

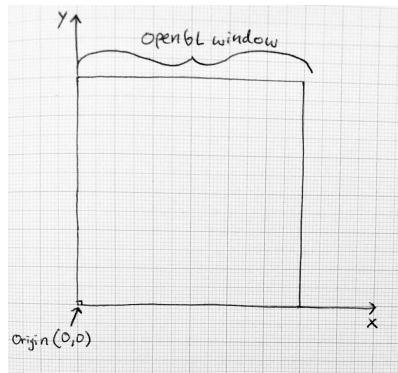
Derivation and OpenGL Implementation of Bresenham's Algorithm for Rendering Circles

Prepared for: NYU Tandon School of Engineering CS-GY 6533: Interactive Computer Graphics

Instructions:

- Run the executable file 'opengl_bresenham_algorithm' and follow prompts.
- To view implementation / source code: `opengl_bresenham_algorithm.cpp`

Derivation:



$$F(x, y) = x^2 + y^2 - r^2 = 0$$

If $F(M) < 0$: M inside circle, choose N
 If $F(M) \geq 0$: M outside circle, choose NW

$$D = F(M)$$

$$D_{start} = F(x_i - \frac{1}{2}, y_i + 1)$$

$$= (x_i - \frac{1}{2})^2 + (y_i + 1)^2 - r^2$$

$$= F(x_i - \frac{1}{2}, y_i)$$

$$= (x_i - \frac{1}{2})^2 + y_i^2 - r^2$$

$$= r^2 - r + \frac{1}{4} + r^2$$

$$= \frac{5}{4} - r \approx 1 - r$$

CASE 1: $D < 0$, choose N

$$\Delta D = D_{new} - D_{old} \quad M' = (x_i - \frac{1}{2}, y_i + 2)$$

$$= F(M') - F(M)$$

$$= F(x_i - \frac{1}{2}, y_i + 2) - F(x_i - \frac{1}{2}, y_i + 1)$$

$$= (x_i - \frac{1}{2})^2 + (y_i + 2)^2 - (x_i - \frac{1}{2})^2 - (y_i + 1)^2$$

$$= x_i^2 - x_i + \frac{1}{4} + y_i^2 + 4y_i + 4 - x_i^2 - x_i + \frac{1}{4} - y_i^2 - 2y_i - 1$$

$$= 2y_i + 3$$

CASE 2: $D \geq 0$, choose NW

$$\Delta D = D_{new} - D_{old} \quad M'' = (x_i - \frac{3}{2}, y_i + 2)$$

$$= F(M'') - F(M)$$

$$= F(x_i - \frac{3}{2}, y_i + 2) - F(x_i - \frac{1}{2}, y_i + 1)$$

$$= (x_i - \frac{3}{2})^2 + (y_i + 2)^2 - (x_i - \frac{1}{2})^2 - (y_i + 1)^2$$

$$= x_i^2 - 3x_i + \frac{9}{4} + y_i^2 + 4y_i + 4 - x_i^2 - x_i + \frac{1}{4} - y_i^2 - 2y_i - 1$$

$$= -2x_i + 2y_i + 5$$

$$D_{start} = 1 - r$$

If $D < 0$:

- choose N

$$D_{i+1} = D_i + 2y_i + 3$$

If $D \geq 0$:

- choose NW

$$D_{i+1} = D_i - 2x_i + 2y_i + 5$$