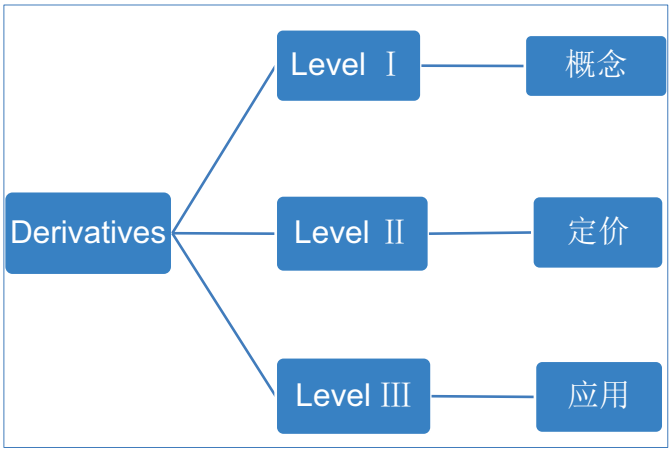
CFA一级培训项目

Derivative Investments

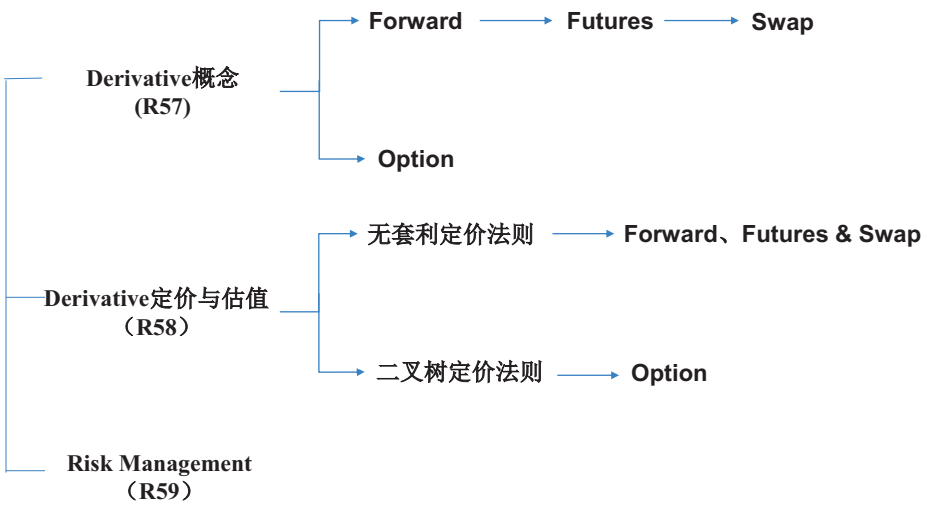
Introduction—Framework



Topic Weightings in CFA Level I

Session NO.	Content	Weightings
Study Session 1	Ethics & Professional Standards	15
Study Session 2-3	Quantitative Analysis	12
Study Session 4-6	Economics	10
Study Session 7-10	Financial Reporting and Analysis	20
Study Session 11	Corporate Finance	7
Study Session 12	Portfolio Management	7
Study Session 13-14	Equity Investment	10
Study Session 15-16	Fixed Income	10
Study Session 17	**Derivatives**	**5**
Study Session 18	Alternative Investments	4

Derivative框架结构



Framework

- **R57：Derivative markets and instruments**
  - **基本概念**
    1. Derivative的定义
    2. 四大类衍生产品介绍
    3. 衍生产品分类
       - Exchange-traded & Over-the-counter的区别
       - Forward commitment & Contingent claim的区别
       - Derivatives underlyings
    4. Derivatives的优缺点
    5. Risk free arbitrage
  - **四大类产品**
    1. Forward
    2. Futures
    3. Swap
    4. Option

R.57 Derivative Markets and Instruments

- **Definition:** A **derivative** is a financial instrument (**contract**) that derives its performance from the performance of an underlying asset.
  - ✓ **Buy or Sell Something:**
    - Buy or Sell **now**
    - Buy or Sell sometime **in the future**.
  - ✓ **Example**：
    - 3个月后→3￥/瓶价格→买果汁；
    - 3个月后→15￥/股价格→买股票；
    - 3个月后→4%利息→借￥1million；
    - 3个月后→6.5CNY/USD→换CNY.
- **关键词：**
  - 合约
  - 规避风险Hedge vs. 赚钱 Speculate
  - 合约收益取决于约定的资产价格变化

R.57 Derivative Markets and Instruments

Forward contract | Futures contract | Swap contract | Option contract

➢ **Forward contract:**

- A **forward contract** is an **private agreement** that obligates one party to buy and the other party to sell a specific quantity of an underlying asset, at a set price, at a future date
- 约定未来特定时间以约定价格买卖标的物的合约.
- If the future price of the underlying assets increase, the buyer has a gain, and the seller has a loss.

R.57 Derivative Markets and Instruments

Forward contract | **Futures contract** | Swap contract | Option contract

➢ **A Futures contract** is a forward contract that is **standardized** and **exchange-traded.**

- A forward contract
- Are regulated
- Backed by a clearinghouse
- Require a daily settlement of gains and losses.

R.57 Derivative Markets and Instruments

Forward contract | Futures contract | **Swap contract** | Option contract

➢ **A Swap contract** is **a series of** forward contracts .

- Exchange cash flows on period settlement dates
- Default risk

R.57 Derivative Markets and Instruments

Forward contract | Futures contract | Swap contract | **Option contract**

➢ **An option contract:**

- The owner has **the right,** but not **the obligation** the conduct a transaction
- **四种contract中只有option权利义务不对等要交期权费**

R.57 Derivative Markets and Instruments

Forward contract | Futures contract | Swap contract | **Option contract**

➢ **Basic characteristics of options**

1. An option to buy an asset at a particular price is termed a call option

Buyer of a call	Right to buy	
Seller of a call		Obligation to sell

2. An option to sell an asset at a particular price is termed a put option

Buyer of a put	Right to sell	
Seller of a put		Obligation to buy

R.57 Derivative Markets and Instruments

➢ 衍生品分类方法

- **根据合约特点分类：Forward commitment & Contingent claim**
  - ✓ ***Forward commitment***: is an agreement between two parties in which one party, the buyer, agrees to buy from the other party, the seller, an underlying asset at a future date at a price established at the start → forward, futures and swap contracts
  - ✓ ***Contingent claim***: is derivative in which the payoffs occur if a specific event happens → option contracts

R.57 Derivative Markets and Instruments

➢ 衍生品分类方法

- 根据交易场所分类：**Exchange-traded & Over-the-counter traded**
  - ✓ ***Exchange-traded***：在一个固定的交易所交易。多空双方不直接见面，与清算所交易（A→ Clearinghouse → B）
  - ✓ ***OTC traded***：没有固定交易场所，多空双方直接交易（A→B）
- 两种交易区别：

Exchange-traded	Over-the-counter
Standardized→ Liquid	Customized/Specific needs
Backed by a clearinghouse	Trade with counterparty (default risk)
Trade in the a physical exchange	not trade in organized markets
Regulated	Unregulated

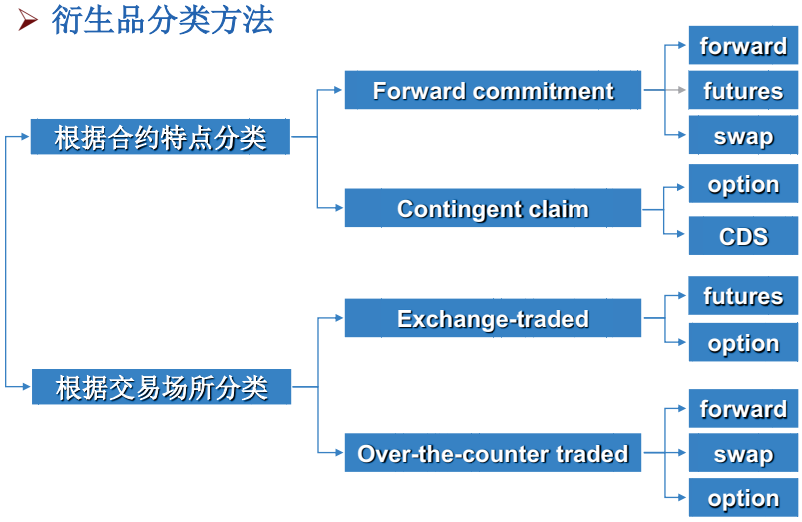
R.57 Derivative Markets and Instruments

➢ 主要术语：

- **Forward commitment**
  - ✓ ***Long***：指**买**标的物
  - ✓ ***Short***：指**卖**标的物
- **Contingent claim**
  - ✓ ***Long***：指**获得**一个权利
  - ✓ ***Short***：指**卖出**一个权利
  - ✓ ***Call***：指**买入**标的物的权利
  - ✓ ***Put***：指**卖出**标的物的权利

R.57 Derivative Markets and Instruments

➢ 衍生品分类方法



Example

- Which of the following is the best example of a derivative'?
  - A. A global equity mutual fund
  - B. A non-callable government bond
  - C. A contract to purchase Apple Computer at a fixed price
- Correct answer: C

- Which of the following statements about derivatives is **not** true?
  - A. They are created in the spot market.
  - B. They are used in the practice of risk management.
  - C. They take their values from the value of something else.
- Correct answer: A

Example

1. The buyer of a call option has the:
   - A. right to buy the underlying asset in the future under certain conditions
   - B. obligation to sell the underlying asset in the future under certain conditions
   - C. right to sell the underlying asset in the future under certain conditions
- Correct answer: A

2. A private agreement between two parties to exchange a series of future cash flows with at least one of the two series of cash flows determined by a later outcome, is best characterized as a(n):
   - A. Swap
   - B. Futures contract
   - C. Exchange-traded contingent claim
- Correct answer: A

R.57 Derivative Markets and Instruments

- **Advantage**:
  - Price discovery
  - Risk management: hedge and speculation
  - Lowering transaction costs
  - Low capital requirement
  - Greater liquidity
  - Ease of going short
  - Enhance market efficiency
- **Disadvantage**:
  - Too risky → High leverage
  - Complex instruments
  - Sometimes likened to gambling
- **考点**:
  - *Always increase risk?* →*No.*

R.57 Derivative Markets and Instruments

➢ **Risk-free arbitrage and no-arbitrage rule:**

- Arbitrage involves earning over the risk-free rate with no risk or earning an immediate gain with no future liabilities
- Arbitrage opportunities: arbitrage occurs when equivalent assets or combinations of assets sell for two different prices
- ***Law of one price***: two securities or portfolios that have identical cash flows in the future, regardless of future events, should have the same price

R.57 Derivative Markets and Instruments

➢ **Risk-free arbitrage and no-arbitrage rule (Cont.):**

- The way of arbitrage: ***sell high, buy low***
- If a portfolio consisting of A and B has a certain payoff, the portfolio should yield the risk-free risk
- The role of arbitrage is to eliminate mispricing and lead to the market efficiency. That is why arbitrage also plays a role in ***pricing.***

Example

➢ Whether these two rules below can restrict the price discover function of the market?

	Restrict sell short system	limit the amount of arbitrage
A.	yes	yes
B.	yes	no
C.	no	yes

➢ Correct answer: A

➢ Solution

- Sell short和arbitrage可以促进市场有效定价，加以限制将影响市场功能.

Framework

- **R57：Derivative markets and instruments**
  - **基本概念**
    1. Derivative的定义
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R.57 Derivative Markets and Instruments—Forward

- **Definition:** A **forward contract** is a bilateral contract that obligates one party to buy and the other party to sell a specific quantity of an underlying asset, at a set price, on a specific date in the future
- **Long and short forward position**
  - ***Long***: buy underlying
  - ***Short***: sell underlying
  - No payments will be made at the inception of a forward contract. So both parties of a forward contract is exposed to potential default risk

R.57 Derivative Markets and Instruments—Forward

- **Forward contracts分类：**
  - *Commodity forward contract*：商品远期合约
  - *Financial forward contract*：金融远期合约
- **Purposes of trading forward contracts：**
  - *Hedge risk*：套期保值，锁定未来交易成本，但不保证一定比不实施套期保值赚钱。存在default risk。
  - *Speculation*：投机，赌未来价格的变化方向。
- **Characteristics of Forward contracts：**
  - *Each party are exposed to **default risk** (or **counterparty risk**).*
  - Zero-sum game.

R.57 Derivative Markets and Instruments—Forward

- **Settling a forward contract *at expiration***
  - ***Physical settlement***: deliver an actual asset，存在储存成本，多用于商品远期
  - ***Cash settlement***: the party that has a position with negative value is obligated to pay that amount to the other party，多用在金融远期
- **Settling a forward contract *prior to expiration***
  - ***Entering into an opposite forward contract***: with an expiration date equal to the time remaining on the original contract
    - ✓ Offsetting with a ***different*** party: some credit risk remains
    - ✓ Offsetting with the ***original*** party: can avoid credit risk

Example

1. Which is the most common way to terminate a forward contract prior to expiration?
   - A. Cash settlement
   - B. Enter into an opposite contract
   - C. Delivers the actual instruments

➢ Correct answer: B

2. How to eliminate the risk on a forward contract: terminate
   - A. enter a opposite trade with same counterparty at same price
   - B. enter a opposite trade with different counterparty for any price
   - C. enter a opposite trade with same counterparty for any price

➢ Correct answer: A

R.57 Derivative Markets and Instruments—Forward

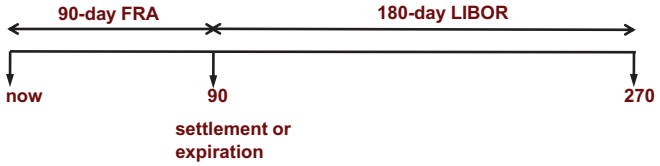
- **LIBOR, Euribor, and FRAs**
  - Eurodollar time deposit.
  - London Interbank Offer Rate (LIBOR).
    - ✓ USD interest rates.
    - ✓ Quoted as an annualized rates based on a 360-day a year
    - ✓ Add-on rate
    - ✓ Single interest

    Example：英国借本金1million的美元，借30天，30天的LIBOR是6%，那30天到期之后我应该还多少钱？
  - Euribor is a similar rate for borrowing and lending in Euros
  - A forward rate agreement (FRA) is a forward contract on an interest rate (LIBOR)

R.57 Derivative Markets and Instruments—Forward

➢ **LIBOR, Euribor, and FRAs（续）**

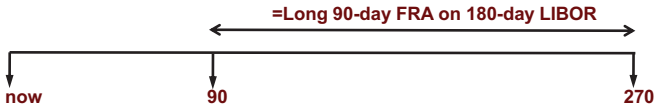
- **FRA定义**：An FRA can be viewed as a forward contract to borrow/lend money at a certain rate at some future date.
  - ✓ **The long position**: is the party that would **borrow** the money
  - ✓ **The short position**: is the party that would **lend** the money
- **FRA期限.**
  - ✓ 常见期限：30、60、90、120天Libor
  - ✓ Off-the-run FRA：非标准周期如45天Libor
- **报价**： Example 3×9FRA



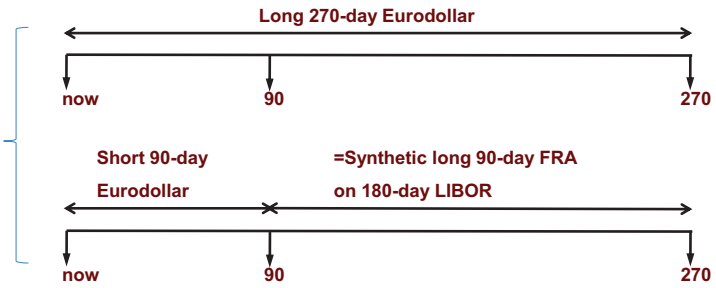
R.57 Derivative Markets and Instruments—Forward

➢ **LIBOR, Euribor, and FRAs（续）**

- **Real FRA**



- **Synthetic FRA**



R.57 Derivative Markets and Instruments—Forward

➢ **LIBOR, Euribor, and FRAs（续）**

交割：settle in cash, but no actual loan is made at the settlement date

- **Payoff定性分析**：
  - ✓ If the reference rate at the expiration date is above the specified contract rate, the long will receive cash payment from the short;
  - ✓ If the reference rate at the expiration date is below the contract rate, the short will receive cash payment from the long
- **Payoff定量分析**

$$(\text{notional principal})\left[\frac{(\text{floating rate at settlement}-\text{forward rate})\left[\frac{\text{days}}{360}\right]}{1+\text{floating rate at settlement}\left[\frac{\text{days}}{360}\right]}\right]$$

Example

- Which of the following best describes the forward rate of an PRA?
  - A. The spot rate implied by the term structure
  - B. The forward rate implied by the term structure
  - C. The rate on a zero-coupon bond of maturity equal to that of the forward contract
- Correct answer: B
- The underlying asset of FRA is
  - A. Bond
  - B. Stock
  - C. Interest rate
- Correct answer: C

R.57 Derivative Markets and Instruments—Futures

- 定义：
  - A **futures contract** is an agreement that obligates one party to buy and the other party to sell a specific quantity of an underlying asset, at a set price, at a future date.
- 与forward contract相似点:
  - Can be either deliverable or cash settlement contracts;
    - ✓ ***Deliverable contracts*** obligate the long to buy and the short to sell a certain quantity of an asset for a certain price on a specified future date.
    - ✓ ***Cash settlement*** contracts are settled by paying the contract value in cash on the expiration date.
  - Are priced to have zero value at the time an investor enters into the contract.

R.57 Derivative Markets and Instruments—Futures

- 与forward区别:

Forwards	Futures
Private contracts	Exchange-traded
Unique customized contracts	Standardized contracts
Little or no regulation	Regulated
Default risk is present	Guaranteed by clearinghouse
Settlement at maturity	Daily settlement (mark to market)
No margin deposit required	Margin required and adjusted

R.57 Derivative Markets and Instruments—Futures

➢ **Standardization:**

- Futures contracts specify the **quality and quantity** of goods that can be delivered, the **delivery time and the manner** of delivery.

➢ **clearinghouse**

- **Each exchange has a clearing house** that guarantees that traders in the futures market will honor their obligations.
- A clearinghouse acts as the **counterparty** to each participant. The clearinghouse is the buyer to every seller and the seller to every buyer.
- There is no need to worry about the **counterparty default risk**.
- Clearinghouse allows either side of the trade to **reverse positions** at a future date.

R.57 Derivative Markets and Instruments—Futures

➢ **Futures contract风险控制方法**

- **Margin;**
- **Daily Price Limit;**
- **Marking to market.**

R.57 Derivative Markets and Instruments—Futures

➢ **Futures contract风险控制方法**

- **方法一：Margin:**
  - ✓ ***Initial margin***: The first deposit is called the initial margin. Initial margin must be posted before any trading takes place;
  - ✓ ***Maintenance margin***: If the margin balance in the trader's account falls below the maintenance margin, the trader will get a margin call
  - ✓ ***Variation margin***: used to bring the margin balance back up to the ***initial margin level.***

➢ 例题

Initial margin=$5/contract, maintenance margin=$2/contract, long 20 contract

Day	Beginning balance	Funds deposited	Futures price	Price change	Gain/Loss	Ending Balance
0	0	100	82			100
1	100	0	84	2	40	140
2	140	0	78	-6	-120	20
3	20	80	73	-5	-100	0
4	0	100	79	6	120	220
5	220	0	82	3	60	280
6	280	0	84	2	40	320

➢ Futures contract风险控制方法（续）

- **Margin** （续）:与股票市场Margin的比较

	期货margin	股票margin
目的	做抵押减少违约风险	借钱给你买股票，举杠杆
现金流方向	现金流出	现金流入
支付利息	不用支付利息	相当于贷款给你，要付利息
补交margin数额	回到initial margin	回到maintenance margin

Example

1. Do “margin” in the stock market and “margin” in the futures market, respectively, mean that an investor has received a loan that reduces the amount of his own money required to complete the transaction?

	“Margin” in the stock market	“Margin” in the future market
A	No	No
B	No	Yes
C	Yes	No

➢ Correct answer: C

2. A futures trader must keep the money in the margin account above the:
   A. initial margin requirement
   B. variation margin requirement
   C. maintenance margin requirement

➢ Correct answer: C

R.57 Derivative Markets and Instruments—Futures

➢ **Futures contract风险控制方法（续）**

- **方法二：Daily Price Limit涨跌停机制:**
  - ✓Price limits are exchanged-imposed limits on how much the contract price can change from the previous day's settlement price;
  - ✓***Limit move***: If traders wish to trade at prices outside these limit---no trades will take place.---the settlement price will be reported upper or lower price limits
  - ✓***Locked limit:*** if trades cannot take place because of a limit move, either up or down, the price is said to be locked limit, since no trades can take place and traders are "locked" into their existing positions.

R.57 Derivative Markets and Instruments—Futures

➢ **Futures contract风险控制方法（续）**

- **方法三：*Marking to market*:** The margin requirement of a futures contract is low because at the end of every day there is a daily settlement process called marking to market

Example

➢ Which of the following statements about futures contracts is **FALSE**?

A. The futures clearinghouse allows traders to reverse their positions without having to contract the other side of the initial trade.

B. To safeguard the clearinghouse, the exchange requires traders to post margin and settle their accounts on a weekly basis.

C. Offsetting trades rather than exchanges for physicals are used to close most futures contracts.

➢ Correct answer: B

➢ Which of the' following occurs in the daily settlement of futures contracts?

A. Initial margin deposits are refunded to the two parties.

B. Gains and losses are reported to other market participants.

C. Losses are charged to one party and gains credited to the other.

➢ Correct answer: B

R.57 Derivative Markets and Instruments—Swap

➢ Characteristics of Swap Contracts

- ***Swap contract*** **:** A swap contract obligates two parties to change a series of cash flows on periodic settlement dates over a certain time period.
- ***与Forward相似点:***
  - ✓ No payment required by either party at initiation except the principal values exchanged in currency swaps.
  - ✓ Custom instruments.
  - ✓ Not traded in any organized secondary market.
  - ✓ Largely unregulated.
  - ✓ Default risk is a critical aspect of the contracts.
  - ✓ Institutions dominate

R.57 Derivative Markets and Instruments—Swap

➢ Three types of swap contracts- Interest Rate Swaps

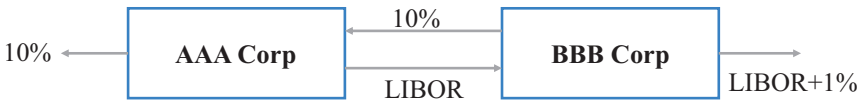
- The **plain vanilla interest rate swap** involves trading fixed interest rate payments for floating-rate payment (paying fixed and receiving floating).
  - ✓ ***Counterparties***: The parties involved in any swap agreement are called the counterparties
  - ✓ ***Pay-fixed side***: The counterparty that wants variable-rate interest agrees to pay fixed-rate interest.
  - ✓ ***Pay-floating side***: The counterparty that receives the fixed payment and agrees to pay variable-rate interest .

R.57 Derivative Markets and Instruments—Swap

➢ Three types of swap contracts- Interest Rate Swaps

- The Comparative Advantage Argument
  - ✓ **AAA Corp**: wants to borrow floating
  - ✓ **BBB Corp**: wants to borrow fixed.

	Fixed	Floating
AAA Corp	10.00%	6-month LIBOR + 0.30%
BBB Corp	11.20%	6-month LIBOR + 1.00%



- ✓ AAA Corp：LIBOR，节省0.3%
- ✓ BBB Corp：11%，节省0.2%

R.57 Derivative Markets and Instruments—Option

➢ Basic Concepts

- **Option定义：** An option gives its owner the right, but not the obligation, to buy or sell an underlying asset on or before a future date (the expiration date) at a predetermined price (the exercise price or strike price)
  - ✓ **Call option：** Long call & Short call
  - ✓ **Put option：** Long put & short put
  - ✓ The seller or short position in an options contract is sometimes referred to as the **writer of the option**
- **价格：**
  - ✓ **期权价格：** option premium paid by the buyer of option;
  - ✓ **执行价格：** Strike price (X) represents the exercise price specified in the contract.

R.57 Derivative Markets and Instruments—Option

Credit default swap (CDS)

- ➢ ***Credit default swaps (CDS)*** is essentially an insurance contract for the reference, the reference obligation is the fixed income security on which the swap is written-usually a bond but potentially also a loan.
  - Protection buyer receives a payment from the protection seller if default occurs on the reference entity.
  - The protection buyer pays the seller a premium. The default swap premium is also referred to as the ***CDS spread***.
- ➢ ***Credit spread option:*** a call option that is based on a bond's yield spread relative to a benchmark.
  - If the bond's credit quality decreases, its yield spread will increase
  - The bondholder will collect a payoff on the option.

R.57 Derivative Markets and Instruments—Option

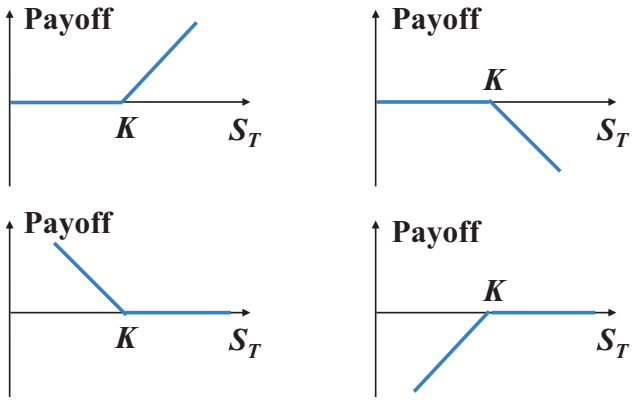
➢ Moneyness（价值状态）:定性看long是否赚钱

- **Moneyness：**
  - ✓ **In the money**: Immediate exercise would generate a positive payoff
  - ✓ **At the money** : Immediate exercise would generate no payoff
  - ✓ **Out of the money** : Immediate exercise would generate no payoff
- The following table summarizes the moneyness of options based on the stock's current price, S, and the option's exercise strike price, X.

Moneyness	Call option	Put Option
In-the-money	S＞X	S＜X
At-the-money	S = X	S = X
Out-the-money	S＜X	S＞X

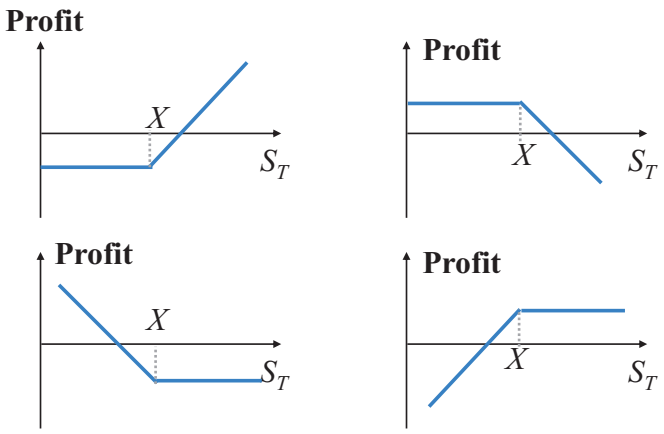
R.57 Derivative Markets and Instruments—Option

➢ **Intrinsic Value（内在价值）:定量看long赚多少钱**



R.57 Derivative Markets and Instruments—Option

➢ **Gain/Loss**



R.57 Derivative Markets and Instruments—Option

➢ **Intrinsic Value（内在价值）:定量看long赚多少钱**

- **Intrinsic Value:** the amount that it is in the money, and zero otherwise
  - ✓ Intrinsic value of call option: C=max[0, S-X]
  - ✓ Intrinsic value of put option: P=max[0, X-S]
- **Time Value:**
  - ✓ The difference between the price of an option (called its premium) and its intrinsic value is due to its time value
  - ✓ **Option value=intrinsic value + time value**
    - 到期日之前：option value>intrinsic value
    - 到期日：option value=intrinsic value
    - Price of the option is **more volatile** than prices of underlying stock

Example

1. Which of the following statements about call options at expiration is **TRUE?**
   - A. The profit potential to the buyer of the option is unlimited.
   - B. The call buyer's maximum loss is the call option's premium.
   - C. All of the answers are correct.

➢ Correct answer: C

2. Which of the below positions is the most risky, in the sense of having the largest potential losses?
   - A. A long position in call options.
   - B. A short position in put options.
   - C. A short (written) position in call options.

➢ Correct answer: C

Example

3. Consider a put option on Deter, Inc., with an exercise price of $45. The current stock price of Deter is $52. What is the intrinsic value of the put option, and is the put option at-the-money or out-of-the-money?

	Intrinsic Value	Moneyness
A.	$7	At-the-money
B.	$0	Out-of-the-money
C.	$0	At-the-money

➢ Correct answer: B

4. Which statement about option valuation is **FALSE**?
   - A. Prior to maturity, out-of-the-money options have no value.
   - B. The value of an option is its time value plus its intrinsic value.
   - C. The buyer of a call option contract can never lose more than the initial premium.

➢ Correct answer: A

R.57 Derivative Markets and Instruments—Option

Put call parity

➢ *Put call parity*.

$$c + X/\left(1+R_f\right)^T = S + p$$

$$或 c + K/\left(1+R_f\right)^T = S + p$$

➢ *Positions replicating*

- *Condition A* $-s = -c + p - X/\left(1+R_f\right)^T$
- *Condition B* $p = c + X/\left(1+R_f\right)^T - S$
- *Condition C* $c = p + S - X/\left(1+R_f\right)^T$
- *Condition D* $-p = -c + S - X/\left(1+R_f\right)^T$
- *Condition E* $-c = -p + X/\left(1+R_f\right)^T - S$

R.57 Derivative Markets and Instruments—Option

- A fiduciary call is a portfolio consisting of:
  - A long position in a European call option with an exercise price of X that maturities in T years on a stock.
  - A long position in a pure-discount riskless bond that pays X in T years.
- The cost a fiduciary call is the cost of the call (C_0) plus the cost of the bond (the present value of X). The payoff to a ***fiduciary call*** will be X if the call is out-of-the-money and S_T if the call is in-the-money, as shown in the following:

	$S_T \le X$ (*Call is out-of or at-the-money*)	$S_T > X$ (*Call is in-the-money*)
Long call payoff	0	$S_T - X$
Long bond payoff	X	X
Total payoff	X	S_T

R.57 Derivative Markets and Instruments—Option

- A protective put is a portfolio consisting of :
  - A long position in a European put option with an exercise price of X that maturities in T years on a stock.
  - A long position in the underlying stock.
- The cost of a protective put is the cost of the put (P_0) plus the cost of the stock(S_0). The payoff to a ***protective put*** is X if the put is in-the-money and S_T if the put is out-of-the-money. as shown in the following:

	$S_T < X$ (put is in-the-money)	$S_T \ge X$ (put is out-of or at-the-money)
Long put payoff	$X - S_T$	0
Long stock payoff	S_T	S_T
Total payoff	X	S_T

Example

- As with all arbitrage trades, you want to "buy low and sell high." if put-call parity doesn't hold (if the cost of a fiduciary call does not equal the cost of a protective put), then you buy (go long in) the underpriced position and sell (go short) in the overpriced position.
- **Example: Exploit violations of put-call parity**
  - 90-day European call and put options with a strike price of $45 is priced at $7.50 and $3.70. The underlying is priced at $48 and makes no cash payments during the life of the options. The risk-free rate is 5%. Calculate the no-arbitrage price of the call option, and illustrate how to earn an arbitrage profit.

Example

Answer:

- $C_0 = P_0 + S_0 - X/(1+R_f)^T = \$3.70 + \$48 - \$45/1.05^{90/365} = \$7.24 < \$7.5$
- Since the call is overpriced
  - we should sell the call for \$7.50 and buy the synthetic call for \$7.24.
  - To buy the synthetic call, buy the put for \$3.70, buy the underlying for \$48, and issue (sell short) a 90-day zero-coupon bond with a face value of \$45.
  - The transaction will generate an arbitrage profit of \$0.26 today.

R.57 Derivative Markets and Instruments—Option

- **Minimum and Maximum Option Values（文字描述）**
  - ***Lower bound***. Theoretically, no option will sell for less than its intrinsic value and no option can take on a negative value.
  - ***Upper bound for call options***. The maximum value of either an American or a European call option at any time t is the time-t share price of the underlying stock.
  - ***Upper bound for put options***.
    - ✓ The price for an American put option cannot be more than its strike price.
    - ✓ The maximum value is the present value of the exercise price discounted at the risk-free rate.

R.57 Derivative Markets and Instruments—Option

- **Minimum and Maximum Option Values（公式）**

Min value and Max value of options without dividend

Option	Min Value	Max Value
European call	$\text{Max}[0, S_t - X/(1+R_f)^{T-t}]$	S_t
American call	$\text{Max}[0, S_t - X/(1+R_f)^{T-t}]$	S_t
European put	$\text{Max}[0, X/(1+R_f)^{T-t} - S_t]$	$X/(1+R_f)^{T-t}$
American put	$P_t \geq \text{Max}[0, X - S_t]$	X

R.57 Derivative Markets and Instruments—Option

➢ **Early Exercise of American Options**

- **American call options**
  - ✓ when the underlying makes no cash payments, no reason to exercise the call early, $C_0 = c_0$,
  - ✓ when the underlying makes cash payments during the life of the option, early exercise can happen, $C_0 >= c_0$
- **American put options**
  - ✓ P0 > p0 , nearly always true,

    as long as there is a possibility of bankruptcy , P_0 always > p_0

    (consider an American put on a bankrupt company, stock →0, cannot go any lower, then put option holder may exercise it)

Example

➢ A European stock index call option has a strike price of $1160 and a time to expiration of 0.25 years. Given a risk-free rate of 4%, if the underlying index is trading at $1,200 and has a multiplier of 1, then the lower bound for the option price is closest to:

A. $ 0.00.
B. $28.29.
C. $51.32.

➢ Correct answer: C

➢ **Solution**

- The lower bound on a European call is either zero or the underlying price minus the present value of the exercise price, whichever is greater. $1200 - ($1160 / 1.04^0.25) = $51.32.

Framework

➢ **R58：Basics of Derivative Pricing and Valuation**

1. Arbitrage, replication, and risk neutrality
2. Forward Markets and Contracts
   - Price and Value
3. Futures Contracts & forward contracts
4. Swap Markets and Contracts
5. Option Markets and Contracts
   - Binomial Model

R58. Basics of Derivative Pricing and Valuation

- The ***price*** is the predetermined price in the contract that the long should pay to the short to buy the underlying asset at the settlement date
- The contract ***value*** is zero to both parties at initiation
- The ***no-arbitrage principle***: there should not be a riskless profit to be gained by a combination of a forward contract position with position in other asset.
  - Two assets or portfolios with identical future cash flows, regardless of future events, should have same price
- **Risk neutrality**
  - Risk-neutral investors are willing to buy risky investments for which they expect to earn only the risk-free rate. They do not expect to earn a premium for bearing risk.
  - The expected payoff of the derivative can be discounted at the risk-free rate. And should yield the ***risk-free rate of return***, if it generates certain payoffs

R58. Pricing and Valuation

- ***Pricing*** a forward contract is the process of determining the *no-arbitrage* price that will make the value of the contract be zero to both sides at the initiation of the contract

Forward Price = price that would not permit profitable riskless arbitrage in frictionless markets

- FP=S_0＋Carrying Costs－Carrying Benefits
- ***Valuation*** of a forward contract means determining the value of the contract to the long (or the short) at some time during the life of the contract.

R58. Forwards Pricing :No-Arbitrage Principle

- ***Cash-and-Carry Arbitrage*** **When the Forward Contract is Overpriced**
  - If FP >$S_0 \times (1+R_f)^T$

At initiation	At settlement date
• Borrow S0 at the risk-free rate • Use the money to buy the • Underlying bond • Short a forward contract	• Deliver the underlying to the long • Get FP from the long • Repay the loan amount of
	Profit= FP- $S_0 \times (1+R_f)^T$

R58. Forwards Pricing :No-Arbitrage Principle

➢ ***Reverse Cash-and-Carry Arbitrage*** **when the Forward Contract is Under-priced**

- If $FP < S_0 \times (1+R_f)^T$

At initiation	At settlement date
• Short sell the underlying bond to get S_0 • Invest S_0 at the risk-free rate • Long a forward contract	• Pay the short FP to get the underlying bond • Close out the short position by delivering the bond • Receive investment proceeds
	Profit=$S_0 \times (1+R_f)^T$-*FP*

R58. Pricing and Valuation

➢ ***T-bill (zero-coupon bond) forwards***

- buy a T-bill today at the spot price (S_0) and short a T-month T-bill forward contract at the forward price (FP)

$$FP = S_0 \times (1+R_f)^T$$

➢ ***Forward value of long position at initiation, during the contract life, and at expiration***

Time	Forward Contract Valuation
t=0	*Zero*, because the contract is priced to prevent arbitrage
t=t	$V_{long} = S_t - \frac{FP}{(1+R_f)^{T-t}}$ $V_{short} = -V_{long} = \frac{FP}{(1+R_f)^{T-t}} - S_t$
t=T	S_T-FP

R58. Forward Pricing and Valuation

➢ ***T-bill (zero-coupon bond) forwards***

- buy a T-bill today at the spot price (S_0) and short a T-month T-bill forward contract at the forward price (FP)

$$FP = S_0 \times (1+R_f)^T$$

➢ ***Equity Forward (forward contracts on a dividend-paying stock)***

➢ ***Bond Forward***

R58. Pricing and Valuation with cost and benefit

➢ Forward contracts on a dividend-paying stock

- **Price:**

$$FP = (S_0 - PVD_0) \times (1 + R_f)^T$$

- **Value:**

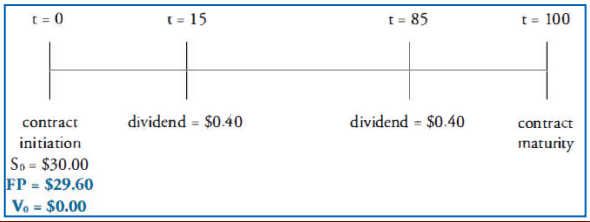
$$V_{long} = S_t - PVD_t - \frac{FP}{(1 + R_f)^{T-t}}$$

R58. Equity Forward Contracts

➢ Example

- Calculate the no-arbitrage forward price for a 100-day forward on a stock that is currently priced at $30.00 and is expected to pay a dividend of $0.40 in 15 days, $0.40 in 85 days, and $0.50 in 175 days. The annual risk-free rate is 5%, and the yield curve is flat.
  - ✓ Ignore the dividend in 175 days because it occurs after the maturity of the forward contract.
  - ✓ $PVD = \frac{\$0.4}{1.05^{15/365}} + \frac{\$0.4}{1.05^{85/365}} = \$0.7946$

$FP = (\$30 - \$0.7946) \times 1.05^{100/365} = \$29.6$

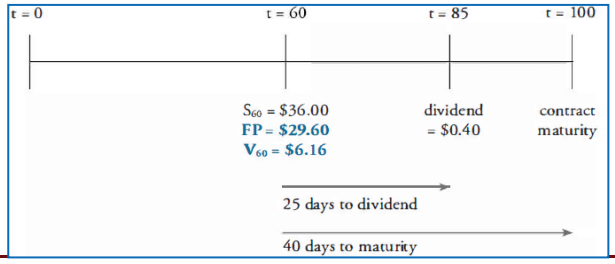


R58. Equity Forward Contracts

➢ Example

- After 60 days, the value of the stock in the previous example is $36.00. Calculate the value of the equity forward contract on the stock to the long position, assuming the risk-free rate is still 5% and the yield curve is flat.
  - ✓ There's only one dividend remaining (in 25 days) before the contract matures (in 40 days) as shown below, so:
  - ✓ $PVD_{60} = \frac{\$0.4}{1.05^{25/365}} = \$0.3987$

V_{60} (long position) = $(\$36 - \$0.3987) - \frac{\$29.6}{1.05^{40/365}} = \$6.16$



4.4 Forward Contracts on Coupon Bonds

➢ **Coupon bonds**

- Similar to dividend-paying stocks, but the cash flows are coupons
- **Price:**

$$FP = (S_0 - PVC_0) \times (1 + R_f)^T$$

- **Value:**

$$V_{long} = (S_t - PVC_t) - \frac{FP}{(1 + R_f)^{T-t}}$$

➢ **General Equation**

- $\boldsymbol{FP = (S_0 - PVD_0 + PVC_0) \times (1 + R_f)^T}$
- $\boldsymbol{FP = S_0(1 + R_f)^T - (PVD_0 + PVC_0)(1 + R_f)^T}$

R58. Pricing and Valuation with cost and benefit

➢ ***Carrying Costs, Cost of Storage, θ***

- One cost incurred in owning commodity. e.g., corn, live cattle, and gold.

➢ ***Carrying Benefits, γ***

- Monetary benefits: dividends, coupons, interest, etc
- Non-monetary benefits: convenience yield
  - ***Convenience yield*** are primarily associated with commodities and generally exist as a result of difficulty in either shorting the commodity or unusually tight supplies.
- $\boldsymbol{FP = (S_0 - \gamma + \theta) \times (1 + R_f)^T}$ ***or*** $\boldsymbol{FP = S_0(1 + R_f)^T - (\gamma - \theta)(1 + R_f)^T}$

- The net cost and benefit is often referred to by the term ***carry***, or sometimes ***cost of carry***. (Benefit – Cost, $\boldsymbol{\gamma - \theta}$)

R58. Arbitrage, replication, and risk neutrality

➢ **It's more easy to conceive of derivative that would produce identical payoffs than many investments .**

- The payoffs for most derivatives come directly from the value of the underlying at the expiration of the derivative.
- The value of the derivative at expiration is certain.
- The price of the derivative is tied to the price of the underlying.
- The derivative can be used to hedge the underlying.

➢ **Limits to Arbitrage**

- Transaction costs.
- Borrow unlimited amounts of money at risk-free rate.
- Transactions require additional capital to maintain position.
- Gains from an offsetting position might not be liquid.
- One position can not be perfect hedged in practice.

R58. Arbitrage, replication, and risk neutrality

- **Replication: replicate the payoffs on one asset or portfolio with those of a different asset or portfolio.**
  - A risk-free asset (or portfolio) can be created from a position in the underlying asset that is hedged with a position in a derivative security.
    - ✓ Asset + Derivative = Risk-free asset
  - Or
    - ✓ Asset - Risk-free asset = - Derivative
    - ✓ Derivative - Risk-free asset = - Asset

Examples

- Which of the following does not represent a benefit of holding an asset?
  - A. The convenience yield
  - B. An optimistic expected outlook for the asset
  - C. Dividends if the asset is a stock or interest if the asset is a bond
  - Correct answer: B
- Which of the following best describes an arbitrage opportunity? It is an opportunity to:
  - A. earn a risk premium in the short run.
  - B. buy an asset at less than its fundamental value.
  - C. make a profit at no risk with no capital invested.
  - Correct answer: C

Examples

- An investor who requires no premium to compensate for the assumption of risk is said to be which of the following?
  - A. Risk seeking
  - B. Risk averse
  - C. Risk neutral
  - Correct answer: C

Examples

- Which of the following best describes the difference between the price of a forward contract and its value?
  - A. The forward price is fixed at the start, and the value starts at zero and then changes.
  - B. The price determines the profit to the buyer, and the value determines the profit to the seller.
  - C. The forward contract value is a benchmark against which the price is compared for the purposes of determining whether a trade is advisable.
  - Correct answer: A
- Which of the following best describes the value of the forward contract at expiration? The value is the price of the underlying:
  - A. minus the forward price.
  - B. divided by the forward price.
  - C. minus the compounded forward price.
  - Correct answer: A

R58. Futures Pricing and Valuation

- ***Prices of Futures vs. Forward Contracts***

If the correlation between the underlying asset value and interest rate is…	Investors will…
Positive	Prefer to go long in a futures contract, and the futures price will be greater than the price of an otherwise comparable forward contract.
Zero	Have no preference
Negative	Prefer to go long in a forward contract, and the forward price will be greater than the price of an otherwise comparable futures contract.

R58. Futures Pricing and Valuation

- ***Pricing and valuation of Futures Contracts***
  - The value of a futures contract is zero at contract inception.
  - Futures contracts are marked to market daily, the value just after marking to market is reset to zero.
  - Between the times at which the contract is marked to market, the value can be different from zero.
  - *V (long) = current futures price − futures price at the last mark-to-market time*.
  - Another view of futures: settle previous futures, and then open another new futures with same date of maturity.

R58. Swap Pricing and Valuation

- A ***swap contract*** is an agreement between two parties to exchange a series of future cash flows. There are three kinds of swaps: interest rate swaps, currency swaps and equity swaps.
- ***A plain vanilla swap*** is an interest rate swap in which one party pays a fixed rate and the other pays a floating rate. The terms of the long and short are not used here, instead we say the fixed-rate payer and floating-rate (variable-rate) payer.
- The price is just the fixed rate (called the ***swap rate***) that makes the contract value zero to both parties at initiation. After some days the market situation changes, one party will make money and the other lose money. The contract value is no longer zero to both parties.

R58. Swap Pricing and Valuation

- ***Equivalence of swaps to bonds:***
  - An interest rate swap is identical to issuing a fixed-rate bond and using the proceeds to buy a floating-rate bond.
  - A currency swap is identical to issuing a fixed- or floating-rate bond in one currency, converting the proceeds to another currency, and using the proceeds to buy a floating- or fixed-rate bond in another currency.
  - An equity swap is identical to issuing a fixed- or floating-rate bond and using the proceeds to buy a stock or an index.

R58. Swap Pricing and Valuation

- ***Equivalence of swaps to forward contracts (FRA):***
  - A forward contract is an agreement to exchange future cash flows once, so a swap can be viewed as a series of forward contracts.
  - An interest rate swap, currency swap and equity swap are identical to a series of FRAs, currency forwards and equity forwards, respectively.
  - There are, however, some differences between swaps and forwards.
    - ✓ A series of FRAs will not all have the same forward rates, unless the yield curve is flat. So we often refer to a swap as a series of off-market forwards.
    - ✓ In addition, in interest rate swaps, the next payment is known one period ahead. This is not the case for an FRAs.

R58. Swap Pricing and Valuation

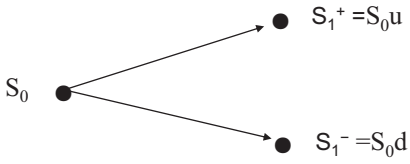
- ***Example:***
- 1. A swap is equivalent to a series of:
  - A. forward contracts, each created at the swap price.
  - B. long forward contracts, matched with short futures contracts.
  - C. forward contracts ,each created at their appropriate forward prices.
- Correct answer: A
  - Each implicit forward contract is said to be off-market, because it is created at the swap price, not the appropriate forward price, which would be the price created in the forward market.

Examples

- The price of a swap typically:
  - A. is zero at initiation.
  - B. fluctuates over the life of the contract.
  - C. is obtained through a process of replication.
  - Correct answer: C
- The value of a swap is equal to the present value of the:
  - A. fixed payments from the swap.
  - B. net cash flow payments from the swap.
  - C. underlying at the end of the contract.
  - Correct answer: B

R58. Option Pricing and Valuation

- A ***binomial model*** is based on the idea that, over the next period, some value will change to one of two possible values (binomial). To construct a binomial model, we need to know the beginning asset value, the size of the two possible changes, and the probabilities of each of these changes occurring.
- We start off by having only one binomial period, which means that the underlying price moves to two new prices at option expiration. We let S_0 be the price of the underlying stock now. One period later, the stock price can move up to S_1^+ or down to S_1^-. We then identify a factor, u, as the up move on the stock and d as the down move. Thus, $S_1^+ = S_0u$ and $S_1^- = S_0d$. We further assume that $u = 1/d$.



R58. Option Pricing and Valuation

- Risk-neutral probability of an up move is π_u ; Risk-neutral probability of an down move is π_d=1- π_u;

$$\pi_u = \frac{1+R_f-d}{u-d}$$

- We start with a call option. If the stock goes up to S_1^+ , the call option will be worth C_1^+. If the stock goes down to S_1^-, the call option will be worth C_1^-. We know that the value of a call option will be its intrinsic value on expiration date. Thus we get: $C_1^+ = \text{Max}(0, S_1^+ - X)$; $C_1^- = \text{Max}(0, S_1^- - X)$

$$\text{value of an option: } c = \left[\pi_u C_1^+ + \pi_d C_1^-\right] \times \frac{1}{(1+R_f)^T}$$

- Hedge ratio :

$$\text{Delta} = \frac{C^+ - C^-}{S^+ - S^-}(shares\ per\ option)$$

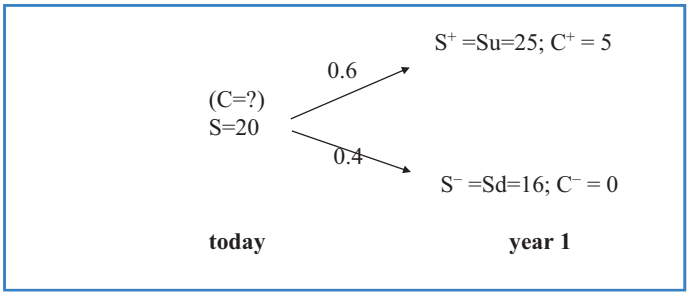
R58. Option Pricing and Valuation

Example: Calculate the value today of a 1-year call option on the stock with the strike price of $20. The price of the stock is $20 now, and the size of an up-move is 1.25. The risk-free rate is 7%.

- **Answer:**
  - **Step 1: Calculate the parameters:**
    - ✓ u=1.25 ; d=1/u=0.8 ; Su=20×1.25=25; Sd=20×0.8=16
    - ✓ $C^+ = \text{Max}(0, 25-20) = 5$; $C^- = \text{Max}(0, 16-20) = 0$
  - **Step 2: Calculate risk-neutral probabilities,** π_u and π_d =1− π_u :
    - ✓ $\pi_u = (1+0.07-0.8)/(1.25-0.8) = 0.6$ π_d =1− π_u = 0.4

R58. Option Pricing and Valuation

- **Step 3: Draw the one-period binomial tree:**



R58. Option Pricing and Valuation

- Pricing a put option is similar to that of a call. The only difference is that $P^+ = \text{Max}(0, X - S^+)$ and $P^- = \text{Max}(0, X - S^-)$.

Example: Use the information in the previous example to calculate the value today of a put on the same stock with the strike price of $20

Answer:

$P^+ = \text{Max}(0, 20-25) = 0$; $P^- = \text{Max}(0, 20-16) = 4$

$P = (0.6 \times 0 + 0.4 \times 4)/1.07 = 1.6/1.07 = 1.50$

R58. Option Pricing and Valuation

- ***Factors affect the value of an option***

Sensitivity Factor	Calls	Puts
Underlying price	Positively related	Negatively related
Volatility	Positively related	Positively related
Risk-free rate	Positively related	Negatively related
Time to expiration	Positively related	Positively related*
Strike price	Negatively related	Positively related
Payments on the underlying	Negatively related	Positively related
Carrying cost	Positively related	Negatively related

* There is an exception to the general rule that European put option thetas are negative. The put value may increases as the option approaches maturity if the option is deep in-the-money and close to maturity.

Framework

- **R59： Risk management applications of option strategies（Option Markets and Contracts的延伸）★★**

  Call/put/covered call/ protective put → value/profit/shape等

R.59 Risk management applications of option strategies

Basic Concepts

- The key here is your ability to interpret option payoff diagrams and calculate profit/loss diagrams
- Option positions
  - ✓ Buyer of a call option — long position.
  - ✓ Writer (seller) of a call option — short position.
  - ✓ Buyer of a put option — long position.
  - ✓ Writer (seller) of a put option — short position.

R.59 Risk management applications of option strategies

Call

- **Buying a call**
  - **Value at expiration** of buying a call**:** max(0,S-X)
  - **Profit** from buying a call: value at expiration minus option premium, max(0,S-X)-c
  - Maximum profit: infinite
  - Maximum loss: option premium (c)
  - Breakeven underlying price at expiration: exercise price plus option premium (X+c)
- When **selling a call**, these results are reversed
  - **Value at expiration** of selling a call**: -**max(0,S-X)
  - **Profit** from selling a call: option premium minus value at expiration, -max(0,S-X)+c
  - Maximum profit: option premium (c)
  - Maximum loss: infinite
  - Breakeven underlying price at expiration: exercise price plus option premium (X+c)

R.59 Risk management applications of option strategies

Put

- **Buying a put**
  - **Value at expiration** of buying a put**:** max(0,X-S)
  - **Profit** from buying a put: value at expiration minus option premium, max(0,X-S)-p
  - Maximum profit: exercise price minus option premium (X-p)
  - Maximum loss: option premium (p)
  - Breakeven underlying price at expiration: exercise price minus option premium (X-p)
- When **selling a put**, these results are reversed
  - **Value at expiration** of selling a put**: -**max(0,X-S)
  - **Profit** from selling a put: option premium minus value at expiration, -max(0,X-S)+p
  - Maximum profit: option premium (p)
  - Maximum loss: exercise price minus option premium (X-p)
  - Breakeven underlying price at expiration: exercise price minus option premium (X-p)

R.59 Risk management applications of option strategies

➢ Covered Call

- A covered call is the combination of a long stock and a short call

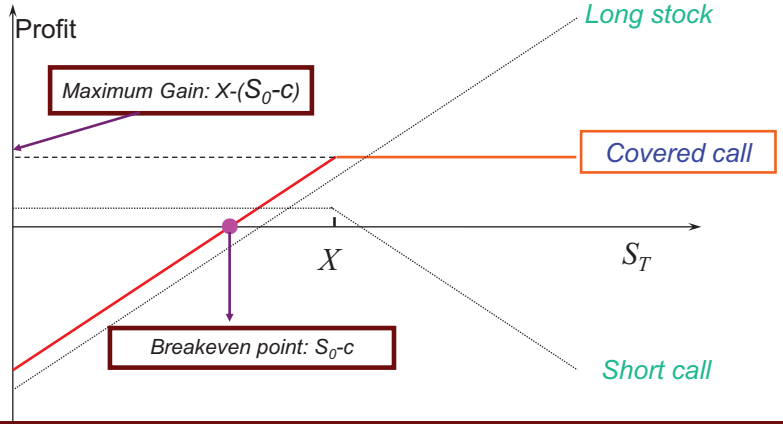
Covered call=S－C

- The term covered means that the stock covers the inherent obligation assumed in writing the call
- Why would you write a covered call? You feel the stock's price will not go up any time soon, and you want to increase your income by collecting some call option premiums.
- This strategy for enhancing income is not without risk. The call writer is trades the stock's upside potential, above the strike price, for the call premium. .

R.59 Risk management applications of option strategies

➢ Covered Call

$$\text{Covered Call} = -c+S = -p+\frac{X}{(1+r_f)^{T-t}}$$



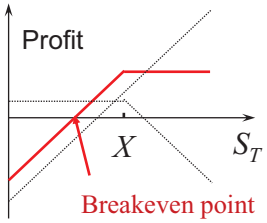
R.59 Risk management applications of option strategies

➢ Breakeven Point-Covered Call

$S_t<X$ Profit= { Stock: S_t-S_0 ; Option: C } ⇨ Total Profit=S_t-S_0+C

$S_t>=X$ Profit= { Stock: S_t-S_0 ; Option: C-(S_t-X) } ⇨ Total Profit=X-S_0+C

最大收益



R.59 Risk management applications of option strategies

➢ Protective Put

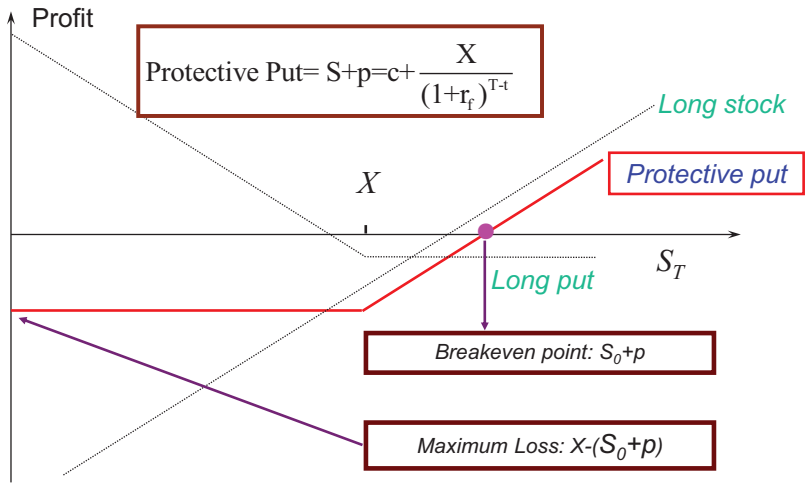
- A protective put is constructed by buying a stock and a put option on that stock

Protective put=S+P

- A protective put is an investment management technique designed to protect a stock from a decline in value
- If the stock price is above the strike price, you make money on the stock's appreciation but the gain is reduced by the put premium paid
- If the stock price decreases, the loss on the stock is offset by the gain on the put. The loss on the position is the put premium and any amount that the strike price is below the original stock price

R.59 Risk management applications of option strategies

➢ Protective Put



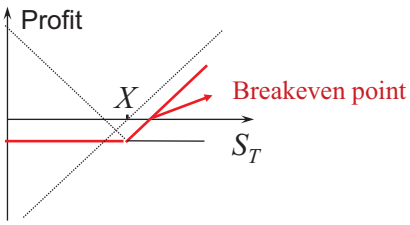
R.59 Risk management applications of option strategies

➢ Breakeven Point- Protective Put

$S_t<X$ Profit= { Stock: S_t-S_0 ; Option: $X-S_t-P$ } ⇨ Total Profit=$X-S_0-P$

最大损失

$S_t>=X$ Profit= { Stock: S_t-S_0 ; Option: $-P$ } ⇨ Total Profit=S_t-S_0-P



Example

- An investor has purchased a share of stock for $190. A call option on this stock, expiring in seven months and with an exercise price of $200, is priced at $11.40. If the investor enters into a covered call now, the profit on this strategy if the stock price at expiration is $215 is closest to:

  A. -$3.60.

  B. $21.40.

  C. $28.60.

- **B is correct.**
  - The profit on a covered call is calculated as follows:
  - $\pi = S_T - S_0 - \max(0, S_T - X) + c_0$
  - Ð = $215 -$190 - max(0, $215 - $200) + $11.40 = $21.40.

Example

- An investor simultaneously purchases an equity priced at €11 and a put option on the same equity, with an exercise price of €10 at a premium of €0.80. The breakeven price at expiration of this strategy is closest to:

  A. €10.80.

  B. €11.80.

  C. €10.20.

- Correct Answer: B
  - The breakeven price is defined as the sum of purchase price of the underlying and the put premium. In this case, €11 + €0.80 = €11.80.

R.59 Risk management applications of option strategies

- **Important Summary of risk management applications**
  - ***Covered call***
    - ✓ **Consists of**: *short call and long stock*
    - ✓ **Equivalent to**: *short put and long bond*
    - ✓ **Similar to**: *Short put*
    - ✓ **Breakeven point**: S_0-c
    - ✓ **Maximum Gain**: X-(S_0-c)
  - ***Protective put***
    - ✓ **Consists of**: *long stock and long put*
    - ✓ **Equivalent to**: *long call and long bond*
    - ✓ **Similar to**: *long call*
    - ✓ **Breakeven point**: S_0+p
    - ✓ **Maximum Loss**: X-(S_0+p)

It’s not the end but just the beginning.

Never stop smiling, not even when you're sad, someone might fall in love with your smile.

永远都不要停止微笑，即使是在你难过的时候，说不定有人会因为你的笑容而爱上你。

Thank you!