ROME Example:

Portfolio Allocation under the Entropic Satisficing Measure

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1 Introduction

The example is adopted from Brown and Sim [2] and Brown, De Giorgi and Sim [3]. The entropic (prospective) satisficing measure or EPSM is defined as

$$\rho(X) \triangleq \sup\left\{k \ : \ \frac{1}{k}\ln\mathbb{E}\left[\exp(-Xk)\right] \leq 0\right\}.$$

Note that EPSM is reciprocal to the riskiness index introduced by Aumann and Serrano [1]. In this example, we obtain the portfolio weights that optimize the EPSM objective using ROME. In particular, we illustrate the use of `expcone` function provided in ROME (see ROME manual), which is useful in modeling convex constraints involving exponential, geometric or entropic functions.

2 Model Description

Consider n assets with independent random returns V_i, $i = 1, \ldots, n$ that are discretely distributed as follows:

$$\mathbb{P}\left\{V_i = v\right\} = \begin{cases} p_i & \text{if } v = \underline{v}_i \\ 1 - p_i & \text{if } v = \overline{v}_i \end{cases}$$

We then consider the following asset allocation problem

$$\begin{aligned} \sup \quad & \rho\left(\sum_{i=1}^{n} w_i V_i - \tau\right) \\ \text{s.t.} \quad & \sum_{i=1}^{n} w_i = 1 \\ & w_i \geq 0, \qquad i = 1, \ldots, n. \end{aligned}$$

where τ is a given target return such that $\tau < \max_i \mathbb{E}[V_i]$. We solve the following optimization problem

$$\begin{aligned}
\max \quad & k \\
\text{s.t.} \quad & \sum_{i=1}^{n} \frac{1}{k} \ln(p_i \exp(-w_i k \underline{v}_i) + q_i \exp(-w_i k \overline{v}_i)) \leq -\tau \\
& \sum_{i=1}^{n} w_i = 1 \\
& k > 0, w_i \geq 0, \qquad i = 1, \ldots, n,
\end{aligned}$$

where $q_i = 1 - p_i$. By replacing k with $1/a$, we can convert to the following convex optimization problem

$$\begin{aligned}
\min \quad & a \\
\text{s.t.} \quad & \sum_{i=1}^{n} a \ln(p_i \exp(-w_i \underline{v}_i / a) + q_i \exp(-w_i \overline{v}_i / a)) \leq -\tau \\
& \sum_{i=1}^{n} w_i = 1 \\
& a > 0, w_i \geq 0, \qquad i = 1, \ldots, n.
\end{aligned}$$

Finally, using the `expcone` function in ROME, we can transform the problem to

$$\begin{aligned}
\min \quad & a \\
\text{s.t.} \quad & a \geq p_i \underbrace{a \exp((-z_i - w_i \underline{v}_i)/a)}_{\texttt{expcone(-z(i)-vlo(i)*w(i),a)}} + q_i a \exp((-z_i - w_i \overline{v}_i)/a)) \quad i = 1, \ldots, n \\
& \sum_{i=1}^{n} z_i \leq -\tau \\
& \sum_{i=1}^{n} w_i = 1 \\
& a > 0, w_i \geq 0, z_i \text{ free} \qquad i = 1, \ldots, n.
\end{aligned}$$

References

[1] Aumann, R., and R. Serrano (2008): An Economic Index of Riskiness, *Journal of Political Economy*, 116(5), 810-836.

[2] Brown, D. B. and M. Sim (2009): Satisficing Measures for Analysis of Risky Positions, *Management Science*, 55(1), 71-84.

[3] Brown, D. B., De Giorgi, E. and M. Sim (2009): A Satisficing Alternative to Prospect Theory, *Working Paper*, Available at SSRN: http://ssrn.com/abstract=1406399.