A Composable View of Verifiable Homomorphic Encryption in Multi-Party Settings

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March 31, 2025

Motivation

- On-the-Fly MPC [LTV12]

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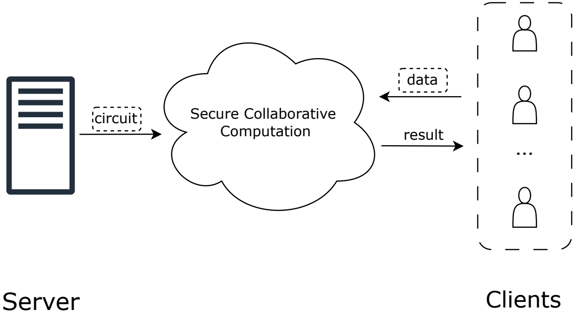


Motivation

– Homomorphic Encryption (HE) is a good candidate...

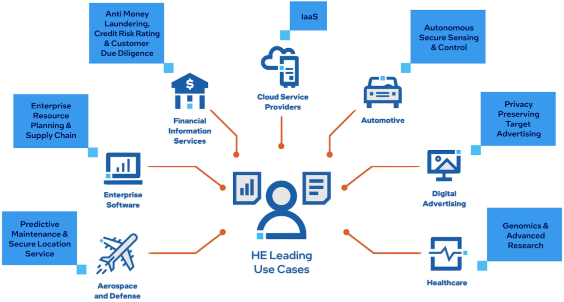
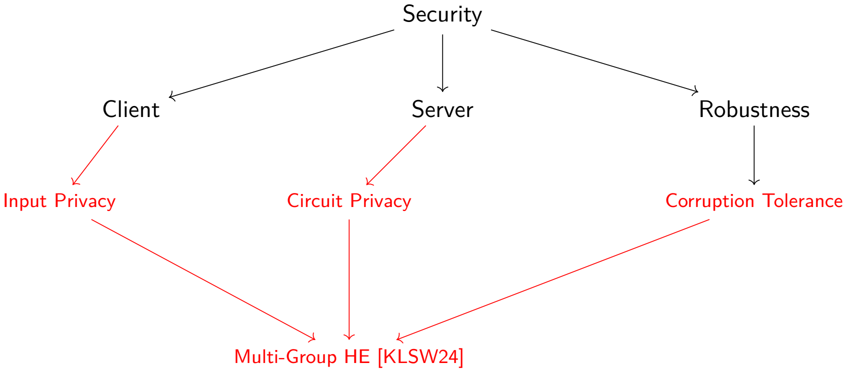


Figure: Use cases of HE [Int].

Goal


Security
Client
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Robustness
Input Privacy
Circuit Privacy
Corruption Tolerance
Multi-Group HE [KLSW24]

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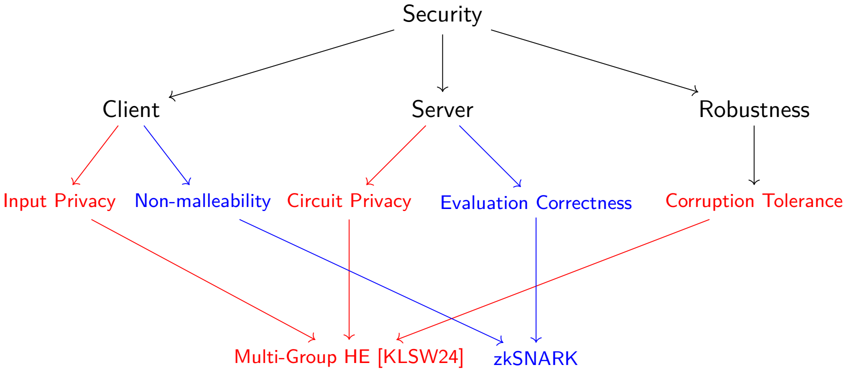
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zkSNARK

Contribution

– Formalism

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  - New game-based notions for (Multi-Key, Threshold, Multi-Group) HE in multi-party setting.

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  - UC functionality for HE in multi-party setting.
- Construction
  - UC-secure MPC via verifiable multi-group HE.

Multi-Group HE (MGHE) [KLSW24]

- Hybrid approach between Threshold HE and Multi-Key HE

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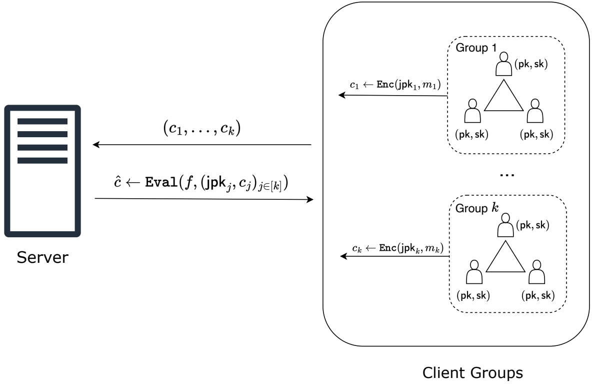
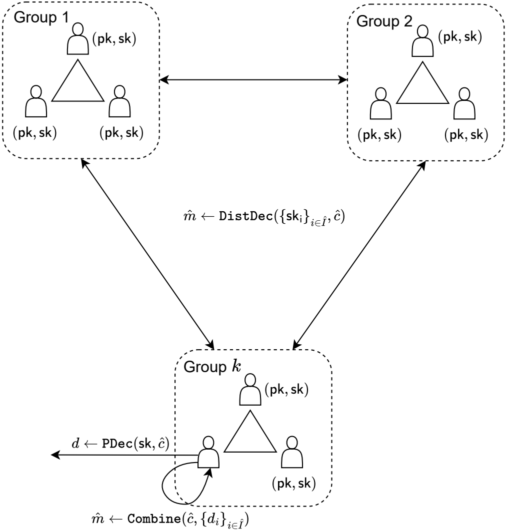


Multi-Group HE

EPFL



Confidentiality with Multi-Group HE

$$\mathcal{G}_{\mathsf{KRK}} \xrightarrow{\Pi_{\mathsf{MGHE}}} \mathcal{F}_{\mathsf{MGHE}}$$

if MGHE satisfies

$$\begin{aligned} & \text{IND-CPA}^{\text{pD}} \\ \wedge\ & \{\text{IND}, \text{SIM}\}\text{-CIRC} \\ \wedge\ & \text{SIM-PDEC} \\ \wedge\ & \text{Decryption Consistency (DC)} \end{aligned}$$

Client Side: IND-CPA$^{\mathrm{pD}}$ Security

IND-CPA
For PKE

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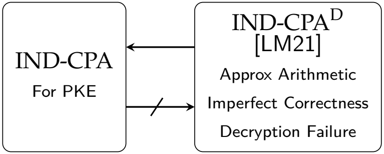

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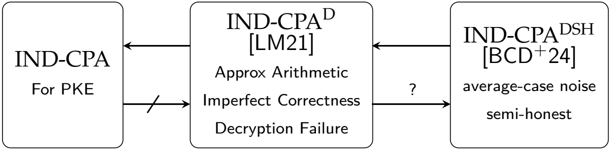

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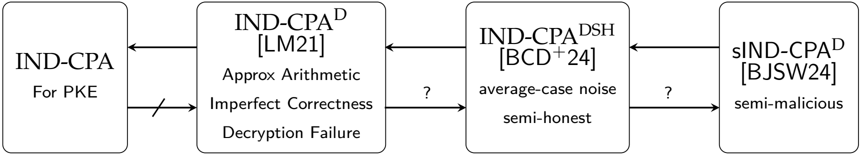

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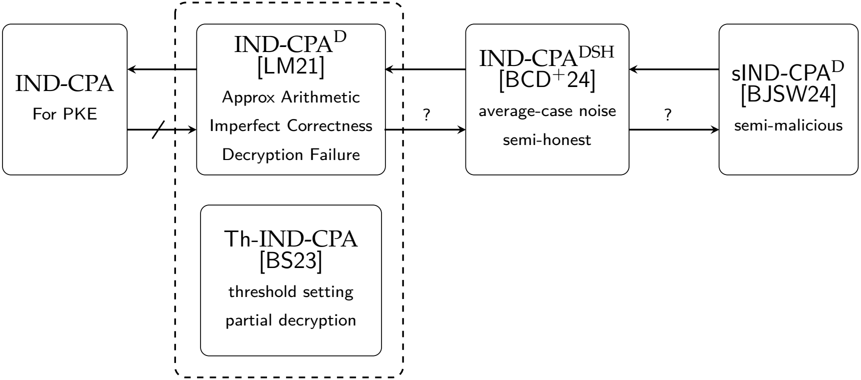

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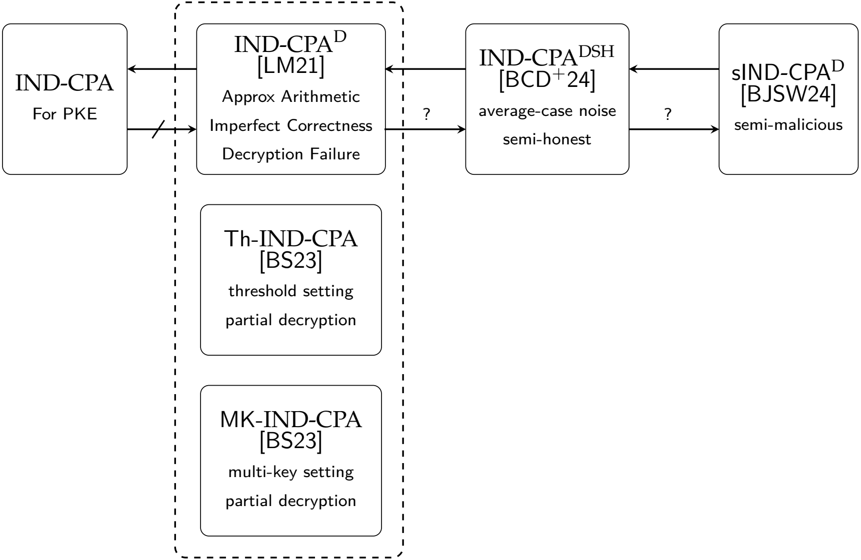

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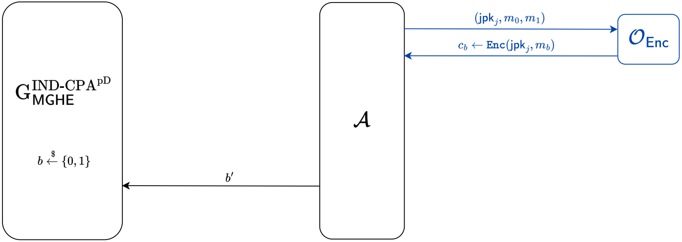

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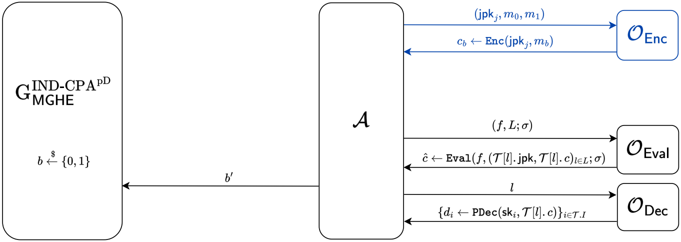

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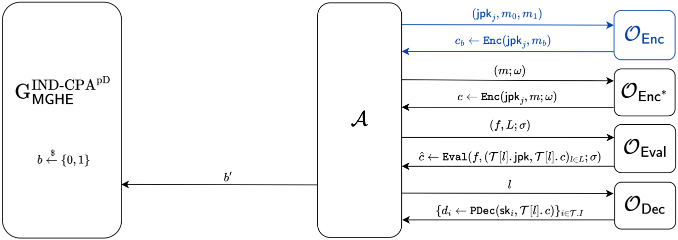

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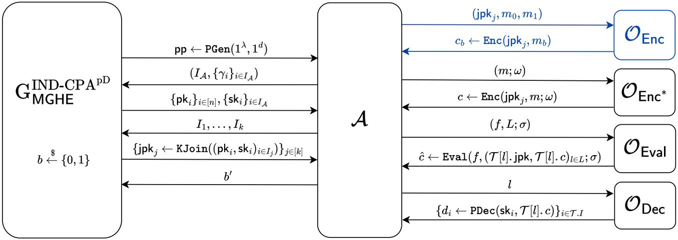

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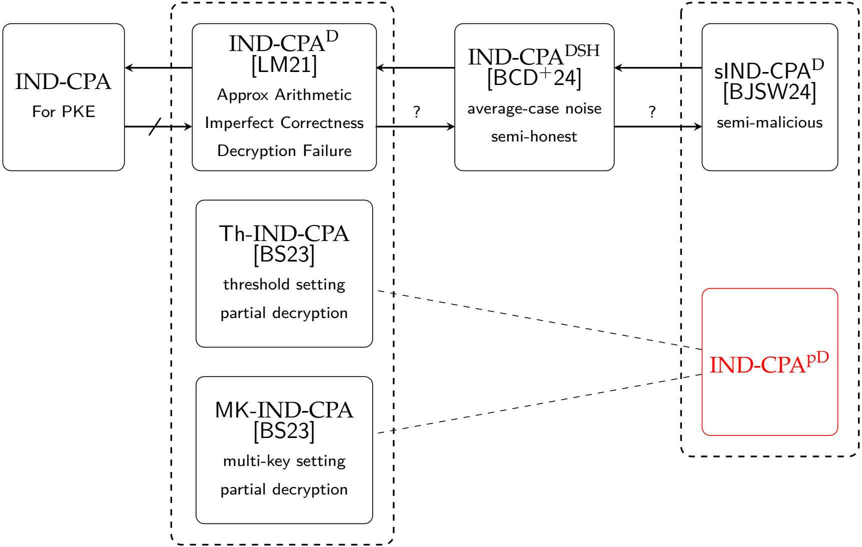

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$$\hat{c} \leftarrow \mathsf{MGHE}.\texttt{Eval}(f, (\mathsf{jpk}_j, c_j)_{j \in [\ell]})$$

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– Stronger security with *statistical indistinguishability*.

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$$\hat{m} = f(m_1, \ldots, m_\ell)$$

$$\hat{m} + \varepsilon \leftarrow \mathsf{MGHE}.\mathtt{Dec}(\hat{c})$$

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EPFL

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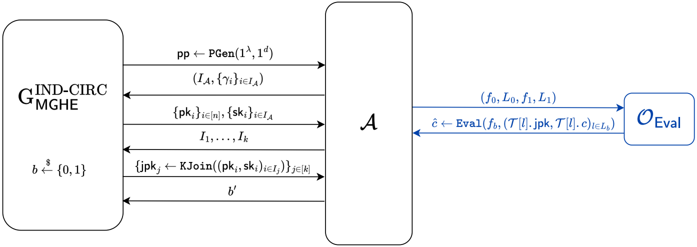

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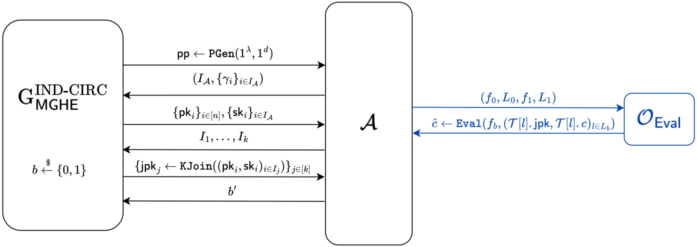

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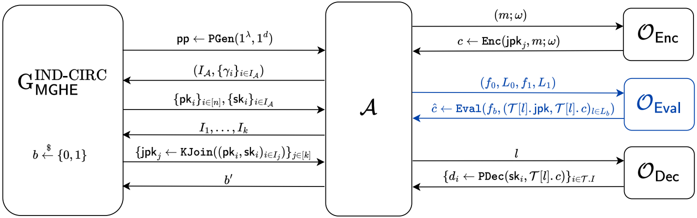

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- Security of secret key sk_j of honest client i.e., $j \notin I_{\mathcal{A}}$

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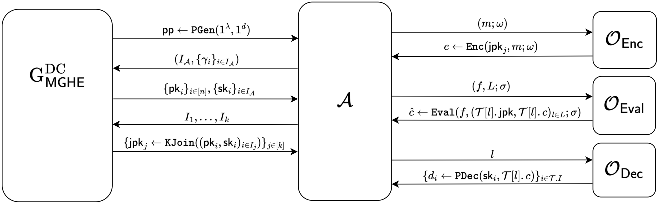

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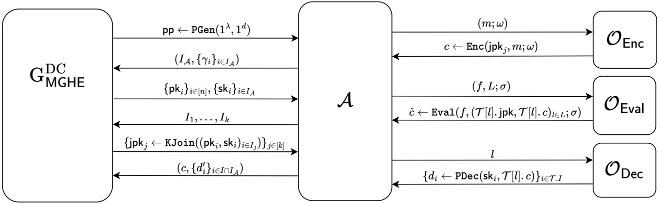

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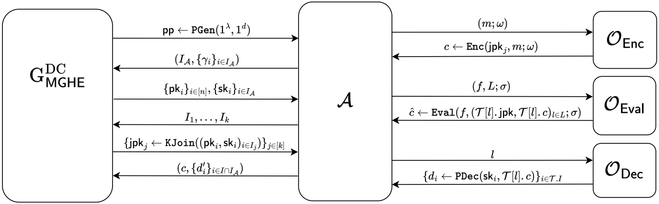
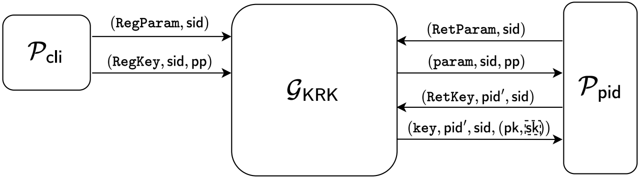


– $\mathcal{A}$ wins if

$$m \neq \mathtt{Combine}(c, \{d_i\}_{i \in I \setminus I_\mathcal{A}} \cup \{d'_i\}_{i \in I \cap I_\mathcal{A}})$$

UC: Global Key Registry $\mathcal{G}_{\mathsf{KRK}}$

– Global subroutine for key management taken from [BCNP04].



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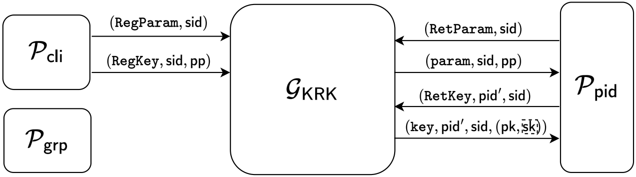

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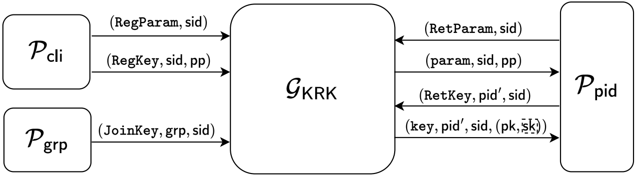
- Key aggregation for groups (equivalent to $\mathcal{F}_{\mathsf{MPC}}$).



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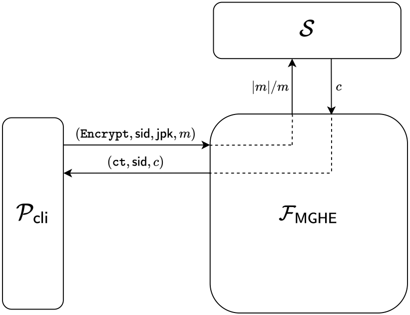

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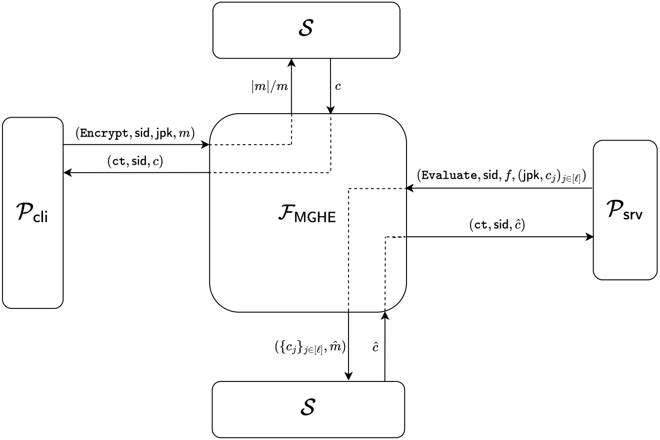

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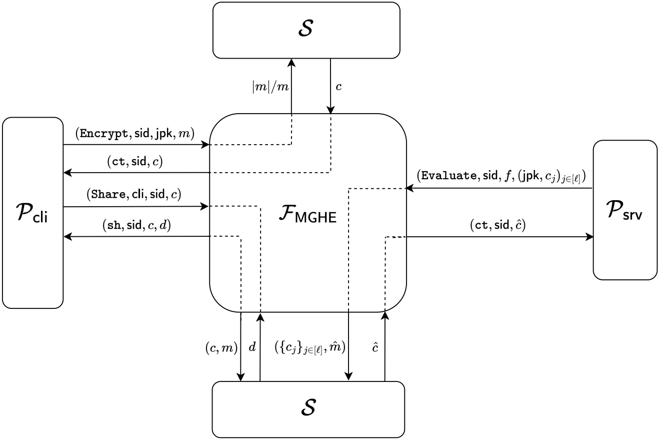

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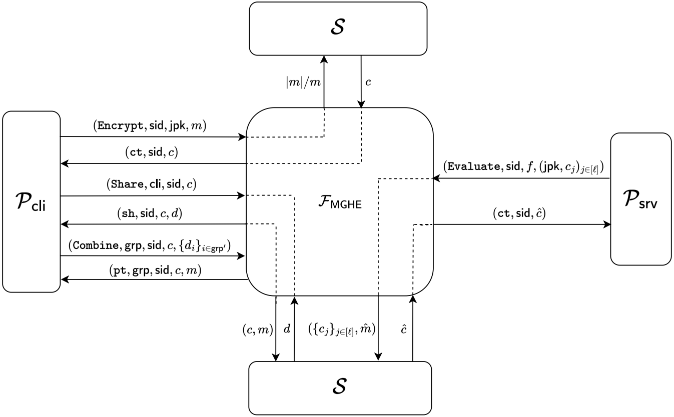

Realization of $\mathcal{F}_{\mathsf{MGHE}}$

EPFL

Theorem 1

Π_{MGHE} UC-realizes $\mathcal{F}_{\mathsf{MGHE}}$ against a semi-malicious adversary in presence of $\mathcal{G}_{\mathsf{KRK}}$ if MGHE *is* $\mathrm{IND\text{-}CPA}^{\mathrm{pD}}$, $\mathrm{IND\text{-}CIRC}$ ($\mathrm{SIM\text{-}CIRC}$)*, and* $\mathrm{SIM\text{-}PDEC}$ *secure under the static corruption of clients in a group up to the threshold and possibly the server.*

Realization of $\mathcal{F}_{\mathsf{MGHE}}$

$$
\begin{tikzcd}
\boxed{\mathbf{REAL}^{\mathcal{G}_{\mathsf{KRK}}}_{\Pi_{\mathsf{MGHE}},\mathcal{A},\mathcal{Z}}} \arrow[r, bend right, "\text{DC}"'] & \boxed{\mathbf{HYB}^{1\,\mathcal{G}_{\mathsf{KRK}}}_{\Pi_{\mathsf{MGHE}},\mathcal{A},\mathcal{Z}}} \arrow[rr, bend left, "\text{IND-CPA}^{\text{pD}}"] & & \boxed{\mathbf{HYB}^{3\,\mathcal{G}_{\mathsf{KRK}}}_{\Pi_{\mathsf{MGHE}},\mathcal{A},\mathcal{Z}}} \arrow[r, bend right, "\text{SIM-PDEC}"'] & \boxed{\mathbf{IDEAL}^{\mathcal{G}_{\mathsf{KRK}}}_{\mathcal{F}_{\mathsf{MGHE}},\mathcal{S},\mathcal{Z}}}
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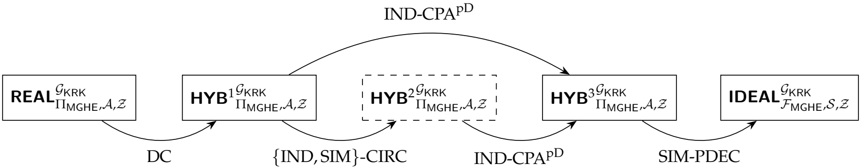

Integrity via Verifiability

MGHE $\Rightarrow$ Security against *semi-malicious* adversary

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MGHE + zkSNARK $\Rightarrow$ Security against *(full) malicious* adversary

zkSNARK in ROM

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zkSNARK		[CF24]	[BFKT24]	[GKO^{+}23]
NIZK	[BS21]	[LR22]		[LR22]
	CRS	ROM	ROM-AGM	CRS-ROM

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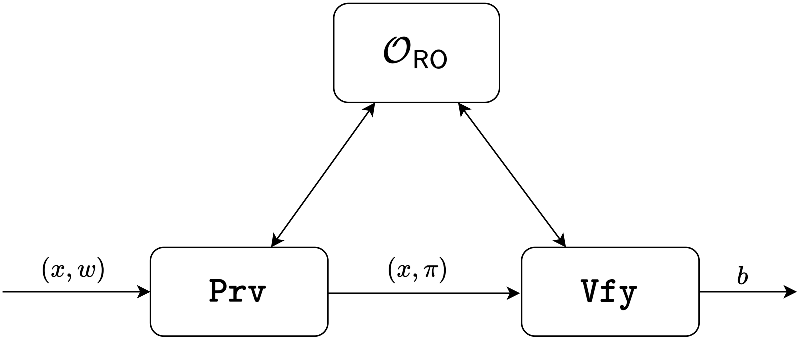

Properties of zkSNARK

- *Completeness*: Valid arguments must be accepted.

$$\forall (x,w) \in R,\ \Pr\left[\mathtt{Vfy}^{\mathcal{O}_{\mathsf{RO}}}(x,\pi) = 1 \;\middle|\; \begin{array}{l} \mathcal{O}_{\mathsf{RO}} \leftarrow \mathcal{U}(\lambda) \\ \pi \leftarrow \mathtt{Prv}^{\mathcal{O}_{\mathsf{RO}}}(x,w) \end{array}\right] = 1.$$

Properties of zkSNARK

- *Zero-Knowledge*: Arguments do not disclose information about witness.

$$\left\{ \mathsf{out} \;\middle|\; \begin{array}{l} \mathcal{O}_{\mathsf{RO}} \leftarrow \mathcal{U}(\lambda) \\ (x, w, \mathsf{aux}) \leftarrow \mathcal{A}^{\mathcal{O}_{\mathsf{RO}}} \\ \pi \leftarrow \mathtt{Prv}^{\mathcal{O}_{\mathsf{RO}}}(x, w) \\ \mathsf{out} \leftarrow \mathcal{A}^{\mathcal{O}_{\mathsf{RO}}}(\mathsf{aux}, \pi) \end{array} \right\} \approx \left\{ \mathsf{out} \;\middle|\; \begin{array}{l} \mathcal{O}_{\mathsf{RO}} \leftarrow \mathcal{U}(\lambda) \\ (x, w, \mathsf{aux}) \leftarrow \mathcal{A}^{\mathcal{O}_{\mathsf{RO}}} \\ (\pi, \mathsf{pg}) \leftarrow \mathtt{Sim}^{\mathcal{O}_{\mathsf{RO}}}(x) \\ \mathsf{out} \leftarrow \mathcal{A}^{\mathcal{O}_{\mathsf{RO}}[\mathsf{pg}]}(\mathsf{aux}, \pi) \end{array} \right\}$$

Properties of zkSNARK

- *Simulation Soundness*: Non-malleability of arguments.

$$\Pr\left[\begin{array}{l}|x| \leq n \\ \wedge\, x \notin \mathcal{L}(R) \\ \wedge\, \texttt{Vfy}^{\mathcal{O}_{\mathsf{RO}}}(x, \pi) = 1\end{array} \;\middle|\; \begin{array}{l}\mathcal{O}_{\mathsf{RO}} \leftarrow \mathcal{U}(\lambda) \\ (x, \pi) \leftarrow \mathcal{A}^{\mathcal{O}_{\mathsf{RO}}}(\texttt{Sim})\end{array}\right] \leq \text{negl.}$$

Properties of zkSNARK

- *Succinctness*: Argument is efficient.

$$|\pi| \ll |w|$$

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- Three-phase protocol

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    - the clients in a group up to the threshold

Phase 1: Data Uploading

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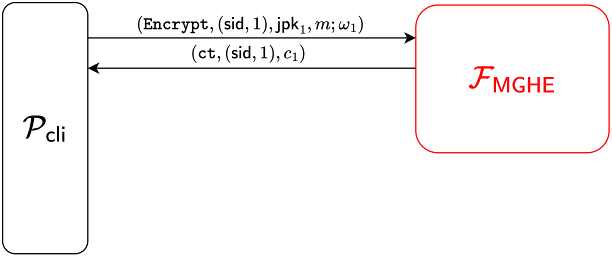

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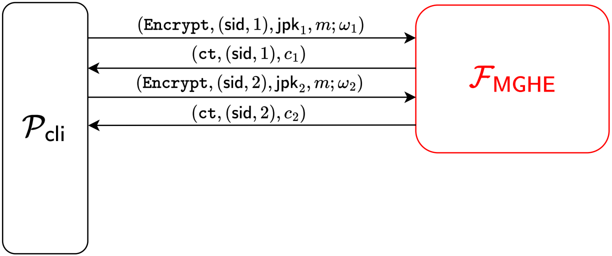

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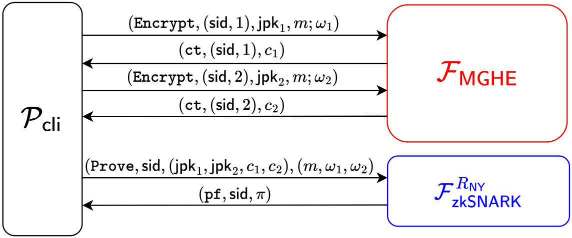

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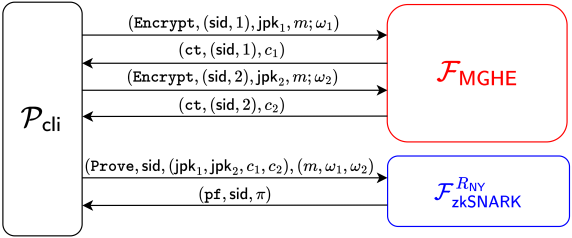

$$R_{\mathsf{NY}} = \left\{ \left(\begin{pmatrix} \mathsf{jpk}_1, c_1, \\ \mathsf{jpk}_2, c_2 \end{pmatrix}, (m, \omega_1, \omega_2) \right) \middle| \begin{array}{l} c_1 = \mathsf{MGHE}.\texttt{Enc}(\mathsf{jpk}_1, m; \omega_1) \\ \wedge \\ c_2 = \mathsf{MGHE}.\texttt{Enc}(\mathsf{jpk}_2, m; \omega_2) \end{array} \right\}$$

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  - Must know m to generate valid ciphertext tuple.

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Naor-Yung + Simulation Soundness

Phase 1: Data Uploading

Naor-Yung + Simulation Soundness

or

One-Pass + Simulation Extractability

Phase 1: Data Uploading

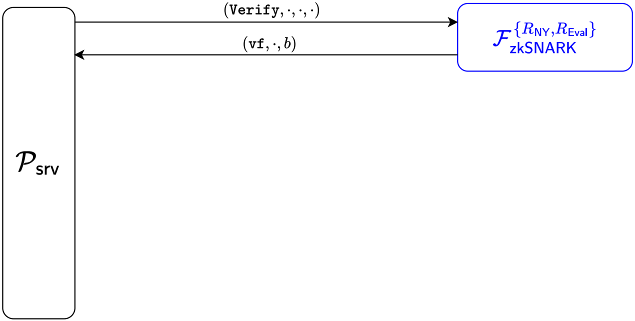
Naor-Yung + Simulation Soundness

or

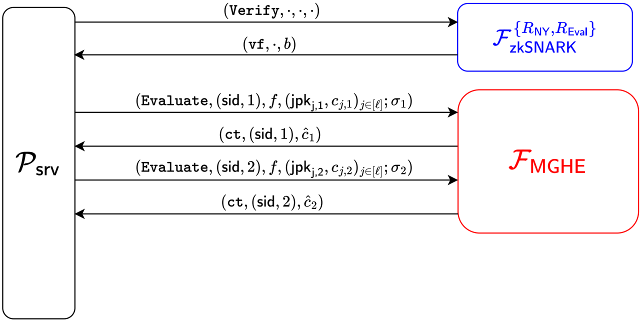
One-Pass + Simulation Extractability

$$R_{\mathsf{NY}} = \{(\mathsf{jpk}, c), (m, \omega)) \quad c = \mathsf{MGHE}.\mathtt{Enc}(\mathsf{jpk}, m; \omega)\}$$

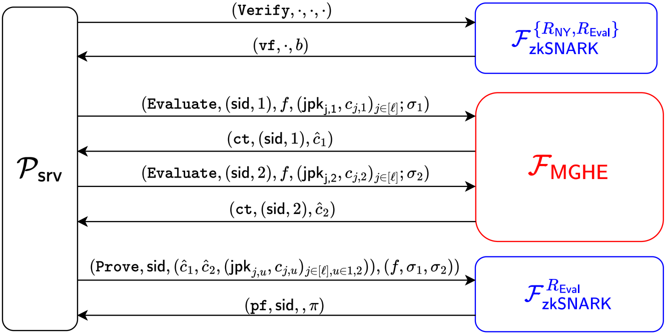
Phase 2: Circuit Evaluation



Phase 2: Circuit Evaluation



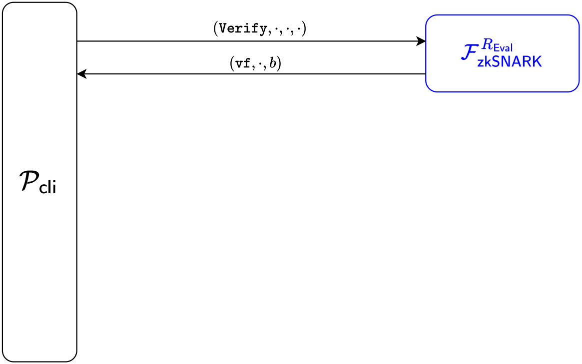
Phase 2: Circuit Evaluation



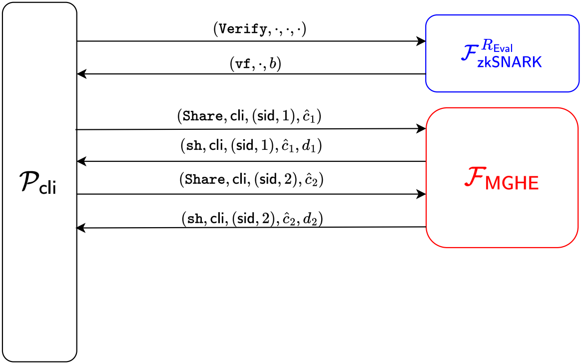
Phase 2: Circuit Evaluation

$$R_{\mathsf{Eval}} = \left\{ \begin{array}{c|c} \left(\begin{pmatrix} \hat{c}_1, (\mathsf{jpk}_{j,1}, c_{j,1})_{j \in [\ell]}, \\ \hat{c}_2, (\mathsf{jpk}_{j,2}, c_{j,2})_{j \in [\ell]} \end{pmatrix}, (f, \sigma_1, \sigma_2) \right) & \\ & \begin{array}{l} \hat{c}_1 = \mathsf{MGHE}.\mathtt{Eval}(f, (\mathsf{jpk}_{j,1}, c_{j,1})_{j \in [\ell]}; \sigma_1) \\ \wedge \\ \hat{c}_2 = \mathsf{MGHE}.\mathtt{Eval}(f, (\mathsf{jpk}_{j,2}, c_{j,2})_{j \in [\ell]}; \sigma_2) \end{array} \end{array} \right\}.$$

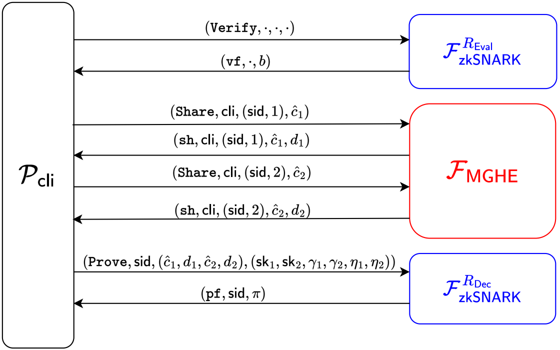
Phase 3: Result Retrieval - Partial Decryption



Phase 3: Result Retrieval - Partial Decryption

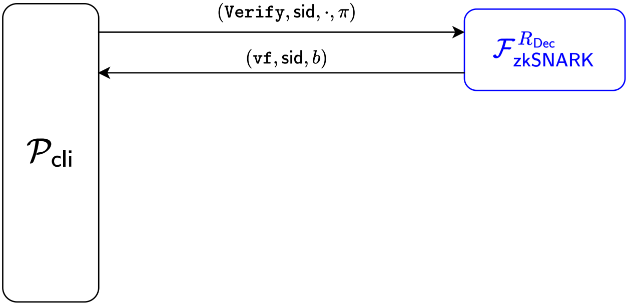


Phase 3: Result Retrieval - Partial Decryption

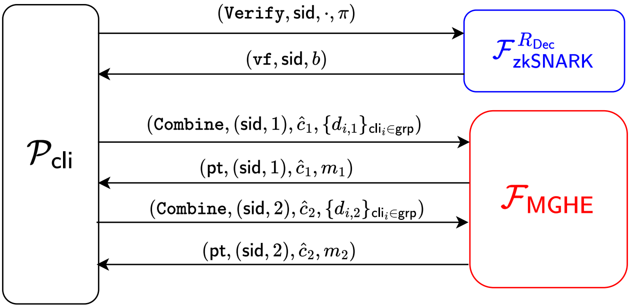


$$R_{\mathsf{Dec}} = \left\{ \begin{pmatrix} \mathsf{pp}_1, \mathsf{pk}_1, c_1, d_1, \\ \mathsf{pp}_2, \mathsf{pk}_2, c_2, d_2 \end{pmatrix}, \begin{pmatrix} \mathsf{sk}_1, \gamma_1, \eta_1 \\ \mathsf{sk}_2, \gamma_2, \eta_2 \end{pmatrix} \middle| \begin{array}{l} \forall u \in \{1, 2\} : \\ \quad d_u = \mathsf{MGHE.PDec}(\mathsf{sk}_u, c_u; \eta_u) \\ \quad \wedge\, \mathsf{pk}_u = \mathsf{PKGen}(\mathsf{pp}_u, \mathsf{sk}_u; \gamma_u) \end{array} \right\}.$$

Phase 3: Result Retrieval - Reconstruction

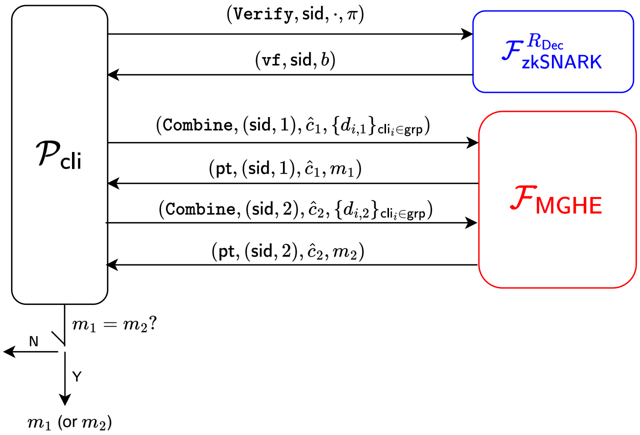


Phase 3: Result Retrieval - Reconstruction



Phase 3: Result Retrieval - Reconstruction

EPFL



Realization of On-the-Fly MPC

Theorem 2

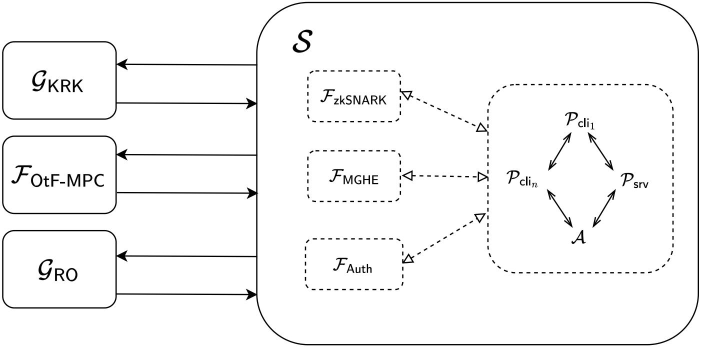
$\Pi_{\mathsf{OtF\text{-}MPC}}$ *UC-realizes* $\mathcal{F}_{\mathsf{OtF\text{-}MPC}}$ *in* $[\mathcal{F}_{\mathsf{MGHE}}, \mathcal{F}_{\mathsf{zkSNARK}}, \mathcal{F}_{\mathsf{Aut}}]$*-hybrid model in presence of* $\mathcal{G}_{\mathsf{KRK}}$ *and* $\mathcal{G}_{\mathsf{RO}}$.

Realization of On-the-Fly MPC

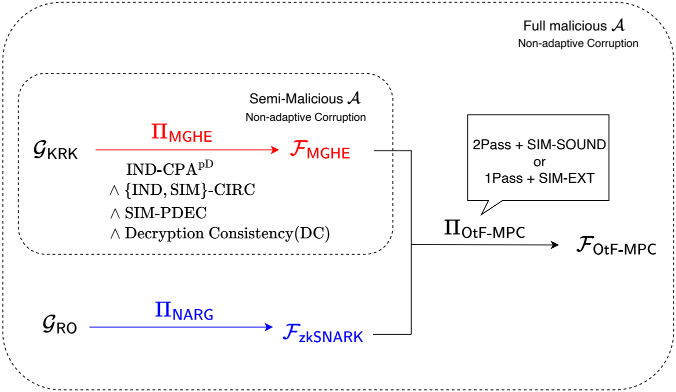
EPFL

Theorem 3

$\Pi_{\mathsf{OtF\text{-}MPC}}$ UC-realizes $\mathcal{F}_{\mathsf{OtF\text{-}MPC}}$ in $[\mathcal{F}_{\mathsf{MGHE}}, \mathcal{F}_{\mathsf{zkSNARK}}, \mathcal{F}_{\mathsf{Aut}}]$-hybrid model in presence of $\mathcal{G}_{\mathsf{KRK}}$ and $\mathcal{G}_{\mathsf{RO}}$.

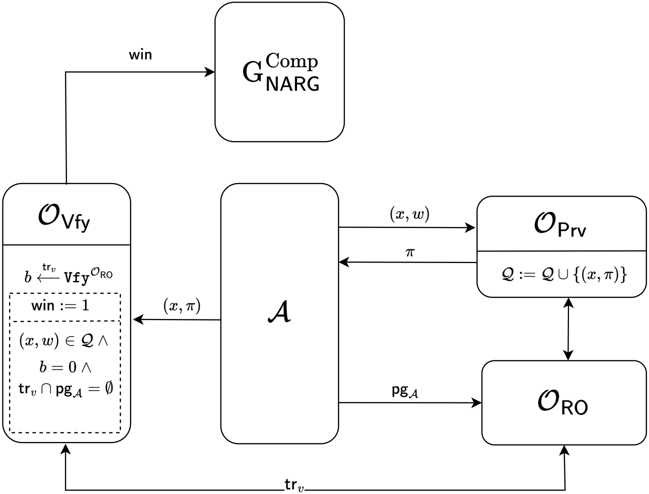


Conclusion


Full malicious $\mathcal{A}$
Non-adaptive Corruption
Semi-Malicious $\mathcal{A}$
Non-adaptive Corruption
$\mathcal{G}_{\mathsf{KRK}}$
Π_{MGHE}
$\mathcal{F}_{\mathsf{MGHE}}$
$\text{IND-CPA}^{\text{pD}}$
$\wedge$ {IND, SIM}-CIRC
$\wedge$ SIM-PDEC
$\wedge$ Decryption Consistency(DC)
2Pass + SIM-SOUND
or
1Pass + SIM-EXT
$\Pi_{\mathsf{OtF\text{-}MPC}}$
$\mathcal{F}_{\mathsf{OtF\text{-}MPC}}$
$\mathcal{G}_{\mathsf{RO}}$
Π_{NARG}
$\mathcal{F}_{\mathsf{zkSNARK}}$

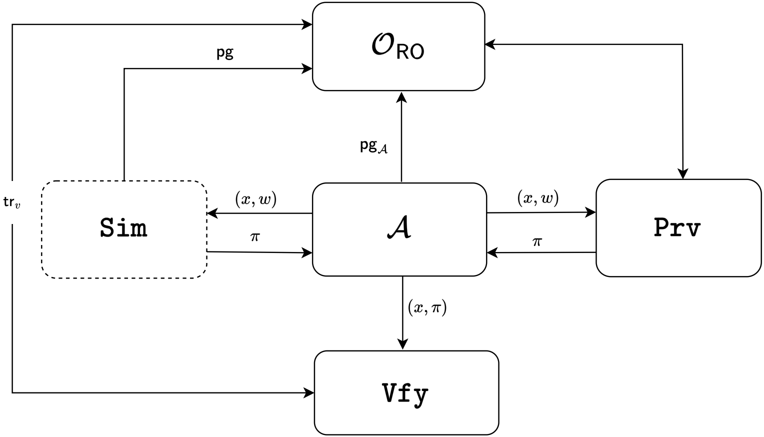
Appendix: Completeness

– Valid arguments must be accepted.



Appendix: Zero-Knowledge

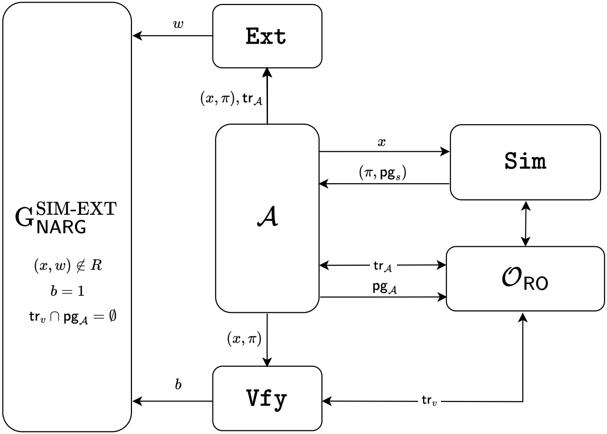
- Arguments does not disclose information about witness.



Appendix: sSIM-EXT

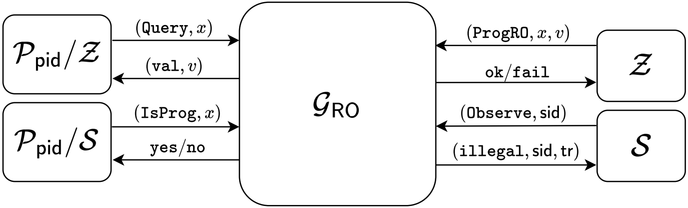
EPFL

- Stronger version for *Knowledge Soundness*.
- *Non-malleability* for UC-security.

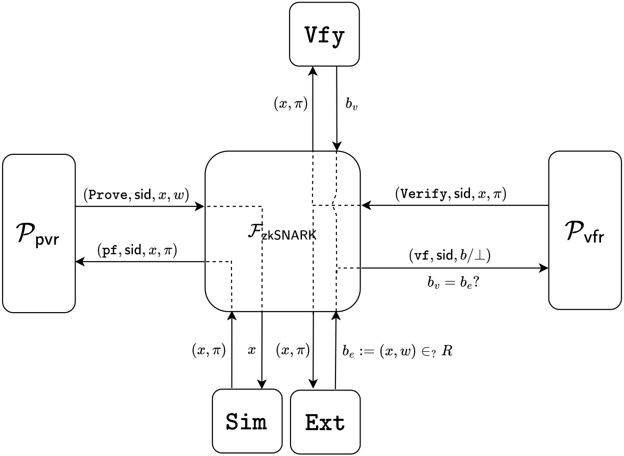


Appendix: Global Random Oracle $\mathcal{G}_{\mathsf{RO}}$

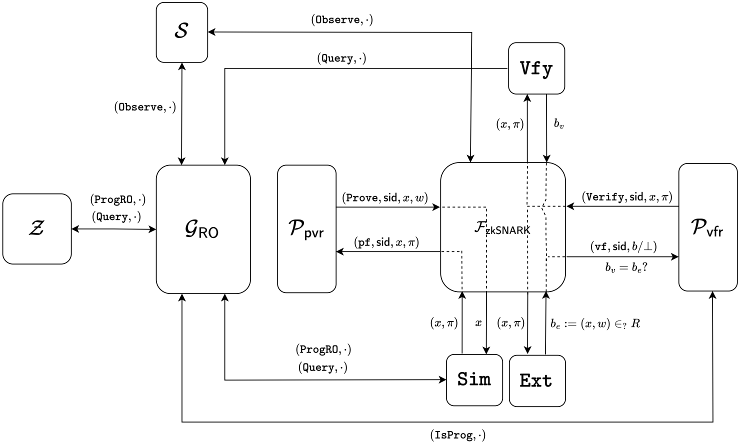
- Global random oracle with restricted *programming* and *observability* [CDG+18].



Appendix: Functionality $\mathcal{F}_{\mathsf{zkSNARK}}$ [CF24]



Appendix: $\mathcal{F}_{\mathsf{zkSNARK}} + \mathcal{G}_{\mathsf{RO}}$



Appendix: Realization of $\mathcal{F}_{\mathsf{zkSNARK}}$

Theorem 4[?, TCC:ChiFen24

]Π_{NARG} securely realizes $\mathcal{F}_{\mathsf{zkSNARK}}$ in $\mathcal{G}_{\mathsf{RO}}$-hybrid model if NARG *satisfies completeness,* $s\mathrm{SIM\text{-}EXT}$*, and zero-knowledge.*

$$
\begin{tikzcd}
\boxed{\mathbf{REAL}^{\mathcal{G}}_{\Pi,\mathcal{A},\mathcal{Z}}} \arrow[r, bend right, "\text{Completeness}"'] & \boxed{\mathbf{HYB}^{1\,\mathcal{G}}_{\Pi,\mathcal{A},\mathcal{Z}}} \arrow[r, bend right, "\text{ZK}"'] & \boxed{\mathbf{HYB}^{2\,\mathcal{G}}_{\Pi,\mathcal{A},\mathcal{Z}}} \arrow[r, bend right, "s\mathrm{SIM\text{-}EXT}"'] & \boxed{\mathbf{IDEAL}^{\mathcal{G}}_{\mathcal{F},\mathcal{S},\mathcal{Z}}}
\end{tikzcd}
$$

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