

Boolean Sign    1 = True   0 = False	Logical Sign    T = True    F = False
Multiplication *	AND ( ^ )
Addition +	OR ( V )
Complement (bar) $\bar{0} = 1$	NOT ( ~ )
Circuit <b>Closed</b> = 1                      => Electric <b>can</b> flow through Circuit <b>Open</b> = 0                        => Electric <b>can't</b> flow through	

Idempotent laws:	$x + x = x$	$x \cdot x = x$
Associative laws:	$(x + y) + z = x + (y + z)$	$(xy)z = x(yz)$
Commutative laws:	$x + y = y + x$	$xy = yx$
Distributive laws:	$x + yz = (x + y)(x + z)$	$x(y + z) = xy + xz$
Identity laws:	$x + 0 = x$	$x \cdot 1 = x$
Domination laws:	$x + 1 = 1$	$x \cdot 0 = 0$
Double complement law:	$\overline{\overline{x}} = x$	
Complement laws:	$x + \overline{x} = 1$ $\overline{0} = 1$	$x\overline{x} = 0$ $\overline{1} = 0$
De Morgan's laws:	$\overline{x + y} = \overline{x}\overline{y}$	$\overline{xy} = \overline{x} + \overline{y}$
Absorption laws:	$x + (xy) = x$	$x(x + y) = x$

**Minterms** are included for the rows in which the function evaluates to 1. New row is (+) - Or  
**Find expressions of Minterm** , if  $x = 1$  then  $x \text{ Bar} = 0$ .

**Maxterm** is similar to Minterms but the negation ( bar ) of the variable will be flipped.  
**Find expressions of Minterm** , if  $x = 0$  then  $x \text{ Bar} = 1$ .

Table representation of a boolean function with input variables x, y, z

Find an equivalent boolean expression for f

Find each row where  $f(x, y, z) = 1$

x	y	z	f(x, y, z)
0	0	0	0
0	0	1	0
0	1	0	0
0	1	1	1
1	0	0	1
1	0	1	1
1	1	0	0
1	1	1	1

$\bar{x}yz = 1$  if and only if  $x = 0, y = 1, z = 1$

Now add them together

$f(x, y, z) = \bar{x}yz + x\bar{y}\bar{z} + x\bar{y}z + xyz$

$$f(x, y, z) = \bar{x}yz + x\bar{y}\bar{z} + x\bar{y}z + xyz.$$

Captions ^

1. To find an equivalent Boolean expression for function f expressed by a table, first find the rows in which the value of f is 1.
2. f is 1 when  $x = 0, y = 1, z = 1$ .  $\bar{x}yz = 1$  if and only if  $x = 0, y = 1, z = 1$ .
3. The row 100 (representing  $x = 1, y = 0, z = 0$ ), corresponds to  $x\bar{y}\bar{z}$ .
4. 101 corresponds to  $x\bar{y}z$  and 111 corresponds to  $xyz$ . Now add all the terms together.
5.  $f(x, y, z) = \bar{x}yz + x\bar{y}\bar{z} + x\bar{y}z + xyz$ .

Fi

**Conj = (A\*B) Disj = (A+B)**

**DNF Form: Conj-Disj-Conj-...**  
 $xyz + xy + w$

**CJF Form: Dish-Conj-Disj-...**  
 $(x+y+z)*xy*w*(x+w)$

**Example:** How many different Boolean functions of degree  $n$  are there?

**Solution:** By the product rule for counting, there are  $2^n$  different  $n$ -tuples of 0s and 1s. Because a Boolean function is an assignment of 0 or 1 to each of these different  $n$ -tuples, by the product rule there are  $2^{2^n}$  different Boolean functions of degree  $n$ . The example tells us that there are 16 different Boolean functions of degree two. We display these in Table 3.

**TABLE 4** The Number of Boolean Functions of Degree  $n$ .

Degree	Number
1	4
2	16
3	256
4	65,536
5	4,294,967,296
6	18,446,744,073,709,551,616

**TABLE 3** The 16 Boolean Functions of Degree Two.

$x$	$y$	$F_1$	$F_2$	$F_3$	$F_4$	$F_5$	$F_6$	$F_7$	$F_8$	$F_9$	$F_{10}$	$F_{11}$	$F_{12}$	$F_{13}$	$F_{14}$	$F_{15}$	$F_{16}$
1	1	1	1	1	1	1	1	1	1	0	0	0	0	0	0	0	0
1	0	1	1	1	1	0	0	0	0	1	1	1	1	0	0	0	0
0	1	1	1	0	0	1	1	0	0	1	1	0	0	1	1	0	0
0	0	1	0	1	0	1	0	1	0	1	0	1	0	1	0	1	0

- Degree is how many variables we have. For example  $x, y$  are degrees of 2.

$2^{2^2} = 2^{2^2}$  equals 16. That mean we're having 16 combination of Zeros (0) and Ones (1) from  $x, y$

The NAND Operator mean Not And ( Opposite with And )

Symbol: Arrow Up

The NOR Operator mean Not OR ( Opposite with Or )

Symbol: Arrow Down

Elimination of addition (+) in DNF

De Morgan's law with three terms  $a + b + c = \overline{\overline{a} \cdot \overline{b} \cdot \overline{c}}$

is applied with  $a = \overline{x} \overline{y}$ ,  $b = \overline{xy}$ , and  $c = xy$ .

Another Morgan Law

$$x + y = \overline{\overline{x} \cdot \overline{y}}$$

The NAND operation (which stands for "not and") is denoted by the symbol  $\uparrow$ . The expression  $x \uparrow y$  is equivalent to  $xy$ . The NOR operation (which stands for "not or") is denoted by the symbol  $\downarrow$ . The expression  $x \downarrow y$  is equivalent to  $x + y$ .

Some others:

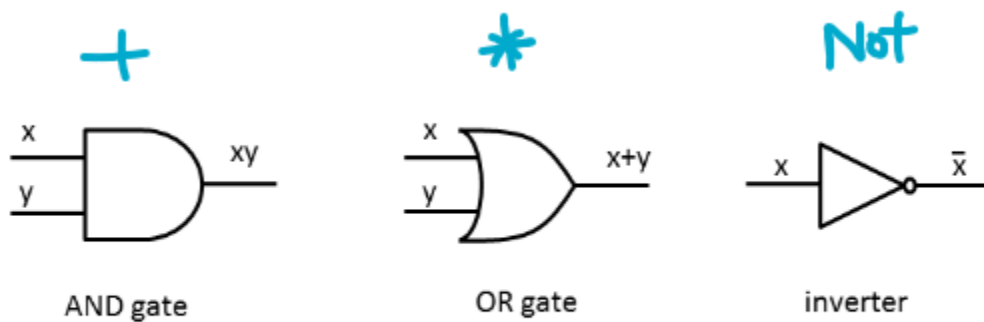
$$\overline{a} = a \uparrow a$$

$$xy = \overline{x \uparrow y} = (x \uparrow y) \uparrow (x \uparrow y)$$

$$\odot \quad x + y = \overline{x \downarrow y}$$

Expression satisfied is when its final produce equal to 1

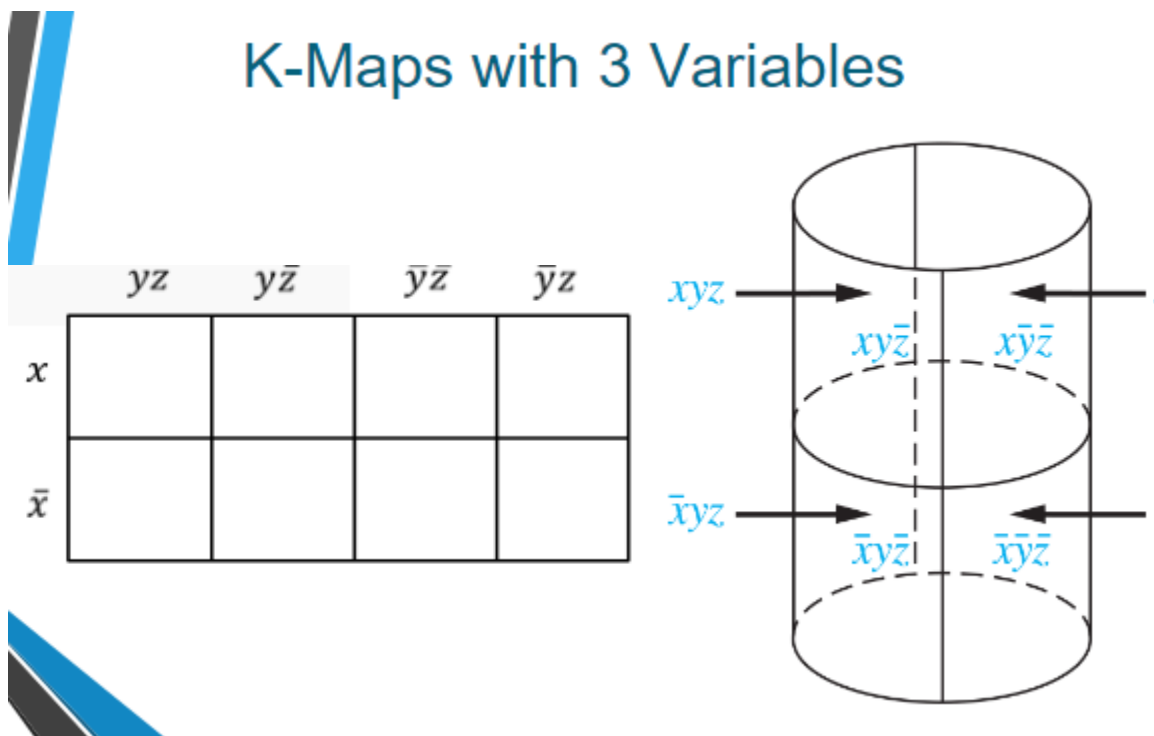
## Digital Circuit



## K MAP

The position is matter so follow template below:

To simplify expression, we put 1 into squares that indicate each element in our expression.



## For example:

Step 1:  $xy\sim z$ , then we put 1 on the row of  $x$  and column that display  $y\sim z$ .

Step 2: circle them but based on an even number, can't be 3, 5 or diagonal line.

Step 3: Because this K map is a cylinder so check the edges.

Step 4: To simplify check if that variable is staying the same or different. if  $x$  +  $x$  then it's  $x$  but if it's  $y + \sim y$ , then it's 0 ( cancelled )

	$yz$	$y\bar{z}$	$\bar{y}\bar{z}$	$\bar{y}z$		$yz$	$y\bar{z}$	$\bar{y}\bar{z}$	$\bar{y}z$
$x$	0	1	1	0	$x$	0	0	0	0
$\bar{x}$	0	1	1	0	$\bar{x}$	1	1	1	1

$xy\bar{z} + x\bar{y}\bar{z} + \bar{x}y\bar{z} + \bar{x}\bar{y}\bar{z} = \bar{z}$ 
 $\bar{x}yz + \bar{x}y\bar{z} + \bar{x}\bar{y}\bar{z} = \bar{x}$

Blue shapes indicate cylinder shape. Simplify them.

$\sim xyz$  and  $\sim x\sim yz \Rightarrow \sim xz$  because  $\sim x$  and  $z$  are not changed.

	$yz$	$y\bar{z}$	$\bar{y}\bar{z}$	$\bar{y}z$
$x$			1	1
$\bar{x}$	1		1	1