## A Composable View of Verifiable Homomorphic Encryption in Multi-Party Settings

Ganyuan Cao



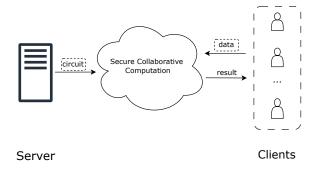
March 31, 2025



- On-the-Fly MPC [LTV12]

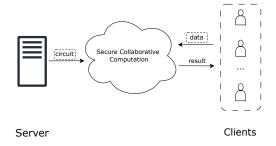


- On-the-Fly MPC [LTV12]





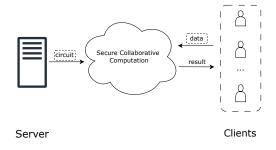
On-the-Fly MPC [LTV12]



- Dynamically joining parties.



- On-the-Fly MPC [LTV12]



- Dynamically joining parties.
- Computation outsourced to untrusted but powerful server.



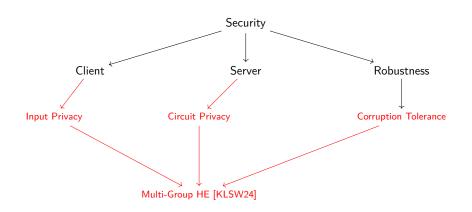
- Homomorphic Encryption (HE) is a good candidate...



Figure: Use cases of HE [Int].

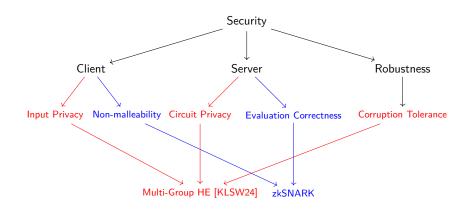
### Goal





#### Goal







- Formalism



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  - New game-based notions for (Multi-Key, Threshold, Multi-Group) HE in multi-party setting.



#### Formalism

- New game-based notions for (Multi-Key, Threshold, Multi-Group) HE in multi-party setting.
- UC functionality for HE in multi-party setting.



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- Construction



#### Formalism

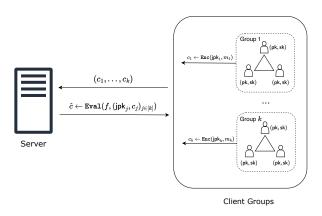
- New game-based notions for (Multi-Key, Threshold, Multi-Group) HE in multi-party setting.
- UC functionality for HE in multi-party setting.

#### Construction

- UC-secure MPC via verifiable multi-group HE.

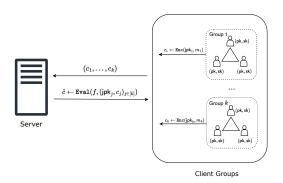
## Multi-Group HE (MGHE) [KLSW24]





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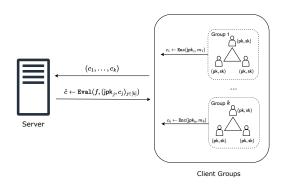




- Hybrid approach between Threshold HE and Multi-Key HE

## Multi-Group HE (MGHE)

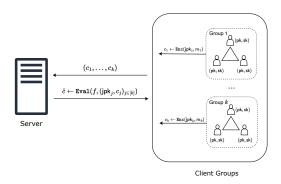




- Hybrid approach between Threshold HE and Multi-Key HE
  - Fewer public keys ⇒ better scalability

## Multi-Group HE (MGHE) [KLSW24]

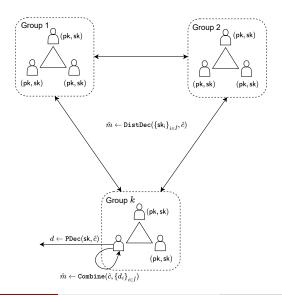




- Hybrid approach between Threshold HE and Multi-Key HE
  - Fewer public keys ⇒ better scalability
  - Allow for dynamically joining parties ⇒ better flexibility

## Multi-Group HE





## Confidentiality with Multi-Group HE



$$\mathcal{G}_{\mathsf{KRK}} \xrightarrow{\Pi_{\mathsf{MGHE}}} \mathcal{F}_{\mathsf{MGHE}}$$

$$if \, \mathsf{MGHE} \, \mathsf{satisfies}$$

$$\mathsf{IND}\text{-}\mathsf{CPAP^D}$$

$$\land \, \{\mathsf{IND},\mathsf{SIM}\}\text{-}\mathsf{CIRC}$$

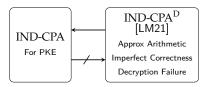
$$\land \, \mathsf{SIM}\text{-}\mathsf{PDEC}$$

$$\land \, \mathsf{Decryption} \, \mathsf{Consistency} \, (\mathsf{DC})$$

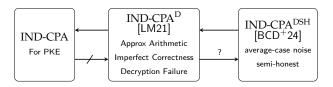


IND-CPA For PKE

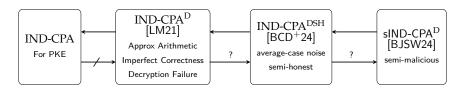




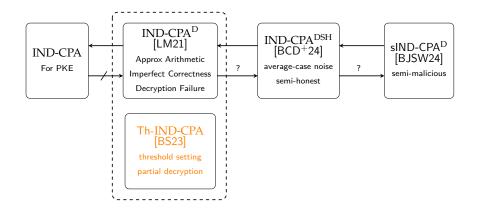




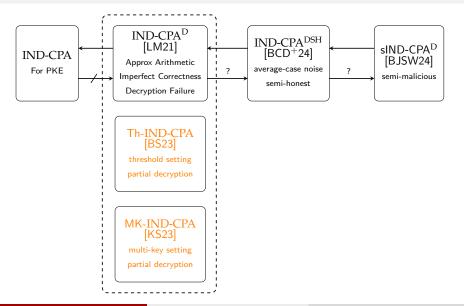




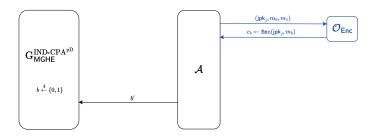




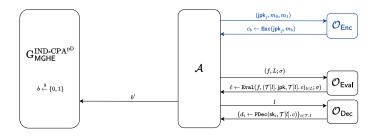




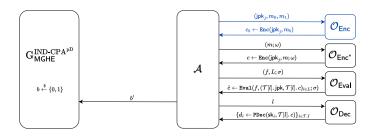




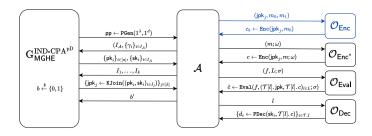




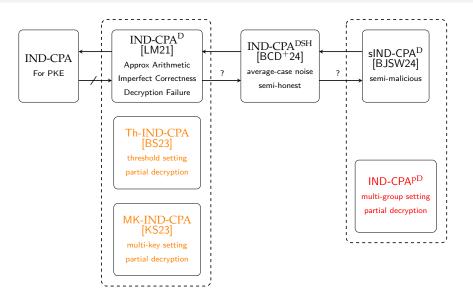














- Usually formalized using Simulation [IP07, Gen09, BdPMW16].



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$$\mathtt{Sim}_{circ}((\mathtt{jpk}_1,\ldots,\mathtt{jpk}_\ell),f(m_1,\ldots,m_\ell))$$
  $\stackrel{s}{pprox}$   $\hat{c} \leftarrow \mathsf{MGHE.Eval}(f,(\mathtt{jpk}_j,c_j)_{j\in[\ell]})$ 



Usually formalized using Simulation [IP07, Gen09, BdPMW16].

$$\begin{split} \mathtt{Sim}_{circ}((\mathtt{jpk}_1,\ldots,\mathtt{jpk}_\ell),f(m_1,\ldots,m_\ell)) \\ &\overset{s}{\approx} \\ \hat{c} \leftarrow \mathsf{MGHE}.\mathtt{Eval}(f,(\mathtt{jpk}_j,c_j)_{j\in[\ell]}) \end{split}$$

- Stronger security with statistical indistinguishability.



Not suitable for schemes with approximate evaluation like [CKKS17].



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$$\hat{m}=f(m_1,\ldots,m_\ell)$$

$$\hat{m} + \varepsilon \leftarrow \mathsf{MGHE.Dec}(\hat{c})$$

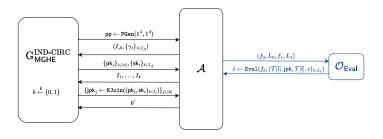


Variant of IND-CIRC security [KS23] in multi-group setting.

### Server Side: Circuit Privacy



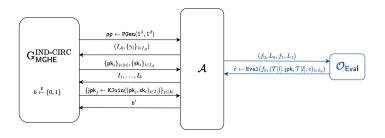
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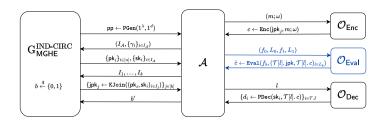
- Challenge with  $(f_0, L_0, f_1, L_1)$  instead s.t.

$$f_0(\{m_j\}_{j\in L_0}) = f_1(\{m_j\}_{j\in L_1})$$

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### Threshold Security: SIM-PDEC Security



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Simulatability of partial decryption

$$\begin{split} \mathtt{Sim}_{th}(c, m, \{\mathsf{sk}_i\}_{i \in I_{\mathcal{A}}}) \\ & \stackrel{s}{\approx} \\ d \leftarrow \mathsf{MGHE}.\mathtt{PDec}(\mathsf{sk}_j, c), j \not \in I_{\mathcal{A}} \end{split}$$

### Threshold Security: SIM-PDEC Security



- Simulatability of partial decryption

$$\mathtt{Sim}_{th}(c, m, \{\mathsf{sk}_i\}_{i \in I_\mathcal{A}})$$
  $\stackrel{s}{pprox}$   $d \leftarrow \mathsf{MGHE.PDec}(\mathsf{sk}_j, c), j \notin I_\mathcal{A}$ 

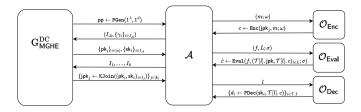
– Security of secret key  $\mathsf{sk}_j$  of honest client i.e.,  $j \not \in I_{\mathcal{A}}$ 



- In a (t,n)-threshold structure, message is reconstructed correctly as long as sufficient partial decryptions have been obtained.

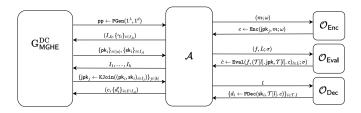


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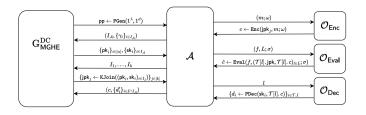


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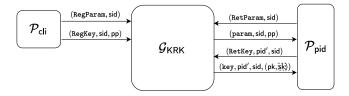
-A wins if

$$m \neq \mathtt{Combine}(c, \{d_i\}_{i \in I \setminus I_{\mathcal{A}}} \cup \{d_i'\}_{i \in I \cap I_{\mathcal{A}}})$$

## UC: Global Key Registry $\mathcal{G}_{KRK}$



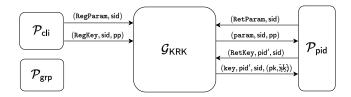
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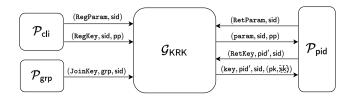
- Global subroutine for key management taken from [BCNP04].
- "Virtual entity"  $\mathcal{P}_{grp}$  for a group  $grp = \{cli_1, cli_2, \dots, cli_n\}$ .



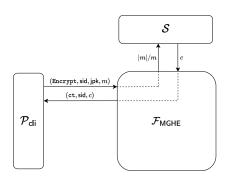
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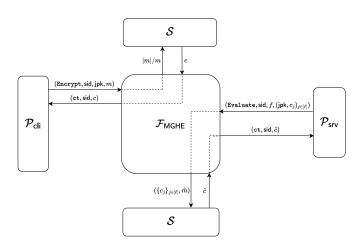
- Global subroutine for key management taken from [BCNP04].
- "Virtual entity"  $\mathcal{P}_{grp}$  for a group  $grp = \{cli_1, cli_2, \ldots, cli_n\}$ .
- Key aggregation for groups (equivalent to  $\mathcal{F}_{\mathsf{MPC}}$ ).



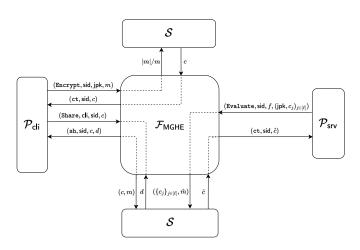




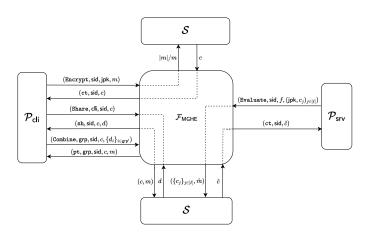












### Realization of $\mathcal{F}_{\mathsf{MGHF}}$



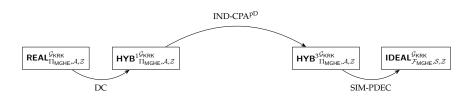
#### Theorem 1

 $\Pi_{\text{MGHE}}$  UC-realizes  $\mathcal{F}_{\text{MGHE}}$  against a semi-malicious adversary in presence of  $\mathcal{G}_{\text{KRK}}$  if MGHE satisfies decryption consistency (DC), is IND-CPA<sup>PD</sup>, IND-CIRC (SIM-CIRC), and SIM-PDEC secure under the static corruption of clients in a group up to the threshold and possibly the server.

### Realization of $\mathcal{F}_{MGHF}$



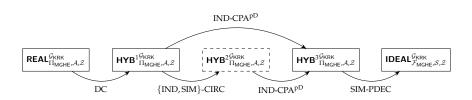
Both server and clients are corrupted.



### Realization of $\mathcal{F}_{\mathsf{MGHF}}$



Only clients are corrupted.





Three-phase protocol



- Three-phase protocol
  - Data Uploading  $\Rightarrow$  MGHE.Enc( $\cdot$ )



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- Three-phase protocol
  - Data Uploading  $\Rightarrow$  MGHE.Enc $(\cdot)$
  - Circuit Evaluation ⇒ MGHE.Eval(·)
  - Result Retrieval  $\Rightarrow$  MGHE.PDec $(\cdot)$  then MGHE.Combine $(\cdot)$

### Integrity via Verifiability



MGHE ⇒ MPC against *semi-malicious* adversary

## Integrity via Verifiability



MGHE ⇒ MPC against *semi-malicious* adversary

MGHE + zkSNARK ⇒ MPC against (full) malicious adversary



UC-secure zkSNARK

### zkSNARK in ROM



UC-secure zkSNARK

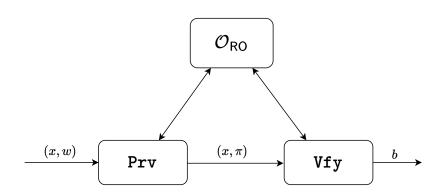
zkSNARK		[CF24]	[BFKT24]	[GKO <sup>+</sup> 23]
NIZK	[Gro06]	[LR22]		[LR22]
	CRS	ROM	ROM-AGM	CRS-ROM



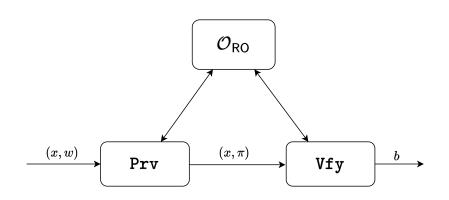
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*Succinctness*:  $|\pi| \ll |w|$ 

### Properties of zkSNARK



Completeness: Valid arguments must be accepted.

$$\forall (x,w) \in R, \; \Pr \left[ \mathtt{Vfy}^{\mathcal{O}_{\mathsf{RO}}}(x,\pi) = 1 \; \middle| \; \begin{matrix} \mathcal{O}_{\mathsf{RO}} \leftarrow \mathcal{U}(\lambda) \\ \pi \leftarrow \mathtt{Prv}^{\mathcal{O}_{\mathsf{RO}}}(x,w) \end{matrix} \right] = 1.$$

### Properties of zkSNARK



Zero-Knowledge: Arguments do not disclose information about witness.

$$\left\{ \mathsf{out} \left| \begin{array}{l} \mathcal{O}_{\mathsf{RO}} \leftarrow \mathcal{U}(\lambda) \\ (x, w, \mathsf{aux}) \leftarrow \mathcal{A}^{\mathcal{O}_{\mathsf{RO}}} \\ \pi \leftarrow \mathsf{Prv}^{\mathcal{O}_{\mathsf{RO}}}(x, w) \\ \mathsf{out} \leftarrow \mathcal{A}^{\mathcal{O}_{\mathsf{RO}}}(\mathsf{aux}, \pi) \end{array} \right\} \approx \left\{ \mathsf{out} \left| \begin{array}{l} \mathcal{O}_{\mathsf{RO}} \leftarrow \mathcal{U}(\lambda) \\ (x, w, \mathsf{aux}) \leftarrow \mathcal{A}^{\mathcal{O}_{\mathsf{RO}}} \\ (\pi, \mathsf{pg}) \leftarrow \mathsf{Sim}^{\mathcal{O}_{\mathsf{RO}}}(x) \\ \mathsf{out} \leftarrow \mathcal{A}^{\mathcal{O}_{\mathsf{RO}}[\mathsf{pg}]}(\mathsf{aux}, \pi) \end{array} \right\}$$

### Properties of zkSNARK



Simulation Soundness: Non-malleability of arguments.

$$\Pr\begin{bmatrix} |x| \leq n \\ \land x \not\in \mathcal{L}(R) \\ \land \texttt{Vfy}^{\mathcal{O}_{\mathsf{RO}}}(x,\pi) = 1 \end{bmatrix} \begin{vmatrix} \mathcal{O}_{\mathsf{RO}} \leftarrow \mathcal{U}(\lambda) \\ (x,\pi) \leftarrow \mathcal{A}^{\mathcal{O}_{\mathsf{RO}}}(\mathtt{Sim}) \end{bmatrix} \leq \mathsf{negl}.$$

### MPC with MGHE + zkSNARK



MGHE + zkSNARK ⇒ MPC against (full) malicious adversary

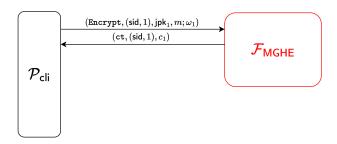
## Phase 1: Data Uploading



Naor-Yung Double Encryption Paradigm.

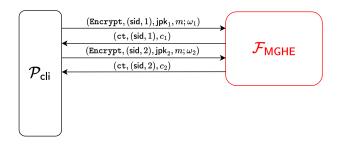


Naor-Yung Double Encryption Paradigm.



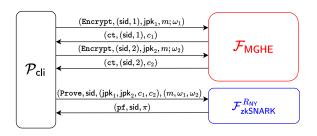


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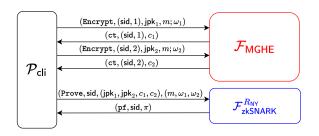




- Naor-Yung Double Encryption Paradigm.



#### Naor-Yung Double Encryption Paradigm.



$$R_{\mathsf{NY}} = \left\{ \begin{pmatrix} \left( \mathsf{jpk}_1, c_1, \\ \mathsf{jpk}_2, c_2 \right), (m, \omega_1, \omega_2) \right) \middle| \begin{array}{l} c_1 = \mathsf{MGHE}.\mathsf{Enc}(\mathsf{jpk}_1, m; \omega_1) \\ \wedge \\ c_2 = \mathsf{MGHE}.\mathsf{Enc}(\mathsf{jpk}_2, m; \omega_2) \end{array} \right\}$$



CCA1-secure HE as in [LMSV12, BSW12, CRRV17].



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  - Tampering with  $c_1$  or  $c_2 \Rightarrow$  Verification fails.



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  - Tampering  $c_1$  or  $c_2 \Rightarrow$  Verification fails.
  - Must know m to generate valid ciphertext tuple.



Naor-Yung + Simulation Soundness



 ${\sf Naor-Yung} + {\sf Simulation} \ {\sf Soundness}$ 

or

 ${\sf One\text{-}Pass} + {\sf Simulation} \ {\sf Extractability}$ 



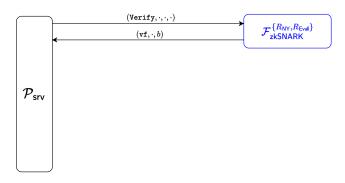
 ${\sf Naor-Yung} + {\sf Simulation} \; {\sf Soundness}$ 

or

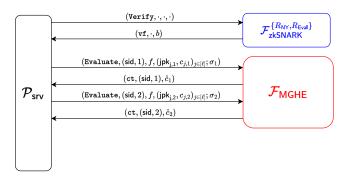
One-Pass + Simulation Extractability

$$R_{\mathsf{NY}} = \{(\mathsf{jpk}, c), (m, \omega)) \ c = \mathsf{MGHE}.\mathsf{Enc}(\mathsf{jpk}, m; \omega)\}$$

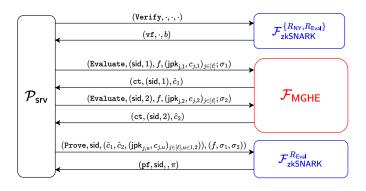










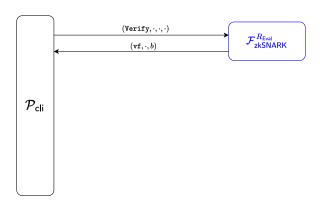




$$R_{\mathsf{Eval}} = \left\{ \begin{pmatrix} \left( \hat{c}_1, (\mathsf{jpk}_{j,1}, c_{j,1})_{j \in [\ell]}, \\ \hat{c}_2, (\mathsf{jpk}_{j,2}, c_{j,2})_{j \in [\ell]} \right), (f, \sigma_1, \sigma_2) \end{pmatrix} \middle| \\ \\ \hat{c}_1 = \mathsf{MGHE}.\mathsf{Eval}(f, (\mathsf{jpk}_{j,1}, c_{j,1})_{j \in [\ell]}; \sigma_1) \\ \\ \\ \hat{c}_2 = \mathsf{MGHE}.\mathsf{Eval}(f, (\mathsf{jpk}_{j,2}, c_{j,2})_{j \in [\ell]}; \sigma_2) \end{pmatrix}. \right.$$

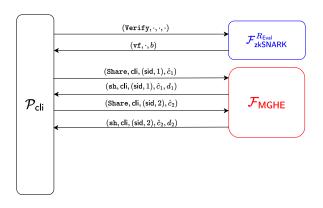
# Phase 3: Result Retrieval - Partial Decryption **EPFL**





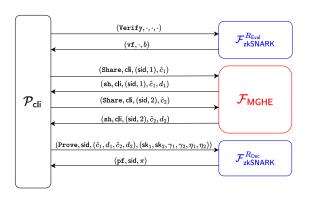
# Phase 3: Result Retrieval - Partial Decryption **EPFL**





### Phase 3: Result Retrieval - Partial Decryption

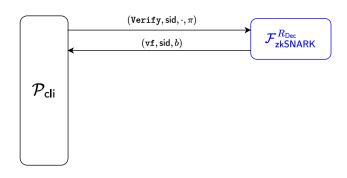




$$R_{\mathsf{Dec}} = \left\{ \begin{pmatrix} \mathsf{pp}_1, \mathsf{pk}_1, c_1, d_1, \\ \mathsf{pp}_2, \mathsf{pk}_2, c_2, d_2 \end{pmatrix}, \begin{pmatrix} \mathsf{sk}_1, \gamma_1, \eta_1 \\ \mathsf{sk}_2, \gamma_2, \eta_2 \end{pmatrix} \middle| \begin{array}{l} \forall u \in \{1, 2\} : \\ d_u = \mathsf{MGHE.PDec}(\mathsf{sk}_u, c_u; \eta_u) \\ \land \mathsf{pk}_u = \mathsf{PKGen}(\mathsf{pp}_u, \mathsf{sk}_u; \gamma_u) \end{array} \right\}.$$

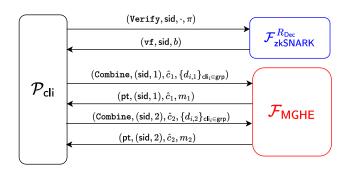
#### Phase 3: Result Retrieval - Reconstruction





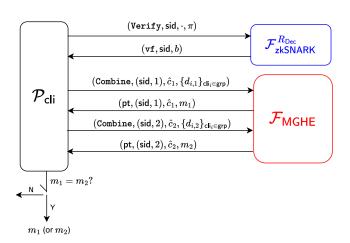
### Phase 3: Result Retrieval - Reconstruction





### Phase 3: Result Retrieval - Reconstruction





### Realization of On-the-Fly MPC



#### Theorem 2

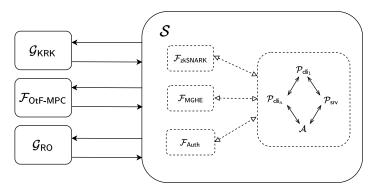
 $\Pi_{\text{OtF-MPC}}$  UC-realizes  $\mathcal{F}_{\text{OtF-MPC}}$  in [ $\mathcal{F}_{\text{MGHE}}$ ,  $\mathcal{F}_{\text{zkSNARK}}$ ,  $\mathcal{F}_{\text{Aut}}$ ]-hybrid model in presence of  $\mathcal{G}_{\text{KRK}}$  and  $\mathcal{G}_{\text{RO}}$ .

### Realization of On-the-Fly MPC



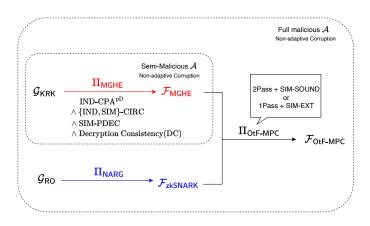
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#### Conclusion

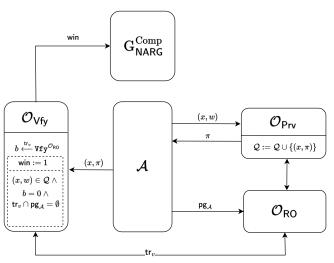




### Appendix: Completeness



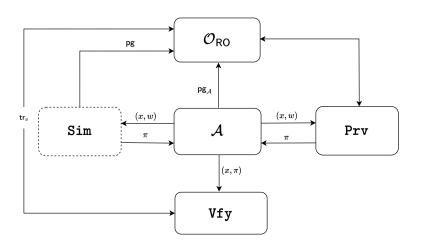
- Valid arguments must be accepted.



### Appendix: Zero-Knowledge



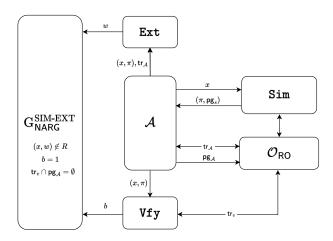
- Arguments does not disclose information about witness.



### Appendix: sSIM-EXT



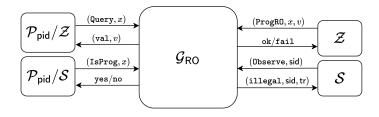
- Stronger version for *Knowledge Soundness*.
- Non-malleability for UC-security.



### Appendix: Global Random Oracle $\mathcal{G}_{RO}$

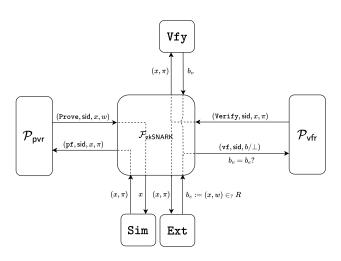


 Global random oracle with restricted programming and observability [CDG<sup>+</sup>18].



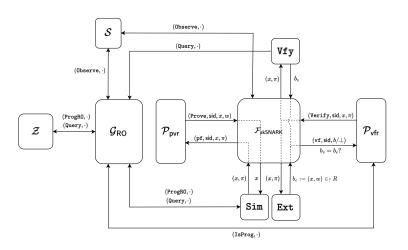
## Appendix: Functionality $\mathcal{F}_{zkSNARK}$ [CF24]





## Appendix: $\mathcal{F}_{zkSNARK} + \mathcal{G}_{RO}$



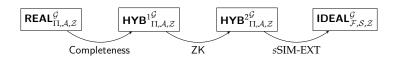


### Appendix: Realization of $\mathcal{F}_{zkSNARK}$



#### Theorem 3[CF24]

 $\Pi_{NARG}$  UC-realizes  $\mathcal{F}_{zkSNARK}$  in presence of  $\mathcal{G}_{RO}$  if and only if NARG satisfies completeness, sSIM-EXT, and zero-knowledge.





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