

# THE DESIGN AND ANALYSIS OF PD RADAR AID BY GPS/INS ULTRA-TIGHTLY COUPLED NAVIGATION

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## Abstract

GPS/INS ultra-tightly coupled navigation system has been widely used for the tracking and positioning of high dynamic aircraft. It has the advantages of high positioning accuracy and low power consumption in high dynamic environment. The algorithm of the airborne PD radar's DPCA (Displaced Phase Center Antenna) technique needs a high accuracy measurement of the aircraft's speed. Hence, applying the GPS/INS ultra-tightly coupled navigation to the measurement of the radar carrier's speed will clearly improve the property of the airborne PD radar. In the paper, a airborne PD radar assisted by GPS/INS ultra-tightly coupled navigation is designed and analyzed and gain the conclusion that the performance of the coupled system is better than the radar with the traditional aircraft velocity measurement method.

**Keywords:** GPS/INS, coupled navigation, DPCA.

## 1 Introduction

GPS/INS coupled navigation has been widely used for the tracking and positioning of high dynamic target. It makes good use of the complementary relationship of the error propagation characteristics between GPS and INS and improves the performance of navigation and positioning. The GPS/INS ultra-tightly coupled system[3][4] not only uses the initial measurement including carrier phase, pseudo range to assist INS, but also directly assists the loop in GPS receiver. So the robust of the system and the accuracy of the measurement will be greatly improved. As the GPS/INS ultra-tightly coupled navigation system is put into the airborne PD radar system, the velocity of the aircraft is measured more accurately which makes the performance of DPCA algorithm substantial increase a lot<sup>[2]</sup>.

However, a perfect data fusion algorithm and the excellent real-time performance are both in need if the data from every transducer is comprehensively utilized. The velocity measurement method though GPS/INS system is to compute the carrier Doppler frequency and the carrier phase'

derivative. As the measurement of Doppler frequency, carrier phase and INS measurement are instantaneous values at some time, the way how to apply the transducers' data and assist the radar system will more or less decide the performance of the coupled system. In this paper, a new design is put out to finish the synchronization of the three different systems and it gives out the method how to get and utilize the GPS/INS information to regulate the error of the radar.

The first section introduces the GPS/INS ultra-tightly coupled navigation's working method. The second section gives out the coupled system design and the combination method. In the final, a simulation is made and the then it analyzes the performance of the new system and gain the conclusion that the new system has better performance compared with the tradition method.

## 2 GPS/INS integrated navigation

Inertial Navigation System (INS) calculates the position and the velocity of the carrier by measuring the carrier's acceleration with the help of inertial devices such as. gyroscope. It has the advantages of being able to complete positioning when the velocity, acceleration of the carrier is big. However, the INS may accumulate the error of the measurement. Once the error is so much that the INS cannot work normally. The GNSS such as GPS has the superiority of high accuracy velocity measurement, fast positioning and low consumption. But it has to receive satellite signal so that the system is subject to the environment and it also cannot work when the carrier is of high dynamic because of the limitation of digital system. Consequently, the GPS/INS integrated Navigation has both the superiorities of the two navigation system and the performance of the GPS/INS integrated Navigation is better than any one of the two navigation systems.

As the GPS/INS ultra-tightly coupled navigation system showed in fig 1, the phase of the signal is sent into both the INS and the GPS loop. The GPS loop will get the phase error between the input signal and the local replica by the discriminator. The loop filter is a low pass filter and the bandwidth of the filter is properly the loop system's

bandwidth. The magnitude of the bandwidth will affect the stability of the loop and the measuring accuracy. The

discriminator's output is filtered and then combined with the INS output to control the NCO. As the INS can eliminate the

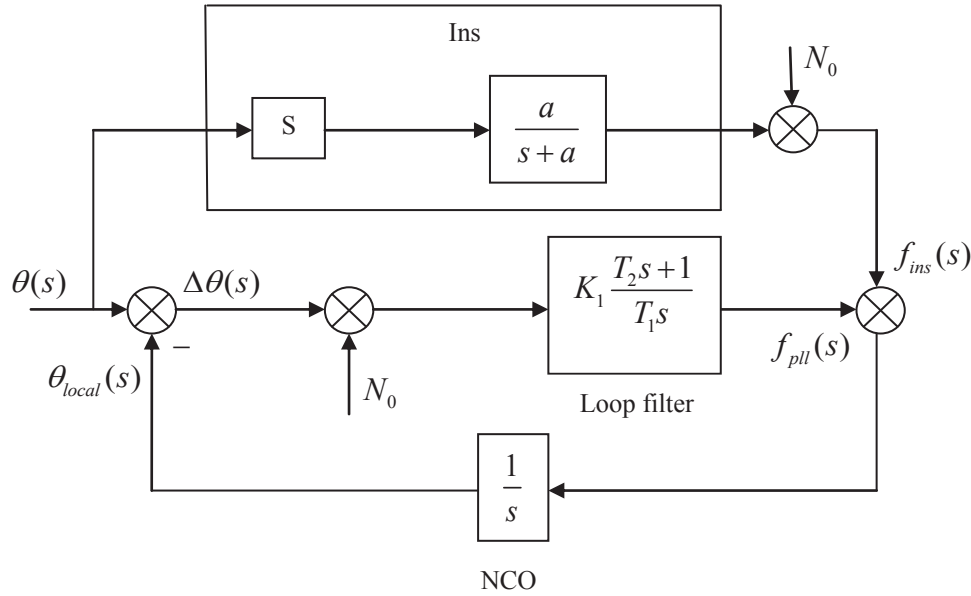


Fig1 the PLL model assisted by INS

dynamic stress error, the second-order carrier loop which is absolutely stable is chosen to be the carrier loop in the GPS/INS ultra-tightly coupled navigation system. The transfer function of the second-order carrier loop is as the equation (2-1)

$$H(s) = \frac{2\xi\omega_n s + \omega_n^2}{s^2 + 2\xi\omega_n s + \omega_n^2} \quad (2-1)$$

In the equation,  $\xi$  is the damping coefficient,  $\omega_n$  is the system characteristic frequencies, the output error function is the equation (2-2)

$$H_e(s) = \frac{s^2}{s^2 + 2\xi\omega_n s + \omega_n^2} \quad (2-2)$$

There is a steady state error in the second-order loop when a signal with acceleration is tracking. And if there is higher step velocity component in the signal the loop tries to track, the loop will more or less lost. However, if the INS system is pulled in the loop, the dynamic stress error will be removed by the INS information. In fact, pulling the INS system into the loop is properly introducing a feedforward to improve the reaction speed of the system. The transfer function of the loop assisted by INS is (2-3)

$$H_a(s) = \frac{2\xi\omega_n s + \omega_n^2 + \frac{\alpha_{IMU}s^2}{s + \alpha_{IMU}}}{s^2 + 2\xi\omega_n s + \omega_n^2} \quad (2-3)$$

When  $\alpha_{IMU} \rightarrow \infty$ , in other words, the INS provide a nearly infinite bandwidth, the transfer function of the whole system is close to 1. At the same time, the carrier loop's bandwidth can be infinitesimal. When it is in the environment of high dynamic, the loop is still able to keep in stably tracking and the accuracy is much higher, the influence of noise is poorer than a single carrier loop. In real environment, the carrier loop bandwidth is decided by the frequency error outside.

### 3 System Design

The performance of the DPCA algorithm depends on the accuracy of the velocity measurement to a certain extent and the GPS/INS ultra-tightly coupled navigation system will successfully completes velocity correction. However, the velocity measurement must be strictly synchronized with the radar pulse so that the local matched filter can be as same as the radar echo.

The way that makes use of the 1pps to synchronize the GPS and INS has been depict in [5][6]. In this paper, a way which uses the satellite 1pps to synchronize the GPS receiver's measurement output and the PD radar is proposed. At the same time, the GPS and INS are still synchronized by the outside 1pps.

When GPS receiver completes positioning, it will correct the local clock error and control the local TIC to run as the satellite's clock. At that time, the receiver 1pps is more or less the same as the satellite 1pps and it can be used as a

frequency standard because of the stability of the satellite clock and the minority of the 1pps error. The system is designed as fig 2

In this design, the GPS loop and INS are synchronized by the local clock and the measurement picking time and the radar pulse are synchronized by the satellite 1pps. Although the system will be easier if the three parts are synchronized by the same local clock, the local clock drift will not be able to diminish and the accuracy cannot be really high. However, if the GPS/INS position in a high frequency and the clock error is corrected a little every time. The difference between the local corrective 1pps and the satellite 1pps is less than  $10^{-1}$  ns[6]. As the satellite clock is an atomic clock and the satellite clock error information can be got in data, the clock is stable and accurate. Moreover, to make use of the satellite 1pps can synchronize measurement and the radar pulse better and the velocity information will help the DPCA better. It is sure that the velocity measuring result is the right time the pulse is sent and received so that the result of the measurement is just the velocity of the aircraft during the radar pulse.

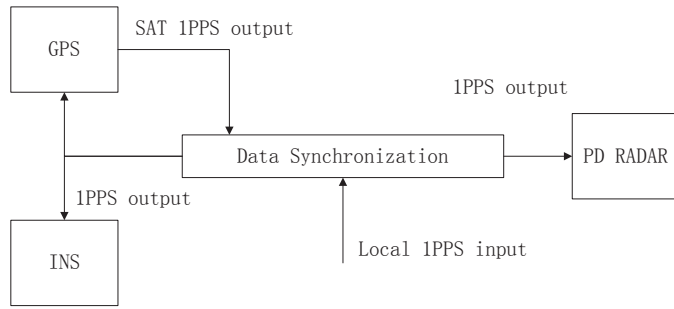


Fig2 the design of the data synchronization system

If the measurement especially the carrier phase is picked at edge of the radar pulse, the real-time velocity information may be gotten for any pulse's echo and make use of it to create the local matched filter so that the performance will greatly improved. The only clock error of the system is the 1pps correcting error and it will affect the precision of the velocity the radar use to make the matched filter. In the following, the DPCA performance is analyzed with the 1pps error and the GPS/INS velocity measurement.

#### 4. System Analysis

As the designed system is usually used on high dynamic carrier on earth such as airplane and missile, it is suitable to assume an airborne model to describe and analyze the integrate system. In the airborne model, it is properly to assume that there are two transmitting antennas at two points of  $S_1$  and  $S_2$  along the flying direction. The airborne radar sends the pulse at the point of  $S_2$  and the two point of the antenna received the signal at the same time and the receiving signal model is as (4-1)

$$\begin{aligned} R_1(t) &= R_1 + Vt + \frac{1}{2} At^2 \\ R_2(t) &= R_2 + Vt + \frac{1}{2} At^2 \end{aligned} \quad (4-1)$$

In the equation,  $R_1$  and  $R_2$  is the distance vector, the acceleration vector and the velocity vector of the radar relative to the target on the ground at the time respectively. And the instantaneous frequency can be expressed as the equations (4-2)

$$\begin{aligned} f_1 &= -(f_{d_1} + f_{r_1} \cdot t) \\ f_2 &= -(f_{d_2} + f_{r_2} \cdot t) \end{aligned} \quad (4-2)$$

It is clearly that the echo signal Azimuth frequency has the property of liner frequency modulation. The azimuth expressions of the two antenna apertures after pulse compression in range direction is as the equation (4-3)

$$\begin{aligned} S_1(t) &= G \cdot e^{\{-j2\pi\{f_{d_2} \cdot t + \frac{1}{2} f_{r_2} \cdot t^2\}\}} \\ S_2(t) &= G \cdot e^{\{-j2\pi\{\frac{D^2 - 2X_0 L}{2\lambda R}\}\}} \cdot e^{\{-j2\pi\{f_{d_1} \cdot t + \frac{1}{2} f_{r_1} \cdot t^2\}\}} \cdot e^{\{j2\pi\frac{V_a L}{\lambda R} \cdot t\}} \end{aligned} \quad (4-3)$$

In the equation,  $D$  is the antenna spacing,  $X_0$  is the azimuth location of the point target on the ground,  $V_a$  is the velocity along the flight path. The first part of the second equation of (4-3) is the distance error which is brought by the antenna azimuth position error. The third part of the equation is the centre Doppler error.

However, as the aircraft velocity vector may deviate away from the flight path or affected by wind, the velocity of the aircraft is more than a component along the flight path. There are also another two velocity components horizontal perpendicular to the flight path and longitudinal vertical to the flight path respectively. Besides that, there is velocity measurement error. If any one of the components is neglected or the velocity measurement error is intolerable, the performance of the radar will be poorly deteriorated.

The GPS/INS ultra-tightly coupled navigation system can successfully solve the problem. The velocity measurement formulation is as the equation (4-4)

$$-v \cdot I^{(n)} + \delta f_u = (\dot{\rho}^{(n)} - v^{(n)} \cdot I^{(n)} + \delta f^{(n)}) - \varepsilon_{\rho}^{(n)} \quad (4-4)$$

Where  $v$  is the velocity vector to be tested. There are three components in the vector including  $x$ ,  $y$  and  $z$  which represent the three direction in the geocentric coordinate system.  $\delta f_u$  is satellite clock error.  $I^{(n)}$  is the current location.  $\rho^{(n)}$  is the carrier phase error between two adjacent epochs. When the integrate navigation receiver receives the signal form more than four satellite, more than four formulation can be gotten

and the velocity can be calculated on the approach of least squares.

In this way, the 3D velocity can be gotten and the velocity error can be corrected by the measurement because of the high accuracy of the measurement. If  $V_h$  is the velocity horizontal perpendicular to the flight path,  $V_a$  is the velocity longitudinal vertical to the flight path,  $V_l$  is the velocity along the flight path after coordinate transformation, the liner frequency model slope is as equation (4-5), the frequency error is as (4-6)

$$f_{fm_1} \approx f_{fm_2} = \frac{2[(V_a - V_x)^2 + (V_l - V_y)^2 + (V_h - V_z)^2]}{\lambda R} \quad (4-5)$$

$$f_{d_2} = \frac{2V_y}{\lambda} \quad (4-6)$$

In considerate of the 1pps error, there is a additional error in the local matched filer so that the frequency module is  $f_{r1} = f_{mf1} + \Delta f_{1pps}$ ,  $f_{r2} = f_{mf2} + \Delta f_{1pps}$ . In the equation,  $f_{mf(n=1,2)}$  is the real estimating frequency modulation slope and  $\Delta f_{1pps}$  is the 1pps error which can be treated as a frequency modulation error.

When the condition of DPCA is fulfilled, the receiving signal model is as the equation (4-7)

$$S_1(t) = G \cdot e^{\{-j2\pi[f_{d1} \cdot t + \frac{1}{2}f_{r1} \cdot t^2]\}} \quad (4-7)$$

$$S_1(t + \Delta t) = G \cdot e^{\{-j2\pi[f_{d1} \cdot t + \frac{1}{2}f_{r1} \cdot t^2]\}} \cdot e^{\{-j\frac{2\pi}{\lambda} \frac{L}{\sqrt{V_a^2 + V_h^2}} V_y\}}$$

Where  $\Delta t = L/2V_a$ ,  $V_y$  is the radial velocity of the target,  $(V_a^2 + V_h^2)^{1/2}$  is the horizontal velocity of the plane. If considering the delay produced by the antenna spacing and the frequency module component brought by the carrier velocity, the frequency expression of the receiving signal module is as (4-8)

$$S_1(f) = G \cdot \sum_{n=0}^{N-1} \{e^{\{-j2\pi f_{d2} \cdot nT\}} \cdot e^{\{-j\pi[f_{r2} - \frac{2V^2}{\lambda R_0}](nT)^2\}} \cdot e^{\{-j2\pi f_{nT}\}}\}$$

$$S_2(f) = G \cdot e^{\{-j\frac{2\pi}{\lambda} \frac{L}{\sqrt{V_a^2 + V_h^2} + \Delta V_{1PPS}} V_y\}} \cdot \sum_{n=0}^{N-1} \{e^{\{-j2\pi f_{d2} \cdot nT\}} \cdot e^{\{-j\pi[f_{r2} - \frac{2V^2}{\lambda R_0}](nT)^2\}} \cdot e^{\{-j2\pi f_{nT}\}}\} \quad (4-8)$$

Where  $V$  is the velocity of the aircraft. The expression of  $V$  is (4-9)

$$V = \sqrt{V_a^2 + V_l^2 + V_h^2 + \Delta V_{1PPS}} \quad (4-9)$$

The result of the cancellation between the two antenna aperture after simplified is (4-10)

$$|S_{12}(f)| = G \sqrt{\frac{\lambda R}{2V^2}} \left| 2 \sin \left( \frac{\pi}{\lambda} \frac{V_y D}{\sqrt{V_a^2 + V_h^2} + \Delta V_{1PPS}} \right) \right| \quad (4-10)$$

Another problem is the PRF (pulse repeating frequency) of the radar is high, so the velocity measuring frequency has to be matched with the PRF. As the aircraft fly in stratosphere, the signal to noise ratio is high, the loop control frequency can be high and the integration accumulation length can be shorten. In this way, for the reason that the GPS receiver is aid by INS, the carrier loop has no need to adapt to dynamic and the bandwidth is small which makes the loop tracks with low jitter and the carrier phase accuracy is high. Hence, if the carrier phase information is picked up twice at the time of sending and receiving the signal, the velocity measurement result can be used to aid the radar and the accuracy can reach 0.03m/s without 1pps error.

When the carrier velocity error is  $\Delta V_R$  which contain 1pps error and velocity error,  $V_R = (V_a^2 + V_h^2)^{1/2}$  and  $\omega$  is the Doppler frequency, the detecting statistic can be written as equation(4-11)

$$AMP = G \sqrt{\frac{\lambda R}{2V^2}} \left| 2 \sin \left( \frac{\pi}{\lambda} \frac{V_y D}{V_R (1 + \Delta V_R / V_R)} - \frac{d\Delta V_R / V_R}{2V_R (1 + \Delta V_R / V_R)} \omega \right) \right| \quad (4-11)$$

The expression of the clutter suppression error is as equation (4-12)

$$AMP_{error} = G \sqrt{\frac{\lambda R}{2V^2}} \left| 2 \sin \left( \frac{D\Delta V_R / V_R}{2V_R (1 + \Delta V_R / V_R)} \omega \right) \right| \quad (4-12)$$

So we can define the impact factor of the system without consideration the inner carrier influence after clutter suppression is as equation (4-13)

$$IF = \left| \frac{\sin \left( \frac{\pi}{\lambda} \frac{V_y D}{V_R (1 + \Delta V_R / V_R)} - \frac{D\Delta V_R / V_R}{2V_R (1 + \Delta V_R / V_R)} \omega \right)}{\sin \left( \frac{D\Delta V_R / V_R}{2V_R (1 + \Delta V_R / V_R)} \omega \right)} \right| \quad (4-13)$$

The equation can be expressed as fig 3. We can get the conclusion that the impact factor will be close to 0 dB if the velocity measurement error exceeds 1 m/s which is 30dB lower than the max point. So when the velocity measurement accuracy reaches 0.1m/s, the performance of clutter suppression is better. Consequently, the GPS/INS integrate system is made to aid the radar system.

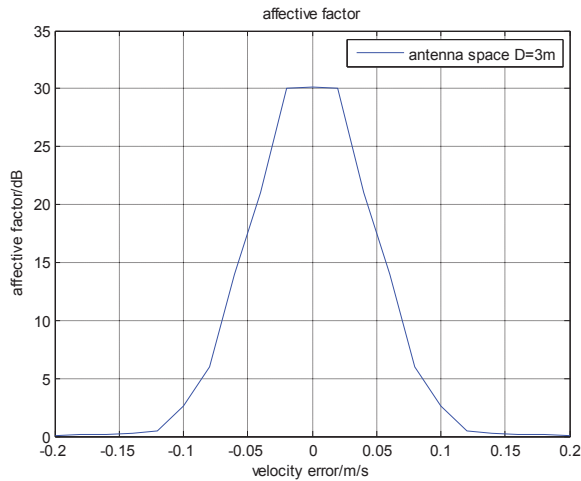


Fig3 the relationship between the velocity error and impact factor

## 5. Simulation

Here is the simulation result that the velocity measurement accuracy without 1pps error in the fig4. In the fig, the aircraft's accelerator is 100g, the jerks of the aircraft is 30m/s<sup>3</sup>. The velocity is gotten from the carrier phase. In the simulation, the loop is aided by INS with a clock error and the carrier loop bandwidth is 0.1Hz.

As the measuring of the velocity depends on the carrier phase and the carrier phase is gotten at the same time the pulse is sent and received, so the aiding is at the right time if there is no error in 1PPS. So the aiding is only affected by the 1pps error. Simulation result of the real 1pps correction error and the influence of the error on the velocity measurement are illustrated in fig 5 and fig 6 respectively. As the fig5, the 1pps is corrected at the time the receiver complete positioning and the local clock will run away from the standard clock in one direction so that the 1pps is deviate in the direction. As consequence, the error is increasing as the 1pps error as illustrated in fig 6. The result indicates that when the 1pps is corrected, there is a big error in the result and the velocity error comes from the fixed 1pps error at any other time. So if the pulse is sent out at times expect the time 1pp is corrected, the 1pps error can be less than several 10<sup>-9</sup>s so that the velocity error brought by it is only 0.002m/s. So it will help the radar system a better performance.

The simulation result of the affective factor is in fig7. In the simulation, only system clock synchronization error and the 1pps error is considered. The horizontal velocity is Gaussian distribution with mean and RMS being 0 and 2m/s respectively. The Longitudinal velocity is Gaussian distribution with mean and RMS being 0 and 2m/s respectively. It is the result of the affective factor of the

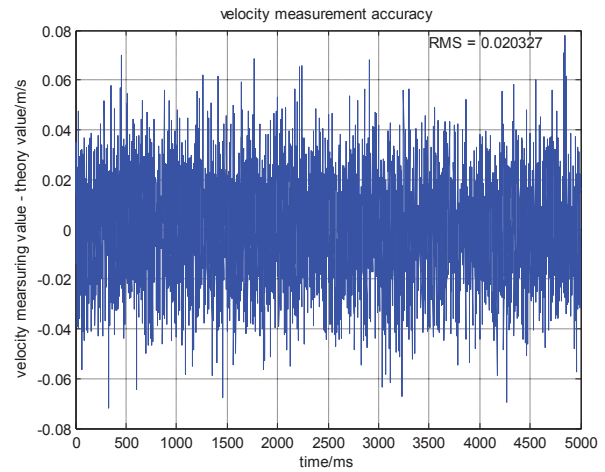


Fig4 velocity measurement accuracy in the simulation

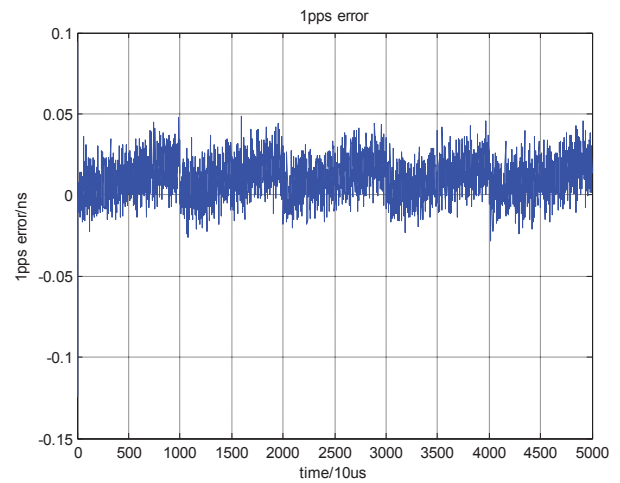


Fig5 velocity measurement accuracy in the simulation

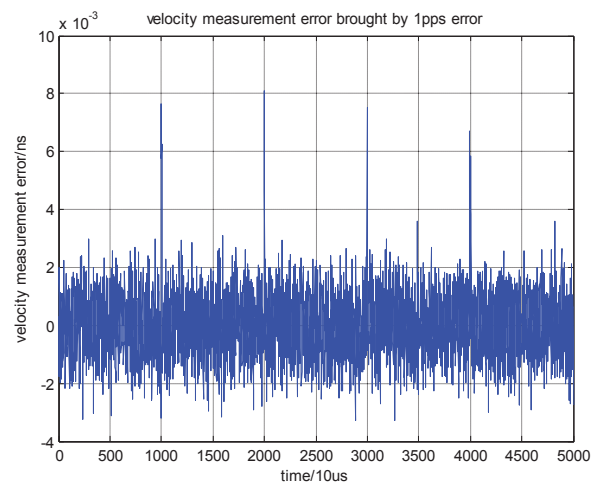


Fig6 velocity measurement error brought by 1pps error in the simulation



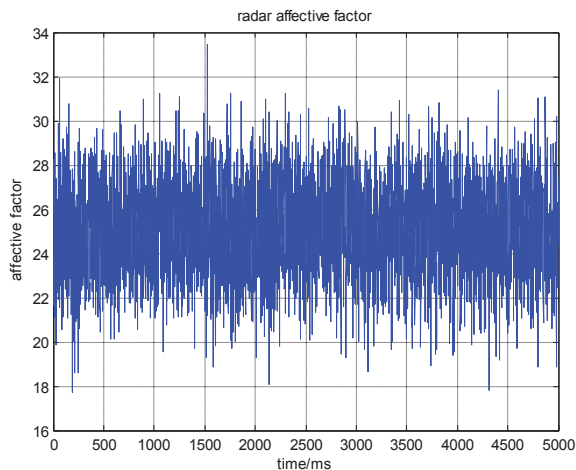


Fig7 the simulation result of the whole system

system when the 1pps error is  $10^{-1}$  ns which brings a 0.003m/s in velocity error.

We can get the conclusion from the simulation that the affective factor is more or less 30dB when the radar is aid by GPS/INS in velocity measurement for the reason that the velocity measurement accurate is 0.1m/s.

## 6 conclusion

In this paper, a novel airborne PD radar system aid by GPS ultra-tightly coupled navigation is proposed and analyzed. The use of the 1pps of the navigation system excellently solved the problem of clock synchronization. The performance of the radar is improved for the reason that the GPS/INS ultra-tightly coupled navigation can provide a 3D velocity measurement and the accuracy is high. Consequently it gains the conclusion that the integrate system has a great advantages in clutter suppression and target detection.

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