Control Systems

G V V Sharma*

Contents

1 Laplace Transform

1.1. An input p(t) = sin(t) is applied to the system

$$G(s) = \frac{s-1}{s+1} \tag{1.1.1}$$

The corresponding steady state output is

$$y(t) = sin(t+\varphi) \tag{1.1.2}$$

where the phase φ (in degrees), when restricted to

$$0^\circ \le \varphi \le 360^\circ, \tag{1.1.3}$$

is ?

Solution: We have p(t) = sin(t)

We know that Laplace Transform of p(t) = $\mathcal{L}$(p(t)) = P(s) . So ,

$$P(s) = \frac{1}{s^2+1} \tag{1.1.4}$$

And we are given the steady state output

$$y_s(t) = sin(t+\varphi) \tag{1.1.5}$$

So,

$$y_s(t) = sin(t)cos(\varphi) + cos(t)sin(\varphi) \tag{1.1.6}$$

Hence , Laplace transform of y(t) in steady state is :

$$\mathcal{L}(y_s(t)) = Y_s(s) = \mathcal{L}(sin(t))cos(\varphi) + \mathcal{L}(cos(t))sin(\varphi) \tag{1.1.7}$$

Since ,

$$\mathcal{L}(sin(t)) = \frac{1}{s^2+1}; \mathcal{L}(cos(t)) = \frac{s}{s^2+1} \tag{1.1.8}$$

*The author is with the Department of Electrical Engineering, Indian Institute of Technology, Hyderabad 502285 India e-mail: gadepall@iith.ac.in.

So,

$$Y_s(s) = \frac{cos\varphi + s(sin\varphi)}{s^2+1} \tag{1.1.9}$$

Hence , the output of the system in s-domain is Y(s) = P(s)G(s) . So,

$$Y(s) = \frac{1}{s^2+1}.\frac{s-1}{s+1} \tag{1.1.10}$$

For solving this we can use the partial fractions:

$$Y(s) = \frac{As+B}{s^2+1} + \frac{C}{s+1}.i.e, \tag{1.1.11}$$

$$Y(s) = \frac{(A+C)s^2 + (A+B)s + (B+C)}{(s^2+1)(s+1)} \tag{1.1.12}$$

Hence by comparing the coefficients,we get A+C=0 , A+B=0 , B+C=-1.So,
After solving these above equations,we get A=1 , B=0 , C=-1.

$$Y(s) = \frac{s}{s^2+1} - \frac{1}{s+1} \tag{1.1.13}$$

Now , we know that Laplace transform of e^{-t}u(t) is

$$\mathcal{L}(e^{-t}u(t)) = \frac{1}{s+1} \tag{1.1.14}$$

i.e,

$$\mathcal{L}^{-1}(\frac{1}{s+1}) = e^{-1}u(t) \tag{1.1.15}$$

2 Steady State Analysis

As for steady state analysis , we put t $\to \infty$,therefore $e^{-1}u(t)$ will disappear while taking inverse laplace transform . Hence in steady state , only $\frac{s}{s^2+1}$ term will appear in laplace transform of y(t) as t $\to \infty$.

Hence , in steady state,

$$Y(s) = \frac{s}{s^2 + 1} = Y_s(s). \tag{2.1.1}$$

So ,

$$\frac{s}{s^2 + 1} = \frac{cos(\varphi) + s(sin(\varphi))}{s^2 + 1} \tag{2.1.2}$$

By comparing coefficients of s and constants , we get $cos(\varphi) = 0$ and $sin(\varphi) = 1$.
So, because $0^\circ \leq \varphi \leq 360^\circ$, therefore $\varphi = 90^\circ$

Plot obtained for verification in python :
(You can download code from codes/ee18btech11016.py)

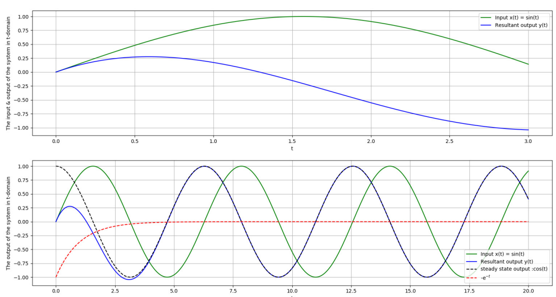

Fig. 2.1

In above plot , black color plot is of cos(t).And blue color plot is the plot of resultant y(t) .So , we can see from above plots that black and blue color plots are coinciding after t=3 . Hence y(t) = sin(t + 90°) in steady state.