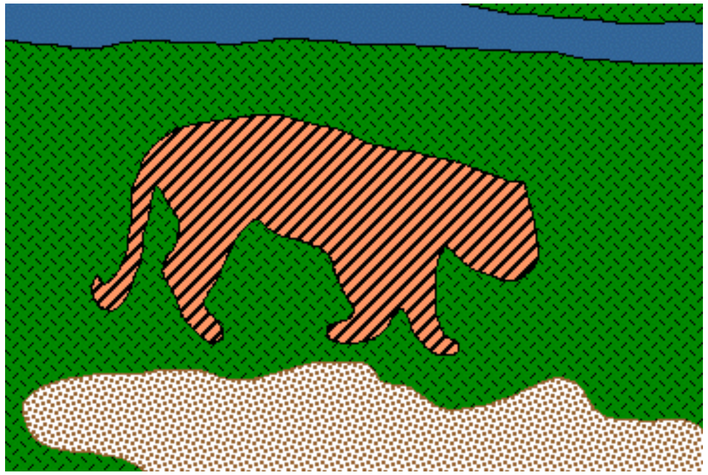
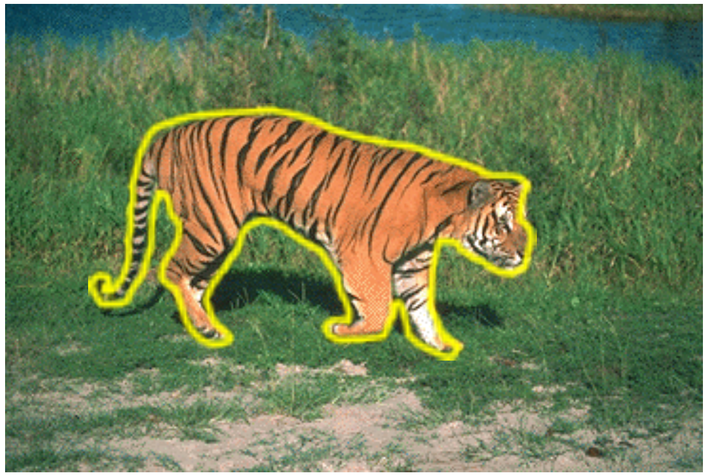
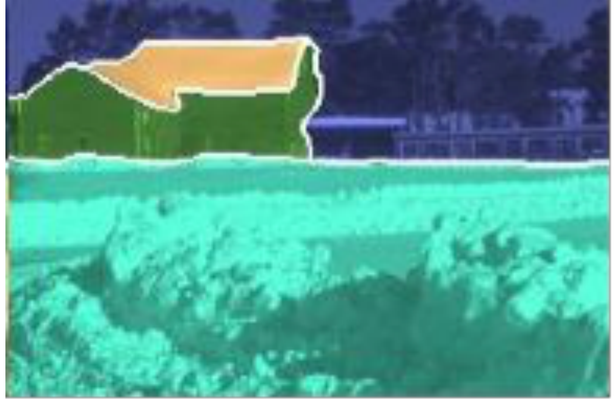
Segmentation

Gary Overett (Slides adapted from CMU 16-720 2014)







$k = 4$

$nc = .0017$

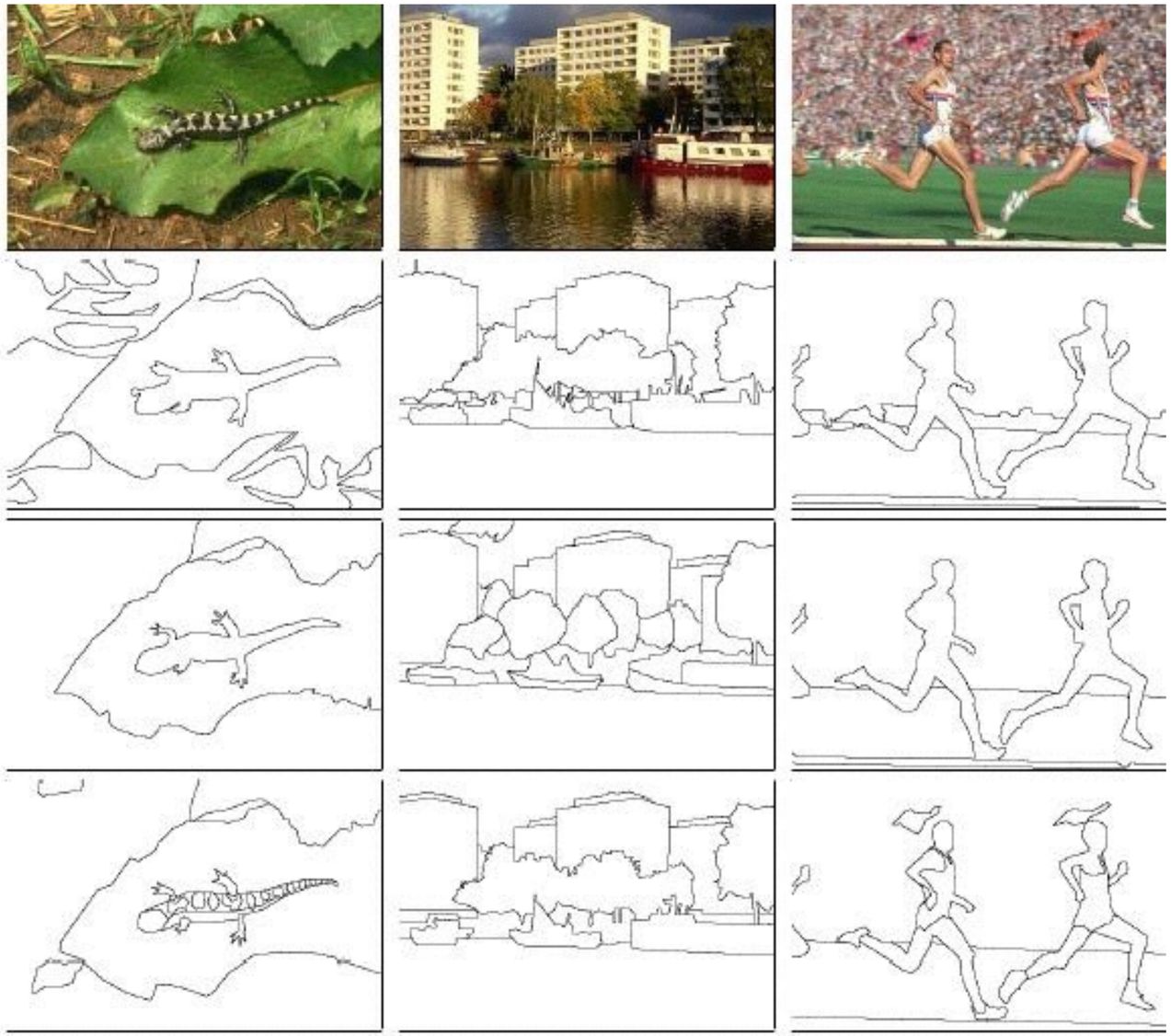
$k = 5$

$nc = .0060$

What is a “good” segmentation?

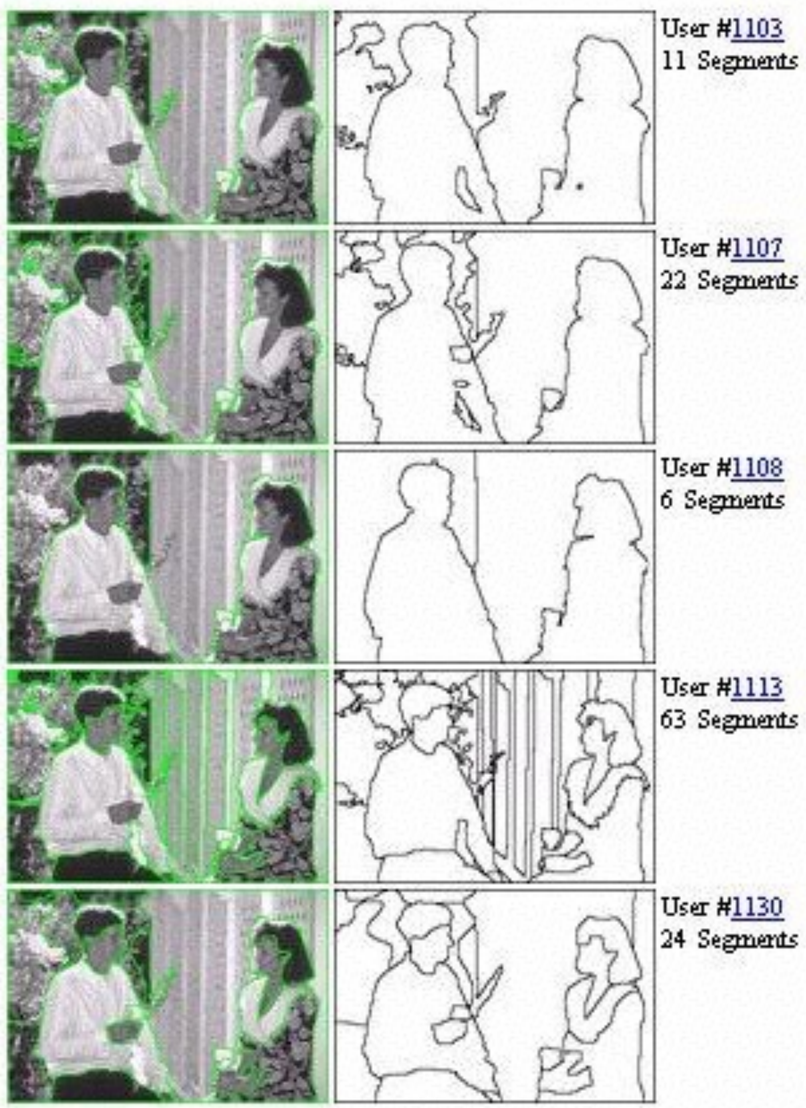
Key idea: Compare to human segmentation or to "ground truth"

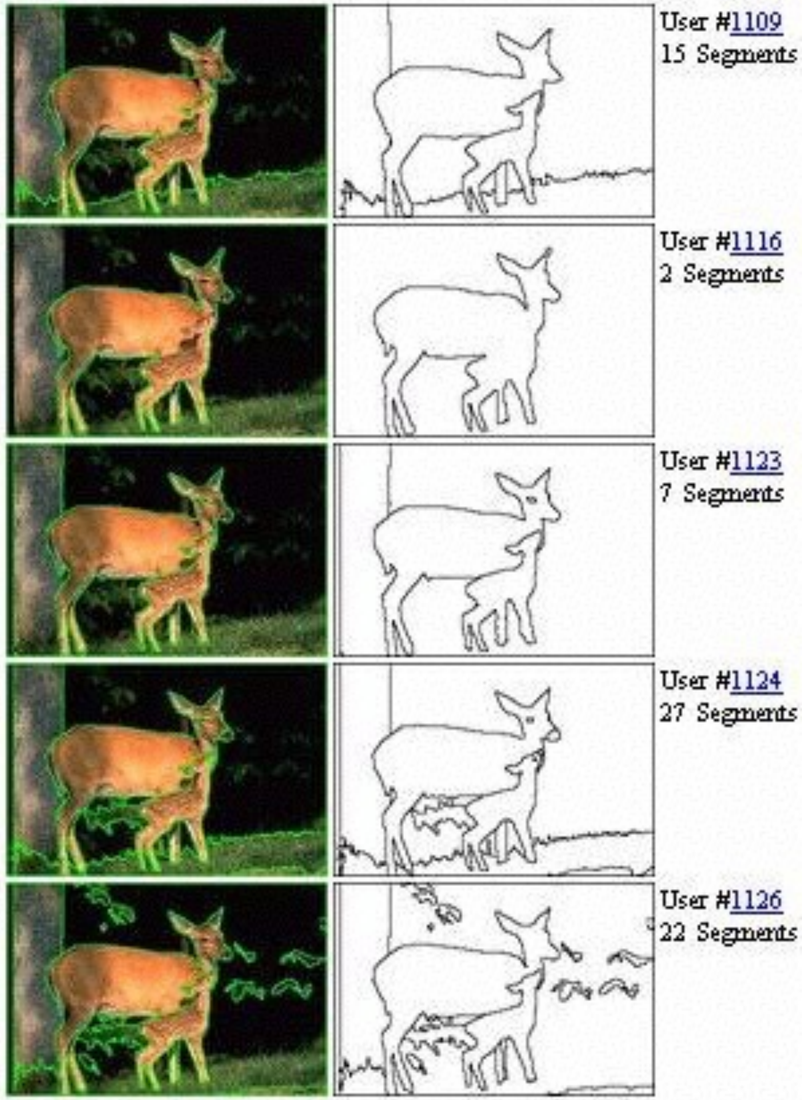
No objective definition of segmentation!



http://www.eecs.berkeley.edu/Research/Proje cts/CS/vision/grouping/resources.html

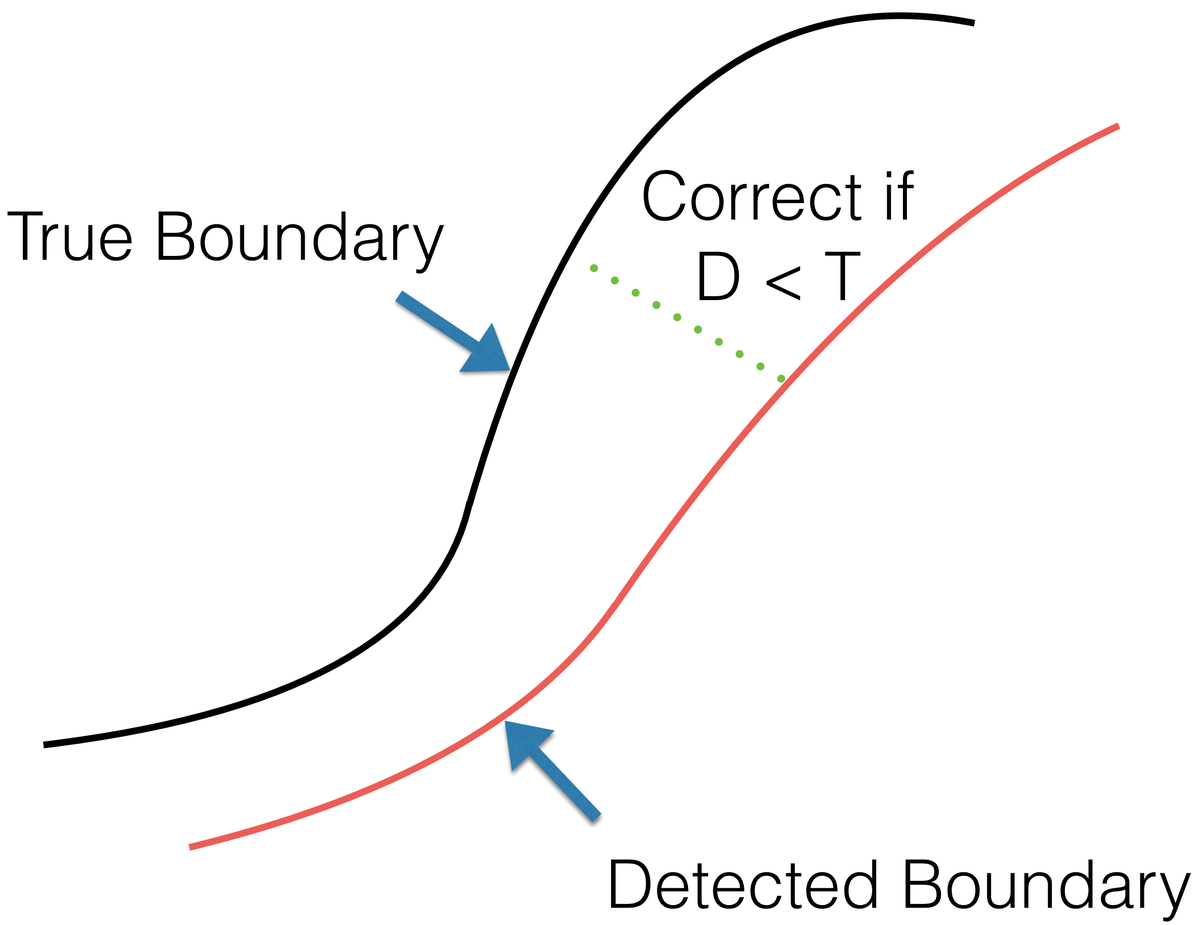
No objective definition of segmentation!

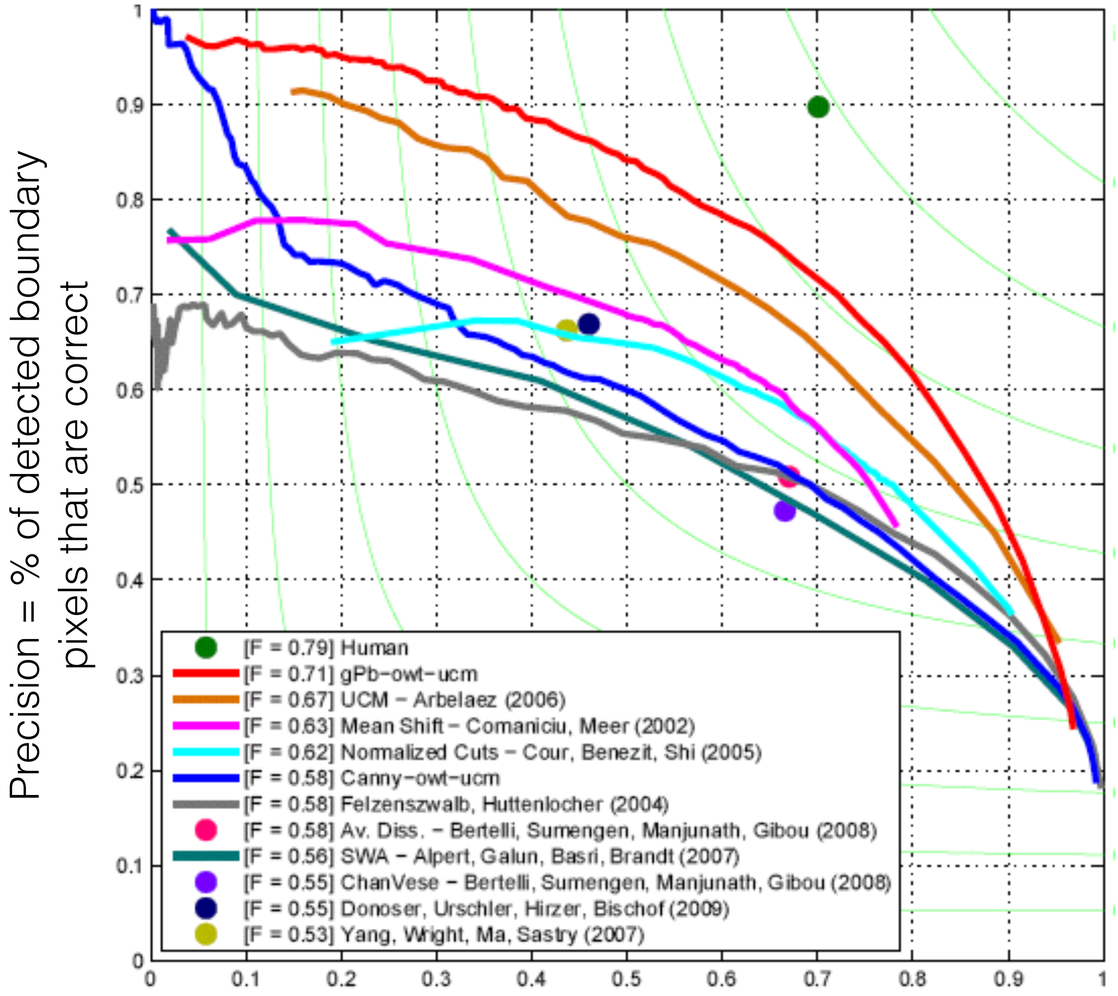




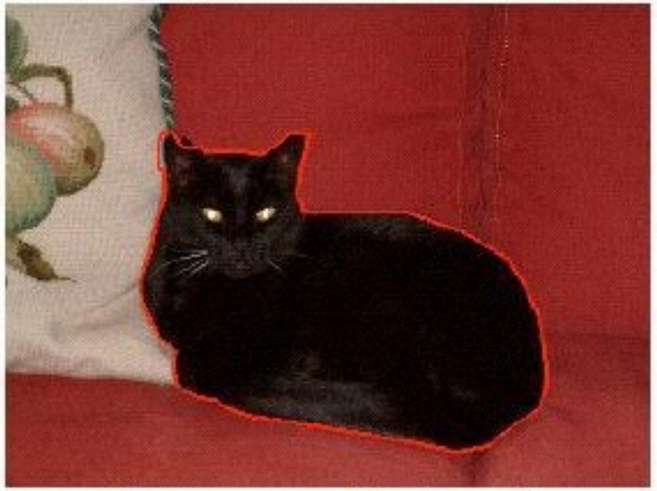
http://www.eecs.berkeley.edu/Research/Projects/CS/vision/bsds/BSDS300/html/dataset/images/color/317080.html

Evaluation : Boundary Agreement





Evaluation : Intersection over Union



Ground Truth

Segment #1

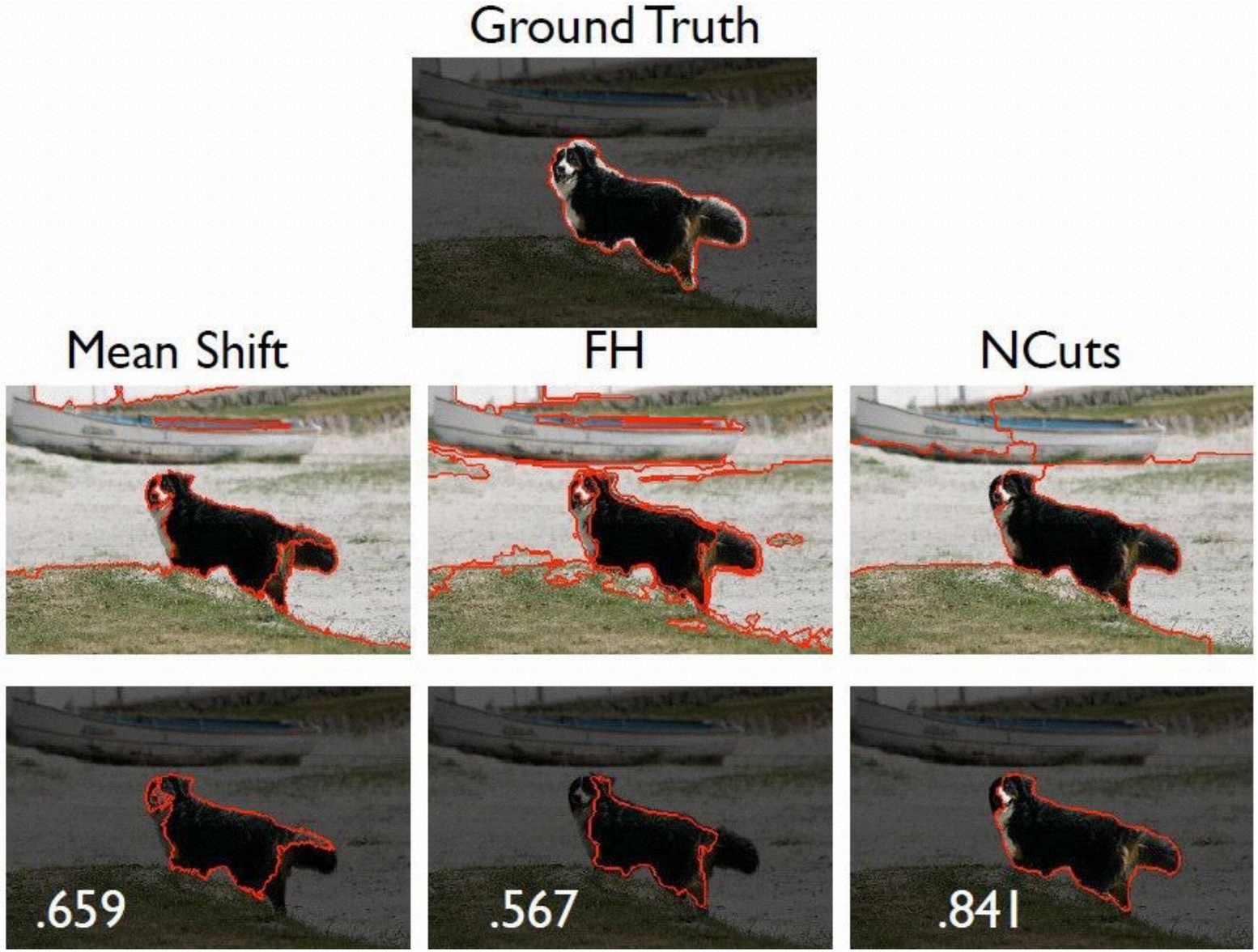
.825

Segment #2

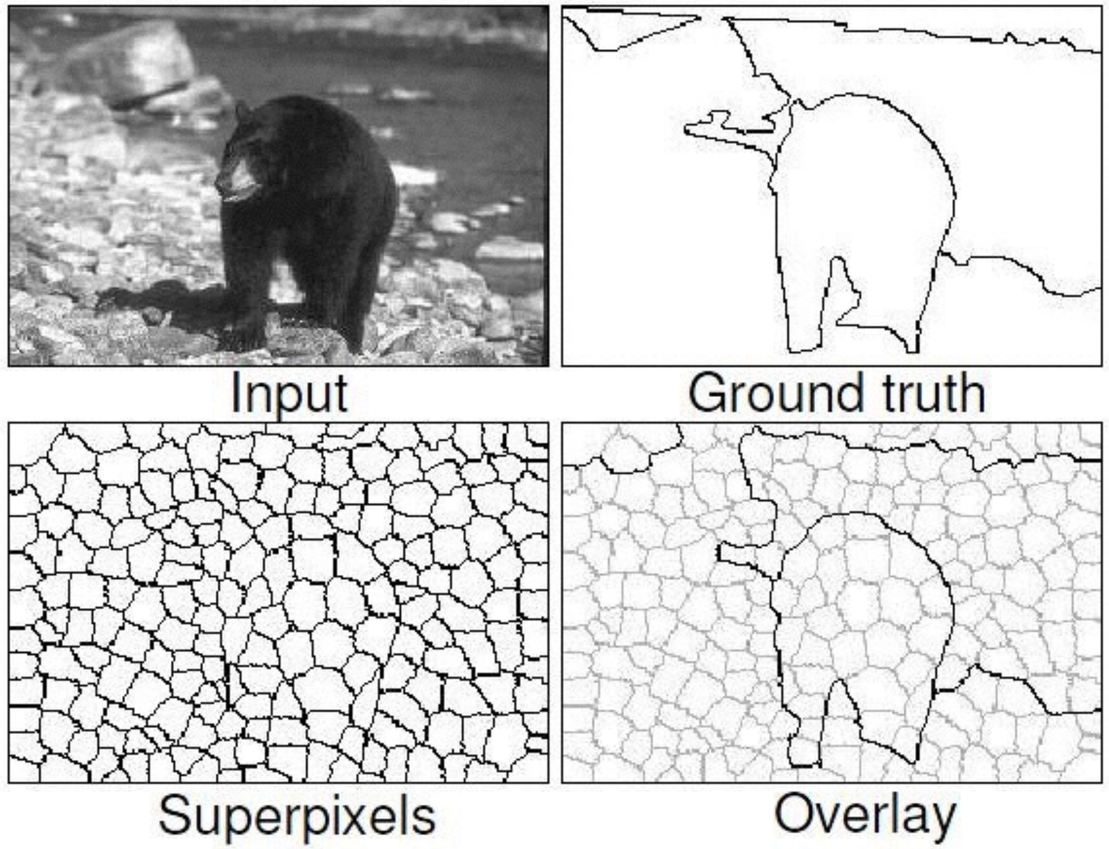
.892

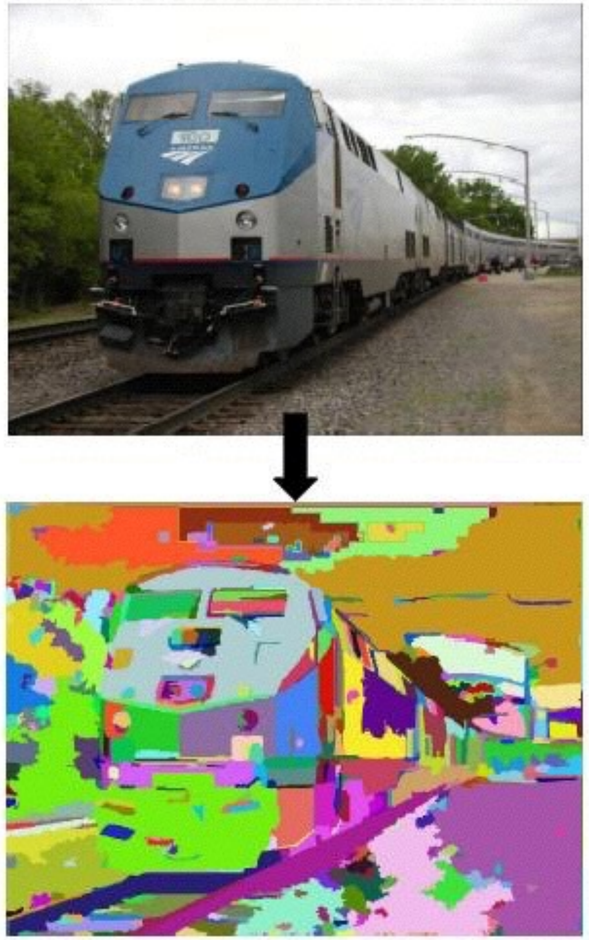
$$OS(S,G) = \frac{|S \cap G|}{|S \cup G|}$$

Evaluation : Intersection over Union



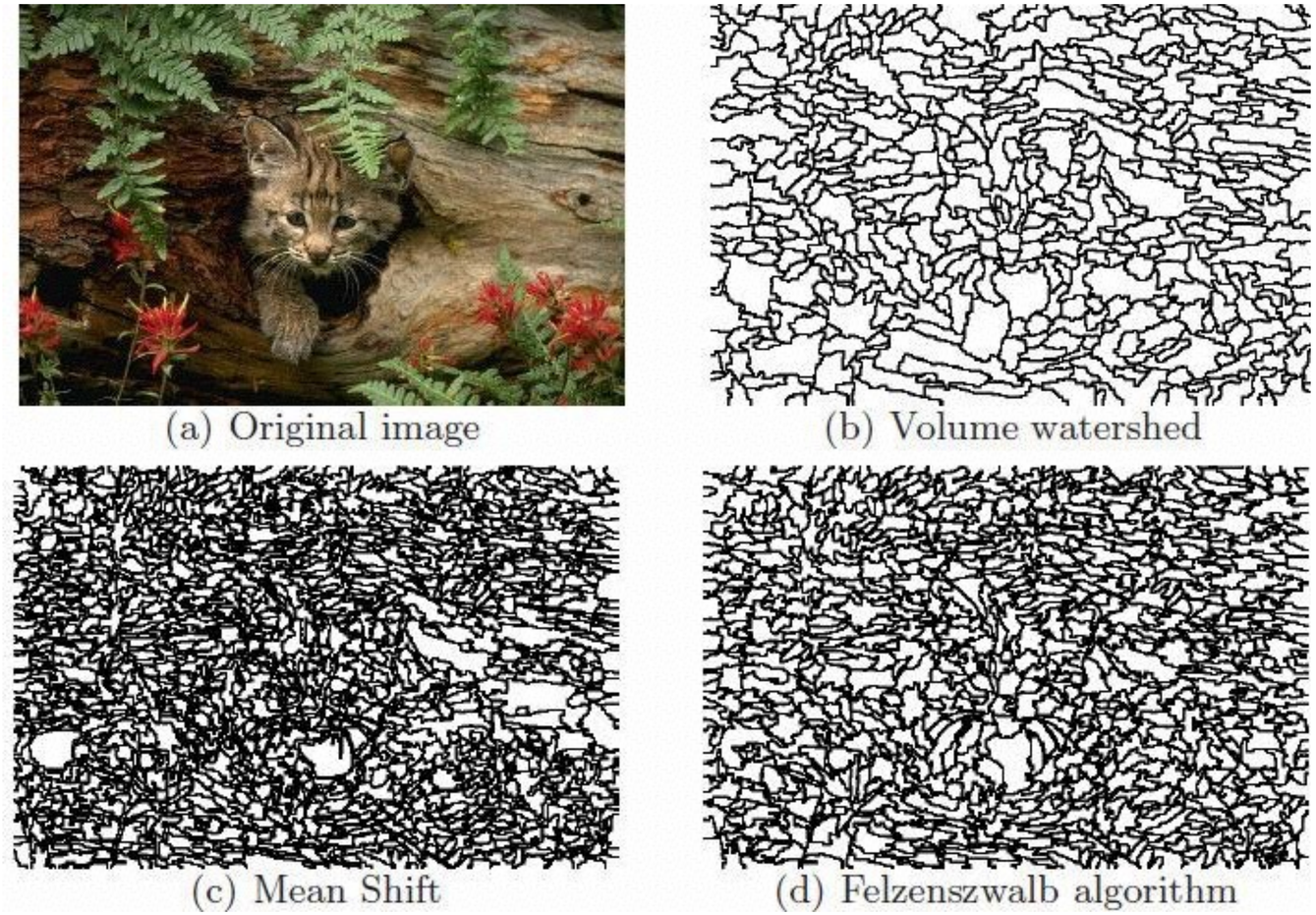
Key Idea : Superpixels





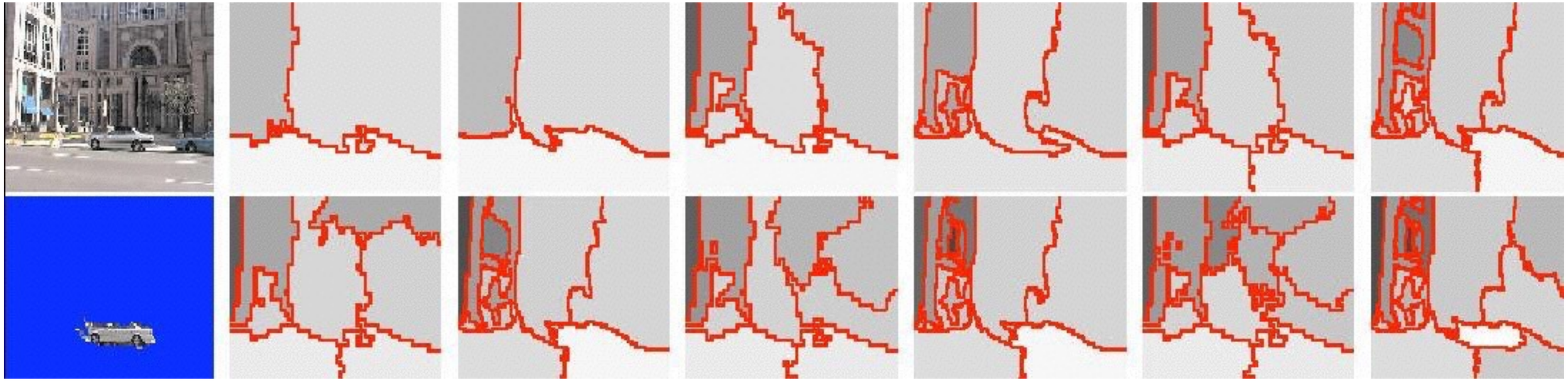
- Let's not even try to compute a "correct" segmentation
- Let's be content with an oversegmentation in which each region is very likely (formal guarantees are hard) to be uniform

Key Idea : Superpixels



- Example from: How Do Superpixels Affect Image Segmentation?
- Progress in Pattern Recognition, Image Analysis and Applications. Springer LNCS. Volume 5197/2008.

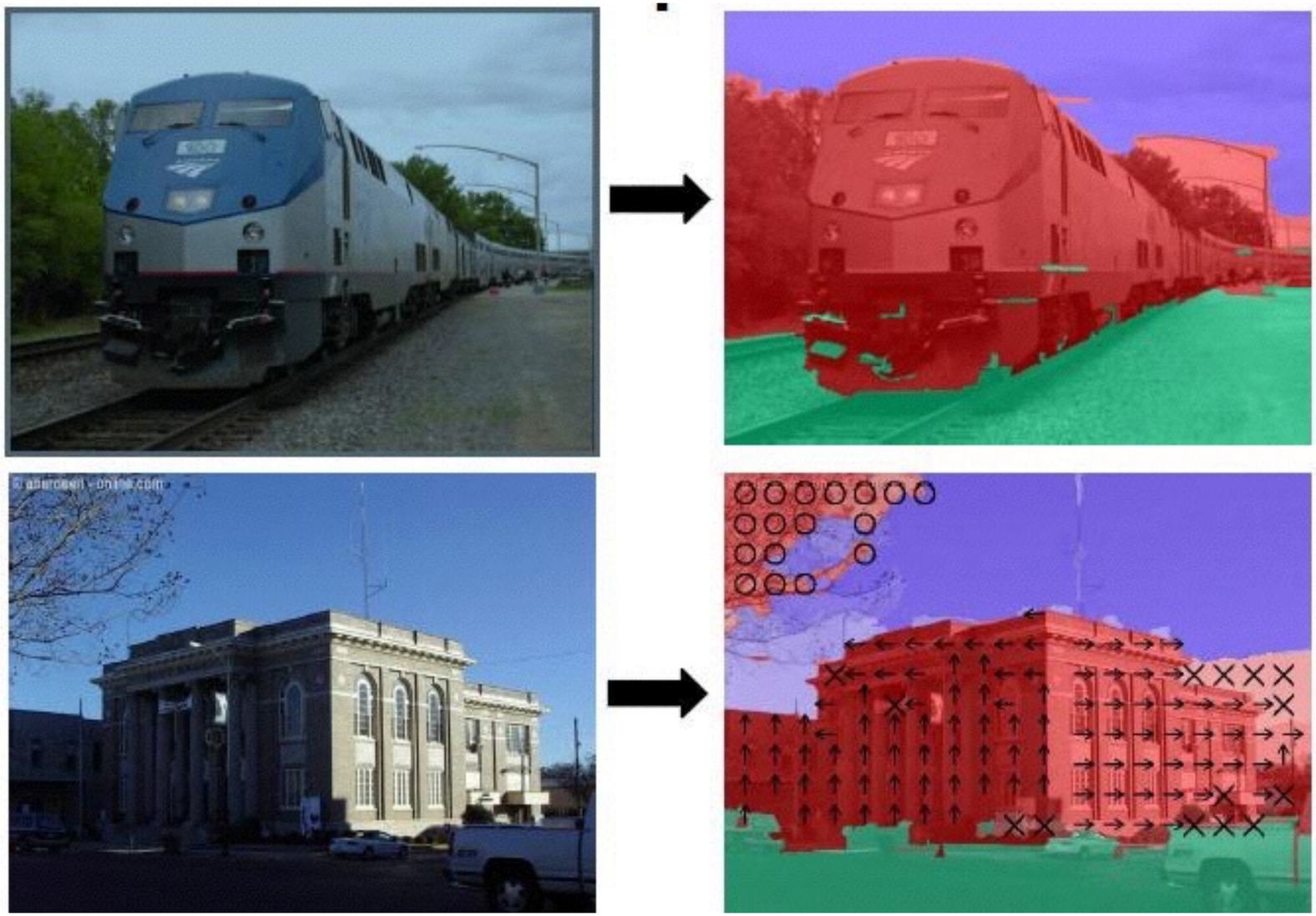
Key Idea : Multiple Segmentations



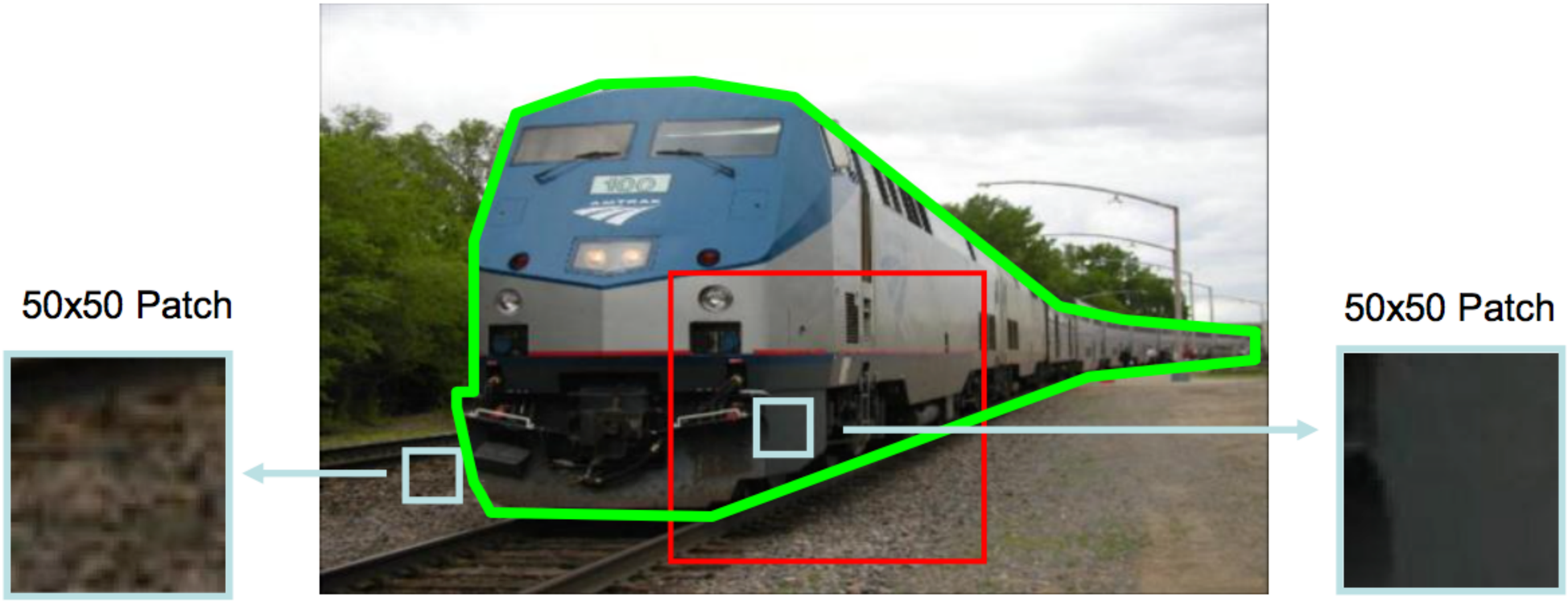
- Generate many segmentations of the same image
- Even though many regions are “wrong”, some consensus should emerge

Example: Improving Spatial Support for Objects via Multiple Segmentations Tomasz Malisiewicz and Alexei A. Efros. British Machine Vision Conference (BMVC), September, 2007.

Multiple Segmentations : Example

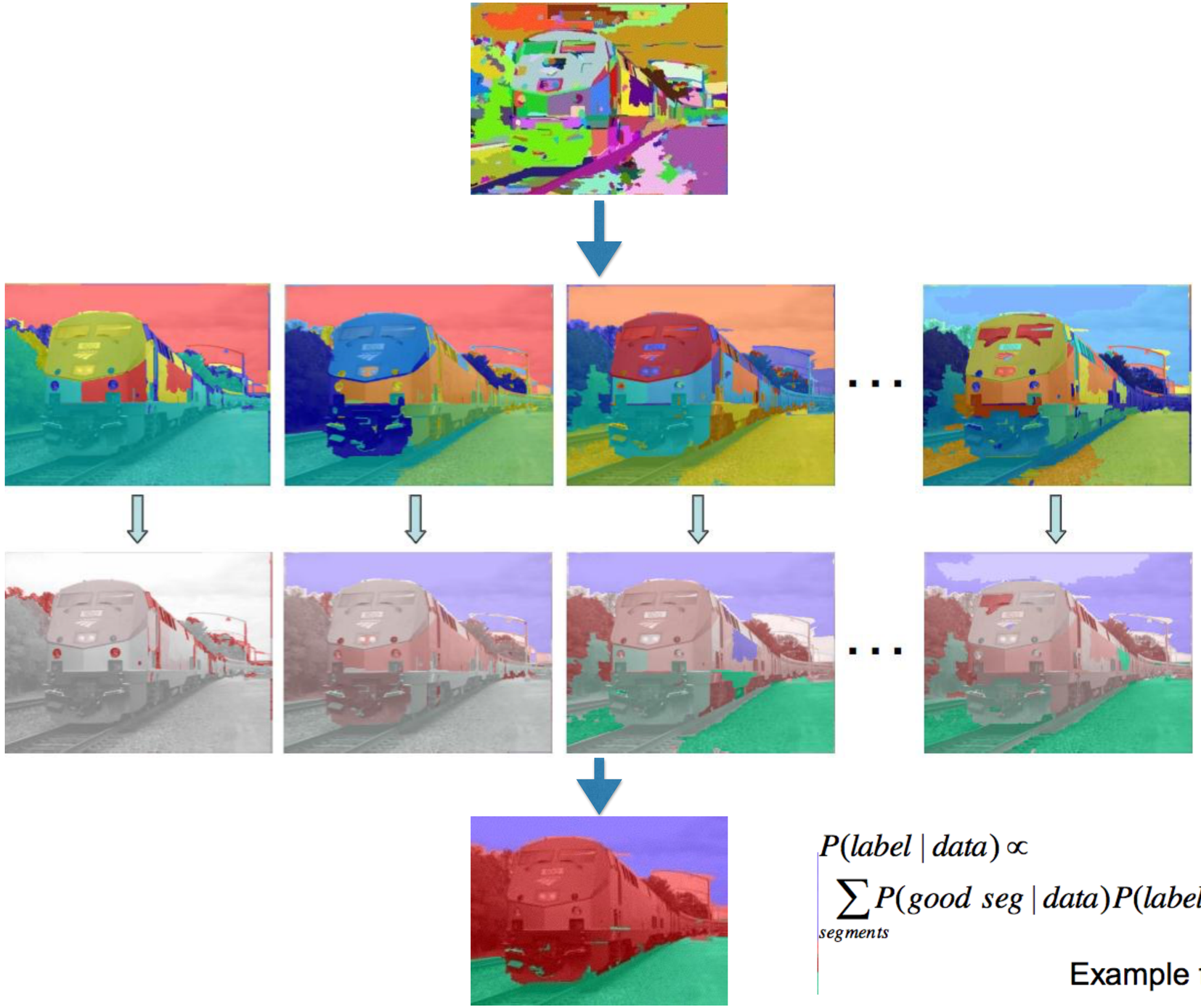


- Task: Regions -> Features -> Labels (horizontal, vertical, sky, etc.)



- Chicken and egg problem:
  - If we knew the regions, we could compute the features and label the right regions
  - But to know the right regions we need to know the labels!
- Solution:
  - Generate lots of segmentations
  - Combine the classifications to get consensus

Recovering Surface Layout from an Image. D. Hoiem, A.A. Efros, and M. Hebert. IJCV, Vol. 75, No. 1, October 2007.



$$P(label \mid data) \propto \sum_{segments} P(good\ seg \mid data) P(label \mid good\ seg, data$$

Example from D. Hoiem

Summary so far…

- Match ground truth (no objective definition)
- Superpixels = oversegmentation
- Using multiple segmentations

Main Approaches

- Clustering
- Mean shift
- Spectral techniques
- Graph-based techniques
- Segmentation as boundary detection

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Segmentation as Clustering (Informally)

- Given a set of features $\{x_i\}$
- Given a "distance" between features $d(x,x')$
- Find a partition $\{C_j\}$ such that:
  - $d(x_i,x_j)$ is "small" if x_i and x_j belong to the same group
  - $d(x_i,x_j)$ is large if x_i and x_j belong to different groups

- Examples
  - Color segmentation: x_i is color of pixel i. Subsets are connected regions of "similar" color.
  - Texture segmentation: x_i is vector of responses to texture filters at pixel i. Subsets are regions of "similar" texture. $x_i = [F_1*I,..., F_n*I]$
  - Contour grouping: x_i is edge element. Subsets are sets of edges that form a smooth connected contour.

K-Means Clustering

If the number of clusters is know a-priori:

$$\Phi(\text{clusters}, \text{data}) = \sum_{j\,\text{cluster}} \sum_{i \in \text{cluster } j} \left\| \mathbf{x}_i - \mathbf{c}_j \right\|^2$$

1. Given $\{\mathbf{c}_j\}$, assign each $\mathbf{x}_i$ to the closest j
2. Compute the centers $\mathbf{c}_j = 1/N \, \Sigma \, \mathbf{x}_i$

Image

Clusters on intensity

Feature = intensity

Clusters on color

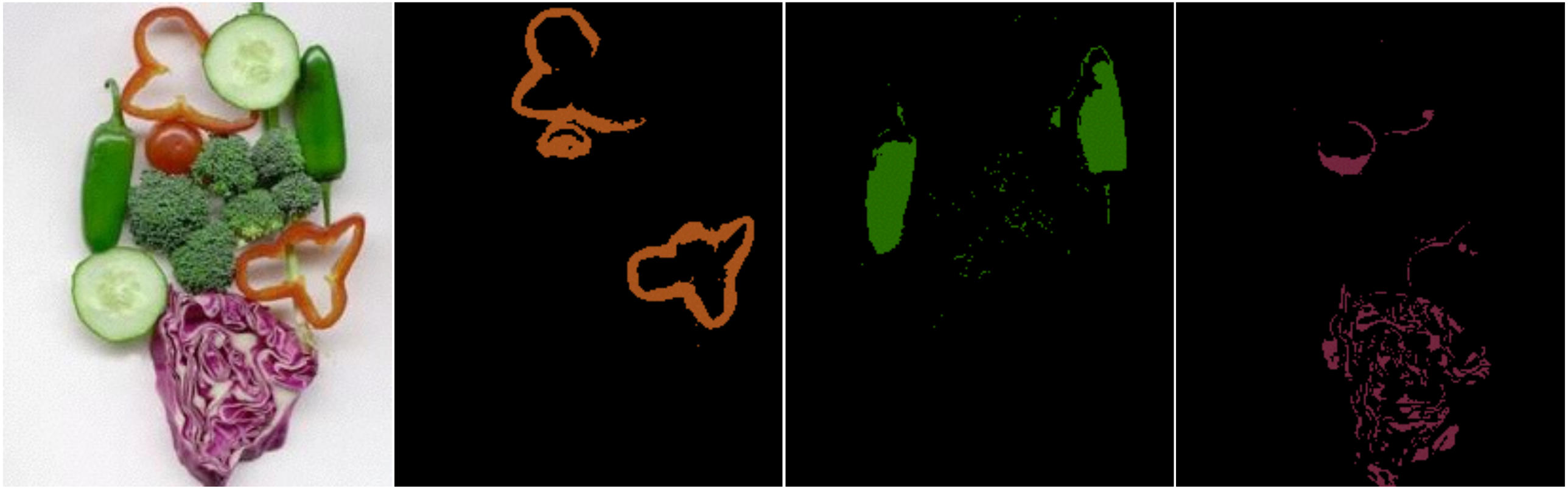
Feature = [r g b]

K-means clustering using intensity alone and color alone

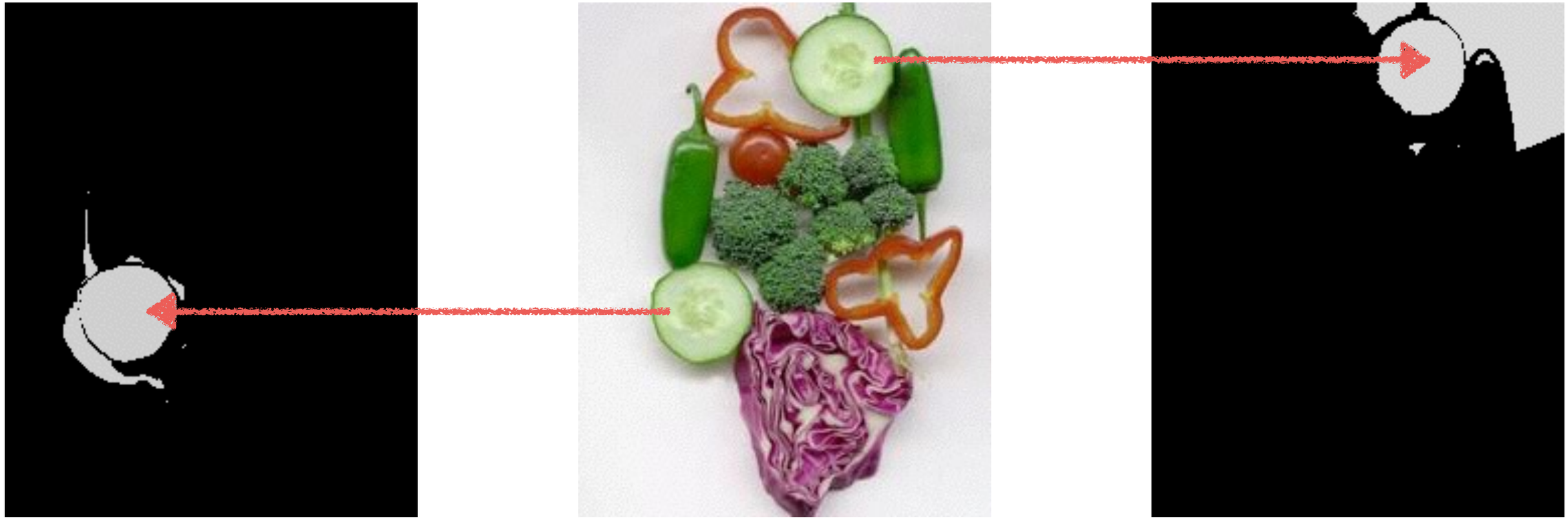
Image

Clusters on color

K-means using color alone, 11 segments



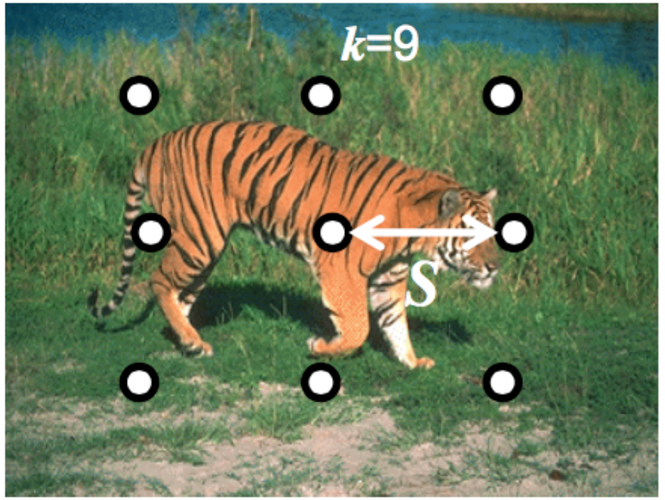
K-means using color alone, 11 segments. Feature = [r g b]



K-means using color and position, 20 segments Feature = [r g b x y]

State of the Art : SLIC Superpixels

- Find k superpixels such that
  – Respect boundaries
  – Are spatially roughly equally sized



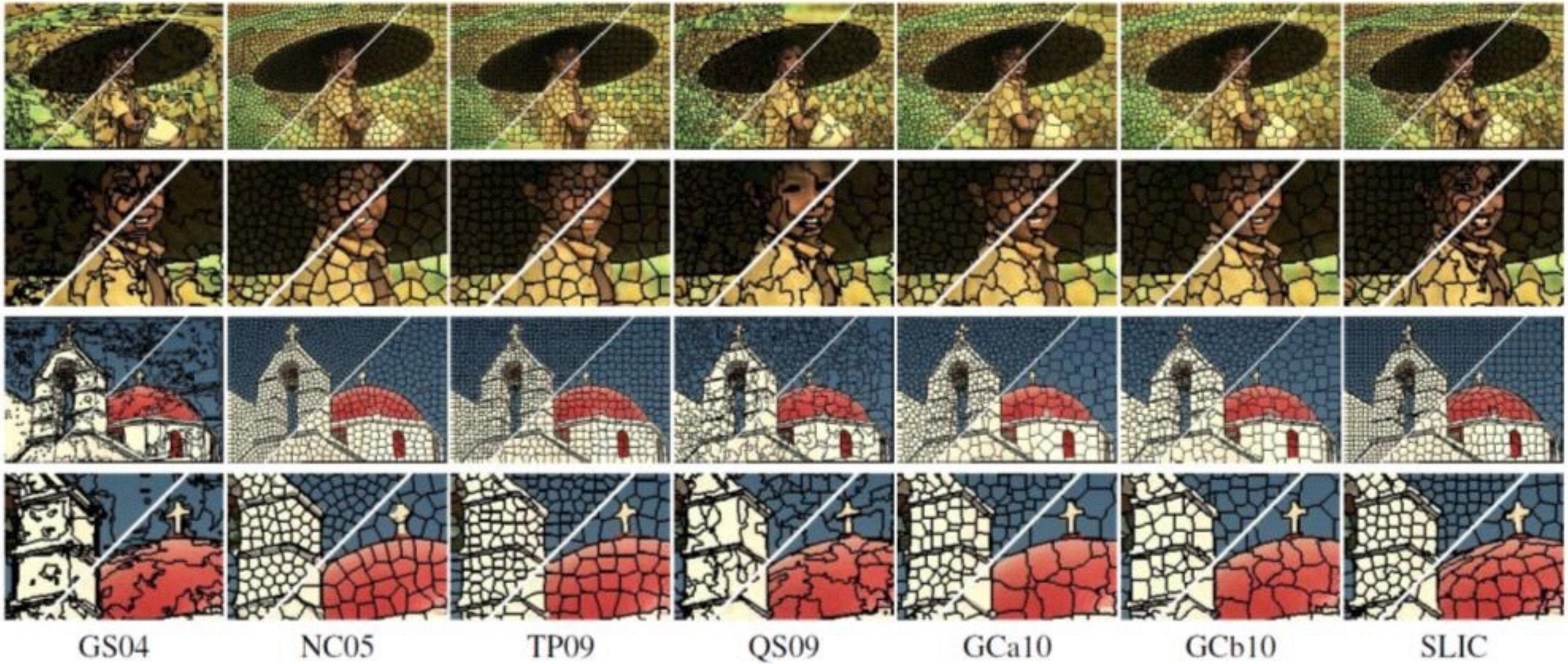
Select $\{\mathbf{c}_j\}$ on a regular grid of k centers

0. Initialize cluster centers $\{\mathbf{c}_j\}$

1. Given $\{\mathbf{c}_j\}$, assign each $\mathbf{x}_i$ to the closest j
2. Compute the centers $\mathbf{c}_j = 1/N \Sigma\, \mathbf{x}_i$

Search only in a neighborhood $2S$x$2S$ neighborhood around i

http://ivrg.epfl.ch/supplementary_material/RK_SLICSuperpixels/index.html

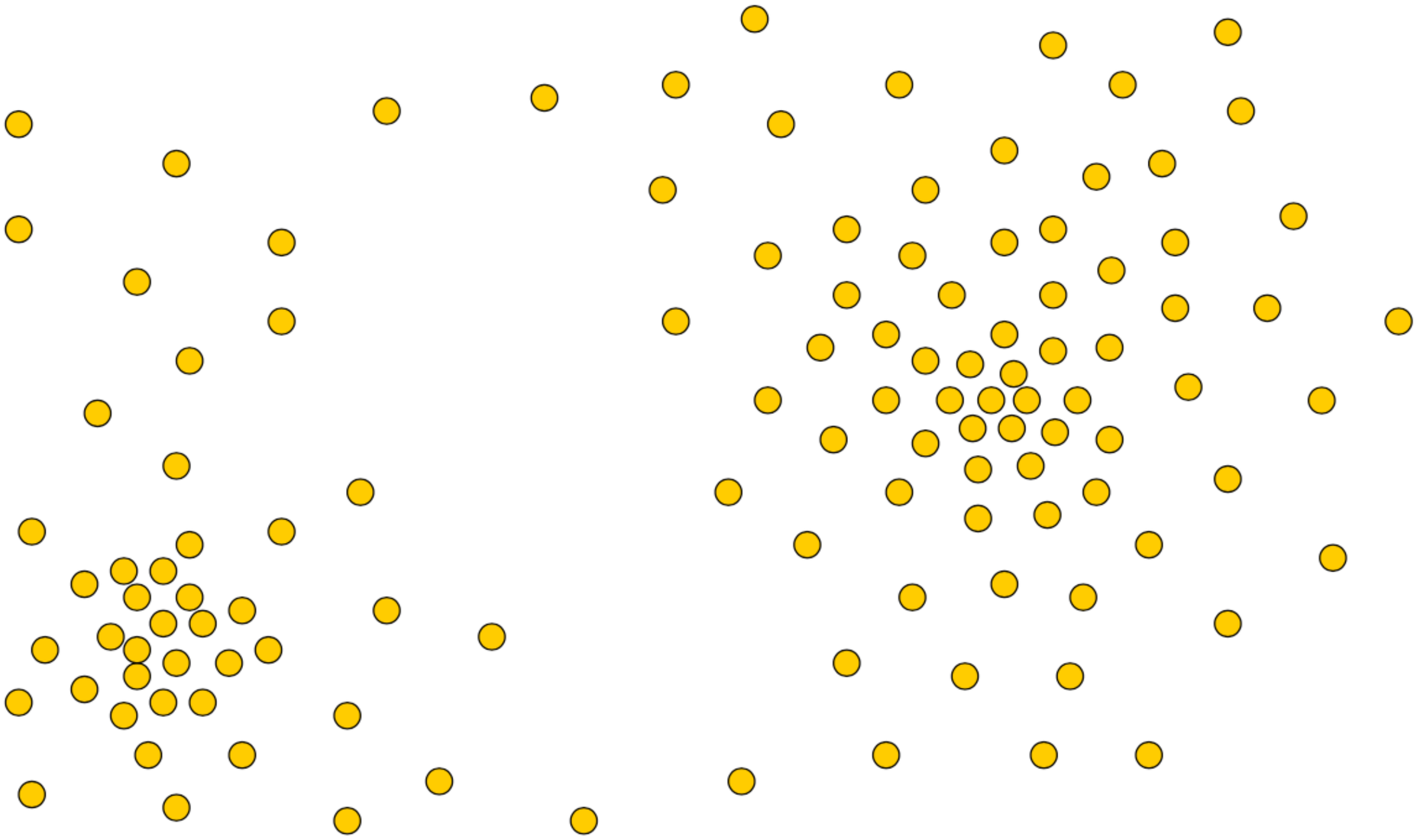


Main Approaches

- Clustering
- Mean shift
- Spectral techniques
- Graph-based techniques
- Segmentation as boundary detection

Segmentation: The Mean-Shift Algorithm (hw3 warning)

Segmentation as finding places of high density in feature space



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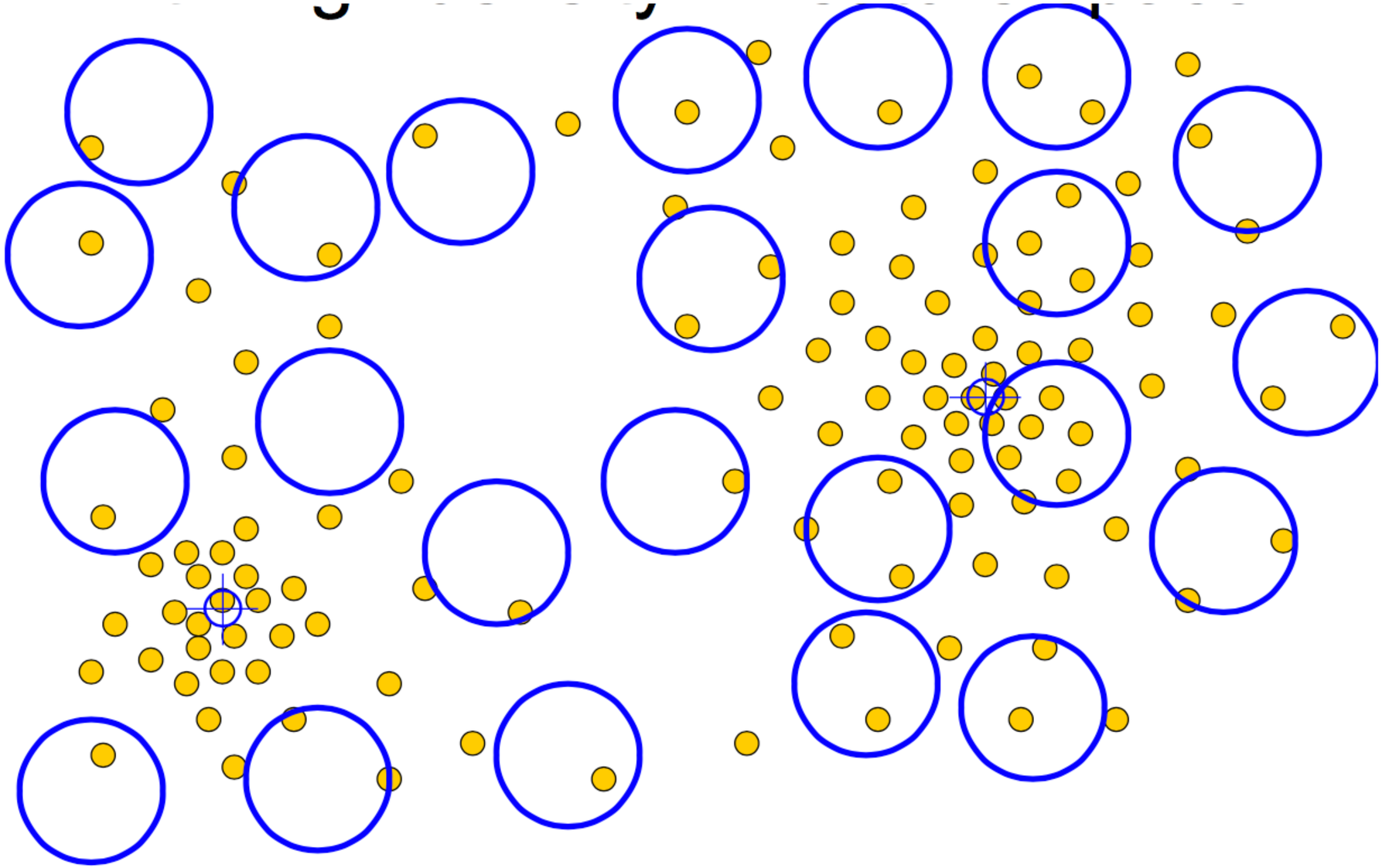
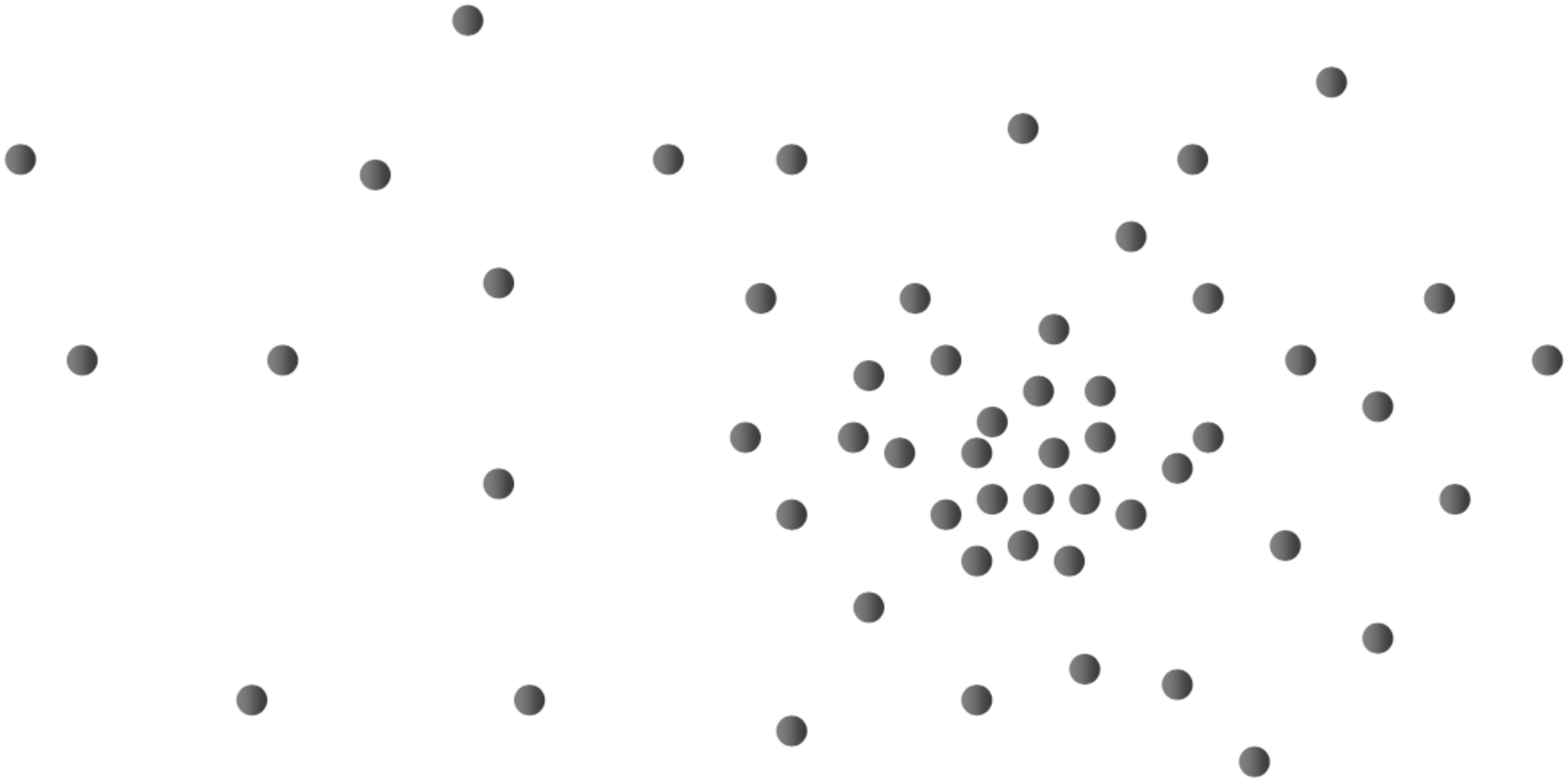
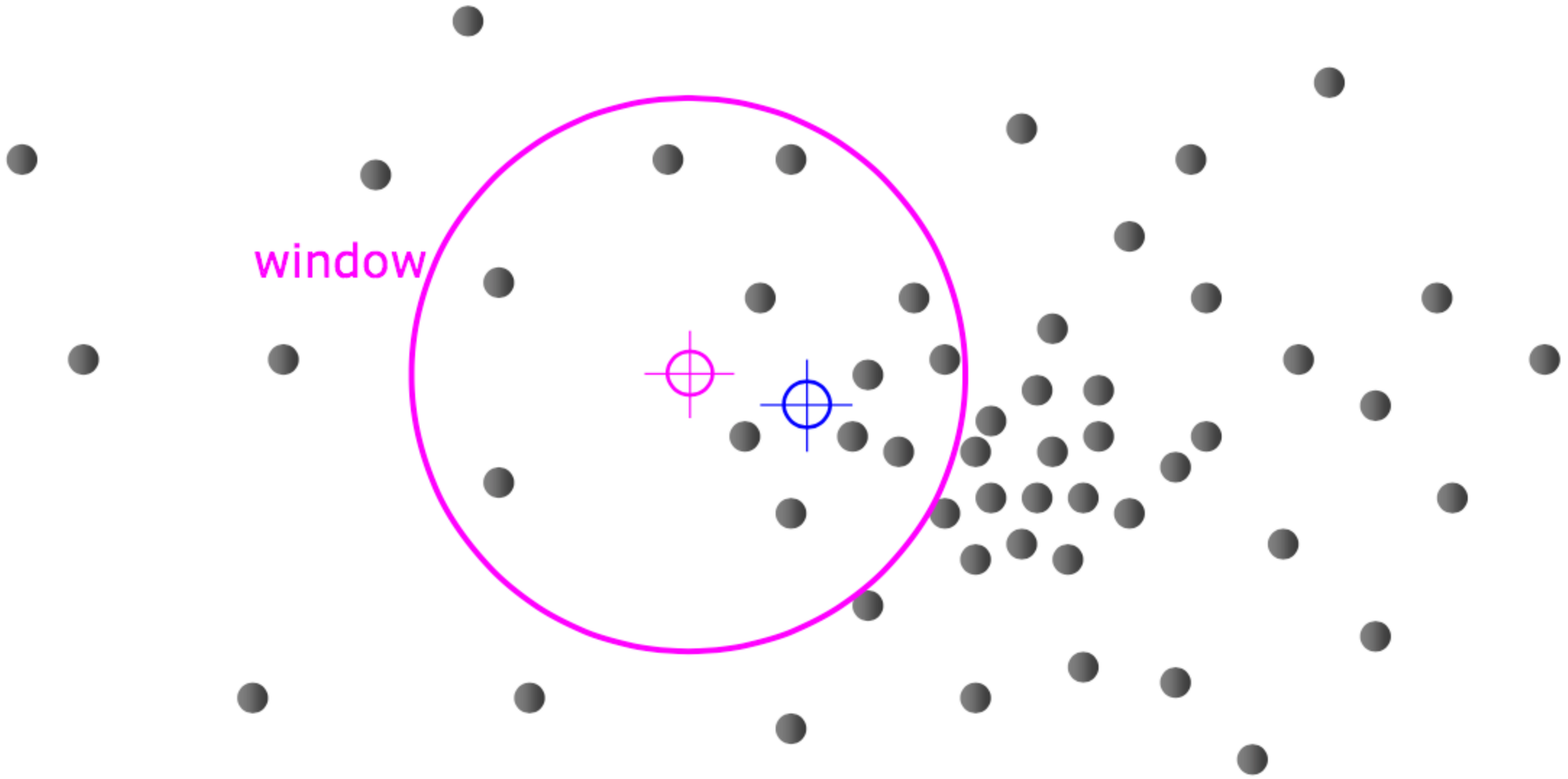


Segmentation as finding places of high density in feature space



Example from: Ukrainitz&Sarel, Weizmann

Segmentation as finding places of high density in feature space



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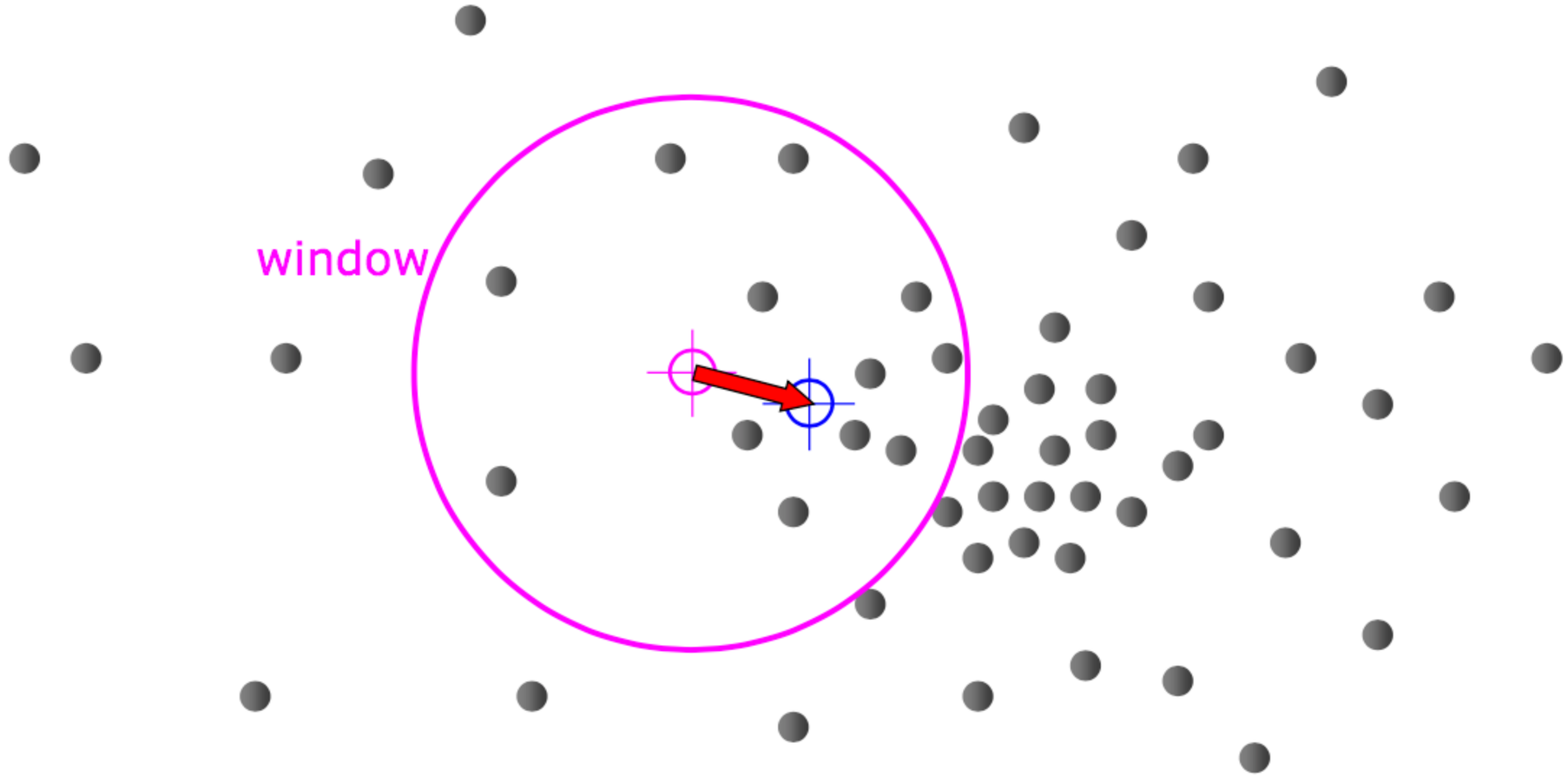

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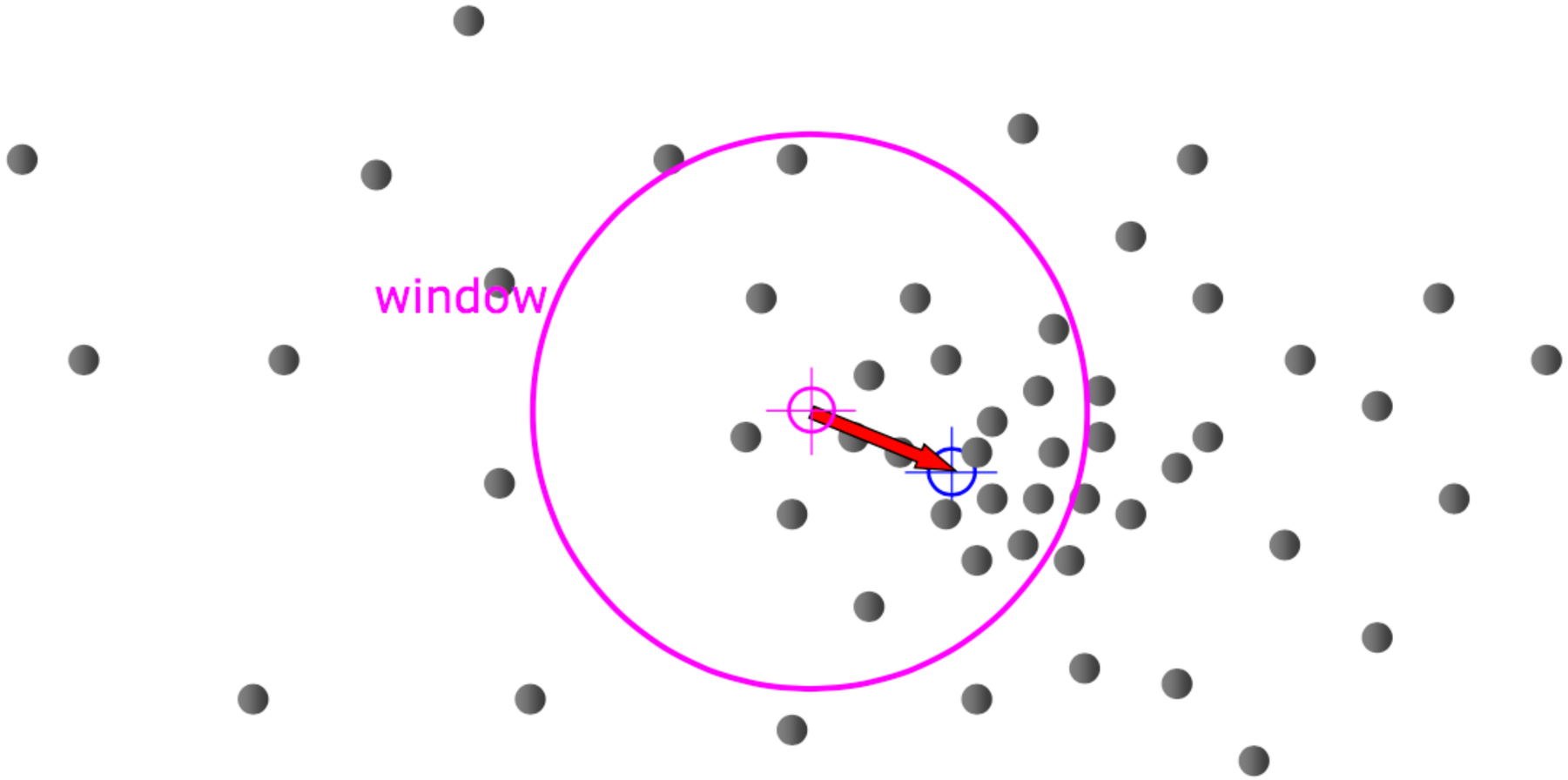

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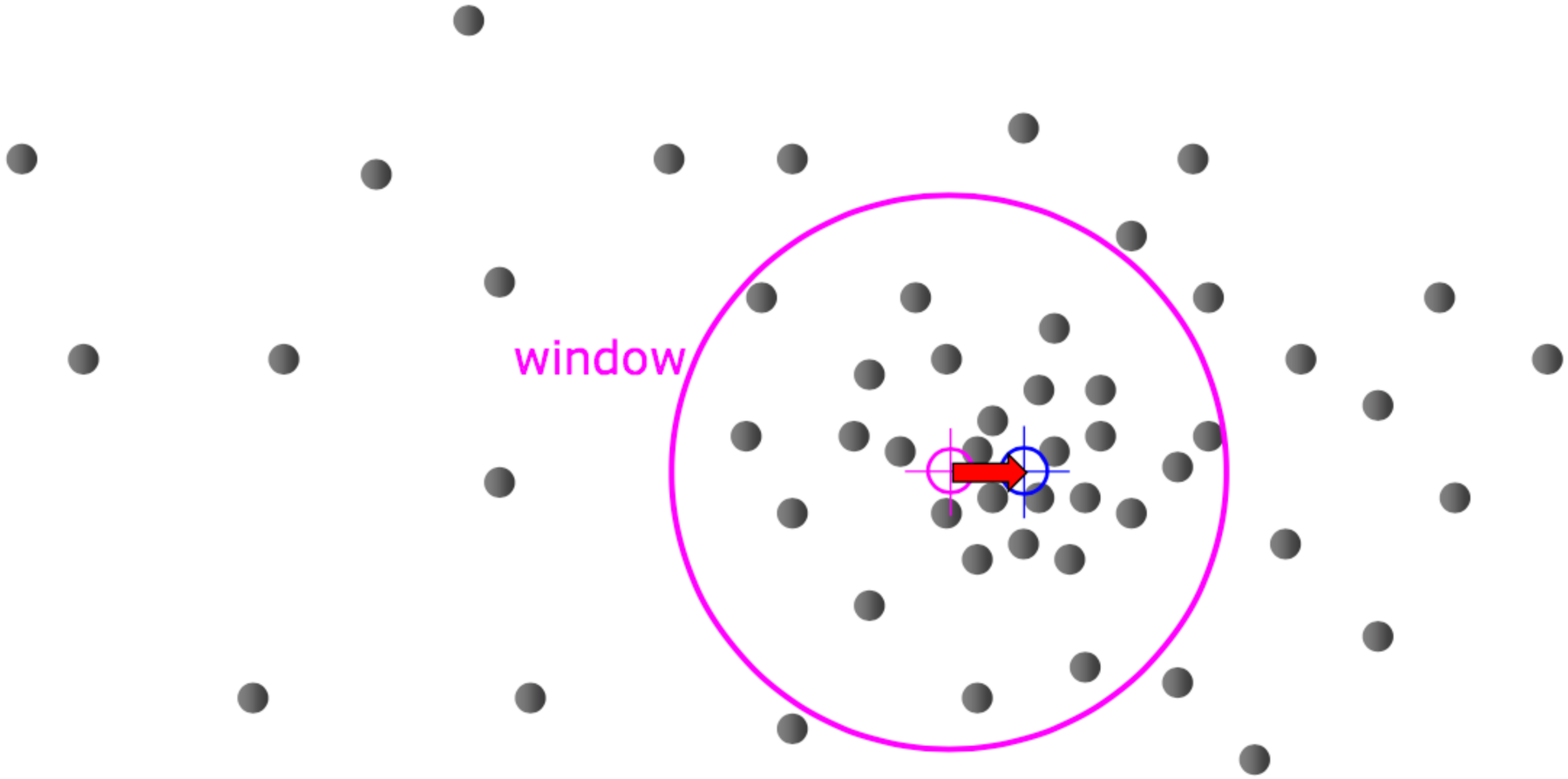

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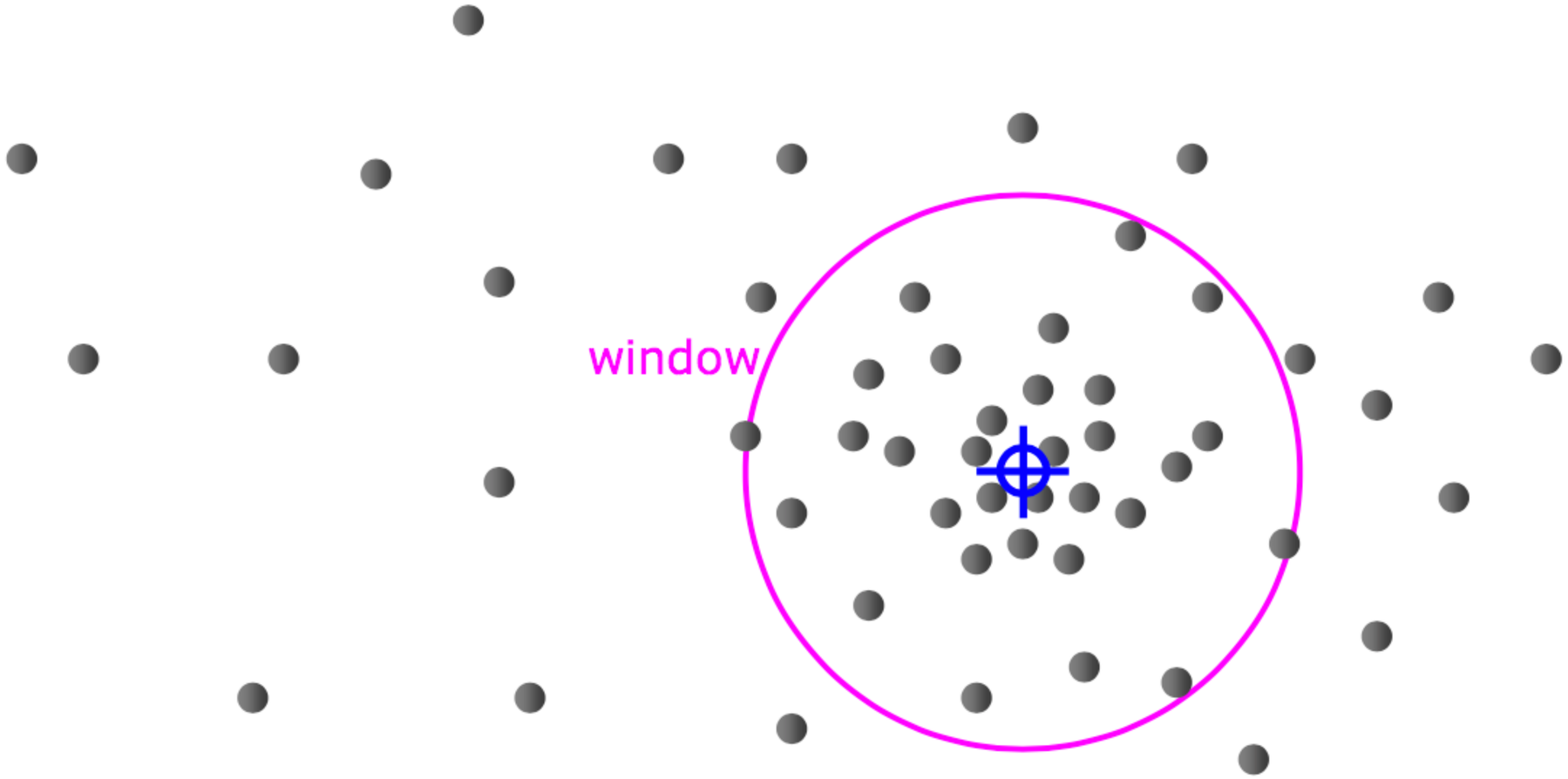
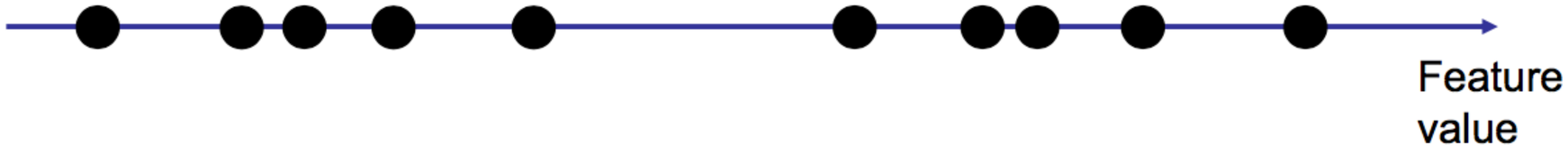
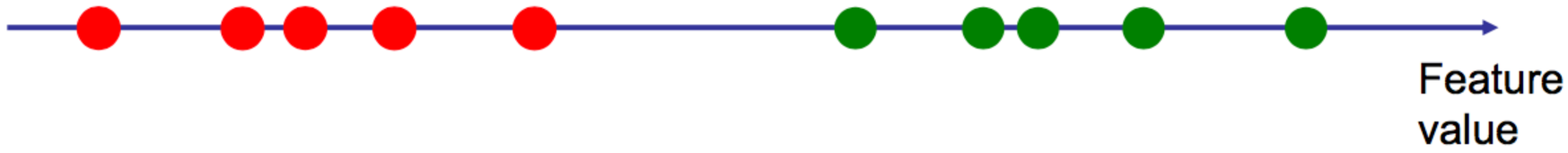


A 1-D Example



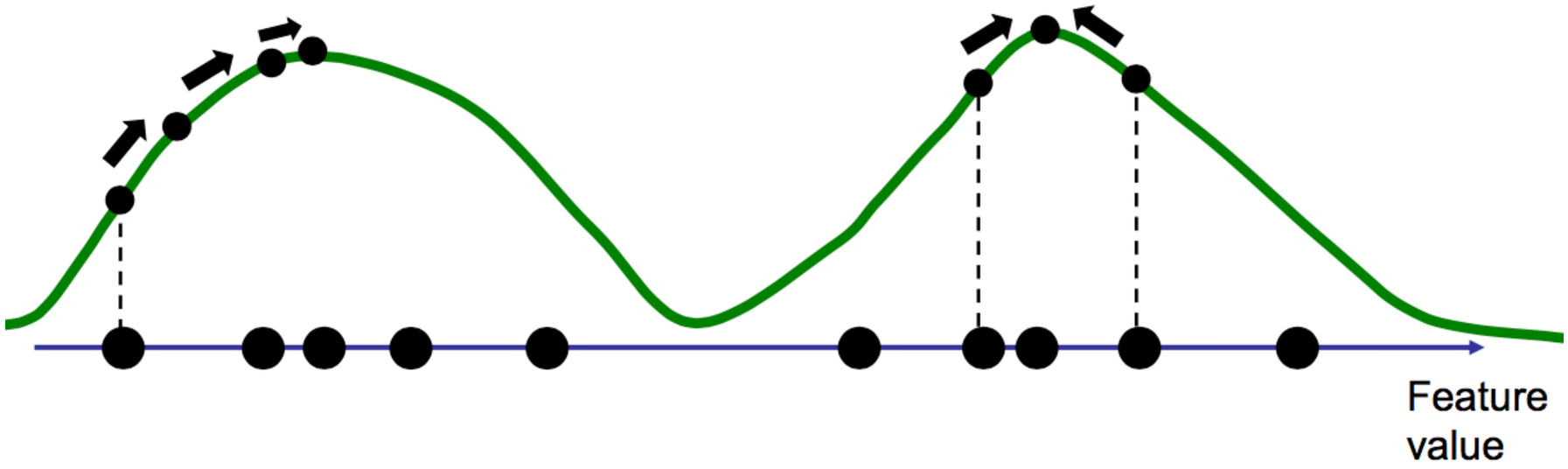
- Consider a set of points in a boring, one-dimensional feature space

A 1-D Example



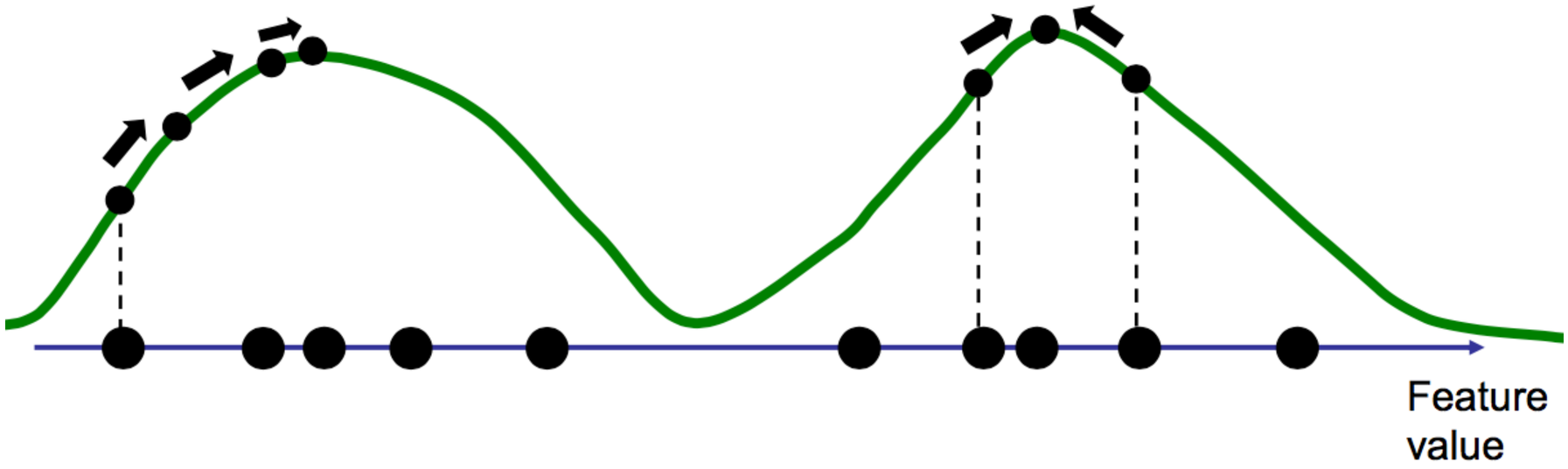
- Obviously, we would like to generate two groups, corresponding to the two parts of the feature space in which we have a *high density* of points
- How can we capture this notion of "high density" → *kernel density estimation*

A 1-D Example



- If we had a continuous function instead of a bunch of data points, we could find the maxima by gradient ascent.
- How can we convert our set of points to a continuous function?

A 1-D Example



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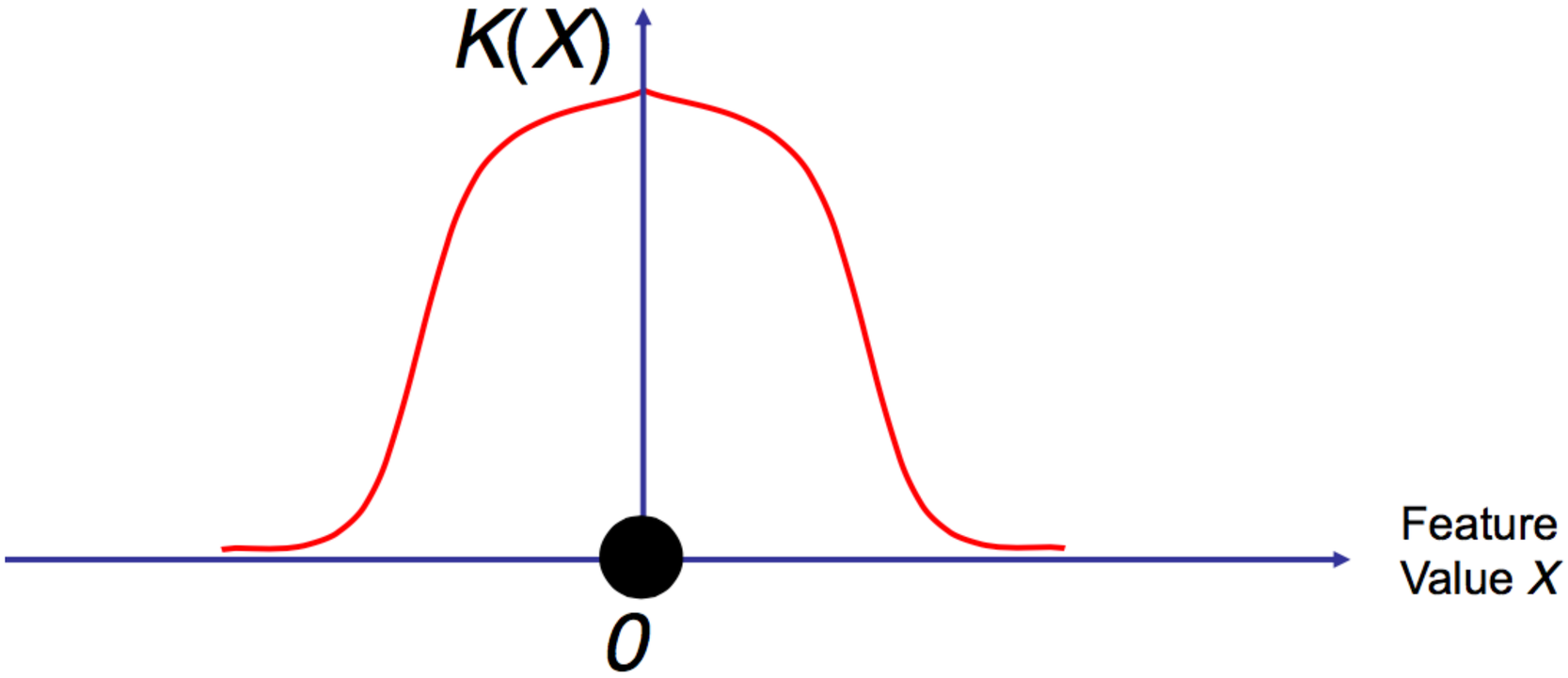
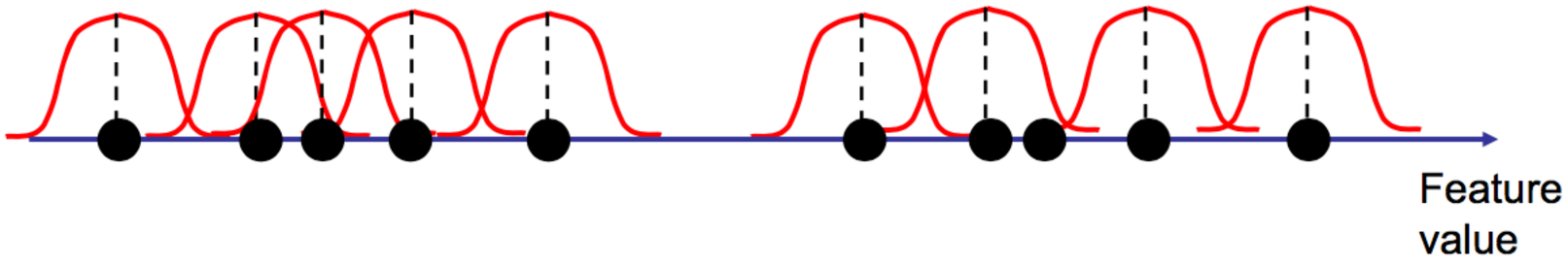


- Let us define a kernel function: $K(X)$, with the properties:
- K decays to zero far from 0
- K is maximum at 0
- K is symmetric

A 1-D Example

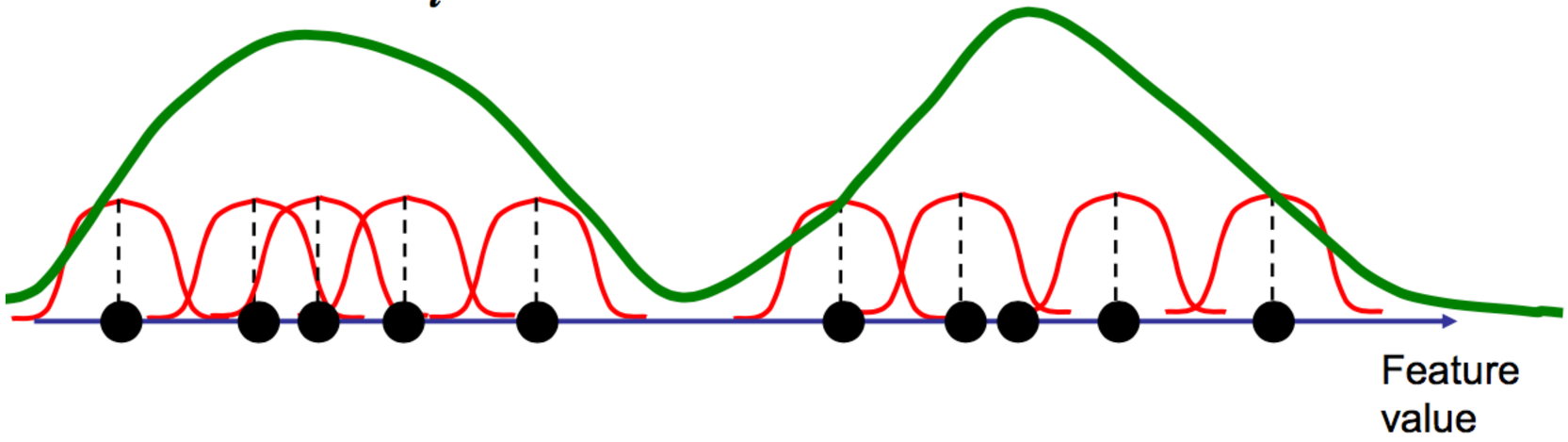


- We can define the kernel at each data point and sum up the result into a single function:

$$f(X) = \frac{1}{N} \sum_i K(X - X_i)$$

A 1-D Example

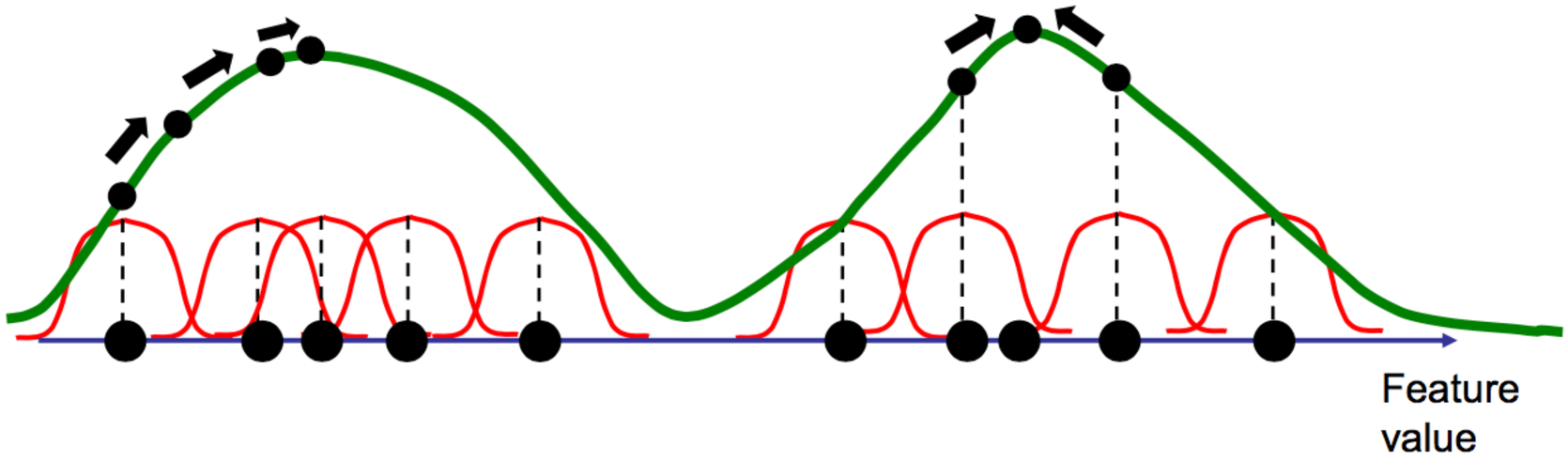
$$f(X) = \frac{1}{N}\sum_i K(X - X_i)$$



- *V* is a normalization term
- *f(X) approximates the probability that feature X is observed given the data points*
- The maxima of *f* (the modes of the pdf) correspond to the clusters in the data

A 1-D Example

$$f(X) = \frac{1}{N}\sum_i K(X - X_i)$$



- If we move each point in the direction of the gradient, we will converge to the closest mode
- How can we do this efficiently?

A 1-D Example

- For $i = 1,..,N$

  $$\boldsymbol{X} \leftarrow \boldsymbol{X}_i$$

  – Repeat

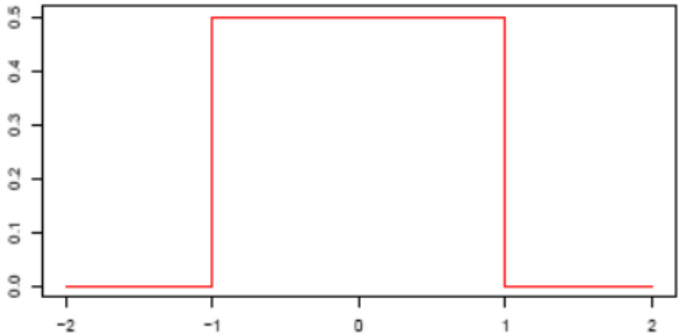
  $$\boldsymbol{X} \leftarrow \boldsymbol{X}_i + \nabla \boldsymbol{f}(\boldsymbol{X}) = \boldsymbol{X}_i + \frac{1}{\boldsymbol{N}} \sum_{\boldsymbol{i}} \nabla \boldsymbol{K}(\boldsymbol{X} - \boldsymbol{X}_i)$$

  – Until X does not change

Example kernels

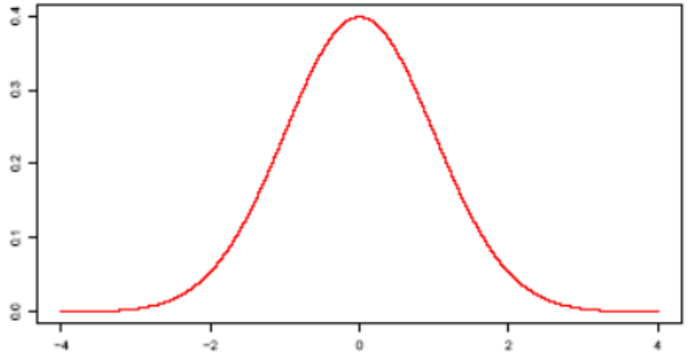
Uniform:

$$K_U(\mathbf{x}) = \begin{cases} c & \|\mathbf{x}\| \le 1 \\ 0 & \text{otherwise} \end{cases}$$



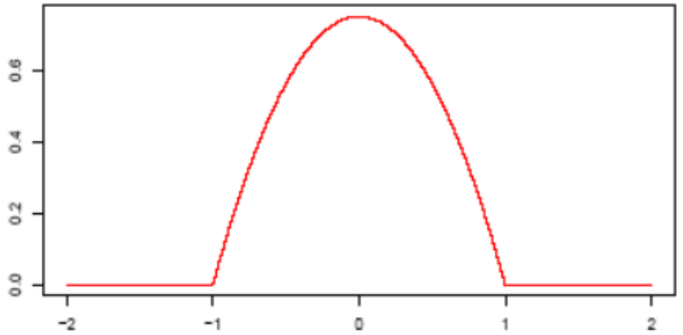
Gaussian:

$$K_N(\mathbf{x}) = c \cdot \exp\left(-\frac{1}{2} \|\mathbf{x}\|^2 \right)$$



Epanechnikov:

$$K_E(\mathbf{x}) = \begin{cases} c\left(1 - \|\mathbf{x}\|^2\right) & \|\mathbf{x}\| \le 1 \\ 0 & \text{otherwise} \end{cases}$$



Bandwidth

- Kernel is defined as:

$$K(X) = ck\left(\left\|\frac{X}{h}\right\|^2\right)$$

- *h* is the bandwith of the kernel
- *k*(.) is:
  - For Gaussian: $k(t) = e^{-t/2}$
  - For Epanechnikov: $k(t) = \begin{cases} (1-t) \text{ if } |t| < 1 \\ 0 \end{cases}$

Bandwidth

- Kernel is defined as:

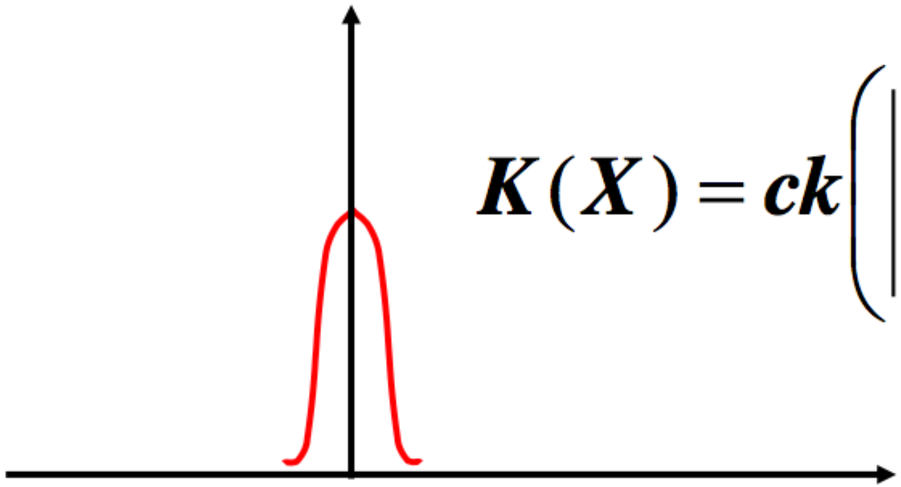
$$K(X) = ck\left(\left\|\frac{X}{h}\right\|^2\right)$$

- *h* is the bandwith of the ke
- *k*( ) is:

Bandwidth *h* controls the radius of influence of each data point.

- Too small: Overfits the data points
- Too large: Smoothes out the details of the data

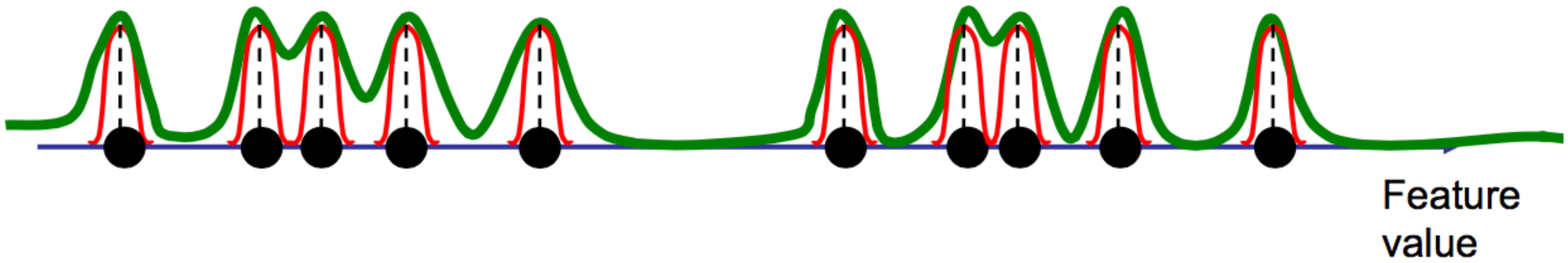
$$k(t) = \begin{cases} \\ 0 \end{cases}$$

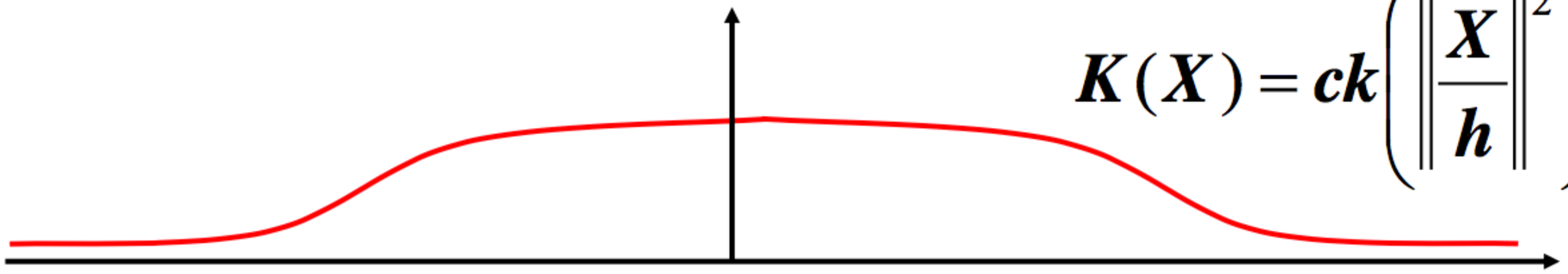


$$K(X) = ck\left(\left\|\frac{X}{h}\right\|^2\right)$$

h too small: The pdf overfits the noise in the data → Too many modes

$$f(X) = \sum_i ck\left(\left\|\frac{X - X_i}{h}\right\|^2\right)$$



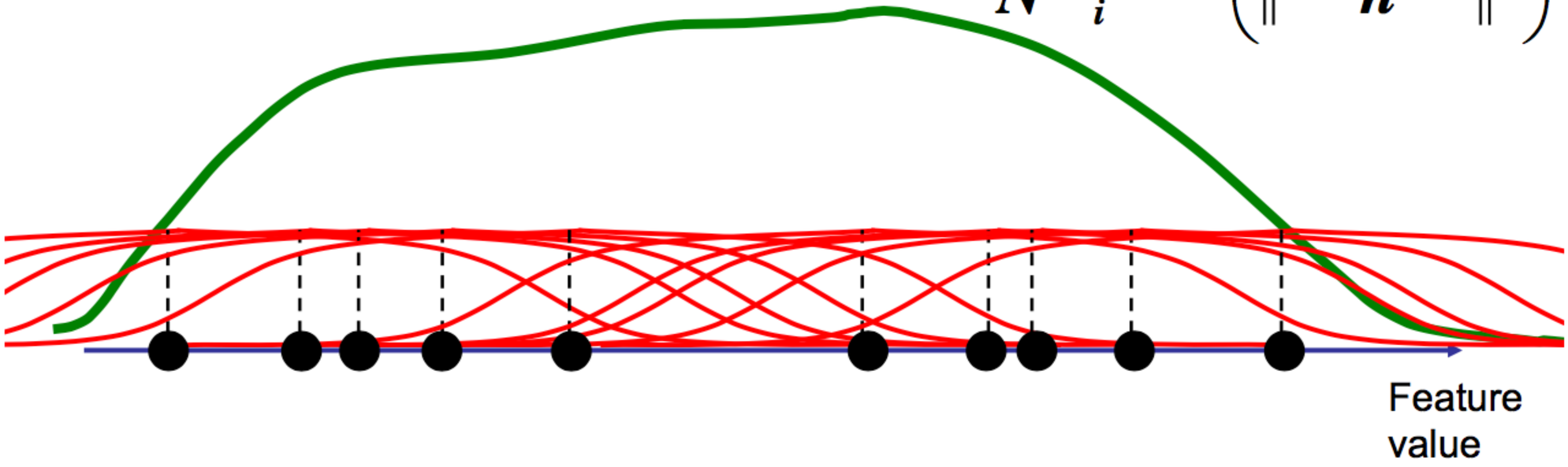


$$K(X) = ck\left(\left\|\frac{X}{h}\right\|^2\right)$$

h too large: The details of the initial data are smoothed out

→ Too few modes

$$f(X) = \frac{1}{N}\sum_i ck\left(\left\|\frac{X - X_i}{h}\right\|^2\right)$$



Choice of Kernel

- The kernel must satisfy a few technical conditions (*aka Parzen windows*).
- Integrates to 1 so that *f*(.) is a pdf: $\int_{R^d} K(\mathbf{x})d\mathbf{x} = 1$
- Symmetric
- Decays quickly (exponentially) as |*x*| increases: $\lim_{\|\mathbf{x}\|\to\infty} \|\mathbf{x}\|^d K(\mathbf{x}) = 0$
- The extent of the kernel is the same along all the dimensions: $\int_{R^d} \mathbf{x}\mathbf{x}^T K(\mathbf{x})d\mathbf{x} = c\mathbf{I}$

Computing the Gradient

- Now we have a representation of the pdf from which, in principle, we can find the modes by following the gradient.
- How can we do this efficiently?
- Notations:

$$g(t) = -k'(t)$$

- Gradient of each individual entry in the sum defining $f(.)$:

$$\nabla K(X - X_i) = \nabla\left(ck\left(\frac{\|X - X_i\|^2}{h^2}\right)\right) = \frac{2c}{h^2}(X_i - X)g\left(\frac{\|X - X_i\|^2}{h^2}\right)$$

Computing the Gradient

- Gradient of the entire pdf:

$$\nabla f(X) = \frac{1}{N}\sum_i \nabla K(X - X_i) = \frac{2c}{Nh^2}\sum_i (X_i - X) g\left(\frac{\|X - X_i\|^2}{h^2}\right)$$

$$\Downarrow$$

$$\nabla f(X) = \left(\frac{2c}{Nh^2}\sum_i g\left(\frac{\|X - X_i\|^2}{h^2}\right)\right)\left(\frac{\sum_i X_i g\left(\frac{\|X - X_i\|^2}{h^2}\right)}{\sum_i g\left(\frac{\|X - X_i\|^2}{h^2}\right)} - X\right)$$

$$\nabla f(X) = \left(\frac{2c}{Nh^2} \sum_i g\left(\frac{\|X - X_i\|^2}{h^2} \right) \right) \underbrace{\left(\frac{\sum_i X_i g\left(\frac{\|X - X_i\|^2}{h^2} \right)}{\sum_i g\left(\frac{\|X - X_i\|^2}{h^2} \right)} - X \right)}$$

Mean shift vector, $M(X)$ = Difference between X and the mean of the data points weighted by $g(.)$ (points further from X count less)

- Key result: The mean shift vector points in the same direction as the gradient
- Solution: Iteratively move in the direction of the mean shift vector

The Mean Shift Algorithm

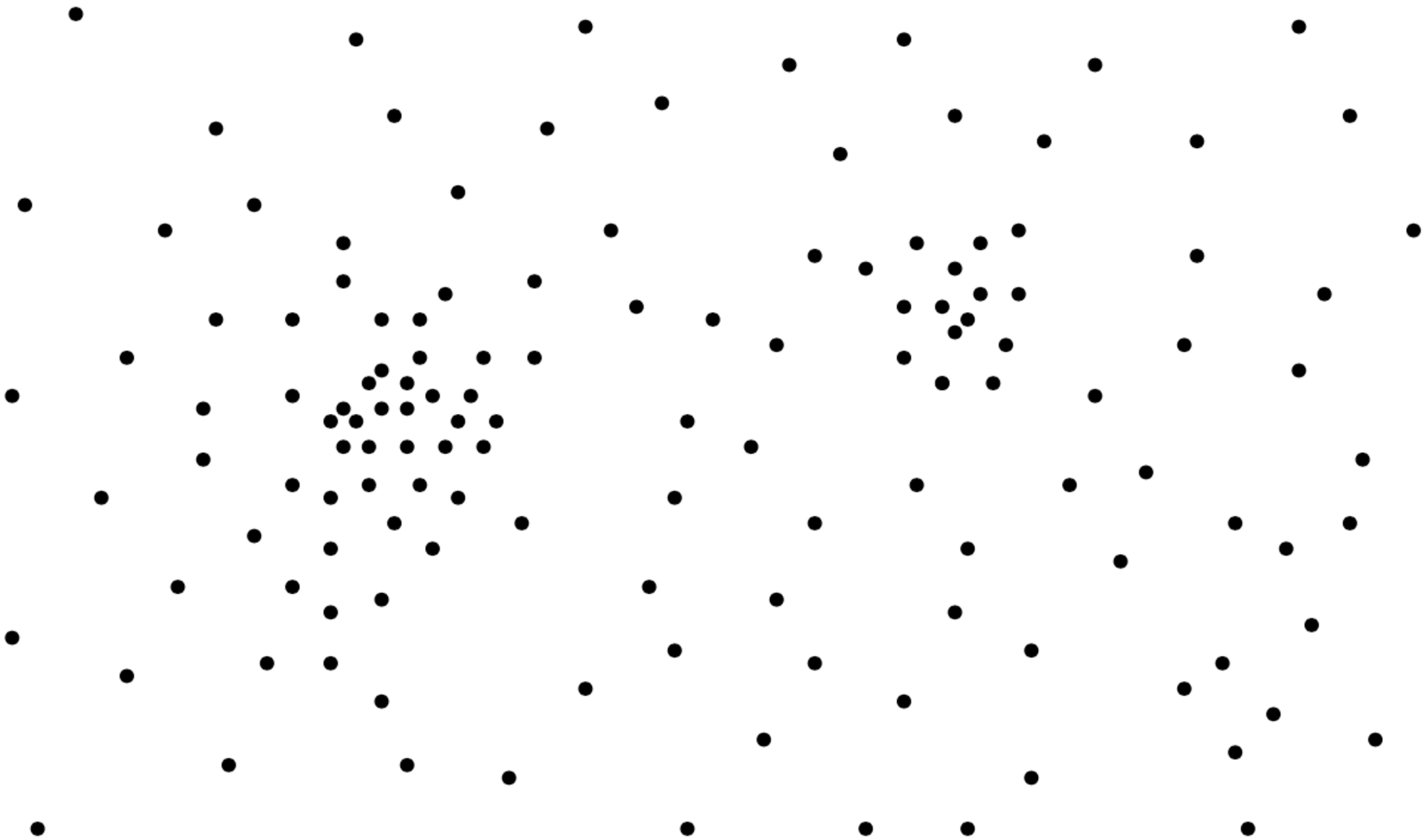
- Initialize: Set X to the value of the point to classify
- Repeat:
  - Move X by the corresponding mean shift vector:

$$X \leftarrow X + M(X) = \frac{\sum_i X_i g\left(\frac{\|X - X_i\|^2}{h^2}\right)}{\sum_i g\left(\frac{\|X - X_i\|^2}{h^2}\right)}$$

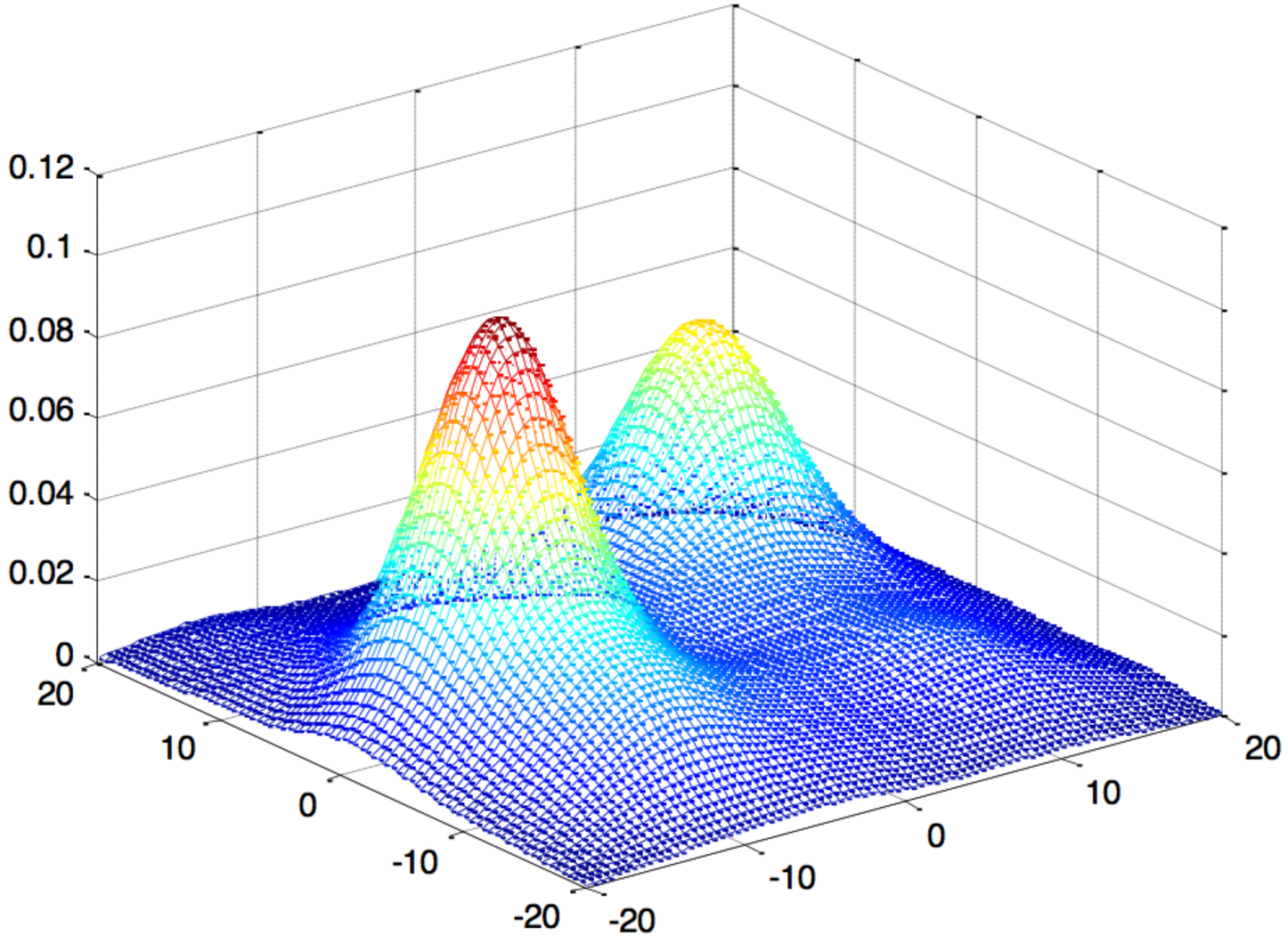
- Until X converges
- Note: Convergence is guaranteed.

2-D Example

$$f(X) = \frac{c}{N}\sum_{i=1}^{N} k\left(\left\|\frac{X - X_i}{h}\right\|^2\right)$$

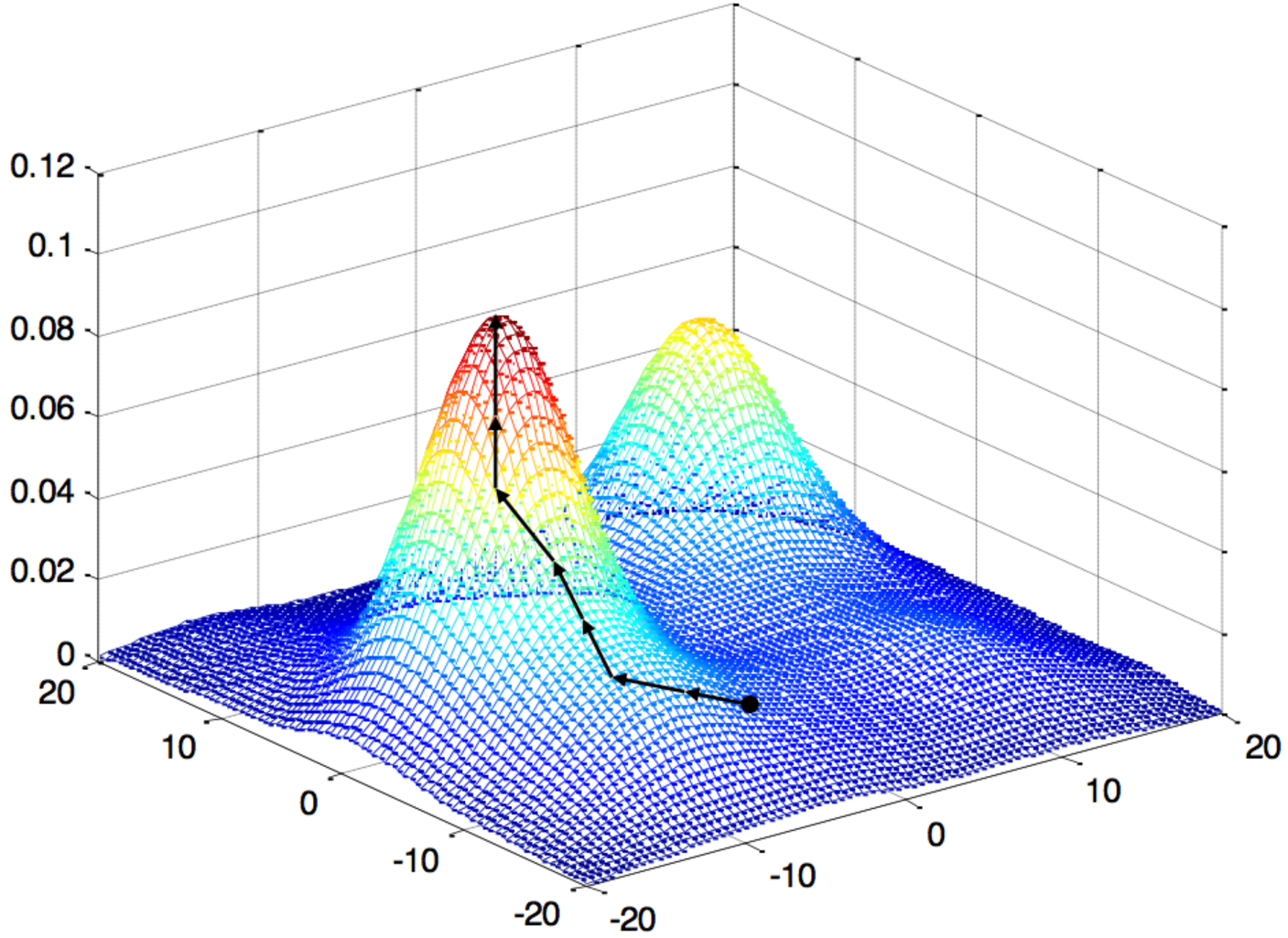


Estimated PDF: $$f(X) = \frac{c}{N}\sum_{i=1}^{N} k\left(\left\|\frac{X - X_i}{h}\right\|^2\right)$$



The trajectory of locations for finding modes

$$f(X) = \frac{c}{N}\sum_{i=1}^{N} k\left(\left\|\frac{X - X_i}{h}\right\|^2\right)$$



The Reality

- This is all much simpler than it looks!!
- For Epanechnikov:
  - $k(t) = (1-t)$ if $|t| < 1$, 0 otherwise
  - $g(t) = 1$ if $|t| < 1$, 0 otherwise
- So, the "mean" part of $M(X)$ is:

$$\boxed{\frac{\sum_i X_i g\left(\frac{\|X - X_i\|^2}{h^2}\right)}{\sum_i g\left(\frac{\|X - X_i\|^2}{h^2}\right)} = \frac{\sum_{\|X - X_i\| < h} X_i}{\sum_{\|X - X_i\| < h} 1} = \frac{\sum_{\|X - X_i\| < h} X_i}{N_h(X)}}$$

The Reality

This is *simply* the *average* of the data points within a radius *h* of *X*!!!

$$\frac{\sum_i X_i g\left(\frac{\|X - X_i\|^2}{h^2}\right)}{\sum_i g\left(\frac{\|X - X_i\|^2}{h^2}\right)} = \boxed{\frac{\sum_{\|X - X_i\| < h} X_i}{N_h(X)}}$$

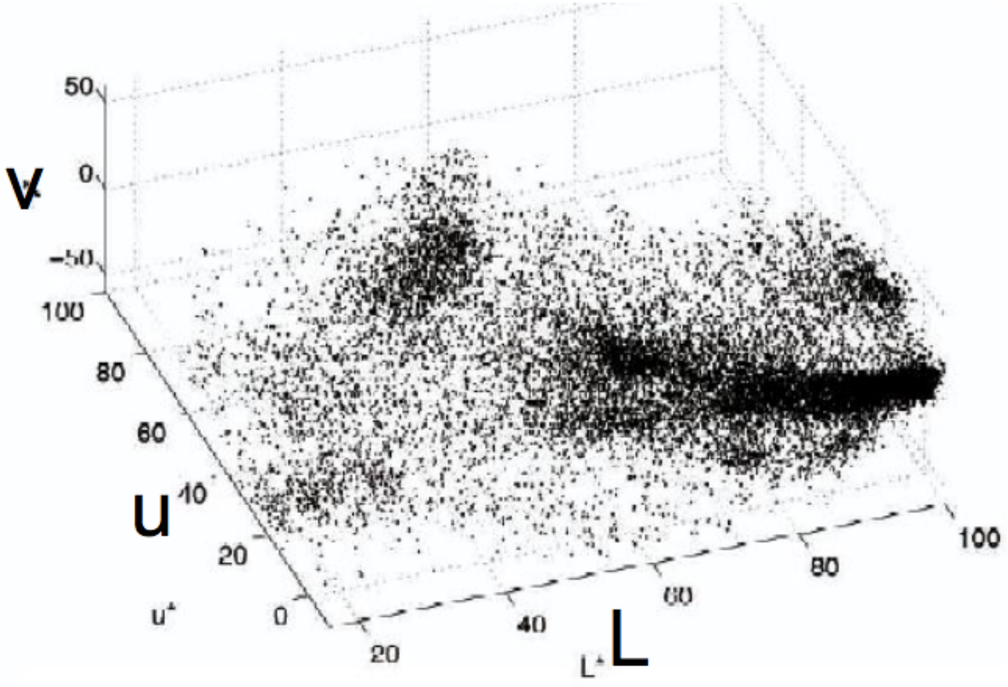
Number of data points within a radius *h* of *X*

Example : Color Segmentation

- Feature space: (L,u,v,x,y) → Intensity + (u,v) color channels + Position in image (x,y)
- Apply meanshift in the 5-dimensional space
- For each pixel (x_i,y_i) of intensity L_i and color (u_i,v_i), find the corresponding mode c_k
- All of the pixel (x_i,y_i) corresponding to the same mode c_k are grouped into a single region

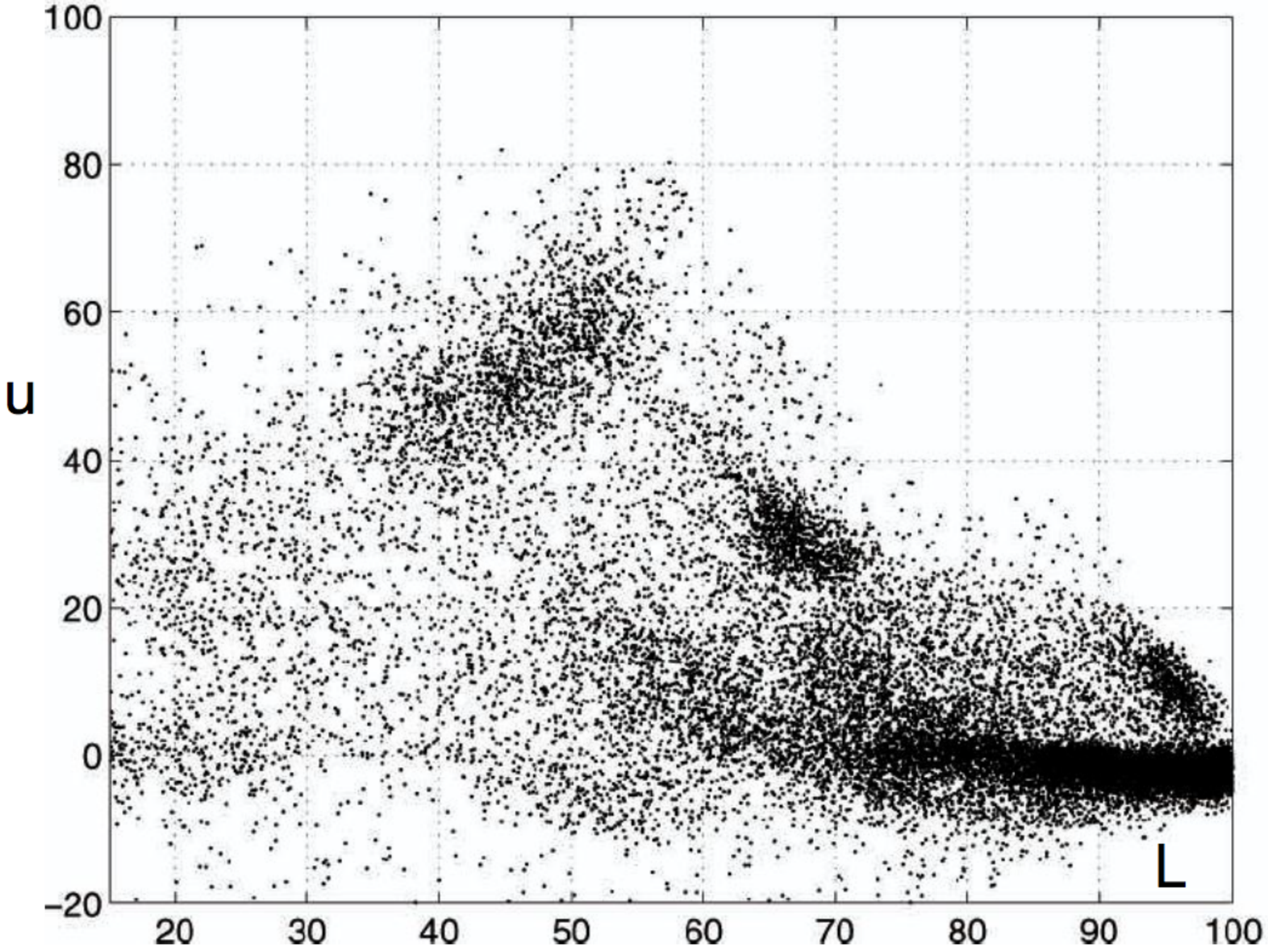
Example : Color Segmentation

Input Image



Luv Space ()

Example from D. Comaniciu and P. Meer, "Mean Shift: A Robust Approach Toward Feature Space Analysis".



110,400 data points.

$$K_{h_{pos}h_{col}}(X) = ck\left(\frac{\|X_{pos}\|^2}{h_{pos}^{\ 2}}\right)k\left(\frac{\|X_{col}\|^2}{h_{col}^{\ 2}}\right)$$

Kernel on position (*x*,*y*)

Kernel on color (*L*,*u*,*v*)

- *Good news:* We don't need to know the number of regions (modes, clusters).
- *Bad news:* We need to choose the bandwidths h_{pos} and h_{col}

The Mean Shift Process

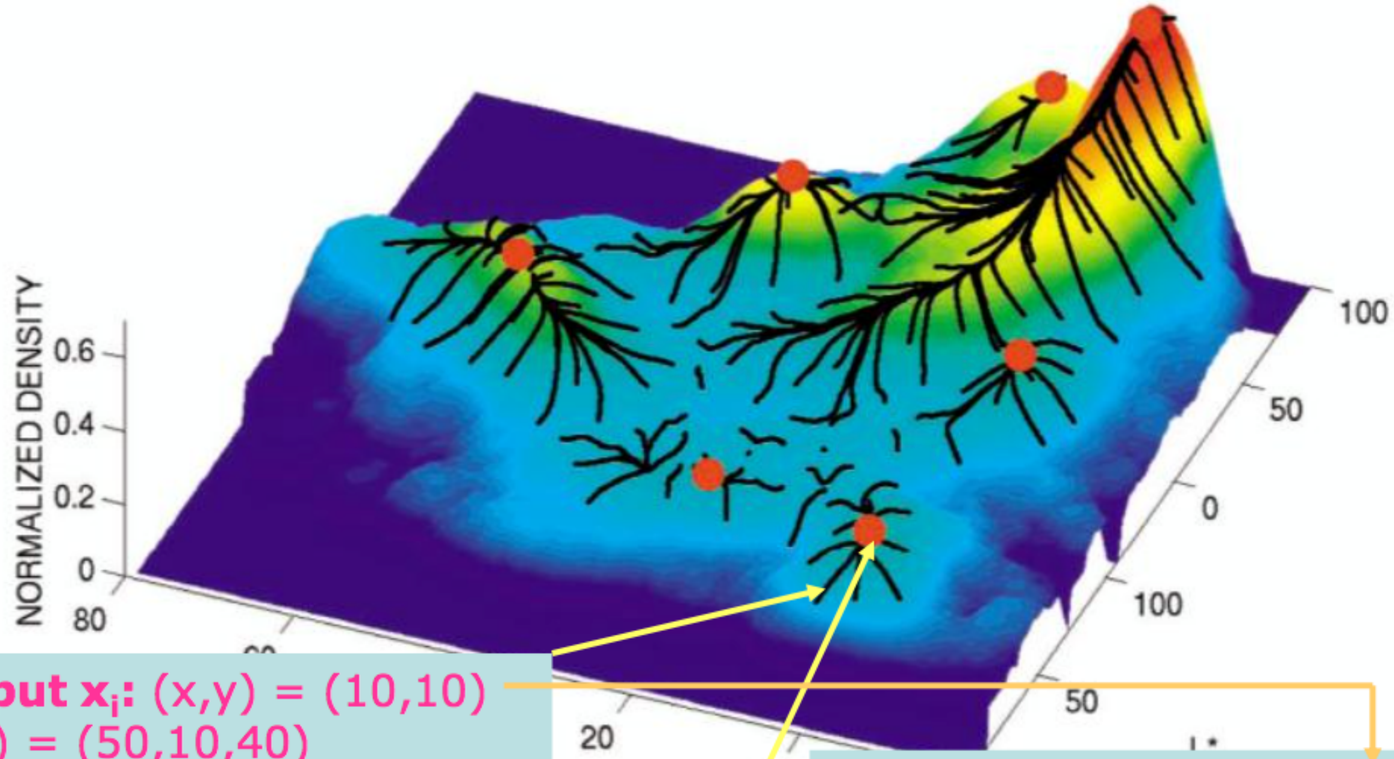
Notes:

- If we do not apply the last step, we get "smoothing"
→ Replacing each color by the closest mode

- The "color" part of the feature can be replaced by other things like texture (bank of filter outputs) or other values (multispectral). The only change is to increase the dimension *p* of the feature space

- The fundamental operation to compute the kernels is to find the neighbors within some radius (defined by *h*). This can be very expensive in high dimension with lots of points → Need smart "nearest-neighbor" data structures.

Example Color



1) Input x_i: (x,y) = (10,10)
(L,u,v) = (50,10,40)

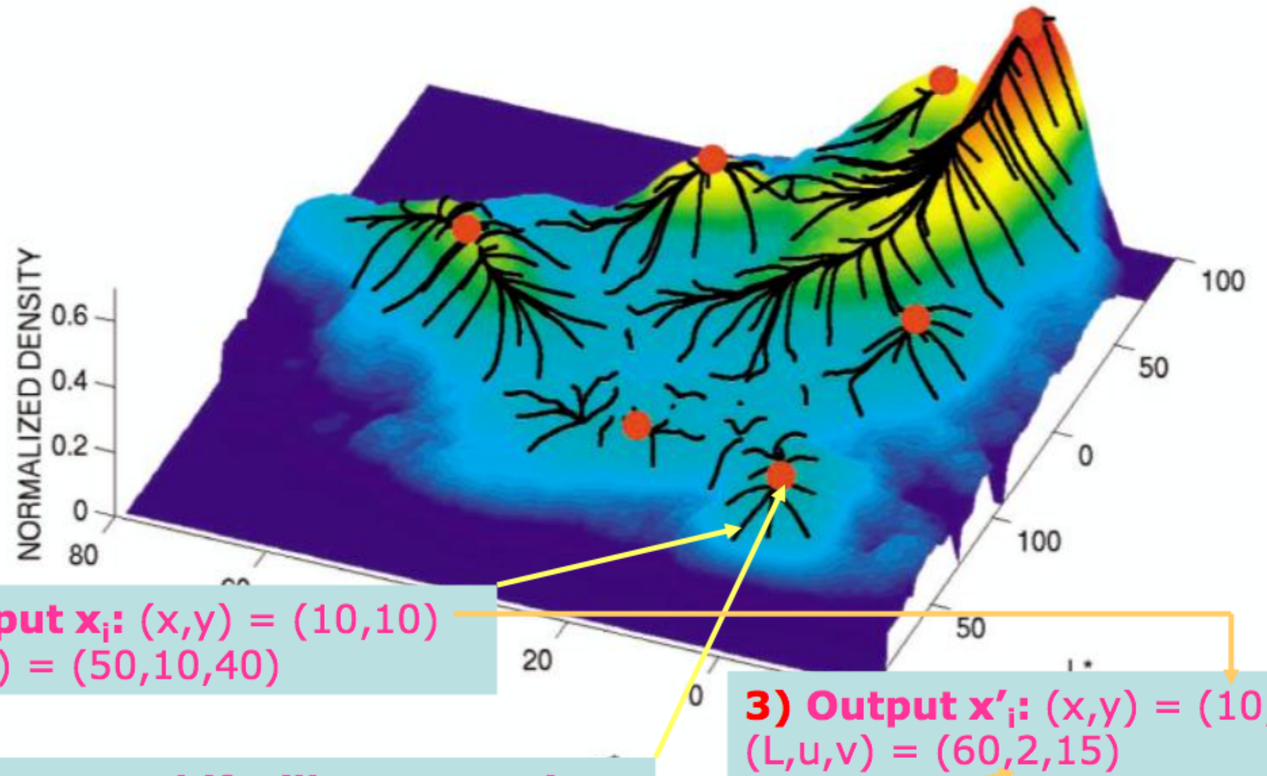
2) Apply mean shift till converged

c_i: (x,y) = (15,20) (L,u,v) = (60,2,15)

3) Output x'_i: (x,y) = (10,10)
(L,u,v) = (60,2,15)

D. Comaniciu and P. Meer, "Mean Shift: A Robust Approach Toward Feature Space Analysis".

Example Color



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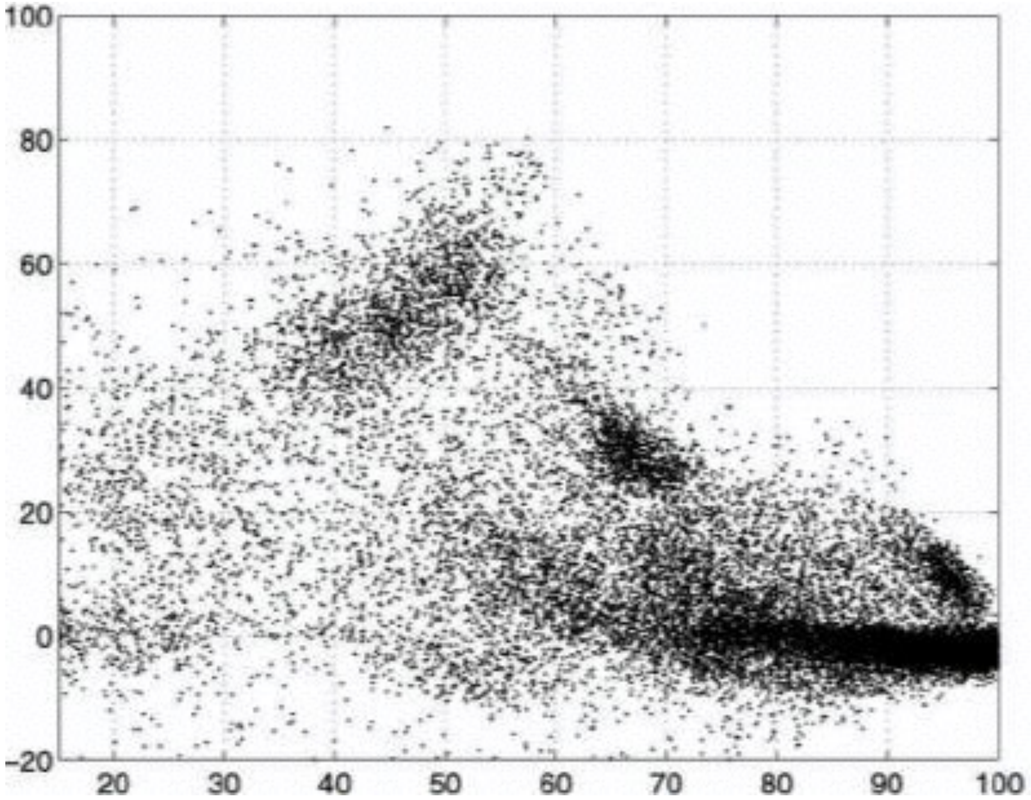
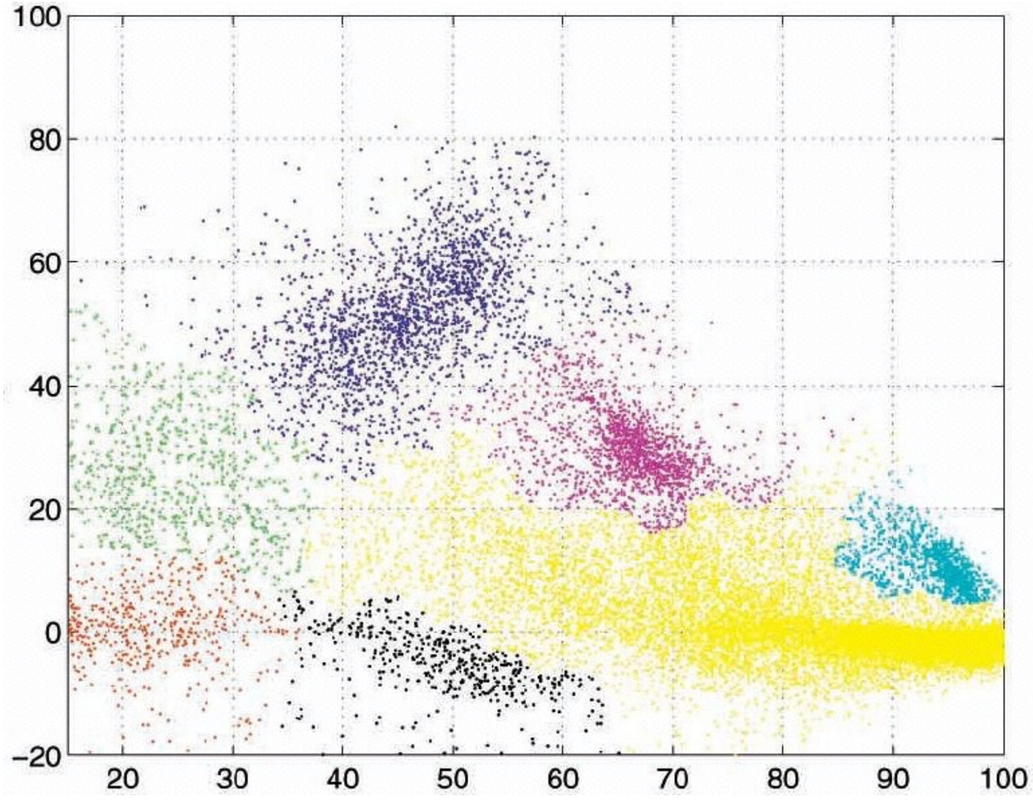
3) Output x'_i: (x,y) = (10,10)
(L,u,v) = (60,2,15)

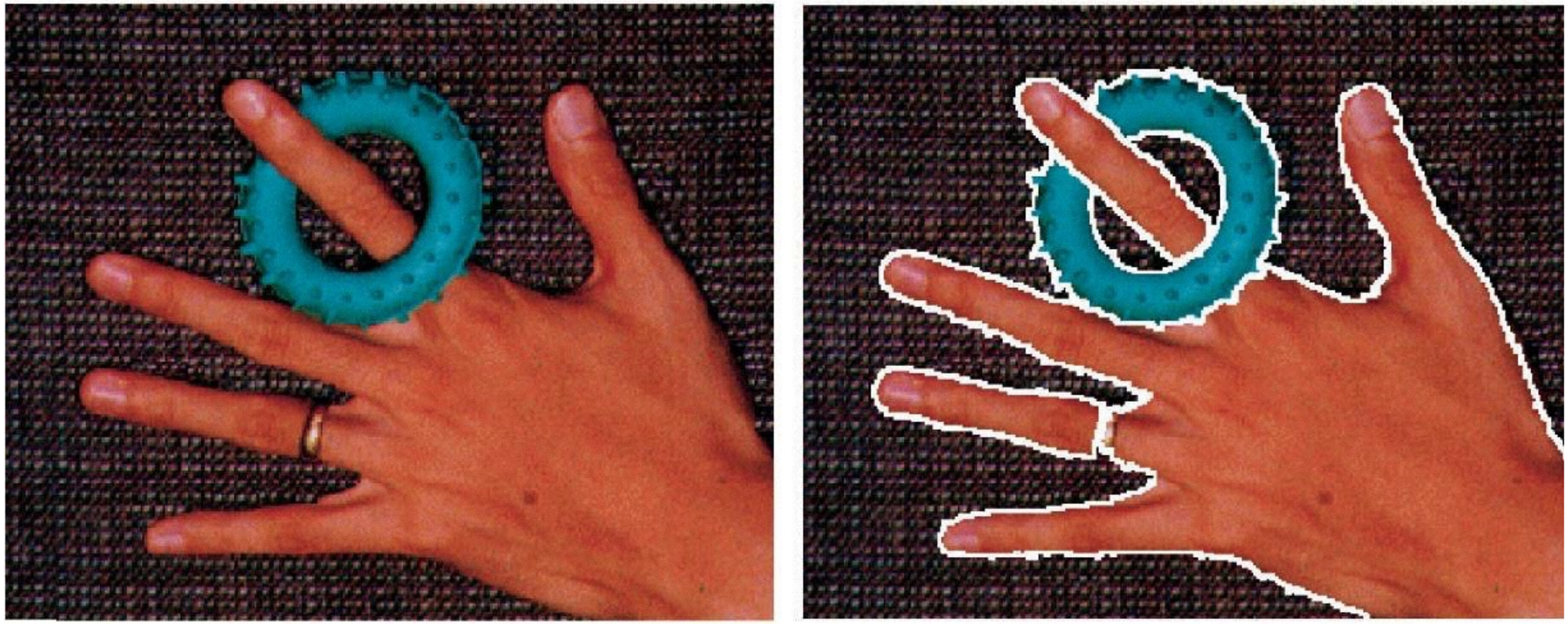
Note: In practice, all points may not converge to the same mode -→ Need an additional (easy) clustering step to group the converged locations to the location

Example Color

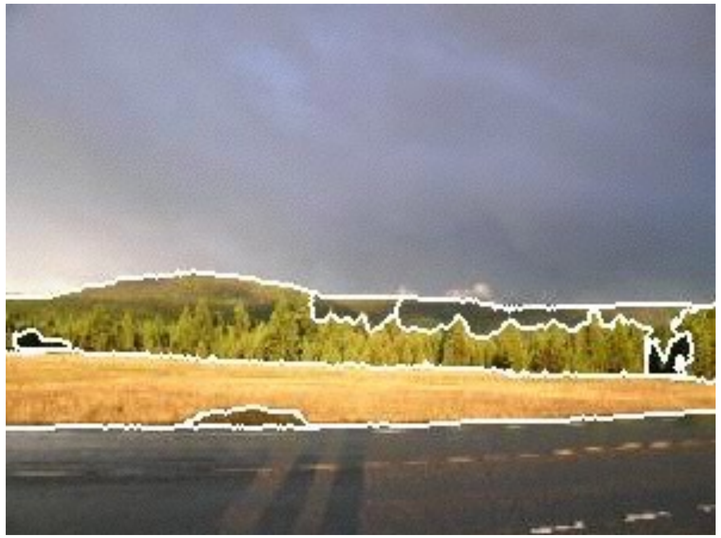


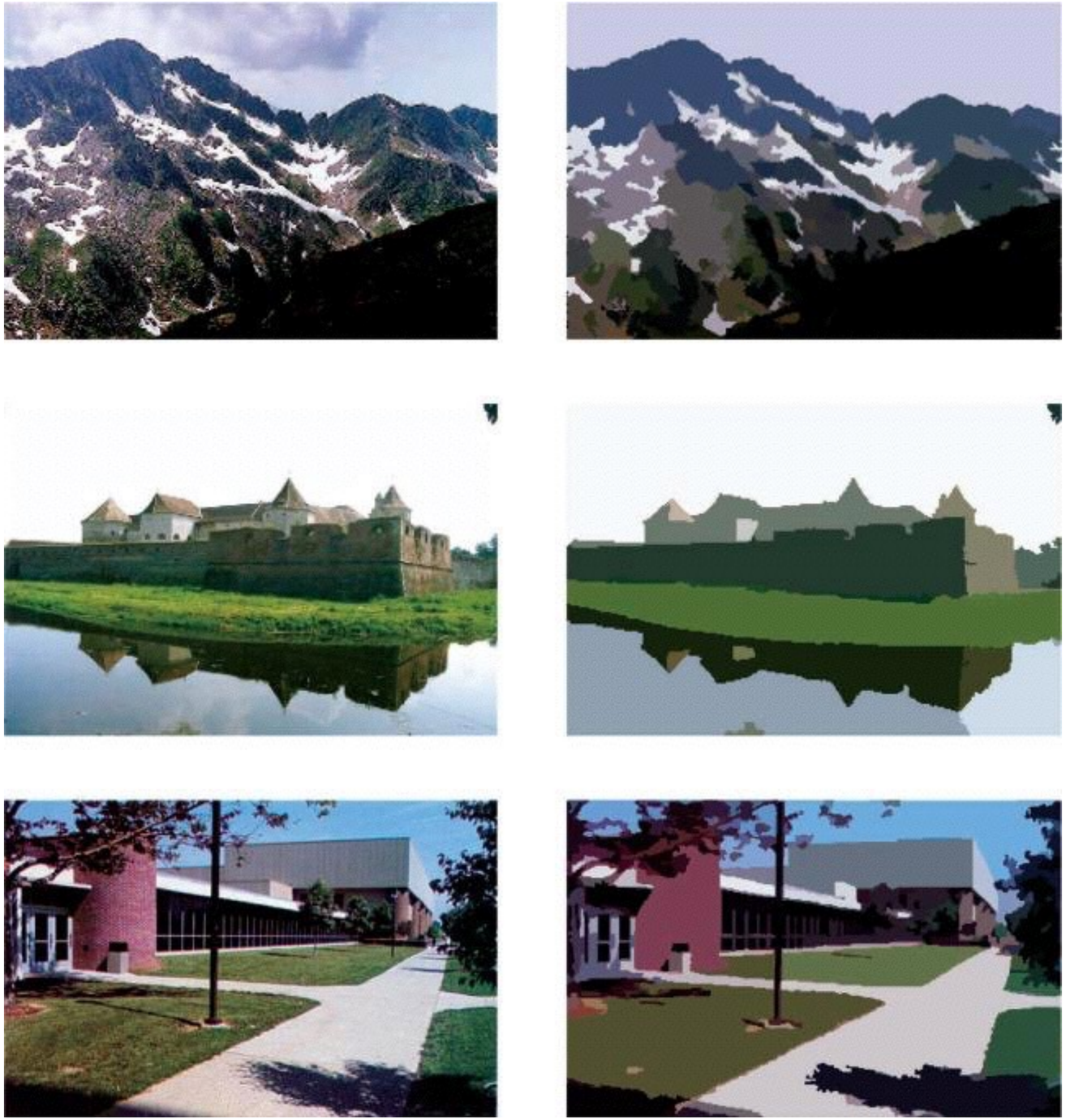


Experimental Results



Experimental Results





Main Approaches

- Clustering
- Mean shift
- Spectral techniques (skipped)
- Graph-based techniques
- Segmentation as boundary detection

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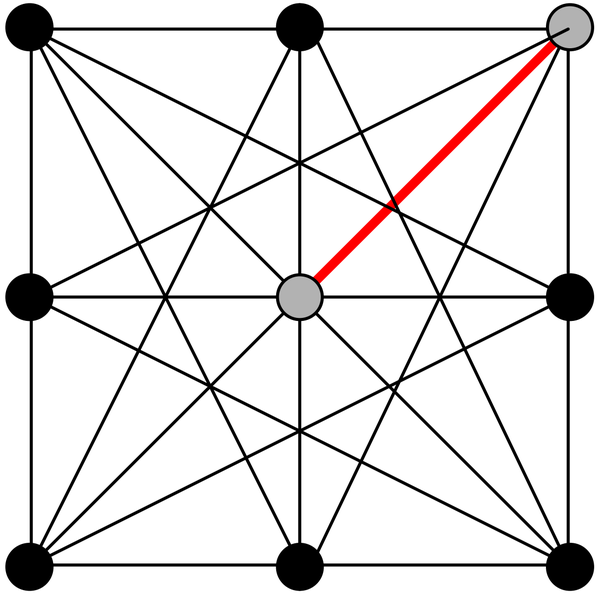
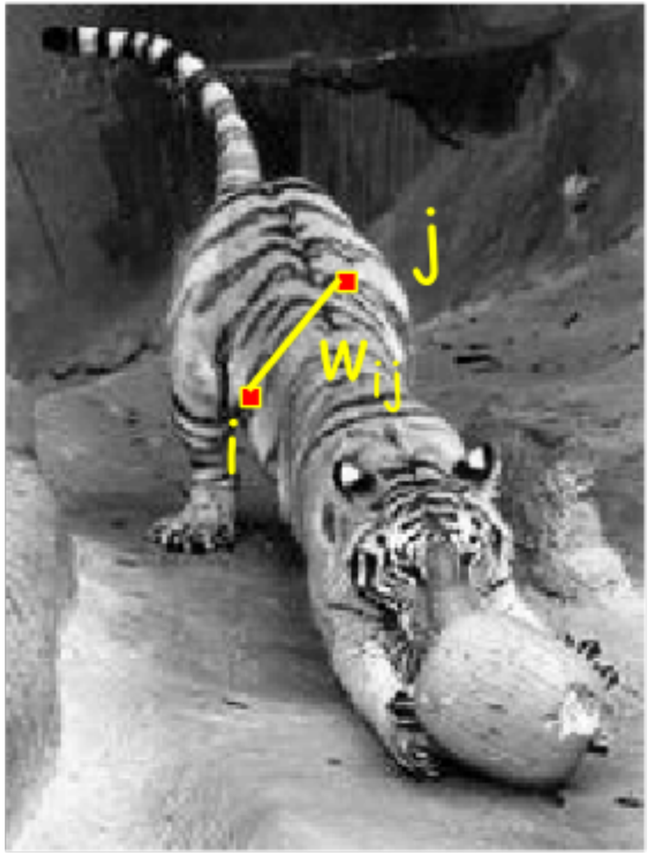
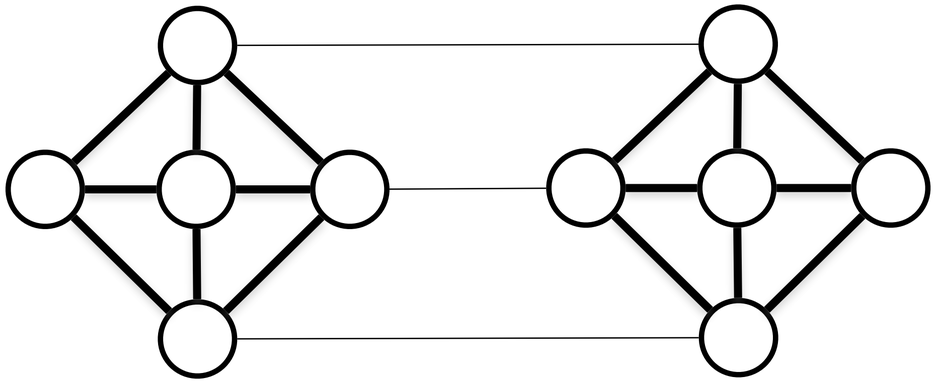
Images as Graphs





- Node for every pixel
- Edges between “neighbors”
- Edge weights/capacities are some measure of similarity

Connected Components



- Define similarity w_{ij} between nodes i,j
  - If similar, $w_{ij} \rightarrow 0$

Connected Components

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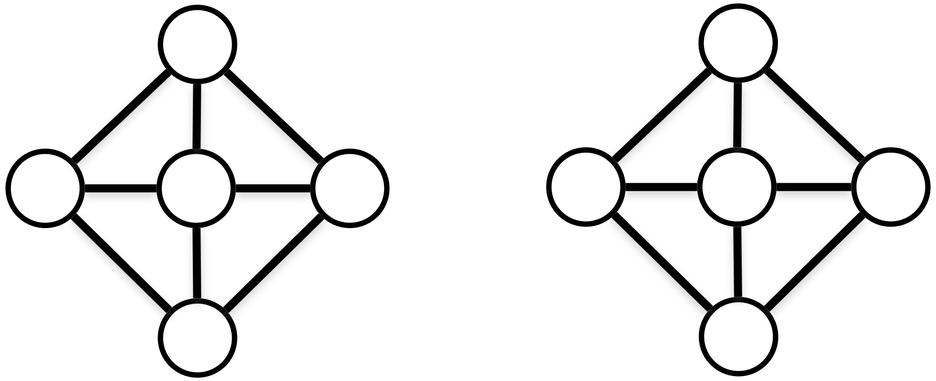

- Remove edges with $w_{ij} \geq \tau$

Connected Components

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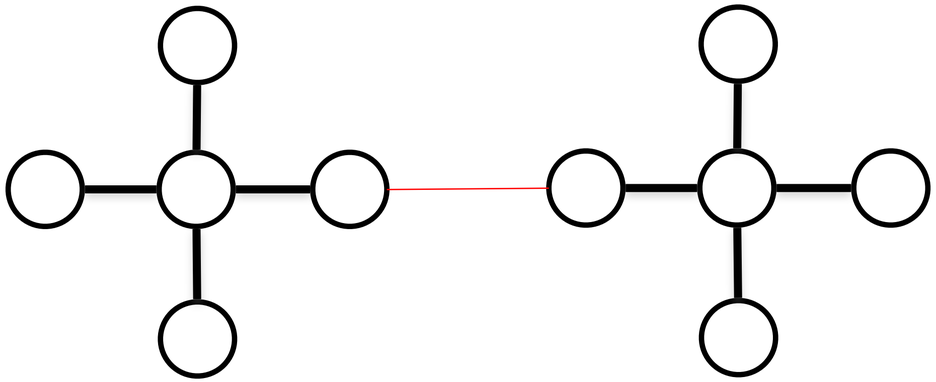
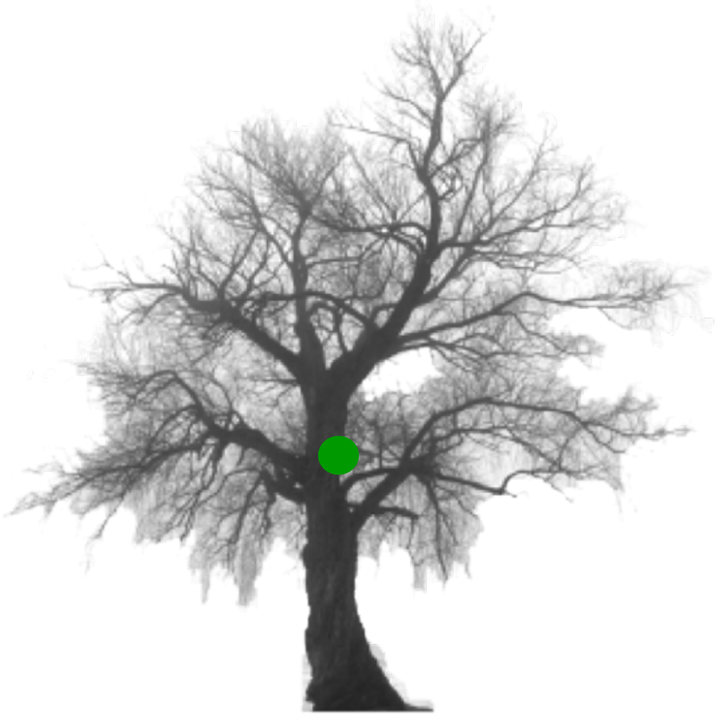


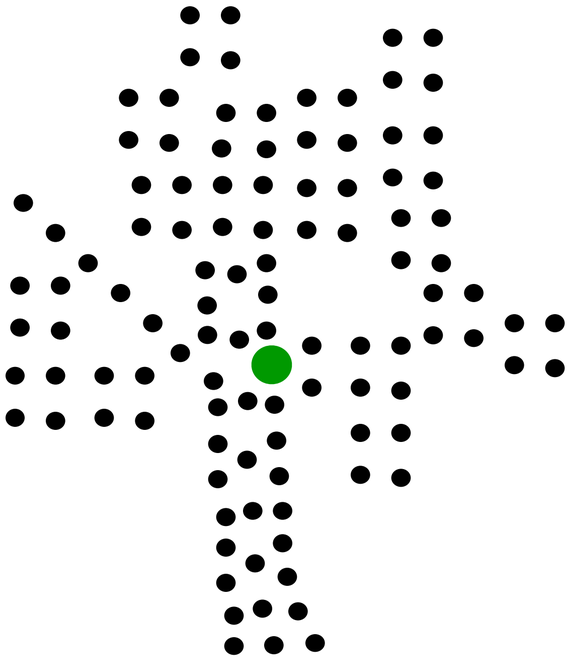
- Remove edges with $w_{ij} \geq \tau$
- Easily done by pruning the minimum spanning tree

Minimum Spanning Forests

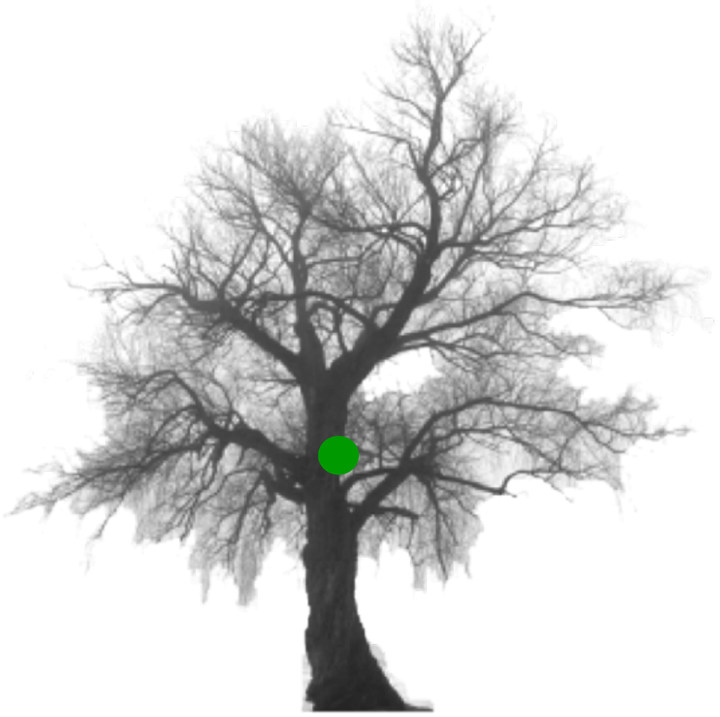
- Initialize each node as a component
- Iterate over w_{ij} in increasing order
  - If $w_{ij} \geq \tau$, STOP
  - If component(i) != component(j)
    - Merge(component(i), component(j))
- Problem: τ very sensitive
  - 1 incorrect merge can lead to creating a huge segment

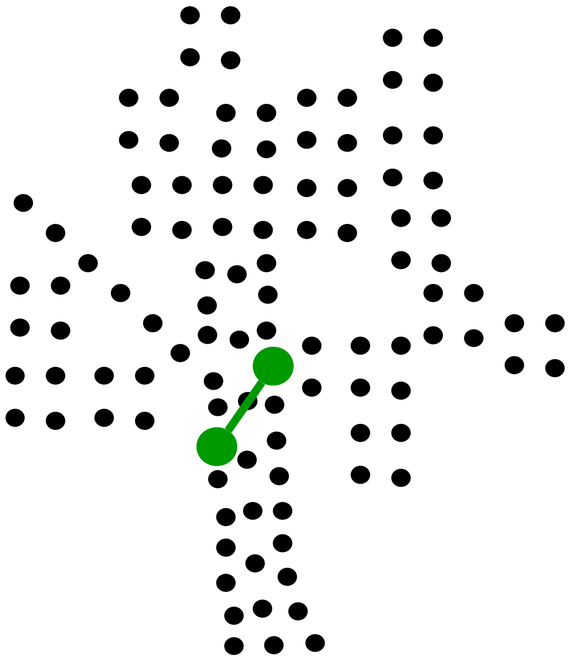
Graphical Approach : Higher Order Potentials





$$\mathbf{y}^* = \arg\max_{\mathbf{y}} \sum_{i=1}^{N} \boxed{\phi_i(y_i, \mathbf{x})} + \sum_{(ij)\in E} \phi_{ij}(y_i, y_j, \mathbf{x}) + \sum_{c\in S} \phi_c(\mathbf{y}_c, \mathbf{x})$$





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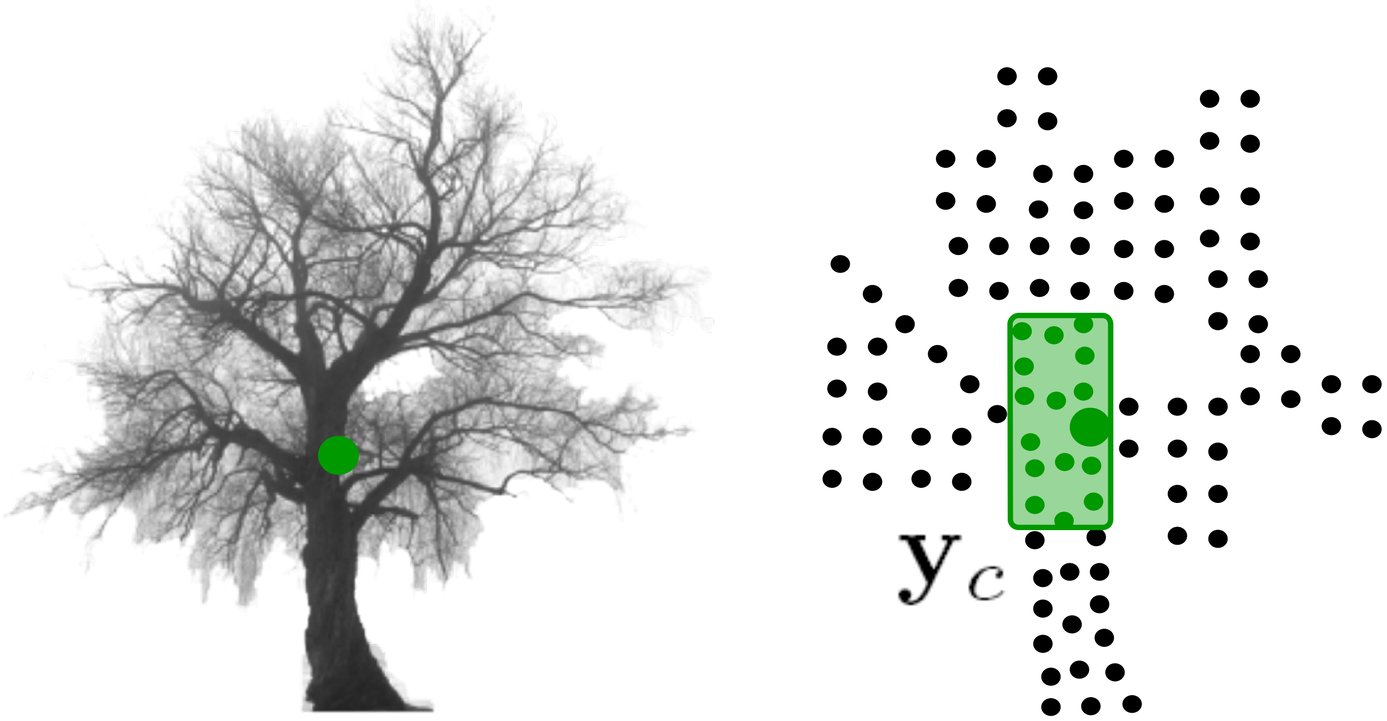

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- Inference: Kohli et al., “Robust Higher Order Potentials for Enforcing Label Consistency”, IJCV 2009
- Learning: Munoz et al., “Contextual Classification with Functional Max-Margin Markov Networks”, CVPR 2009

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