{X}}{2} - \frac{1}{2} \cdot \bar{X} + \frac{\lambda \cdot \bar{X}^2 \cdot N_S}{2} - \frac{N_S}{2} \cdot \bar{X}$$

$$\bar{T} \cong \frac{\lambda \cdot \bar{X}^2 \cdot N_S}{2} - \frac{N_S}{2} \cdot \bar{X}$$

The model demonstrates that T is linearly proportional to both the arrival rate, and number served (run time). Results are in the appendix:

Max Server Utilization mean delay model.

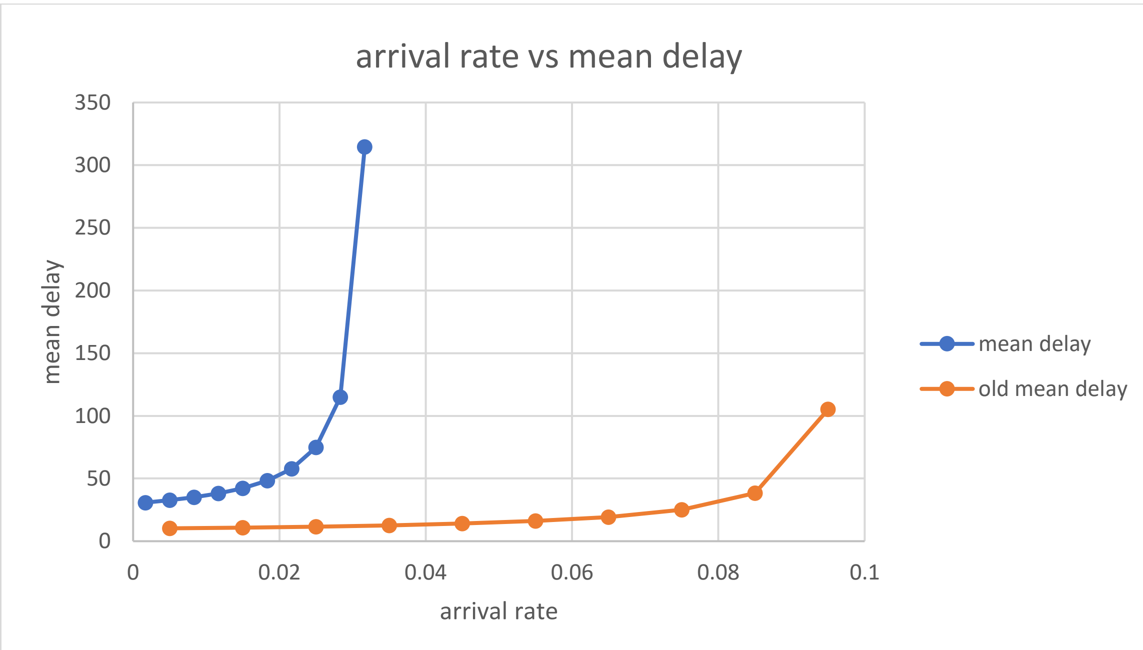
This queue exploding to infinity only occurs when server utilization is 100%. New customers will arrive faster than they can be served. That is why we **need to stay within the range** specified in the lab manual.

$$0 < \texttt{ARRIVAL\_RATE} \times \texttt{SERVICE\_TIME} < 1$$

Arrival time x Service rate is the server utilization, and if it exceeds 100% then the queue buffer will explode to infinity.

Section 4

The arrival rates were ranged over the interval [0.001667, 0.031666]. The raw data is in the appendix: mean delay vs. ARRIVAL RATE Data. (Part 4).



This run was done with mean delay -> SERVICE_TIME = 30, old mean delay -> SERVICE_TIME = 10.

This graph shows that as the SERVICE_TIME **increases**, the arrival rate asymptote for server utilization = 100% will **decrease.**

This means that as service time increases, the arrival rate must also decrease, to keep server Utilization low.

Section 5

Using $\overline{T} = \overline{W} + \overline{X}$ and $\overline{W} = \frac{\lambda \overline{X^2}}{2(1-\lambda\overline{X})}$, we can compare the experimental results against the statistical models.

In a constant distribution $\overline{X^2} = \overline{X}^2$. Therefore, the M/G/1 is equivalent to the M/D/1 model for a constant distribution.

Using a model of the form:

$$\overline{T}(\lambda) = \frac{\lambda \cdot \overline{X^2}}{2 \cdot (1 - \lambda \cdot \overline{X})} + \overline{X}$$

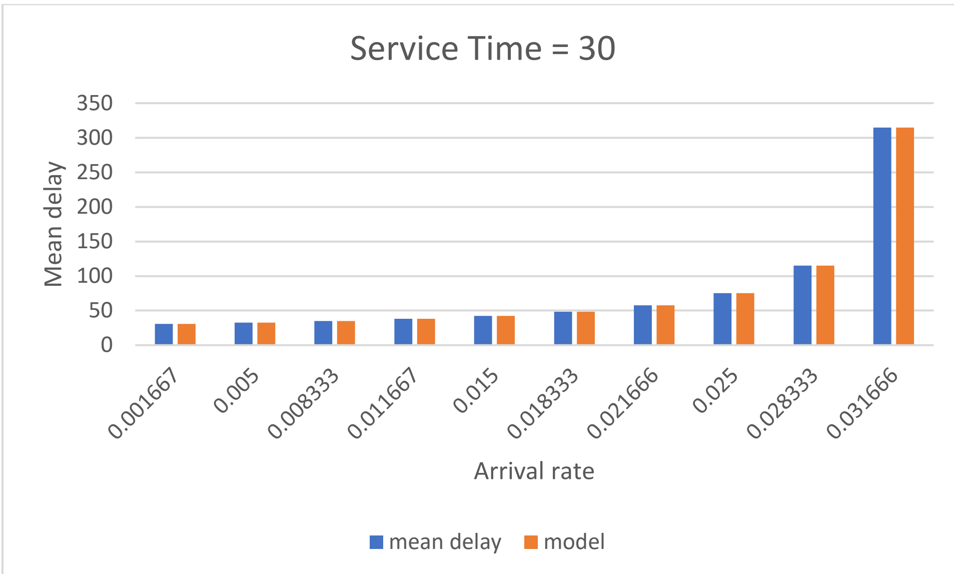
Which is equivalent to:

$$\bar{T}(\lambda) = \frac{\bar{X} \cdot (2 - \lambda \cdot \bar{X})}{2 \cdot (1 - \lambda \cdot \bar{X})}$$

When $\overline{X^2} = \bar{X}^2$.

Where $\bar{X}$ is the service rate.

We get the following results. Model data is in the appendix.



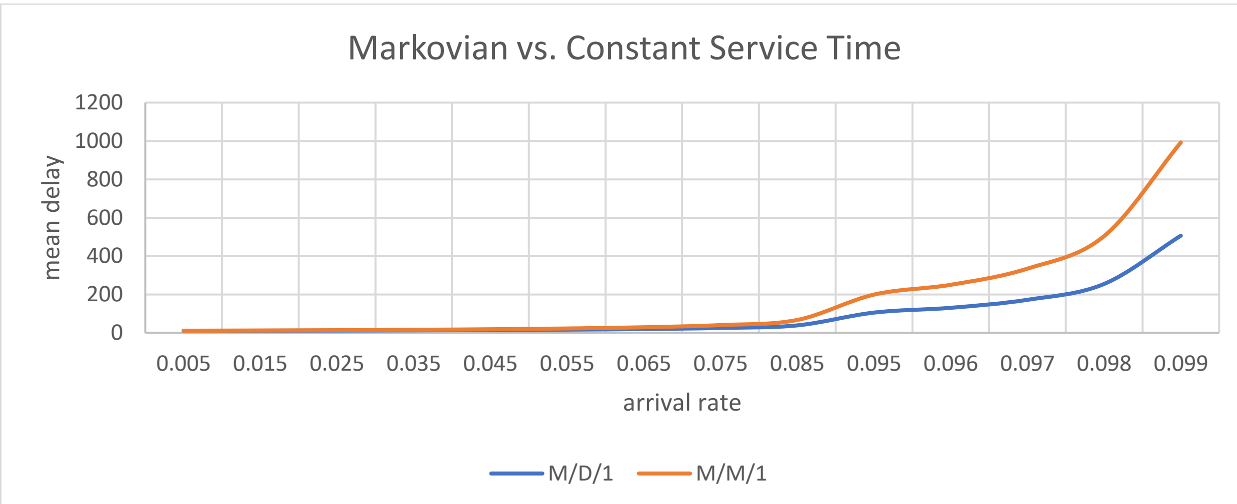
Appendix: AVG: mean delay vs. ARRIVAL RATE Data. (Part4)

Appendix: AVG: mean delay vs. ARRIVAL RATE Data.

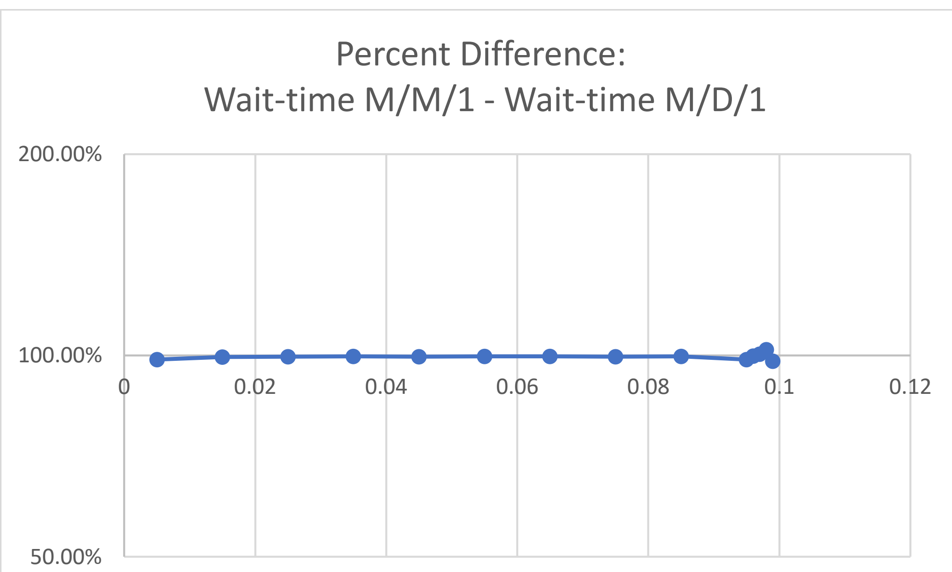
Section 6

When the service time is changed to an exponential distribution, we see that the **mean delay** for both cases follow a similar model. The mean delay increases as the arrival rate increases, and it trends to infinity as the 1/mean_service_rate value is approached. It is also apparent that the **waiting (queuing) time** ($\overline{W}$) for the M/D/1 is half of the waiting time for M/M/1.

We know that $\overline{T} = \overline{W} + \overline{X}$, and $\overline{X} = 10$ in both cases. Both graphs have a horizontal asymptote at 10. The waiting time ($\overline{W}$) is the difference between $\overline{T}$ and 10. We can see in the following graph, that the mean delay for the M/D/1 is nearly half of the M/M/1. This becomes more noticeable as the arrival rate increases.



When calculating a percentage difference between the waiting time for M/M/1 and M/D/1, the doubling becomes even clearer.



We know that

$$\bar{T}(\lambda) = \frac{\lambda \cdot \overline{X^2}}{2 \cdot (1 - \lambda \cdot \bar{X})} + \bar{X}$$

Where $\bar{X}$ is the service rate.

We know that for a Markovian (exponential distribution):

$$\bar{X} = \frac{1}{\mu}$$

$$\sigma = \frac{1}{\mu}$$

$$Var = \sigma^2 = \overline{X^2} - \bar{X}^{\,2}$$

$$\therefore \overline{X^2} = \frac{2}{\mu^2}$$

$$\mu = 0.1$$

$$\therefore \overline{X^2} = 200$$

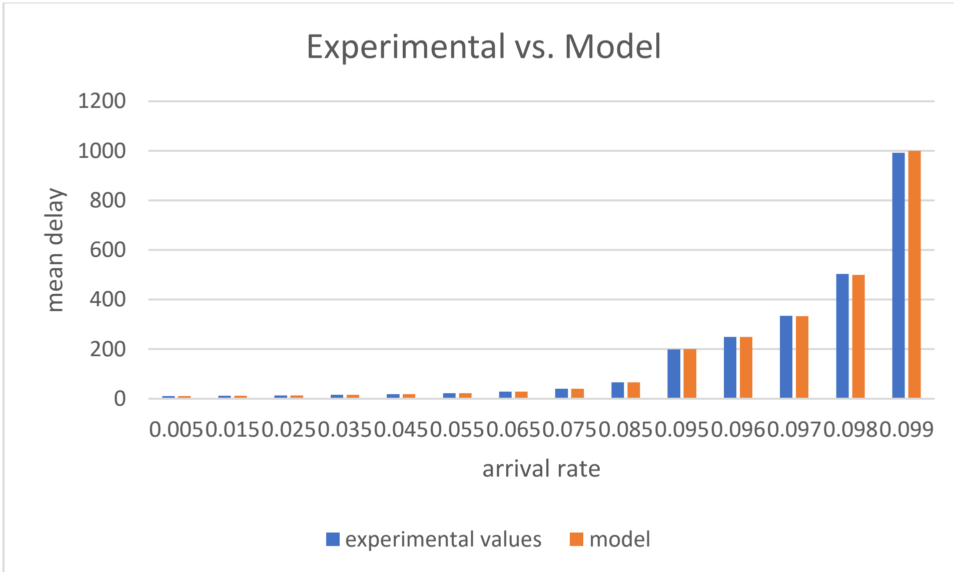
$$\therefore \bar{T}(\lambda) = \frac{\lambda \cdot 200}{2 \cdot (1 - \lambda \cdot 10)} + 10 = \frac{1}{\mu - \lambda}$$

When comparing this to the model for M/D/1 derived in section 2 (where $\overline{X^2} = \bar{X}^2$), we see that the waiting time is in fact double for the M/M/1

$$\bar{T}(\lambda) = \frac{\lambda \cdot 100}{2(1 - \lambda \cdot 10)} + 10 \qquad (M/D/1)$$

This is because in the exponential distribution, the mean **residual** time for a service event to occur is $\frac{1}{\mu}$ whereas in the constant distribution, the mean residual time is $\frac{1}{2\mu}$.

Comparing the experimental data against the above model, we get the following results.

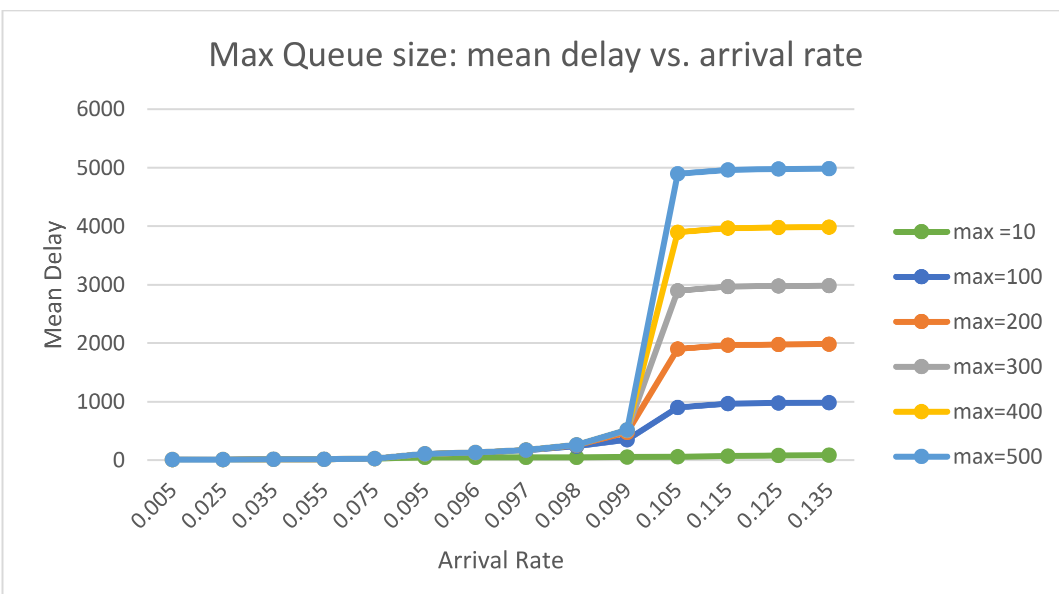


AVG: mean delay vs. ARRIVAL RATE Data. (Part6)

Section 7

When a MAX_QUEUE_SIZE is added, it seems that there are two observed behaviours of the mean delay: **before** the asymptote, and **after** the asymptote.

After the asymptote, the mean delay trends to MAX_QUEUE_SIZE x mean_service_time. This is because any customer that is not rejected will have to wait for the queue size x mean_service time, and queue size will equal MAX_QUEUE_SIZE.



Appendix: Max Queue size: mean delay vs. arrival rate

We can see that the asymptotic mean delay values approach

$$buffer_{max} \cdot mean\ service\ time$$

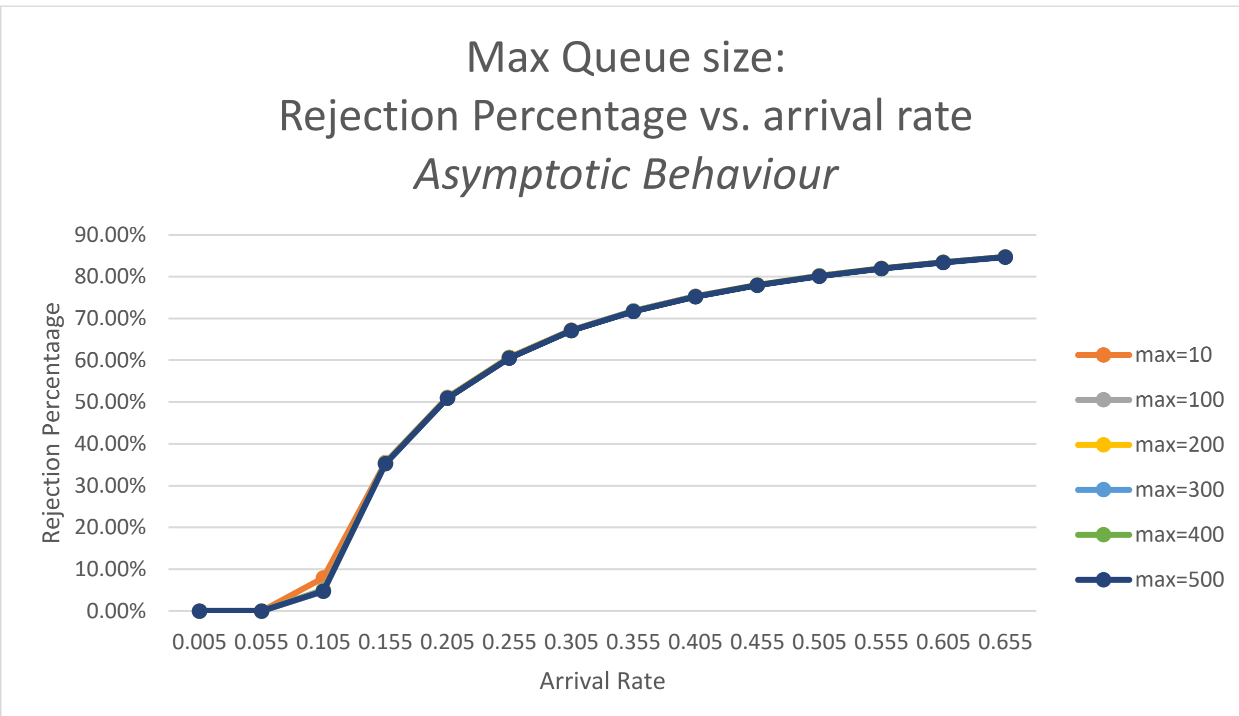
This is **different from the infinite queue case.** In the infinite case, the mean delay would continue to increase to infinity as the arrival rate increased. This is because an arrival would get serviced, after waiting an unbounded amount of time. This meant that the mean delay was unbounded, and dependant only on the arrival rate and total number served.

In the **finite queue case**, when the system is unstable the majority of new customers will be rejected: only those who are not rejected will be serviced, and their waiting time will **always** be MAX_QUEUE_SIZE x mean_service_time, because the queue is full.

Rejection Percentage

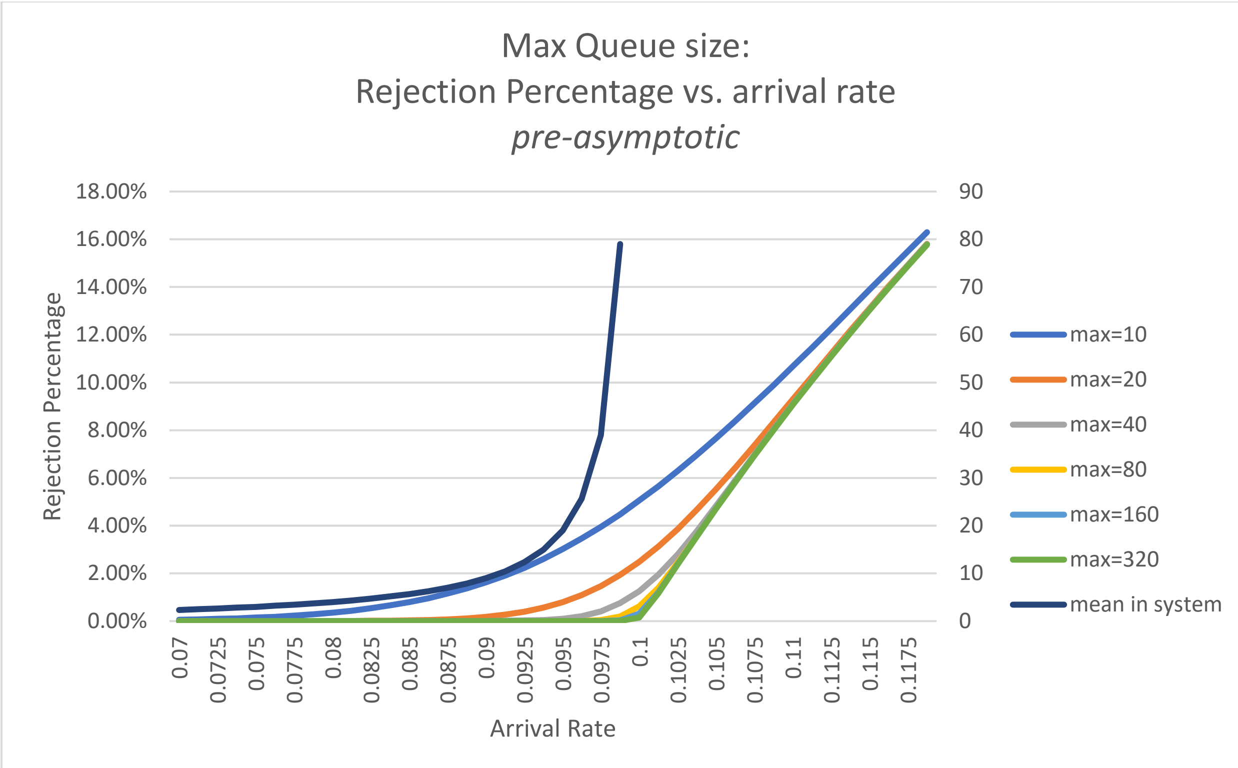
We define the rejection percentage as the number of rejected customers, divided by the total number of customers. The following is a plot of the asymptotic behavior, for large values of arrival rate. We can see that the MAX_QUEUE_SIZE has little impact at high values of arrival rate, as most customers will be rejected due to the utilization being at 100%.

The following graph demonstrated how for large values of arrival rate; the rejection percentage trends towards 100% for all values of MAX_QUEUE_SIZE.



Appendix: Max Queue size: Rejection Percentage vs. arrival rate *Asymptotic Behaviour*

The **pre-asymptotic** behaviour is different. In the following graph, we see that the smaller values of max see an increased rejection percentage.

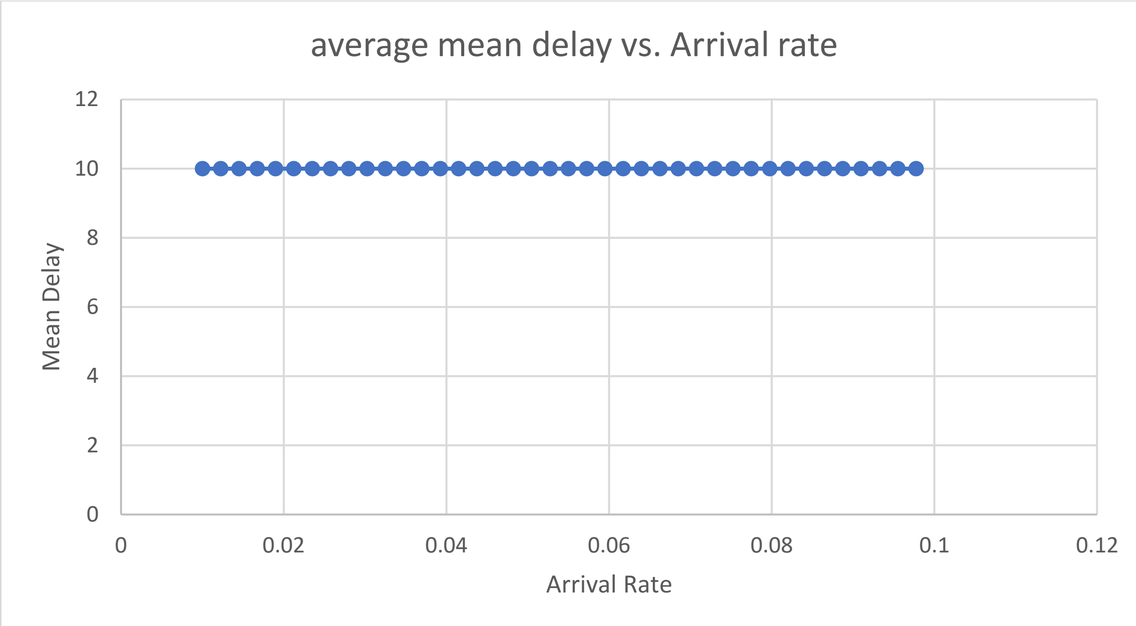


Appendix: Max Queue size: Rejection Percentage vs. arrival rate *pre-asymptotic*

This is because the mean number of customers in the system approaches the max buffer **faster** when the max buffer is smaller. A smaller buffer has a higher probability of reaching the max, at a given arrival rate. We can see how a buffer size could be design based on the expected arrival rate. The rejection probability stays very low, it only increases as the mean in the system approaches the max buffer size.

Section 8

When the arrival rate and the service rate are both constant distributions, the graph of arrival rate vs mean delay becomes constant for a stable system.



Appendix: *average mean delay vs. Arrival rate Part 8*

At **low** arrival rates and **high** arrival rates, the mean delay is a constant 10. We see this constant 10, because the arrivals are now fixed at a constant rate. It is no longer possible for several arrivals to occur in a short succession, which would increase the waiting time $\overline{W}$. This is because the **variance** has decreased. Using Kingman's approximation for a G/G/1 mean waiting time, we can see that as the co-efficients of variance (c_a, c_s) go to 0, the mean waiting time goes to 0.

$$\mathbb{E}(W_q) \approx \left(\frac{\rho}{1-\rho}\right)\left(\frac{c_a^2 + c_s^2}{2}\right)\tau$$

This leaves mean delay as:

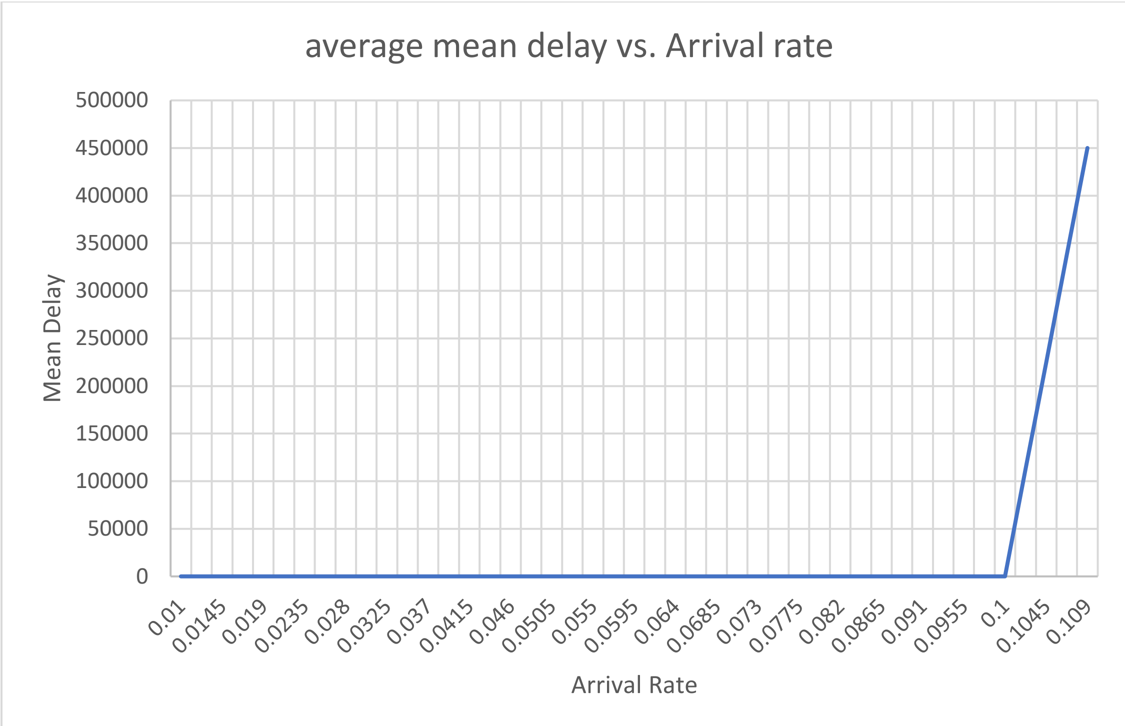
$$\overline{T} = \overline{W} + \overline{X}$$

$$\overline{T} = 0 + \overline{X}$$

$$\therefore \overline{T} = \overline{X}$$

This only applies to a stable system.

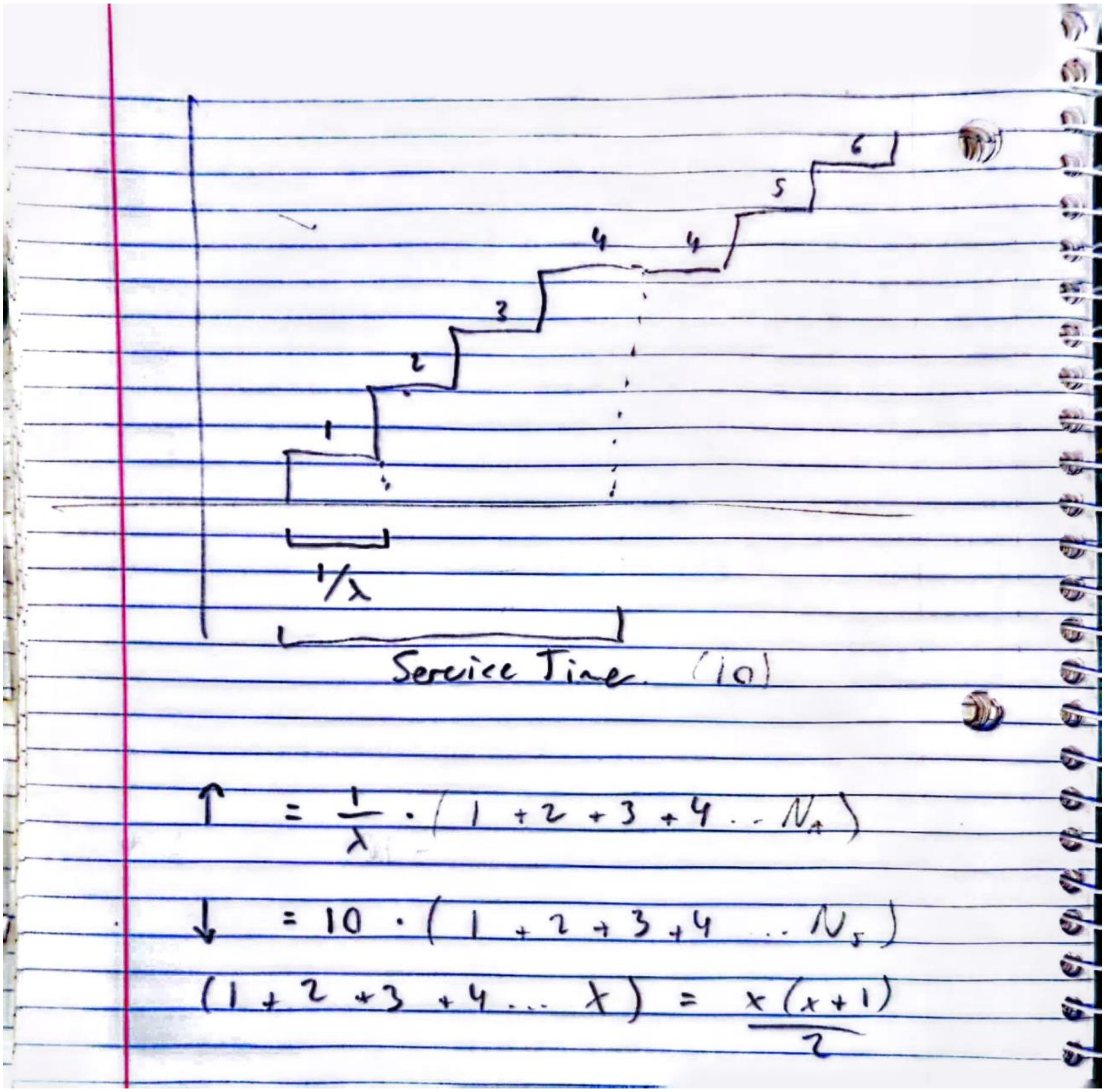
As soon as the arrival rate **exceeds** 1/service_rate, the mean delay explodes to infinity.



Appendix: *average mean delay vs. Arrival rate Asymptote with model Part 8*

I used the model derived in *Section 3* to predict the values observed for a given arrival rate that is greater than 0.1. The model was quite accurate, it seems exactly equal to the waiting time $\overline{W}$.

This model is derived by looking at the number in the system vs. time.



Every $\frac{1}{\lambda}$ we have an arrival, every $\frac{1}{\mu}$ we have a departure. This means that if we know the total number of arrivals and departures, we can calculate the area under the curve. We know that the area under the curve is the total delay for that run (I). We can get the mean delay, as $\bar{T} = \frac{I}{N_S}$.

This ultimately results in the model

$$\bar{T} \cong \frac{\lambda \cdot \bar{X}^2 \cdot N_S}{2} - \frac{N_S}{2} \cdot \bar{X}$$

Extended derivation is shown in *Section 3.*

This graph of N(t) vs. Time shows that the **shape of the curve** is continuing to expand to infinity, as arrivals occur faster than departures. This results in a utilization of 100%, and an unstable system. The results are the mean delay exploding to infinity, **or** if there is a buffer max, it will explode to a fixed value.

Modifications to code

For-loops were used to allow a range of arrival rates, seeds, number to serve, and max queue sizes to be generated: to collect a fine range of data.

Data was exported en-masse to a .csv file, where the data was manipulated in excel to create the plots.

The code was modified throughout the lab, to enable the ability to have a max buffer size, as well as to modify the service time and arrival time distributions.

Appendix

mean delay vs. ARRIVAL RATE Data.

Student number is 400137271.

arrival rate	mean delay	random seed
0.005	10.26321	5259140
0.005	10.263419	400137271
0.005	10.262991	124125
0.005	10.26294	4863905
0.005	10.262885	1946482
0.005	10.26301	73449993
0.005	10.262881	113513516
0.005	10.262769	84593432
0.005	10.26325	11552266
0.005	10.262806	34578944
0.015	10.882994	5259140
0.015	10.883382	400137271
0.015	10.88206	124125
0.015	10.881988	4863905
0.015	10.881623	1946482
0.015	10.882499	73449993
0.015	10.881316	113513516
0.015	10.881514	84593432
0.015	10.88297	11552266
0.015	10.881283	34578944
0.025	11.668134	5259140
0.025	11.667898	400137271
0.025	11.666353	124125
0.025	11.666115	4863905
0.025	11.666128	1946482
0.025	11.667119	73449993

0.025	11.665365	113513516
0.025	11.666139	84593432
0.025	11.667474	11552266
0.025	11.665405	34578944
0.035	12.693893	5259140
0.035	12.694215	400137271
0.035	12.693184	124125
0.035	12.691347	4863905
0.035	12.692295	1946482
0.035	12.692916	73449993
0.035	12.69007	113513516
0.035	12.692875	84593432
0.035	12.692639	11552266
0.035	12.690265	34578944
0.045	14.093676	5259140
0.045	14.095436	400137271
0.045	14.093402	124125
0.045	14.088875	4863905
0.045	14.090992	1946482
0.045	14.091624	73449993
0.045	14.087382	113513516
0.045	14.092228	84593432
0.045	14.090435	11552266
0.045	14.08758	34578944
0.055	16.11665	5259140
0.055	16.120261	400137271
0.055	16.116261	124125
0.055	16.108001	4863905
0.055	16.110627	1946482
0.055	16.112469	73449993
0.055	16.106762	113513516
0.055	16.112466	84593432
0.055	16.111925	11552266
0.055	16.106567	34578944
0.065	19.300901	5259140
0.065	19.302736	400137271
0.065	19.293921	124125
0.065	19.278559	4863905
0.065	19.284307	1946482
0.065	19.285934	73449993
0.065	19.282664	113513516
0.065	19.286512	84593432
0.065	19.289379	11552266

0.065	19.27998	34578944
0.075	25.020947	5259140
0.075	25.02291	400137271
0.075	25.007514	124125
0.075	24.97699	4863905
0.075	24.999669	1946482
0.075	24.99927	73449993
0.075	24.996776	113513516
0.075	24.999157	84593432
0.075	25.014616	11552266
0.075	24.985374	34578944
0.085	38.35902	5259140
0.085	38.358463	400137271
0.085	38.333843	124125
0.085	38.270771	4863905
0.085	38.329979	1946482
0.085	38.364151	73449993
0.085	38.309869	113513516
0.085	38.322338	84593432
0.085	38.415986	11552266
0.085	38.27795	34578944
0.095	104.483032	5259140
0.095	105.630895	400137271
0.095	105.253244	124125
0.095	104.833254	4863905
0.095	104.978166	1946482
0.095	105.809409	73449993
0.095	104.930784	113513516
0.095	105.024328	84593432
0.095	105.675779	11552266
0.095	104.897435	34578944

AVG: mean delay vs. ARRIVAL RATE Data.

arrival rate	mean service time	model values
0.005	10.2630161	10.26315789
0.015	10.8821629	10.88235294
0.025	11.666613	11.66666667
0.035	12.6923699	12.69230769
0.045	14.091163	14.09090909
0.055	16.1121989	16.11111111
0.065	19.2884893	19.28571429
0.075	25.0023223	25

0.085	38.334237	38.33333333
0.095	105.1516326	105

Max Server Utilization mean delay model

arrival rate	number served	actual mean delay	model	corr
0.225	100000	626989.9	625000	0.999999951
0.235	100000	676885	675000	
0.245	100000	726724.1	725000	
0.255	100000	776551	775000	
0.265	100000	826384.3	825000	
0.275	100000	876252.3	875000	
0.285	100000	926131.6	925000	
0.295	100000	976026.4	975000	
0.305	100000	1025939	1025000	
0.315	100000	1075865	1075000	

mean delay vs. ARRIVAL RATE Data. (Part 4)

Student number is 400137271.

arrival rate	mean delay	random seed
0.001667	30.78704	34578944
0.005	32.64359	34578944
0.008333	34.9965	34578944
0.011667	38.07022	34578944
0.015	42.26247	34578944
0.018333	48.3245	34578944
0.021666	57.84352	34578944
0.025	74.98934	34578944
0.028333	115.0362	34578944
0.031666	316.4332	34578944
0.001667	30.79025	400137271
0.005	32.65012	400137271
0.008333	35.00363	400137271
0.011667	38.08252	400137271
0.015	42.28608	400137271
0.018333	48.36037	400137271
0.021666	57.90741	400137271
0.025	75.06693	400137271
0.028333	115.0697	400137271

0.031666	316.8345	400137271
0.001667	30.78885	1367137
0.005	32.64249	1367137
0.008333	34.9925	1367137
0.011667	38.06703	1367137
0.015	42.25385	1367137
0.018333	48.30384	1367137
0.021666	57.8186	1367137
0.025	74.93884	1367137
0.028333	115.0784	1367137
0.031666	316.0808	1367137
0.001667	30.78875	9750342
0.005	32.64546	9750342
0.008333	34.99687	9750342
0.011667	38.07352	9750342
0.015	42.26998	9750342
0.018333	48.32571	9750342
0.021666	57.85694	9750342
0.025	75.03043	9750342
0.028333	115.03	9750342
0.031666	312.2466	9750342
0.001667	30.78792	1317178
0.005	32.6422	1317178
0.008333	34.99471	1317178
0.011667	38.07076	1317178
0.015	42.25962	1317178
0.018333	48.31695	1317178
0.021666	57.84784	1317178
0.025	74.98838	1317178
0.028333	115.0166	1317178
0.031666	314.8653	1317178
0.001667	30.79024	83539228
0.005	32.64633	83539228
0.008333	34.99948	83539228
0.011667	38.07474	83539228
0.015	42.2646	83539228
0.018333	48.31865	83539228
0.021666	57.8378	83539228
0.025	74.99704	83539228
0.028333	114.8795	83539228
0.031666	312.4023	83539228
0.001667	30.78819	32582424
0.005	32.64728	32582424

0.008333	34.99796	32582424
0.011667	38.07311	32582424
0.015	42.27002	32582424
0.018333	48.32993	32582424
0.021666	57.86454	32582424
0.025	75.04544	32582424
0.028333	115.0226	32582424
0.031666	315.0148	32582424
0.001667	30.79023	35932147
0.005	32.64774	35932147
0.008333	34.99777	35932147
0.011667	38.07215	35932147
0.015	42.26802	35932147
0.018333	48.32749	35932147
0.021666	57.84298	35932147
0.025	74.94727	35932147
0.028333	114.7821	35932147
0.031666	314.051	35932147
0.001667	30.78842	994627257
0.005	32.6451	994627257
0.008333	34.99749	994627257
0.011667	38.07485	994627257
0.015	42.27332	994627257
0.018333	48.33646	994627257
0.021666	57.86142	994627257
0.025	75.0066	994627257
0.028333	114.9527	994627257
0.031666	314.8691	994627257
0.001667	30.79012	12345611
0.005	32.64838	12345611
0.008333	35.00103	12345611
0.011667	38.07546	12345611
0.015	42.26439	12345611
0.018333	48.31932	12345611
0.021666	57.84396	12345611
0.025	74.98614	12345611
0.028333	114.7829	12345611
0.031666	312.3396	12345611

AVG: mean delay vs. ARRIVAL RATE Data. (Part4)

arrival rate	mean delay	model
0.001667	30.7890003	30.78964

0.005	32.6458679	32.64706
0.008333	34.9977939	34.99973
0.011667	38.0734348	38.07728
0.015	42.2672358	42.27273
0.018333	48.3263225	48.33259
0.021666	57.8525	57.85469
0.025	74.9996404	75
0.028333	114.9650645	114.9933
0.031666	314.5137167	314.88

AVG: mean delay vs. ARRIVAL RATE Data. (Part6)

arrival rate	M/D/1 mean delay	M/M/1 mean delay	M/M/1 model data
0.005	10.26307871	10.52259314	10.52631579
0.015	10.88239636	11.76035114	11.76470588
0.025	11.66702771	13.3269235	13.33333333
0.035	12.69272129	15.37669186	15.38461538
0.045	14.09105007	18.16627236	18.18181818
0.055	16.11050671	22.20012229	22.22222222
0.065	19.28362457	28.53997614	28.57142857
0.075	24.99651857	39.93534357	40
0.085	38.34577393	66.61775664	66.66666667
0.095	105.1058584	198.8905461	200
0.096	130.0680039	249.926739	250
0.097	171.7434834	334.1772574	333.3333333
0.098	254.4460249	503.8206946	500
0.099	506.0808052	992.3412267	1000

Max Queue size: Rejection Percentage vs. arrival rate *Asymptotic Behaviour*

arrival rate	mean delay	seed	rejection percentage	max buffer size
0.005	10.254398	400137271	0.04905	1
0.055	13.326692	400137271	0.422759	1
0.105	17.716101	400137271	0.650429	1
0.155	23.940914	400137271	0.787713	1
0.205	33.026785	400137271	0.871235	1
0.255	46.321339	400137271	0.921922	1
0.305	65.857822	400137271	0.952554	1
0.355	95.206807	400137271	0.971258	1
0.405	139.317525	400137271	0.982583	1
0.455	205.69864	400137271	0.98943	1

0.505	308.012138	400137271	0.993613	1
0.555	463.521173	400137271	0.996128	1
0.605	699.398276	400137271	0.997642	1
0.655	1083.423699	400137271	0.998593	1
0.005	10.267528	400137271	0	10
0.055	16.174638	400137271	0.00002	10
0.105	60.909485	400137271	0.079615	10
0.155	89.506052	400137271	0.355419	10
0.205	93.959071	400137271	0.511541	10
0.255	95.642575	400137271	0.606971	10
0.305	96.519315	400137271	0.672107	10
0.355	97.093119	400137271	0.718171	10
0.405	97.487093	400137271	0.75308	10
0.455	97.780604	400137271	0.780192	10
0.505	98.012698	400137271	0.802036	10
0.555	98.19338	400137271	0.819884	10
0.605	98.339072	400137271	0.834737	10
0.655	98.472904	400137271	0.847385	10
0.005	10.267528	400137271	0	100
0.055	16.179246	400137271	0	100
0.105	896.362551	400137271	0.050646	100
0.155	988.65345	400137271	0.354795	100
0.205	993.509286	400137271	0.511102	100
0.255	995.334012	400137271	0.606617	100
0.305	996.285155	400137271	0.671812	100
0.355	996.904672	400137271	0.717917	100
0.405	997.329571	400137271	0.752858	100
0.455	997.645812	400137271	0.779994	100
0.505	997.894646	400137271	0.801858	100
0.555	998.088297	400137271	0.819722	100
0.605	998.244468	400137271	0.834588	100
0.655	998.386827	400137271	0.847248	100
0.005	10.267528	400137271	0	200
0.055	16.179246	400137271	0	200
0.105	1870.609537	400137271	0.049697	200
0.155	1986.100514	400137271	0.354151	200
0.205	1992.165191	400137271	0.510613	200
0.255	1994.422322	400137271	0.606224	200
0.305	1995.59858	400137271	0.671484	200
0.355	1996.355331	400137271	0.717636	200
0.405	1996.868305	400137271	0.752611	200
0.455	1997.247797	400137271	0.779774	200
0.505	1997.544654	400137271	0.80166	200

0.555	1997.775829	400137271	0.819542	200
0.605	1997.962218	400137271	0.834423	200
0.655	1998.129559	400137271	0.847095	200
0.005	10.267528	400137271	0	300
0.055	16.179246	400137271	0	300
0.105	2819.682146	400137271	0.048749	300
0.155	2981.912177	400137271	0.353506	300
0.205	2989.888645	400137271	0.510125	300
0.255	2992.876547	400137271	0.605831	300
0.305	2994.420399	400137271	0.671156	300
0.355	2995.397316	400137271	0.717354	300
0.405	2996.065355	400137271	0.752364	300
0.455	2996.558902	400137271	0.779554	300
0.505	2996.941489	400137271	0.801462	300
0.555	2997.239392	400137271	0.819361	300
0.605	2997.479047	400137271	0.834258	300
0.655	2997.690182	400137271	0.846942	300
0.005	10.267528	400137271	0	400
0.055	16.179246	400137271	0	400
0.105	3753.613365	400137271	0.0478	400
0.155	3976.087407	400137271	0.352862	400
0.205	3986.727886	400137271	0.509636	400
0.255	3990.688975	400137271	0.605438	400
0.305	3992.759535	400137271	0.670828	400
0.355	3994.068167	400137271	0.717072	400
0.405	3994.957269	400137271	0.752117	400
0.455	3995.606421	400137271	0.779335	400
0.505	3996.105374	400137271	0.801264	400
0.555	3996.495133	400137271	0.819181	400
0.605	3996.808294	400137271	0.834093	400
0.655	3997.079788	400137271	0.84679	400
0.005	10.267528	400137271	0	500
0.055	16.179246	400137271	0	500
0.105	4668.232807	400137271	0.046852	500
0.155	4968.72956	400137271	0.352217	500
0.205	4982.685899	400137271	0.509148	500
0.255	4987.944652	400137271	0.605045	500
0.305	4990.6507	400137271	0.6705	500
0.355	4992.360393	400137271	0.71679	500
0.405	4993.519573	400137271	0.75187	500
0.455	4994.367584	400137271	0.779115	500
0.505	4995.01801	400137271	0.801066	500
0.555	4995.526173	400137271	0.819001	500

0.605	4995.93466	400137271	0.833927	500
0.655	4996.284927	400137271	0.846637	500

Max Queue size: Rejection Percentage vs. arrival rate *pre-asymptotic*

arrival rate	mean delay	seed	rejection percentage	buffer max	mean in system
0.07	21.490908	400137271	0.000568	10	2.333333
0.07125	22.153942	400137271	0.00073	10	2.478261
0.0725	22.862839	400137271	0.000932	10	2.636364
0.07375	23.619544	400137271	0.001172	10	2.809524
0.075	24.427836	400137271	0.001473	10	3
0.07625	25.294253	400137271	0.001846	10	3.210526
0.0775	26.217428	400137271	0.00232	10	3.444444
0.07875	27.202882	400137271	0.002879	10	3.705882
0.08	28.261375	400137271	0.003535	10	4
0.08125	29.368492	400137271	0.00438	10	4.333333
0.0825	30.531858	400137271	0.005407	10	4.714286
0.08375	31.796407	400137271	0.006563	10	5.153846
0.085	33.135992	400137271	0.007959	10	5.666667
0.08625	34.538168	400137271	0.009624	10	6.272727
0.0875	36.02205	400137271	0.01156	10	7
0.08875	37.56695	400137271	0.01375	10	7.888889
0.09	39.182718	400137271	0.016279	10	9
0.09125	40.856343	400137271	0.019114	10	10.42857
0.0925	42.590296	400137271	0.022327	10	12.33333
0.09375	44.323577	400137271	0.026032	10	15
0.095	46.094575	400137271	0.030171	10	19
0.09625	47.893212	400137271	0.034617	10	25.66667
0.0975	49.737509	400137271	0.039482	10	39
0.09875	51.527959	400137271	0.044731	10	79
0.1	53.346057	400137271	0.050533	10	
0.10125	55.203654	400137271	0.056507	10	
0.1025	56.986871	400137271	0.062875	10	
0.10375	58.721007	400137271	0.069578	10	
0.105	60.426308	400137271	0.076654	10	
0.10625	62.077308	400137271	0.083809	10	
0.1075	63.687483	400137271	0.091359	10	
0.10875	65.20882	400137271	0.098934	10	
0.11	66.701391	400137271	0.106751	10	
0.11125	68.12492	400137271	0.114585	10	
0.1125	69.476173	400137271	0.122644	10	
0.11375	70.773808	400137271	0.130662	10	
0.115	72.02147	400137271	0.1388	10	
0.11625	73.142422	400137271	0.146852	10	

0.1175	74.276051	400137271	0.1549	10	
0.11875	75.302657	400137271	0.16296	10	
0.07	21.673498	400137271	0	20	
0.07125	22.397723	400137271	0	20	
0.0725	23.185197	400137271	0.000001	20	
0.07375	24.044862	400137271	0.000004	20	
0.075	24.980476	400137271	0.00001	20	
0.07625	26.014062	400137271	0.000016	20	
0.0775	27.165569	400137271	0.000025	20	
0.07875	28.444928	400137271	0.000038	20	
0.08	29.866151	400137271	0.00006	20	
0.08125	31.460678	400137271	0.000096	20	
0.0825	33.261793	400137271	0.000149	20	
0.08375	35.312386	400137271	0.000229	20	
0.085	37.680227	400137271	0.000341	20	
0.08625	40.385789	400137271	0.000512	20	
0.0875	43.501237	400137271	0.000773	20	
0.08875	47.055528	400137271	0.001168	20	
0.09	51.071842	400137271	0.001798	20	
0.09125	55.65745	400137271	0.002668	20	
0.0925	60.776531	400137271	0.003935	20	
0.09375	66.452188	400137271	0.005649	20	
0.095	72.718906	400137271	0.00796	20	
0.09625	79.690571	400137271	0.010944	20	
0.0975	87.069283	400137271	0.01468	20	
0.09875	94.943471	400137271	0.019311	20	
0.1	102.872811	400137271	0.024884	20	
0.10125	110.615155	400137271	0.031311	20	
0.1025	118.367981	400137271	0.038574	20	
0.10375	125.734356	400137271	0.046644	20	
0.105	132.462161	400137271	0.055238	20	
0.10625	138.503758	400137271	0.064229	20	
0.1075	144.053596	400137271	0.073665	20	
0.10875	148.975231	400137271	0.083266	20	
0.11	153.276298	400137271	0.092921	20	
0.11125	157.112766	400137271	0.102497	20	
0.1125	160.431331	400137271	0.112133	20	
0.11375	163.338402	400137271	0.1216	20	
0.115	165.834067	400137271	0.130937	20	
0.11625	168.172575	400137271	0.140258	20	
0.1175	170.121381	400137271	0.149192	20	
0.11875	171.91315	400137271	0.158086	20	
0.07	21.673498	400137271	0	40	

0.07125	22.397723	400137271	0	40	
0.0725	23.185898	400137271	0	40	
0.07375	24.0481	400137271	0	40	
0.075	24.988759	400137271	0	40	
0.07625	26.028574	400137271	0	40	
0.0775	27.190149	400137271	0	40	
0.07875	28.486831	400137271	0	40	
0.08	29.936286	400137271	0	40	
0.08125	31.573582	400137271	0	40	
0.0825	33.443772	400137271	0	40	
0.08375	35.601727	400137271	0.000001	40	
0.085	38.130276	400137271	0.000005	40	
0.08625	41.102519	400137271	0.000013	40	
0.0875	44.645211	400137271	0.00002	40	
0.08875	48.914796	400137271	0.00003	40	
0.09	54.22456	400137271	0.000061	40	
0.09125	60.977956	400137271	0.000127	40	
0.0925	69.575687	400137271	0.000263	40	
0.09375	81.012156	400137271	0.000499	40	
0.095	96.419067	400137271	0.000986	40	
0.09625	116.889716	400137271	0.002013	40	
0.0975	141.147737	400137271	0.004125	40	
0.09875	168.919031	400137271	0.007492	40	
0.1	200.256838	400137271	0.012563	40	
0.10125	231.869351	400137271	0.019456	40	
0.1025	261.156418	400137271	0.027944	40	
0.10375	286.43818	400137271	0.037701	40	
0.105	306.69273	400137271	0.048292	40	
0.10625	322.068706	400137271	0.058988	40	
0.1075	333.544107	400137271	0.069808	40	
0.10875	342.17463	400137271	0.080466	40	
0.11	348.943195	400137271	0.090955	40	
0.11125	354.239022	400137271	0.101122	40	
0.1125	358.524576	400137271	0.111157	40	
0.11375	362.021298	400137271	0.120906	40	
0.115	364.978414	400137271	0.130472	40	
0.11625	367.539638	400137271	0.139924	40	
0.1175	369.719639	400137271	0.148962	40	
0.11875	371.667851	400137271	0.157931	40	
0.07	21.673498	400137271	0	80	
0.07125	22.397723	400137271	0	80	
0.0725	23.185898	400137271	0	80	
0.07375	24.0481	400137271	0	80	

0.075	24.988759	400137271	0	80	
0.07625	26.028574	400137271	0	80	
0.0775	27.190149	400137271	0	80	
0.07875	28.486831	400137271	0	80	
0.08	29.936286	400137271	0	80	
0.08125	31.573582	400137271	0	80	
0.0825	33.443772	400137271	0	80	
0.08375	35.602839	400137271	0	80	
0.085	38.137964	400137271	0	80	
0.08625	41.127253	400137271	0	80	
0.0875	44.690443	400137271	0	80	
0.08875	48.996059	400137271	0	80	
0.09	54.388325	400137271	0	80	
0.09125	61.362828	400137271	0	80	
0.0925	70.628136	400137271	0	80	
0.09375	83.545395	400137271	0.000005	80	
0.095	103.108384	400137271	0.000019	80	
0.09625	135.853291	400137271	0.000101	80	
0.0975	191.305709	400137271	0.000478	80	
0.09875	281.263027	400137271	0.001956	80	
0.1	393.335098	400137271	0.006189	80	
0.10125	517.25582	400137271	0.013956	80	
0.1025	611.726893	400137271	0.024459	80	
0.10375	665.268724	400137271	0.035839	80	
0.105	697.847097	400137271	0.047301	80	
0.10625	717.954192	400137271	0.058517	80	
0.1075	731.563537	400137271	0.06954	80	
0.10875	741.187418	400137271	0.080294	80	
0.11	748.364443	400137271	0.090852	80	
0.11125	753.868701	400137271	0.101048	80	
0.1125	758.276364	400137271	0.111089	80	
0.11375	761.832363	400137271	0.120858	80	
0.115	764.845368	400137271	0.130432	80	
0.11625	767.434252	400137271	0.13989	80	
0.1175	769.612068	400137271	0.148928	80	
0.11875	771.565923	400137271	0.157897	80	
0.07	21.673498	400137271	0	160	
0.07125	22.397723	400137271	0	160	
0.0725	23.185898	400137271	0	160	
0.07375	24.0481	400137271	0	160	
0.075	24.988759	400137271	0	160	
0.07625	26.028574	400137271	0	160	
0.0775	27.190149	400137271	0	160	

0.07875	28.486831	400137271	0	160	
0.08	29.936286	400137271	0	160	
0.08125	31.573582	400137271	0	160	
0.0825	33.443772	400137271	0	160	
0.08375	35.602839	400137271	0	160	
0.085	38.137964	400137271	0	160	
0.08625	41.127253	400137271	0	160	
0.0875	44.690443	400137271	0	160	
0.08875	48.996059	400137271	0	160	
0.09	54.388325	400137271	0	160	
0.09125	61.362828	400137271	0	160	
0.0925	70.628136	400137271	0	160	
0.09375	83.577667	400137271	0	160	
0.095	103.256242	400137271	0	160	
0.09625	137.260562	400137271	0	160	
0.0975	209.002117	400137271	0.000036	160	
0.09875	367.434477	400137271	0.000289	160	
0.1	783.189926	400137271	0.003111	160	
0.10125	1200.802979	400137271	0.01218	160	
0.1025	1395.017162	400137271	0.023913	160	
0.10375	1461.095947	400137271	0.035661	160	
0.105	1496.258669	400137271	0.047203	160	
0.10625	1516.956438	400137271	0.058441	160	
0.1075	1530.718121	400137271	0.069465	160	
0.10875	1540.456061	400137271	0.080221	160	
0.11	1547.701384	400137271	0.09078	160	
0.11125	1553.252907	400137271	0.100976	160	
0.1125	1557.701651	400137271	0.111018	160	
0.11375	1561.291689	400137271	0.120788	160	
0.115	1564.334662	400137271	0.130362	160	
0.11625	1566.953547	400137271	0.139821	160	
0.1175	1569.160774	400137271	0.14886	160	
0.11875	1571.141472	400137271	0.15783	160	
0.07	21.673498	400137271	0	320	
0.07125	22.397723	400137271	0	320	
0.0725	23.185898	400137271	0	320	
0.07375	24.0481	400137271	0	320	
0.075	24.988759	400137271	0	320	
0.07625	26.028574	400137271	0	320	
0.0775	27.190149	400137271	0	320	
0.07875	28.486831	400137271	0	320	
0.08	29.936286	400137271	0	320	
0.08125	31.573582	400137271	0	320	

0.0825	33.443772	400137271	0	320	
0.08375	35.602839	400137271	0	320	
0.085	38.137964	400137271	0	320	
0.08625	41.127253	400137271	0	320	
0.0875	44.690443	400137271	0	320	
0.08875	48.996059	400137271	0	320	
0.09	54.388325	400137271	0	320	
0.09125	61.362828	400137271	0	320	
0.0925	70.628136	400137271	0	320	
0.09375	83.577667	400137271	0	320	
0.095	103.256242	400137271	0	320	
0.09625	137.260562	400137271	0	320	
0.0975	210.768929	400137271	0	320	
0.09875	401.33498	400137271	0.000022	320	
0.1	1486.427746	400137271	0.001463	320	
0.10125	2746.774207	400137271	0.011703	320	
0.1025	2980.935161	400137271	0.023756	320	
0.10375	3051.332819	400137271	0.035506	320	
0.105	3088.418859	400137271	0.04705	320	
0.10625	3110.72709	400137271	0.058291	320	
0.1075	3125.694294	400137271	0.069317	320	
0.10875	3136.361089	400137271	0.080073	320	
0.11	3144.354739	400137271	0.090634	320	
0.11125	3150.367135	400137271	0.100832	320	
0.1125	3155.131383	400137271	0.110876	320	
0.11375	3158.944062	400137271	0.120647	320	
0.115	3162.186734	400137271	0.130223	320	
0.11625	3164.968355	400137271	0.139683	320	
0.1175	3167.305675	400137271	0.148724	320	
0.11875	3169.405331	400137271	0.157695	320	

Max Queue size: mean delay vs. arrival rate

arrival rate	mean delay	seed	rejection percentage	buffer max
0.005	10.263419	400137271	0	100
0.025	11.667898	400137271	0	100
0.035	12.694215	400137271	0	100
0.055	16.120261	400137271	0	100
0.075	25.02291	400137271	0	100
0.095	105.612649	400137271	0.000002	100
0.096	130.760523	400137271	0.000017	100
0.097	170.62508	400137271	0.000103	100
0.098	237.148862	400137271	0.000528	100

0.099	348.348997	400137271	0.00217	100
0.105	899.447459	400137271	0.049763	100
0.115	965.236499	400137271	0.175993	100
0.125	978.515784	400137271	0.361993	100
0.135	984.236849	400137271	0.594512	100
0.005	10.263419	400137271	0.802688	200
0.025	11.667898	400137271	0.802688	200
0.035	12.694215	400137271	0.802688	200
0.055	16.120261	400137271	0.802688	200
0.075	25.02291	400137271	0.802688	200
0.095	105.630895	400137271	0.802687	200
0.096	131.153051	400137271	0.802686	200
0.097	173.981482	400137271	0.802686	200
0.098	257.738317	400137271	0.802689	200
0.099	474.397211	400137271	0.802742	200
0.105	1899.13831	400137271	0.812306	200
0.115	1965.15936	400137271	0.872171	200
0.125	1978.46364	400137271	1.002505	200
0.135	1984.19795	400137271	1.187557	200
0.005	10.263419	400137271	1.603396	300
0.025	11.667898	400137271	1.603396	300
0.035	12.694215	400137271	1.603396	300
0.055	16.120261	400137271	1.603396	300
0.075	25.02291	400137271	1.603396	300
0.095	105.630895	400137271	1.603393	300
0.096	131.153051	400137271	1.603392	300
0.097	173.981482	400137271	1.603391	300
0.098	259.084951	400137271	1.603391	300
0.099	509.569123	400137271	1.603374	300
0.105	2898.62904	400137271	1.574613	300
0.115	2965.01726	400137271	1.568135	300
0.125	2978.37982	400137271	1.64282	300
0.135	2984.13488	400137271	1.780419	300
0.005	10.263419	400137271	2.403857	400
0.025	11.667898	400137271	2.403857	400
0.035	12.694215	400137271	2.403857	400
0.055	16.120261	400137271	2.403857	400
0.075	25.02291	400137271	2.403857	400
0.095	105.630895	400137271	2.403853	400
0.096	131.153051	400137271	2.403851	400
0.097	173.981482	400137271	2.40385	400
0.098	259.084951	400137271	2.40385	400
0.099	517.355575	400137271	2.403844	400

0.105	3897.96835	400137271	2.336857	400
0.115	3964.80676	400137271	2.264041	400
0.125	3978.25229	400137271	2.283082	400
0.135	3984.04765	400137271	2.373232	400
0.005	10.263419	400137271	3.204252	500
0.025	11.667898	400137271	3.204252	500
0.035	12.694215	400137271	3.204252	500
0.055	16.120261	400137271	3.204252	500
0.075	25.02291	400137271	3.204251	500
0.095	105.630895	400137271	3.204246	500
0.096	131.153051	400137271	3.204244	500
0.097	173.981482	400137271	3.204243	500
0.098	259.084951	400137271	3.204242	500
0.099	518.271381	400137271	3.204239	500
0.105	4897.11455	400137271	3.099061	500
0.115	4964.50064	400137271	2.959911	500
0.125	4978.08589	400137271	2.92331	500
0.135	4983.92709	400137271	2.966014	500

average mean delay vs. Arrival rate Part 8

arrival rate	average mean delay
0.01	10
0.01225	10
0.0145	10
0.01675	10
0.019	10
0.02125	10
0.0235	10
0.02575	10
0.028	10
0.03025	10
0.0325	10
0.03475	10
0.037	10
0.03925	10
0.0415	10
0.04375	10
0.046	10
0.04825	10
0.0505	10
0.05275	10
0.055	10

0.05725	10
0.0595	10
0.06175	10
0.064	10
0.06625	10
0.0685	10
0.07075	10
0.073	10
0.07525	10
0.0775	10
0.07975	10
0.082	10
0.08425	10
0.0865	10
0.08875	10
0.091	10
0.09325	10
0.0955	10
0.09775	10
0.1	10
0.10225	112510
0.1045	225010
0.10675	337510
0.109	450010
0.11125	562510
0.1135	675010
0.11575	787510
0.118	900010
0.12025	1012510
0.1225	1125010
0.12475	1237510
0.127	1350010
0.12925	1462510
0.1315	1575010
0.13375	1687510
0.136	1800010
0.13825	1912510
0.1405	2025010
0.14275	2137510
0.145	2250010
0.14725	2362510
0.1495	2475010
0.15175	2587510

0.154	2700010
0.15625	2812510
0.1585	2925010
0.16075	3037510
0.163	3150010
0.16525	3262510
0.1675	3375010
0.16975	3487510
0.172	3600010
0.17425	3712510
0.1765	3825010
0.17875	3937510
0.181	4050010
0.18325	4162510
0.1855	4275010
0.18775	4387510

average mean delay vs. Arrival rate Asymptote with model Part 8

arrival rate	average mean delay	model
0.10225	112510	112500
0.1045	225010	225000
0.10675	337510	337500
0.109	450010	450000
0.11125	562510	562500
0.1135	675010	675000
0.11575	787510	787500
0.118	900010	900000
0.12025	1012510	1012500
0.1225	1125010	1125000
0.12475	1237510	1237500
0.127	1350010	1350000
0.12925	1462510	1462500
0.1315	1575010	1575000
0.13375	1687510	1687500
0.136	1800010	1800000
0.13825	1912510	1912500
0.1405	2025010	2025000
0.14275	2137510	2137500
0.145	2250010	2250000
0.14725	2362510	2362500
0.1495	2475010	2475000
0.15175	2587510	2587500

0.154	2700010	2700000
0.15625	2812510	2812500
0.1585	2925010	2925000
0.16075	3037510	3037500
0.163	3150010	3150000
0.16525	3262510	3262500
0.1675	3375010	3375000
0.16975	3487510	3487500
0.172	3600010	3600000
0.17425	3712510	3712500
0.1765	3825010	3825000
0.17875	3937510	3937500
0.181	4050010	4050000
0.18325	4162510	4162500
0.1855	4275010	4275000
0.18775	4387510	4387500