Closed loop gain and Phase Margin

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An amplifier has a dc gain of 10^5 and poles at 10^5 Hz , 3.16×10^5 Hz and 10^6 Hz . Find the value of β,and the corresponding closed-loop gain , for which a phase margin of 45° is obtained.

1. Find the transfer function of the three pole OPAMP.
   Solution: For a 3-pole amplifier open loop transfer function is

$$G(s) = \frac{G_0}{\left(1 + \frac{s}{P_1}\right)\left(1 + \frac{s}{P_2}\right)\left(1 + \frac{s}{P_3}\right)} \quad (1.1)$$

   where the Gain and Poles are listed in Table 1.

Parameters	Value
P_1	$2\pi \times 10^5$ rad/sec
P_2	$2\pi(3.16 \times 10^5)$ rad/sec
P_3	$2\pi \times 10^6$ rad/sec
G_0	10^5

TABLE 1

Poles are at $f_1 = 10^5$ and $f_2 = 3.16 \times 10^5$ and $f_3 = 10^6$

$$G(f) = \frac{G_0}{\left(1 + \jmath\frac{f}{f_1}\right)\left(1 + \jmath\frac{f}{f_2}\right)\left(1 + \jmath\frac{f}{f_3}\right)}$$

$$= \frac{10^5}{\left(1 + \jmath\frac{f}{10^5}\right)\left(1 + \jmath\frac{f}{3.16\times10^5}\right)\left(1 + \jmath\frac{f}{10^6}\right)} \quad (1.2)$$

2. Find the loop gain expression $(G(s)H)$ (H is constant in this question).
   Solution:

$$GH = \frac{10^5}{\left(1 + \jmath\frac{f}{10^5}\right)\left(1 + \jmath\frac{f}{3.16\times10^5}\right)\left(1 + \jmath\frac{f}{10^6}\right)}.H \quad (2.1)$$

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3. Find the PM and the crossover frequency.
   Solution: The phase margin = 180° - $\phi(f_c)$ where f_c is the frequency where $|G(f)H| = 1$.It is required that the phase margin is 45° , so that :

$$45° = 180° - \phi(f_c) \implies \phi(f_c) = -135°. \quad (3.1)$$

   From (2.1)

$$-135° = -\tan^{-1}\left(\frac{f_c}{10^5}\right) - \tan^{-1}\left(\frac{f_c}{3.16 \times 10^5}\right) - \tan^{-1}\left(\frac{f_c}{10^6}\right) \quad (3.2)$$

   After solving the above equation , we get f_c = 315KHz.

4. Verify your result using a Bode plot.
   Solution: The following code id used to verify the value of f_c Fig. 4
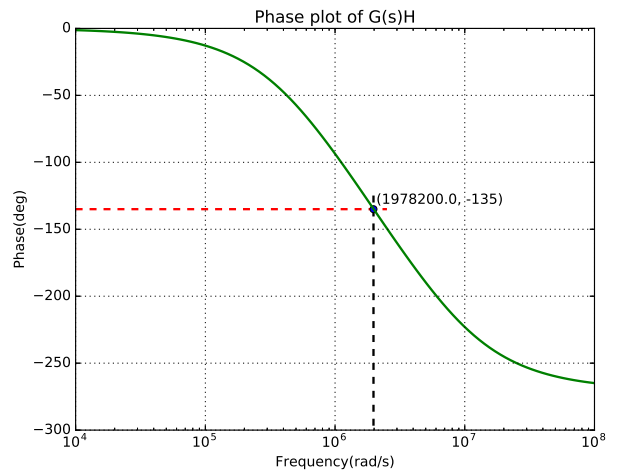
   codes/ee18btech11016/ee18btech11016_1.py

Fig. 4

5. Find the value of H.
   Solution: From (2.1),The magnitude of the loop gain at this frequency f_c is given by $|G(f_c)H|$:

$$H\left(\frac{10^5}{\sqrt{1 + (\frac{315\times10^3}{10^5})^2}\ \sqrt{1 + (\frac{315\times10^3}{3.16\times10^6})^2}\ \sqrt{1 + (\frac{315\times10^3}{10^6})^2}}\right)$$

$$(5.1)$$

which is equal to $H \times (28.8588 \times 10^3)$. So,

$$|G(f_c)H| = H \times (28.8588 \times 10^3) \qquad (5.2)$$

Setting $|G(f_c)H| = 1$, Solving for β (here β is equal to H) yields

$$\beta = 34.651 \times 10^{-6} \qquad (5.3)$$

Or

$$H = 34.651 \times 10^{-6} \qquad (5.4)$$

The following code provides the method to calculate the unit step response and the values of H,G(fc).

```
codes/ee18btech11016/
    ee18btech11016_verifyingvalues.py
```

6. Find the corresponding closed-loop gain for which a phase margin of $45°$ is obtained.
Solution: The closed loop dc gain is given as

$$A_f = \frac{G_0}{1 + HG_0} = \frac{10^5}{1 + 34.651 \times 10^{-6}(10^5)} \qquad (6.1)$$

$$A_f = 22.3957 \times 10^3 \qquad (6.2)$$

7. Realise the above system with $PM = 45°$ using a feedback circuit.
Solution:
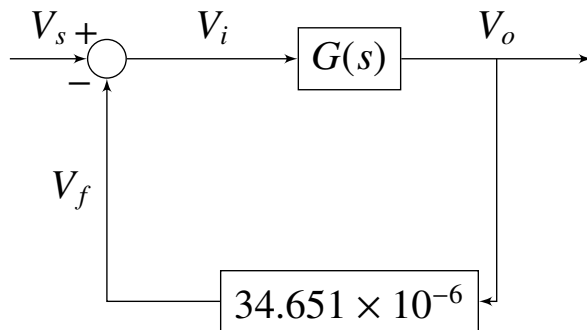
Fig. 7

The transfer function of OPAMP is

$$G(s) = \frac{10^5}{\left(1 + \frac{s}{2\pi\times10^5}\right)\left(1 + \frac{s}{2\pi\times3.16\times10^5}\right)\left(1 + \frac{s}{2\pi\times10^6}\right)}$$

$$(7.1)$$

8. For the feedback gain H
Solution:
Choose a resistance network such that

$$H = \frac{V_f}{V_o} = \frac{R_{f_1}}{R_{f_1} + R_{f_2}} \approx 34.651 \times 10^{-6} \qquad (8.1)$$
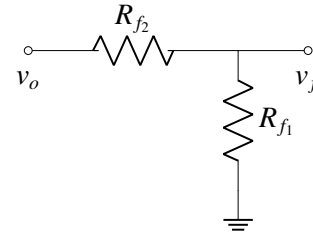
Fig. 8

Choose R_{f_1} and R_{f_2} as

$$R_{f_1} = 100\Omega \qquad (8.2)$$
$$R_{f_2} = 4.057M\Omega \qquad (8.3)$$

$$H = \frac{R_{f_1}}{R_{f_1} + R_{f_2}} = \frac{100}{100 + 4.057 \times 10^6} \approx 34.651 \times 10^{-6}$$

$$(8.4)$$

9. Feedback Circuit for $PM = 45°$
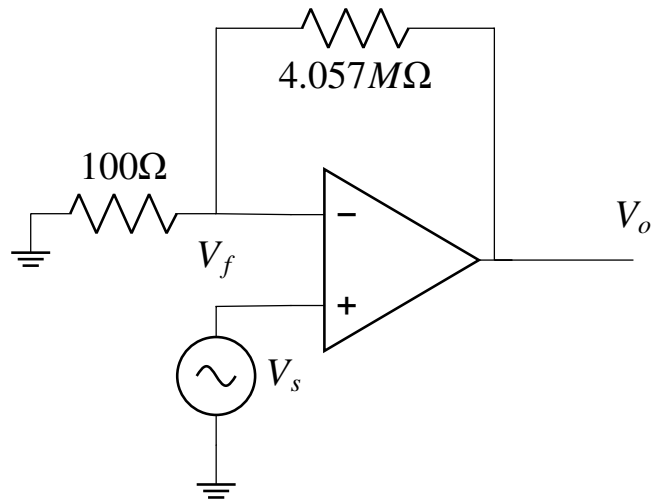**Solution:**

Fig. 9

10. Simulate the circuit in ngspice.
    Solution: For $H = 34.651 \times 10^{-6}$ the closed loop response is

$$|T| \approx \frac{1}{H} = 28.8588 \times 10^3 \qquad (10.1)$$

which can be observable from graph also.

The following code provides instructions about the simulation.

codes/ee18btech11016/spice/README.md

The following netlist simulates the unity feedback system.

codes/ee18btech11016/spice/ ee18btech1016_sim.net

The step response in spice is plotted using the following code in Fig. 10

codes/ee18btech11016/spice/ ee18btech11016_simulation.py

Fig. 10

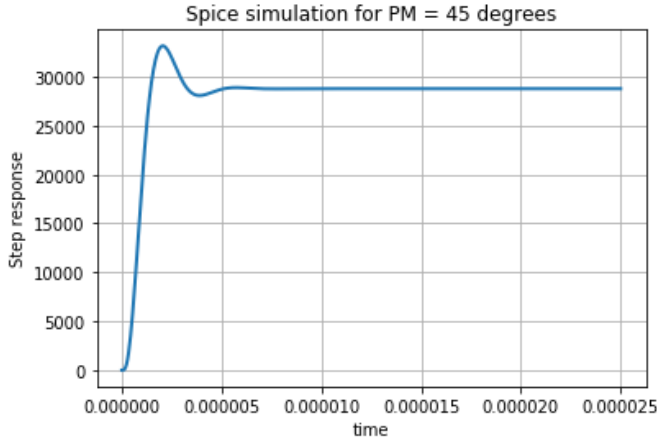| Elements	Value
G_1	$10^{-1}(V_+ - V_-)A/V$
G_2	$10^{-6}A/V$
G_3	$10^{-6}A/V$
R_1	$1M\Omega$
R_2	$1M\Omega$
R_3	$1M\Omega$
C_1	$1.59pF$
C_2	$0.503pF$
C_3	$0.159pF$
R_{IN}	$1000M\Omega$
R_{OUT}	100Ω
R_{f_1}	100Ω
R_{f_2}	$4.057M\Omega$
R_s	$1M\Omega$

TABLE 11

11. Overview of implementation.
    Solution: Fig. 11 shows how the circuit is actually implemented in spice using the parameters in Table 11

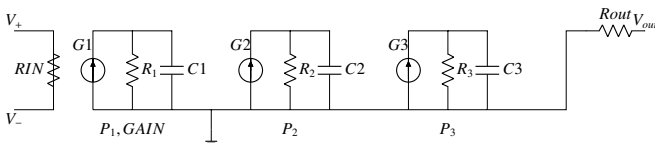
Fig. 11: Circuit resembling G(s)