Problem 1: A thin-walled cylindrical pressure vessel of the diameter $d = 250$ mm and the wall thickness $t = 5$ mm is rigidly attached to a wall, forming a cantilever as shown in Figure 1. The following loads are applied: internal pressure $p = 1.2$ MPa, torque $T = 3$ kN·m, and the direct force $P = 20$ kN. Determine the maximum shear stresses and the associated normal stresses at point A of the cylindrical wall. Show the results on a properly oriented element.

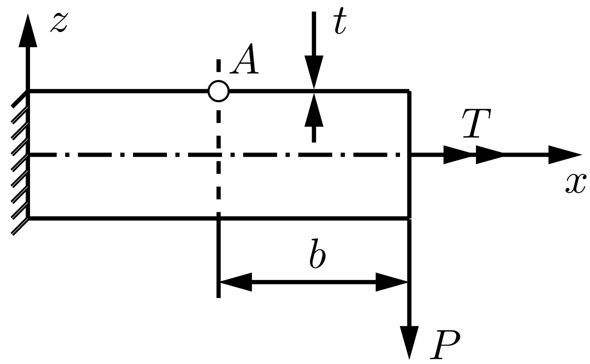

Figure 1: Thin-walled vessel.

Solution: The internal force resultants on a transverse section through point A are found from the equilibrium conditions of the free-body diagram shown in Figure 2. They are $V = 20$ kN, $M = P \cdot b = 20 \cdot 0.4 = 8$ kN·m, and $T = 3$ kN·m.

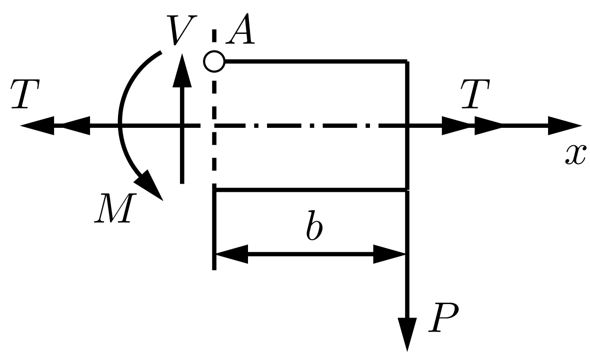

Figure 2: Free-body diagram of a segment.

In Figure 3, the combined axial, tangential, and shear stresses are shown on small element at point A. The stresses are

$$\sigma_b = \frac{Mr}{I_x} = \frac{Mr}{\pi r^3 t} = \frac{M}{\pi r^2 t} = \frac{8 \cdot 10^3}{\pi (0.25/2)^2 (0.005)} = 32.6\,\text{MPa} \tag{1}$$

$$\tau_t = -\frac{Tr}{J_c} = -\frac{Tr}{2\pi r^3 t} = -\frac{T}{2\pi r^2 t} = \frac{3 \cdot 10^3}{2\pi (0.25/2)^2 (0.005)} = -6.112\,\text{MPa} \tag{2}$$

$$\sigma_a = \frac{pr}{2t} = \frac{1.2 \cdot 10^6 (0.25/2)}{2\pi (0.005)} = 15\,\text{MPa} \tag{3}$$

$$\sigma_\theta = 2\sigma_a = 2 \cdot 15 = 30\,\text{MPa} \tag{4}$$

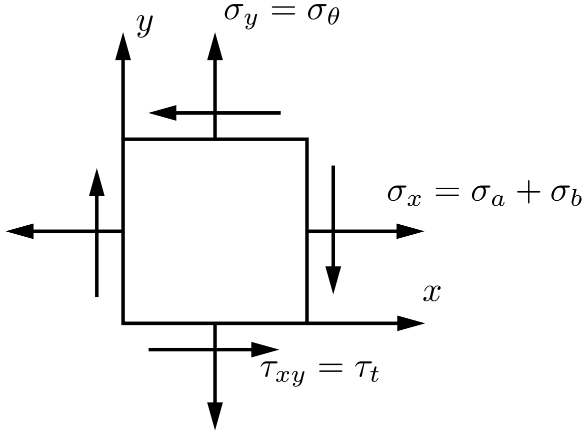

Figure 3: Stresses on a the element at point A (view from top).

Thus, we have $\sigma_x = \sigma_a + \sigma_b = 15 + 32.6 = 47.6$ MPa, $\sigma_y = \sigma_\theta = 30$ MPa, $\tau_{xy} = \tau_t = -6.112$ MPa. Note that for element A, we have $Q = 0$. Hence, the direct shear stress $\tau_d = \tau_x z = VQ/(Ib) = 0$. The maximum shear stress are given by

$$\tau_{max} = \pm\sqrt{\left(\frac{\sigma_x - \sigma_y}{2}\right)^2 + \tau_{xy}^2} = \pm\sqrt{\left(\frac{47.6 - 30}{2}\right)^2 + (-6.112)^2} = \pm 10.71\,\text{MPa} \tag{5}$$

Normal stress acting on the planes of maximum shear stress is given by

$$\sigma^{'} = \sigma_{ave} = \frac{1}{2}(\sigma_x + \sigma_y) = \frac{1}{2}(47.6 + 30) = 38.8\,\text{MPa} \tag{6}$$

To locate the maximum shear planes, we use

$$\tan 2\theta_s = -\frac{\sigma_x - \sigma_y}{2\tau_{xy}}, \tag{7}$$

from which we can derive

$$\theta_s = \frac{1}{2}\tan^{-1}\left[-\frac{\sigma_x - \sigma_y}{2\tau_{xy}}\right] = \frac{1}{2}\tan^{-1}\left[-\frac{47.6 - 30}{2(-6.112)}\right] = 27.6^\circ \text{ and } 117.6^\circ \tag{8}$$

Now we obtain $\tau_{x'y'}$ using the transformation equation

$$\begin{aligned} \tau_{x'y'} &= -\frac{1}{2}(\sigma_x - \sigma_y)\sin 2\theta + \tau_{xy}\cos 2\theta = \\ &= -\frac{1}{2}(47.6 - 30)\sin(55.2^\circ) + (-6.112)\cos(55.2^\circ) = -10.71\,\text{MPa} \end{aligned} \tag{9}$$

The stresses are shown in their proper directions in Figure 4.

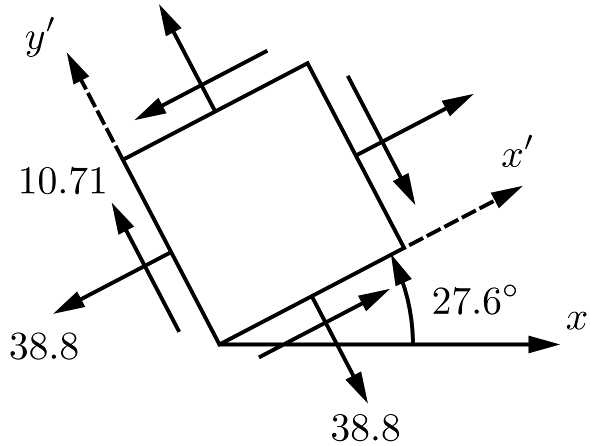

Figure 4: Stresses on a the properly oriented element at point A (view from top).

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