金程教育 GOLDEN FUTURE

专业·领先·增值

专业来自101%的投入!

CFA一级培训项目

Quantitative Methods

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Topic Weightings in CFA Level I

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Quantitative Methods

- Time Value Calculation
  - R5 The Time Value of Money
  - R6 Discounted Cash Flow Applications
- Probability & Statistics
  - R7 Statistical Concepts and Market Returns
  - R8 Probability Concepts
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- Inferential statistics
  - R10 Sampling and Estimation
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Time Value

Time Value of Money

- **Required rate of return** is
  - affected by the supply and demand of funds in the market;
  - the return that investors and savers require to get them to willingly lend their funds;
  - usually for particular investment.
- **Discount rate** is
  - the interest rate we use to discount payments to be made in the future.
  - usually used interchangeably with the interest rate.
- **Opportunity cost** is
  - also understood as a form of interest rate. It is the value that investors forgo by choosing a particular course of action.

Time Value of Money

- **Decompose required rate of return:**
  - Nominal risk-free rate = real risk-free rate + expected inflation rate
  - Required interest rate on a security

    = nominal risk-free rate + default risk premium + liquidity risk premium + maturity risk premium

Time Value of Money

EAR calculation:

$$\text{EAR}=(1+\text{periodic rate})^{m}-1 \quad \longleftrightarrow \quad 1+EAR=\left(1+\frac{r}{m}\right)^{m}=e^{r}$$

- **那么如果是semi, m=2; 如果是quarterly, m=4**
- **如果是连续复利，公式则变为EAR = e $^{\text{annual int}}$-1**

➢ **定性**（EAR和计息次数有关）

- The greater the compounding frequency, the greater the EAR will be in comparison to the stated rate

Time Value of Money

- Future value (**FV**): Amount to which investment grows after one or more compounding periods.
- Present value (**PV**): Current value of some future cash flow
- **Annuities**: is a stream of equal cash flows that occurs at equal intervals over a given period

➢ **内容：**

- N = number of periods
- I/Y = interest rate per period
- PV = present value
- PMT = amount of each periodic payment
- FV= future value

Time Value of Money

An example of ordinary annuities（后付年金）:

Example: What's the FV of an ordinary annuity that pays 100 per year at the end of each of the next 3 years, given the discount rate is 10%

Solutions: enter relevant data for calculate.

N=3, I/Y=10, PMT=-100, PV=0, CPT→FV=331

Time Value of Money

- **About an annuity due（先付年金）**
  - **Definition:** an annuity where the annuity payments occur at the beginning of each compounding period.
  - **Calculation:**
    - ✓**Measure 1:** put the calculator in the BGN mode and input relevant data.
    - ✓**Measure 2:** treat as an ordinary annuity and simply multiple the resulting PV by (1+I/Y)

Example: Time Value of Money

1. A company plans to borrow $50,000 for five years. The company's bank will lend the money at a rate of 9% and requires that the loan be paid off in five equal end-of-year payments. Calculate the amount of the payment that the company must make in order to fully amortize this loan in five years.

- Answer:
- N=5, I/Y=9, PV=50,000, FV=0; CPT: PMT=-12,854.62

2. Using the loan described in the preceding example, determine the payment amount if the bank requires the company to make quarterly payments.

- Answer:
- N=5×4=20, I/Y=9/4=2.25, PV=50,000, FV=0; CPT: PMT=-3,132.10

Example: 房贷月供问题

- 张女士买了一套价值100万的房子，首付比例30%，她从银行贷款70万，贷款的年利率为6.2%，期限为20年。她每月月末需向银行还款多少钱？
- 利用TVM功能：
- N=20*12=240, I/Y=6.2/12, PV=700,000, FV=0
- CPT PMT=-5096.12

- 还完第一个月以后，还有多少本金余额没有还？第一个月中本金还了多少？利息还了多少？
- 利用AMORT功能：（[2ND] [PV]）
- P1=1，P2=1，BAL=698,520.5485，PRN=-1,479.4515，INT=-3,616.6667

Example: 养老问题

- 张女士今年60岁，她从今天开始即将退休。如果从退休开始每年年初都要支取10万块钱的养老金的话，假设回报率为4%，张女士现在需要准备多少钱来支持她未来20年的生活？

- BGN模式：([2ND] [BGN], [2ND] [SET], [2ND] [QUIT])
- 利用TVM功能：
- N=20, I/Y=4, PMT=100,000 FV=0
- CPT PV=-1,413,393.94

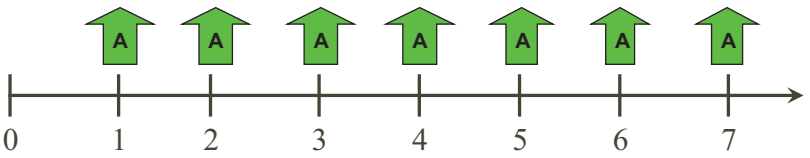
Example: 教育金问题

- 张女士的女儿今年要上大学，她现在开始每年年初都要给女儿支付5万块钱的教育金，一直持续到女儿大学毕业，为期4年，如果现在的市场利率为4%，张女士现在需要为女儿准备多少教育金？

- BGN模式：([2ND] [BGN], [2ND] [SET], [2ND] [QUIT])
- 利用TVM功能：
- N=4, I/Y=4, PMT=50,000, FV=0
- CPT PV=-188,754.55

Time Value of Money

- **About perpetuity**
  - **Definition:** A perpetuity is a financial instruments that pays a fixed amount of money at set intervals over an **infinite** period of time.



$PV_1 = A/(1+r)$ ←

$PV_2 = A/(1+r)^2$ ←——

$PV_3 = A/(1+r)^3$ ←——

$PV_4 = A/(1+r)^4$ ←——

etc. ←——

etc.

$$PV = \frac{A}{1+r} + \frac{A}{(1+r)^2} + \frac{A}{(1+r)^3} + \cdots \quad (1) \qquad (1+r)PV = A + \frac{A}{1+r} + \frac{A}{(1+r)^2} + \cdots \quad (2)$$

$$(2)-(1) \qquad r \times PV = A \Rightarrow PV = \frac{A}{r}$$

Discounted Cash Flow Applications

$$NPV = CF_0 + \frac{CF_1}{(1+r)^1} + \frac{CF_2}{(1+r)^2} + \ldots + \frac{CF_N}{(1+r)^N} = \sum_{t=0}^{N} \frac{CF_t}{(1+r)^t}$$

$$NPV = 0 = CF_0 + \frac{CF_1}{(1+IRR)^1} + \frac{CF_2}{(1+IRR)^2} + \ldots + \frac{CF_N}{(1+IRR)^N} = \sum_{t=0}^{N} \frac{CF_t}{(1+IRR)^t}$$

IRR（Internal Rate of Return）

- When NPV= 0, the discount rate.

Example: Discounted Cash Flow Applications

- 张女士花了10万块钱买了一份理财产品，期限为5年。这份理财产品在这5年里每年年末分别可以给张女士带来的收益为：3万、2万、2万、3万、4万。如果市场利率为5%，张女士购买这份理财产品净赚多少钱？内部收益率是多少？

Example: Discounted Cash Flow Applications

按键	解释	显示
[CF] [2ND] [CLR WORK]	清除CF功能中的存储记忆	CF0=0.0000
10[+/-][ENTER]	期初投入	CF0=-10.0000
[↓] 3 [ENTER]	第一期现金流	C01=3.0000
[↓] [↓] 2 [ENTER]	第二期现金流	C02=2.0000
[↓] [↓] 2 [ENTER]	第三期现金流	C03=2.0000
[↓] [↓] 3 [ENTER]	第四期现金流	C04=3.0000
[↓] [↓] 4 [ENTER]	第五期现金流	C05=4.0000
[NPV] 5 [ENTER]	折现率5%	I=5.0000
[↓] [CPT]	计算NPV	NPV=2.0011
[IRR] [CPT]	计算IRR	IRR=11.5156

Discounted Cash Flow Applications

HPR

- **Define:** the holding period return is simply the percentage change in the value of an investment over the period it is hold.

- **Calculate:**

$$HPR = \frac{P_1 - P_0 + CF_1}{P_0}$$

Example: Discounted Cash Flow Applications

- A stock is purchased for $30 and is sold for $33 six months later, during which time it paid $0.50 in dividends. Calculate the holding period return.

- Answer:

$$HPR = \frac{P_1 - P_0 + CF_1}{P_0} = \frac{33 - 30 + 0.5}{30} = 11.67\%$$

Statistical Concepts

Statistical Concepts

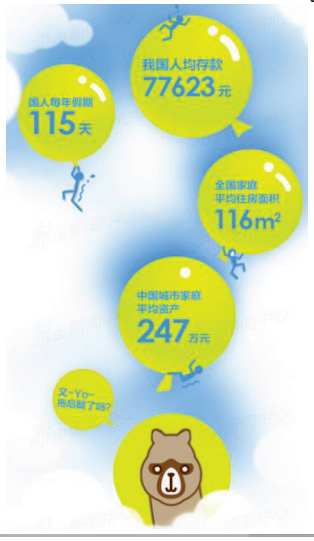
- Descriptive statistics
  - Summarize the important characteristics of large data sets.
- Inferential statistics
  - Make forecasts, estimates, or judgments about a large set of data on the basis of the statistical characteristics of a smaller set (a sample)

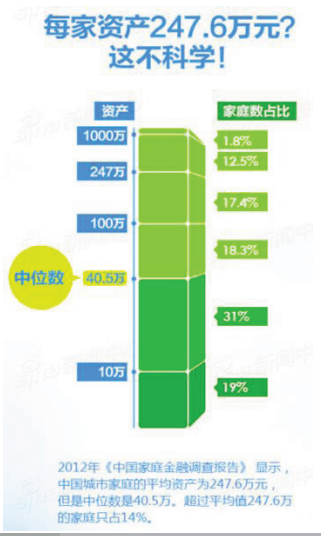
Statistical Concepts

- A measure used to describe a characteristic of a population is referred to as a **parameter**.
- In the same manner that a parameter may be used to describe a characteristic of a population, a **sample statistic** is used to measure a characteristic of a sample.

Statistical Concepts

- **Measures of central tendency: mode, median, mean**





Statistical Concepts

The arithmetic mean: $\overline{X} = \frac{\sum_{i=1}^{N} X_i}{n}$

The weighted mean: $\overline{X_W} = \sum_{i=1}^{n} w_i X_i = (w_1 X_1 + w_2 X_2 + \cdots + w_n X_n)$

The geometric mean: $G = \sqrt[N]{X_1 X_2 X_3 .. X_N} = (\prod_{i=1}^{N} X_i)^{1/N}$

The harmonic mean: $\overline{X_H} = \frac{n}{\sum_{i=1}^{n} (1/X_i)}$

Statistical Concepts

Range = maximum value – minimum value

$MAD = \frac{\sum_{i=1}^{N} |X_i - \overline{X}|}{n}$

For population: $\sigma^2 = \frac{\sum_{i=1}^{N} (X_i - \mu)^2}{N}$

For sample: $s^2 = \frac{\sum_{i=1}^{n} (X_i - \overline{X})^2}{n-1}$

- Standard deviation
- XYZ Corp. Annual Stock Prices

2003	2004	2005	2006	2007	2008
22%	5%	-7%	11%	2%	11%

1. Assuming that the distribution of XYZ stock returns is a population, what is the population variance?
   A. 6.8%² B. 7.7%² C. 80.2%²
2. Assuming that the distribution of XYZ stock returns is a sample, the sample variance is closet to?
   A. 5.0%² B. 72.4%² C. 96.3%²

Statistical Concepts

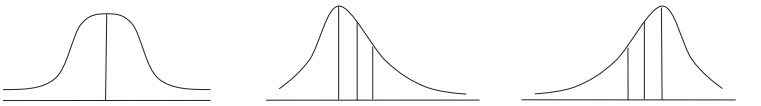
- **Coefficient of variation** measures the amount of dispersion in a distribution relative to the distribution's mean. (relative dispersion)

$$CV = \frac{S_x}{\overline{X}}$$

- **The sharp ratio** measures excess return per unit of risk.

$$\text{Sharp ratio} = \frac{R_P - R_f}{\sigma_P}$$

Statistical Concepts



Mean=Median=Mode | Mode<Median<Mean | Mean<Median<Mode

Symmetrical | **Positive (right) skew** | **Negative (left) skew**

Positive skewed：Mode<median<mean, having a right fat tail

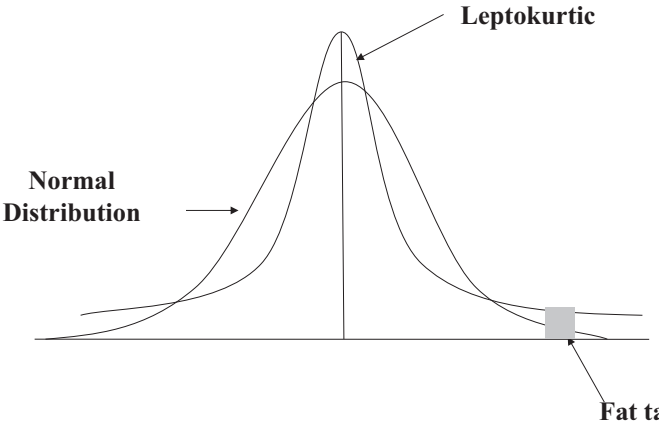
Negative skewed：Mode>media>mean, having a left fat tail

Statistical Concepts

- **Leptokurtic vs. platykurtic**
  - It deals with whether or not a distribution is more or less "peaked" than a normal distribution

	leptokurtic	Normal distribution	platykurtic
Sample kurtosis	>3	=3	<3

Statistical Concepts



A leptokurtic return distribution has more frequent extremely large deviations from the mean than a normal distribution.

Probability Concepts

17 世纪，欧洲贵族之间盛行赌博之风。我们的主人公，德·梅勒（De Mere），就是其中一位法国贵族。

历史没有留下他的画像，但我们可以知道，他是一位充满魅力的、有能力和经验的，赌徒，类似于这样...

顺治帝

故事发生时，中国正处于清朝顺治时期

德·梅勒

1651 年的一天，梅勒和朋友玩这么一个游戏

- 每个人出 30 金币；
- 两个人各自选择一个点数掷骰子；
- 谁选择的点数首先被掷出 3 次，谁就赢得全部赌注

德·梅勒

我选择 5

我选择 3

梅勒之友

哗啦哗啦！

哗啦哗啦！

形势大好时，梅勒由于一件紧急的事情必须离开，赌局不得不中止

你妈喊你回家吃饭！

德·梅勒

赌局进行了一会后，梅勒选择的“5”出现了两次，而他朋友选择的“3”只出现一次

德·梅勒

?

?

梅勒之友

问题来了，这 60 枚土豪金该如何划分？！

很傻很天真！

我们最多还需要两次结果就可以分出胜负，这两次结果无非有 4 种可能：

德·梅勒

1. 都是梅勒赢

2. 梅勒和朋友各赢一局

再赌下去，我还要赢 2 局才能取胜，而你只要 1 次。

那，做人呢，最重要的就是开心，这样好了，2：1，你拿 40 个金币，我拿 20 个。

梅勒之友

3. 梅勒和朋友各赢一局

4. 都是朋友赢

4 种可能中，前三种情况发生我都会取胜，所以我理应分得 3/4，即 45 个金币

德·梅勒

很黄很暴力……

梅勒之友

Probability Concepts

- **Basic Concepts**
  - **Random variable** is uncertain quantity/number.
  - **Outcome is** an observed value of a random variable.
  - **Event**
    - ✓ Mutually exclusive events—can not both happen at the same time.
    - ✓ Exhaustive events—include all possible outcomes.
- **Two Defining Properties of Probability**
  - **$0 \le P(E) \le 1$**
  - **$P(E1)+ P(E2)+\ldots\ldots+ P(En)=1$**

Probability Concepts

- Unconditional Probability (marginal probability): P(A)
- Conditional probability : P(A|B)

Probability Concepts

- **Joint probability : P(AB)**
  - **Multiplication rule:**
    - ✓P(AB)=P(A|B)×P(B)= P(B|A)×P(A)
  - If A and B are mutually exclusive events, then:

    P(AB)=P(A|B)=P(B|A)=0
- **Probability that at least one of two events will occur:**
  - **Addition rule:**
    - ✓P(A or B)=P(A)+P(B)-P(AB)
  - If A and B are mutually exclusive events, then:

    P(A or B)=P(A)+P(B)

Probability Concepts

- **The occurrence of A has no influence of on the occurrence of B**
  - P(A|B)=P(A) or P(B|A)=P(B)
  - P(AB)=P(A)×P(B)
  - P(A or B)=P(A)+P(B)-P(AB)
- **Independence and Mutually Exclusive are quite different**
  - If exclusive, must not independence;
  - Cause exclusive means if A occur, B can not occur, A influents B.
    - ✓P(AB)=P(A)×P(B)

Probability Concepts

- Expected value: $E(X)=\sum P(X_i)X_i$

$$E(X)=\sum x_i * P(x_i)=x_1 * P(x_1)+x_2 * P(x_2)+\cdots+x_n * P(x_n)$$

$$\sigma=\sqrt{\sigma^2} \qquad \sigma^2=\sum_{i=1}^{N} P_i(X_i-EX)^2$$

Common Probability Distributions

Common Probability Distributions

- **Probability Distribution**
  - Describe the probabilities of all the possible outcomes for a random variable.
- **Discrete and continuous random variables**
  - Discrete random variables: the number of possible outcomes can be counted, and for each possible outcome, there is a measurable and positive probability.
  - Continuous variables: the number of possible outcomes is infinite, even if lower and upper bounds exist.
    - ✓P (x)=0 even though x can occur.
    - ✓P (x1<X<x2)

Common Probability Distributions

- **Probability function:** $p(x)=P(X=x)$
  - For discrete random variables
  - $0 \le p(x) \le 1$
  - $\Sigma p(x)=1$
- **Probability density function (p.d.f) :** f(x)
  - For continuous random variable commonly

Common Probability Distributions

➢ **Discrete uniform**

- A discrete uniform random variable is one for which the probabilities for all possible outcomes for a discrete random variable are equal.
- For example, consider the discrete uniform probability distribution defined as X={1,2,3,4,5}, p(x)=0.2.
  - ✓Here, the probability for each outcome is equal to 0.2 [i.e., p(1)=p(2)=p(3)=p(4)=p(5)=0.2].

Common Probability Distributions

➢ **Continuous Uniform Distribution**

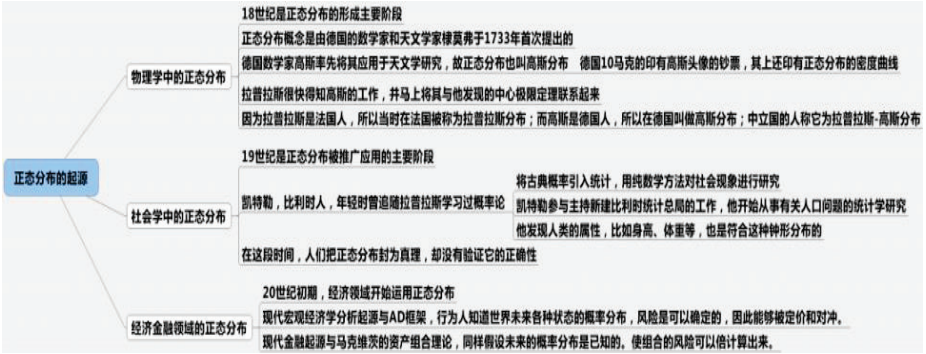
---- is defined over a range that spans between some lower limit, a, and upper limit, b, which serve as the parameters of the distribution.

➢ Properties of Continuous uniform distribution

- For all a ≤x1< x2 ≤b
- P (X<a or X>b) = 0
- $P(x_1 \le X \le x_2) = (x_2 - x_1)/(b-a)$

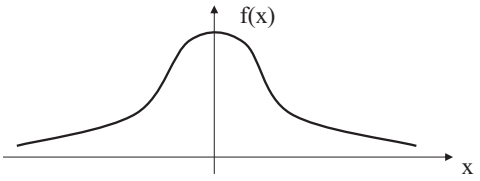
Common Probability Distributions

➢ 正态分布的前世今生



Common Probability Distributions

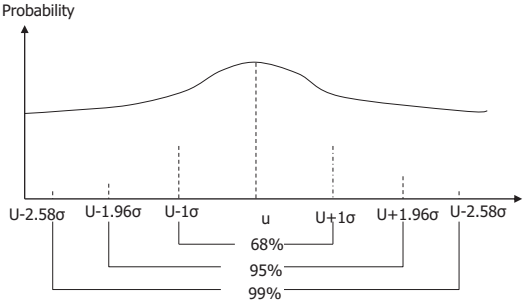
- **The shape of the density function**



- **Properties**:
  - X~N(μ , σ²)
  - Symmetrical distribution: skewness=0; kurtosis=3
  - The tails get thin and go to zero but extend infinitely, asympotic（渐近）

Common Probability Distributions

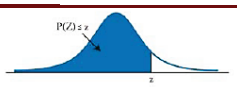
- **The confidence intervals**
  - 68% confidence interval is $[\mu-\sigma,\mu+\sigma]$
  - 90% confidence interval is $[\mu-1.65\sigma,\mu+1.65\sigma]$
  - 95% confidence interval is $[\mu-1.96\sigma,\mu+1.96\sigma]$
  - 99% confidence interval is $[\mu-2.58\sigma,\mu+2.58\sigma]$



Common Probability Distributions

- **Standard normal distribution**
  - N(0,1) or Z
  - Standardization: if X~N(μ , σ²), then $Z=\frac{X-\mu}{\sigma}\sim N(0,1)$
  - Z-table
- F(-z)=1-F(z)
- P(Z>z) = 1 –F(z)

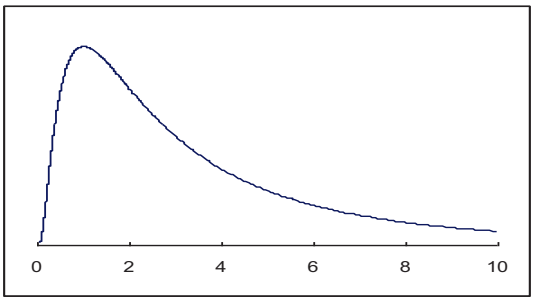
Cumulative Z-Table



Standard Normal Distribution
$P(Z \le z) = N(z)$ for $z \ge 0$

z	0.00	0.01	0.02	0.03	0.04	0.05	0.06	0.07	0.08	0.09
0.0	0.5000	0.5040	0.5080	0.5120	0.5160	0.5199	0.5239	0.5279	0.5319	0.5359
0.1	0.5398	0.5438	0.5478	0.5517	0.5557	0.5596	0.5636	0.5675	0.5714	0.5753
0.2	0.5793	0.5832	0.5871	0.5910	0.5948	0.5987	0.6026	0.6064	0.6103	0.6141
0.3	0.6179	0.6217	0.6255	0.6293	0.6331	0.6368	0.6406	0.6443	0.6480	0.6517
0.4	0.6554	0.6591	0.6628	0.6664	0.6700	0.6736	0.6772	0.6808	0.6844	0.6879
0.5	0.6915	0.6950	0.6985	0.7019	0.7054	0.7088	0.7123	0.7157	0.7190	0.7224
0.6	0.7257	0.7291	0.7324	0.7357	0.7389	0.7422	0.7454	0.7486	0.7517	0.7549
0.7	0.7580	0.7611	0.7642	0.7673	0.7704	0.7734	0.7764	0.7794	0.7823	0.7852
0.8	0.7881	0.7910	0.7939	0.7967	0.7995	0.8023	0.8051	0.8078	0.8106	0.8133
0.9	0.8159	0.8186	0.8212	0.8238	0.8264	0.8289	0.8315	0.8340	0.8365	0.8389
1.0	0.8413	0.8438	0.8461	0.8485	0.8508	0.8531	0.8554	0.8577	0.8599	0.8621
1.1	0.8643	0.8665	0.8686	0.8708	0.8729	0.8749	0.8770	0.8790	0.8810	0.8830
1.2	0.8849	0.8869	0.8888	0.8907	0.8925	0.8944	0.8962	0.8980	0.8997	0.9015
1.3	0.9032	0.9049	0.9066	0.9082	0.9099	0.9115	0.9131	0.9147	0.9162	0.9177
1.4	0.9192	0.9207	0.9222	0.9236	0.9251	0.9265	0.9279	0.9292	0.9306	0.9319

- **Definition:** If lnX is normal, then X is lognormal, which is used to describe the price of asset
- **Right skewed**
- **Bounded from below by zero,** so it is useful for modeling asset Prices



Sampling and Estimation

Sampling and Estimation

- Sampling and estimation
  - Simple random sampling
  - Stratified random sampling: to separate the population into smaller groups based on one or more distinguishing characteristics. Stratum and cells=M*N
- Sampling error: sampling error of the mean= sample mean- population mean

Sampling and Estimation

- Time-series data
  - consist of observations taken over a period of time at specific and equally spaced time intervals.
- Cross-sectional data
  - a sample of observations taken at a single point in time.

Time-series data	Cross-sectional data
a collection of data recorded over a period of time	a collection of data taken at a single point of time.

Sampling and Estimation

- **Point estimate:** the statistic, computed from sample information, which is used to estimate the population parameter
- **Interval estimate:** a range of values constructed from sample data so the parameter occurs within that range at a specified probability.

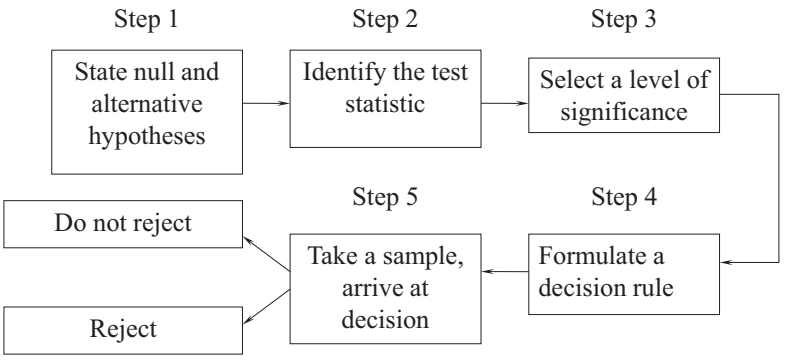
Sampling and Estimation

- **Data-mining bias**
  - Refers to results where the statistical significance of the pattern is overestimated because the results were found through data mining.
- **Sample selection bias**
  - Some data is systematically excluded from the analysis, usually because of the lack of availability.
- **Survivorship bias**
  - Usually derives from sample selection for only the existing portfolio are included
- **Look-ahead bias**
  - Occurs when a study tests a relationship using sample data that was not a available on the test date.
- **Time-period bias**
  - Time period over which the data is gathered is either too short or too long. If the time period is too short, research results may reflect phenomena specific to that time period, or perhaps even data mining.

Hypothesis Testing

Hypothesis Testing

- Hypothesis testing
  - The steps of hypothesis testing



Technical Analysis

Technical Analysis

- **Principles:**
  - Prices are determined by the interaction of supply and demand.
  - Only participants who actually trade affect prices, and better-informed participants tend to trade in greater volume.
  - Price and volume reflect the collective behavior of buyers and sellers.
- **Assumptions:**
  - Market prices reflect both rational and irrational investor behavior.
    - ✓ Investor behavior is reflected in trends and patterns that trend to repeat and can be identified and used for forecasting prices.
    - ✓ Efficient markets hypothesis dose not hold.

Technical Analysis

- The differences among technicians, fundamentalists and Efficient market followers.
  - Fundamental analysis of a firm attempts to determine the intrinsic value of an asset by using the financial statements and other information.
  - Technical analysis uses only the firm's share price and trading volume data, and it is not concerned with identifying buyers' and sellers' reasons for trading, but only with the trades that have occurred.
  - Fundamentalists believe that prices react quickly to changing stock values, while technicians believe that the reaction is slow. Technicians look for changes in supply and demand, while fundamentalists look for changes in value.

Technical Analysis

- **Advantages of technical analysis:**
  - Actual price and volume data are observable.
  - Technical analysis itself is objective (although require subjective judgment), while much of the data used in fundamental analysis is subject to assumptions or restatements.
  - It can be applied to the prices of assets that do not produce future cash flows, such as commodities.
  - It can also be useful when financial statement fraud occurs.
- **Disadvantage:**
  - The usefulness is limited in markets where price and volume data might not truly reflect supply and demand, such as in illiquid markets and in markets that are subject to outside manipulation.

It's not the end but just the beginning.

Life is short. If there was ever a moment to follow your passion and do something that matters to you, that moment is now.

生命苦短，如果你有一个机会跟随自己的激情去做你认为重要的事，那么这个机会就是现在。