

Robust Choice Model

Sep 4 2015

$|N|$ = number of purchases made by ONE customer
 $|J|$ = number of product choices
dimension of β = number of attributes
dimension of α = $|J|$

Let $\hat{j}_n = \sum_{j \in J} j \times y_{nj}$, where $y_{nj} \in \{0, 1\}$ denotes the observed choice. We further denote $\mathbf{Z}^j = (z_{1j}, z_{2j}, \dots, z_{|N|j})'$ be a vector in $\mathbb{R}^{|N|}$ and matrix $\mathbf{Z} = (\mathbf{Z}^j)_{j \in J}$.

$$\max_{\alpha, \beta} \min_{\mathbf{Z} \in \mathcal{Z}(\Gamma)} \sum_{n \in N} \sum_{j \in J} z_{nj} \ln \frac{\exp(\alpha_j + x'_{nj}\beta)}{\sum_{\ell \in J} \exp(\alpha_\ell + x'_{n\ell}\beta)} \quad (1)$$

where

$$\mathcal{Z}(\Gamma) = \left\{ \mathbf{Z} \in \mathbb{R}^{|N| \times |J|} \left| \begin{array}{l} \sum_{j \in J} z_{nj} = 1, \quad n \in N, \\ \sum_{n \in N} z_{nj} \geq |N| - \Gamma, \\ z_{nj} \geq 0, \quad n \in N, j \in J \end{array} \right. \right\}.$$

The above problem is equivalent to

$$\max_{\alpha, \beta} G(\alpha, \beta). \quad (2)$$

and

$$\begin{aligned} G(\alpha, \beta) = & \min \sum_{n \in N} \sum_{j \in J} z_{nj} \ln \frac{\exp(\alpha_j + x'_{nj}\beta)}{\sum_{\ell \in J} \exp(\alpha_\ell + x'_{n\ell}\beta)} \\ \text{s.t. } & \mathbf{Z} \in \mathcal{Z}(\Gamma). \end{aligned} \quad (3)$$

Problem (3) can be written as

$$\begin{aligned} \min & \sum_{n \in N} \sum_{j \in J} z_{nj} \left(-\ln \sum_{\ell \in J} \exp(\alpha_\ell - \alpha_j + (x_{n\ell} - x_{nj})'\beta) \right) \\ \text{s.t. } & \sum_{j \in J} z_{nj} = 1, \quad n \in N, \\ & \sum_{n \in N} z_{nj} \geq |N| - \Gamma, \\ & z_{nj} \geq 0, \quad n \in N, j \in J. \end{aligned} \quad (4)$$

The dual of problem (4) is

$$\begin{aligned}
& \max \quad \sum_{n \in N} a_n + b(|N| - \Gamma) \\
& \text{s.t.} \quad a_n + b \leq -\ln \sum_{\ell \in J} \exp(\alpha_\ell - \alpha_{\hat{j}_n} + (x_{n\ell} - x_{n\hat{j}_n})' \boldsymbol{\beta}), \quad n \in N, \\
& \quad a_n \leq -\ln \sum_{\ell \in J} \exp(\alpha_\ell - \alpha_j + (x_{n\ell} - x_{nj})' \boldsymbol{\beta}), \quad j \in J \setminus \{\hat{j}_n\}, \quad n \in N, \\
& \quad b \geq 0.
\end{aligned} \tag{5}$$