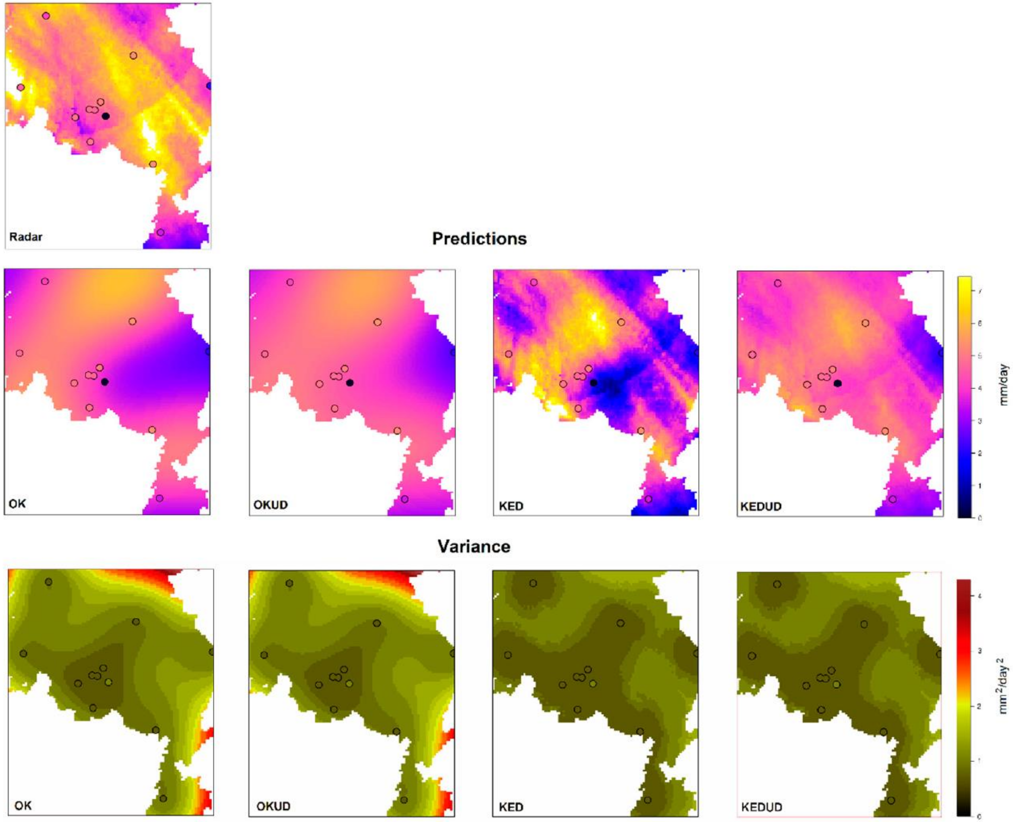
Statistical Methods

- **Kriging (Simple, Ordinary, Universal)**
- **Conditional Simulation**



Cecinati et al.: Considering Rain Gauge Uncertainty Using Kriging for Uncertain Data, Atmosphere (2018)

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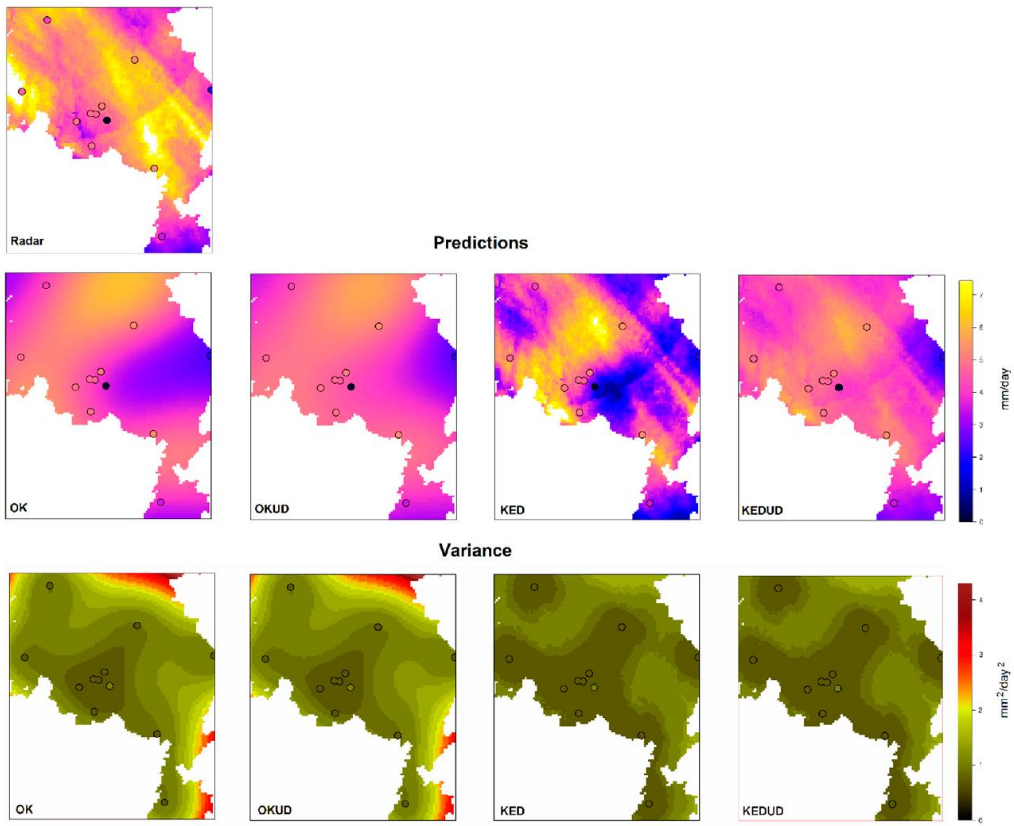

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Kriging

The measurement Z_i at location $x_i \in \mathbb{R}^n$ is interpreted as a **random variable** with **expected value**

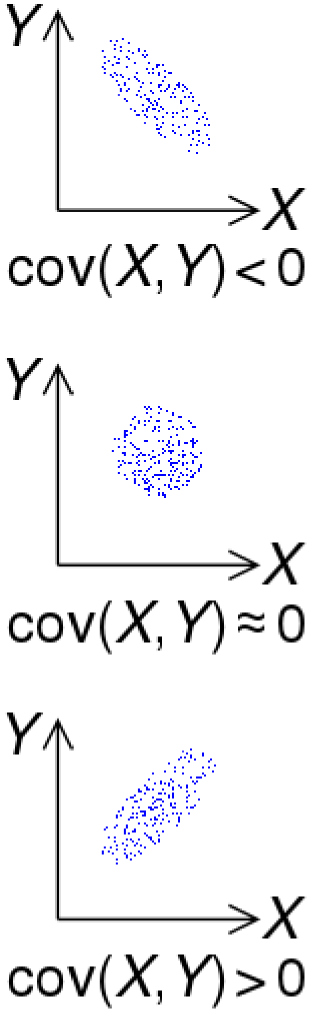
$$E[Z_i] = \int_\Omega z \, d\mu_{Z_i}(z)$$

and **variance**

$$Var(Z_i) = E[(Z_i - E[Z_i])^2].$$

The **covariance** of two random variables is given by

$$Cov(Z_i, Z_j) = E[(Z_i - E[Z_i])(Z_j - E[Z_j])].$$



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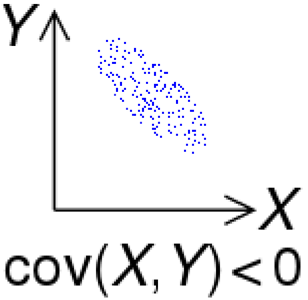
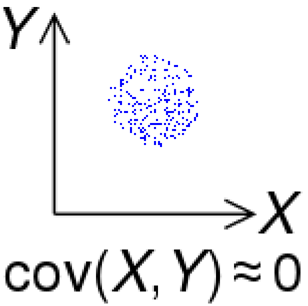
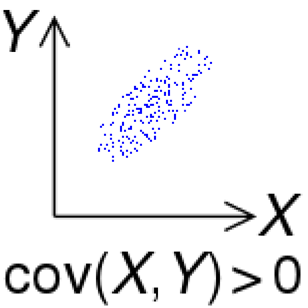
A crucial assumption in geostatistics is that of **stationarity**:

$$E[Z(x)] = m, \quad Cov(Z(x+h), Z(x)) = C(h).$$

The covariance depends only on the increment $x - x_i$ between to locations, not the location itself!







Kriging

An approximation Z^* of Z is given by

$$Z^*(x) = \lambda_0 + \sum_{i=1}^{N} \lambda_i(x) Z_i,$$

with unknown weights $\lambda_i(x)$ such that

$$Z^*(x_i) = z_i, \quad i = 1, \ldots, N.$$

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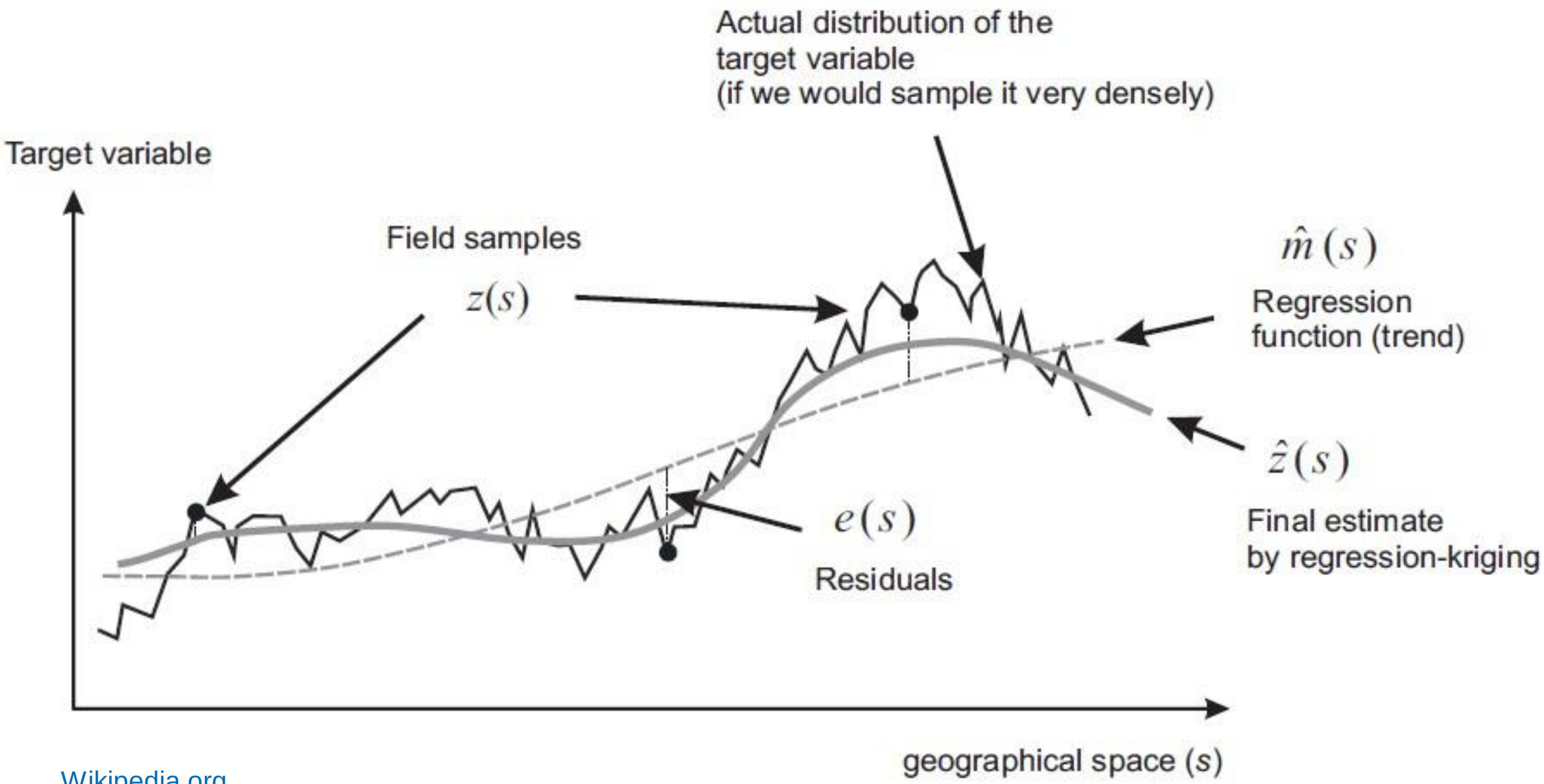
How to find the weights $\lambda_i(x)$? **Unbiasedness**

$$E[Z^*(x)] = E[Z(x)]$$

and **variance minimizing**

$$\min_{Z^*} Var(Z^*(x) - Z(x)).$$

Kriging



Wikipedia.org

Kriging

Simple Kriging: Expectation $m_0 = E[Z(x)]$ is known: Unbiasedness yields

$$\lambda_0 = m_0 - \sum_{i=1}^{n} \lambda_i m_0.$$

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Variance minimization leads to

$$\sum_{i=1}^{n} \lambda_i(x) Cov(Z_i, Z_k) = Cov(Z_k, Z(x)), \quad k = 1, \ldots, n.$$

Note that stationarity implies $Cov(Z_k, Z(x)) = C(x_k - x) \longrightarrow$ Relation to radial basis functions.

$$\sigma_{SK}^2 = Var[Z^* - Z_0] = C(0) - \sum_{k=1}^{n} \lambda_k^{SK} C(x_k - x_0)$$

Kriging

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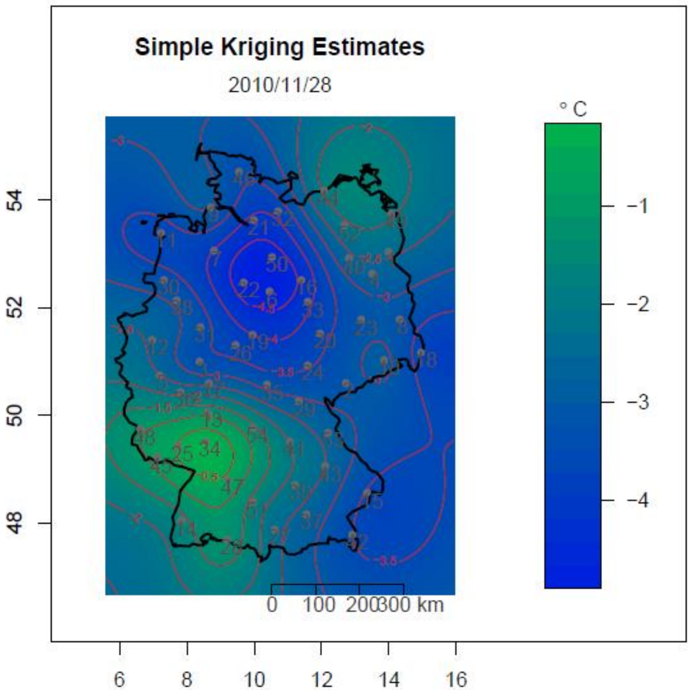

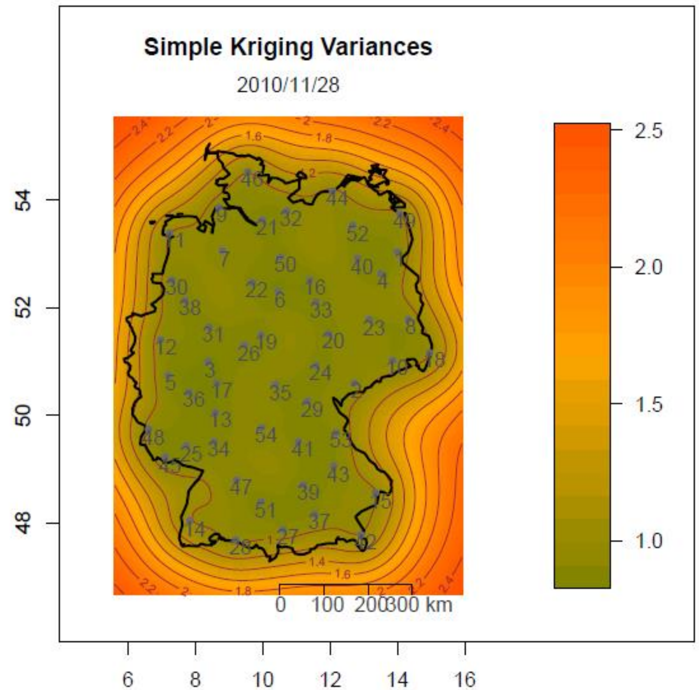

Example from the Bachelor thesis ”Kriging methods in spatial statistics”, Andreas Lichtenstern, TU München

Kriging

A crucial role is given to the **variogramm**

$$\gamma(h) = \frac{1}{2} Var(Z(x) - Z(x+h)) = C(0) - C(h).$$

It can be estimated from the data (under certain conditions).

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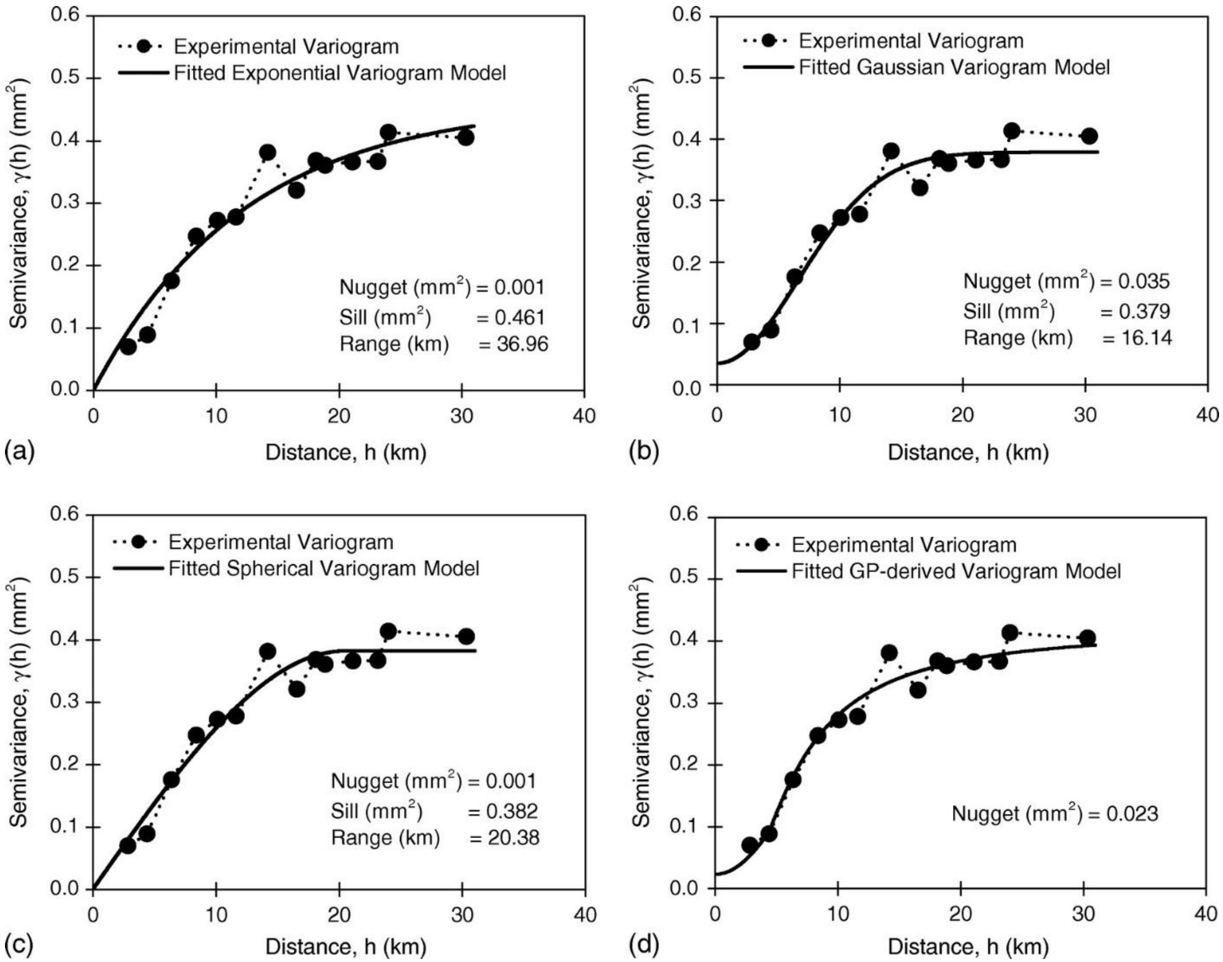
Empirical variogram

$$\hat{\gamma}(h) = \frac{1}{N_h} \sum_{x_i - x_j = h} (Z_i - Z_j)^2.$$

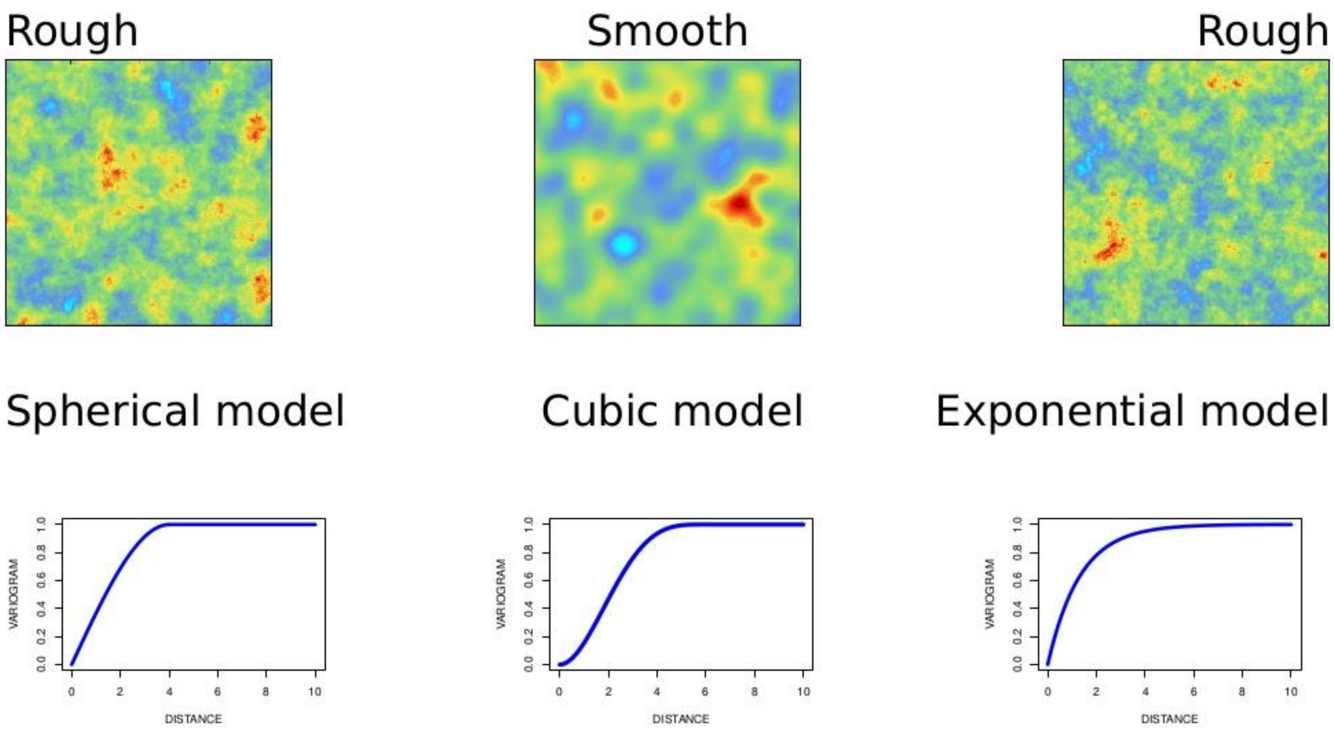
After the empirical variogram has been computed, a theoretical model needs to be fit to it. This model needs to satisfy certain properties, e.g. conditional negative definiteness.

Kriging



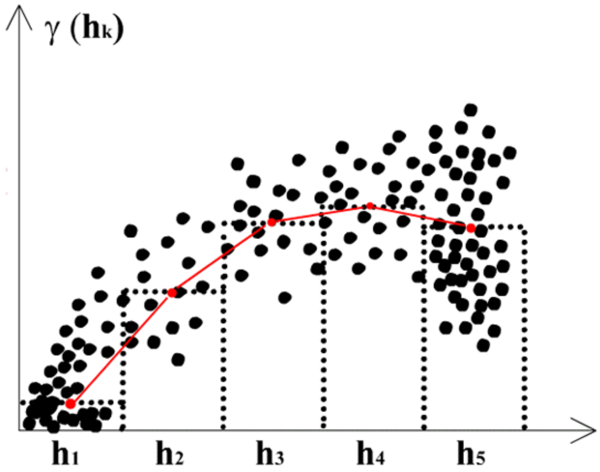
Experimental Variogram
Fitted Exponential Variogram Model
Nugget (mm²) = 0.001
Sill (mm²) = 0.461
Range (km) = 36.96
Semivariance, γ(h) (mm²)
Distance, h (km)
(a)
Experimental Variogram
Fitted Gaussian Variogram Model
Nugget (mm²) = 0.035
Sill (mm²) = 0.379
Range (km) = 16.14
Semivariance, γ(h) (mm²)
Distance, h (km)
(b)
Experimental Variogram
Fitted Spherical Variogram Model
Nugget (mm²) = 0.001
Sill (mm²) = 0.382
Range (km) = 20.38
Semivariance, γ(h) (mm²)
Distance, h (km)
(c)
Experimental Variogram
Fitted GP-derived Variogram Model
Nugget (mm²) = 0.023
Semivariance, γ(h) (mm²)
Distance, h (km)
(d)

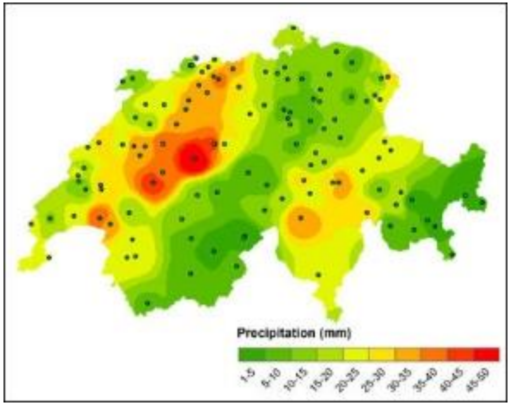
Kriging



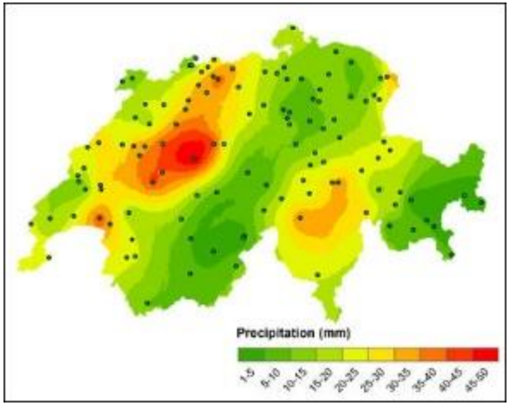
From a presentation by H. Wackernagel

Example IDW, Kriging





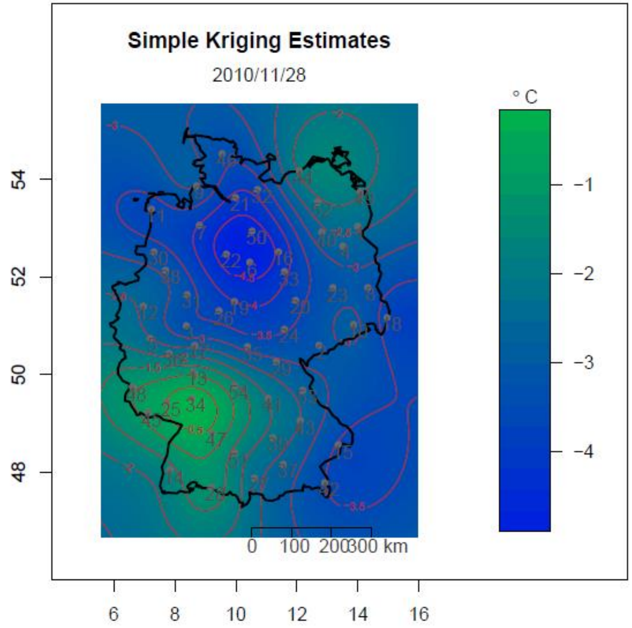
IDW

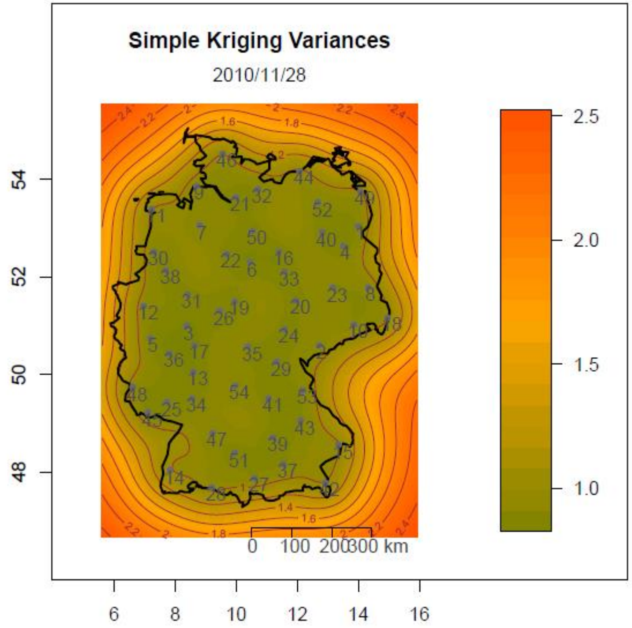


Kriging

gitta.info

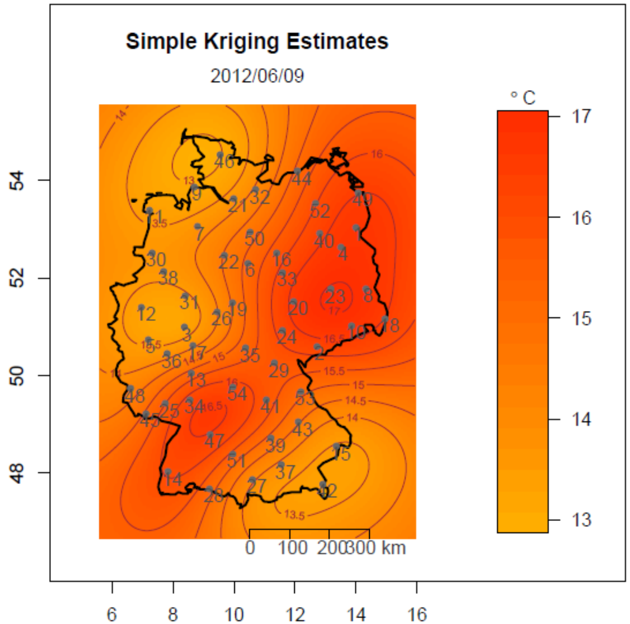
Example Kriging

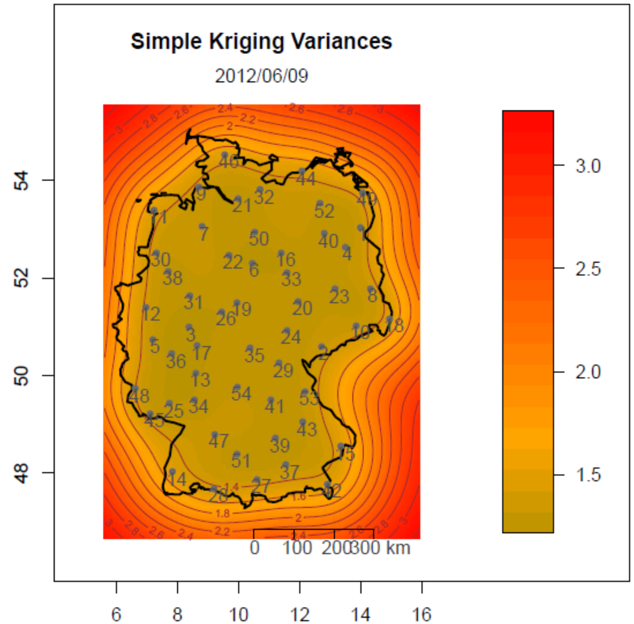




Example from the Bachelor thesis ”Kriging methods in spatial statistics”, Andreas Lichtenstern, TU München

Example Kriging





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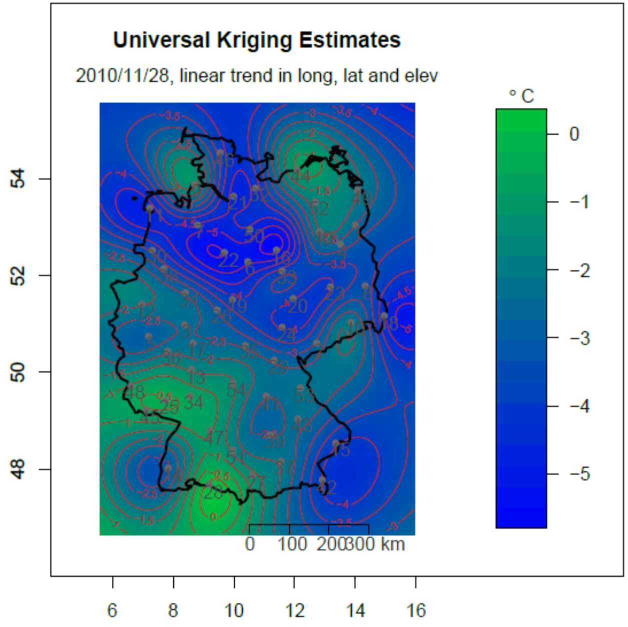

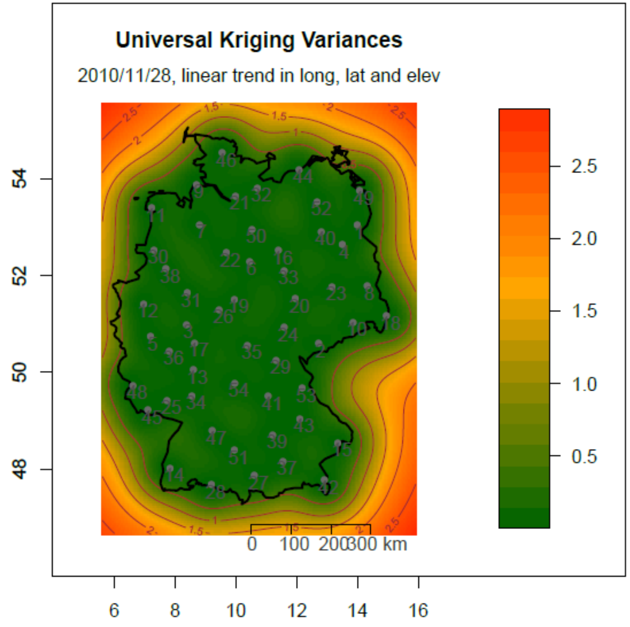

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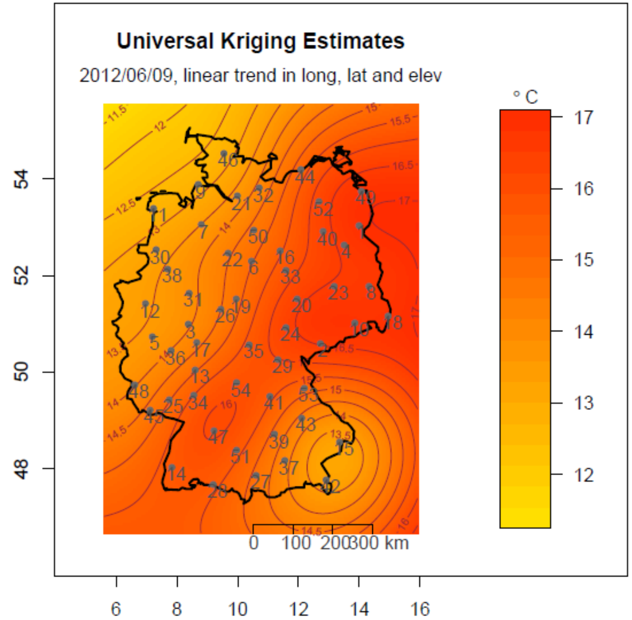

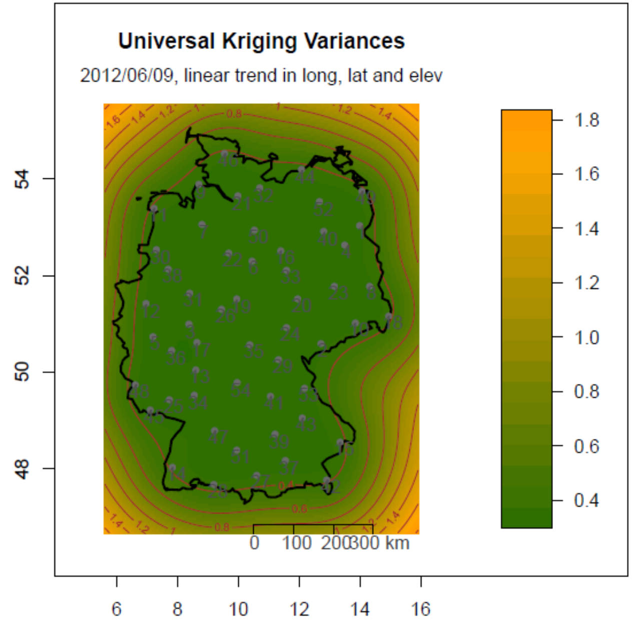

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