Closed loop gain and Phase Margin

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An amplifier has a dc gain of 10^5 and poles at 10^5 Hz , 3.16×10^5 Hz and 10^6 Hz . Find the value of β,and the corresponding closed-loop gain , for which a phase margin of 45° is obtained.

1. Find the transfer function of the three pole OPAMP.
Solution: For a 3-pole amplifier open loop transfer function is

$$G(s) = \frac{G_0}{\left(1+\frac{s}{P_1}\right)\left(1+\frac{s}{P_2}\right)\left(1+\frac{s}{P_3}\right)} \tag{1.1}$$

where the Gain and Poles are listed in Table 1.

Parameters	Value
P_1	$2\pi\times10^5$ rad/sec
P_2	$2\pi(3.16\times10^5)$ rad/sec
P_3	$2\pi\times10^6$ rad/sec
G_0	10^5

TABLE 1

Poles are at $f_1 = 10^5$ and $f_2 = 3.16\times10^5$ and $f_3 = 10^6$

$$G(f) = \frac{G_0}{\left(1+\jmath\frac{f}{f_1}\right)\left(1+\jmath\frac{f}{f_2}\right)\left(1+\jmath\frac{f}{f_3}\right)} \tag{1.2}$$

$$= \frac{10^5}{\left(1+\jmath\frac{f}{10^5}\right)\left(1+\jmath\frac{f}{3.16\times10^5}\right)\left(1+\jmath\frac{f}{10^6}\right)} \tag{1.3}$$

2. Find the loop gain expression ($G(s)H$) (H is constant in this question).
Solution:

$$GH = \frac{10^5}{\left(1+\jmath\frac{f}{10^5}\right)\left(1+\jmath\frac{f}{3.16\times10^5}\right)\left(1+\jmath\frac{f}{10^6}\right)}.H \tag{2.1}$$

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3. Find the PM and the crossover frequency.
Solution: The phase margin = 180° - $\phi(f_c)$ where f_c is the frequency where $|G(f)H| = 1$.It is required that the phase margin is 45° , so that :

$$45° = 180° - \phi(f_c) \tag{3.1}$$
$$\phi(f_c) = -135°. \tag{3.2}$$

From (2.1)

$$-135° = -tan^{-1}\left(\frac{f_c}{10^5}\right) - tan^{-1}\left(\frac{f_c}{3.16\times10^5}\right) - tan^{-1}\left(\frac{f_c}{10^6}\right) \tag{3.3}$$

After solving the above equation , we get f_c = 315KHz.

4. Verify your result using a Bode plot.
Solution: The following code id used to verify the value of f_c Fig. 4

```
codes/ee18btech11016/ee18btech11016_1.py
```

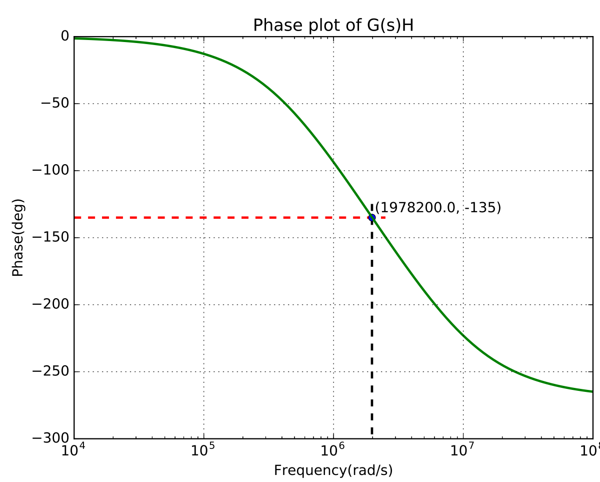

Fig. 4

5. Find the value of H.
Solution: From (2.1),The magnitude of the loop gain at this frequency f_c is given by $|G(f_c)H|$:

$$H\left(\frac{10^5}{\sqrt{1+(\frac{315\times10^3}{10^5})^2}\sqrt{1+(\frac{315\times10^3}{3.16\times10^6})^2}\sqrt{1+(\frac{315\times10^3}{10^6})^2}}\right) \tag{5.1}$$

which is equal to $H \times (20.04 \times 10^3)$. So,

$$|G(f_c)H| = H \times (20.04 \times 10^3) \tag{5.2}$$

Setting $|G(f_c)H| = 1$, Solving for β (here β is equal to H) yields

$$\beta = 48.9 \times 10^{-6} \tag{5.3}$$

Or

$$H = 48.9 \times 10^{-6} \tag{5.4}$$

6. Find the corresponding closed-loop gain for which a phase margin of 45° is obtained.
Solution: The closed loop dc gain is given as

$$A_f = \frac{G_0}{1+HG_0} = \frac{10^5}{1+48.9\times10^{-6}(10^5)} \tag{6.1}$$

$$A_f = 17 \times 10^3 \tag{6.2}$$

7. Realise the above system with $PM = 45°$ using a feedback circuit.
Solution:

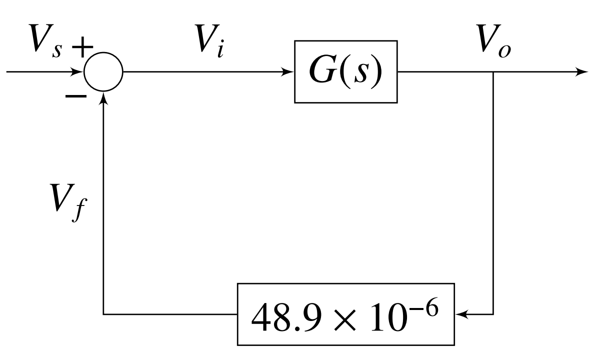

Fig. 7

The transfer function of OPAMP is

$$G(s) = \frac{10^5}{\left(1+\frac{s}{2\pi\times10^5}\right)\left(1+\frac{s}{2\pi\times3.16\times10^5}\right)\left(1+\frac{s}{2\pi\times10^6}\right)} \tag{7.1}$$

8. For the feedback gain H
Solution:

Choose a resistance network such that

$$H = \frac{V_f}{V_o} = \frac{R_{f_1}}{R_{f_1}+R_{f_2}} \approx 48.9\times10^{-6} \tag{8.1}$$

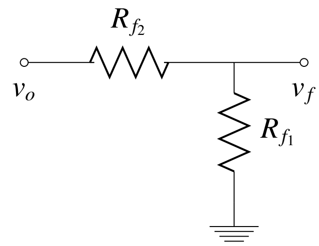

Fig. 8

Choose R_{f_1} and R_{f_2} as

$$R_{f_1} = 100\Omega \tag{8.2}$$

$$R_{f_2} = 2.045M\Omega \tag{8.3}$$

$$H = \frac{R_{f_1}}{R_{f_1}+R_{f_2}} = \frac{100}{100+2.045\times10^6} \approx 48.9\times10^{-6} \tag{8.4}$$

9. Feedback Circuit for $PM = 45°$
Solution:

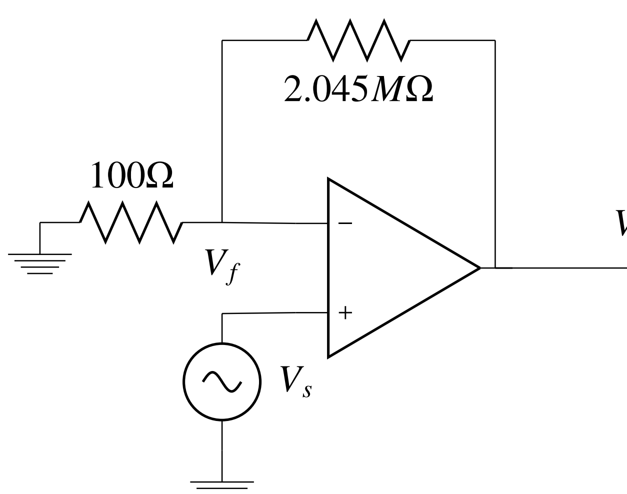

Fig. 9

10. Simulate the circuit in ngspice.
Solution: The following code provides instructions about the simulation.

```
codes/ee18btech11016/spice/README.md
```

The following netlist simulates the unity feedback system.

codes/ee18btech11016/spice/ee18btech1016_sim.net

The step response in spice is plotted using the following code in Fig. 10

codes/ee18btech11016/spice/ee18btech11016_simulation.py

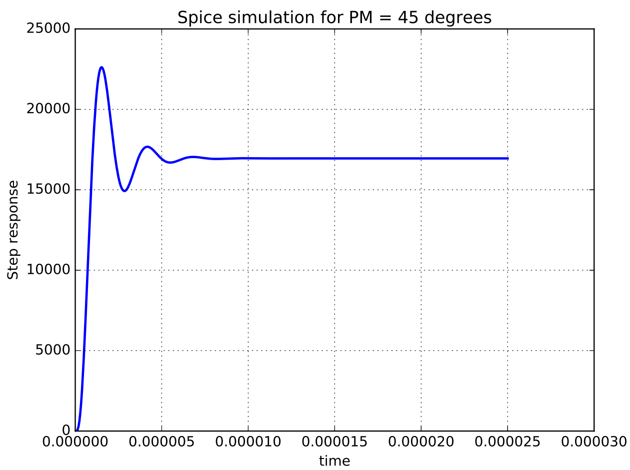

Fig. 10

11. Overview of implementation.
Solution: Fig. 11 shows how the circuit is actually implemented in spice using the parameters in Table 11

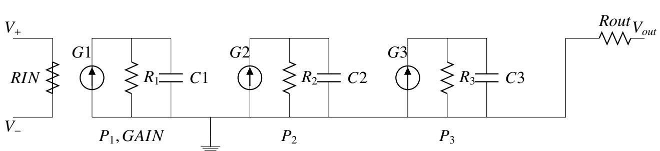

Fig. 11: Circuit resembling G(s)

Elements	**Value**
G_1	$10^{-1}(V_+ - V_-)A/V$
G_2	$10^{-6}A/V$
G_3	$10^{-6}A/V$
R_1	$1M\Omega$
R_2	$1M\Omega$
R_3	$1M\Omega$
C_1	$1.59pF$
C_2	$0.503pF$
C_3	$0.159pF$
R_{IN}	$1000M\Omega$
R_{OUT}	100Ω
R_{f_1}	100Ω
R_{f_2}	$2.045M\Omega$
R_s	$1M\Omega$

TABLE 11