Motion, Tracking & Optical Flow II

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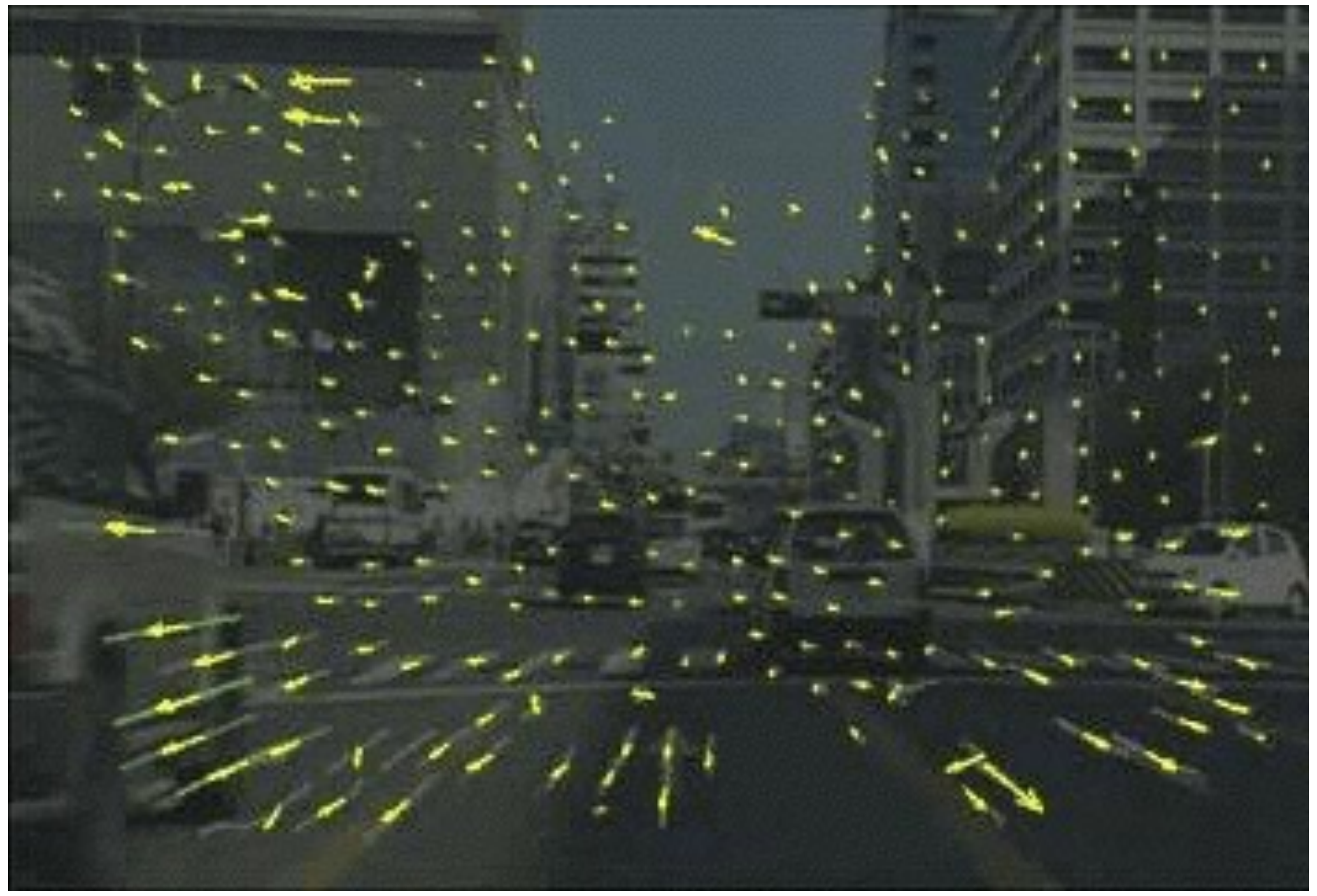
Szeliski Chapter 8

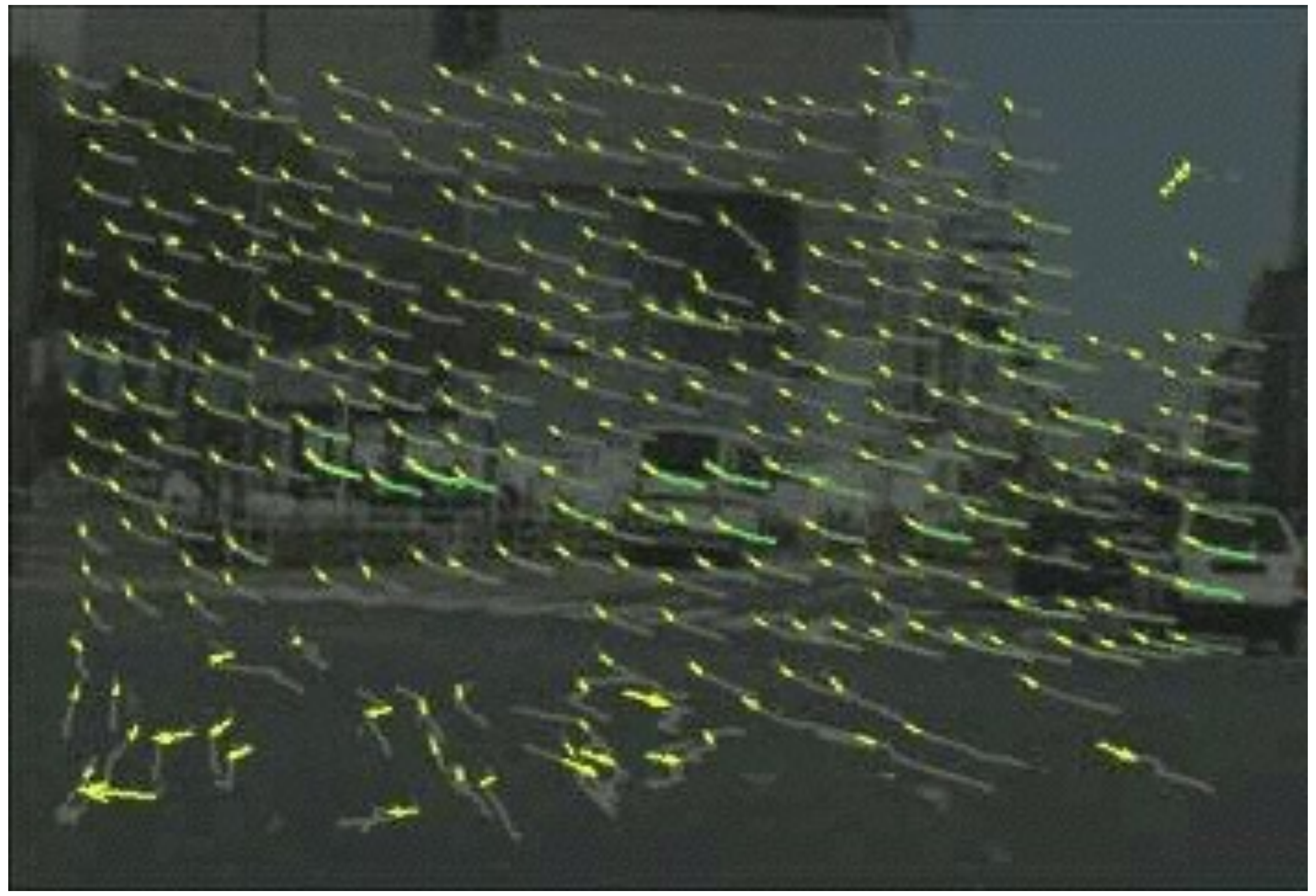
Summary so far…

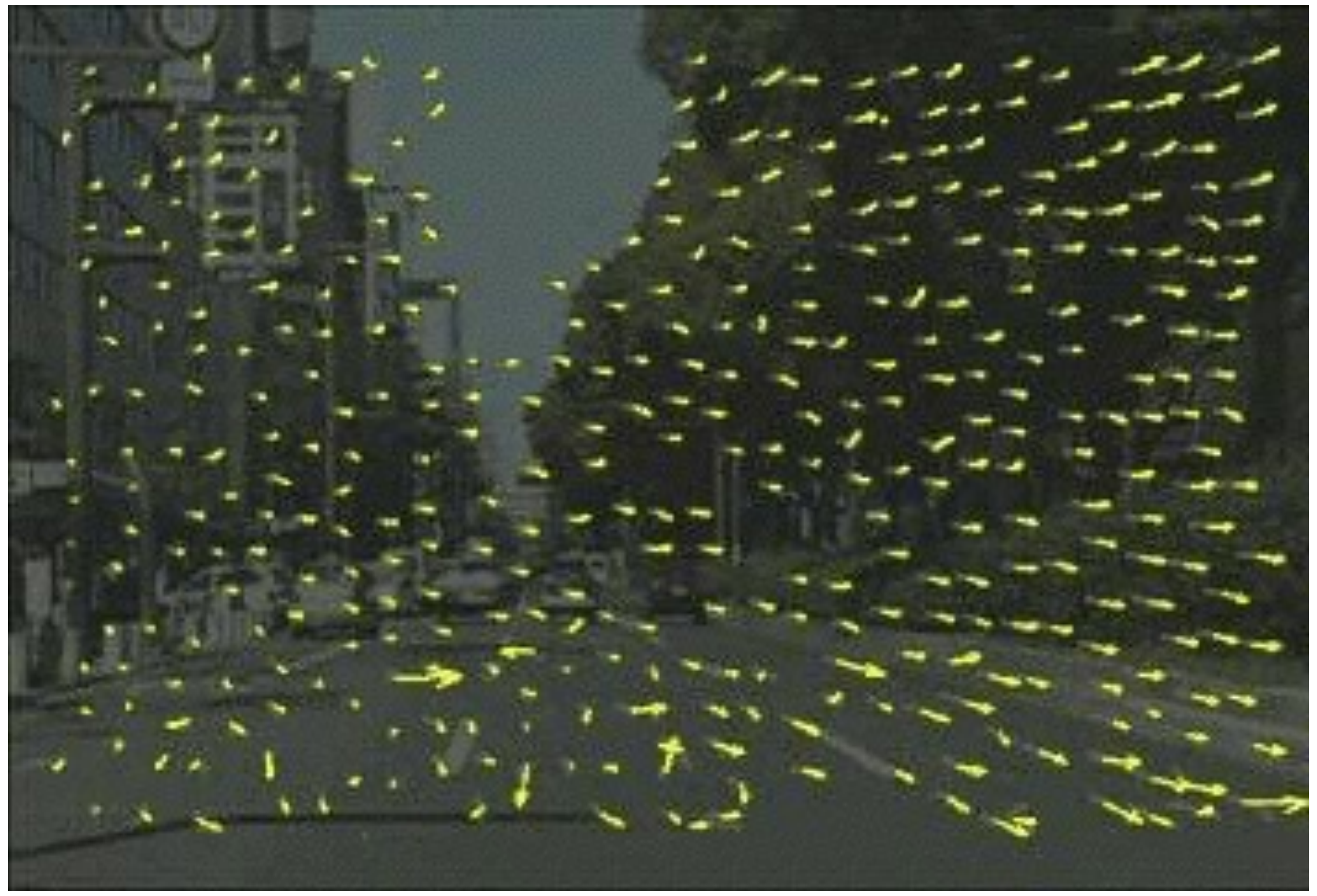
- Motion under constant brightness assumption
- Resolving ambiguities (constant flow, affine motion, planar motion)
- Lucas-Kanade Motion and Tracking
- Mean Shift Tracking
- Background Subtraction, Layer Motion Models etc.
- NOW: optical flow

Optical Flow

Estimate (u,v) at every pixel (x,y)







Optical Flow

- Possible approach: minimize

$$E_{intensity} = \sum_{(x,y)} \left(I_2\left(x + u(x,y), y + v(x,y)\right) - I_1(x,y)\right)^2$$

- Small motions:

$$E_{intensity}$$

$$= \sum_{(x,y)} \left(u(x,y)I_x + v(x,y)I_y\,(x,y) + I_t(x,y)\right)^2$$

Optical Flow

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$$E_{intensity} = \sum_{(x,y)} \left(I_2(x + u(x,y), y + v(x,y)) - I_1(x,y)\right)^2$$

- Small motions:

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- Problems:
  - Underconstrained, No consistency enforced across pixels, small motions only

Optical Flow - Smoothness

Estimate (u, v) at every pixel (x, y)

In principle, minimize $E_{intensity}$:

$$\sum_{(x,y)} \left\| I_2(x + u(x,y), y + v(x,y)) - I_1(x,y) \right\|^2$$

Subject to

Smoothness constraints E_{smooth}

$$u(x,y) \approx u(x + \Delta x, y + \Delta \mathrm{y})$$
$$v(x,y) \approx v(x + \Delta x, y + \Delta \mathrm{y})$$

Consistent flow field

$$\frac{\partial v}{\partial x} \approx \frac{\partial u}{\partial y}$$

Optical Flow - Smoothness

- Introduce smoothness constraint

$$E_{intensity} + \alpha(\|\nabla u\|^2 + \|\nabla v\|^2)$$

$$\left(\frac{\partial u}{\partial x}\right)^2 + \left(\frac{\partial u}{\partial y}\right)^2$$

- Two problems:
  - Susceptible to noise
  - Assumes only smooth flow → What about discontinuities?

Horn and Schunck. Determining optical flow. Artificial Intelligence. 16. 1981

Sensitive to noise & discontinuity

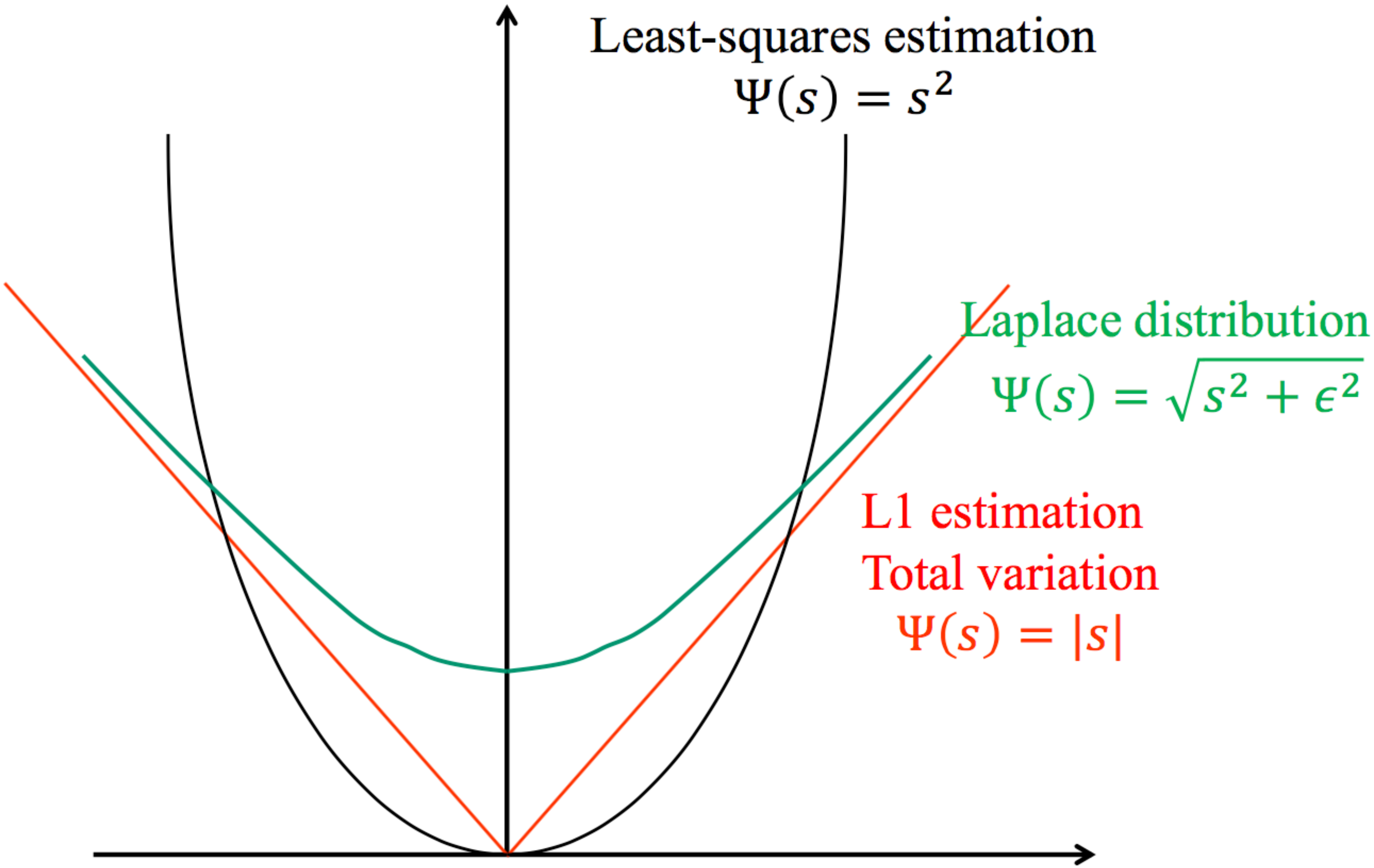
$$Min_{u(.,.),v(.,.)} \sum_{(x,y)} \left(I_2(x+u(x,y), y+v(x,y)) - I_1(x,y)\right)^2$$

- General problem:

$$Min_\theta \sum_X (E(X,\theta))^2$$

- Fundamental issue:
  - Outliers (data *X* that does not fit the model well) is going to overwhelm the objective function because of the quadratic growth
  - Idea: Replace $(.)^2$ with a "better behaved" function $\Psi(.)$ (slower increase on outliers) → Robust estimator

Primer - Robust Estimation



Sparsity

- Simple example:
  - Same error value $\sum_X \Psi(E(X,\theta)) \quad =1$
  - Much sparser solution with L1 (and Laplace) than L2

$$E_1 = \begin{bmatrix} \sqrt{1/d} \\ \vdots \\ \sqrt{1/d} \end{bmatrix} \quad E_2 = \begin{bmatrix} 1 \\ \vdots \\ 0 \end{bmatrix}$$

Smooth with sparse discontinuities

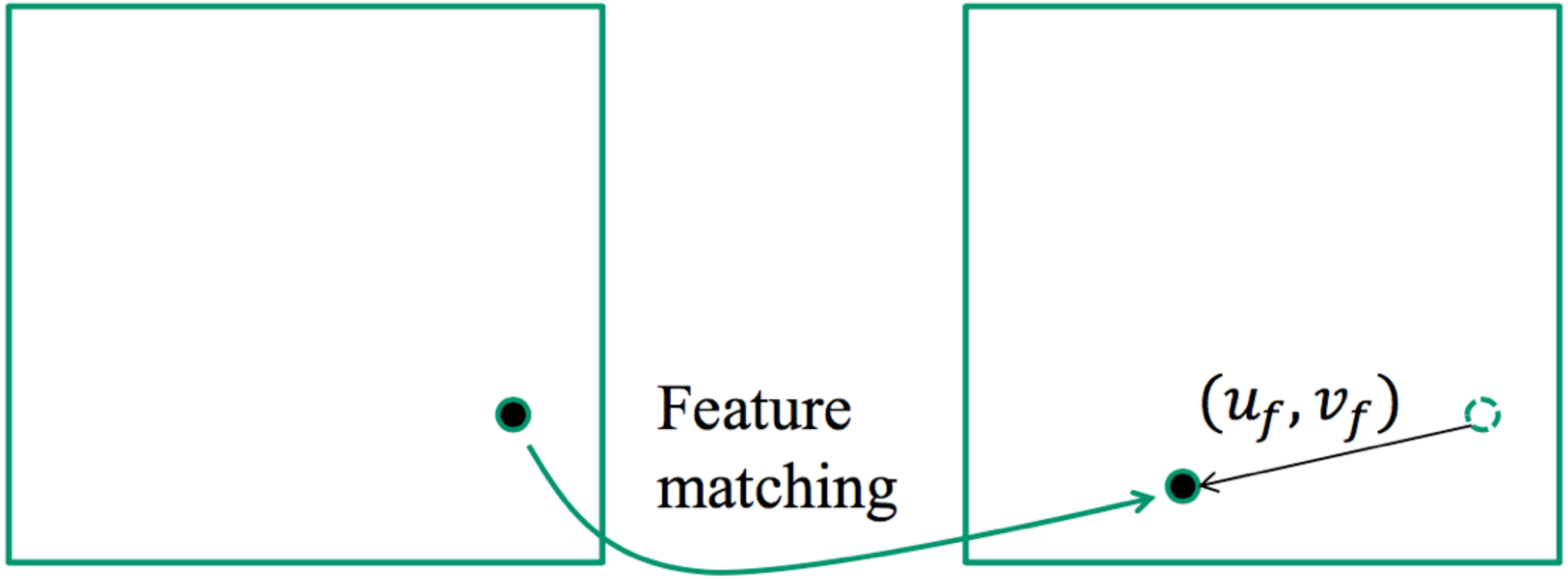
Robust to outliers

$$\sum_{x,y} \Psi\left(uI_x + vI_y + I_t\right) + \alpha(\Psi(\|\nabla u\|) + \Psi(\|\nabla v\|))$$

Allows sparse discontinuities

Zach, Pock, Bischof. A duality based approach for realtime TV-L1 optical flow. DAGM07. 2007

What about large motions?



- Differential motion *(u,v)* for places where motion is small
- Motion from feature matching (u_f, v_f) (e.g., SIFT, SURF, etc.), including where there is large motion
- Enforce agreement between the two where they overlap $(u, v) \approx (u_f, v_f)$

Large Motions

- Add two terms to the objective function:
- Error on flow from feature matching
- Agreement between features and differential flow

Idea:

Combine with motion estimated from feature matches

(e.g., SIFT) $E_{feature}$

$$\sum_{Feature\ locations\ (x,y)} \left\| I_2\left(x + u_f(x,y), y + v_f(x,y)\right) - I_1(x,y) \right\|^2$$

Feature matches can deal with large motion

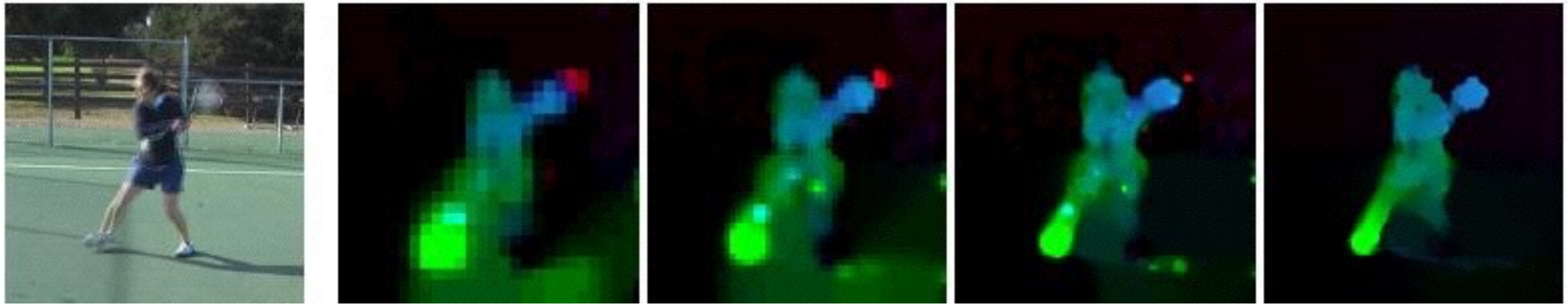
Combine the two motion estimated by adding new consistency term $E_{consistent}$

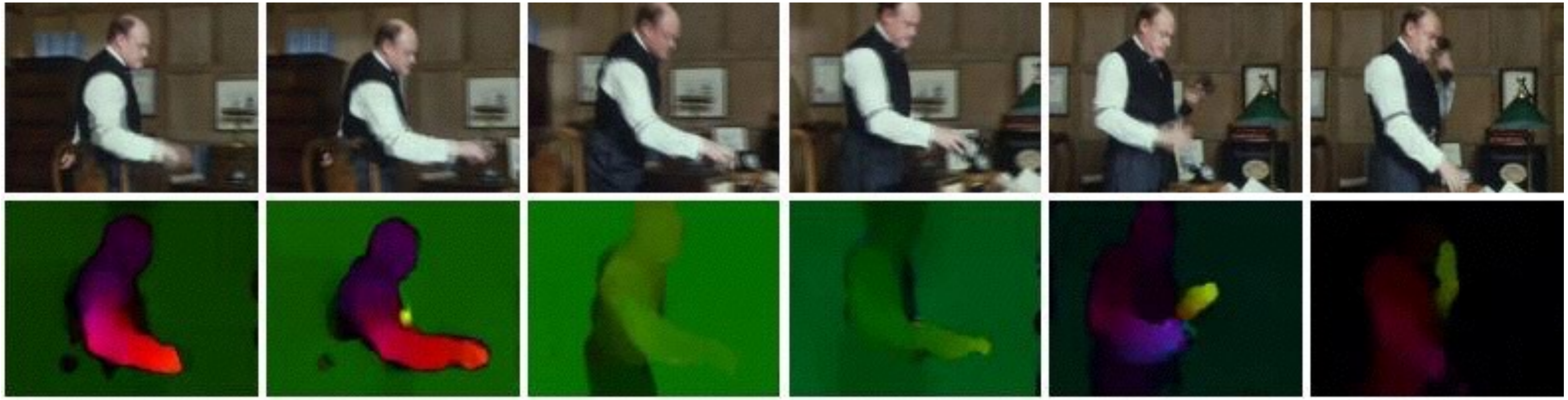
$$\sum \left\| \begin{bmatrix} u(x,y) \\ v(x,y) \end{bmatrix} - \begin{bmatrix} u_f(x,y) \\ v_f(x,y) \end{bmatrix} \right\|^2$$

Final form:

$$E_{intensity} + \alpha E_{smooth} + \beta E_{feature} + \gamma E_{consistent}$$

Example





T. Brox, J. Malik. Large Displacement Optical Flow: Descriptor Matching in Variational Motion Estimation. IEEE Trans. PAMI. 2010.

Example Flow with CNN's!