

Control Systems

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Abstract—The objective of this manual is to introduce control system design at an elementary level.

Download python codes using

svn co <https://github.com/gadepall/school/trunk/control/kean/codes>

1 STABILITY

1.1 Second order System

2 ROUTH HURWITZ CRITERION

3 COMPENSATORS

4 NYQUIST PLOT

4.1. Using Nyquist criterion, find out the range of K for which the closed loop system will be stable.

$$G(s) = \frac{K(s+2)(s+4)}{s^2-3s+10}; H(s) = \frac{1}{s} \quad (4.1.1)$$

4.2. The system flow can be described by 4.2

4.3. Find the open loop transfer function $G(s)H(s)$

Solution: From (4.1.1),

$$G(s)H(s) = \frac{K(s+2)(s+4)}{s(s^2-3s+10)} \quad (4.3.1)$$

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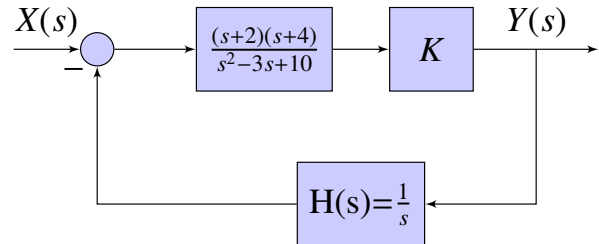


Fig. 4.2

$$G(j\omega)H(j\omega) = \frac{K(j\omega+2)(j\omega+4)}{j\omega((10-\omega^2)-3j\omega)} \quad (4.3.2)$$

$$\text{Re}\{G(j\omega)H(j\omega)\} = \frac{K(84\omega^2 - 9\omega^4)}{\omega^6 - 11\omega^4 + 100\omega^2} \quad (4.3.3)$$

$$\text{Im}\{G(j\omega)H(j\omega)\} = \frac{K(-\omega^5 + 36\omega^3 - 80\omega)}{\omega^6 - 11\omega^4 + 100\omega^2} \quad (4.3.4)$$

4.4. Sketch the Nyquist plot.

Solution: The Nyquist plot is a graph of $\text{Re}\{G(j\omega)H(j\omega)\}$ vs $\text{Im}\{G(j\omega)H(j\omega)\}$. Let's take $K=1$ and draw the nyquist plot .

The following python code generates the Nyquist plot in Fig. 4.4

codes/ee18btech11016.py

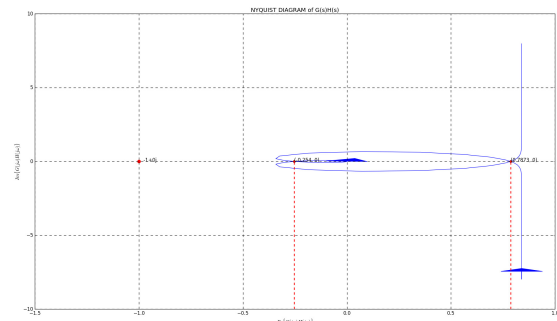


Fig. 4.4

Note that this nyquist plot is plotted when $K=1$.

- 4.5. Use the Nyquist Stability criterion to determine the value of K since we know that the system in (4.1.1) is stable.

Solution: Nyquist criterion-For the stable system :

$$Z = P + N = 0, \quad (4.5.1)$$

Variable	Description
Z	Poles of $\frac{G(s)}{1+G(s)H(s)}$ in right half of s plane
N	No of encirclements of $G(s)H(s)$ about $-1+j0$ in the Nyquist plot
P	Poles of $G(s)H(s)$ in right half of s plane

TABLE 4.5

Since from the equation (4.3.1), $P = 2$ as the number of poles on right hand side of s-plane is equal to 2 .So, for Z to be equal to 0 ,we have to choose the range of K such that N should be equal to -2.

- 4.6. Find the range of K from Nyquist criterion.

Solution: If we consider the Nyquist plot with K term i.e. of equation (4.3.1) , then it will cut x-axis at $x = -0.254K$, $x = 0$ and at $x = 0.7873K$ (as we have nyquist graph at $K=1$, now we just need to multiply the intersected coordinates on x-axis by K).

So, we have to make sure that $(-1 + j0)$ should be included in between $x = -0.254K$ to $x = 0$, because then only $N = -2$ (as the no. of encirclements are 2 in anticlockwise direction in this case so $N=-2$)

$$-0.254K < -1 < 0 \quad (4.6.1)$$

So,

$$K > \frac{1}{0.254} \quad (4.6.2)$$

i.e.

$$K > 3.937 \quad (4.6.3)$$

Hence $K > 3.937$ ensures that the system is stable as no. of poles on the right hand side of s-plane (in this case) is 0.