MI-KRY – Advanced Cryptology

Meet-in-the-middle

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Lecture outline

- Brute force vs. meet-in-the-middle attack
- Meet-in-the-middle attack on 3DES
- Meet-in-the-middle attack on RSA without padding

Brute force attack

Function $f : \{1, 2, \ldots, N\} \mapsto \{1, 2, \ldots, N\}$ is fixed. Attacker knows y, where $y = f(x)$, and needs to find x.

- Find a preimage of a hash function f
- Find the key for a fixed plaintext P, ciphertext C:

$$f(k) = C = E_k(P)$$

- Find the secret message M knowing the ciphertext C and public key PK:

$$f(M) = C = E_{PK}(M)$$

Assume it is difficult to find f^{-1}, so that $f^{-1}(y) = x$.

Brute force attack: Search exhaustively for x until $y = f(x)$. Complexity $O(N) = O(2^n)$, memory negligible.

Brute force attack on double encryption

Let us have an encryption transformation $c = E_K(p)$ that is composed of two subsequent transfomations E'_{K_1} and E''_{K_2}, so that $c = E_K(p) = E''_{K_2}(E'_{K_1}(p))$.

The subkeys are defined as $K_1 \in \{0,1\}^n$, $K_2 \in \{0,1\}^m$.

Decryption: $p = D_K(c) = D'_{K_1}(D''_{K_2}(c))$

Goal: Recover K_1 and K_2, knowing p and c (known plaintext attack).

Brute force attack: Exhaustive search for K_1 and K_2, time $O(2^{n+m})$ encryptions.

Meet-in-the-middle attack

We can exploit the double-encryption composition to find a **collision** in the middle of the computation:

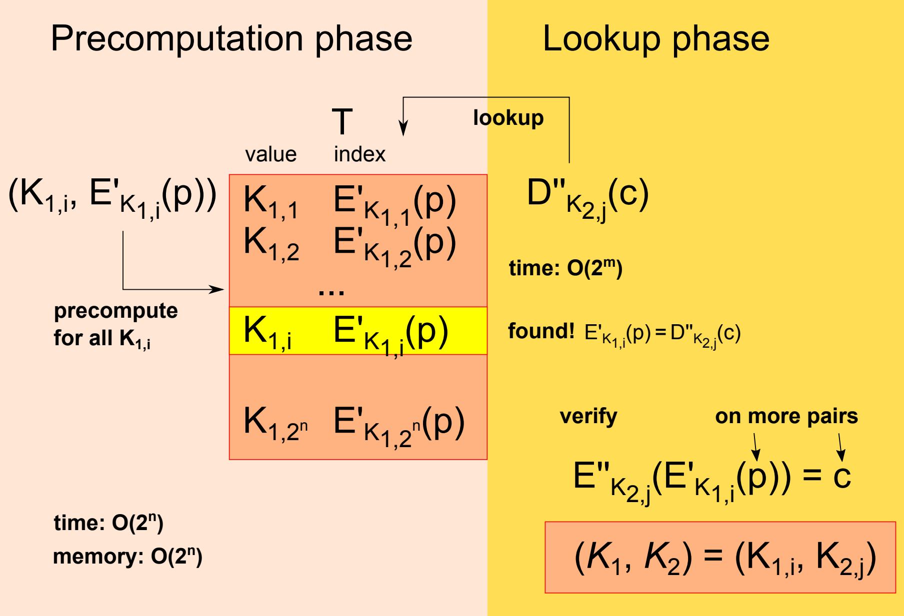
$$\begin{aligned} c &= E''_{K_2}(E'_{K_1}(p)) \\ D''_{K_2}(c) &= D''_{K_2}(E''_{K_2}(E'_{K_1}(p))) \\ D''_{K_2}(c) &= E'_{K_1}(p) \end{aligned}$$

Meet-in-the-middle

- First, precompute all possible $E'_{K_1}(p)$ for the plaintext and store the pairs $(K_1, E'_{K_1}(p))$ in a lookup table T.
- Then, find K_2 such that $D''_{K_2}(c)$ is in T, i.e. $\exists K_1, (K_1, D''_{K_2}(c)) \in T$.
- Found (K_1, K_2) is the key of $c = E''_{K_2}(E'_{K_1}(p))$.
- Verify (K_1, K_2) on other ciphertexts, continue searching if incorrect.

The precomputation phase requires $O(2^n)$ encryptions and $O(2^n)$ memory. The lookup phase requires $O(2^m)$ encryptions and lookups. The resulting time complexity is $O(2^n + 2^m)$, as opposed to $O(2^{n+m})$ for the simple exhaustive search.

Meet-in-the-middle attack



Meet-in-the-middle attack example: 3DES-EDE

Let E_K and D_K be DES encryption and decryption, respectively, with $K \in \{0,1\}^{56}$. Consider the usual EDE variant of $3DES_{K_{123}}$, where $K_{123} = (K_1|K_2|K_3)$: $c = 3DES_{K_{123}}(p) = E_{K_3}(D_{K_2}(E_{K_1}(p)))$.

$$\begin{aligned} c &= E_{K_3}(D_{K_2}(E_{K_1}(p))) \\ D_{K_3}(c) &= D_{K_2}(E_{K_1}(p)) \\ E_{K_2}(D_{K_3}(c)) &= E_{K_1}(p) \end{aligned}$$

Meet-in-the-middle: First, precompute all possible $E_{K_1}(p)$ for the plaintext and store them in a lookup table T. Then, find (K_2, K_3) such that $\exists K_1 : (K_1, E_{K_2}(D_{K_3}(c))) \in T$. Then, (K_1, K_2, K_3) is likely the correct key of $c = E_{K_3}(D_{K_2}(E_{K_1}(p)))$.

The precomputation phase requires $O(2^{56})$ encryptions and $O(2^{56})$ memory. The lookup phase requires $O(2^{112})$ encryptions and lookups. The resulting time complexity approx. $O(2^{112})$, as opposed to $O(2^{168})$ for the simple exhaustive search. Therefore we must consider 3DES to have 112-bit security even though $K_1 \neq K_2 \neq K_3$.

Meet-in-the-middle: RSA without padding

Consider a randomly chosen 64-bit message M (for example, a session key) that is encrypted using RSA with a public key (n, e). Assume e is large enough to hinder the attack using integer e-th root.

$C = |M^e|_n, \; M < 2^m$

Goal: Recover the m-bit secret message M, knowing C, n, e.

With high probability, the random M is composite, so that $M = M_1 M_2$, where $M_1 < 2^{m_1}$, $M_2 < 2^{m_2}$. For example, if $m = 64$, then M is composed of 32-bit numbers with 18% probability, and of at most 33-bit numbers with 29% probability.

Meet-in-the-middle: RSA without padding 2

Due to the multiplicative homomorphism of RSA,

$C = |M^e|_n = |(M_1 M_2)^e|_n = |M_1^e M_2^e|_n$, therefore
(assuming $\gcd(M_1, n) = 1$)

$|C\ M_2^{-e}|_n = |M_1^e|_n$

We can search for a collision, looking for M_1 and M_2 separately. The attack is not guaranteed to always succeed, but it will succeed with high probability.

Meet-in-the-middle: RSA without padding 3

Attack (meet-in-the-middle):

- Precompute all possible $|M_1^e|_n$ and store the pairs $(M_1, |M_1^e|_n)$ in a lookup table T.
- Then, find M_2 such that $|C\,M_2^{-e}|_n$ is in T, i.e. $\exists M_1 : (M_1, |C\,M_2^{-e}|_n) \in T$.
- Found (M_1, M_2) likely gives the secret $M = M_1 M_2$.

We do not need to store the whole $|M_1^e|_n$ in T, it suffices to store enough bits to distinguish between different numbers. $2\max(m_1, m_2)$ least significant bits should be enough.

The attack requires $2^{m_1+1}\max(m_1, m_2)$ bits of memory and takes $2^{m_1} + 2^{m_2}$ modular exponentiations (including precomputation).

Similar attacks

Meet-in-the-Middle is a generic space-time tradeoff attack. Other similar attacks include:

- Time-memory tradeoff attack on block, stream ciphers – Hellman
- Time-memory tradeoff attack on hash function preimages
  - Original attack by Martin Hellman
  - Rainbow tables attack by Philippe Oechslin
- Baby step – giant step algorithm for DLP

For similar time-memory tradeoff attacks on hash functions and stream ciphers, including rainbow tables, see the MI-BHW (Security and Hardware) course, Lecture 10.

Bibliography

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