A Composable View of Verifiable Homomorphic Encryption in Multi-Party Settings

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March 31, 2025



- On-the-Fly MPC [LTV12]



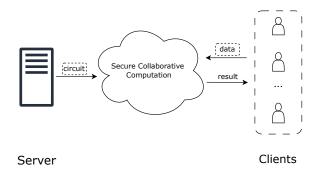
- On-the-Fly MPC [LTV12]
 - Dynamically joining parties.



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 - Computation outsourced to untrusted but powerful server.



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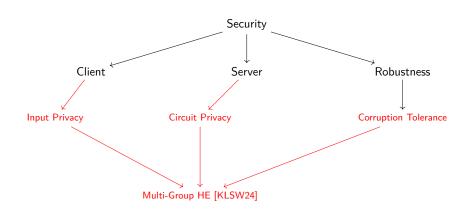
- Homomorphic Encryption (HE) is a good candidate...



Figure: Use cases of HE [Int].

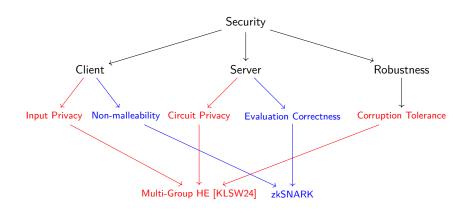
Goal





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- Formalism



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 - New game-based notions for (Multi-Key, Threshold, Multi-Group) HE in multi-party setting.



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- UC functionality for HE in multi-party setting.



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Construction

- UC-secure MPC via verifiable multi-group HE.

Multi-Group HE (MGHE) [KLSW24]



Hybrid approach between Threshold HE and Multi-Key HE

Multi-Group HE (MGHE)



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 - Fewer public keys \Rightarrow better scalability

Multi-Group HE (MGHE) [KLSW24]

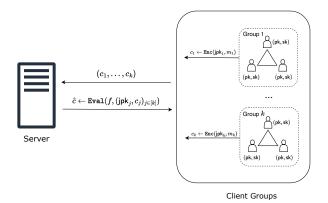


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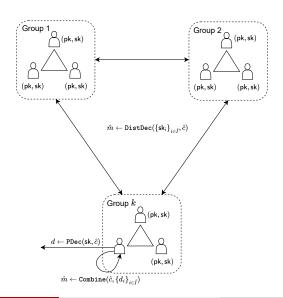


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Multi-Group HE





Confidentiality with Multi-Group HE



$$\mathcal{G}_{\mathsf{KRK}} \xrightarrow{\Pi_{\mathsf{MGHE}}} \mathcal{F}_{\mathsf{MGHE}}$$

$$if \, \mathsf{MGHE} \, \mathsf{satisfies}$$

$$\mathsf{IND}\text{-}\mathsf{CPAP^D}$$

$$\land \, \{\mathsf{IND},\mathsf{SIM}\}\text{-}\mathsf{CIRC}$$

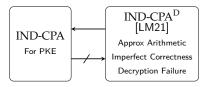
$$\land \, \mathsf{SIM}\text{-}\mathsf{PDEC}$$

$$\land \, \mathsf{Decryption} \, \mathsf{Consistency} \, (\mathsf{DC})$$

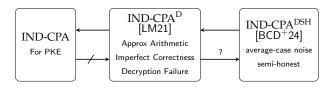


IND-CPA For PKE

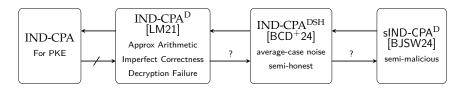




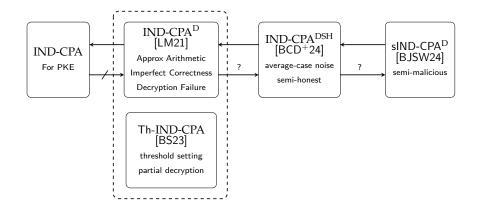




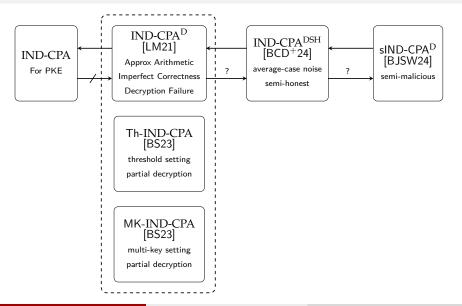




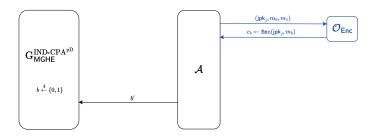




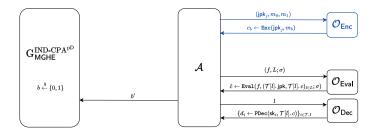




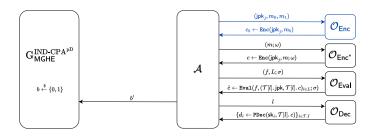




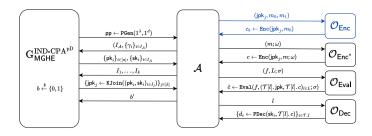




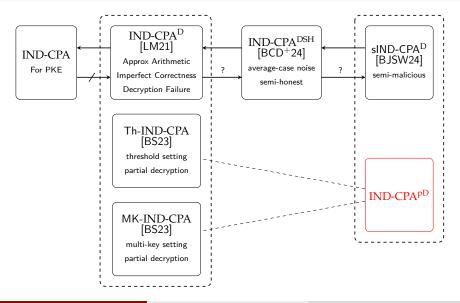














- Usually formalized using Simulation [IP07, Gen09, BdPMW16].



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$$\mathtt{Sim}_{circ}((\mathtt{jpk}_1,\ldots,\mathtt{jpk}_\ell),f(m_1,\ldots,m_\ell))$$
 $\stackrel{s}{pprox}$ $\hat{c} \leftarrow \mathsf{MGHE.Eval}(f,(\mathtt{jpk}_j,c_j)_{j\in[\ell]})$



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$$\begin{split} \mathtt{Sim}_{circ}((\mathtt{jpk}_1,\ldots,\mathtt{jpk}_\ell),f(m_1,\ldots,m_\ell)) \\ &\overset{s}{\approx} \\ \hat{c} \leftarrow \mathsf{MGHE}.\mathtt{Eval}(f,(\mathtt{jpk}_j,c_j)_{j\in[\ell]}) \end{split}$$

- Stronger security with statistical indistinguishability.



Not suitable for schemes with approximate evaluation like [CKKS17].



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$$\hat{m}=f(m_1,\ldots,m_\ell)$$

$$\hat{m} + \varepsilon \leftarrow \mathsf{MGHE.Dec}(\hat{c})$$

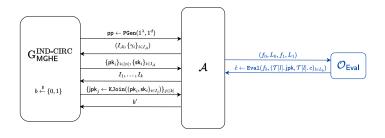


Variant of IND-CIRC security [KS23] in multi-group setting.

Server Side: Circuit Privacy



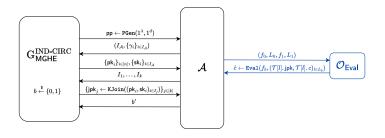
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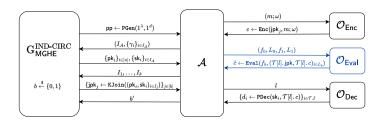
- Challenge with (f_0, L_0, f_1, L_1) instead s.t.

$$f_0(\{m_j\}_{j\in L_0}) = f_1(\{m_j\}_{j\in L_1})$$

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Threshold Security: SIM-PDEC Security



Simulatability of partial decryption

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Simulatability of partial decryption

$$\begin{split} \mathtt{Sim}_{th}(c, m, \{\mathsf{sk}_i\}_{i \in I_{\mathcal{A}}}) \\ & \stackrel{s}{\approx} \\ d \leftarrow \mathsf{MGHE}.\mathtt{PDec}(\mathsf{sk}_j, c), j \not \in I_{\mathcal{A}} \end{split}$$

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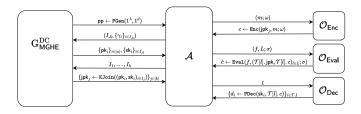
– Security of secret key sk_j of honest client i.e., $j \not \in I_{\mathcal{A}}$



- In a (t, n)-threshold structure, message is reconstructed correctly as long as sufficient partial decryptions have been obtained.

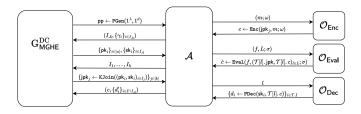


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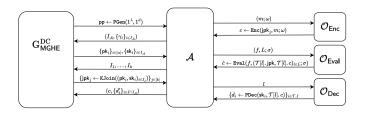


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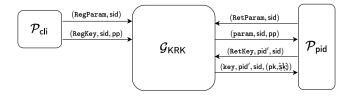
-A wins if

$$m \neq \mathtt{Combine}(c, \{d_i\}_{i \in I \setminus I_{\mathcal{A}}} \cup \{d_i'\}_{i \in I \cap I_{\mathcal{A}}})$$

UC: Global Key Registry \mathcal{G}_{KRK}



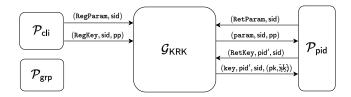
- Global subroutine for key management taken from [BCNP04].



UC: Global Key Registry \mathcal{G}_{KRK}



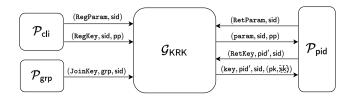
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- "Virtual entity" \mathcal{P}_{grp} for a group $grp = \{cli_1, cli_2, \dots, cli_n\}$.



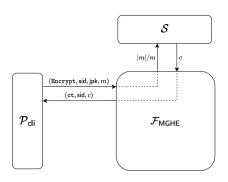
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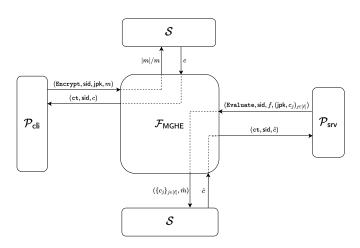
- Global subroutine for key management taken from [BCNP04].
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- Key aggregation for groups (equivalent to \mathcal{F}_{MPC}).



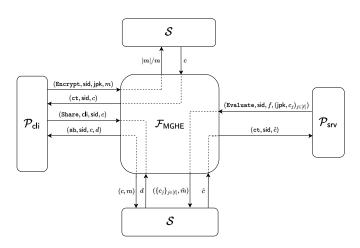




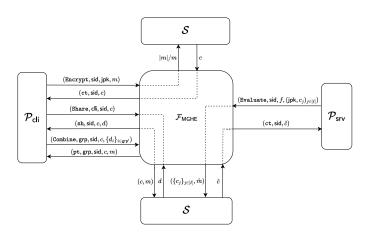












Realization of $\mathcal{F}_{\mathsf{MGHF}}$



Theorem 1

 Π_{MGHE} UC-realizes $\mathcal{F}_{\text{MGHE}}$ against a semi-malicious adversary in presence of \mathcal{G}_{KRK} if MGHE is IND-CPA^{pD}, IND-CIRC (SIM-CIRC), and SIM-PDEC secure under the static corruption of clients in a group up to the threshold and possibly the server.

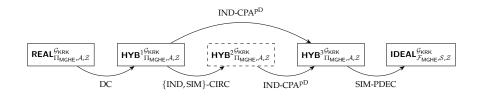
Realization of $\mathcal{F}_{\mathsf{MGHE}}$





Realization of $\mathcal{F}_{\mathsf{MGHE}}$





Integrity via Verifiability



MGHE ⇒ Security against *semi-malicious* adversary

Integrity via Verifiability



MGHE ⇒ Security against *semi-malicious* adversary

 $MGHE + zkSNARK \Rightarrow Security against (full) malicious adversary$



UC-secure zkSNARK



UC-secure zkSNARK

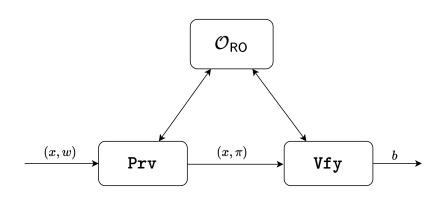
zkSNARK		[CF24]	[BFKT24]	[GKO ⁺ 23]
NIZK	[BS21]	[LR22]	 	[LR22]
	CRS	ROM	ROM-AGM	CRS-ROM



UC-secure zkSNARK

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- Completeness: Valid arguments must be accepted.

$$\forall (x,w) \in R, \; \Pr \left[\mathtt{Vfy}^{\mathcal{O}_{\mathsf{RO}}}(x,\pi) = 1 \; \middle| \; \begin{matrix} \mathcal{O}_{\mathsf{RO}} \leftarrow \mathcal{U}(\lambda) \\ \pi \leftarrow \mathtt{Prv}^{\mathcal{O}_{\mathsf{RO}}}(x,w) \end{matrix} \right] = 1.$$



Zero-Knowledge: Arguments do not disclose information about witness.

$$\left\{ \mathsf{out} \left| \begin{array}{l} \mathcal{O}_{\mathsf{RO}} \leftarrow \mathcal{U}(\lambda) \\ (x, w, \mathsf{aux}) \leftarrow \mathcal{A}^{\mathcal{O}_{\mathsf{RO}}} \\ \pi \leftarrow \mathsf{Prv}^{\mathcal{O}_{\mathsf{RO}}}(x, w) \\ \mathsf{out} \leftarrow \mathcal{A}^{\mathcal{O}_{\mathsf{RO}}}(\mathsf{aux}, \pi) \end{array} \right\} \approx \left\{ \mathsf{out} \left| \begin{array}{l} \mathcal{O}_{\mathsf{RO}} \leftarrow \mathcal{U}(\lambda) \\ (x, w, \mathsf{aux}) \leftarrow \mathcal{A}^{\mathcal{O}_{\mathsf{RO}}} \\ (\pi, \mathsf{pg}) \leftarrow \mathsf{Sim}^{\mathcal{O}_{\mathsf{RO}}}(x) \\ \mathsf{out} \leftarrow \mathcal{A}^{\mathcal{O}_{\mathsf{RO}}[\mathsf{pg}]}(\mathsf{aux}, \pi) \end{array} \right\}$$



- Simulation Soundness: Non-malleability of arguments.

$$\Pr\begin{bmatrix} |x| \leq n \\ \land x \not\in \mathcal{L}(R) \\ \land \mathsf{Vfy}^{\mathcal{O}_{\mathsf{RO}}}(x,\pi) = 1 \end{bmatrix} \begin{vmatrix} \mathcal{O}_{\mathsf{RO}} \leftarrow \mathcal{U}(\lambda) \\ (x,\pi) \leftarrow \mathcal{A}^{\mathcal{O}_{\mathsf{RO}}}(\mathtt{Sim}) \end{bmatrix} \leq \mathsf{negl}.$$



- Succinctness: Argument is efficient.

$$|\pi| \ll |w|$$



Three-phase protocol



- Three-phase protocol
 - Data Uploading



- Three-phase protocol
 - Data Uploading
 - Circuit Evaluation



- Three-phase protocol
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 - Result Retrieval



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 - against *malicious* adversary.

On-the-Fly MPC



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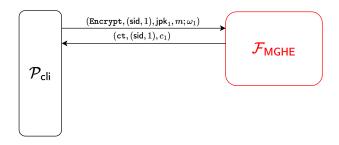
On-the-Fly MPC



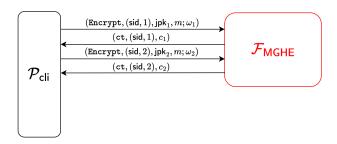
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 - * the server
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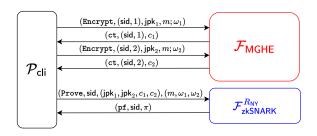


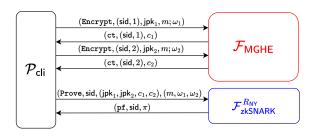












$$R_{\mathsf{NY}} = \left\{ \begin{pmatrix} \left(\mathsf{jpk}_1, c_1, \\ \mathsf{jpk}_2, c_2 \end{pmatrix}, (m, \omega_1, \omega_2) \right) \middle| \begin{array}{l} c_1 = \mathsf{MGHE}.\mathsf{Enc}(\mathsf{jpk}_1, m; \omega_1) \\ \land \\ c_2 = \mathsf{MGHE}.\mathsf{Enc}(\mathsf{jpk}_2, m; \omega_2) \end{array} \right\}$$



CCA1-secure HE as in [LMSV12, BSW12, CRRV17].



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 - Tampering with c_1 or $c_1 \Rightarrow$ Verification fails.



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 - Tampering c_1 or $c_1 \Rightarrow$ Verification fails.
 - Must know m to generate valid ciphertext tuple.



Naor-Yung + Simulation Soundness



 ${\sf Naor-Yung} + {\sf Simulation} \; {\sf Soundness}$

or

 ${\sf One\text{-}Pass} + {\sf Simulation} \ {\sf Extractability}$



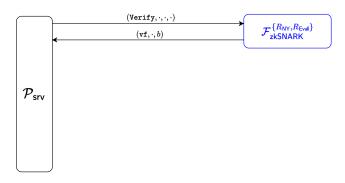
 ${\sf Naor-Yung} + {\sf Simulation} \; {\sf Soundness}$

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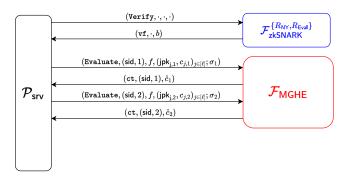
One-Pass + Simulation Extractability

$$R_{\mathsf{NY}} = \{(\mathsf{jpk}, c), (m, \omega)\} \ c = \mathsf{MGHE}.\mathsf{Enc}(\mathsf{jpk}, m; \omega)\}$$

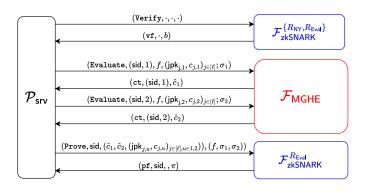










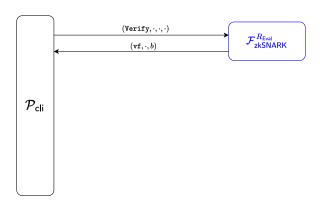




$$R_{\mathsf{Eval}} = \left\{ \begin{pmatrix} \left(\hat{c}_1, (\mathsf{jpk}_{j,1}, c_{j,1})_{j \in [\ell]}, \\ \hat{c}_2, (\mathsf{jpk}_{j,2}, c_{j,2})_{j \in [\ell]} \right), (f, \sigma_1, \sigma_2) \end{pmatrix} \middle| \\ \\ \hat{c}_1 = \mathsf{MGHE}.\mathsf{Eval}(f, (\mathsf{jpk}_{j,1}, c_{j,1})_{j \in [\ell]}; \sigma_1) \\ \\ \\ \hat{c}_2 = \mathsf{MGHE}.\mathsf{Eval}(f, (\mathsf{jpk}_{j,2}, c_{j,2})_{j \in [\ell]}; \sigma_2) \end{pmatrix}. \right.$$

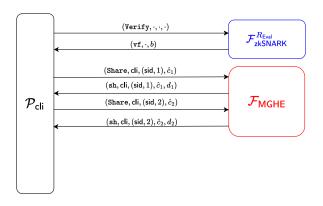
Phase 3: Result Retrieval - Partial Decryption **EPFL**





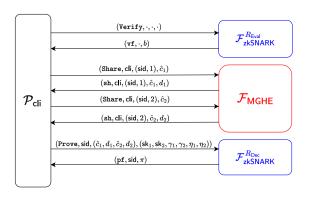
Phase 3: Result Retrieval - Partial Decryption **EPFL**





Phase 3: Result Retrieval - Partial Decryption

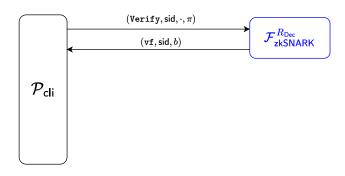




$$R_{\mathsf{Dec}} = \left\{ \begin{pmatrix} \mathsf{pp}_1, \mathsf{pk}_1, c_1, d_1, \\ \mathsf{pp}_2, \mathsf{pk}_2, c_2, d_2 \end{pmatrix}, \begin{pmatrix} \mathsf{sk}_1, \gamma_1, \eta_1 \\ \mathsf{sk}_2, \gamma_2, \eta_2 \end{pmatrix} \middle| \begin{array}{l} \forall u \in \{1, 2\} : \\ d_u = \mathsf{MGHE.PDec}(\mathsf{sk}_u, c_u; \eta_u) \\ \land \mathsf{pk}_u = \mathsf{PKGen}(\mathsf{pp}_u, \mathsf{sk}_u; \gamma_u) \end{array} \right\}.$$

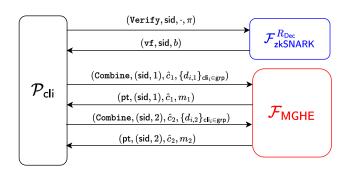
Phase 3: Result Retrieval - Reconstruction





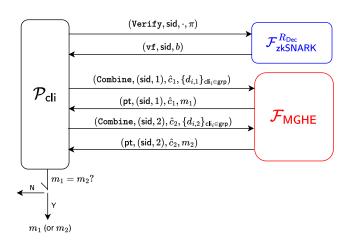
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Phase 3: Result Retrieval - Reconstruction



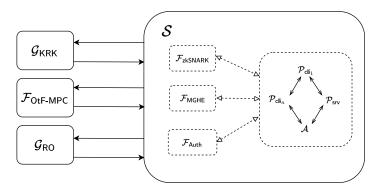


Realization of On-the-Fly MPC



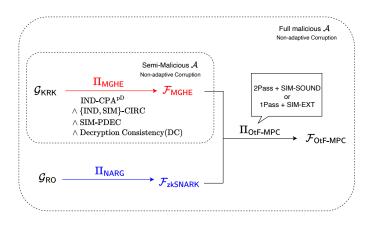
Theorem 2

 $\Pi_{\text{OtF-MPC}}$ UC-realizes $\mathcal{F}_{\text{OtF-MPC}}$ in $[\mathcal{F}_{\text{MGHE}}, \mathcal{F}_{\text{zkSNARK}}, \mathcal{F}_{\text{Aut}}]$ -hybrid model in presence of \mathcal{G}_{KRK} and \mathcal{G}_{RO} .



Conclusion

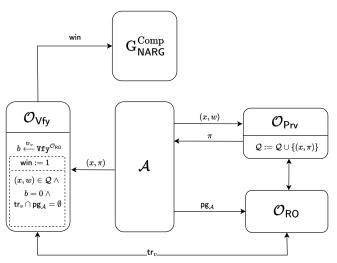




Appendix: Completeness



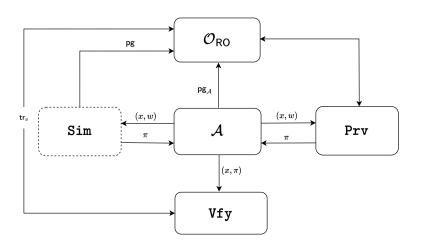
- Valid arguments must be accepted.



Appendix: Zero-Knowledge



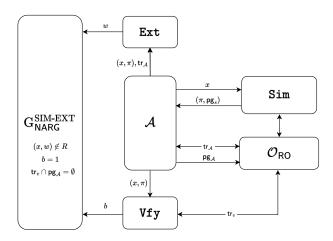
- Arguments does not disclose information about witness.



Appendix: sSIM-EXT



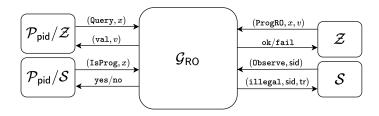
- Stronger version for *Knowledge Soundness*.
- Non-malleability for UC-security.



Appendix: Global Random Oracle \mathcal{G}_{RO}

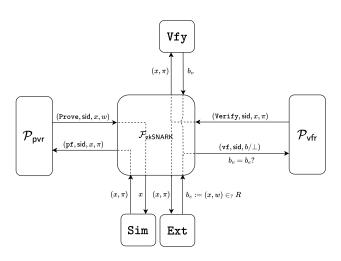


 Global random oracle with restricted programming and observability [CDG⁺18].



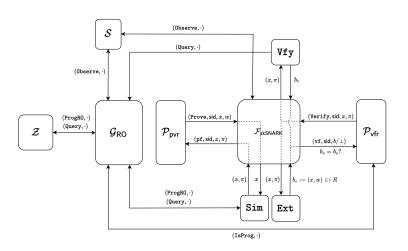
Appendix: Functionality $\mathcal{F}_{zkSNARK}$ [CF24]





Appendix: $\mathcal{F}_{zkSNARK} + \mathcal{G}_{RO}$



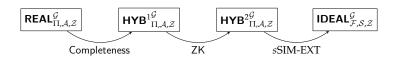


Appendix: Realization of $\mathcal{F}_{zkSNARK}$



Theorem 3[?, TCC:ChiFen24

 $J\Pi_{NARG}$ securely realizes $\mathcal{F}_{zkSNARK}$ in \mathcal{G}_{RO} -hybrid model if NARG satisfies completeness, sSIM-EXT, and zero-knowledge.





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