COL702 Quiz-1

Ganraj Borade

TOTAL POINTS

5 / 10

QUESTION 1

1 Q1 **5 / 5**

+ 0 pts Incorrect/Not Attempted

✓ **+ 1 pts** *Correct Answer*

✓ **+ 4 pts** *Correct Proof*

QUESTION 2

2 Q2 **0 / 5**

+ 5 pts Correct Proof

✓ **+ 0 pts** *Incorrect Proof/Not_attempted*

+ 3 pts Partially correct proof

1 2/(log2(5))<1

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There are 2 questions for a total of 10 points.

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1. (5 points) Consider the following recursive function that takes as input a positive integer.

```
F(n)
  · if (n = 1) return
  · if (n is odd) F(n − 1)
  · else
      · print("Hello World")
      · F(n/2)
```

Give the **exact** expression, in terms of n, for the number of times "Hello World" is printed when a call to $F(n)$ is made. Argue the correctness of your expression using mathematical induction.

2. (5 points) Prove or disprove: $5^{\log_2 n}$ is $O(n^2)$.

represent no. of times Hello world is printed

① $T(n) = \begin{cases} T(n-1) & \text{if } n = \text{odd} \\ T(\frac{n}{2})+1 & \text{if } n = \text{even} \end{cases}$ and $T(1) = 0$

$n = 2k+1$ (odd) s.t $k \in \mathbb{Z}$ $\Rightarrow T(2k+1) = T(2k) = T(k)+1$ as $2k$ = even

and $T(2k) = T(k)+1$ $\therefore$ $T(2k+1) = T(2k)$

Let's ~~just~~ solve $\cancel{T(2k) = T(k)+1}$

from this if $2^k \le n < 2^{k+1}$ then $T(n) = k$.

$k \le \log n < k+1 \Rightarrow \lfloor \log n \rfloor = k$

$\therefore \boxed{T(n) = \lfloor \log n \rfloor}$

$T(1)=0$
$T(2)=1$
$T(3)=1$
$T(4)=2$
$T(5)=2$
$T(6)=2$
$T(7)=2$
$T(8)=3$
$\vdots$
$T(15)=3$

Proof :-

① Basis step :- if $n=1$ $\Rightarrow$ by logic we can see that $T(1) = 0$

and by above expression $\Rightarrow T(1) = \lfloor \log 1 \rfloor = 0$

② Inductive step :- Assume $T(1), T(2), \ldots T(k)$ is true.

$\cancel{T(k+1)} = \cancel{\lfloor \log(k+1) \rfloor}$

1) Let $k+1$ = even $\therefore T(k+1) = 1 + T\left(\frac{k+1}{2}\right) = 1 + \left\lfloor \log\left(\frac{k+1}{2}\right) \right\rfloor$

$\hookrightarrow k+1 = 2m$ $\therefore T(k+1) = \cancel{1+\lfloor \log \ldots} \left\lfloor \log 2 + \log\left(\frac{k+1}{2}\right) \right\rfloor$

$T(k+1) = \lfloor \log(k+1) \rfloor$ Hence proved when $k+1$ = even

2) let $k+1$ = odd $\Rightarrow k+1 = 2m+1$ $\therefore k = 2m$ | $T(k+1) = T(k) = \lfloor \log k \rfloor = \lfloor \log(k+1) \rfloor$

let $2^p \le k < 2^{p+1}$

$p \le \log k < p+1$

Since k = even $\therefore$ max^m of $k = 2^{p+1} - 2$

$\therefore$ $2^p \le k+1 < 2^{p+1}$

$\therefore p < \log(k+1) < p+1$

Hence i.e $\boxed{\lfloor \log(k+1) \rfloor = p}$ (It can never be $p+1$)

$= \lfloor \log k \rfloor$

② $5^{\log_2 n} = O(n^2)$

→ To Prove ⇒ we need to find c, n_0 s.t

$5^{\log_2 n} \le cn^2 \quad \forall n \ge n_0$

Applying log on both sides :-

$\log_2 n \cdot \log 5 \le \log c + 2\log n$ as → both RHS & LHS are positive

$\log_2 n \le \frac{\log c}{\log 5} + \frac{2}{\log 5}\log_2 n$

Take $c = 1$ ⇒

$\log_2 n \le \frac{2}{\log 5}\log_2 n$ which is true for ~~for~~ all $n \ge 1$

∴ $c = 1$ and $n_0 = 1$.

Hence, we proved that $5^{\log_2 n} = O(n^2)$.

Continuing 1^{st} question:-

⇒ Since for $k+1$ = odd ⇒ $\lfloor \log k \rfloor = \lfloor \log(k+1) \rfloor$

∴ $T(k+1) = \lfloor \log k \rfloor = \lfloor \log(k+1) \rfloor$

Hence we have proved this too.