Assignment-1

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Download all python codes from

https://github.com/ganrajborade/EE3025/tree/main/A1/codes

and latex-tikz codes from

https://github.com/ganrajborade/EE3025/tree/main/A1

1 Problem

1.1. Let

$$x(n) = \left\{\underset{\uparrow}{1}, 2, 3, 4, 2, 1\right\} \tag{1.1.1}$$

$$y(n) + \frac{1}{2}y(n-1) = x(n) + x(n-2) \tag{1.1.2}$$

1.2. Compute

$$X(k) \triangleq \sum_{n=0}^{N-1} x(n)e^{-j2\pi kn/N}, \quad k = 0, 1, \dots, N-1 \tag{1.2.1}$$

and $H(k)$ using h(n).

1.3. Compute

$$Y(k) = X(k)H(k) \tag{1.3.1}$$

2 Solution

2.1. The Impulse Response of the LTI system is the output of the system when Unit Impulse Signal is given as input to the system. Impulse response h(n) can be found from given difference equation as follows (h(n) is IIR Filter)

$$h(n) + \frac{1}{2}h(n-1) = \delta(n) + \delta(n-2) \tag{2.1.1}$$

2.2. DFT of a Input Signal $x(n)$ is

$$X(k) \triangleq \sum_{n=0}^{N-1} x(n)e^{-j2\pi kn/N}, \quad k = 0, 1, \dots, N-1 \tag{2.2.1}$$

2.3. Let $W_N = e^{-j2\pi/N}$

We can express X as Matrix Multiplication of DFT Matrix and x.

$$X = \left[W_N^{ij}\right]_{N\times N} x, \quad i, j = 0, 1, \dots, N-1 \tag{2.3.1}$$

2.4. In the given problem, we have N = 6

$$\implies W_6 = e^{-j2\pi/6} = \frac{1}{2} - \frac{\sqrt{3}}{2}j \tag{2.4.1}$$

$$\begin{bmatrix} X(0)\\ X(1)\\ X(2)\\ X(3)\\ X(4)\\ X(5) \end{bmatrix} = \begin{bmatrix} W_6^0 & W_6^0 & W_6^0 & W_6^0 & W_6^0 & W_6^0\\ W_6^0 & W_6^1 & W_6^2 & W_6^3 & W_6^4 & W_6^5\\ W_6^0 & W_6^2 & W_6^4 & W_6^6 & W_6^8 & W_6^{10}\\ W_6^0 & W_6^3 & W_6^6 & W_6^9 & W_6^{12} & W_6^{15}\\ W_6^0 & W_6^4 & W_6^8 & W_6^{12} & W_6^{16} & W_6^{20}\\ W_6^0 & W_6^5 & W_6^{10} & W_6^{15} & W_6^{20} & W_6^{25} \end{bmatrix} \begin{bmatrix} x(1)\\ x(2)\\ x(3)\\ x(4)\\ x(5)\\ x(6) \end{bmatrix} \tag{2.4.2}$$

Using $x(n)$ from Eq(1.1.1), we get

$$\begin{bmatrix} X(0)\\ X(1)\\ X(2)\\ X(3)\\ X(4)\\ X(5) \end{bmatrix} = \begin{bmatrix} W_6^0 & W_6^0 & W_6^0 & W_6^0 & W_6^0 & W_6^0\\ W_6^0 & W_6^1 & W_6^2 & W_6^3 & W_6^4 & W_6^5\\ W_6^0 & W_6^2 & W_6^4 & W_6^6 & W_6^8 & W_6^{10}\\ W_6^0 & W_6^3 & W_6^6 & W_6^9 & W_6^{12} & W_6^{15}\\ W_6^0 & W_6^4 & W_6^8 & W_6^{12} & W_6^{16} & W_6^{20}\\ W_6^0 & W_6^5 & W_6^{10} & W_6^{15} & W_6^{20} & W_6^{25} \end{bmatrix} \begin{bmatrix} 1\\ 2\\ 3\\ 4\\ 2\\ 1 \end{bmatrix} \tag{2.4.3}$$

$$\implies \begin{bmatrix} X(0)\\ X(1)\\ X(2)\\ X(3)\\ X(4)\\ X(5) \end{bmatrix} = \begin{bmatrix} 13 + 0j\\ -4 - \sqrt{3}j\\ 1 + 0j\\ -1 + 0j\\ 1 + 0j\\ -4 + \sqrt{3}j \end{bmatrix} \tag{2.4.4}$$

2.5. DFT of a Impulse Response $h(n)$ is

$$H(k) \triangleq \sum_{n=0}^{N-1} h(n)e^{-j2\pi kn/N}, \quad k = 0, 1, \dots, N-1 \tag{2.5.1}$$

Similarly, converting the above expression in matrix form to find $H(k)$

$$\begin{bmatrix} H(0) \\ H(1) \\ H(2) \\ H(3) \\ H(4) \\ H(5) \end{bmatrix} = \begin{bmatrix} W_6^0 & W_6^0 & W_6^0 & W_6^0 & W_6^0 & W_6^0 \\ W_6^0 & W_6^1 & W_6^2 & W_6^3 & W_6^4 & W_6^5 \\ W_6^0 & W_6^2 & W_6^4 & W_6^6 & W_6^8 & W_6^{10} \\ W_6^0 & W_6^3 & W_6^6 & W_6^9 & W_6^{12} & W_6^{15} \\ W_6^0 & W_6^4 & W_6^8 & W_6^{12} & W_6^{16} & W_6^{20} \\ W_6^0 & W_6^5 & W_6^{10} & W_6^{15} & W_6^{20} & W_6^{25} \end{bmatrix} \begin{bmatrix} 1 \\ -0.5 \\ 1.25 \\ -0.625 \\ 0.3125 \\ -0.15625 \end{bmatrix} \tag{2.5.2}$$

$$\implies \begin{bmatrix} H(0) \\ H(1) \\ H(2) \\ H(3) \\ H(4) \\ H(5) \end{bmatrix} = \begin{bmatrix} 1.28125 + 0j \\ 0.51625 - 0.5142j \\ -0.07813 + 1.1096j \\ 3.84375 + 0j \\ -0.07183 - 1.1096j \\ 0.51625 + 0.5142j \end{bmatrix} \tag{2.5.3}$$

2.6. We can now compute $Y(k)$ using Eq (2.6.1)

$$Y(k) = X(k)H(k) \tag{2.6.1}$$

So, $Y(k)$ is obtained element wise multiplication of $X(k)$ and $H(k)$

$$\begin{bmatrix} Y(0) \\ Y(1) \\ Y(2) \\ Y(3) \\ Y(4) \\ Y(5) \end{bmatrix} = \begin{bmatrix} X(0) \cdot H(0) \\ X(1) \cdot H(1) \\ X(2) \cdot H(2) \\ X(3) \cdot H(3) \\ X(4) \cdot H(4) \\ X(5) \cdot H(5) \end{bmatrix} \tag{2.6.2}$$

Computing the above expression we get,

$$\begin{bmatrix} Y(0) \\ Y(1) \\ Y(2) \\ Y(3) \\ Y(4) \\ Y(5) \end{bmatrix} = \begin{bmatrix} 16.6562 + 0j \\ -2.95312 + 1.16372j \\ -0.07812 + 1.10959j \\ -3.84375 - 9.27556j \\ -0.07812 - 1.10959j \\ -2.95312 - 1.16372j \end{bmatrix} \tag{2.6.3}$$

2.7. The following code computes Y and generates magnitude and phase plots of X, H, Y

https://github.com/ganrajborade/EE3025/blob/main/A1/codes/EE18BTECH11016.py

2.8. The following plots are obtained