Joint Institute of Engineering

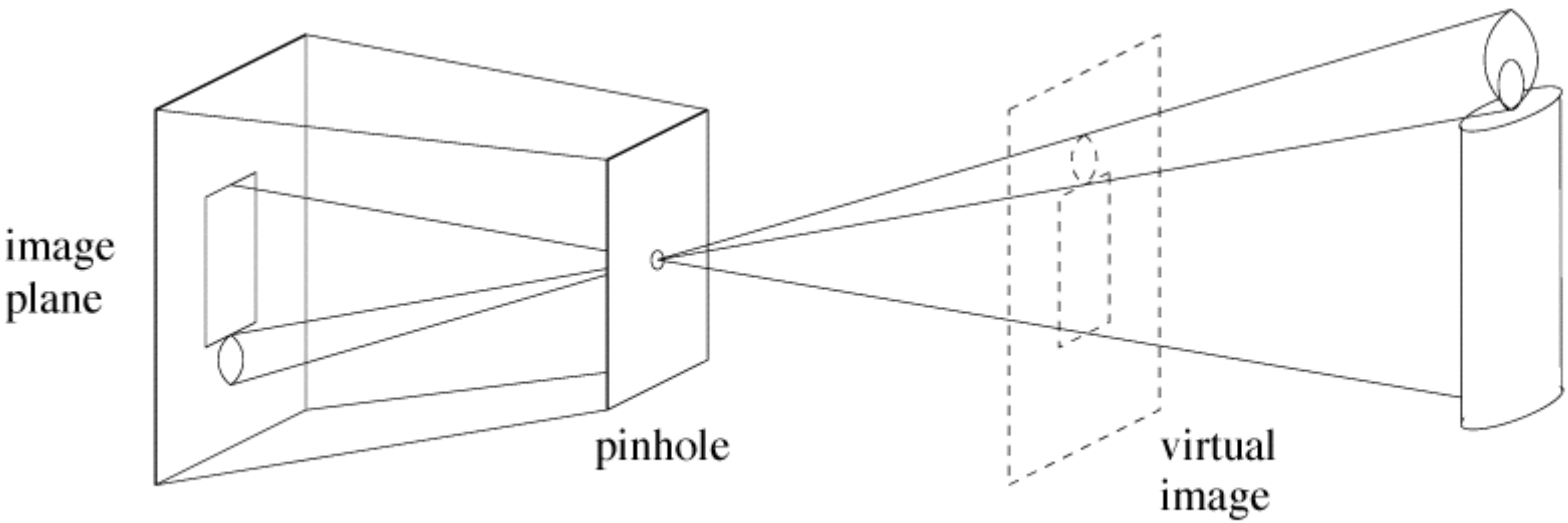
SUN YAT-SEN UNIVERSITY

Carnegie Mellon University

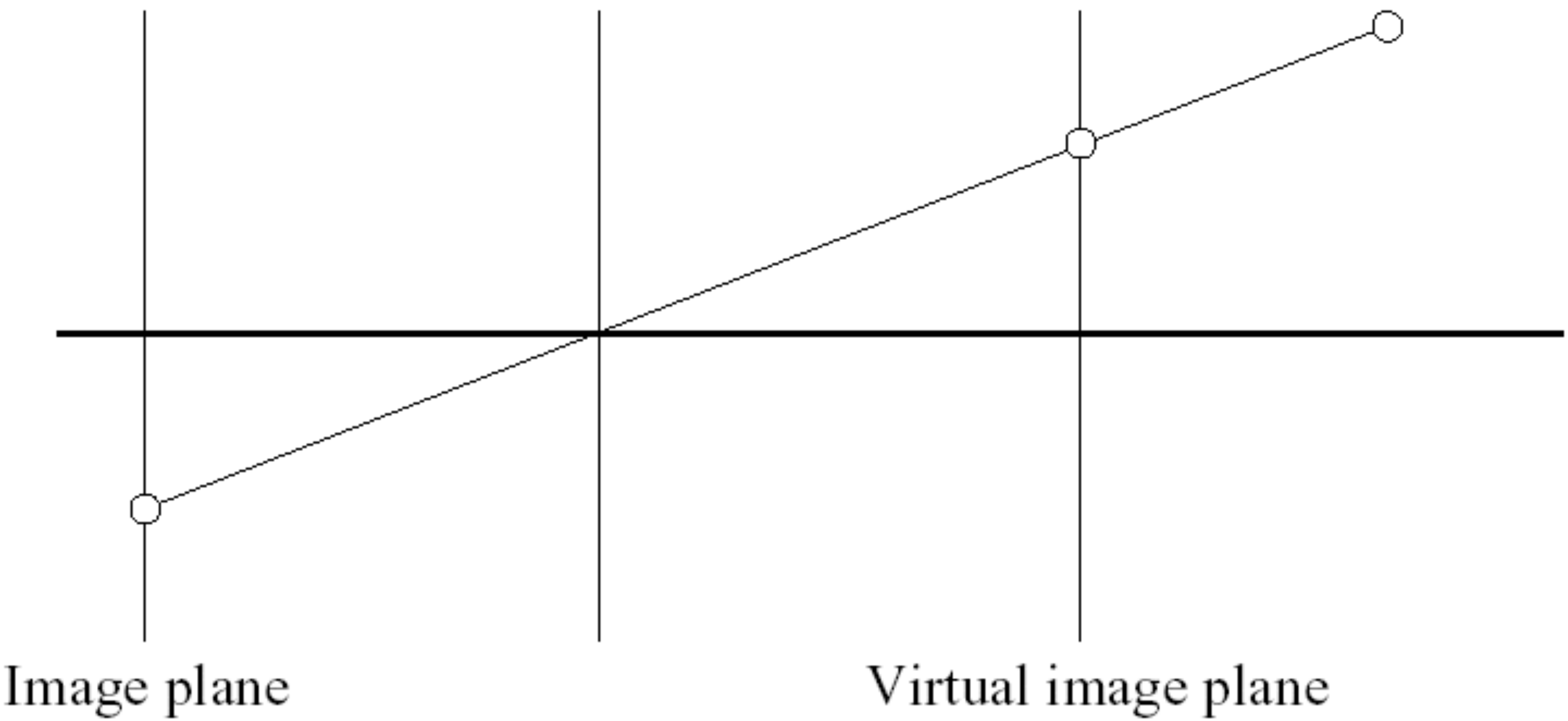
Cameras and Camera Geometry

Forsyth&Ponce: Chap. 1,2,3
Szeliski: Chap. 2.1

Pinhole Cameras



Equivalent Model with Virtual Image Plane

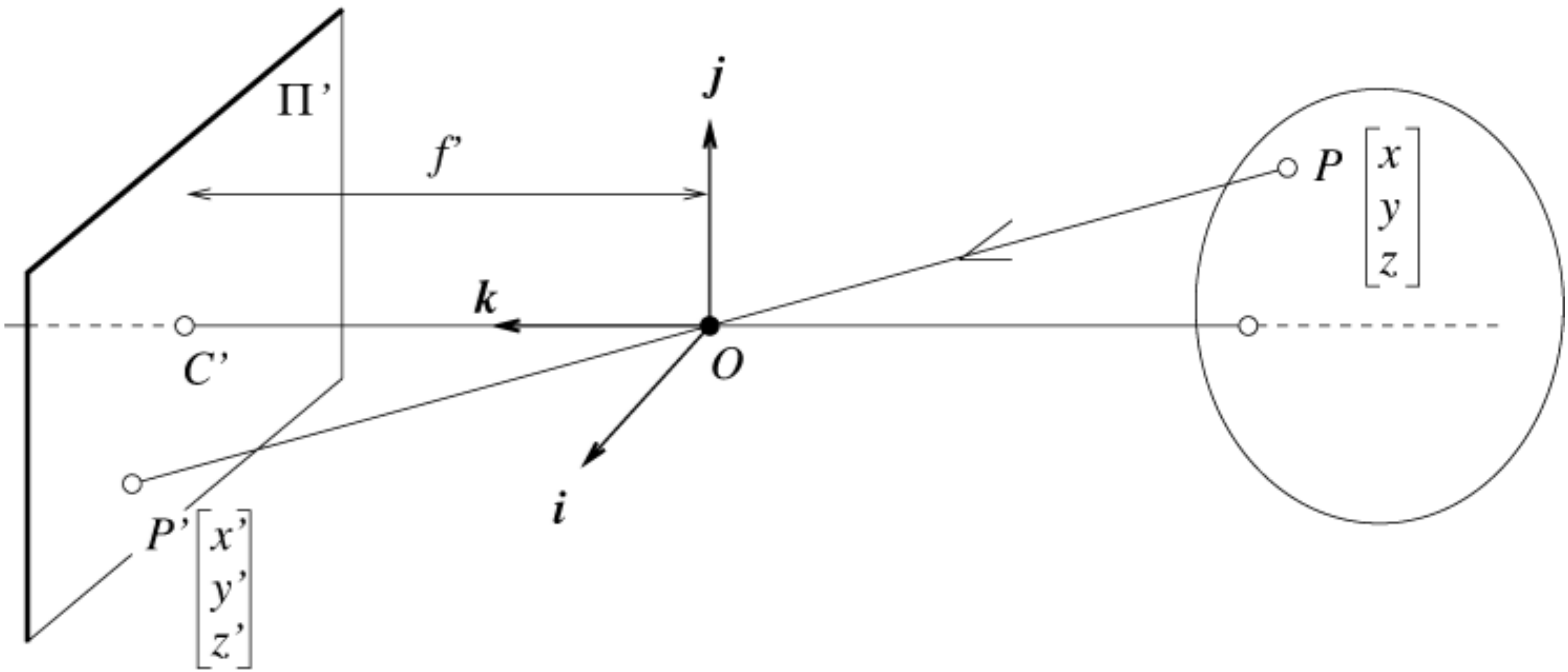


Basic Geometric Properties

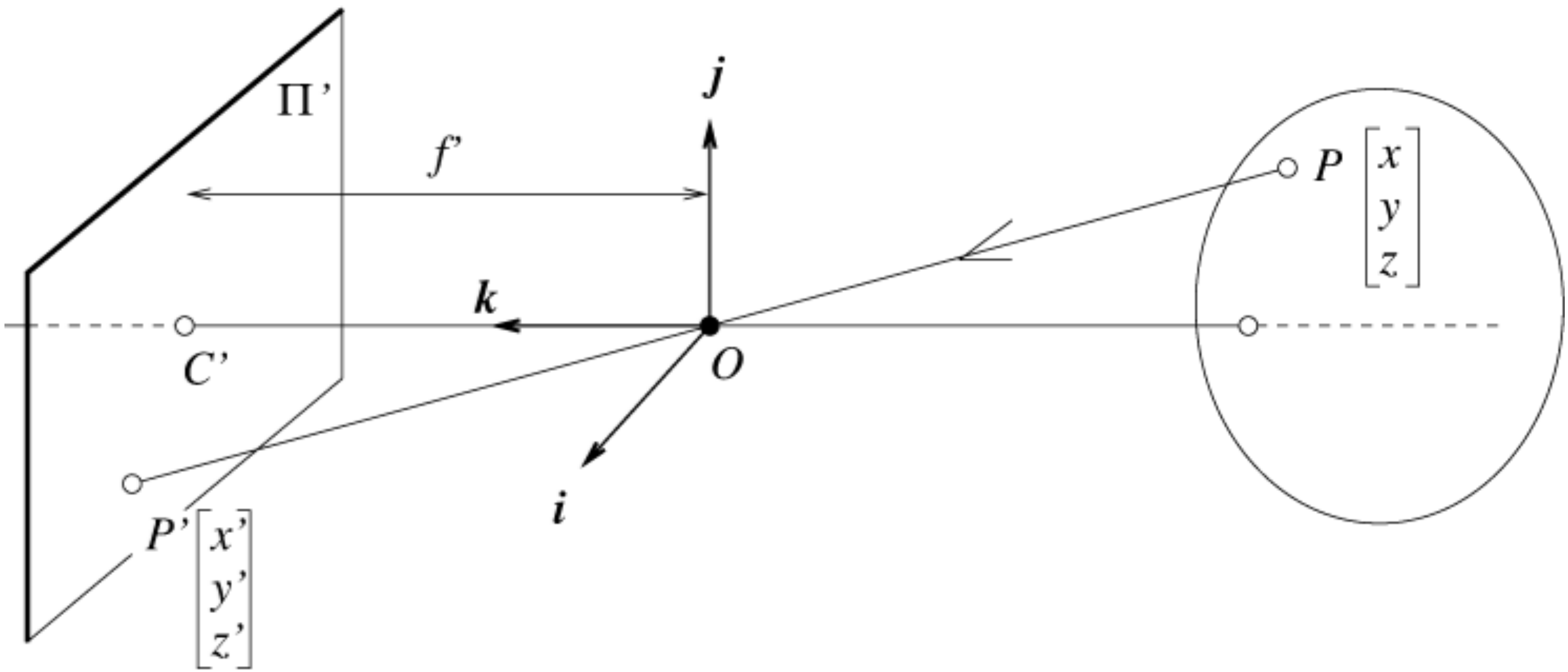
- Distant objects are smaller
- Lines project to lines
- The projection of parallel lines meet at a single vanishing point
- Vanishing points of coplanar sets of lines are collinear, form the vanishing line of the plane (horizon)

Road Scene Example

Perspective Projection

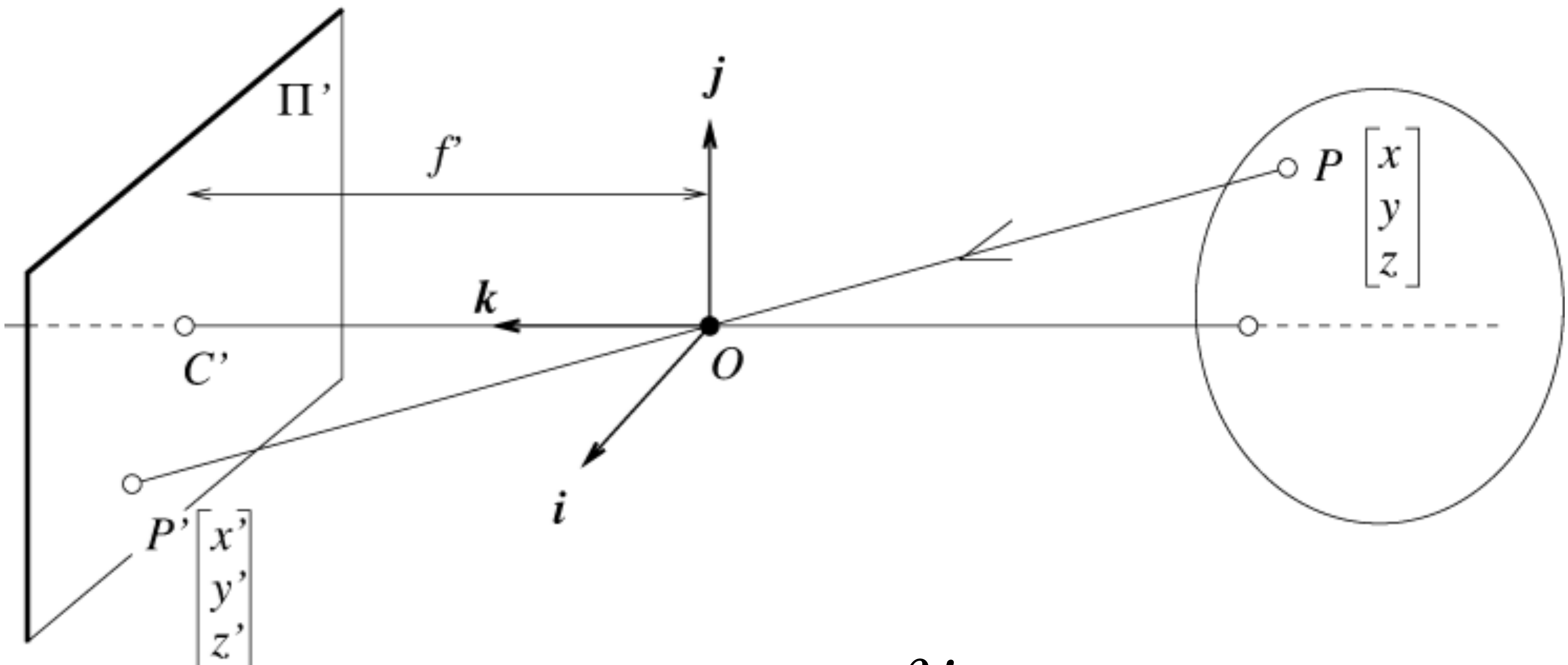


Perspective Projection



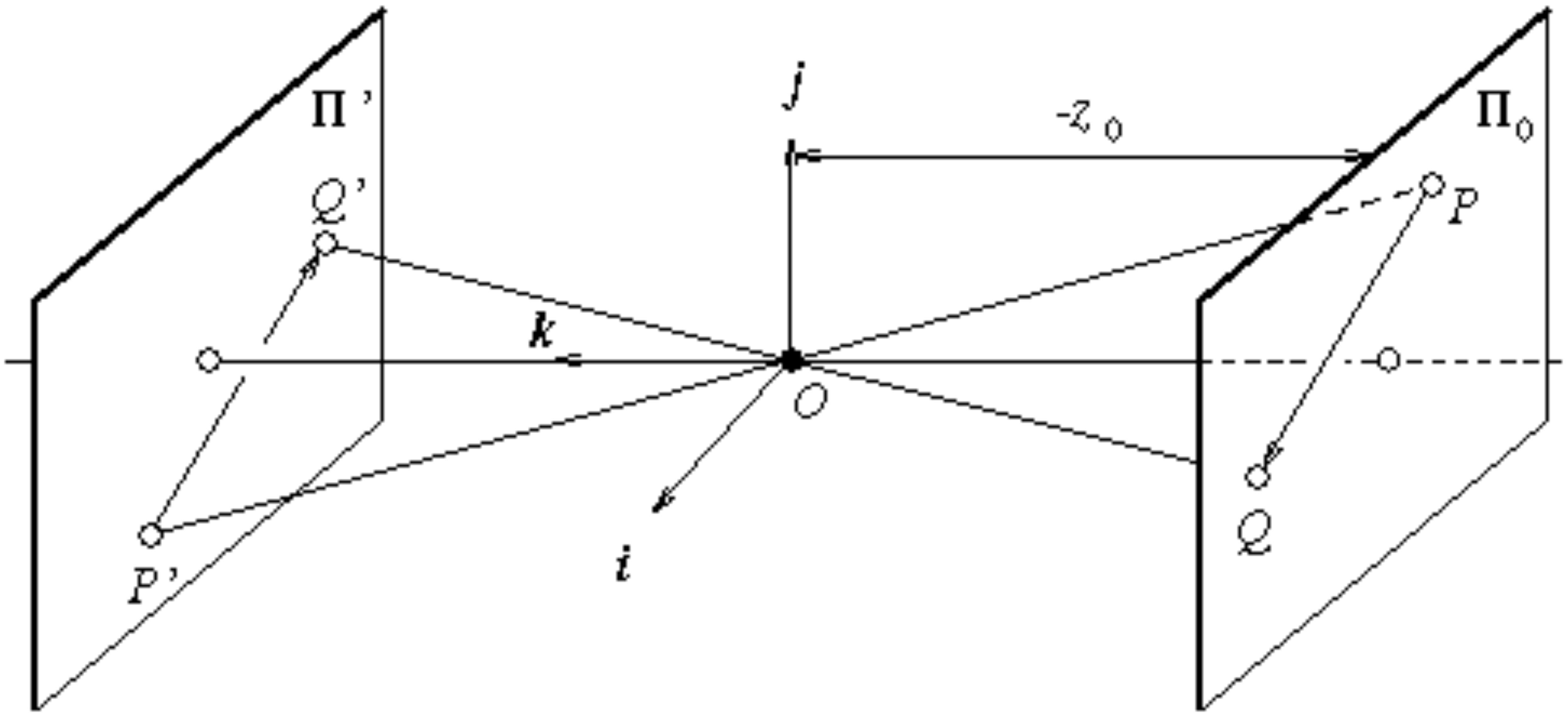
What is y' in terms of x,y,z,f?

Perspective Projection



$$y' = f\frac{y}{z}$$

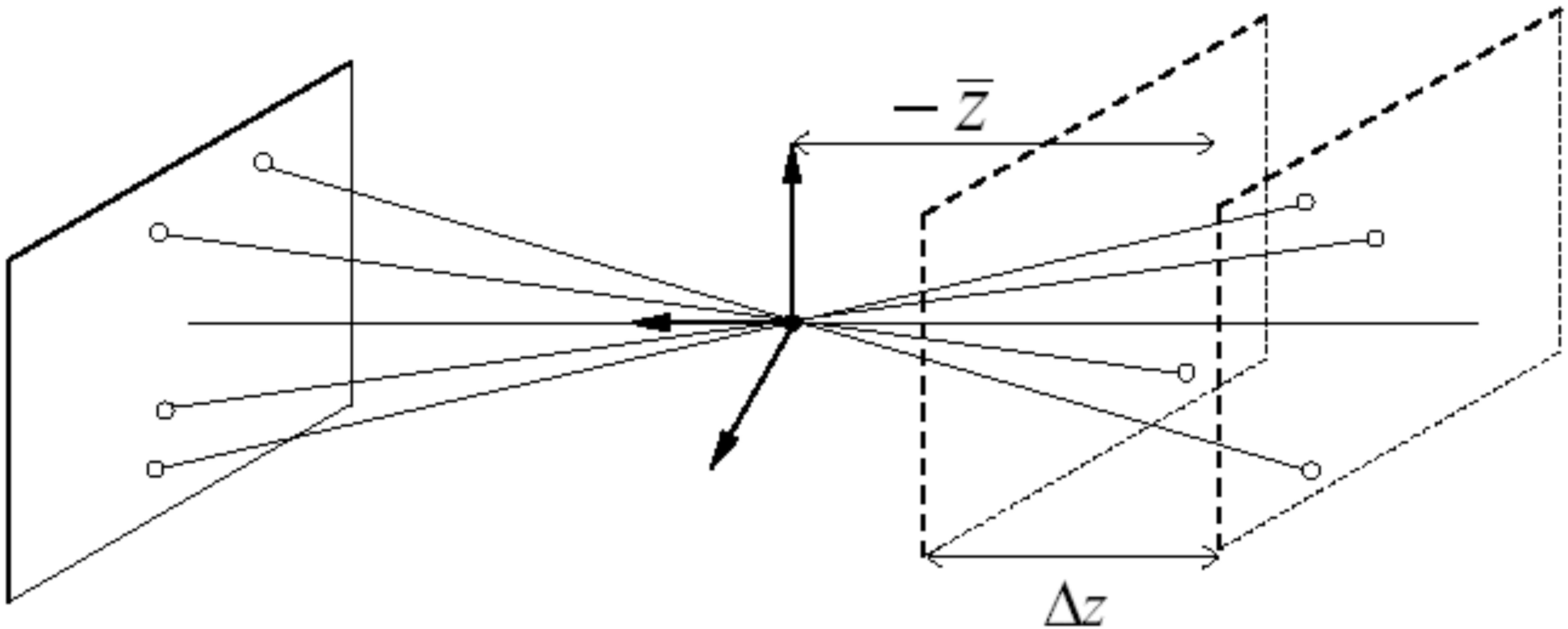
Special Case: Weak Perspective (Affine Projection)



$$x' \approx -mx \qquad y' \approx -my \qquad m = -\frac{f'}{z_o}$$

If scene points are in a plane, projections are simply magnified by m

Special Case: Weak Perspective (Affine Projection)



If $\Delta z << -\bar{z}$: $\begin{matrix} x' \approx -mx \\ y' \approx -my \end{matrix} \quad m = -\dfrac{f'}{\bar{z}}$

Justified if scene depth is small relative to average distance from camera

Strong Perspective

- Angles are NOT preserved
- The projections of parallel lines intersect at one point

Strong vs Weak Perspective

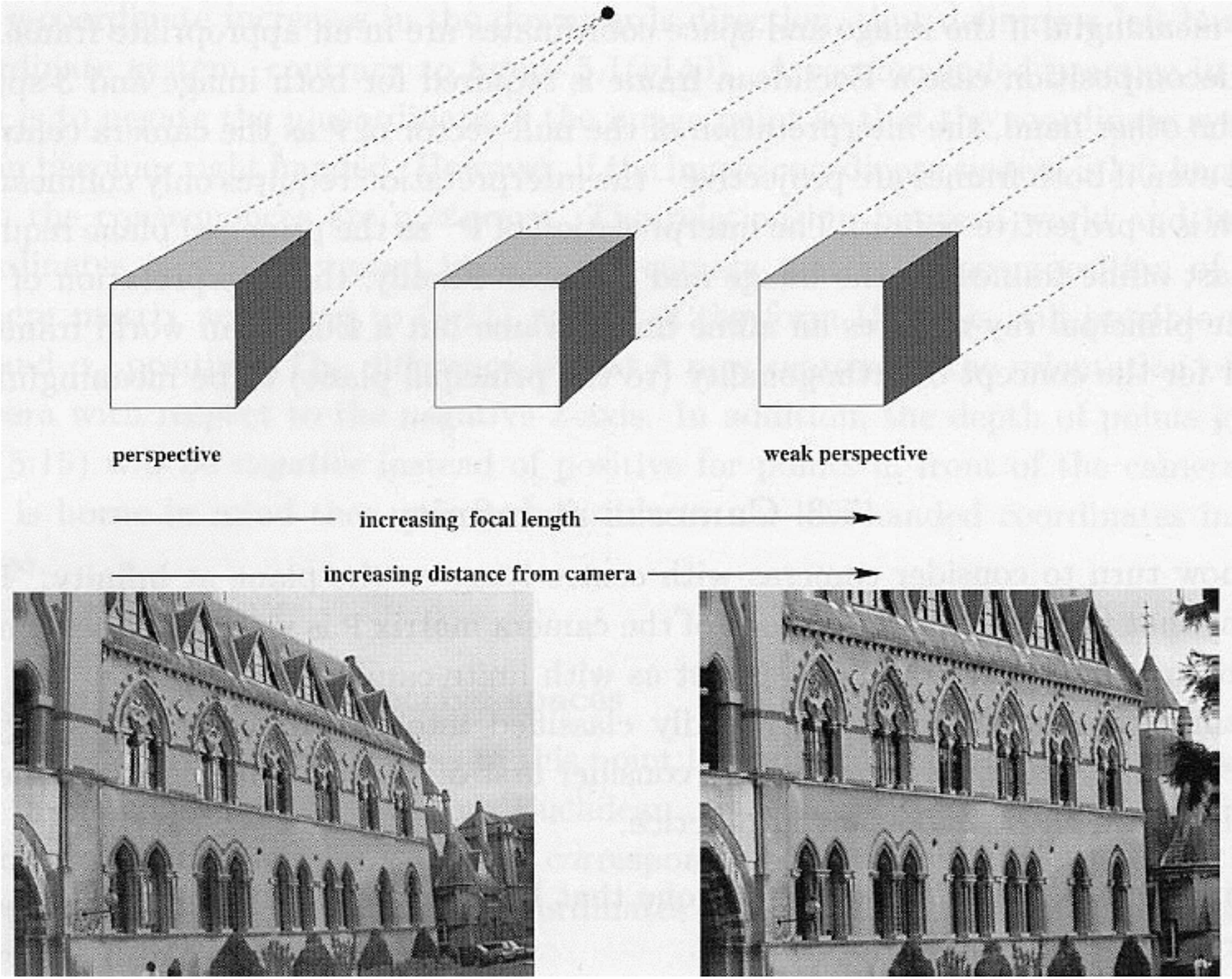


image credit: Zisserman & Hartley

Strong vs Weak Perspective

Strong Perspective:

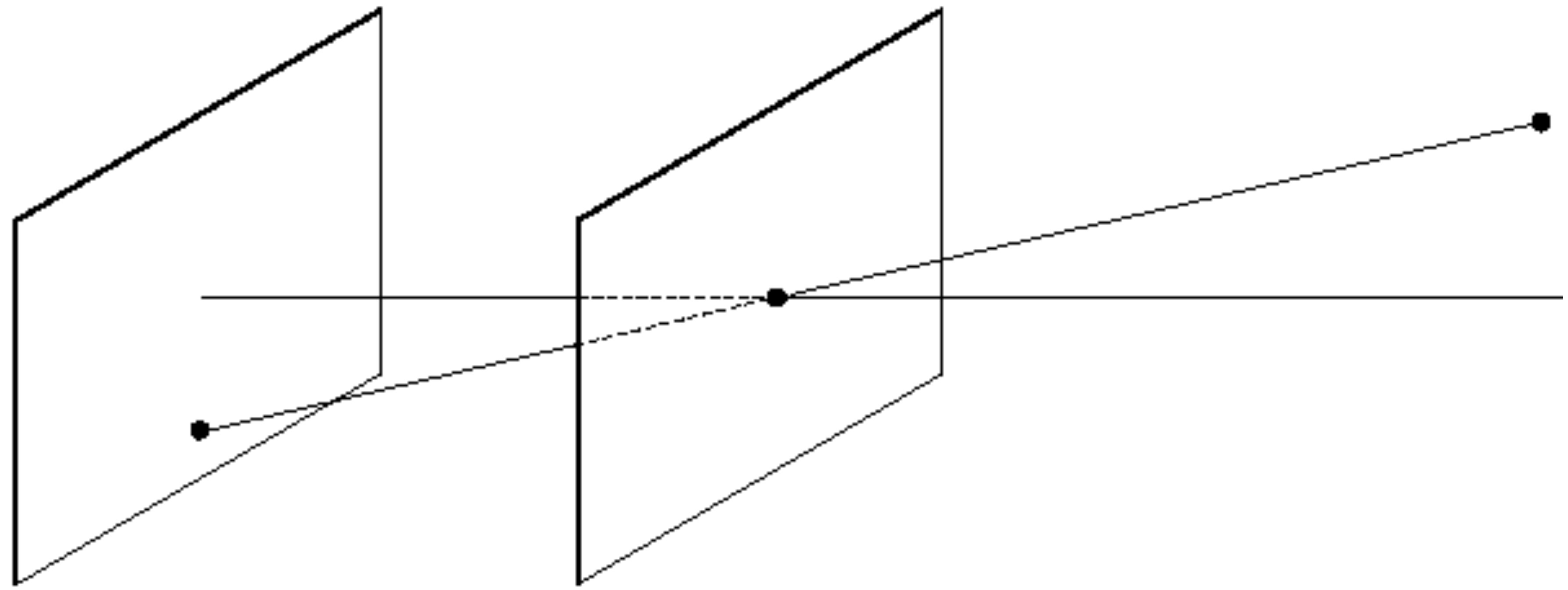
- Angles are NOT preserved
- The projections of parallel lines intersect at one point

Weak Perspective:

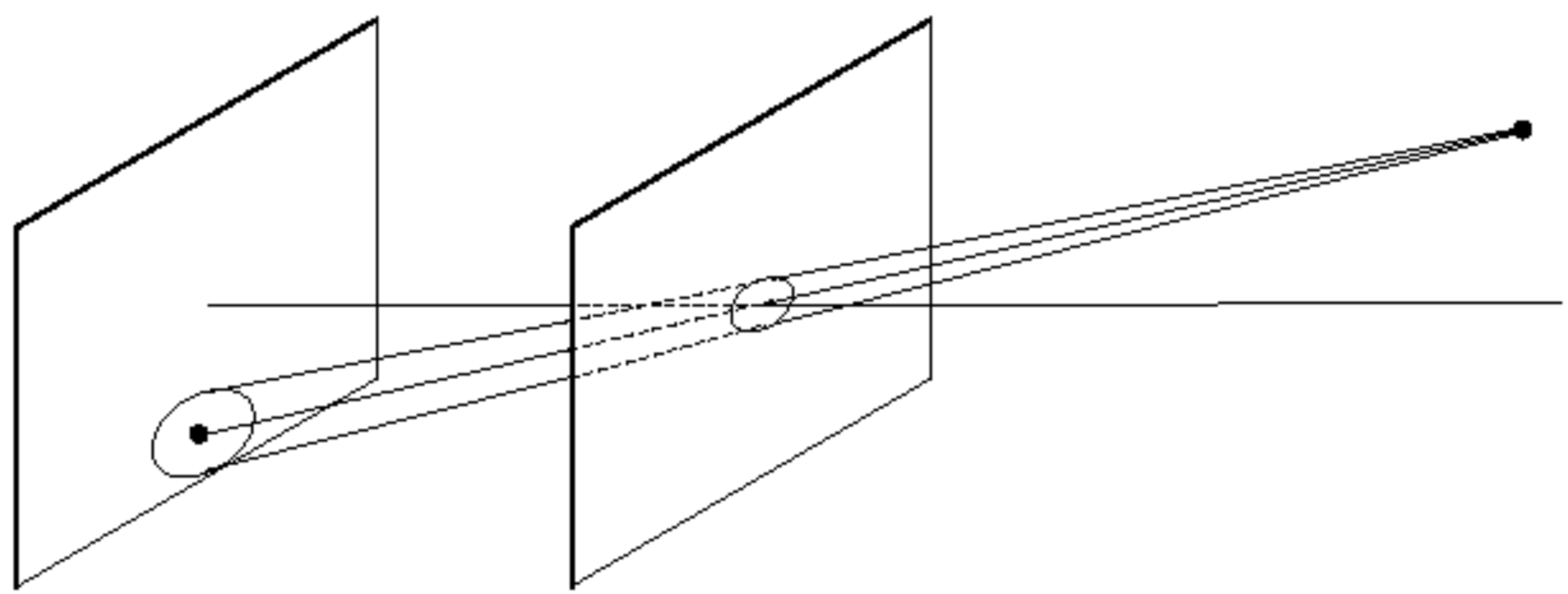
- Angles are better preserved
- The projections of parallel lines are (almost) parallel

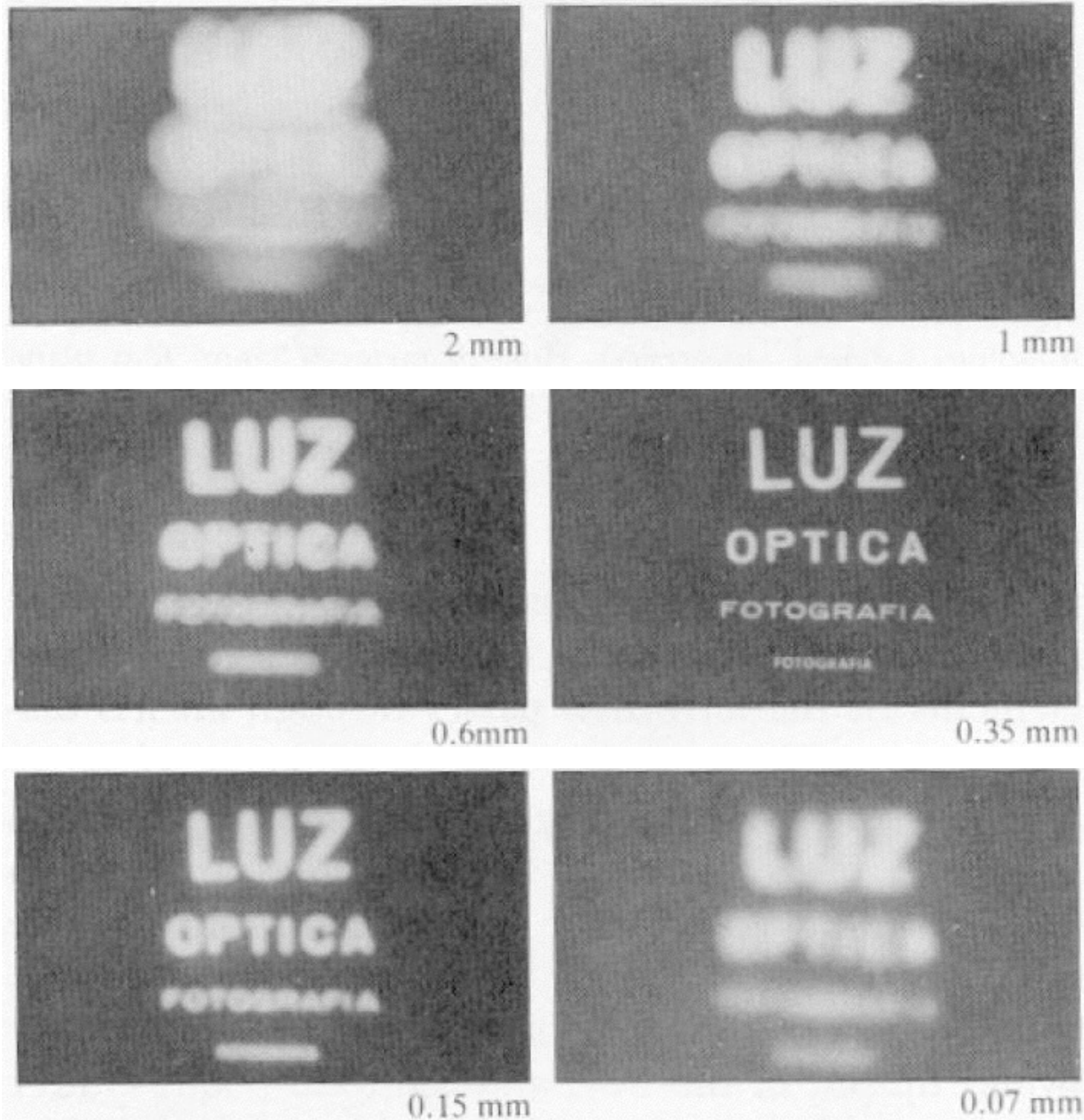
Limitations of the Pinhole Model

Ideal pinhole:
Single scene point
generates single image
but:
Diffraction
Low light level

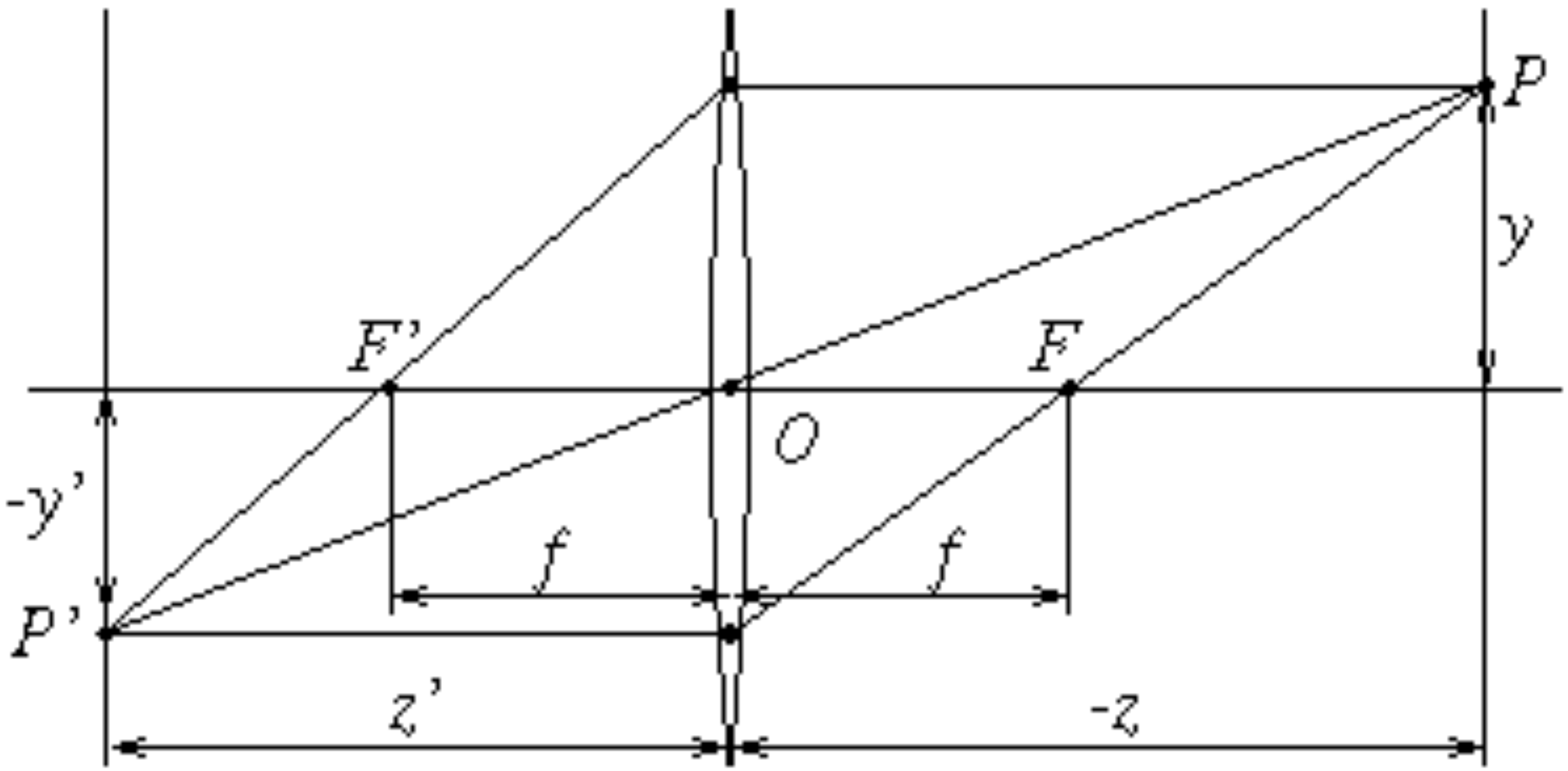


Finite-size pinhole:
Single scene point
generates extended
image.
Resulting image is
blurry




2 mm
1 mm
LUZ
OPTICA
0.6mm
LUZ
OPTICA
FOTOGRAFIA
0.35 mm
LUZ
OPTICA
0.15 mm
0.07 mm

Thin Lens Model



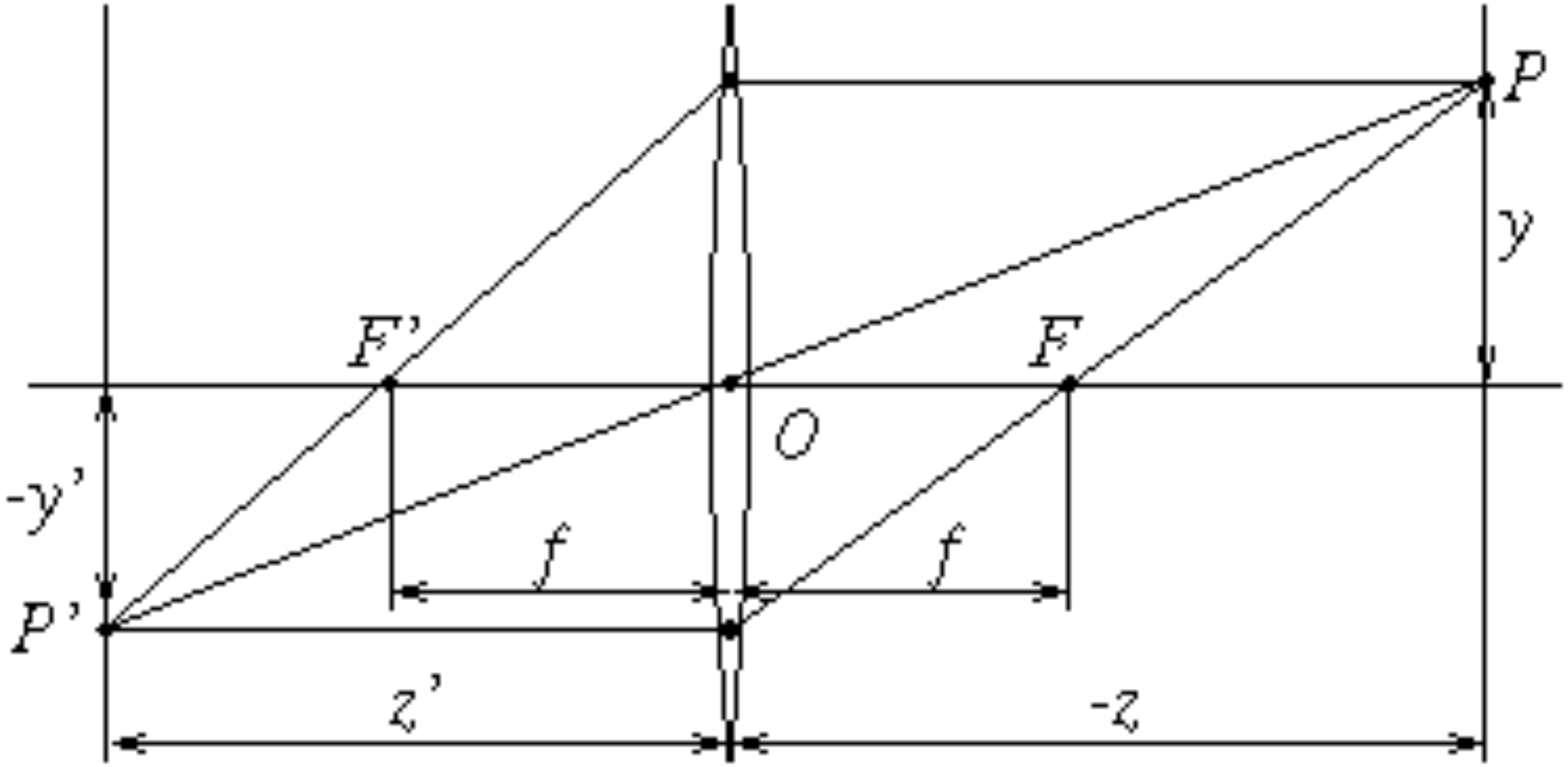
All rays emanating from ***P*** converge to a single point ***P'***

$$\frac{1}{z'} - \frac{1}{z} = \frac{1}{f}$$

Points at infinity are focused on plane $z' = f$

Ideal because: infinite aperture
infinite field of view
infinitely small distance between surfaces

Thin Lens Model



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infinite field of view
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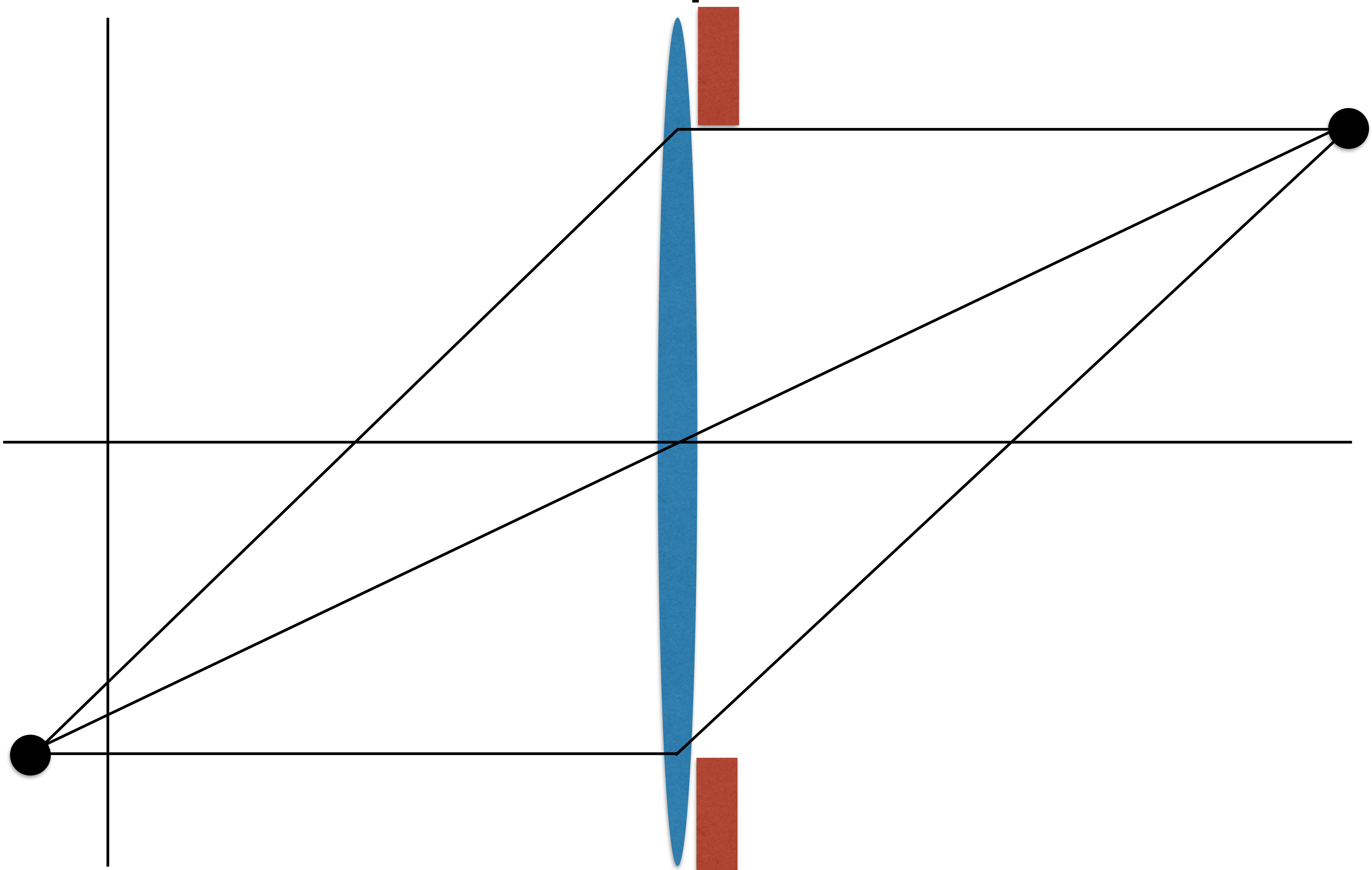
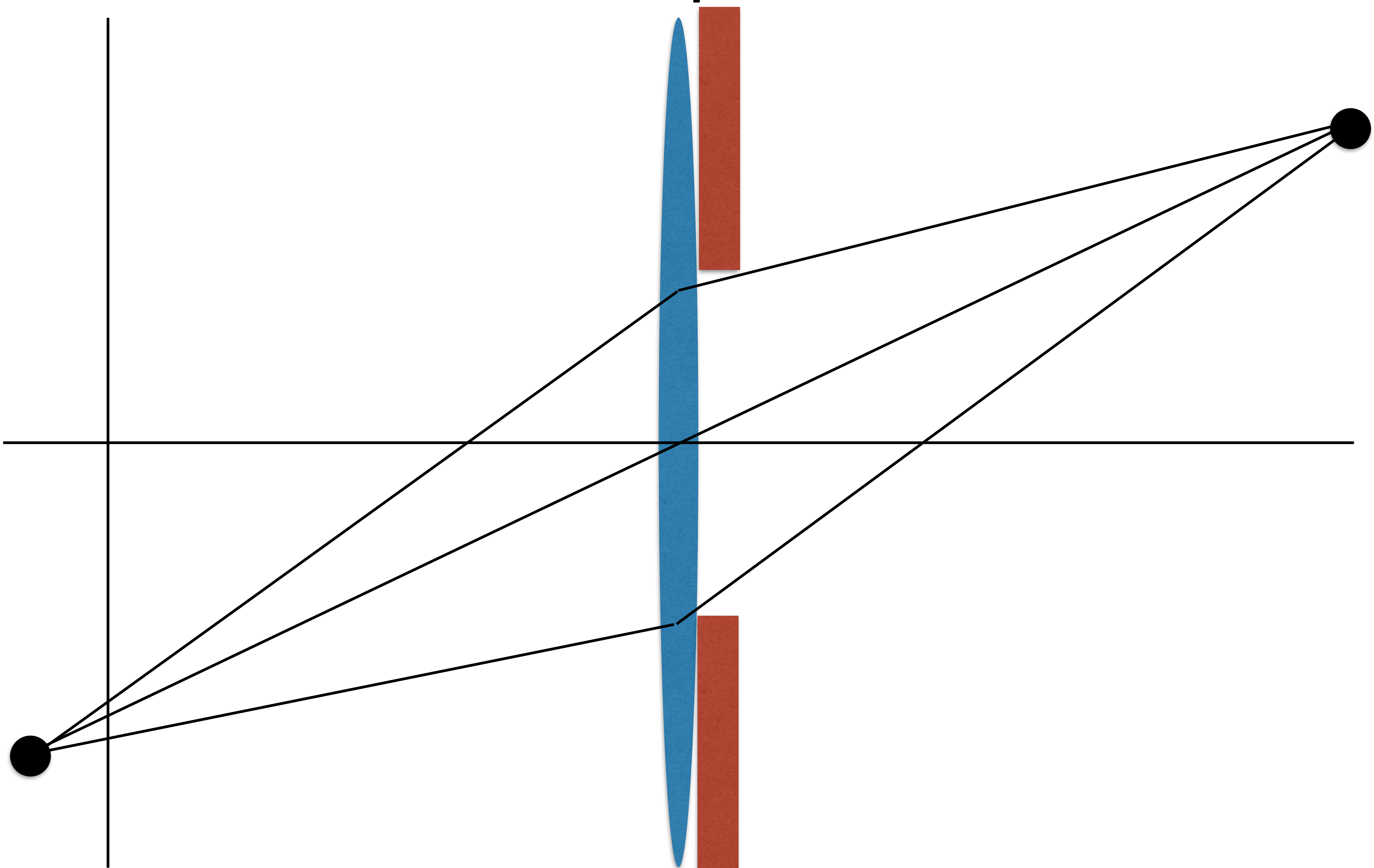
Can you convince yourself of this geometrically and mathematically?

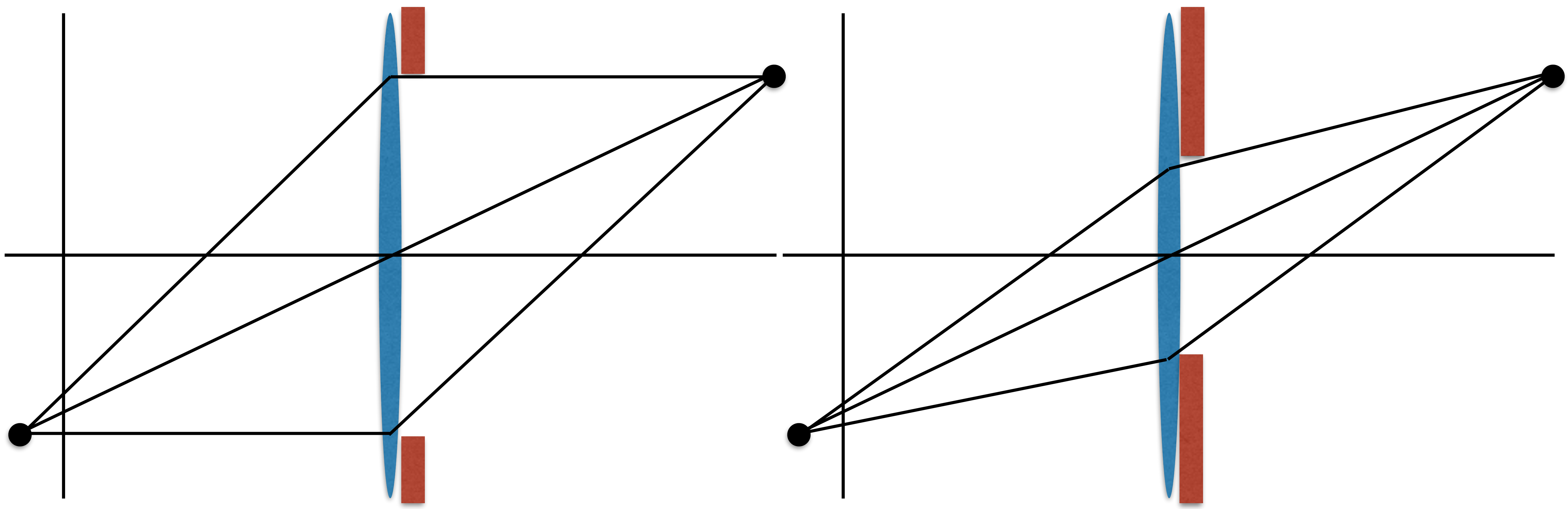
Finite Aperture



Finite Aperture



Finite Aperture



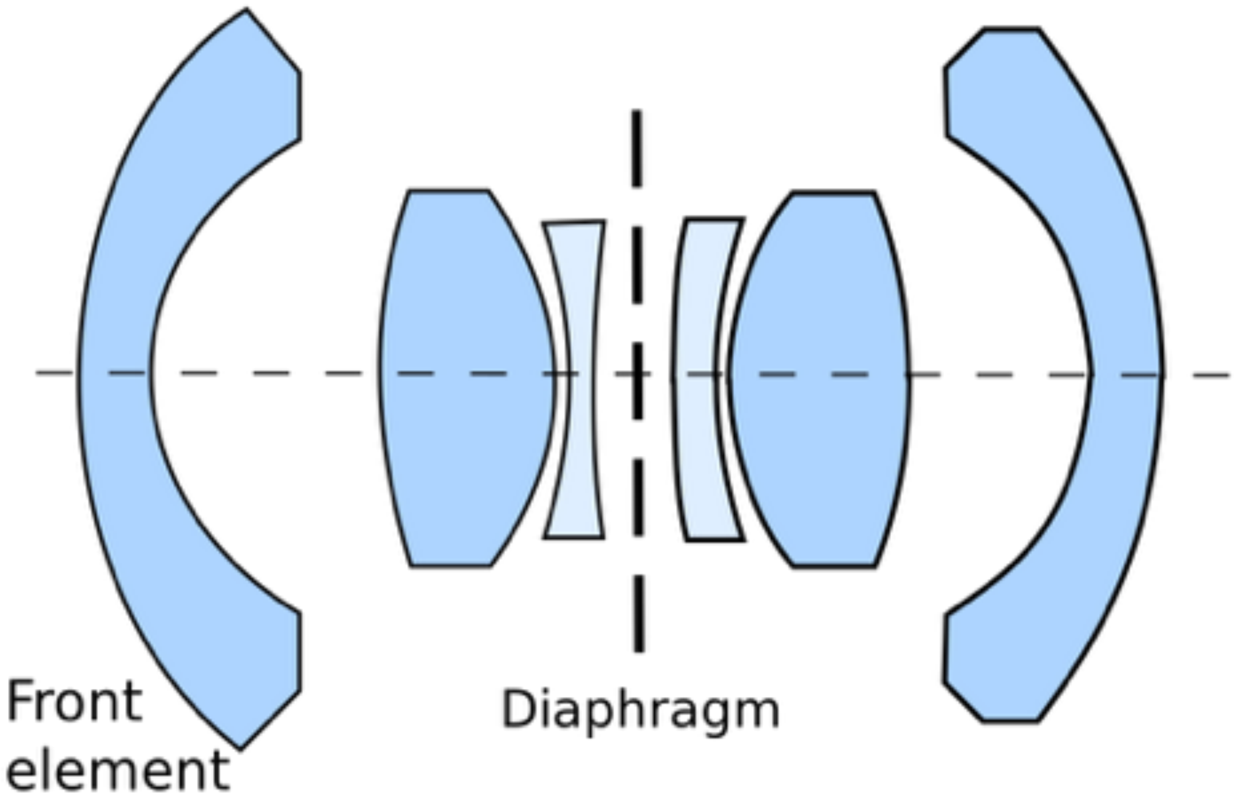
Ideal case: Only the points on one plane are in perfect focus

Finite aperture: points within a region of depth D (depth of field) are in focus.

For a given f, the larger the aperture, the smaller D

Depth of field controlled by f/a

Meanwhile: Real Lenses



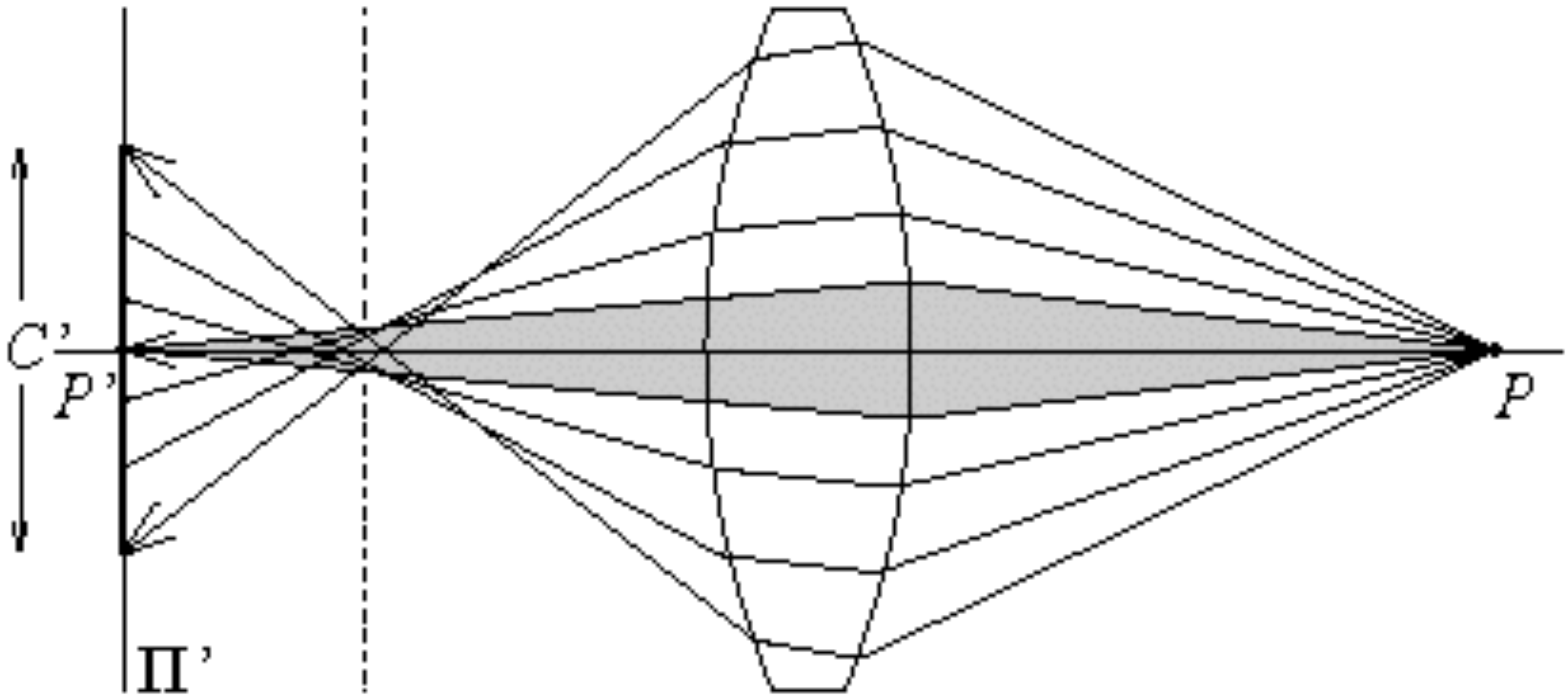
"Symetrical" by Rama. Inspired from en:File:Symmetrical.png by en:User:Anoneditor, made from scratch by Rama - Own work. Licensed under CC BY-SA 2.0 fr via Commons - https://commons.wikimedia.org/wiki/File:Symetrical.svg#/media/File:Symetrical.svg

Previous approximation is incorrect:

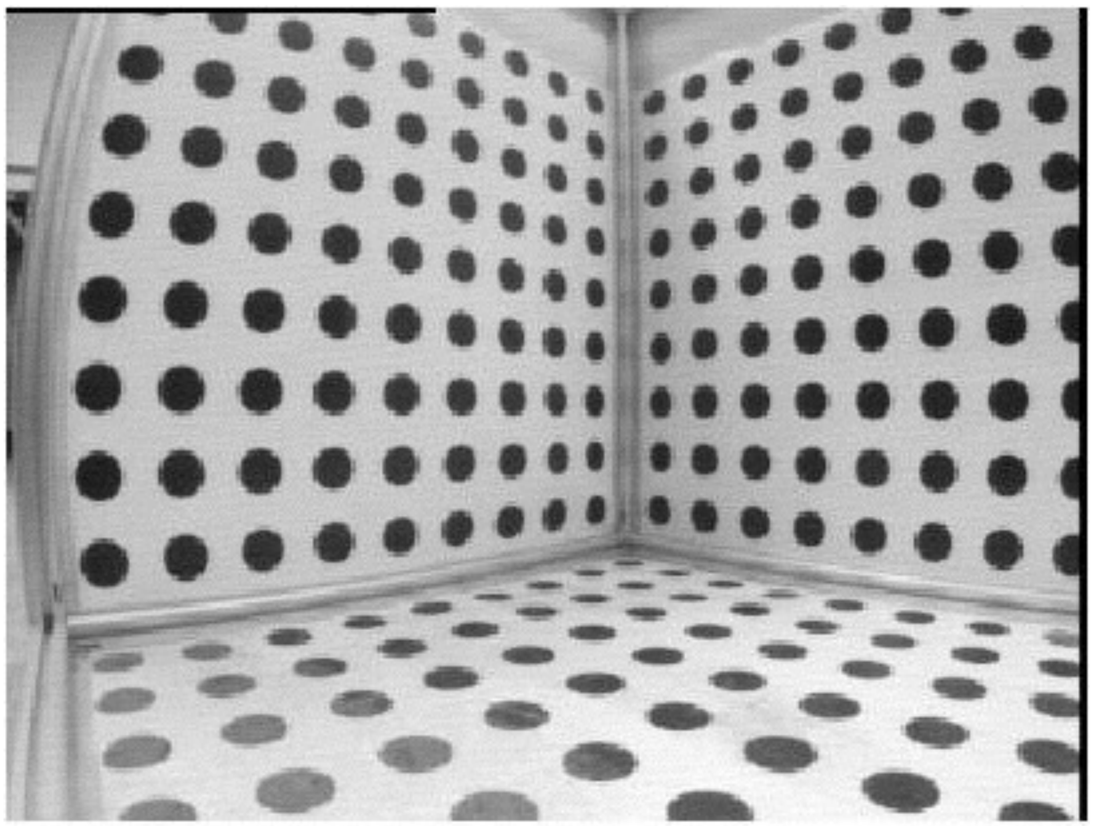
- Aberrations and distortions
- Blurring and incorrect shape in the image

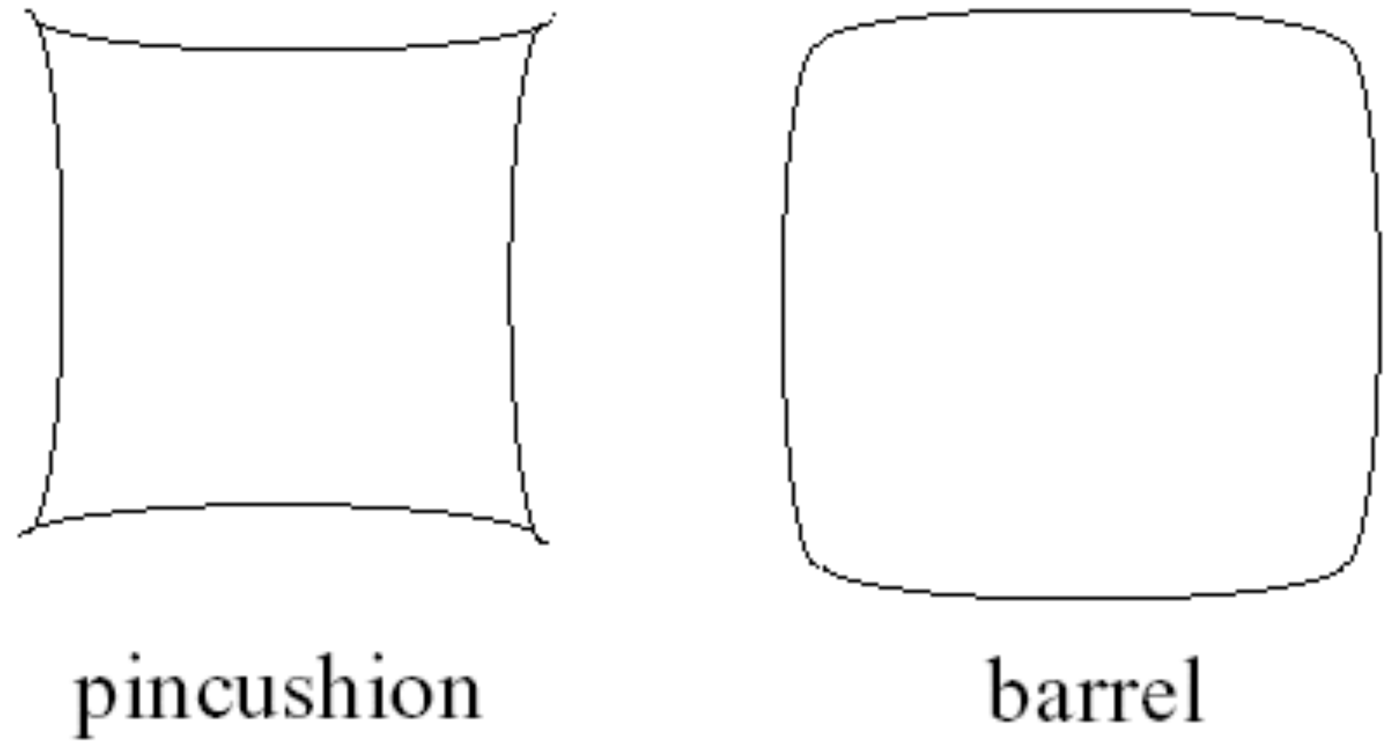
Spherical Aberrations

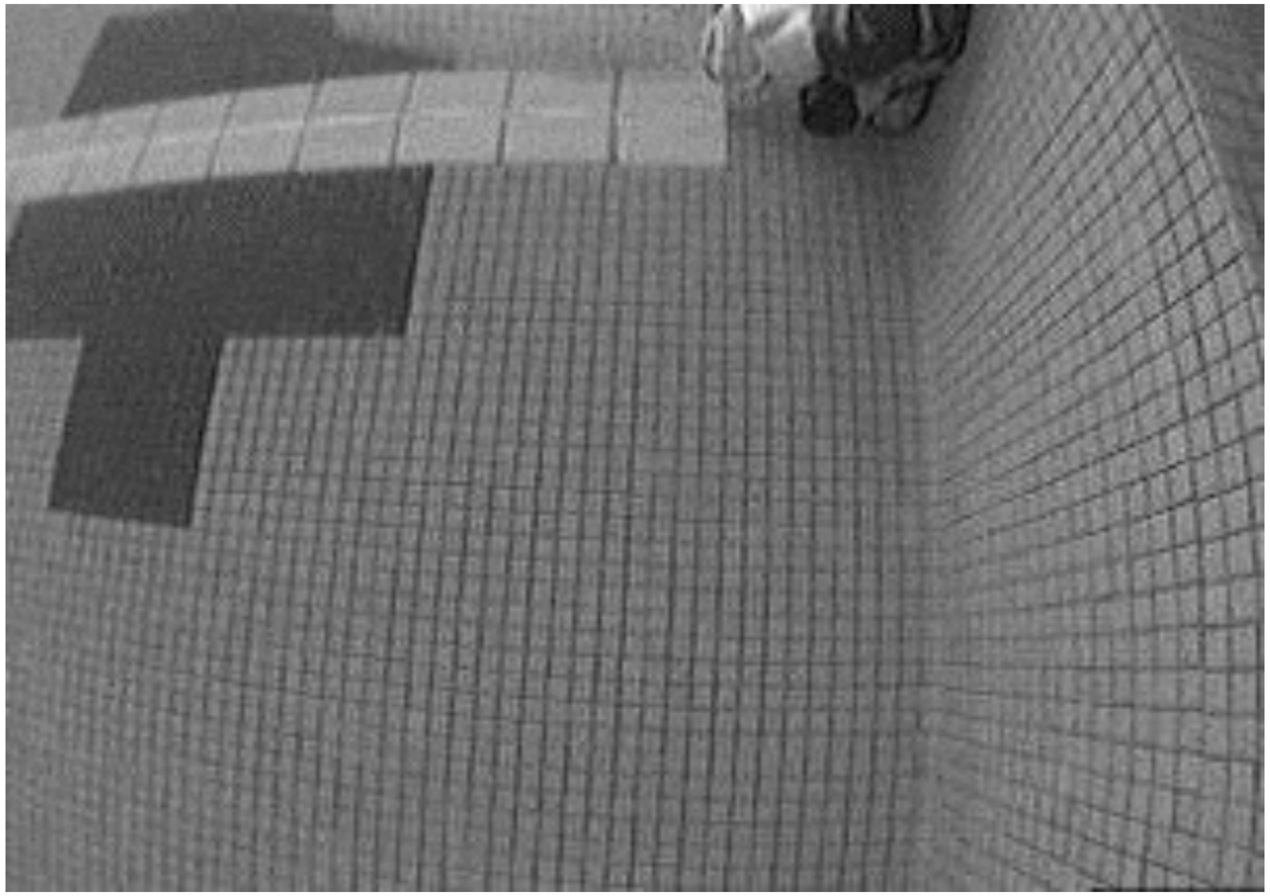
Rays further from the optical axis are focused closer to the lens

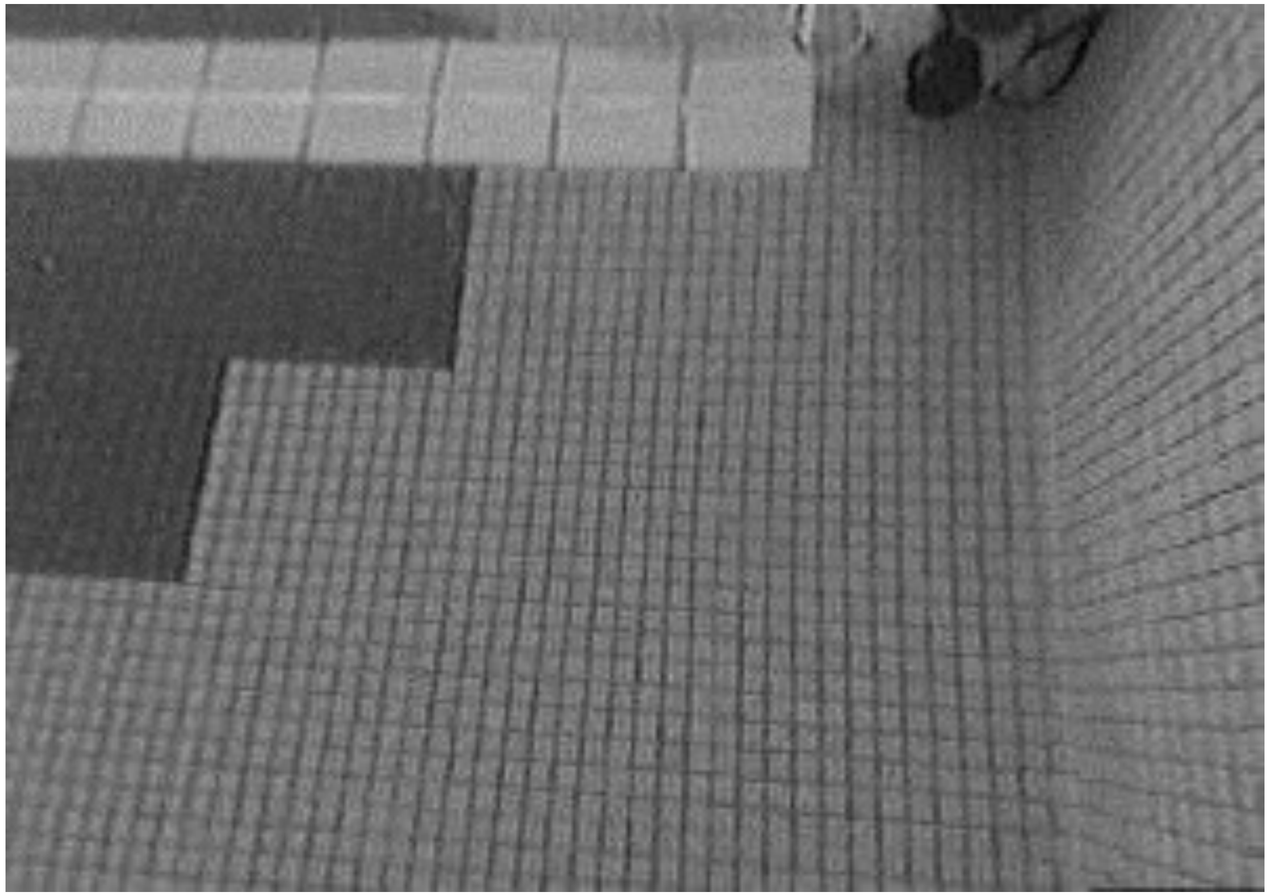


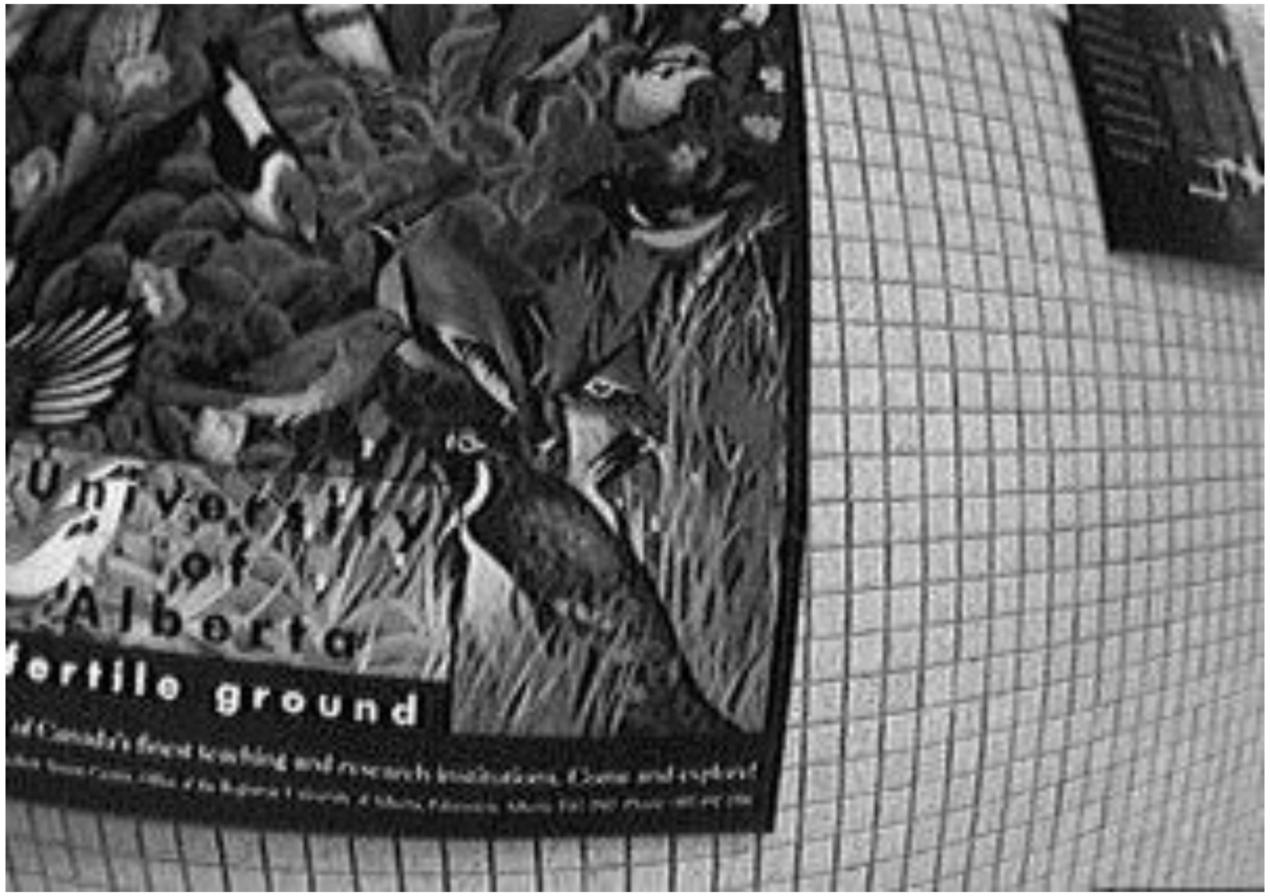
Geometric Distortion







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fertile ground


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of
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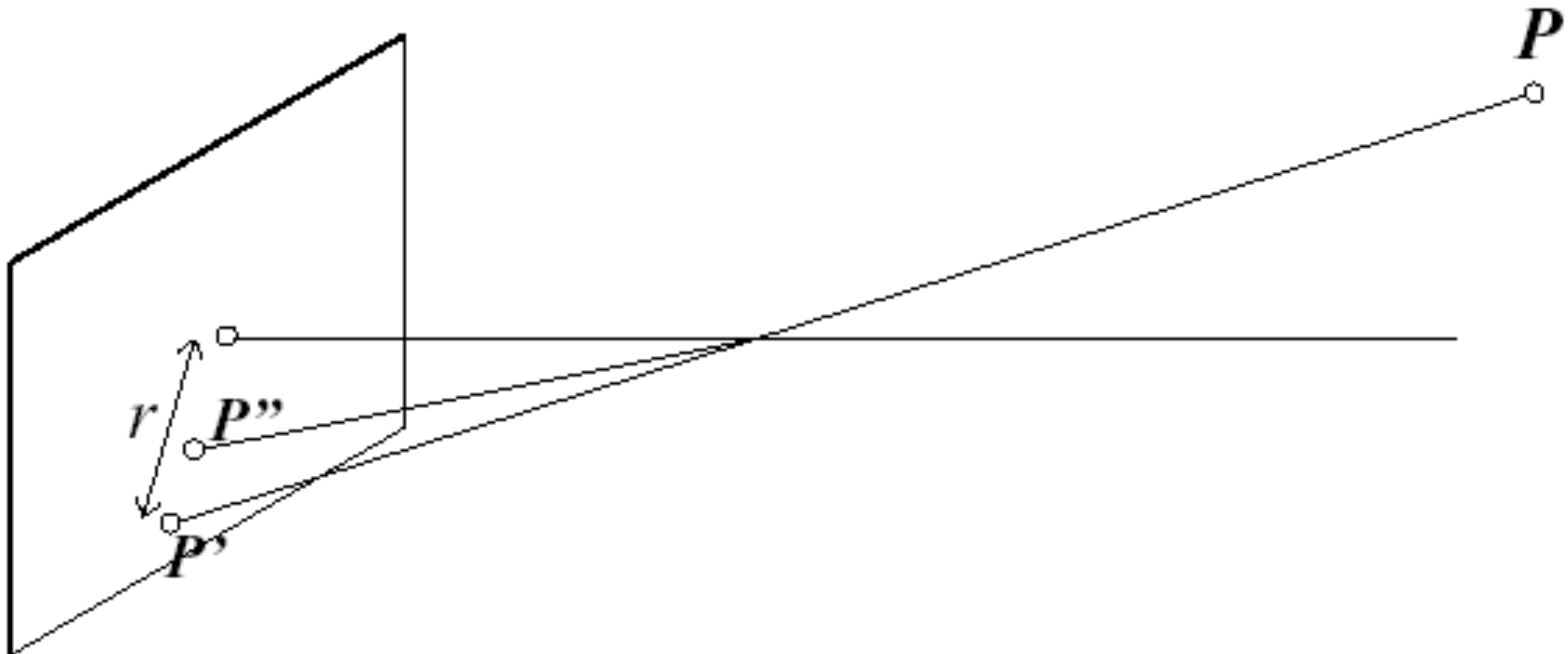
tile ground

Radial Distortion Model



Ideal:

$$x' = f\frac{x}{z}$$

$$y' = f\frac{y}{z}$$

Distorted:

$$x'' = \frac{1}{\lambda}x'$$

$$y'' = \frac{1}{\lambda}y'$$

$$\lambda = 1 + k_1 r^2 + k_2 r^4 + \cdots$$

Fun Facts to Remember!

Perspective Projection	$x' = f\frac{x}{z}$ $y' = f\frac{y}{z}$	x,y: World coordinates x',y': Image coordinates f: pinhole-to-retina distance
Weak-Perspective Projection (Affine)	$x' \approx -mx$ $y' \approx -my$ $m = -\frac{f}{\bar{z}}$	x,y: World coordinates x',y': Image coordinates m: magnification
Orthographic Projection (Affine)	$x' \approx x$ $y' \approx y$	x,y: World coordinates x',y': Image coordinates
Common distortion model	$x'' = \frac{1}{\lambda}x'$ $y'' = \frac{1}{\lambda}y'$ $\lambda = 1 + k_1 r^2 + k_2 r^4 + \cdots$	x',y': Ideal image coordinates x'',y'': Actual image coordinates