EE1390

Matrix Project

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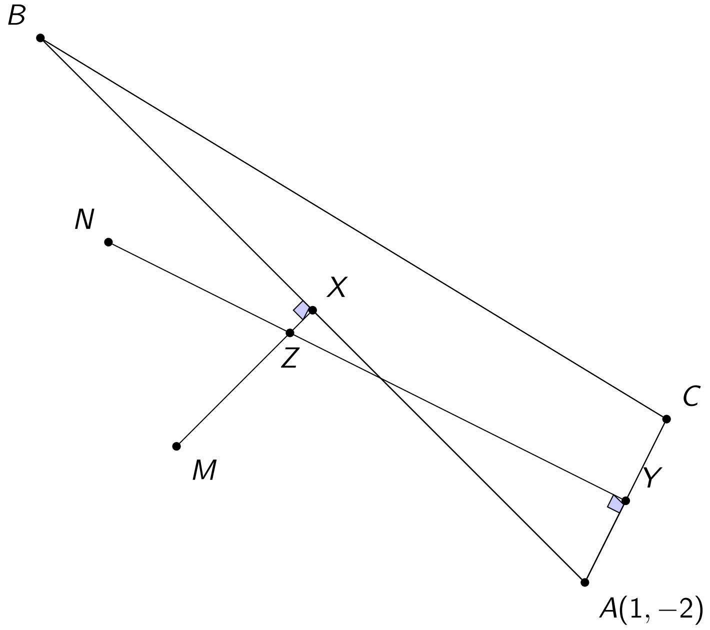
Question

The equation of perpendicular bisectors of sides AB and AC of a triangle ABC are $x-y+5=0$ and $x+2y=0$ respectively. If a point A is $(1,-2)$ then the equation of line BC is

[IIT 1986]


B
N
X
Z
M
C
Y
A(1, −2)

Solution(Approach 1)

Converting the problem in matrix form:

Let the perpendicular bisector of AB and AC are XZ and ZY respectively.
Equation of XZ and ZY in matrix form are respectively:

$$\begin{pmatrix}1 & -1\end{pmatrix}\mathbf{x} + 5 = 0$$
$$\begin{pmatrix}1 & 2\end{pmatrix}\mathbf{x} = 0$$

Since the equation in matrix form of line is

$$n^T x = p;$$

where n is normal vector to the line.

Hence the normal vector of XZ $= \binom{1}{-1}$ and the normal vector of ZY $= \binom{1}{2}$

So The direction vector of m_1 of XZ is $\binom{1}{1}$ and the direction vector of m_2 of XZ is $\binom{-2}{1}$. Hence, the equation of AB matrix form is :

$$(1 \;\; 1)\mathbf{x}=p_1$$

Now since AB passes through A=$\binom{1}{-2}$, so

$$(1 \;\; 1)\binom{1}{-2}= p_1$$

$$p_1 = \text{-}1$$

Therefore equation of AB is $(1 \;\; 1)\mathbf{x} = -1$

Similarly, the equation of AC matrix form is :

$$(\text{-}2 \;\; 1)\mathbf{x}=p_2$$

Now since AC passes through $\binom{1}{-2}$, so

$$(\text{-}2 \;\; 1)\binom{1}{-2}= p_2$$

$$p_2 = \text{-}4$$

Therefore equation of AC is $(\text{-}2 \;\; 1)\mathbf{x} = -4$

Now Let's find out the point of intersection of AB and XZ

$$(1 \;\; \text{-}1)\mathbf{x} = -5 \quad (XZ)$$

$$(1 \;\; 1)\mathbf{x} = -1 \quad (AB)$$

Stacking both equations: $\begin{pmatrix} 1 & -1 \\ 1 & 1 \end{pmatrix} \mathbf{x} = \begin{pmatrix} 5 \\ -1 \end{pmatrix}$

$$\mathbf{x} = \begin{pmatrix} -3 \\ 2 \end{pmatrix}$$

Similarly,the point of intersection of ZY and AC:

$$(\text{-}2 \;\; 1)\mathbf{x} = \text{-}4 \quad (AC)$$

$$(1 \;\; 2)\mathbf{x} = 0 \quad (ZY)$$

Stacking both equations: $\begin{pmatrix} 1 & 2 \\ -2 & 1 \end{pmatrix} \mathbf{x} = \begin{pmatrix} 0 \\ -4 \end{pmatrix}$

$$\mathbf{x} = \begin{pmatrix} 1.6 \\ -0.8 \end{pmatrix}$$

By mid point formula X = (A+B)/2 and Y = (A+C)/2

so, B = 2X - A and C = 2Y - A

$B = 2\binom{-3}{2} - \binom{1}{-2}$ and $C = 2\binom{1.6}{-0.8} - \binom{1}{-2}$

$$B = \binom{-7}{6} \text{ and } C = \binom{2.2}{0.4}$$

Hence the direction vector of BC is $\binom{9.2}{-5.6}$

so normal vector is $\binom{-5.6}{-9.2}$

Equation of BC is $n^T\ \mathbf{x} = n^T B$

or it can be also written as $n^T\ \mathbf{x} = n^T C$. For simplicity in calculation,we are considering B.

$$n^T = (-5.6\ \ -9.2)\ .\ \text{Because n} = \binom{-5.6}{-9.2}$$

So equation of BC is $(-5.6\ \ -9.2)\mathbf{x} = -16$

$$(14\ \ 23)\mathbf{x} = 40$$

Approach 2(Generalized way)

If we are given a line $(m \;\; -1)\mathbf{x} = -c$, then the mirror image of a point of a point P=$\mathbf{x}$ is given by

$$\mathbf{x'} = {}^{1}/_{(1+\,m^2)} \begin{pmatrix} 1-m^2 & 2m \\ 2m & m^2-1 \end{pmatrix} \mathbf{x} - {}^{1}/_{(1+\,m^2)} \begin{pmatrix} 2mc \\ -2c \end{pmatrix}$$

In our case, image of A about $(1 \;\; -1)\mathbf{x} = -5$ is B. (Because we are taking the image across the perpendicular bisector)

And image of A about $(1 \;\; 2)\mathbf{x} = 0$ is C.

So for $(1 \;\; -1)\mathbf{x} = -5$, m=1 and c=5,Hence

$$\mathbf{x'} = \begin{pmatrix} 0 & 1 \\ 1 & 0 \end{pmatrix} \begin{pmatrix} 1 \\ -2 \end{pmatrix} - \begin{pmatrix} 5 \\ -5 \end{pmatrix}$$

$$\mathbf{x'} = \begin{pmatrix} -7 \\ 6 \end{pmatrix}$$

Therefore,B $= \begin{pmatrix} -7 \\ 6 \end{pmatrix}$

Similarly for $(1 \;\; 2)\mathbf{x} = 0$, m=-1/2 and c $= 0$,Hence

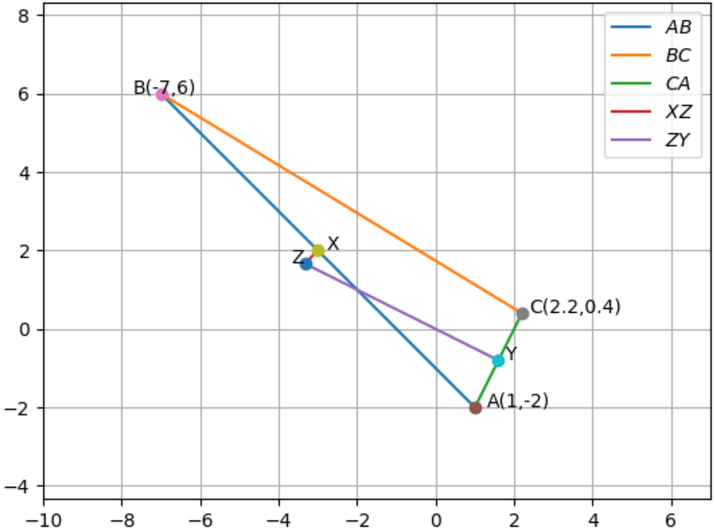
$\mathbf{x'} = \begin{pmatrix} 3/5 & -4/5 \\ -4/5 & 3/5 \end{pmatrix} \begin{pmatrix} 1 \\ -2 \end{pmatrix}$ - $\begin{pmatrix} 0 \\ 0 \end{pmatrix}$

$\mathbf{x'} = \begin{pmatrix} 11/5 \\ 2/5 \end{pmatrix}$

Therefore,C $= \begin{pmatrix} 11/5 \\ 2/5 \end{pmatrix}$

Since we have the two coordinates B and C, the equation of BC is

$$(14 \;\; 23)\mathbf{x} = 40$$


AB
BC
CA
XZ
ZY
B(-7,6)
X
Z
C(2.2,0.4)
Y
A(1,-2)
8
6
4
2
0
−2
−4
−10
−8
−6
−4
−2
0
2
4
6