Control Systems

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Contents

***Abstract*—The objective of this manual is to introduce control system design at an elementary level.**

Download python codes using

```
svn co https://github.com/gadepall/school/trunk/
    control/ketan/codes
```

1 Stability

1.1 Second order System

2 Routh Hurwitz Criterion

3 Compensators

4 Nyquist Plot

4.1. Using Nyquist criterion, find out the range of K for which the closed loop system will be stable.

$$G(s) = \frac{K(s+2)(s+4)}{s^2-3s+10}; H(s) = \frac{1}{s} \tag{4.1.1}$$

4.2. The system flow can be described by Fig. 4.2

4.3. Find the open loop transfer function G(s)H(s)

Solution: From (4.1.1),

$$G(s)H(s) = \frac{K(s+2)(s+4)}{s(s^2-3s+10)} \tag{4.3.1}$$

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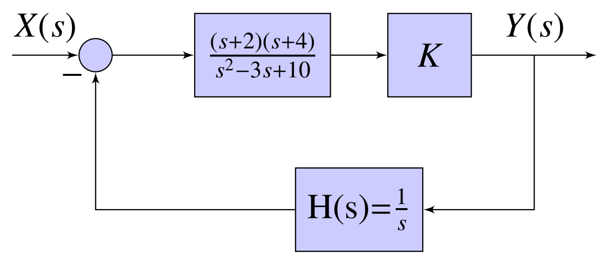

Fig. 4.2

$$G(\jmath\omega)H(\jmath\omega) = \frac{K(j\omega+2)(j\omega+4)}{j\omega((10-\omega^2)-3j\omega)} \tag{4.3.2}$$

$$\text{Re}\{G(\jmath\omega)H(\jmath\omega)\} = \frac{K(84\omega^2-9\omega^4)}{\omega^6-11\omega^4+100\omega^2} \tag{4.3.3}$$

$$\text{Im}\{G(\jmath\omega)H(\jmath\omega)\} = \frac{K(-\omega^5+36\omega^3-80\omega)}{\omega^6-11\omega^4+100\omega^2} \tag{4.3.4}$$

4.4. Sketch the Nyquist plot.

Solution: The Nyquist plot is a graph of $\text{Re}\{G(\jmath\omega)H(\jmath\omega)\}$ vs $\text{Im}\{G(\jmath\omega)H(\jmath\omega)\}$. Let's take K =1 and draw the nyquist plot .

The following python code generates the Nyquist plot in Fig. 4.4

```
codes/ee18btech11016.py
```

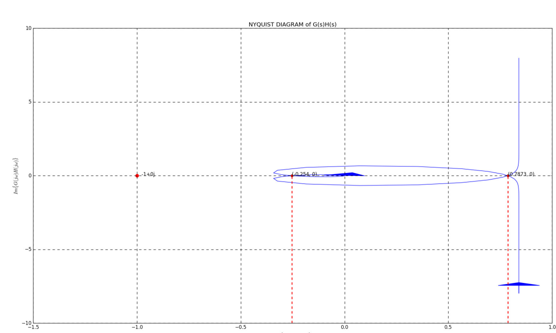

Fig. 4.4

Note that this nyquist plot is plotted when K=1.

4.5. Use the Nyquist Stability criterion to determine the value of K since we know that the system in (4.1.1) is stable.
Solution: Nyquist criterion-For the stable system :

$$Z = P + N = 0, \tag{4.5.1}$$

Variable	Description
Z	Poles of $\frac{G(s)}{1+G(s)H(s)}$ in right half of s plane
N	No of encirclements of $G(s)H(s)$ about -1+j0 in the Nyquist plot
P	Poles of $G(s)H(s)$ in right half of s plane

TABLE 4.5

Since from the equation (4.3.1), P = 2 as the number of poles on right hand side of s-plane is equal to 2 .So, for Z to be equal to 0 ,we have to choose the range of K such that N should be equal to -2.

4.6. Find the range of K from Nyquist criterion. **Solution:** If we consider the Nyquist plot with K term i.e. of equation (4.3.1) , then it will cut x-axis at x = -0.254K , x = 0 and at x = 0.7873K (as we have nyquist graph at K=1, now we just need to multiply the intersected coordinates on x-axis by K).

So, we have to make sure that $(-1 + j0)$ should be included in between x = -0.254K to x = 0, because then only N = -2 (as the no. of encirclements are 2 in anticlockwise direction in this case so N=-2)

$$-0.254K < -1 < 0 \tag{4.6.1}$$

So,

$$K > \frac{1}{0.254} \tag{4.6.2}$$

i.e.

$$K > 3.937 \tag{4.6.3}$$

Hence K > 3.937 ensures that the system is stable as no. of poles on the right hand side of s-plane (in this case) is 0.